# DIFFERENTIAL GEOMETRY I SUMMARY

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**Abstract**. This is a rough summary of the course *Differential Geometry I* held at *ETH Zurich* by *Prof. Dr. William J. Merry* in autumn 2018. The main focus of this summary is to give a neat preparation for the oral exam.

## Contents



## The Category of Smooth Manifolds

**Definition 1.1 (Topological Manifold).** *Let $n \in \mathbb{N}$. A topological space $M$ is said to be a* ***topological manifold of dimension n****, iff*

(i) *$M$ is locally Euclidean of dimension $n$, that is, for every $x \in M$ there exist an open subset $U \subseteq M$ and a function $\varphi : U \to \mathbb{R}^n$ such that $\varphi(U) \subseteq \mathbb{R}^n$ is open and $\varphi : U \to \varphi(U)$ is a homeomorphism.*

(ii) *$M$ is Hausdorff and has at most countably many connected components.*

(iii) *$M$ is paracompact, that is, every open cover of $M$ admits a locally finite open refinement.*

**Proposition 1.1 (Chain Rule).** *Let $U \subseteq \mathbb{R}^n$ and $V \subseteq \mathbb{R}^m$ be open, $F \in C^1(U, V)$ and $G \in C^1(V, \mathbb{R}^p)$. Then $G \circ F \in C^1(U, \mathbb{R}^p)$ and*

$$\frac{\partial(G^i \circ F)}{\partial x^j}(x) = \frac{\partial G^i}{\partial y^k}\big(F(x)\big)\frac{\partial F^k}{\partial x^j}(x)$$

*holds for all $x \in U$.*

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