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EPD629: Powertrain Systems and Controls
Lesson 03 Assignment
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Assignment:

1) Complete chapter 2 textbook problem 1a, 1b, 1c, and 1d. (4 points)
2) Complete chapter 2 textbook problem 2a, 2b, and 2c. (3 points)
3) Continuation of MATLAB/Simulink model problem. (submitted seperately)

(Problem Source:
https://bcs.wiley.com/he-bcs/Books?action=mininav&bcsId=11568&itemId=1119474221&assetId=473646&resourceId=45905&newwindow=true)

Problems and Attempt:

1. Derive the Laplace transform for the following time functions: [Section: 2.2]
   - u(t)
     - Taking u(t) as
     - $u(t) = f(x) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$
     - The laplace transform is then
     - $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-s*t} * f(t) * dt$
     - $\mathcal{L}\{u(t)\} = \int_0^\infty e^{-s*t} * u(t) * dt$
     - $\mathcal{L}\{u(t)\} = \int_0^\infty e^{-s*t} * 1 * dt$
     - $\mathcal{L}\{u(t)\} = \int_0^\infty e^{-s*t} * dt$
     - $\mathcal{L}\{u(t)\} = \frac{1}{-s} * e^{-s*t}\Big|_0^\infty$
     - $\mathcal{L}\{u(t)\} = -\frac{1}{s} * [(e^{-s*(\infty)}) - (e^{-s*(0)})]$
     - $\mathcal{L}\{u(t)\} = -\frac{1}{s} * [(e^{-s*(\infty)}) - 1]$
     - $\mathcal{L}\{u(t)\} = -\frac{1}{s} * [\frac{1}{e^{s*(\infty)}} - 1]$
     - $\mathcal{L}\{u(t)\} = -\frac{1}{s} * [0 - 1]$
     - $\mathcal{L}\{u(t)\} = -\frac{1}{s} * (-1)$
     - $\mathcal{L}\{u(t)\} = \frac{1}{s}$

   - t*u(t)f
     - $\mathcal{L}\{t * u(t)\} = \int_0^\infty e^{-s*t} * t * u(t) * dt$
     - $\mathcal{L}\{t * u(t)\} = \int_0^\infty e^{-s*t} * t * 1 * dt$
     - $\mathcal{L}\{t * u(t)\} = \int_0^\infty e^{-s*t} * t * dt$
     - $\mathcal{L}\{t * u(t)\} = -\frac{t*e^{-s*t}}{s}\Big|_0^\infty - \int_0^\infty \frac{e^{-s*t}}{-s} dt$
     - $\mathcal{L}\{t * u(t)\} = -\frac{t*e^{-s*t}}{s}\Big|_0^\infty + \frac{1}{s}\int_0^\infty e^{-s*t} dt$
     - $\mathcal{L}\{t * u(t)\} = -\frac{t*e^{-s*t}}{s}\Big|_0^\infty - \frac{1}{s^2} * e^{-s*t}\Big|_0^\infty$
     - $\mathcal{L}\{t * u(t)\} = (-\frac{t*e^{-s*t}}{s} - \frac{1}{s^2} * e^{-s*t})\Big|_0^\infty$

- $\mathcal{L}\{t*u(t)\} = \left(-\frac{\infty*e^{-s*\infty}}{s} - \frac{1}{s^2}*e^{-s*\infty}\right) - \left(-\frac{0*0}{s} - \frac{1}{s^2}*e^{-s*0}\right)$
- $\mathcal{L}\{t*u(t)\} = \left(-\frac{\infty*e^{-s*\infty}}{s} - \frac{1}{s^2}*\frac{1}{e^{s*\infty}}\right) - \left(0 - \frac{1}{s^2}*1\right)$
- $\mathcal{L}\{t*u(t)\} = \left(-0 - \frac{1}{s^2}*0\right) + \frac{1}{s^2}$
- $\mathcal{L}\{t*u(t)\} = (0) + \frac{1}{s^2}$
- $\mathcal{L}\{t*u(t)\} = \frac{1}{s^2}$

* sin(ω*t)*u(t)
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \int_0^\infty e^{-s*t}*\sin(\omega*\mathrm{t})*u(t)\,dt$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \int_0^\infty e^{-s*t}*\sin(\omega*\mathrm{t})*1\,dt$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \int_0^\infty e^{-s*t}*\sin(\omega*\mathrm{t})\,dt$
  - Let $s = \sigma + j*\omega$
  - Let $\sin(\omega*t) = \frac{e^{j*\omega*t}-e^{-j*\omega*t}}{2*j}$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \int_0^\infty e^{-s*t}*\frac{e^{j*\omega*t}-e^{-j*\omega*t}}{2*j}dt$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \frac{1}{2*j}\left[\int_0^\infty e^{-s*t}*e^{j*\omega*t}dt - \int_0^\infty e^{-s*t}*e^{-j*\omega*t}dt\right]$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \frac{1}{2*j}\left[\int_0^\infty e^{-(s-j*\omega*t}dt - \int_0^\infty e^{-(s+j*\omega*t}dt\right]$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \frac{1}{2*j}\left[\frac{1}{s-j*\omega}*e^{-(s-j*\omega*t}\Big|_0^\infty + \frac{1}{s-j*\omega}*e^{-(s+j*\omega*t}\Big|_0^\infty\right]$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \frac{1}{2*j}\left[\frac{-1}{s-j*\omega}(0-1) + \frac{-1}{s+j*\omega}(0-1)\right]$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \frac{1}{2*j}\left[\frac{1}{s-j*\omega} + \frac{1}{s+j*\omega}\right]$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \frac{1}{2*j}\left[\frac{s+j*\omega-(s-j*\omega)}{(s+j*\omega)*(s-j*\omega)}\right]$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \frac{1}{2*j}\left[\frac{2*j*\omega}{s^2-j^2*\omega^2}\right]$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \left[\frac{\omega}{s^2-j^2*\omega^2}\right]$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \left[\frac{\omega}{s^2-(-1)*\omega^2}\right]$
  - $\mathcal{L}\{\sin(\omega*\mathrm{t})*u(t)\} = \left[\frac{\omega}{s^2+*\omega^2}\right]$

