1)

a) $fl\left(\left(1+2^{-54}\right)-1\right)=\underline{0}$ (since 2^{-52} is the smallest number such that $1+2^{-52}>1$)

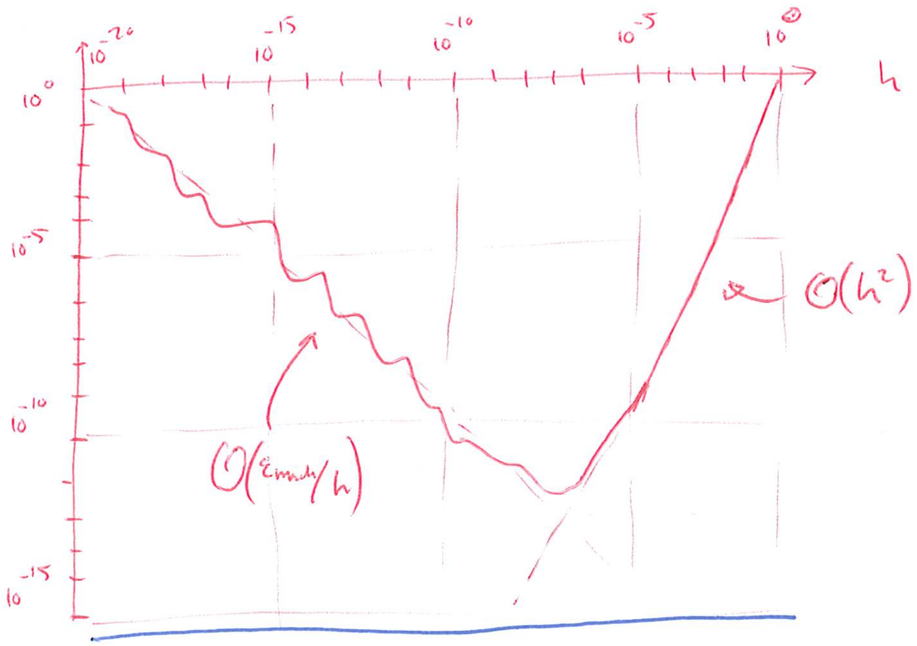
b) $2^{-60}+2^{-75}=\underline{2^{-60}+2^{-75}}$ (since $2^{-60}+2^{-75}=2^{-60}(1+2^{-15})$ — 2^{-15} larger than 2^{-52})

c) for $x \approx 0$ we have substraction of nearly equal numbers. Reformulating as

$$\frac{1}{1+x}-\frac{1}{1-x}=\frac{1-x}{1-x^2}-\frac{1+x}{1-x^2}=\underline{\frac{-2x}{1-x^2}}$$

avoids the problem

d)



---

2)

a)

	x_0	conv?	to?	rate?	(why not?)
i)	1	yes	to $\approx$ 0.75	rate 1st order (double root)	
ii)	2	yes	$x\approx 2$	2nd order	–
iii)	3	probably not			division by almost zero
iv)	4	divergence to $x\to\infty$			

mention double root ok

b) $x^3+6x=3x^2+11$

$f(x)=x^3+6x-3x^2-11$

$f'(x)=3x^2+6-6x$

$x=0$: $0 < 1$

$x=1$: $7 < 14$

$x=2$: $20 < 23$

$x=3$: $45 > 38$

startguessing: $x_0=2$

$x_1=2.5$

$x_2=2.41025641$

$x_3=2.40629501$

$x_4=2.40628758$

$\Delta=0.00001<0.5\cdot 10^{-4}$

ans: 2.4063

3)

E.g. Lagrange

$$l_1 = \frac{(x-3)(x-4)}{(1-3)(1-4)} = \frac{x^2-7x+12}{6}$$

$$l_2 = \frac{(x-1)(x-4)}{(3-1)(3-4)} = \frac{x^2-5x+4}{-2}$$

$$l_3 = \frac{(x-1)(x-3)}{(4-1)(4-3)} = \frac{x^2-4x+3}{3}$$

$$p(x) = 2l_1 + 6l_2 + 5l_3 = \frac{x^2-7x+12}{3} - \cancel{8}\,9\,\frac{x^2-5x+4}{3} + 5\,\frac{x^2-4x+3}{3}$$

$4-12+5$

$$= \frac{x^2(1-9+5) + x(-7+45-20) + 12-36+15}{3}$$

$$= \underline{-x^2+6x-3}$$

test: $p(1) = -1+6-3 = 2$
$p(3) = -9+18-3 = 6$
$p(4) = -16+24-3 = 5$

$p'(x) = -2x+6 = \underline{2(3-x)}$

a) $-x^2+6x-3$

b) $p(2) = -4+12-3 = \underline{5}$ rimlig

c) $R(x) = f(x) - P(x) = (x-x_0)(x-x_1)\cdots(x-x_n)\dfrac{f^{n+1}(\xi)}{(n+1)!}$

3 punkter: $\Rightarrow n = 2$

behöver f'''

