

# Visualization of relativistic phenomena

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## Abstract

*Special relativity describes many phenomena, not observable in everyday life. This article presents an educational application, which helps to understand these effects. It models vision of objects travelling close to the speed of light. It visualizes length contraction, time dilatation, relativistic Doppler effect and more. It considers the path of light between the object and our eye, but it also presents an opportunity to visualize events happening simultaneously as well. It allows to switch between Lorentz and Galilean transformation, so we can compare Einstein's and Newton's model. It serves with a three-dimensional space-time diagram, on which we have a chance to further analyse the movement of objects.*

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Visualization of relativistic phenomena

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## 1. Introduction

Theory of relativity is a key part of modern physics. There are many use cases of this theory. Despite of it's importance there is very little chance for people to experience relativistic phenomena in their everyday life, thus differences between Isaac Newton's classical and Albert Einstein's modern model remain unexplored. The application described in this article offers a way to visualise the most well-known phenomena associated with special relativity. The goal of this article is to showcase the algorithmic challenges surrounding this program and introduce solutions for these problems.

## 2. Previous Work

This article is a direct follow up of the work <sup>1</sup>. It explains previously neglected details, e.g. implementation of Wigner-rotation.

## 3. Our proposal in detail

### 4. Implementation of physical models

The rules of simulation are based on Einstein's postulates <sup>2</sup>. These suggest that

- an observer can not differentiate between different inertial frames of reference by taking measurements in each frame of reference.
- the velocity of propagation of any physical effect can not exceed the speed of light in vacuum:  $c = 299792458 \frac{m}{s} \sim 3 \cdot 10^8 \frac{m}{s}$ .

These two constraints do not provide sufficient tools for the programmer on their own. However, all the required formulas can be derived from these two postulates.

### 4.1. Usage of the term „absolute frame”

To simplify representation of world lines every event and velocity is described in the same frame of reference. In code this frame of reference is called absolute frame. This helps circumvent confusion when transforming between different observer's proper frames. Each value is stored as it is measured in absolute frame. When visualising a given value, than it gets transformed to a particular observers proper frame.

### 4.2. Representation of physical bodies

To model the journey of physical objects through space-time, it is convenient to assign a world line to each object. To be more precise, this is going to be the world line of the object's origin. For practical purposes this origin point is selected to be the same as the origin point in model space. Such world line can be stored as a set of events. Between the events we assume geodetic world lines. At this point it is necessary to note, that we also assume flat space-time. This method allows us to represent world lines consisting of few geodetic sections easily. World lines of accelerating objects appear curved in space-time diagrams. We can approximate these lines by a set of short geodetic sections. This approximation is useful, because along the small sections we can use the

simple formulas of Lorentz transformation. The downside of this method is, that many calculations require iteration over the small sections. E. g. determining the age of an object without a priory information requires summing of ageing of the object along the geodetic sections. The other obvious downside of such approximation is, that the resulting world line is going to have low resolution. This is particularly noticeable, when we –as a viewer– place ourselves in the proper frame of such accelerating object. In this case the violent jumping nature of the movement of surroundings is much more noticeable.

## 5. View modes

The application supports multiple view modes. The two main modes are the *real time* three dimensional view and the *diagram view*. In the *real time* mode the scene is shown as regular three dimensional scene, changing over time. We can look around with the virtual camera to observe the moving objects. The shape and colour of objects gets distorted in accordance with the relativistic effects.

In the *diagram view* we can observe the world lines of objects present in the scene. This provides additional insight into the behaviour of objects. As a recent addition to the project now the user can switch to orthographic projection from the standard perspective projection. This improves the readability of the space-time diagram, and it's also useful in the *real time* view.

In order to help the user spot the differences between classic and relativistic physics, there is a way to switch between Galilean<sup>4</sup> and Lorentz transformation. A viewer can see the incoming light from an object, thus we can only see images showing the past. This is due to the finiteness of the speed of light. The application allows to toggle between visualisation of present event and past events in accordance with the incoming light beams.

### 5.1. Switching between observers

To compare the observations in different inertial frames of reference, we can toggle between multiple observers. In this implementation observers are independent from physical objects, however it's possible to make an observer follow an object. Every observer has it's own world line. These lines are also visible in *diagram view*.

When switching between frames of reference the origin of the frame also changes. The  $\vec{O} = (0, 0, 0)$  in current proper frame gets assigned to the space coordinate vector of an event along the world line of the observer. The question is, how do we select the time component. In other words which event will be selected along the new world line. This implementation chooses the event from the new observers world line, which happened simultaneously with the event of changing observers according to the previous observers.

This method introduces a somewhat more realistic approach, however in real life still no object can travel from one event's location to another's if these two events appear to be simultaneous. In defence of this approach in this computer simulation neither the user's perspective nor an observer is bound to a physical object.

## 6. Digitising Errors

Like most cartographic algorithms, the Douglas–Peucker algorithm does not fully address the issue of digitising errors. When estimating truth values, it is usually assumed that the true line (in this case the analogue line) lies within the error band of the digitised line. This band is also known as the Perkal epsilon band. In his review on issues relating to the accuracy of spatial databases, Goodchild<sup>10</sup> indicated that researchers have proposed uniform, normal and even bimodal distributions of error across this band. This concept provides some basis for estimating the position of the true line at locations between digitised points. Here, we are merely concerned with the accuracy of digitised points. Whilst it is probable that operators digitise points along high curvatures more carefully than at intermediate positions, there is at present no sound basis for modelling the distribution of error along the line. As in the Circular Map Accuracy Standard, it is usual to assume a bivariate normal distribution of error when estimating the position of the true point. In the context of line simplification, absolute positional accuracy is less important than the relative position of points describing the shape of features along the line.

