

SOLUTIONS SHEET 6

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Exercise 1.

(a)

Lemma 1.1. *Let $U : \mathbf{Grp} \rightarrow \mathbf{Set}$ be the forgetful functor and $F : \mathbf{Set} \rightarrow \mathbf{Grp}$ be the free functor. Then $F \dashv U$.*

Proof. Let $S \in \mathbf{ob}(\mathbf{Set})$ and $G \in \mathbf{ob}(\mathbf{Grp})$. Let $\varphi \in \mathbf{Grp}(F(S), G)$. Define a function $\bar{\varphi} \in \mathbf{Set}(S, U(G))$ by $\bar{\varphi}(s) := \varphi(\iota(s))$, where $\iota : S \hookrightarrow F(S)$ is the natural injection (see [Lee11, p. 240]). Moreover, for $f \in \mathbf{Set}(S, U(G))$ define $\bar{f} \in \mathbf{Grp}(F(S), G)$ to be the unique extension of f (see [Lee11, p. 240]). Clearly $\bar{\bar{f}} = f$ and $\bar{\bar{g}} = g$. \square

References

- [Lee11] John M. Lee. *Introduction to Topological Manifolds*. Second Edition. Springer Science+Business Media, 2011.