L2 Sets and mathematical proofs

1ma406/1ma901 Linear algebra

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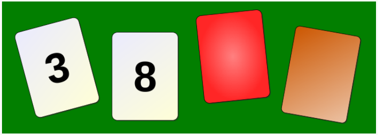
Below you see a famous problem called the Wason selection task. The exact formulation in this slide is taken from Wikipedia.

You are shown a set of four cards placed on a table, each of which has a number on one side and a colored patch on the other side. The visible faces of the cards show 3, 8, red and brown. Which card(s) must you turn over in order to test the truth of the proposition that if a card shows an even number on one face, then its opposite face is red?



Reading mathematics

- Skim
- Identify what is important
- Ask questions
- Careful reading
- Stop periodically to review
- Read statements first – proofs later

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Proof by induction. Left to a course in discrete mathematics.

Direct proofs

i) In the direct method, to show $A \implies B$ we assume A and proceed to show B.

ii) This is often done in smaller steps where each conclusion is direct, for instance

$$A \implies A_1, \quad A_1 \implies A_2, \quad \ldots \quad A_{k-1} \implies A_k, \quad A_k \implies B.$$

If and only if proofs

i) State what you will be proving, *e.g.* $A \iff B$

ii) State the assumption A and prove that this implies B, *i.e.* $A \implies B$.

iii) State the assumption B and prove that this implies A, *i.e.* $B \implies A$.

Case proofs

i) State that you will make a proof by cases.

ii) Conclude what cases you will use, *e.g.* Case 1, . . . , Case n

iii) Conclude the proof for Case 1

iv) Conclude the proof for Case 2

...) ...

n + ii) Conclude the proof for Case n

Contradiction proofs

i) State that you are assuming the statement is false. Seasoned mathematicians will recognize that the proof will be by contradiction.

ii) Write out what the statement being false means using negation.

iii) Work out what this would imply until you find a contradiction.

iv) Announce that a contradiction has been found.

Contraposition proofs

i) State that you will make a proof by contraposition of the statement, *e.g.* we want to prove that $\neg B \implies \neg A$.

ii) Use any of the previous mentioned techniques to prove the implication.

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If $x^2 - 6x + 5$ is even then x is odd.

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Suppose a, b be integers. If $a + b \geq 19$, then $a \geq 10$ or $b \geq 10$.

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Give a proof by cases of the following proposition:

Theorem
Let n be an integer. Then $n^3 - n$ is a multiple of 3.

Sets and notation

Definition
A *set* is an unordered collection of objects, called *elements* or *members* of the set. A set is said to *contain* its elements. We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that A is not an element of the set A.

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The set builder notation reads 'A is the set of all $x \in \mathbb{N}$ such that $x \leq 10$.'

Facts: $5 \in A$ and $11 \notin A$

These are also sets $\{\text{red}, \text{blue}\}$, $\{1, 2, \text{rain}, \pi\}$, $\{1, 2, \{2, 3\}, 3, \{3\}\}$.

However, we are primarily interested in number sets.

Important sets

Notation	set	elements
$\emptyset$	empty set	the set that contains no elements
$\mathbb{Z}$	integers	$\{\ldots, -2, 1-, 0, 1, 2, \ldots\}$
$\mathbb{Z}^+$	non-negative integers	$\{0, 1, 2, 3\ldots\}$
$\mathbb{N}$	natural numbers	$\{1, 2, 3\ldots\}$
$\mathbb{Q}$	rational numbers	$\{\frac{p}{q} \| p \in \mathbb{Z}, q \in \mathbb{Z}, \text{ and } q \neq 0\}$
$\mathbb{R}$	real numbers	'decimal numbers' like $1, -\pi, 0.33, e, \sqrt{2}$
$\mathbb{R}_{>0}$	positive real numbers	$\{x \in \mathbb{R} \| x > 0\}$
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Intervals on the real line. When a and b are real numbers such that $a < b$ we write

$[a, b] = \{x \in \mathbb{R} \| a \leq x \leq b\}$	the ***closed interval*** from a to b
$[a, b) = \{x \in \mathbb{R} \| a \leq x < b\}$	half open interval
$(a, b] = \{x \in \mathbb{R} \| a < x \leq b\}$	half open interval
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The set $\mathcal{C}[a, b]$ of all continuous real valued functions on the closed interval $[a, b]$.

Comparing Sets – Subsets

Definition (Subset and set equality)

- A set A is said to be a ***subset*** of a set B, denoted $A \subseteq B$, if every element appearing in A also appears in B. That is,

$$x \in A \subseteq B \iff x \in A \implies x \in B.$$

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- A set A is said to be a ***proper subset*** of a set B, denoted $A \subset B$, if every element appearing in A also appears in B but $A \neq B$. That is,

  $$x \in A \subset B \quad \Longleftrightarrow \quad (x \in A \implies x \in B) \text{ AND } (A \neq B).$$

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Cardinality

Let A be a set. If A contains exactly n distinct elements where n is a nonnegative integer, we say that A is a ***finite set*** and that n is the ***cardinality*** of A, denoted $|A|$. That is $|A| = n$.

Cardinality and Power Sets

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Given a set A, the ***power set*** of A is the set of all subsets of A, denoted by $\mathcal{P}(A)$. So

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$$\mathcal{P}(X) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}.$$

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Proposition
Let A be a finite set of cardinality n. Then the cardinality of its power set $|\mathcal{P}(A)| = 2^n$.

Ordered n-tuples

A ***ordered** n-**tuple*** $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, ..., and a_n as its nth element. We say that two ordered n-tuples $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ are equal if and only if $a_i = b_i$ for every $i = 1, 2 \ldots, n$.

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Cartesian product of two sets

Let A and B be two sets. The ***Cartesian product*** of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. That is

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Example 1: $S = \{1, 2\}$ and $T = \{a, b, c\}$. Then

$$S \times T = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

The Cartesian coordinate plane $\mathbb{R}^2$ and in $\mathbb{R}^3$

Let $\mathbb{R}$ be the set of real numbers and put $\mathbb{R}^2 := \mathbb{R} \times \mathbb{R}$. That is

$$\mathbb{R}^2 := \{(a, b) | a \in \mathbb{R} \wedge b \in \mathbb{R}\}.$$

Further, we have $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$, *i.e.*

$$\mathbb{R}^3 := \{(a, b, c) | a \in \mathbb{R} \wedge b \in \mathbb{R} \wedge c \in \mathbb{R}\}.$$

Sets of the form $\mathbb{R}^n$ for some positive integer n is the corner stone in our course in Linear algebra.

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- Given a set A and some universal set U such that $A \subseteq U$, the ***complement*** of A in U, denoted A^c, consists of all elements in $U - A$. That is,

  $$A^c := U - A.$$

  So, $X^c = \{4, 5\}$ is the complement of X in $U = \{1, 2, 3, 4, 5\}$.

Venn diagram