Traffic Engineering of Unicast Services

Modelação e Desempenho de Redes e Serviços

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Traffic engineering of unicast services

A unicast service is defined by a set of point-to-point traffic flows on a given telecommunication network.

- Consider a network composed by a set of point-to-point links and supporting one unicast service defined by a set of traffic flows T, such that the packets of all flows have the same statistics.
  - The network is modelled by a graph $G=(N,A)$. Set N is the set of network nodes. Set A is the set of network links: the arc $(i,j) \in A$ represents the link between nodes $i \in N$ and $j \in N$ from i to j whose capacity is given by c_{ij} in bps (usually $c_{ij} = c_{ji}$).
  - Each traffic flow $t \in T$ is defined by its origin node o_t, destination node d_t, average throughput from origin to destination b_t (in bps) and average throughput from destination to origin $\underline{b}_t$ (in bps).
  - For each flow $t \in T$, P_t is the set of the candidate routing paths in graph G from its origin node o_t to its destination node d_t.

The traffic engineering task is the task of choosing for each flow $t \in T$ the percentage of its average throughput that must be routed through each of its candidate routing paths of P_t in each direction.

Traffic engineering with single path routing

- In single path routing, each traffic flow must be routed through one single path (no flow bifurcation is allowed).
- Symmetrical routing might be required or not; when required, the routing path from a node $j \in N$ to a node $i \in N$ must use the same links as the routing path from node $i \in N$ to node $j \in N$.

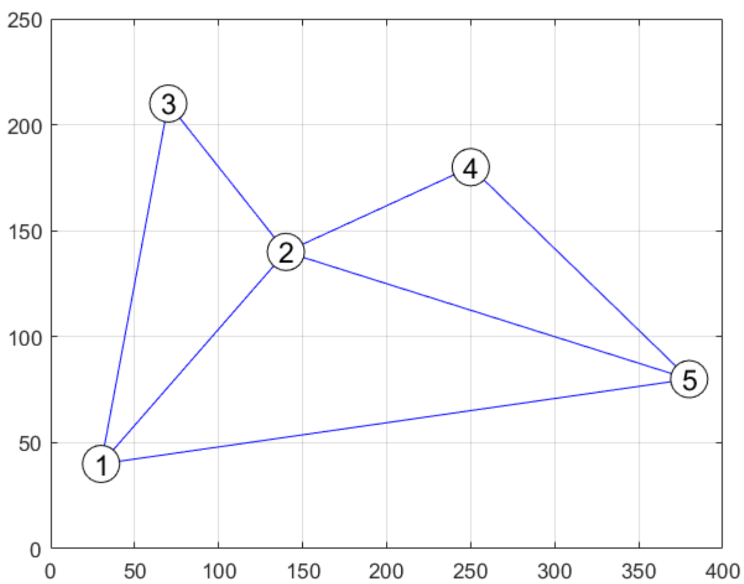
Consider a binary variable x_{tp} associated to each traffic flow $t \in T$ and each routing path $p \in P_t$ that, when is 1, indicates that traffic flow t is routed through path p.

Any traffic engineering solution with single path routing must be compliant with the following constraints:

- For each flow $t \in T$, one of its associated variables x_{tp} must be 1 and all other associated variables must be 0.
- At each arc $(i,j) \in A$, the sum of the throughput values (either b_t or $\underline{b}_t$) of all flows routed through it cannot be higher than its capacity c_{ij}.

Example

Example network:



All links with 10 Gbps of capacity
(in general, these values can be different)

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

In general, these values are different

Flow 1:
Path 1 = 1 2 4
Path 2 = 1 3 2 4
Path 3 = 1 5 4
Path 4 = 1 2 5 4
Path 5 = 1 3 2 5 4
Path 6 = 1 5 2 4

Flow 2:
Path 1 = 1 5
Path 2 = 1 2 5
Path 3 = 1 2 4 5
Path 4 = 1 3 2 5
Path 5 = 1 3 2 4 5

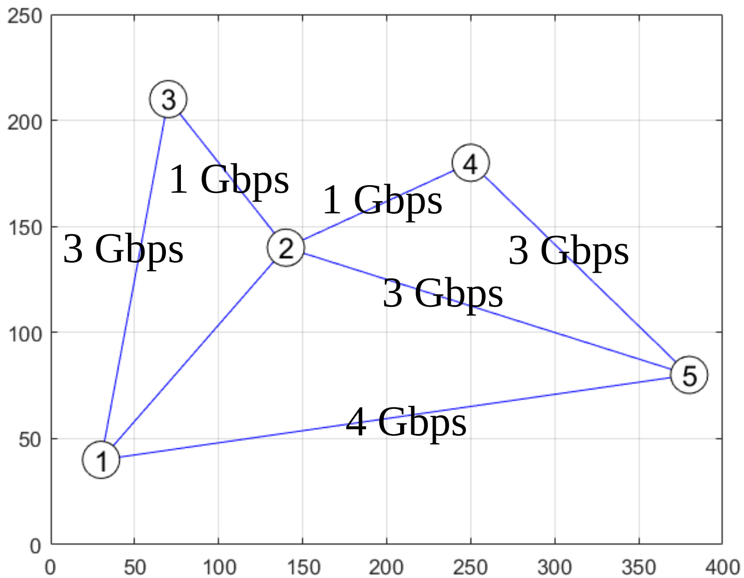
Flow 3:
Path 1 = 2 4
Path 2 = 2 5 4
Path 3 = 2 1 5 4
Path 4 = 2 3 1 5 4

Flow 4:
Path 1 = 3 2 5
Path 2 = 3 2 4 5
Path 3 = 3 1 5
Path 4 = 3 1 2 5
Path 5 = 3 2 1 5
Path 6 = 3 1 2 4 5

Routing paths ordered from shortest to longest lengths

Example: one possible solution

Example network:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 1:
- Path 1 = 1 2 4
- **Path 2 = 1 3 2 4**
- Path 3 = 1 5 4
- Path 4 = 1 2 5 4
- Path 5 = 1 3 2 5 4
- Path 6 = 1 5 2 4

Flow 2:
- **Path 1 = 1 5**
- Path 2 = 1 2 5
- Path 3 = 1 2 4 5
- Path 4 = 1 3 2 5
- Path 5 = 1 3 2 4 5

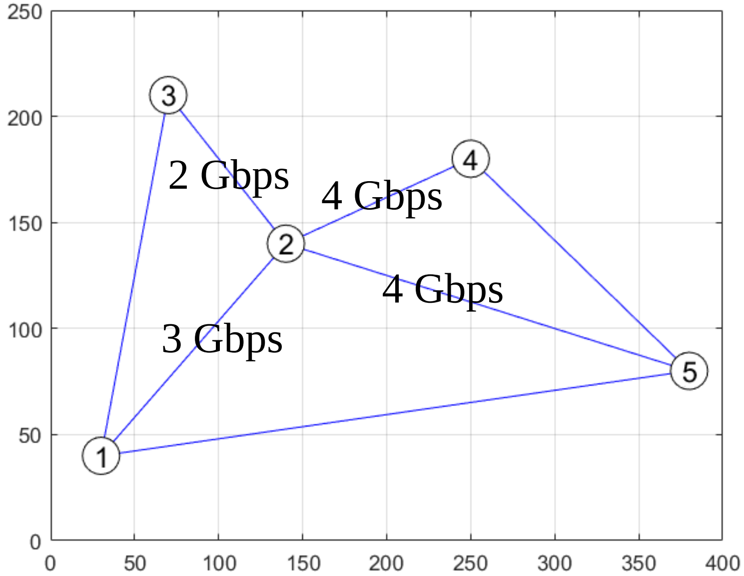
Flow 3:
- Path 1 = 2 4
- **Path 2 = 2 5 4**
- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Flow 4:
- Path 1 = 3 2 5
- Path 2 = 3 2 4 5
- **Path 3 = 3 1 5**
- Path 4 = 3 1 2 5
- Path 5 = 3 2 1 5
- Path 6 = 3 1 2 4 5

One routing path assigned for each traffic flow

Example: another possible solution

Example network:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 1:
- **Path 1 = 1 2 4**
- Path 2 = 1 3 2 4
- Path 3 = 1 5 4
- Path 4 = 1 2 5 4
- Path 5 = 1 3 2 5 4
- Path 6 = 1 5 2 4

Flow 2:
- Path 1 = 1 5
- **Path 2 = 1 2 5**
- Path 3 = 1 2 4 5
- Path 4 = 1 3 2 5
- Path 5 = 1 3 2 4 5

Flow 3:
- **Path 1 = 2 4**
- Path 2 = 2 5 4
- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Flow 4:
- **Path 1 = 3 2 5**
- Path 2 = 3 2 4 5
- Path 3 = 3 1 5
- Path 4 = 3 1 2 5
- Path 5 = 3 2 1 5
- Path 6 = 3 1 2 4 5

Different assignments provide different solutions

Traffic engineering objectives

The traffic engineering task aims to:

- optimize at least one parameter related with either the performance or the operational cost of the network;
- optionally, guarantee (maximum or minimum) values for other parameters.

