ALGEBRAIC TOPOLOGY I

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1. Singular Homology

Proposition 1.1. *There is a functor* $\mathsf{Top} \to \mathsf{Comp}$ *which associates to each topological space* X *the singular chain complex* $(C(X), \partial)$ *and to each* $f \in \mathsf{Top}(X, Y)$ *the chain map* $f^{\#} : C(X) \to C(Y)$.

Proof. The proof is divided into several steps.

Step 1: $(C(X), \partial)$ *is a chain complex.*
Step 2: $f^{\#}$ *is a chain map.*
Step 3: Checking functorial properties.

□

Proposition 1.2. *Let* $n \in \mathbb{Z}$. *Then there exists a functor* $H_n : \mathsf{Comp} \to \mathsf{Ab}$.

Proof. □

Proposition 1.3. *Let* X *be a nonempty, path-connected space. Then* $H_0(X) \cong \mathbb{Z}$. *Moreover, a generator of* $H_0(X)$ *is given by any* $x \in X$.

Proof. □

2. The Homotopy Axiom

Theorem 2.1 (The Homotopy Axiom). *Let* $n \in \mathbb{Z}$. *Then the functor* H_n *induces a functor* $H_n : \mathsf{hTop} \to \mathsf{Ab}$.

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3. The Hurewicz Theorem

Theorem 3.1 (Hurewicz Theorem). *Let X be a path-connected space and $p \in X$. Then $\pi_1(X, p)^{\mathrm{ab}} \cong H_1(X)$.*