* cos(ω*t)*u(t)
  - $\mathcal{L}\{\cos(\omega*\mathrm{t})*u(t)\} = \int_0^\infty e^{-s*t}*\cos(\omega*\mathrm{t})*u(t)\,dt$
  - $\mathcal{L}\{\cos(\omega*\mathrm{t})*u(t)\} = \int_0^\infty e^{-s*t}*\cos(\omega*\mathrm{t})*1\,dt$
  - $\mathcal{L}\{\cos(\omega*\mathrm{t})*u(t)\} = \int_0^\infty e^{-s*t}*\cos(\omega*\mathrm{t})\ dt$
  - Let $u = \cos(\omega*t)$
  - Let $du = -\omega*\sin(\omega*t)dt$
  - Let $dv = \mathrm{e}^{-s*t}dt$
  - Let $v = -\frac{1}{s}*e^{-s*t}$
  - $\mathcal{L}\{\cos(\omega*\mathrm{t})*u(t)\} = \cos(\omega*t)*\left(-\frac{1}{s}\right)*e^{-s*t}[eval\ 0 to \mathrm{inf}] - \int_0^\infty -\frac{1}{s}*e^{-s*t}*(-\omega*\sin(\omega*t))\mathrm{dt}$

- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = \cos(\omega * \infty) * \frac{-1}{s} * e^{-\infty} - \cos(0) * \left(-\frac{1}{s}\right) * e^{0} - \frac{\omega}{s} * \int_0^{\infty} e^{-s*t} * \sin(\omega * t)\, dt$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = 0 - 1 * \left(-\frac{1}{s}\right) * 1 - \frac{\omega}{s} * \int_0^{\infty} e^{-s*t} * \sin(\omega * t)\, dt$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = \frac{1}{s} - \frac{\omega}{s} * \int_0^{\infty} e^{-s*t} * \sin(\omega * t)\, dt$
- Let $u = \sin(\omega * t)$
- Let $du = \omega * \cos(\omega * t)$
- Let $dv = e^{-s*t} dt$
- Let $v = -\frac{1}{s} * e^{-s*t}$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = \frac{1}{s} - \frac{\omega}{s} * \left[\sin(\omega * t) * \frac{-1}{s} * e^{-s*t}\right]\Big|_0^{\infty} - \int_0^{\infty} \frac{-1}{s} * e^{-s*t} * \omega * \cos(\omega * t)\, dt$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = \frac{1}{s} - \int_0^{\infty} \frac{-1}{s} * e^{-s*t} * \omega * \cos(\omega * t)\, dt$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = \frac{1}{s} - \frac{\omega^2}{s^2} * \int_0^{\infty} e^{-s*t} * \cos(\omega * t)\, dt$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = \frac{1}{s} - \frac{\omega^2}{s^2} * \int_0^{\infty} e^{-s*t} * \cos(\omega * t)\, dt$
- Say F(s) = $\int_0^{\infty} e^{-s*t} * \cos(\omega * t)\, dt$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = F(s) = \frac{1}{s} - \frac{\omega^2}{s^2} * \int_0^{\infty} e^{-s*t} * \cos(\omega * t)\, dt$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = F(s) = \frac{1}{s} - \frac{\omega^2}{s^2} * F(s)$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = F(s) + \frac{\omega^2}{s^2} * F(s) = \frac{1}{s}$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = F(s) * \left(1 + \frac{\omega^2}{s^2}\right) = \frac{1}{s}$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = F(s) * \left(\frac{s^2}{s^2} + \frac{\omega^2}{s^2}\right) = \frac{1}{s}$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = F(s) = \frac{s}{s^2+\omega^2}$
- $\mathcal{L}\{\cos(\omega * t) * u(t)\} = \frac{s}{s^2+\omega^2}$

2. Using the Laplace transform pairs of Table 2.1 and the Laplace transform theorems of Table 2.2, derive the Laplace transforms for the following time functions: [Section: 2.2]
   - e^(-a*t) * sin(ω*t) * u(t)
     - Let $f(t) = \sin(\omega * t)$
     - F(s)= $\frac{\omega}{s^2 + *\omega^2}$
     - Combine this with the frequency shift theorem
     - $\mathcal{L}\{e^{-a*t} * \sin(\omega * t)\} = F(s+a)$
     - $\mathcal{L}\{e^{-a*t} * \sin(\omega * t)\} = \frac{\omega}{(s+a)^2 + *\omega^2}$

   - e^(-a*t) * cos(ω*t) * u(t)
     - Let $f(t) = cos(\omega * t)$
     - F(s)= $\frac{s}{s^2 + *\omega^2}$
     - Combine this with the frequency shift theorem
     - $\mathcal{L}\{e^{-a*t} * \cos(\omega * t)\} = F(s+a)$

- $\mathcal{L}\{e^{-a*t} * \cos(\omega * \mathrm{t})\} = \frac{s+a}{(s+a)^2+*\omega^2}$

- t^(3) * u(t)
  - $\mathcal{L}\{t^n * u(t)\} = \frac{n!}{s^{n+1}}$
  - $\mathcal{L}\{t^n * u(t)\} = \frac{(3)*(2)*(1)}{s^{3+1}}$
  - $\mathcal{L}\{t^n * u(t)\} = \frac{6}{s^4}$