# MAT602 - FUNCTIONAL ANALYSIS

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## Contents



## 1. Linear Operators

### 1.1. Continuous Operators.

**Definition 1.1.** *Let $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$ be two normed spaces. An* ***operator*** *is a linear mapping $T : X \to Y$. Moreover, we say that an operator $T : X \to Y$ is* ***bounded*** *if there exists $c > 0$ such that*

$$\|T(x)\|_Y \leq c\|x\|_X \tag{1}$$

*holds for all $x \in X$.*

### 1.2. The Hahn-Banach Theorem.

**Lemma 1.1.** *Let $X$ be a vector space over $\mathbb{R}$, $M \subsetneq X$ a linear subspace, $p : X \to \mathbb{R}$ a sublinear functional, $f : M \to \mathbb{R}$ linear and $x_0 \in X \setminus M$. Moreover, assume that $f \leq p$ on $M$. Then there exists $F : M + \mathbb{R}x_0 \to \mathbb{R}$ linear such that $F \leq p$ on $M + \mathbb{R}x_0$ and $F|_M = f$.*

**Theorem 1.1 (Hahn-Banach, real case).** *Let $X$ be a vector space over $\mathbb{R}$, $M \subseteq X$ a linear subspace and $f : M \to \mathbb{R}$ linear. Moreover, let $p : X \to \mathbb{R}$ be a sublinear functional such that $f \leq p$ on $M$. Then there exists $F : X \to \mathbb{R}$ linear such that $F \leq p$ on $X$ and $F|_M = f$.*

**Theorem 1.2 (Hahn-Banach, complex case).**

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