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SOLUTIONS SHEET 2

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Exercise 1. Let $(\Omega, \mathcal{A}, P)$ be a probability space. Recall, that for $A \in \mathcal{A}$ with $P(A) > 0$ the *conditional probability of B with respect to A* is defined by

$$P(B|A) := \frac{P(B \cap A)}{P(A)}. \tag{1}$$

1. Let $A_1, A_2 \in \mathcal{A}$ with $0 < P(A_2) < 1$. Observe, that by $0 < P(A_2) < 1$ the conditional probability $P(B|A_2^c)$ is well-defined since $P(A_2^c) = 1 - P(A_2) > 0$. Thus

$$\begin{aligned} P(A_1) &= P((A_1 \cap A_2) \cup (A_1 \cap A_2^c)) \\ &= P(A_1 \cap A_2) + P(A_1 \cap A_2^c) \\ &= \frac{P(A_1 \cap A_2)}{P(A_2)} P(A_2) + \frac{P(A_1 \cap A_2^c)}{P(A_2^c)} P(A_2^c) \\ &= P(A_1|A_2)P(A_2) + P(A_1|A_2^c)P(A_2^c). \end{aligned}$$

2. We have

$$P(A_3|A_1 \cap A_2) = \frac{P(A_3 \cap A_1 \cap A_2)}{P(A_1 \cap A_2)} = \frac{P(A_3)P(A_1)P(A_2)}{P(A_1)P(A_2)} = P(A_3).$$

3.

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