Assignment 3

6.5.16 a)

$x_i \geq 1$, using $2 \cdot 6 = 12$ of 29, Hence we can see the problem as solving for $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 17$

Which was got from $29 - 12 = 17$.

Number of solutions are $\binom{6+17-1}{17} = \binom{22}{17} = \underline{26334}$

1 for each x

b) Sum of minimum value for each x_i : $1+2+3+4+6+6 = 22$

Same as solving for $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 7$

7 is taken from $29 - 22 = 7$

Where x_i is nonnegative integer

The solutions are $\binom{6+7-1}{7} = \binom{12}{7} = \underline{792}$

c) Number of solutions without restrictions:

$\binom{6+29-1}{29} = \binom{34}{29} = \underline{278256}$

number solutions violating restrictions $x_1 \geq 6$:

$\binom{6+23-1}{23} = \binom{28}{23} = \underline{98280}$

Therefore the answer is $278256 - 98280 = \underline{179976}$

d) The number of solutions with $x_2 \geq 9$

but not any restriction on x_1

$\binom{6+20-1}{20} = \binom{25}{20} = 53130$

Solutions violating $x_1 \geq 8$ is

$\binom{6+12-1}{12} = \binom{17}{12} = 6188$

Thus the answer is $53130 - 6188 = 46942$

7.2.38

a) We need to assume the observer was instructed to tell us wheter atleast one die came up as 6 and no more information. This assumption has to be made for the analysis to be valid.

We say that a represents the first die and b the second die (a, b)

There are 36 equally likely outcomes.

Let T be the event that at least one die equals 6 and let S be the event that the sum of the dice equals 7.

want $p(S|T) = p(T \cap S)/p(T)$. $(T \cap S)$ outcomes are

$(1,6)$ $(6,1)$

So $p(T \cap S) = \frac{2}{36}$

There are $5^2 = 25$ outcomes in $\bar{T}$ so $p(T) = \frac{36-25}{36} = \frac{11}{36}$

Hence, the answer is $\left(\frac{2}{36}\right) / \left(\frac{11}{36}\right) = \underline{\underline{\frac{2}{11}}}$

b) This is the same as in previous exercise. Same goes for the analysis. Hence, the answer would be $\underline{\underline{\frac{2}{11}}}$

7.3.12

a)

Let R_0 be the event that 0 was received

Let S_0 be the event that 0 was sent

Let S_1 be the event that 1 was sent

Note: $\neg S_0 = S_1$

We are told that:

$p(S_1) = \frac{1}{3}$

$p(S_0) = \frac{2}{3}$

$p(R_0 | S_0) = 0,9$

$p(R_0 | S_1) = 0,2$

$$p(R_0) = p(R_0|S_0) \cdot p(S_0) + p(R_0|S_1)\, p(S_1)$$

$$= 0,9 \cdot \frac{2}{3} + 0,2 \cdot \frac{1}{3}$$

$$= \frac{1,8 + 0,2}{3}$$

$$= \underline{\underline{\frac{2}{3}}}$$

b) Using Bayes' theorem

$$p(S_0 | R_0) = \frac{p(R_0|S_0) \cdot p(S_0)}{p(R_0|S_0) \cdot p(S_0) + p(R_0|S_1) \cdot p(S_1)}$$

$$= \frac{0,9 \cdot \frac{2}{3}}{0,9 \cdot \frac{2}{3} + 0,2 \cdot \frac{1}{3}}$$

$$= \frac{0,9 \cdot \frac{2}{3}}{\frac{2}{3}}$$

$$= \underline{\underline{0,9}}$$