Lab-session week 13 (1MA930/1MA931, VT2024)

Try to do the following exercises in Matlab:

1. Evaluate the function $f(x) = \sin(2x)$ at the point $x = 0.5$.
2. Plot the function $f(x) = \sin(2x)$ on the interval $[-1, 1]$.
3. Plot the functions $f(x) = \sin(2x)$ and $g(x) = e^{-x^2}$, in the same figure, on the interval $[-1, 1]$.
4. Make a crude graphical localization of the zeros of
$$f(x) = x - 4\sin(2x) - 3 - \frac{3}{80},$$
by making appropriate plots of f using Matlab.
5. Sauer, exercise 0.3(4): Find (by hand) the largest integer k for which $fl(19 + 2^{-k}) > fl(19)$ in double precision floating point representation. Check (afterwards!) using Matlab.
6. Sauer, exercise 0.4(2): Find the roots of the equation $x^2 + 3x - 8^{-14} = 0$ with three digit accuracy combining calculations by hand and evaluation in Matlab.
7. Sauer, computer problems 0.4(2a): Find the smallest integer p for which the expression calculated in double precision arithmetic at $x = 10^{-p}$ has no correct significant digits for
$$\frac{\tan x - x}{x^3}.$$
8. Compute the sum $S = \sum_{i=1}^{\infty} \frac{1}{i^3}$ using Matlab, in the following ways:

   a) By computing the sum $S_N = \sum_{i=1}^{N} \frac{1}{i^3}$ for some suitable choice of N and summing the terms in *decreasing* order (from the largest to the smallest, i.e. $S_N = 1/1^3 + 1/2^3 + \cdots + 1/N^3$). How many terms in the above sum will affect the computation (i.e. how big should N be so that $1/N^3$ is the largest term that will contribute to the sum)?

   b) By computing the sum $S_M = \sum_{i=1}^{M} \frac{1}{i^3}$ summing the terms in *increasing* order for some suitable M (i.e. $S_M = 1/M^3 + 1/(M-1)^3 + \cdots + 1/1^3$). Roughly how many terms in the above sum seem to affect the computation in this case? Is the result different from the result in a)? Which one do you think is more correct?

   c) Calculate the sums S_M and S_N using the command 'sum' in Matlab for the numbers M and N that you found in exercise a) and b). Is there any difference in the result? In which order do you thing Matlab does the summation?
9. Show that (0.9) holds, that is that $\frac{|\mathrm{fl}(x)-x|}{|x|} \leq \frac{1}{2}\epsilon_{mach}$ holds. For simplicity, suppose that x is positive and not in the subnormal range.