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In Applications and Examples

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CHAPTER 1

Ordinary Differential Equations

1. Extrapolation and Step Size Control

1.1. The Implicit Trapezoidal Rule. The *implicit trapezoidal rule* is given by

$$y_1 = y_0 + \frac{h}{2}\left(f(x_0, y_0) + f(x_1, y_1)\right) \tag{1}$$

As one observes, the discretization y_1 of $y(x_1)$ is calculated implicitely. There are two simple approaches to handle this impliciteness. First of all, y_1 is a fixed point of the function

$$\Phi(y; x_0, y_0, h) := y_0 + \frac{h}{2}\left(f(x_0, y_0) + f(x_0 + h, y)\right) \tag{2}$$

Hence we may apply a simple *fixed point iteration* of the form

$$y_1^{(k+1)} = \Phi\left(y_1^{(k)}; x_0, y_0, h\right) \qquad k = 0, 1, 2, \ldots \tag{3}$$

until a certain tolerance or a user-specified maximal number of iterations is reached. An implementation of the fixed point iteration and the implicit trapezoidal rule can be found in listing 1 and 2 respectively.

```
function [ x ] = fixediter( phi,x0,tol,maxit )
x1 = phi(x0);
it = 1;
while norm(x1 - x0) > tol && it < maxit
    x0 = x1;
    x1 = phi(x0);
    it = it + 1;
end
x = x0;
end
```

Listing 1. Fixed point iteration.

The second approach would be applying the *Newton iteration*

$$y_1^{(k+1)} = y_0^{(k)} - \left(D\Phi\left(y_1^{(k)}; x_0, y_0, h\right)\right)^{-1} \Phi\left(y_1^{(k)}; x_0, y_0, h\right) \qquad k = 0, 1, 2, \ldots \tag{4}$$

```
function [ x,y ] = ITR( f,x0,xN,y0,N,tol,maxit )
    h = ( xN - x0 )/N;
    x = x0:h:xN;
    y = zeros(length(y0), N + 1);
    y(:,1) = y0;
    for k = 1:N
        phi = @(y1) y(:,k) + h/2 * (f(x(k),y(:,k)) + f(x(k + 1), y1));
        y(:,k + 1) = fixediter( phi,y(:,k),tol );
    end
end
```

Listing 2. Implicit trapezoidal rule.

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