DIFFERENTIAL GEOMETRY I SUMMARY

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Abstract. This is a rough summary of the course *Differential Geometry I* held at *ETH Zurich* by *Prof. Dr. William J. Merry* in autumn 2018. The main focus of this summary is to give a neat preparation for the oral exam.

Contents

The Category of Smooth Manifolds

Definition 1.1 (Topological Manifold). *Let $n \in \mathbb{N}$. A topological space M is said to be a* ***topological manifold of dimension n****, iff*

(i) *M is locally Euclidean of dimension n, that is, for every $x \in M$ there exist an open subset $U \subseteq M$ and a function $\varphi : U \to \mathbb{R}^n$ such that $\varphi(U) \subseteq \mathbb{R}^n$ is open and $\varphi : U \to \varphi(U)$ is a homeomorphism. Every such pair (U, φ) is called a* ***chart on M about x****.*

(ii) *M is Hausdorff and has at most countably many connected components.*

(iii) *M is paracompact, that is, every open cover of M admits a locally finite open refinement.*

Definition 1.2 (Smooth Atlas). *A* ***smooth atlas for a topological manifold M*** *is a collection $(U_\alpha, \varphi_\alpha)_{\alpha \in A}$ of charts on M such that*

(i) *$(U_\alpha)_{\alpha \in A}$ is an open cover for M.*

(ii) *For all $\alpha, \beta \in A$ such that $U_\alpha \cap U_\beta \neq \varnothing$, the function*

$$\varphi_\alpha \circ \varphi_\beta^{-1} : \varphi_\beta(U_\alpha \cap U_\beta) \to \varphi_\alpha(U_\alpha \cap U_\beta)$$

is smooth. The function $\varphi_\alpha \circ \varphi_\beta^{-1}$ is called a ***transition function****.*

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