Course introduction, logic and basic of proofs

1ma406/1ma901 Linear algebra

Jonas Nordqvist

Teachers and staff

	Jonas Nordqvist	Course responsible, Examiner
	Sofia Eriksson	Teacher
	Algot Lindström	TA
	Emeli Wickström	Education administrator

Course literature

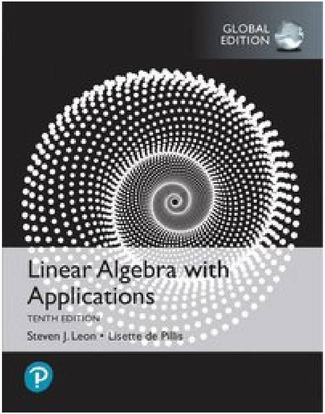

Figure 1: Course literature

We will be using the sections

1.1–1.5, 2, 3, 4, 5.1–5.3, 6.1, 6.3 in Leon.

Course Syllabus

https://kursplan.lnu.se/kursplaner/syllabus-1MA901-2.pdf

Course schedule

For campus students

https://cloud.timeedit.net/lnu/web/staff1/
ri1Y7X86Q9wZ09Qf99098995yXY99Z8XY9554X9Q56199Y9779YY09XY790099QX969777Y9799.html

For distance students

- Monday 9-10
- Wednesday 9-10

Study guide

Available in MyMoodle

Week 36

In this week we will start with the main material of the course: linear algebra, and we start from the beginning with system of linear equations. The following is an example of a linear system consisting of two equations and three unknowns

$$\begin{cases} 3x + 2y - z = 1 \\ 4x - y - z = 2. \end{cases}$$

We say that two systems are *equivalent* if they have the same *solutions set*. Depending on the solution set we may characterize a system as either *consistent* (have solutions) or *inconsistent* (lack solutions). We will use methods such as Gaussian elimination (or Gauss-Jordan reduction) to find the solutions of system of linear equations. It is *crucial* for this course that you understand and can use these methods to solve linear systems.

In the third lecture this week we will talk about matrices, matrix arithmetic and the matrix equation $Ax = b$.

The important results of this week which you should study a bit more detail are: Theorem 1.2.1, Theorem 1.3.1 and Theorem 1.4.1.

Reading instructions:
[A] 1.1--1.4

Recommended exercises:
Section 1.1: 1, 3, 6aceg, 8, 9, Section 1.2: 1, 2, 5,6, 8--10, 16, 17, Section 1.3: 1--4, 7b, 9, 11, 15, Section 1.4: 1, 3, 5, 7, 14, 19--23

Week 37

We will start this week where the last ended, with matrices. We will primarily study inverses of matrices. An inverse to a matrix A is the matrix equivalent of an inverse to a real number a, *i.e.* the inverse of a is a real number b such that $ab = 1$. In order to study inverses we must define the concept of *elementary matrices* and

Duggor

There will be two duggor (for distance these will be digital), which may grant you extra points for the exam and the first re-exam. The points on the duggor translates as follows to points in the exam.

Total score on both duggor	Bonus points on exam
0-12	0
13-18	1
19-25	2
26-30	3

- Dugga I is given on Sep 21
- Dugga II is given on Oct 20

Computer assignment

1.5 of the total credits for this course is devoted to a computer assignment. The computer assignment is due **November 6th 2022**. Instructions and submission details are all found in MyMoodle. The grading for the computer assignment is A-F.

There are 5-6 exercises in the computer assignment

1. Matrices and systems of linear equations (chapter 1)
2. Plotting planes and lines (chapter 5)
3. Computer graphics (chapter 4)
4. The method of least squares (chapter 5)
5. Traffic flow (chapter 1 and 3)

Installing MATLAB

As one of the mandatory parts of the course is a computer assignment using the software MATLAB. We have campus licenses for each student, which means that you can download a version on your own computer. In order to do so use the following instructions.

- Visit the university MATLAB-portal `https://se.mathworks.com/academia/tah-portal/linneuniversitetet-40670975.html`
- Click on *Sign in to get started*
- Create an account using your student-email (this will automatically give you your license)
- Download the installer (R2020b)
- Follow the installation instruction
  - Choose *Student license*
  - Install the selected products
- Activate your license
- Start using MATLAB

Useful resources

- Getting started with MATLAB and other useful tips: `https://se.mathworks.com/help/matlab/`

Exam

The written exam typically consists of eight, five point exercises of various difficulty. To pass, you would need 20 points.

- The first attempt for the exam is **November 1, 2022**
- The second attempt is **December 3, 2022**
- The third attempt is **February 4, 2023**

Exercise sessions

There will typically be two exercise sessions each week given on Tuesdays (13-15) and Thursdays (8-10).

All students are divided into four groups.

Group	Students	Teacher	Language
1	Civilingenjör teknisk matematik, matematikerprogrammet	Sofia Eriksson	Svenska
2	Civilingenjör mjukvaruteknik, fysiker, fristående	Jonas Nordqvist	Svenska
3	International students	Algot Lindström	English
4	Distance*	Jonas and Sofia	English/Svenska

*: not the same time as noted earlier

Course evaluation

- Some mentioned strengths of the course was
  - The teachers
  - The course book and supplementary reading
  - Good organization
  - Students involved in the practice sessions
- Some mentioned suggested improvements of the course was
  - Better handling of distance students
  - Less proofs in class
  - Divide the computer assignment
- These are some of the changes from last year
  - Distance/campus more separated
  - Computer assignment divided into two parts for campus students
  - Two lectures have been removed

Räknestuga

Our TAs have an extra sessions where students can visit and ask questions on first year math courses:

Mondays 17-19 most often in D1140.
`https://mymoodle.lnu.se/course/view.php?id=51896`

Course committee

Each year we form a course committee, consisting of teachers responsible for the programs, the teacher responsible for the course, and students, preferably at least one from each program.

