SOLUTIONS SHEET 1

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Exercise 1.

(a) The pair $(D(X), i)$ has the universal property

$$\begin{tikzcd} X \arrow[r, "i"] \arrow[dr, "\forall \text{ functions } f"'] & D(X) \arrow[d, "\exists! \text{ continuous } \overline{f}"] \\ & \forall (Y, \mathcal{T}_Y). \end{tikzcd}$$

Assume, that there is another pair $(i', D'(X))$ with this property. Thus we get the two commuting diagrams

$$\begin{tikzcd} X \arrow[r, "i"] \arrow[dr, "i'"'] & D(X) \arrow[d, "\overline{i'}"] \\ & D'(X), \end{tikzcd} \qquad \begin{tikzcd} X \arrow[r, "i'"] \arrow[dr, "i"'] & D'(X) \arrow[d, "\overline{i}"] \\ & D(X). \end{tikzcd}$$

Exercise 2.

Exercise 3.

Exercise 4. Let $g, \widetilde{g} : Y \to X$ be inverses of f. Then we have

$$g = g \circ \mathrm{id}_Y = g \circ (f \circ \widetilde{g}) = (g \circ f) \circ \widetilde{g} = \mathrm{id}_X \circ \widetilde{g} = \widetilde{g}.$$

Thus we can unambiguously write $f^{-1} := g$.

Exercise 5. That $h \circ g \circ f$ is an isomorphism immediately follows by

$$\big((g \circ f)^{-1} \circ g \circ (h \circ g)^{-1}\big) \circ (h \circ g \circ f) = \mathrm{id}_X$$
$$(h \circ g \circ f) \circ \big((g \circ f)^{-1} \circ g \circ (h \circ g)^{-1}\big) = \mathrm{id}_W .$$

$$\big((g \circ f)^{-1} \circ g\big) \circ f = (g \circ f)^{-1} \circ (g \circ f) = \mathrm{id}_X$$

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$$\big((h \circ g)^{-1} \circ h\big) \circ g = (h \circ g)^{-1} \circ (h \circ g) = \mathrm{id}_Y$$
$$h \circ \big(g \circ (h \circ g)^{-1}\big) = (h \circ g) \circ (h \circ g)^{-1} = \mathrm{id}_W$$

Exercise 6. Assume $f : X \to Y$ has the left cancellation property. Let $x, y \in X$ such that $f(x) = f(y)$. Now let $Z := \{x, y\}$. Define two functions $c_x, c_y : Z \to X$ by $c_x(z) := x$ and $c_y(z) := y$, respectively. Now

$$f \circ c_x = f(x) = f(y) = f \circ c_y$$

holds by assumption. Thus the left cancellation property implies that $c_x = c_y$, hence $x = y$ and f is injective. Conversly, assume that f is injective. Let $\alpha, \beta : Z \to X$ such that $f \circ \alpha = f \circ \beta$ and $z \in Z$. Then we have that $f(\alpha(z)) = f(\beta(z))$ and thus by injectivity, $\alpha(z) = \beta(z)$.