Examples of optimization parameters:

- the average service packet delay (to minimize the delay performance of the service);
- the worst average packet delay among all traffic flows (to minimize the delay performance fairness among all traffic flows);
- the worst link load (to maximize the robustness of the network to unpredictable traffic growth);
- the energy consumption of the network (to minimize operational costs).

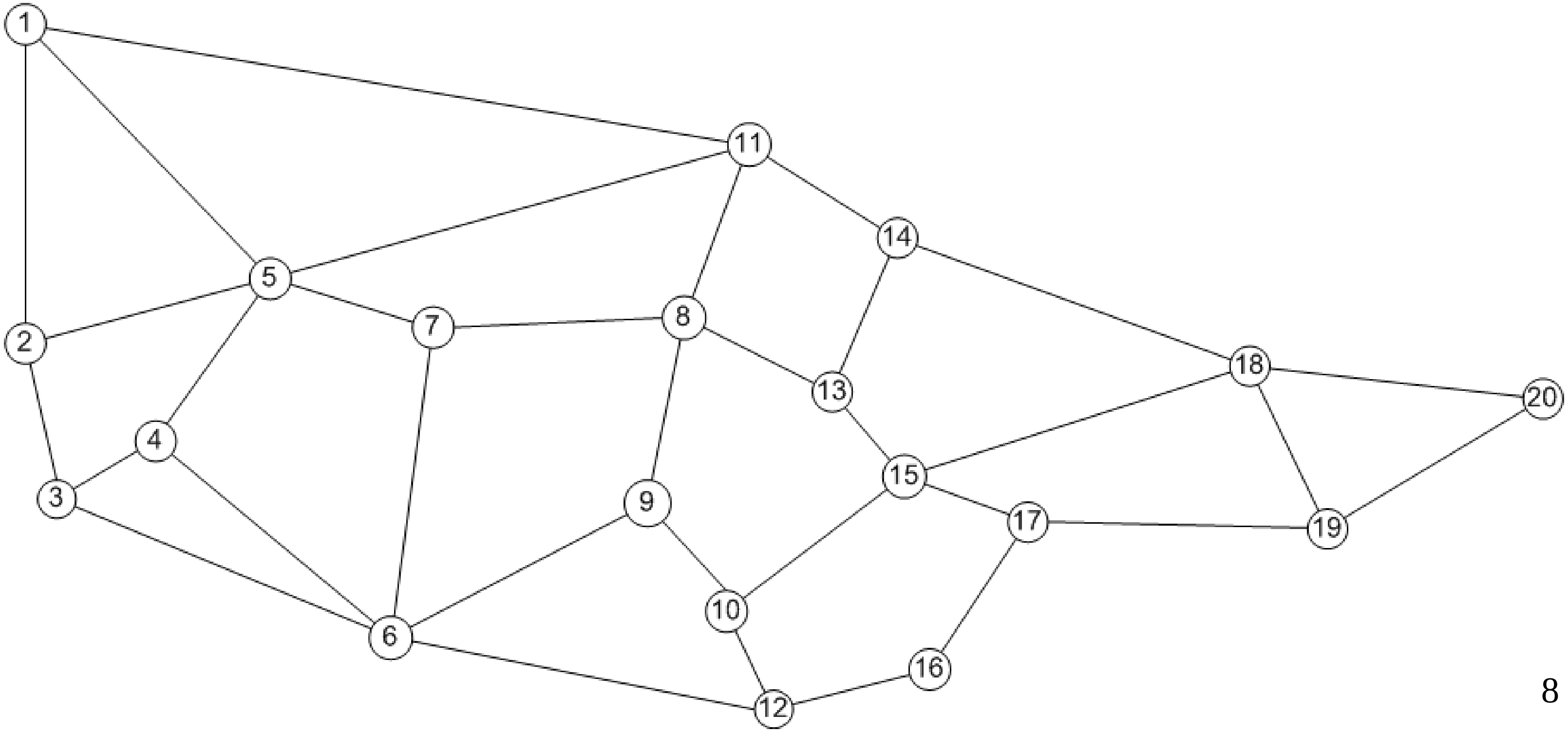
The best traffic engineering solution depends on the optimization objective of interest. Different optimization objectives might be conflicting:

- for example, to reduce the energy consumption, more links must be put in sleeping mode; consequently, the same traffic is concentrated in less links and the worst link load increases.

Example - network

Consider the following network with 20 routers and 33 links where all links have a capacity of 1 Gbps.

The length of the links varies between 88 km (between nodes 10 and 12) and 759 km (between nodes 1 and 11) and the link propagation delay is determined by the speed of light over a optical fibre (approximately 2 x 10^8 meters/s).



Example – traffic flow matrix

Consider the following flow matrix (values in Mbps) where the packets of all flows are exponentially distributed with an average packet size B = 1000 Bytes:

0.0	47.7	64.4	13.6	10.6	45.5	10.6	12.9	9.8	9.8	13.6	11.4	11.4	41.7	12.9	10.6	9.1	12.9	11.4	22.0
47.0	0.0	68.2	32.6	33.3	189.4	59.8	47.0	49.2	38.6	59.1	53.0	38.6	212.1	31.8	48.5	40.9	43.9	54.5	74.2
65.9	69.7	0.0	8.3	13.6	53.0	13.6	14.4	18.9	14.4	11.4	36.4	12.9	63.6	13.6	14.4	11.4	13.6	15.9	22.7
13.6	46.2	13.6	0.0	12.9	31.1	14.4	12.1	11.4	12.9	31.1	12.1	9.1	31.8	11.4	10.6	14.4	12.9	17.4	24.2
7.6	31.8	10.6	14.4	0.0	55.3	11.4	9.1	9.8	12.9	36.4	11.4	12.9	46.2	12.9	7.6	12.1	15.9	16.7	28.0
52.3	174.2	53.8	55.3	38.6	0.0	39.4	47.0	41.7	40.9	53.8	44.7	42.4	212.1	40.9	71.2	62.9	46.2	56.8	72.0
13.6	32.6	11.4	11.4	12.9	54.5	0.0	7.6	12.1	12.9	12.1	14.4	56.8	55.3	9.8	9.1	13.6	15.9	9.8	21.2
13.6	47.0	14.4	11.4	9.8	37.9	10.6	0.0	12.1	9.1	11.4	13.6	12.1	57.6	9.8	10.6	9.8	17.4	12.9	34.1
9.8	33.3	16.7	12.1	14.4	38.6	12.9	9.8	0.0	11.4	13.6	9.1	13.6	56.1	11.4	13.6	12.1	15.2	18.9	18.9
12.9	53.0	10.6	9.8	11.4	46.2	13.6	10.6	10.6	0.0	9.1	9.8	11.4	42.4	10.6	11.4	10.6	15.9	10.6	25.8
9.1	36.4	9.8	31.1	33.3	40.9	9.8	10.6	12.1	14.4	0.0	9.8	10.6	47.7	9.1	11.4	8.3	9.8	12.9	32.6
10.6	61.4	35.6	9.8	9.8	59.8	14.4	8.3	8.3	10.6	12.1	0.0	9.8	33.3	9.8	28.0	9.8	9.1	14.4	25.0
10.6	32.6	9.1	12.1	9.1	35.6	59.8	10.6	9.8	14.4	9.8	13.6	0.0	41.7	11.4	12.9	13.6	15.9	16.7	40.9
40.9	181.8	49.2	56.1	42.4	189.4	55.3	64.4	57.6	31.8	31.8	33.3	46.2	0.0	40.9	57.6	40.2	48.5	51.5	69.7
12.1	37.1	10.6	9.8	10.6	37.1	8.3	14.4	8.3	10.6	9.8	12.1	11.4	47.7	0.0	11.4	10.6	10.6	9.1	28.8
10.6	47.7	9.8	11.4	11.4	44.7	9.8	11.4	10.6	9.1	9.1	12.9	9.1	56.8	14.4	0.0	11.4	9.1	12.9	30.3
13.6	34.1	10.6	10.6	13.6	55.3	12.1	12.9	9.8	11.4	10.6	12.9	9.1	44.7	10.6	9.1	0.0	10.6	9.8	20.5
13.6	40.9	11.4	9.8	18.9	40.9	11.4	18.2	13.6	18.9	12.9	10.6	17.4	40.9	11.4	12.9	9.8	0.0	34.1	24.2
7.6	49.2	18.9	15.9	12.9	53.0	12.1	9.8	15.9	13.6	15.2	10.6	18.9	47.0	12.9	9.8	9.8	30.3	0.0	18.9
23.5	68.2	26.5	28.8	34.1	65.9	25.8	30.3	15.9	22.7	30.3	28.0	34.1	65.2	34.1	25.8	24.2	21.2	15.9	0.0