$x_0 \; x_1 \; x_2$

$f = x^3 - 9x^2 + 25x - 15$
$f' = 3x^2 - 18x + 25$
$f'' = 6x - 18$
$f''' = 6$

$f(2) = 8-36+50-15$
$= 58-36-15$
$= 22-15 = 7$

$R(x) = (2-1)(2-3)(2-4)\dfrac{6}{3!} = 1\cdot(-1)(-2) = \underline{2}$ rimligt

exakt fel också 2 :)

$\frac{180}{36} = \frac{30}{6} = 5$

4a) $\int_1^2 y(x)\,dx \approx \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$

$h = \frac{2-1}{4} = \frac{1}{4}$ $\quad = \frac{1/4}{3}\left(0.1250 + 0.8350 \cdot 4 + 2 \cdot 1.0288 + 4 \cdot 0.5740 - 1.1630\right)$

$= \underline{0,5545}$

b) $\int_1^2 y(x) \approx \frac{2h}{3}\left(y_0 + 4y_2 + y_4\right) = \frac{1/2}{3}\left(0.1250 + 4 \cdot 1.0288 - 1.1630\right)$

$= \underline{0.51233}$

The error in (b) should be $2^4 = 16$ times higher

c) $\int_1^2 y(x) = \frac{2^4 \cdot 0.5545 - 0.51233..}{15} = \underline{0.557311..}$

---

5) Euler forward : $y_{n+1} = y_n + h f(x_n, y_n)$

In our case : $h = 1$

$f(x,y) = x - xy = x(1-y)$

$y_0 = 2$

$\{x_0, x_1, x_2\} = \{0, 1, 2\}$

$y_0 = 2$

$y_1 = y_0 + 1 \cdot (0 - 0 \cdot y_0) = y_0 = 2$

$y_2 = y_1 + 1 \cdot (1 - 1 \cdot y_1) = 2 + (1-2) = \underline{1}$

$h = 0.5$

$\{x_0, x_1, x_2, x_3, x_4\} = \{0, 0.5, 1, 1.5, 2\}$

$y_0 = 2$

$y_1 = y_0 + 0.5(0 - 0) = 2$

$y_2 = y_1 + 0.5(0.5 - 0.5 \cdot 2) = 2 + 0.25(-1) = 1.75$

$y_3 = y_2 + 0.5 \cdot (1 - 1.75) =$

$= 1,75 + 0.5(-0.75) = 1,375$

$y_4 = y_3 + 0.5 \cdot 1.5(1 - 1.375)$

$= \underline{1,09375}$

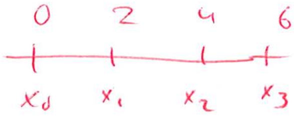
b) $y_{better} = \frac{2^1 \cdot 1.09375 - 1}{2^1 - 1} = \underline{1.1875}$

c) EF $\underbrace{\text{faster}}_{}$ (and easier) but can have stab-issues

EB more complicated but stable

6)

$$y'' = \frac{-15}{x+1}$$



Let w_i approx $y(x_i)$

and $\frac{w_{i+1} - 2w_i + w_{i-1}}{h^2}$ approx y''

$x=0$. Boundary $w_0 = y(0) = -8$

$x = 2$ $\quad \frac{w_2 - 2w_1 + w_0}{2^2} = \frac{-15}{3}$ $\qquad \frac{w_2 - 2w_1 - 8}{4} = -5$

$x = 4$ $\quad \frac{w_3 - 2w_2 + w_1}{2^2} = \frac{-15}{5}$ $\qquad \frac{3 - 2w_2 + w_1}{4} = -3$

$x=6$. Boundary $w_3 = y(6) = 3$

$$w_2 - 2w_1 = -20 + 8 = -12$$
$$-2w_2 + w_1 = -12 - 3 = -15$$

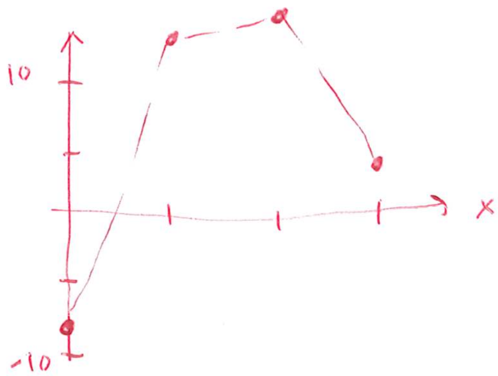
$$\begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} -12 \\ -15 \end{pmatrix}$$

b)

$$\left(\begin{array}{cc|c} -2 & 1 & -12 \\ 1 & -2 & -15 \end{array}\right) \sim \left(\begin{array}{cc|c} -2 & 1 & -12 \\ -3 & 0 & -15-24 \end{array}\right) \sim \left(\begin{array}{cc|c} -2 & 1 & -12 \\ -3 & 0 & -39 \end{array}\right) \sim \left(\begin{array}{cc|c} 0 & 1 & -12+26 \\ 1 & 0 & 13 \end{array}\right) \sim \left(\begin{array}{cc|c} 0 & 1 & 14 \\ 1 & 0 & 13 \end{array}\right)$$

$w_1 = 13$

$w_2 = 14$



~~om -28 -9 kos -8 21.66 15.33 3~~