

Contrary-to-Duty Paradoxes

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Abstract

We will discuss the first problem of Deontic Logic, the so-called Contrary-to-duty (CTD) in this paper. A contrary-to-duty obligation tells us, what we ought to do if something is forbidden. Which is very important in our moral and legal thinking. So we want to discuss the problem with different deontic logic systems, and try to find a way to explain it.

Keywords: contrary-to-duty Paradoxes(CTD), Chisholm's paradox, Standard deontic logic(SDL)

Introduction

What is contrary-to-duty obligation? And why is it a Paradoxes? A contrary-to-duty obligation is an obligation telling us what ought to be the case if something forbidden is true. Here are some examples of sentences that often express such obligations: ‘If she is guilty, she should confess’, ‘If he has hurt his friend, he should apologise to her’, ‘If you are not going to keep your promise to him, you ought to call him’, ‘If the books are not returned by the due date, you must pay a fine’. In this paper we will summarize the first part of the paper “Multiagent Deontic Logic and its Challenges from a Normative Systems Perspective” [1]

1 Chisholm’s paradox

First of all, We are going to introduce first the so-called Chisholm’s paradox [2], which is known as one of the earliest CTD paradox. Chisholm’s paradox consists of the following four sentences:

1. It ought to be that a certain man go to the assistance of his neighbours.
2. It ought to be that if he does go, he tell them he is coming.
3. If he does not go then he ought not to tell them he is coming.
4. He does not go.

Chisholm’s paradox is a contrary-to-duty paradox, since it contains both a primary obligation to go, and a secondary obligation not to tell if the agent does not go. Yet intuitively the natural-language expressions that make up the paradox are consistent and independent from each other: this is why it is called a paradox.

There are also various kinds of scenarios which are similar to Chisholm’s scenario. For example, van Benthem, Grossi and Liu [3] give the following example, in the formulation proposed by Åqvist:

- (1) It ought to be that Smith refrains from robbing Jones.
- (2) Smith robs Jones.
- (3) If Smith robs Jones, he ought to be punished for robbery.
- (4) It ought to be that if Smith refrains from robbing Jones he is not punished for robbery.

Contrary-to-duty obligations turn up in discussions concerning guilt, blame, restoration, reparation, confession, punishment, repentance, retributive justice, compensation, apologies, damage control, etc. Therefore, they are important in our moral and legal thinking. The rationale of a contrary-to-duty obligation is the fact that most of us do neglect our primary duties from time to time and

yet it is reasonable to believe that we should make the best of a bad situation, or at least that it matters what we do when this is the case. Everyone needs to find a way, that tells us what to do, when we don't finish the things we ought to do. In this paper, we will try to find an adequate formal representation of such obligations.

2 Monadic deontic logic

Traditional or 'standard' deontic logic, often referred to as SDL, was introduced by Von Wright[4]

2.1 language

Given two arbitrary formulas ϕ and φ , then

\perp : the empty symbol

$\neg \varphi$: classical negation

$\varphi \vee \phi$: classical disjunction

$\bigcirc \varphi$: φ is obligatory

$\neg \bigcirc \neg \varphi$: φ is forbidden

$\Box \varphi$: φ is necessary

2.2 Semantics

- let ν be a valuation function on AP
- let $M = (W, R)$ A kripke model with on worlds W related via $R : W \times W$
- " $M, s \models \varphi$ " is read a state s satisfies φ in the Model M , we define the satisfaction relation $M, s \models \varphi$ by induction on φ using the clauses:
 - $M, s \models p$ iff $s \in \nu(p)$
 - $M, s \models \neg \varphi$ iff not $M, s \models \varphi$
 - $M, s \models (\varphi \wedge \phi)$ iff $M, s \models \varphi$ and $M, s \models \phi$
 - $M, s \models \bigcirc \varphi$ iff for all t , if Rst then $M, t \models \varphi$
 - $M, s \models \Box \varphi$ iff for all $t \in W$, $M, t \models \varphi$

Axioms for Standard Deontic Logic:

1. Taut: All tautologies are Well formed formulas of the language.
2. *AxiomK* : $O(p \rightarrow q) \rightarrow (O(p) \rightarrow O(q))$
3. *AxiomD* : $O(p) \rightarrow \neg O(\neg p)$
4. *Modus Ponens* : $((p \rightarrow q) \wedge p) \rightarrow q$
5. *Necessity* : $p \rightarrow O(p)$

2.3 Limitations

Now we focus on the Chisholm's paradox with the Monadic deontic logic. We can describe the four sentences of Chisholm's paradox with the Monadic Deontic logic (h : he goes to help his neighbours; t : he tell them), we have four different ways to describe them:

