Lab-session week 16 (1MA930/1MA931, VT2024)

Recall that for help with a MATLAB command, you write `help commandname` *or* `doc` *in the command window. Here* `commandname` *could for example be* `legend` *or* `polyfit`.

1. **Interpolation.**

   a) To get a feeling for the methods used to compute interpolating polynomials, solve (by hand) Exercise 3.1.1(c) and the corresponding 3.1.2 exercise on page 149.

   b) Use Newton's divided differences to find the polynomial $p(x)$ of degree 3 that interpolates the function $f(x) = \sin(x)$ at 4 equally spaced points on $[0, \pi/2]$. *Either done by hand (exact) or in MATLAB (e.g., page 146).*

   c) Plot the functions $f(x)$ and $p(x)$ from 1(b) for $x \in [-\pi, \pi]$ in the same figure window (recall the command `hold on`). Learn how to use the commands `legend`, `xlabel` and `ylabel` to make the figure informative.

   d) Check if your polynomial $p(x)$ is correct by comparing with the book of Sauer (Example 3.7, p. 147-149).

   e) The interpolation error is $f(x) - p(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{n!} f^{(n)}(c)$. Since c is unknown, we use $|f^{(n)}(c)| \leq \max_{x\in[0,\pi/2]} |f^{(n)}(x)|$ to estimate the worst possible error (on $[0, \pi/2]$). Do this (for example) for $x = 1$. Compare with the true error $|f(1) - p(1)|$.

   f) Check if you got it right by comparing with the book of Sauer (p. 152).

2. Test MATLAB's function `polyfit` to find a degree 8 polynomial that fits the data

x	-4	-3	-2	-1	0	1	2	3	4
y	1	1	1	1	2	1	1	1	1

   Evaluate the polynomial for $x \in [-5, 5]$ using the MATLAB function `polyval` and plot the result together with the set of points. Use enough x-values such that the resulting curve looks smooth.

   What happens if you use a smaller degree? A larger? Read the documentation for `polyfit` to understand what it does.

3. **The least square method.**

   a) Fit the data points $(1, 0.6)$, $(0.5, 0.8)$ and $(0.4, -0.9)$ as good as possible (in the least square sense) to the form of an ellipse

   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a, b > 0.$$

   Hint: You need to re-formulate the problem as a (overdetermined) linear system of equations. (Re-formulate by hand, solve it using MATLAB.)

Make a figure in MATLAB, plotting the 3 points together with the ellipse using your best values of a and b using (for example) the commands:

```
plot([1 0.5 0.4],[0.6 0.8 -0.9],'b*')
hold on
theta=2*pi*(0:0.01:1);
plot(a*cos(theta),b*sin(theta),'r-')
```

b) Solve the problem from (a) using QR-factorization. Find Q and R using the command `[Q,R]=qr(A);` Thereafter you can follow the steps from the lecture or Example 4.14 on page 217.

Hint: To pick out the first two rows and columns from R, use the vector notation in MATLAB as `R(1:2,1:2)`. *Alternatively: Write "help qr" to see how you can make MATLAB return the reduced QR-factors $\widetilde{Q}$ and $\widetilde{R}$.*

4. If you have time left, solve computer problems 3.2.3 and 3.2.4.