Example – one possible solution

One possible solution is to route each traffic flow $t \in T$ by the routing path with the shortest length (minimizing, in this way, the propagation delay of each flow).

Using the Kleinrock approximation, we obtain the following performance parameters:

Worst average packet delay = 6.06 ms

Worst link load = 99.3%

Number of active links = 33 out of 33

However, it is possible to obtain better traffic engineering solutions through appropriate optimization algorithms.

Example – optimal solutions

Minimization of the worst average packet delay:

Worst average packet delay = 5.21 ms

Worst link load = 93.6%

Number of active links = 33 out of 33

Minimization of the worst link load:

Worst average packet delay = 8.63 ms

Worst link load = 69.9%

Number of active links = 33 out of 33

Minimization of the number of active links:

Worst average packet delay = 10.54 ms

Worst link load = 82.4%

Number of active links = 26 out of 33

<u>Conclusion</u>:

- Each traffic engineering solution is a different trade-off between the different optimization objectives.
- It is up to the operator to select the best routing solution.

Optimization methods

Exact methods

- Based on mathematical models (for example, Integer Linear Programming)
- In the general case, computationally hard
- Theoretically, they are able to compute the optimal solutions
- Inefficient for large problem instances (they either take too long to even compute feasible solutions or finish due to out-of-memory)

Heuristic methods

- Based on simple programming algorithms
- Easy to implement and quick to find solutions
- Do not guarantee optimality
- For larger runtimes, they find better solutions (than exact methods)
- Efficient for large problem instances

Heuristic method versus heuristic algorithm

Heuristic method: a generic approach to search for good solutions that can be applied to any optimization problem.

Heuristic algorithm: an optimization algorithm that has resulted from applying an heuristic method to a particular optimization problem.

Many heuristic methods (usually, also the simplest ones) are based on two algorithmic strategies:

1. To build a solution starting from the scratch.
   – Examples: *random*, *greedy, greedy randomized, etc…*

2. To get a better solution from a known solution.
   – Examples: *hill climbing, tabu search, simulated annealing, etc...* (we will address only the hill climbing strategy).

Building a solution from the scratch

Building one solution from the scratch (I)

1. Random strategy:

- The solution is built by assigning a random routing path $p \in P_t$ for each flow $t \in T$
- We might obtain better solutions if we consider higher probabilities to routing paths $p \in P_t$ with “better characteristics”
    - For example, paths with a smaller number of links, paths containing links of larger capacity, etc…

2. Greedy strategy:

- Start by considering the network without any routing path
- Then, for each flow $t \in T$:
    - assign the first routing path $p \in P_t$ that, together with the previous assigned routing paths, gives the best objective function value

Building one solution from the scratch (II)

3. Greedy randomized strategy:

The aim is to obtain a different solution on different runs.

First alternative:

- First, choose a random order of the flows $t \in T$
- Then, apply the greedy strategy (previous slide) by the chosen order

Second alternative:

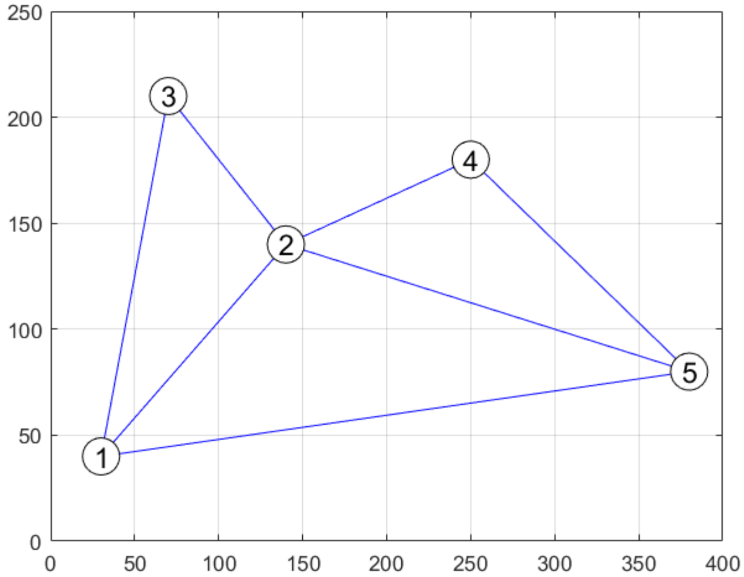
- Start by considering the network without any routing path
- Then, for each flow $t \in T$:
  - compute the α routing paths of P_t that, each one together with the previous assigned routing paths, give the best objective values
    - α is an integer parameter of the algorithm (with $\alpha \geq 2$)
  - assign randomly one of the previous α routing paths to flow $t \in T$

Third alternative:

- To combine the 2 previous alternatives

Minimizing the worst link load: <u>greedy strategy</u>

Example network:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 1:

Path 1 = 1 2 4
Path 2 = 1 3 2 4
Path 3 = 1 5 4
Path 4 = 1 2 5 4
Path 5 = 1 3 2 5 4
Path 6 = 1 5 2 4

Flow 2:

Path 1 = 1 5
Path 2 = 1 2 5
Path 3 = 1 2 4 5
Path 4 = 1 3 2 5
Path 5 = 1 3 2 4 5

Flow 3:

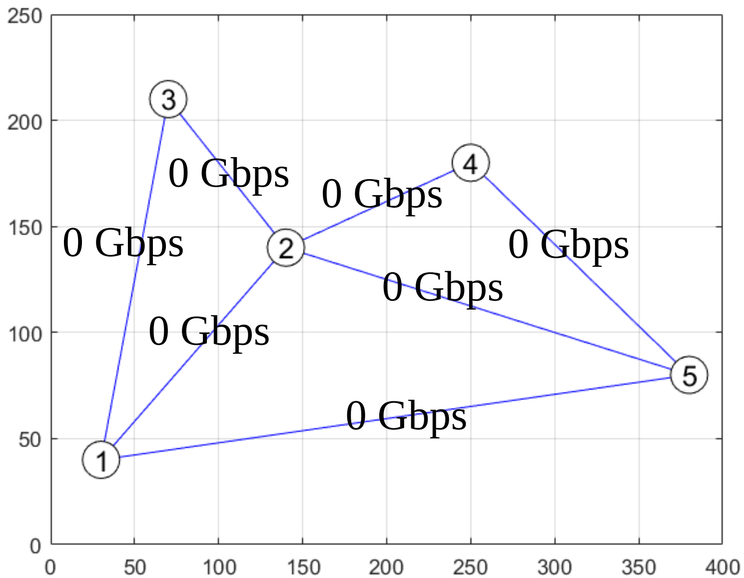
Path 1 = 2 4
Path 2 = 2 5 4
Path 3 = 2 1 5 4
Path 4 = 2 3 1 5 4

