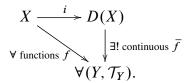
SOLUTIONS SHEET 1

YANNIS BÄHNI

Exercise 1.

(a) The pair (D(X), i) has the universal property



Assume, that there is another pair (i', D'(X)) with this property. Thus we get the two commuting diagrams



Exercise 2.

Exercise 3.

Exercise 4. Let $g, \tilde{g}: Y \to X$ be inverses of f. Then we have

$$g = g \circ id_Y = g \circ (f \circ \widetilde{g}) = (g \circ f) \circ \widetilde{g} = id_X \circ \widetilde{g} = \widetilde{g}.$$

Thus we can unambiguously write $f^{-1} := g$.

Exercise 5. That $h \circ g \circ f$ is an isomorphism immediately follows by

$$((g \circ f)^{-1} \circ g \circ (h \circ g)^{-1}) \circ (h \circ g \circ f) = \mathrm{id}_X$$

$$(h \circ g \circ f) \circ ((g \circ f)^{-1} \circ g \circ (h \circ g)^{-1}) = \mathrm{id}_W.$$

$$((g \circ f)^{-1} \circ g) \circ f = (g \circ f)^{-1} \circ (g \circ f) = \mathrm{id}_X$$

(Yannis Bähni) University of Zurich, Rämistrasse 71, 8006 Zurich *E-mail address*: yannis.baehni@uzh.ch.

$$((h \circ g)^{-1} \circ h) \circ g = (h \circ g)^{-1} \circ (h \circ g) = \mathrm{id}_Y$$

$$h \circ (g \circ (h \circ g)^{-1}) = (h \circ g) \circ (h \circ g)^{-1} = \mathrm{id}_W$$

Exercise 6. Assume $f: X \to Y$ has the left cancellation property. Let $x, y \in X$ such that f(x) = f(y). Now let $Z := \{x, y\}$. Define two functions $c_x, c_y : Z \to X$ by $c_x(z) := x$ and $c_y(z) := y$, respectively. Now

$$f \circ c_x = f(x) = f(y) = f \circ c_y$$

holds by assumption. Thus the left cancellation property implies that $c_x = c_y$, hence x = y and f is injective. Conversly, assume that f is injective. Let $\alpha, \beta : Z \to X$ such that $f \circ \alpha = f \circ \beta$ and $z \in Z$. Then we have that $f(\alpha(z)) = f(\beta(z))$ and thus by injectivity, $\alpha(z) = \beta(z)$.