

## Lab-session week 18 (1MA930/1MA931, VT2024)

### 1. Numerical differentiation

- a) Read/skim Section 5.1.2 (pages 247-249), note in particular equation (5.12). Then do computer problem 5.1.1 (page 254). Try both a "standard plot" and a logarithmic scaling of the axes. Recall that the MATLAB command for the latter is `loglog`. You could also use `plot` in combination with `set(gca, 'XScale', 'log')` and `set(gca, 'YScale', 'log')`.
- b) Re-do computer problem 5.1.1 using the forward finite difference  $D_+$  (5.4). Plot the errors in the same figure as (a). Use logarithmic scaling here and below.
- c) Read through example 5.4 on page 250. Equation (5.16) has been derived using Richardson extrapolation (p. 249). Re-do computer problem 5.1.1 using the formula (5.16). Plot the errors in the same figure as a)-b).
- d) Add plots of  $C_1h$ ,  $C_2h^2$  and  $C_3h^3$  for appropriate choices of positive constants  $C_1, C_2, C_3$ . Here, as above,  $h = 10^{-1}, 10^{-2}, \dots, 10^{-12}$ . What do you observe?
- e) *If you have time:* Use Taylor expansions about  $x$  to find out what the formula

$$\frac{f(x-2h) - 4f(x-h) + 6f(x) - 4f(x+h) + f(x+2h)}{h^4}$$

approximates. What is the order of the approximation?

(Answer on page 102.)

### 2. Numerical integration

- a) Do Exercise 5.2.3, page 263, first (c) and then (a) ("by hand", using MATLAB as a calculator). Recall that using  $m$  panels means using  $2m+1$  points. What do you observe? Read Definition 5.2 on page 258. How does this relate to your observation?
- b) Do Computer exercise 5.2.1(a) (page 264). You can compare with the exact solution or use MATLAB's function `integral`.
- c) Do Computer exercise 5.2.2 (corresponding to 5.2.1(a)).
- d) *If you have time:* Do Computer exercise 5.2.3(a).

### 3. Extrapolation

- a) Do Exercise 5.3.1(c) (by hand, using MATLAB as a calculator).
- b) Do Exercise 5.3.1(c) entirely in MATLAB, copying the code on page 267 (or *if you have time* you can try to implement Romberg's method on your own).
- c) Do Exercise 5.3.1(a) with MATLAB. Check  $R_{11}$  and  $R_{22}$  as well. How does the error behave? Was this expected? Why? (explanation page 268)
- d) *If you have time:* Do Computer exercise 5.3.3.