The DoE/SDD boundary data contain some gross digitising errors. For example, inlet X in Figure 2c does not feature on conventional Ordnance Survey 1:50 000 maps of the area. The data are also not very accurate where coastlines are convoluted. Even if we ignore these and other gross errors, such as spikes, there will always be an element of random error in digitised data. It is reasonable to assume that points digitised from 1:50 000 source material may only be accurate to within  $\pm 5$  metres. This algorithm does not lead to a substantial accumulation of rounding errors, hence the numerical errors discussed earlier tend to be very small compared with digitising errors.

For the purposes of our argument, it is unnecessary to undertake an exhaustive evaluation of the consequences Douglas and Peucker have treated overhangs and closed loops as different problems, and have used different methods to cope with each case.

### 6.1. Numerical Problems

The FORTRAN programs by Douglas, White, and Wade use single precision REALS when computing offsets (see results in Table 1). Whilst double precision accuracy may be attained through the use of compiler options, we are unsure

whether previous research has been based on programs compiled in this manner. Wade's program was so compiled for use in our previous evaluations. Forrest stated that Ramshaw (1982) had to adopt carefully tuned double and single precision floating point arithmetic to compute the intersection of line segments whose end points were defined as integers. Forrest exclaimed "This is an object lesson to us all: constructing geometric objects defined on a grid of points, requiring ten bits for representation can lead to double precision floating point arithmetic!"

Most evaluative studies do not cite the co-ordinates in use. We do not know whether the published test lines were in original digitiser co-ordinates or whether they had been converted to geographic references. British National Grid co-ordinates for the administrative boundaries of England, Scotland and Wales (digitised by the Department of Environment (DoE) and Scottish Development Department (SDD)) are input to one metre accuracy and require seven decimal digits for representation if we include the northern islands of Scotland. At the South West Universities Regional Computer Centre these co-ordinates have been rounded to 10 metre resolution; even this requires six decimal digits. Seamless cartographic files at continental and global scales use much larger ranges of geographic co-ordinates.

Machine	Points	Calculated squares of offset values	
		Single Precision	Double Precision
ICL 3980			
	(C)	28199.351562500	28143.490838958
	(D)	28171.789062500	28143.490838961
VAX 8200			
	(C)	28253.095703125	28143.490838958
	(D)	28165.806640625	28143.490838958
SEQUENT SYMMETRY			
	(C)	28145.100000000	28143.490838961
	(D)	28145.100000000	28143.490838961
SUN 3/60			
	(C)	28253.095703125	28143.490838961
	(D)	28165.806640625	28143.490838961

#### NOTES

*Offsets of points C and D from the anchor-floor line A-B as calculated using Wade's program. Points A, B, C and D are shown in Figure 5. The British National Grid coordinates (in metres) of the points are as follows:*

Point A	238040 (x1)	205470 (y1)	ANCHOR
Point B	237890 (x2)	205040 (y2)	FLOATER
Point C	237810 (x3)	205320 (y3)	
Point d	238120 (x3)	205190 (y3)	

*Note that the above co-ordinates may be used in conjunction with the expression presented in section 3.2.2a to check the tabulated results.*

**Table 1: The Precision of Calculations**

A limited number of papers actually described improved

for new algorithms or methods for visualization<sup>10, 11, 12</sup>. This may be caused by the complexity of the environment in which a method is used; issues of system architecture, user interface, data handling, etc. must be dealt with before a new presentation technique can show its full advantage. But even so, we think the field can use more contributions of this type.

There was also a discussion session on the merits of animation and special effects (such as sound) to support visualization. For example, in the area of flow visualization, it is quite common to use animation, and techniques for video registration have been developed.

## 7. Issues in Visualization

Scientific visualization is an interdisciplinary field, which can only flourish when computer graphics experts cooperate with specialists from application areas, and providers of computing, visualization, and data management facilities. Therefore, it is essential that all of these viewpoints are represented in research projects and also in meetings such as this workshop. It is not enough that suitable display algorithms, data structures, or user interfaces be developed, but also that these be integrated in usable systems and evaluated by expert users. This complex environment, and the complex systems it requires, call for a common language between different parties involved, and therefore a *reference model*, or an abstract description summarizing the entire process of data visualization, is needed.

At the Delft workshop, an attempt was made to continue the meetings of sub-groups as started in Clamart<sup>6</sup>, but it appeared that a useful description of sub-areas or sub-problems should be based on a stable conceptual framework. Except for the flow visualization group, the subgroup definitions were abandoned, and instead it was decided to concentrate on design of an initial reference model; a first attempt is currently being undertaken by Lesley Carpenter and Michel Grave. At the same time, the separate flow visualization subgroup (chaired by Hans-Georg Pagendarm) agreed to design a general model of the flow visualization process! In addition, arrangements were made for the exchange of test data sets for system evaluation, and the exchange of information on and experience with visualization software.

Special discussion sessions were held about the practice the "circle-brush" algorithm. In this algorithm a solid disk is assumed to move along a trajectory in  $R^2$ . This trajectory is then scan-converted into the raster plane, and experience of the Stardent AVS system, and about general evaluation methods for visualization software. There is an obvious need to share experience or even make a formal (comparative) evaluation of systems, but this is also hampered by lack of a common framework, and also by the continuing development of visualization systems.

Interactive visualization was also an interesting subject for discussion, which yielded a lively debate<sup>6</sup>. In a session