Antending this group means you have to sit down on short three meetings throughout the course and say your opinion about the course.

Contact me by e-mail if you are interested.

Some applications of logic

1. Computer programs: Construction of programs and reasoning about the behaviour of programs
2. Logic Circuits and Boolean Algebra
3. Reading, understanding, and using mathematical texts
4. Reading, understanding, and using texts in computer science and technology
5. Technical and mathematical communication; reports, solutions of problems etc.
6. Translating English Sentences
7. System Specifications: Translating sentences into logical expressions is an essential part of specifying both hardware and software systems. System specifications should be **consistent**, that is they should not contain conflicting requirements.
8. Boolean Searches

Statements/Propositions

Definition
A *propostion* or *statement* is a sentence that declares a fact that is either true or false, but not both.

Examples of propositions:

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5. For every nonnegative integer n the value of $f(n) := n^2 + n + 41$ is prime.

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6. Every even integer greater than 2 is the sum of two primes.[1]

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Examples of sentences that are *not* propositions: Shut up! What day is it? $x + 4 = 5$.

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Negation Operator NOT

Definition
Let p be a proposition. The ***negation*** of p, denoted by $\neg p$ is the statement: 'It is not the case that p'. The proposition $\neg p$ is read 'not p'. The truth value of $\neg p$ is the opposite of the truth value of p.

In literature the notation not(p) is also used.

Example
The negation of the statement $1 + 3 = 5$ is $1 + 3 \neq 5$.

Logical Connectives (AND, OR, XOR, IMPLIES)

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Let p and q be propositions. The ***conjunction*** of p and q, denoted by $p \wedge q$ is the proposition'p **and** q'. The conjunction $p \wedge q$ is true when both p and q are true, and is false otherwise.

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In natural language OR is sometimes referred to mean: this OR that but not both!

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Definition
Let p and q be propositions. The ***exclusive or*** of p and q, denoted by $p \oplus q$ is the proposition'p **xor** q', that is true when *exactly one* of p and q is true, and false otherwise.

Example
Let p be the proposition: It is raining outside, and q is the proposition: The date of today is odd. (The lecture is given August 29th, and the news said that it should rain).

- Then $p \wedge q$ is TRUE.
- Then $p \vee q$ is TRUE
- Then $p \oplus q$ is FALSE

Logical Connectives (AND, OR, XOR, IMPLIES)

We can illustrate the logical connectives by means of truth tables.

p	q	$p \wedge q$
1	1	
1	0	
0	1	
0	0	

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We have the following truth table for the implication.

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6. The sum of two even numbers is even.
    - If x and y are even numbers, then $x + y$ is an even number

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Important fact: The **contrapositive**, $\neg q \implies \neg p$, of $p \implies q$ always has the same truth values as $p \implies q$.

$p \implies q \equiv \neg p \vee q$

p	q	$\neg q$	$\neg p$	$\neg q \implies \neg p$	$p \implies q$
1	1	0	0	1	1
1	0	1	0	0	0
0	1	0	1	1	1
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Definition
When two compound propositions always have same truth value we call them *equivalent*.

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$x = -5$ is sufficient for $x < 0$.

Biconditional statements (IF AND ONLY IF, IFF, EQUIVALENCE)

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Note that $p \iff q$ is true exactly when both the conditional statement $p \implies q$ and its converse, $q \implies p$, are true.

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Common ways to express $p \iff q$:

p iff q

if p then q and conversely

p is necessary and sufficient for q

Example
$x^2 - ax + b = 0 \iff x = \frac{a}{2} \pm \sqrt{\frac{a^2}{2} - c}$

Example
Let n be an integer. Then

$$n \text{ is odd} \iff n^2 \text{ is odd.}$$

Examples of important logical equivalences of Boolean Algebra

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When two compound propositions x and y always have same truth value we call them *equivalent* and write $x \equiv y$.

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Implications are logically equivalent to a disjunction:

$p \implies q \equiv \neg p \vee q$

p	q	$\neg p$	$\neg p \vee q$	$p \implies q$
1	1	0	1	1
1	0	0	0	0
0	1	1	1	1
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De Morgan's Laws:

$\neg(p \wedge q) \equiv \neg p \vee \neg q$;

$\neg(p \vee q) \equiv \neg p \wedge \neg q$.

Distributive Laws:

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$;

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

Example: Construct the truth table of $(x \wedge y \wedge z) \vee (x \wedge \neg y \wedge z)$

Truth Tables of Compound Statements and disjunctions

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x	y	z	$\neg y$	$x \wedge y \wedge z$	$x \wedge \neg y \wedge z$	$(x \wedge y \wedge z) \vee (x \wedge \neg y \wedge z)$
1	1	1	0	1	0	1
1	1	0	0	0	0	0
1	0	1	1	0	1	1
1	0	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
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0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	1	1	0	0	0
0	0	0	1	0	0	0

Remark
The compound statement analyzed in the table defines a Boolean function $g(x, y, z) := (x \wedge y \wedge z) \vee (x \wedge \neg y \wedge z)$ *in three variables* x, y, z *that can take the (truth) values* 0 *or* 1*. For example* $g(1, 1, 1) = 1$ *and* $g(1, 1, 0) = 0$.

Exercises for this week

Available on MyMoodle