Lab-session week 19 (1MA930/1MA931, VT2024)

Tasks for ordinary differential equations:

1. Consider the Initial Value Problem in eq. (6.5) om page 283.

   Reproduce the slope field from Figure 6.2 using the following code:

   ```
   [t,y] = meshgrid(0:.1:1,0:.1:2);
   f=t.*y+t.^3;
   l=max(1,sqrt(f.^2)); % to give the arrows similar lengths
   U=1./l;
   V=f./l;

   quiver(t,y,U,V)
   axis equal
   axis([0 1.1 0 2.1])
   ```

2. (a) Apply Euler Forward on the IVP in eq. (6.5). Choose first $n = 5$ and thereafter $n = 10$ time steps, produce a numerical solution and compare to the exact solution from p. 283. See if you can reproduce the tables on page 285. *(It is better to let n define $h = (b-a)/n = 1/n$ than vice versa, since n must be an integer)*

   (b) Plot the results in the same figure as the slope field from exercise 1.

3. Read about the Explicit Trapezoid Method on page 297 and apply it to eq. (6.5). Use $n = 10$ and see if you can reproduce the table on page 298.

4. Apply the Explicit Trapezoid Method again and vary the number of time steps n. Now we are only interested in the error at the last time step, that is $|w_n - y(1)|$. (*Now, we don't need to save the result for all time steps – we save memory by only keeping the solution at the last time-step.*)

   Plot the errors as functions of n in a log-log plot. Start with smaller n, for example $n = 10, 100, \ldots, 10^5$. Can you confirm the expected order of accuracy for the Trapezoid method (page 300)?

   Then try to increase n, but (unless you have very fast computers) don't exceed $n \approx 10^9$. How does the error behave as n increases?

5. Do *one* of the following:

   - Redo Exercise 4 above using the classical fourth order Runge-Kutta method.
   - Read Example 6.13 and solve a system of ordinary differential equations. Either do Computer Problem 6.3.1 where you investigate convergence, or Computer Problem 6.3.10, where you plot a numerical solution to a three-body problem.
   - Implement Euler backward, by replicating example 6.25 and try to reproduce Figure 6.22 on pages 334-335.