Assignment 1

1.7.28. $p: n=2k$ $q: 7n+4$, even

: Prove that if n is a positive integer, then n is even if and only if $7n+4$ is even.

Solution:

$p \Rightarrow q$:

$n \in \mathbb{Z}, n>0, n=2k$ for some integer k.

Then $7n+4 = 7\cdot 2k+4 = 14k+4 = 2(7k+2)$

Hence, $7n+4$ is even.

$q \Rightarrow p$: proof by conversion $\neg p \Rightarrow \neg q$

$n=2k+1$ for some integer k

Then $7n+4 = 7(2k+1)+4 = 14k+7+4 = 14k+11 = 2(7k+10)+1$

Hence, n is odd if and only if $7n+4$ is odd ■

1.7.30.

Prove that $m^2=n^2$ if and only if $m=n$ or $m=-n$

Solution:

$\Rightarrow$

if $m=n$ then $m^2=n^2$ following math principles

if $m=-n$ then $m^2=(-n)^2=n^2$ following math principles

$\Leftarrow$

if $m^2=n^2 \Leftrightarrow \sqrt{m^2}=\sqrt{n^2} = \pm m = \pm n$

Thus, $m=n$ or $-m=n \Leftrightarrow m=-n$ or $m=-n$ or $-m=-n \Leftrightarrow m=n$

Hence, $m^2=n^2$ iff $m=n$ or $m=(-n)$ ■

1.8.32

Prove that there are no solutions in integers x and y to the equation $2x^2+5y^2=14$.

Solution:

Because x and y are squared, negative values for x and y are equal to its positive counterpart.

Because $2x^2$ for $x > 2$ is greater than 14 if $x \in \mathbb{N}$ we conclude that x only could be 1 or 2.

Because $5y^2$ for $y > 1$ is greater than 14 if $y \in \mathbb{N}$, we conclude that y only could be 1.

Proof by cases:

$x = 1$ and $y = 1$

$2 \cdot 1^2 + 5 \cdot 1^2 \neq 14$

$x = 2$ and $y = 1$

$2 \cdot 2^2 + 5 \cdot 1^2 \neq 14$

Hence, there are no solutions to $2x^2+5y^2=14$ if $x \in \mathbb{Z} \wedge y \in \mathbb{Z}$. $\blacksquare$

2.3.26

a)

Prove that a strictly increasing function from $\mathbb{R}$ to itself is one-to-one.

Solution:

~~prove~~ either ($a = b$ and $f(a) = f(b)$ or $a \neq b$ and $f(a) \neq f(b)$)

proving ($a \neq b$ and $f(a) \neq f(b)$)

if $a > b$, then $f(a) < f(b)$

if $a < b$, then $f(a) > f(b)$

Thus if $a \neq b$, then $f(a) \neq f(b)$ ∎

b) Find a function that for example satisfies

$a \neq b$ and $f(a) = f(b)$ or $a = b$ and $f(a) \neq f(b)$

An example function:

$f_1(x) = \lceil x \rceil$

Due to same values having the same image.