Completion of assignment 2 task 3 – Numeriska metoder 1MA930

Jesper Wingren
Emil Ulvagården
Samuel Berg

Task 3.

Task 3

a) Assume A to be a square matrix where the row sums equal to 1.
We want to prove that A has an eigenvalue equal to 1.

Let v denote the vector where each element is equal to 1 such that $v = (1, 1, \dots)^T$

$$Av = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} a_{1j} \\ \sum_{j=1}^{n} a_{2j} \\ \vdots \\ \sum_{j=1}^{n} a_{nj} \end{pmatrix}$$

Since we assumed all row sums equal 1 we can denote $\sum_{j=1}^{n} a_{ij} = 1$ for all i

Which implies $Av = v$ proving v is an eigenvector of A with an eigenvalue of 1.

Q.E.D.

b) Using the fact that a matrix with row sums equal to 1 has eigenvalues whose absolute value is at most 1. Using this result we then use the fact that a stochastic matrix is comparable to a matrix with row sums equal to 1 using transposal.

Let $B = A^T$

A has row sums equal 1 $\Longleftrightarrow A^T$ has column sums equal 1

It is proved that A and A^T has the same eigenvalues.

As proved in a) A with row sum equal to 1 has an eigenvalue of 1 and all eigenvalues is at most the absolute value of 1.

Since A and A^T share eigenvalues this proves a stochastic matrix A with columns sums equal to 1 has all its eigenvalues with absolute value at most 1.

Q.E.D.