1. $(1)O(h); (2)O(h \rightarrow t); (3)O(\neg h \rightarrow \neg t); (4)\neg h$
2. $(1)O(h); (2)O(h \rightarrow t); (3)\neg h \rightarrow O(\neg t); (4)\neg h$
3. $(1)O(h); (2)h \rightarrow O(t); (3)\neg h \rightarrow O(\neg t); (4)\neg h$
4. $(1)O(h); (2)h \rightarrow O(t); (3)O(\neg h \rightarrow \neg t); (4)\neg h$

Now we focus on the above-mentioned four SDL descriptions:

1. From (4) and Axiom 5, we get $O(\neg h)$; From (1) and (2), we get $O(t)$; From $O(\neg h)$ and (3), we get $O(\neg t)$. Obviously, $O(t)$ and $O(\neg t)$ is paradox.
2. From (3) and (4) with Axiom MP, we get $O(\neg t)$; From (2) and Axiom K, we get $O(h) \rightarrow O(t)$; which with (1) and Axiom MP, we get $O(t)$.
3. We know that, $\neg h \rightarrow (h \rightarrow O(t))$ is Taut, which with (4) can we get $h \rightarrow O(t)$. it is identical to (2). Hence, it's redundant.
4. It's redundant with the same reason of 3.

Hence, we can conclude, that under the Monadic Deontic Logic (SDL) the CTD is **inconsistent** and **redundant**. Something must be wrong with our formalisation, with SDL or with our intuitions. In a nutshell, this puzzle is the contrary-to-duty (obligation) paradox.

So how can we overcome the limitation of SDL? Can we extend the semantics of SDL? For example, one can add distinct modal operators for primary and secondary obligations, where a secondary obligation is a kind of reparational obligation. So with the Chisholm's paradox, we regard 1 and 2 as primary obligation (he ought to go), 3 as secondary obligation (he ought to tell his neighbours). So the sentences are now:

- (1) $\bigcirc 1 h$
- (2) $\bigcirc 1 (h \rightarrow t)$
- (3) $\neg h \rightarrow \bigcirc 2 (\neg t)$
- (4) $\neg h$

From 1-4 we can derive only $\bigcirc 1 \neg t \wedge \bigcirc 2 t$, which is consistent.

Seems like we find a good way to solve the limitation of SDL. However, it may not always be easy to distinguish primary from secondary obligations, because it may depend on the context whether an obligation is primary or secondary. It's not easy for us to describe the ideal world with this solution.

3 Dyadic deontic logic

Dyadic deontic logic(or Dyadic Standard Deontic Logic) is the logic for reasoning with dyadic obligations (“it ought to be the case that ... if it is the case that ...”).In this section, we will try to describe CTD with DDL ,and try to find out a .

3.1 Language

Given two arbitrary formulas ϕ and φ , then

\perp :the empty symbol

$\neg \varphi$: classical negation

$\varphi \vee \phi$: classical disjunction

$\bigcirc \varphi$: φ is obligatory

$\neg \bigcirc \neg \varphi$: φ is forbidden

$\Box \varphi$: φ is necessary

$\bigcirc(\varphi \mid \phi)$: It ought to be φ ,given ϕ

$P(\varphi \mid \phi)$: φ is permitted,given ϕ ,as an abbreviation of $\neg \bigcirc (\neg \varphi \mid \phi)$

$\Diamond(\varphi)$:possibly φ ,as an abbreviation of $\neg \Box \neg \varphi$

3.2 Semantics

A preference model $M = (W, \geq, V)$ is a structure where:

- W is a nonempty set of worlds.
- \geq is a reflexive, transitive relation over W satisfying the following limitedness requirement: if $\|\varphi\| \neq \emptyset$, then $x \in \|\varphi\| : (\forall y \in \|\varphi\|) x \geq y \neq \emptyset$. Here $\|\varphi\| = \{x \in W : M, x \models \varphi\}$.
- V is a standard propositional valuation such that for every propositional letter p , $V(p) \subseteq W$.
 - $M, s \models p$ iff $s \in V(p)$
 - $M, s \models \neg \varphi$ iff not $M, s \models \varphi$
 - $M, s \models (\varphi \wedge \phi)$ iff $M, s \models \varphi$ and $M, s \models \phi$
 - $M, s \models \bigcirc(\varphi \mid \phi)$ iff $\forall t ((M, t \models \varphi) \wedge \forall u (M, u \models \phi \Rightarrow t \geq u) \Rightarrow M, t \models \phi)$
 - $M, s \models \Box \varphi$ iff for all $t \in W$, $M, t \models \varphi$

3.3 Limitations

The following example is a variant of the scenario originally phrased by Chisholm in 1963. There is widespread agreement in the literature that, from the intuitive point of view, this set of sentences is consistent, and its members are logically independent of each other.