Flow 4:

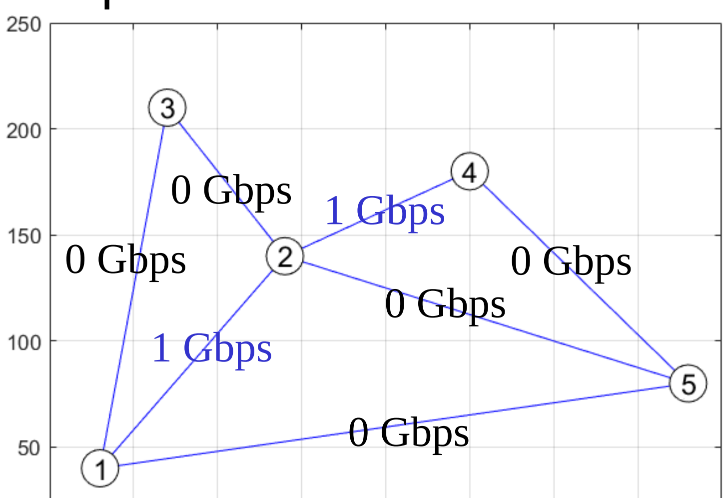
Path 1 = 3 2 5
Path 2 = 3 2 4 5
Path 3 = 3 1 5
Path 4 = 3 1 2 5
Path 5 = 3 2 1 5
Path 6 = 3 1 2 4 5

Greedy strategy: step 1

Current solution:



Updated solution:



Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 1:

Path 1 = 1 2 4
Path 2 = 1 3 2 4
Path 3 = 1 5 4
Path 4 = 1 2 5 4
Path 5 = 1 3 2 5 4
Path 6 = 1 5 2 4

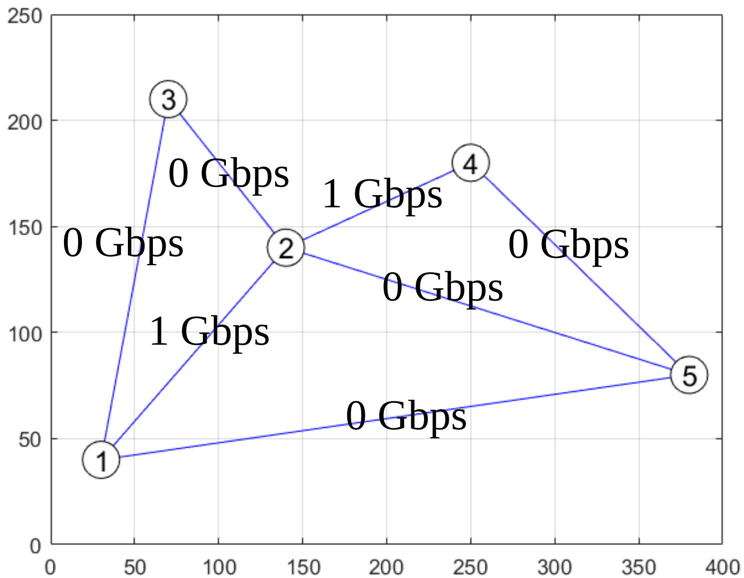
Selected path:

Path 1 = 1 2 4

Path minimizing the worst link load in the updated solution

Greedy strategy: step 2

Current solution:



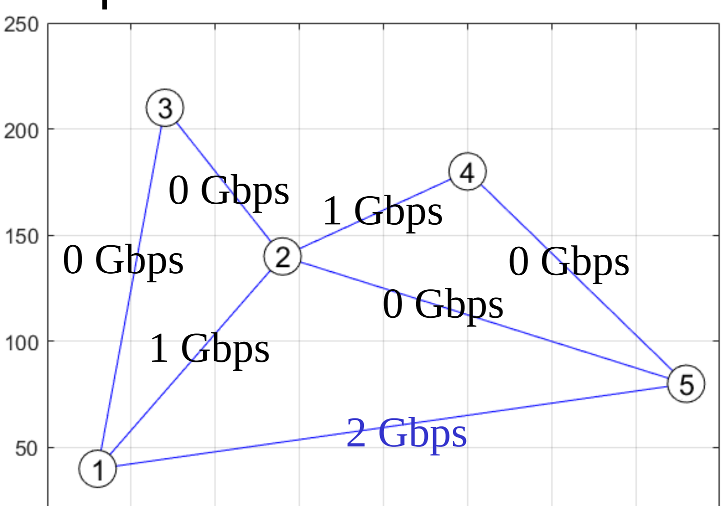
Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 2:

Path 1 = 1 5
Path 2 = 1 2 5
Path 3 = 1 2 4 5
Path 4 = 1 3 2 5
Path 5 = 1 3 2 4 5

Updated solution:

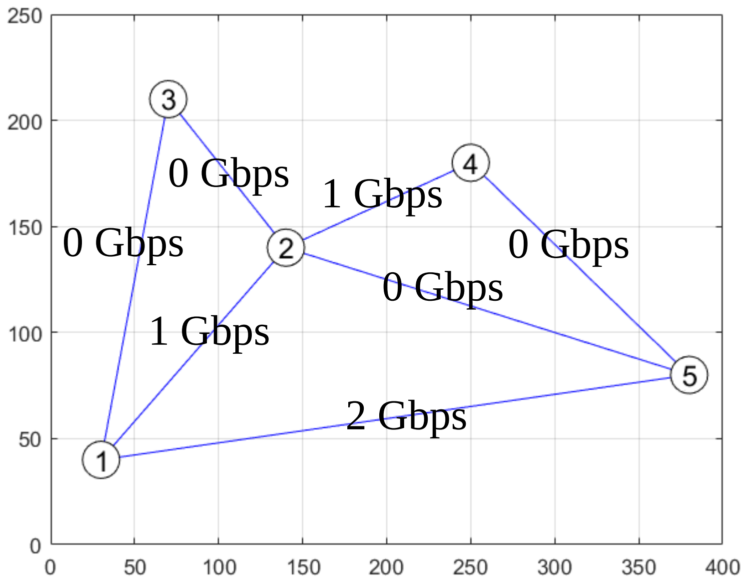


Selected path:
Path 1 = 1 5

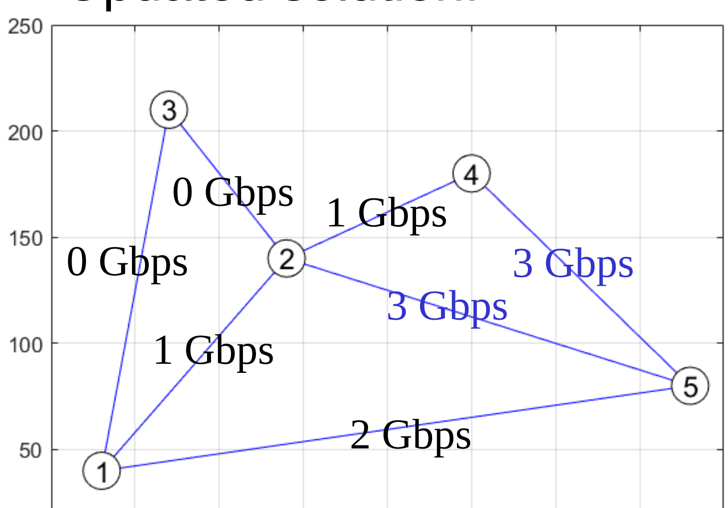
Path minimizing the worst link load in the updated solution

Greedy strategy: step 3

Current solution:



