AN INTRODUCTION TO GENERAL AND ALGEBRAIC TOPOLOGY

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Contents

CHAPTER 1

Constructions

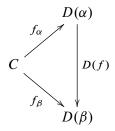
1. Limits

Definition 1.1 (**Diagram**). Let \mathcal{C} be a category and A a small category. A functor $A \to \mathcal{C}$ is called a **diagram in \mathcal{C} of shape A**.

Definition 1.2 (Cone and Limit). Let \mathcal{C} be a category and $D: A \to \mathcal{C}$ a diagram in \mathcal{C} of shape A. A cone on D is a tuple $(C, (f_{\alpha})_{\alpha \in A})$, where $C \in \mathcal{C}$ is an object, called the **vertex** of the cone, and a family of arrows in \mathcal{C}

$$\left(C \xrightarrow{f_{\alpha}} D(\alpha)\right)_{\alpha \in A}. \tag{1}$$

such that for all morphisms $f \in A$, $f : \alpha \to \beta$, the triangle



commutes. A (small) limit of D is a cone $(L, (\pi_{\alpha})_{\alpha \in A})$ with the property that for any other cone $(C, (f_{\alpha})_{\alpha \in A})$ there exists a unique morphism $\overline{f}: A \to L$ such that $\pi_{\alpha} \circ \overline{f} = f_{\alpha}$ holds for every $\alpha \in A$.

Remark 1.1. In the setting of definition 1.2, if $(L, (\pi_{\alpha})_{\alpha \in A})$ is a limit of D, we sometimes reffering to L only as the limit of D and we write

$$L = \lim_{\leftarrow A} D. \tag{2}$$

Definition 1.3 (Product). Let \mathcal{C} be a category and A a set. Define A to be the discrete category with ob(A) := A. Moreover, let D be a diagram in \mathcal{C} of shape A. A (small) product of D is a limit of D.

Remark 1.2. In the setting of definition 1.3, D yields a family $(X_{\alpha})_{\alpha \in A}$ in \mathcal{C} and thus we we speak also of a product of the family $(X_{\alpha})_{\alpha \in A}$. If a product exists in \mathcal{C} , we write

$$\prod_{\alpha \in A} X_{\alpha} := \lim_{\leftarrow A} D. \tag{3}$$

1

CHAPTER 2

The Fundamental Group

1. Homotopies

Proposition 1.1. *Being homotopic is a congruence on* Top.

PROOF. First we show that being homotopic induces an equivalence relation on

 $\bigcup_{(X,Y)\in \mathrm{ob}(\mathsf{Top})\times \mathrm{ob}(\mathsf{Top})} C(X,Y)$