A It ought to be that Jones does not eat fast food for dinner.

- B** It ought to be that if Jones does not eat fast food for dinner, then he does not go to McDonald's.
- C** If Jones eats fast food for dinner, then he ought to go to McDonald's.
- D** Jones eats fast food for dinner.

We try to describe the above-mentioned sentences with DSDL(f :Jones eats fast food for dinner; m :he goes to McDonald's):

- A** $\bigcirc f$
- B** $\bigcirc(\neg m \mid \neg f)$
- C** $\bigcirc(m \mid f)$
- D** f

the dyadic representation A - D highlights the dilemma between factual detachment (FD) and deontic detachment (DD)

FD: $\bigcirc(m \mid f), f \Rightarrow \bigcirc m$

DD: $\bigcirc(\neg m \mid \neg f), \bigcirc \neg f \Rightarrow \bigcirc \neg m$

We cannot have both FD and DD, as we derive a dilemma: $\bigcirc \neg m \wedge \bigcirc m$

Hence, there is also something wrong with the DSDL, when we try to describe CTD paradox. The main drawback of DSDL is that in a monotonic setting, we cannot detach the obligation $\bigcirc m$ from the four sentences.

4 Defeasible Deontic Logic: detachment and constraints

We all know that, Deontic logic is the logic of obligations, i.e. reasoning about what should be the case. Defeasible logic is the logic of default assumptions, i.e. reasoning about what normally is the case. In defeasible deontic logic (DDLs) these two are combined. Defeasible deontic logics (DDLs) use techniques developed in non-monotonic logic. A drawback of the use of non-monotonic techniques is that we often have that violated obligations are no longer derived.

The more details about the defeasible deontic logic will be in Section 8 of the "Multiagent deontic logic and its challenges from a normative systems perspective".

5 Alternative approaches

Carmo and Jones [5] suggest that the representation of the facts is challenging, instead of the representation of the norms. In their approach, depending on the formalisation of the facts various obligations can be detached.

Another approach to Chisholm's paradox is to detach both obligations of the dilemma $\neg m \wedge m$, and represent them consistently using some kind of minimal

deontic logic, for example using techniques from paraconsistent logic. From a practical reasoning point of view, a drawback of this approach is that a dilemma is not very useful as a moral cue for action. Moreover, intuitively it is not clear that the example presents a true dilemma. More about the dilemmas will be in Section 9.

Daniel R  nnedal[6] suggests, we can use the Counterfactual Deontic Logic to solve the Contrary-to-Duty Paradoxes.

A recent representation of Chisholm's paradox is to replace deontic detachment by so-called aggregative deontic detachment (ADD), and to derive from A-D the obligation $(\neg f \wedge \neg m)$ and m , but not $\neg m$.

$\bigcirc(m|f), f \Rightarrow \bigcirc m$ FD

$\bigcirc(\neg m|\neg f), \bigcirc\neg f \Rightarrow \bigcirc(\neg m \wedge \neg f)$ ADD

The limitation of this approach is that we can no longer accept the principle of weakening (also known as inheritance). $\bigcirc(\neg m \wedge \neg f|\top) \Rightarrow \bigcirc(\neg m|\top)$

Nowadays more and more logic researchers focus on the CTD Paradoxes, and build different deontic logic systems. When a new deontic logic is proposed, the traditional contrary-to-duty examples are always the first benchmark examples to be checked.

6 Conclusion

With the limited knowledge about the deontic logic, we cannot find a perfect adequate formal representation of the CTD obligation. In our opinion, the key point of CTD Paradoxes is that how can we find an adequate formula to deal with the relationship between **obligation** and **condition**. Consider the sentence from section 3:

B It ought to be that if Jones does not eat fast food for dinner, then he does not go to McDonald's.

which can be formed as:

- $\bigcirc(\neg f \rightarrow \neg m)$

However, in our opinion, "if Jones does not eat fast food for dinner" is a **condition** (instead of obligation) of "he ought to not go to McDonald's". We cannot lump these two concepts together. So the better form should be:

- $\neg f \rightarrow \bigcirc\neg m$

However, this is also redundant due to $f \models \neg f \rightarrow \bigcirc\neg m$

We always find a limitation, when we try to form CTD with deontic logic, especially SDL. More and more approaches are proposed. Many researchers also build the binary deontic logic to solve them..... When a new deontic logic is proposed, the traditional contrary-to-duty examples are always the first benchmark examples to be checked, which is always the biggest challenge of DL System. What's

more,it is also a challenge for our own moral life and law system. Perhaps there is not an eternal solution to solve the paradoxes entirely.But with the research on all the challenges for multiagent deontic logic,the law system will be more and more perfect,our obligation and violation will be more and more clear...

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