Updated solution:



Traffic flows:

t	o_t	d_t	$\overline{b}_t$ (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 3:

Path 1 = 2 4

Path 2 = 2 5 4

Path 3 = 2 1 5 4

Path 4 = 2 3 1 5 4

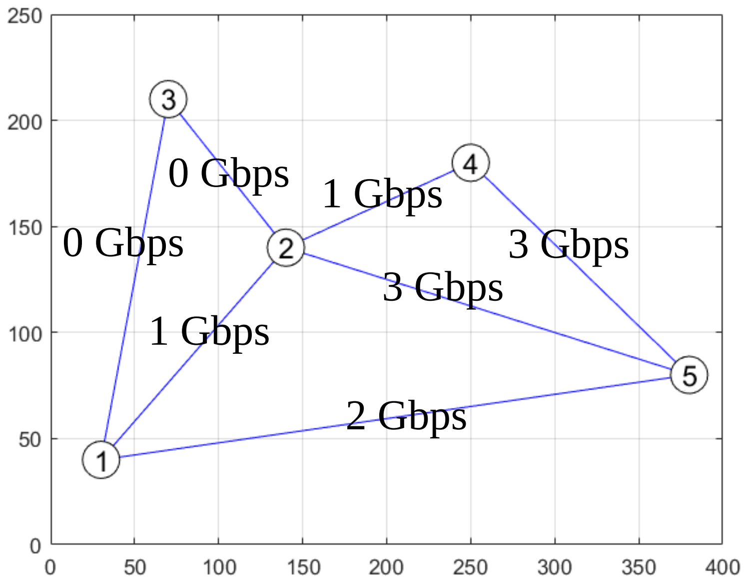
Selected path:

Path 2 = 2 5 4

Path minimizing the worst link load in the updated solution

Greedy strategy: step 4

Current solution:



Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 4:

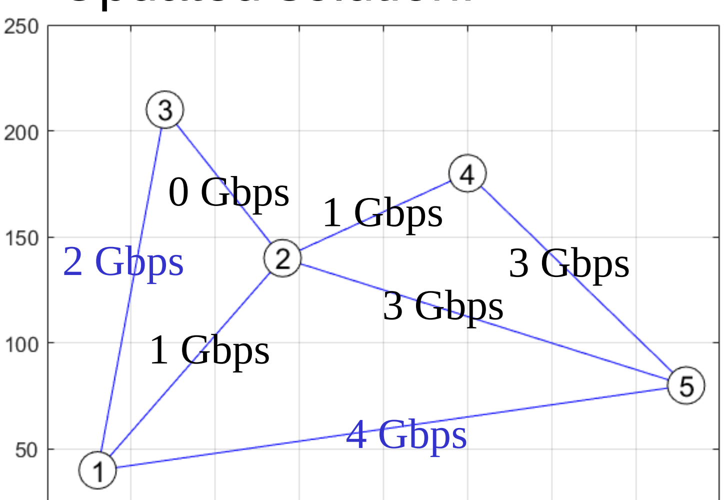
Path 1 = 3 2 5
Path 2 = 3 2 4 5
Path 3 = 3 1 5
Path 4 = 3 1 2 5
Path 5 = 3 2 1 5
Path 6 = 3 1 2 4 5

Selected path:

Path 3 = 3 1 5

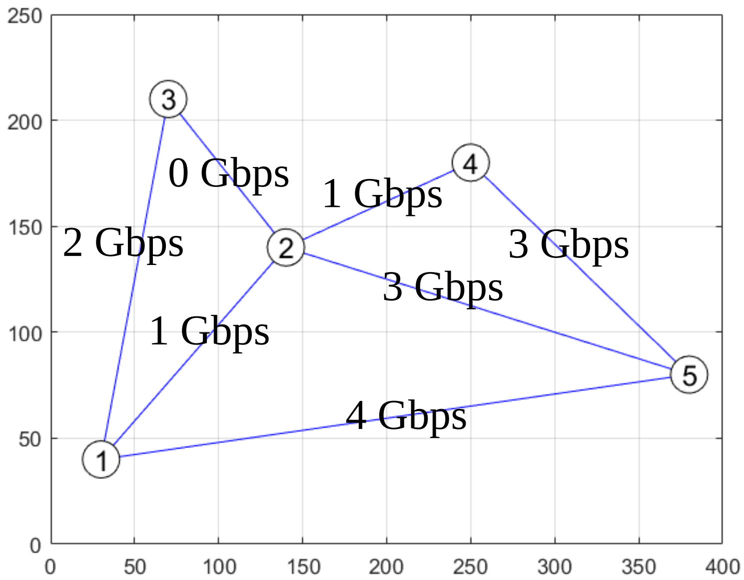
Path minimizing the worst link load in the updated solution

Updated solution:



Greedy strategy: FINAL SOLUTION

Current solution:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	$\overline{b}_t$ (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

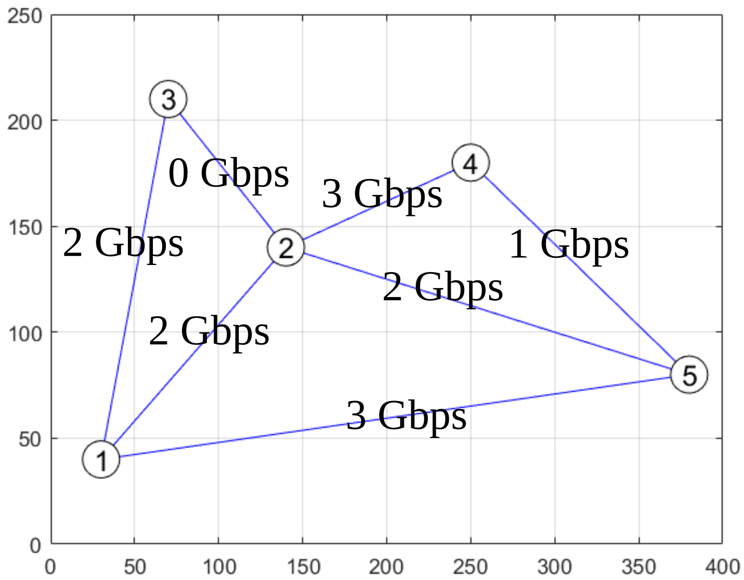
FINAL SOLUTION :

Flow 1:
Path 1 = 1 2 4
Flow 2:
Path 1 = 1 5
Flow 3:
Path 2 = 2 5 4
Flow 4:
Path 3 = 3 1 5

Worst link load → 4 Gbps (= 40%)
Link(s) with the worst link load → (1,5) and (5,1)
Unused links → {2,3}

Greedy randomized strategy

Current solution:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Taking the random order 3 → 1 → 2 → 4 :

Flow 3:
Path 1 = 2 4

Flow 1:
Path 3 = 1 5 4

Flow 2:
Path 3 = 1 2 5

Flow 4:
Path 3 = 3 1 5

Worst link load → 3 Gbps (= 30%)

Link(s) with the worst link load → (1,5), (5,1), (2,4) and (4,2)

Unused links → {2,3}

Optimization algorithm

- In a problem aiming to minimize function $F(x)$, it works as follows:

$f_{best} = +\infty$

repeat

$x = BuildSolution\ ()$

$f = F(x)$

if $f < f_{best}$ **then**

$x_{best} = x$

$f_{best} = f$

endif

until Stopping Criteria is met

Random strategy or
Greedy Randomized strategy

If the aim is to maximize $F(x)$
$f_{best} = -\infty$
$f > f_{best}$

- Examples of Stopping Criteria:
  - Run a predefined time duration
  - Run a predefined number of iterations
  - Run until f_{best} not improving a predefined number of iterations

