Linnaeus University
Department of Mathematics
Karl-Olof Lindahl

Written Exam on Basic Numerical Methods, 1DV519, 5 ECTS
Saturday the 25th of November 2017, 12.00-17.00

The solutions should be complete, mathematically correct, well structured and easy to follow.
If you prefer you may write the answers in Swedish.
Aids: Calculator (you may use a scientific calculator but *not* with internet connection)
Grades: 15p-16p⇒E; 17p-18p⇒D; 19p-23p⇒C; 24p-27p⇒B; 28p-30p⇒A.

1. Given the following set of points (x, y) : $(0.0, -1.0), (2.0, 3.0), (3.0, 4.0), (4.0, 6.0)$; fit the data with a polynomial of degree *one* using the least square method. (5p)

2. Use the Newton-Raphson method to find approximations of all solutions of the equation

$$f(x) = \sin(2x) + xe^x + 1$$

with 4 correct decimals.

(5p)

3. a) Use the trapetzoidal method to calculate approximate values of the integral

$$I = \int_1^2 \ln(x^3)dx,$$

for 3 different step lengths: $h = 1, 0.5, 0.25$. Use 6 correct decimals of function values. (2p)

4. Let $y(x)$ be the solution of $y'(x) = -yx$ for which $y(1) = 1$.

a) Sketch the corresponding vector field. (1p)

b) Is the vector field stable? (1p)

c) Find an approximate value of $y(1.4)$ using Euler forward with step length $h = 0.1$. Use 6 correctly rounded decimals of function values in the written presentation. (2p)

d) Calculate an iterative improvement of the approximate value of $y(1.4)$ obtained in a) using Richardson extrapolation. Also estimate the truncation error. (1p)

5. a) Find the error term and the order for the approximation formula

$$f'(x) \approx \frac{1}{12h}\left(f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)\right).$$

(2p)

b) We have the following correctly rounded function values for f: $f(0.4) = 0.010582$, $f(0.450) = 0.015034$, $f(0.475) = 0.017662$, $f(0.5) = 0.020574$, $f(0.525) = 0.023787$, $f(0.550) = 0.027313$, $f(0.6) = 0.035358$. Use the approximation of $f'(x)$ in a) and Richardson extrapolation to approximate $f'(0.5)$ and the error in the approximation. (2p)

c) Find the roots of the equation $x^2 + 9^{12}x = 3$ with four correct significant digits. (2p)

Please turn page!

d) Let f and g be infinitely many times differentiable real functions. Assume that r is a zero of $f(x)$ and that $r + \Delta r$ is a zero of $f(x) + \epsilon \cdot g(x)$. Then

$$\Delta r \approx -\frac{\epsilon g(r)}{f'(r)}, \quad \text{for } \epsilon \text{ sufficiently smaller than } f'(r). \tag{1}$$

Use this result to estimate the largest root of the equation

$$(x-1)(x-2)(x-3)(x-4)(x-5)(x-6) - 10^{-6}x^7 = 0.$$

(2p)

e) Prove the formula (1) given above. (2p)

Good Luck!