Assignment 1

1.7.28. $P: n = 2k$ $q: 7n+4$, even

: Prove that if n is a positive integer, then n is even if and only if $7n+4$ is even.

Solution:

$P \Rightarrow q$:

$n \in \mathbb{Z}, n > 0, n = 2k$ for some integer k.

Then $7n+4 = 7 \cdot 2k + 4 = 14k + 4 = 2(7k+2)$

Hence, $7n+4$ is even

$q \Rightarrow P$: proof by conversion $\neg p \Rightarrow \neg q$

$n = 2k+1$ for some integer k

Then $7n+4 = 7(2k+1)+4 = 14k+7+4 = 14k+11 = 2(7k+10)+1$

Hence, n is odd if and only if $7n+4$ is odd ■

1.7.30.

Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$

Solution:

$(\Leftarrow)$

if $m^2 = n^2 \Leftrightarrow \sqrt{m^2} = \sqrt{n^2} \Leftrightarrow \pm m = \pm n$

Thus, $m = n$ or $-m = n \Leftrightarrow m = -n$ or $m = -n$ or $-m = -n \Leftrightarrow m = n$

Hence, $m^2 = n^2$ iff $m = n$ or $m = (-n)$ ■

1.8.32

Prove that there are no solutions in integers x and y to the equation $2x^2+5y^2=14$.

Solution:

Because x and y are squared, negative values for x and y are equal to its positive counterpart.

Because $2x^2$ for $x>2$ is greater than 14 if $x \in \mathbb{N}$ we conclude that x only could be 0, 1 or 2.

Because $5y^2$ for $y>1$ is greater than 14 if $y \in \mathbb{N}$ we conclude that y only could be 0 or 1.

Proof by cases:

1. $x=1$ and $y=1$

   $2\cdot1^2+5\cdot1^2 \neq 14$

2. $x=2$ and $y=1$

   $2\cdot2^2+5\cdot1^2 \neq 14$

3. $x=0$ and $y=0$

   $2\cdot0^2+5\cdot0^2 \neq 14$

4. $x=0$ and $y=1$

   $2\cdot0^2+5\cdot1^2 \neq 14$

5. $x=1$ and $y=0$

   $2\cdot1^2+5\cdot0^2 \neq 14$

6. $x=2$ and $y=0$

   $2\cdot2^2+5\cdot0^2 \neq 14$

Hence, there are no solutions to $2x^2+5y^2=14$ if $x \in \mathbb{Z} \wedge y \in \mathbb{Z}$. ∎

2.3.26

a)

Prove that a strictly increasing function from $\mathbb{R}$ to itself is one-to-one.

Solution:

proving ($a \neq b$ and $f(a) \neq f(b)$)

if $a > b$, then $f(a) < f(b)$

if $a < b$, then $f(a) > f(b)$

Thus if $a \neq b$, then $f(a) \neq f(b)$

Hence, a strictly increasing function from $\mathbb{R}$ to itself is not one-to-one

b) Find a function that for example satisfies

$a \neq b$ and $f(a) = f(b)$, or $a \equiv b$ and $f(a) \neq f(b)$

An example function:

$f(x) = \lceil x \rceil$

Due to some values having the same image.