Getting a better solution from a known solution

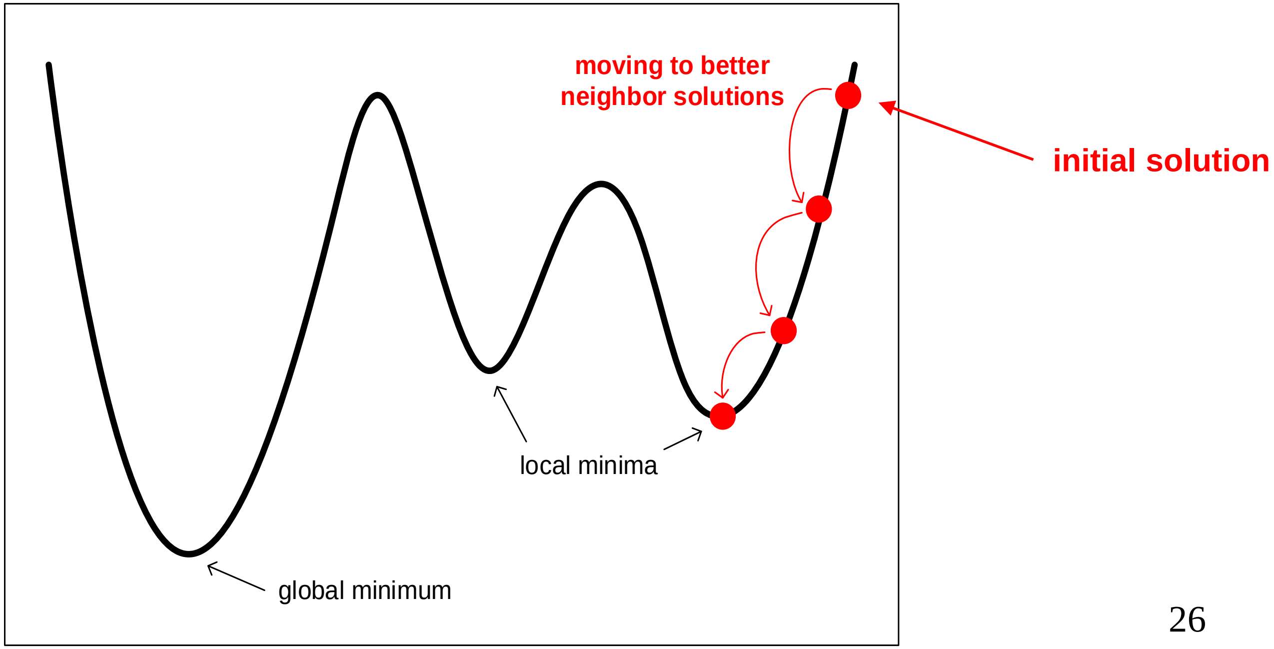
Getting a better solution from a known solution (I)

Hill climbing strategy

Start by an initial solution and try to move to a better solution by making one local change.

- The possible local changes are defined by the neighbour set.
- The moves are repeated until no possible local change produces a better solution (we obtain a local optimum solution).

Minimization problem



Getting a better solution from a known solution (II)

Hill climbing strategy – best neighbour move variant

In this variant, all neighbour solutions are evaluated at each iteration. It provides the best possible improvement on each move.

This strategy is defined as an iterative procedure with the following steps:

1. Set the current solution with the initial solution.
2. For the current solution, evaluate all neighbour solutions and select the best one.
3. If the best neighbour solution is better than the current solution, move to this solution (*i.e.*, set the current solution with the best neighbour solution) and go to step 2 (*i.e.*, a new iteration starts).
4. If not, stop and the current solution is the final solution (we say it is a local optimum solution).

NOTE: If the initial solution is a local optimum solution, the algorithm stops in the first iteration. In this case, the final solution is the initial solution.

Getting a better solution from a known solution (III)

Hill climbing strategy – first best neighbour move variant

If the evaluation of all neighbour solutions is computationally hard (either because each neighbour solution is hard to evaluate or because the neighbour set has too many solutions), the previous variant might be non efficient.

This strategy variant is defined by the following steps:

1. Set the current solution with the initial solution.
2. For the current solution, evaluate the neighbour solutions until a solution better than the current one is found or until all neighbour solutions are evaluated.
3. If the found neighbour solution is better than the current one, set the current solution with the neighbour solution and go to step 2.
4. If not, stop and the current solution is the final solution.

NOTE: Usually, it is more efficient to use the best neighbour move variant, although in some problems it requires a careful definition of the neighbour set.

Hill climbing algorithm

Consider an optimization problem with a solution set S, an optimization function $F(x)$ and a set $V(x)$ of neighbours of each solution $x \in S$

$x' \leftarrow \text{Initial}(x \in S)$

$f' \leftarrow F(x')$

$improved \leftarrow \text{TRUE}$

While *improved* **do**

    $x \leftarrow \text{Best}(x \in V(x'))$

    $f \leftarrow F(x)$

    If f is better than f' **do**

        $x' \leftarrow x$

        $f' \leftarrow f$

    Else do

        $improved \leftarrow \text{FALSE}$

    EndIf

EndWhile

x' is set with an initial known solution and f' is its objective value

the algorithm stops when no improvement can be obtained by a neighbour solution

the <u>best</u> (or <u>first best</u>) neighbour x of solution x' is selected

if $F(x)$ is better than $F(x')$, x becomes the current solution x' and f' its objective value

otherwise, no improvement can be obtained by a neighbor solution

The final result is solution x' whose function value is $f' = F(x')$

Hill climbing strategy: defining the set of neighbour solutions

- The set of neighbour solutions (of a given solution) depends on the addressed optimization problem.
- The neighbour set must be carefully defined in order to allow the algorithm to compute all neighbour solutions in reasonable running time.

Recall that in traffic engineering of telecommunication networks:

- P_t is the set of candidate routing paths of flow $t \in T$
- A solution defines the routing path $p \in P_t$ assigned to traffic flow $t \in T$

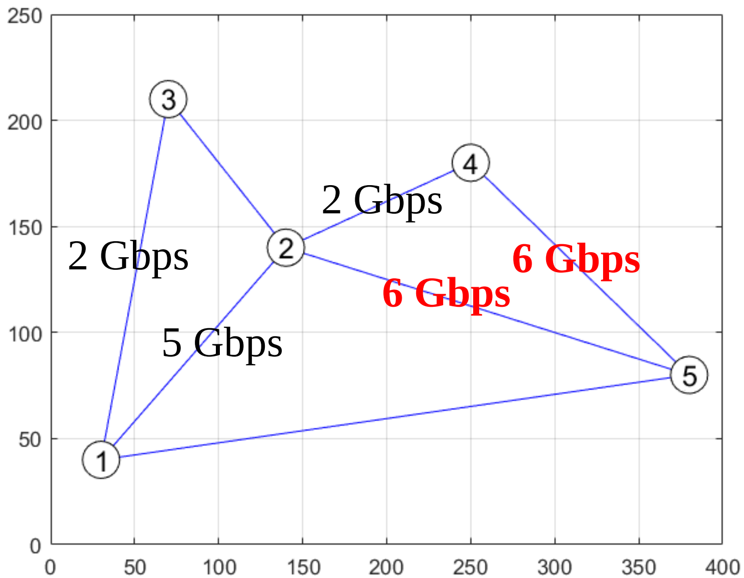
The neighbour set of a current solution is usually defined as:

- all solutions that differ from the given solution in the routing path of a single flow.
- so, the total number of neighbour solutions is: $\sum_{t \in T} (|P_t| - 1)$

  where $|P_t|$ is the number of routing paths of set P_t.

Minimizing the worst link load
Hill climbing: beginning

Example network:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 1:
- Path 1 = 1 2 4
- Path 2 = 1 3 2 4
- Path 3 = 1 5 4
- **Path 4 = 1 2 5 4**
- Path 5 = 1 3 2 5 4
- Path 6 = 1 5 2 4

Flow 2:
- Path 1 = 1 5
- Path 2 = 1 2 5
- **Path 3 = 1 2 4 5**
- Path 4 = 1 3 2 5
- Path 5 = 1 3 2 4 5

Flow 3:
- Path 1 = 2 4
- **Path 2 = 2 5 4**
- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Flow 4:
- Path 1 = 3 2 5
- Path 2 = 3 2 4 5
- Path 3 = 3 1 5
- **Path 4 = 3 1 2 5**
- Path 5 = 3 2 1 5
- Path 6 = 3 1 2 4 5

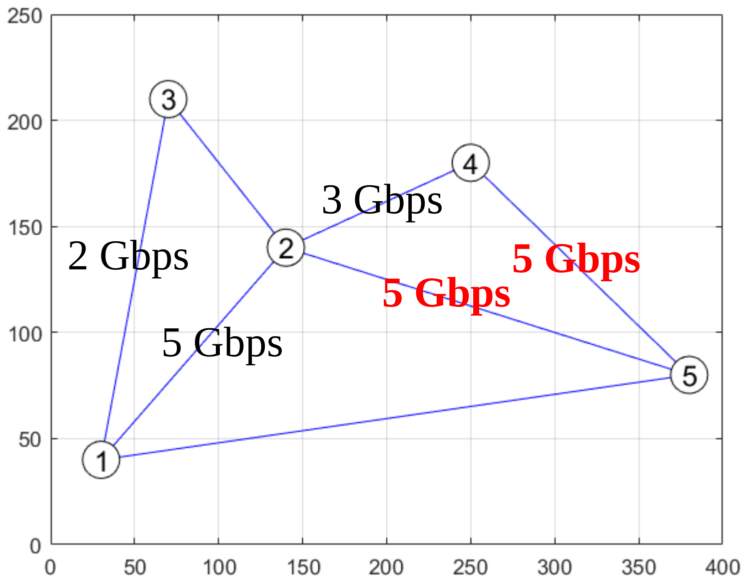
Worst link load: 6 Gbps

Current solution:
Flow 1 – Path 4
Flow 2 – Path 3
Flow 3 – Path 2
Flow 4 – Path 4

Current solution is the initial known solution

Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 3:
- Path 1 = 2 4
- **Path 2 = 2 5 4**
- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

Flow 1:
- **Path 1 = 1 2 4**
- Path 2 = 1 3 2 4
- Path 3 = 1 5 4
- **Path 4 = 1 2 5 4**
- Path 5 = 1 3 2 5 4
- Path 6 = 1 5 2 4

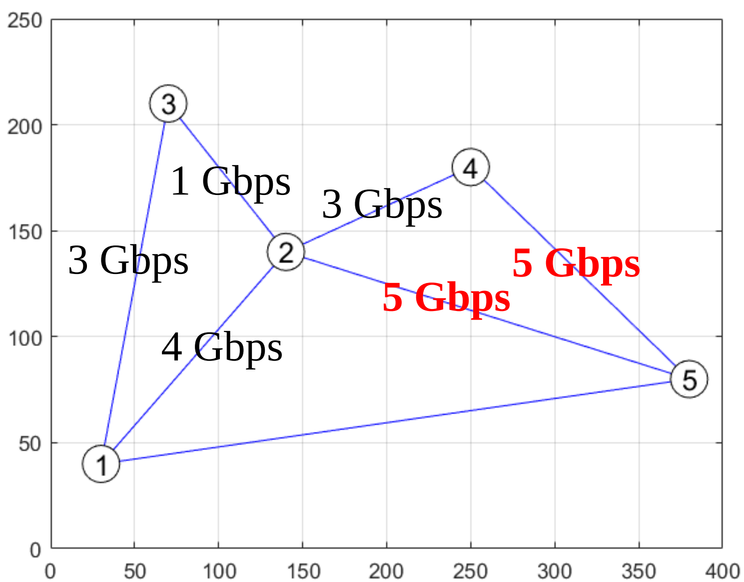
Flow 2:
- Path 1 = 1 5
- Path 2 = 1 2 5
- **Path 3 = 1 2 4 5**
- Path 4 = 1 3 2 5
- Path 5 = 1 3 2 4 5

Flow 4:
- Path 1 = 3 2 5
- Path 2 = 3 2 4 5
- Path 3 = 3 1 5
- **Path 4 = 3 1 2 5**
- Path 5 = 3 2 1 5
- Path 6 = 3 1 2 4 5

Better neighbour solution

Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
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4	3	5	2.0	2.0

Flow 3:
- Path 1 = 2 4
- **Path 2 = 2 5 4**
- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

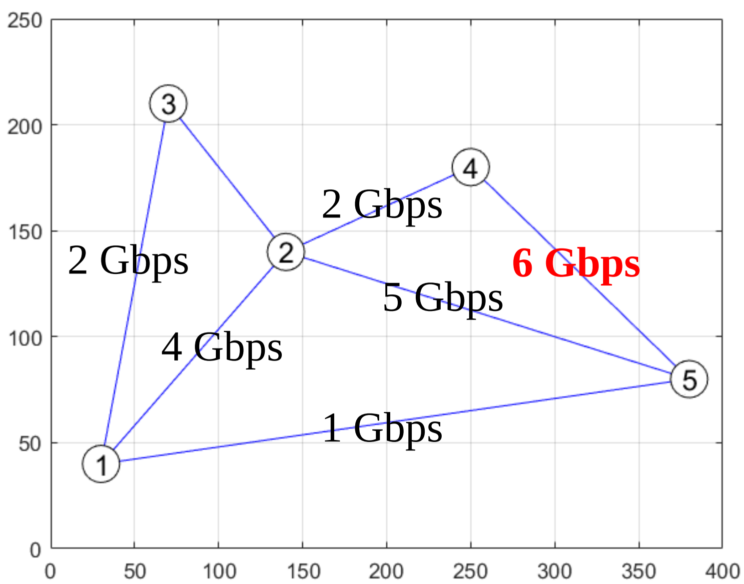
Flow 1:
- Path 1 = 1 2 4
- **Path 2 = 1 3 2 4**
- Path 3 = 1 5 4
- **Path 4 = 1 2 5 4**
- Path 5 = 1 3 2 5 4
- Path 6 = 1 5 2 4

Flow 2:
- Path 1 = 1 5
- Path 2 = 1 2 5
- **Path 3 = 1 2 4 5**
- Path 4 = 1 3 2 5
- Path 5 = 1 3 2 4 5

Flow 4:
- Path 1 = 3 2 5
- Path 2 = 3 2 4 5
- Path 3 = 3 1 5
- **Path 4 = 3 1 2 5**
- Path 5 = 3 2 1 5
- Path 6 = 3 1 2 4 5

Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

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t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
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Flow 3:
- Path 1 = 2 4
- **Path 2 = 2 5 4**
- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

Flow 1:
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- Path 2 = 1 3 2 4
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- Path 5 = 1 3 2 5 4
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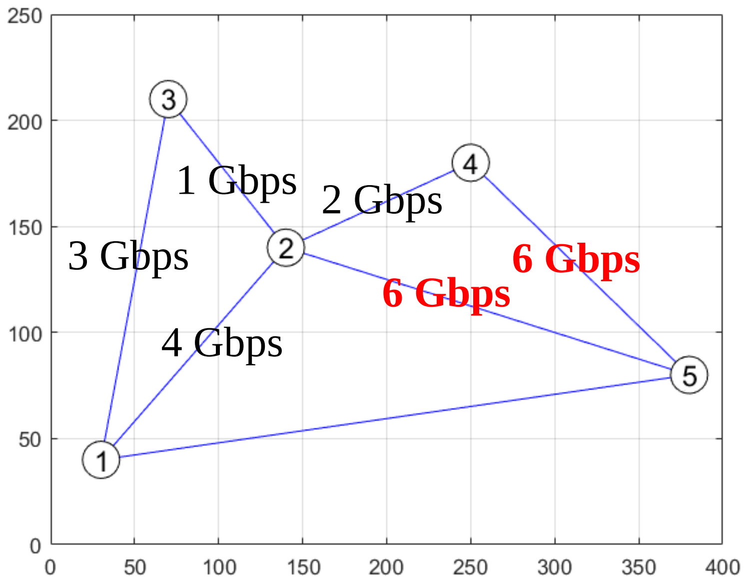
- Path 6 = 3 1 2 4 5

Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

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Flow 3:
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- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

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- **Path 4 = 1 2 5 4**
- **Path 5 = 1 3 2 5 4**
- Path 6 = 1 5 2 4

Flow 2:
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- Path 5 = 1 3 2 4 5

Flow 4:
- Path 1 = 3 2 5
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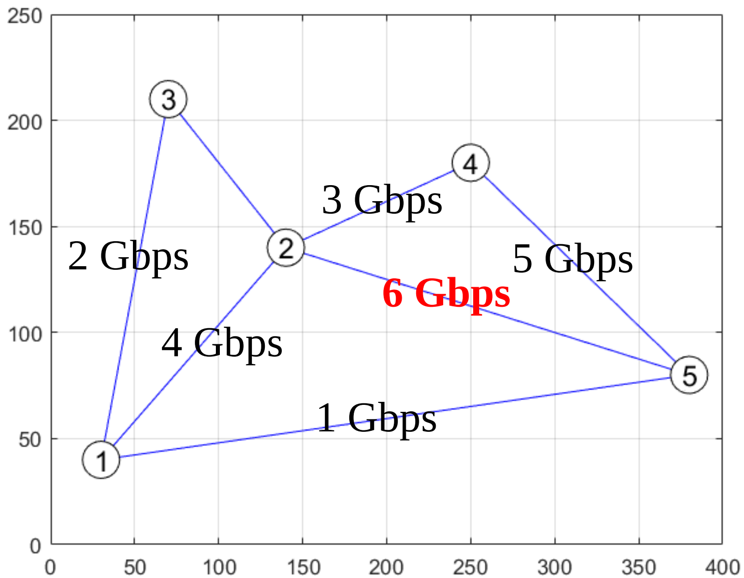
- Path 3 = 3 1 5
- **Path 4 = 3 1 2 5**
- Path 5 = 3 2 1 5
- Path 6 = 3 1 2 4 5

Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

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- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

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- Path 1 = 3 2 5
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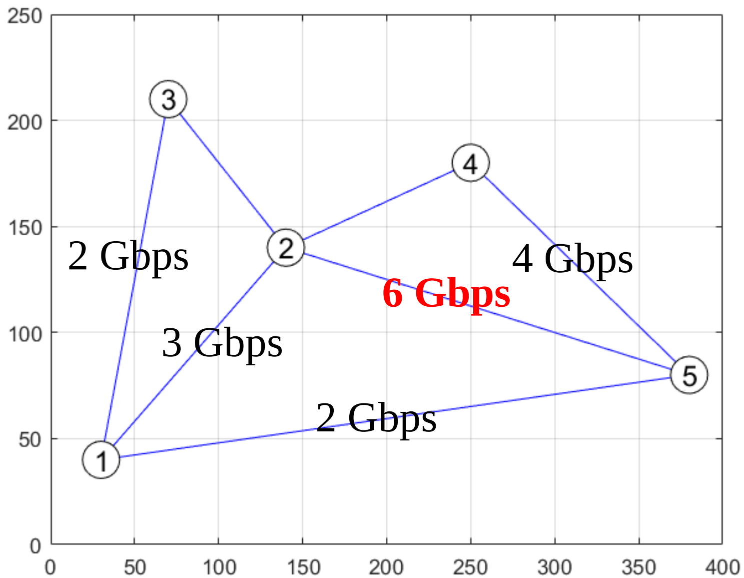
- Path 6 = 3 1 2 4 5

Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

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Flow 3:
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- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

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- Path 5 = 1 3 2 4 5

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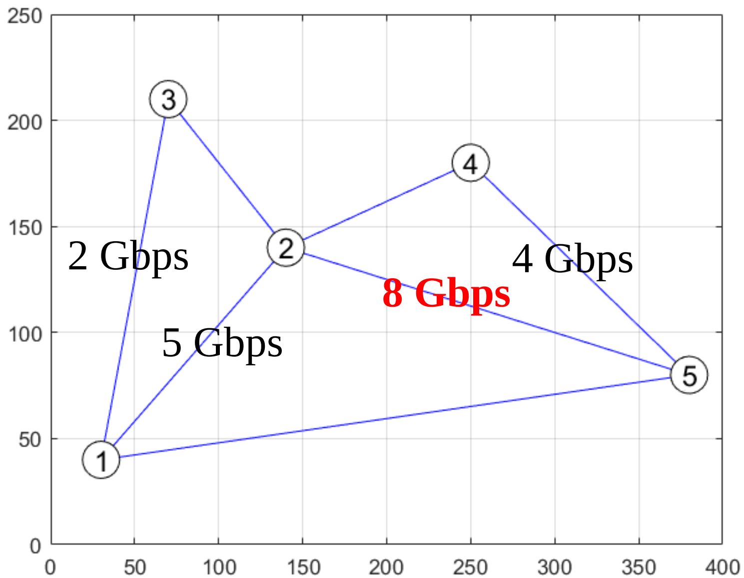
Minimizing the worst link load
Hill climbing: step 1

Example network:



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Worst link load: **5 Gbps**

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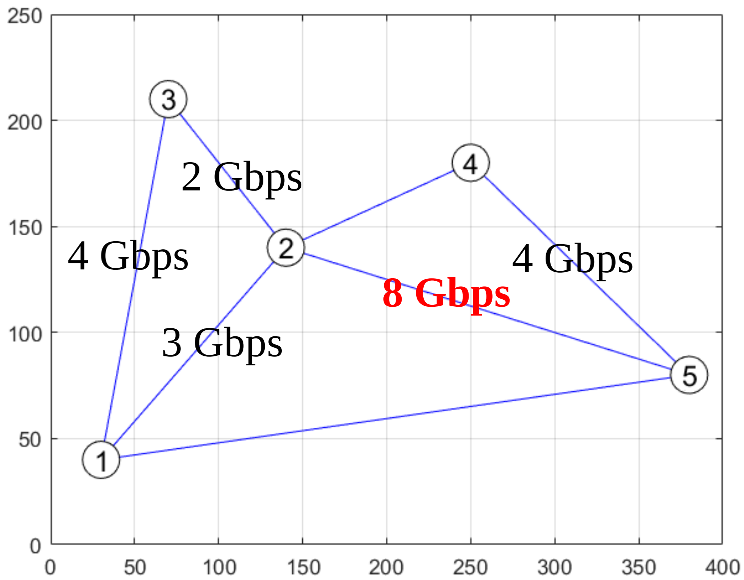
Flow 1: Path 4 →Path1

Minimizing the worst link load
Hill climbing: step 1

Example network:



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Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

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- Path 5 = 3 2 1 5
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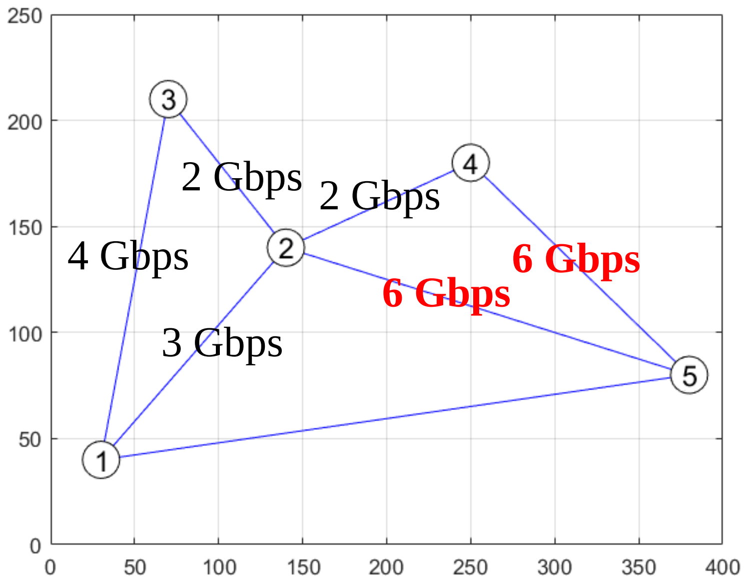
Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

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- Path 1 = 3 2 5
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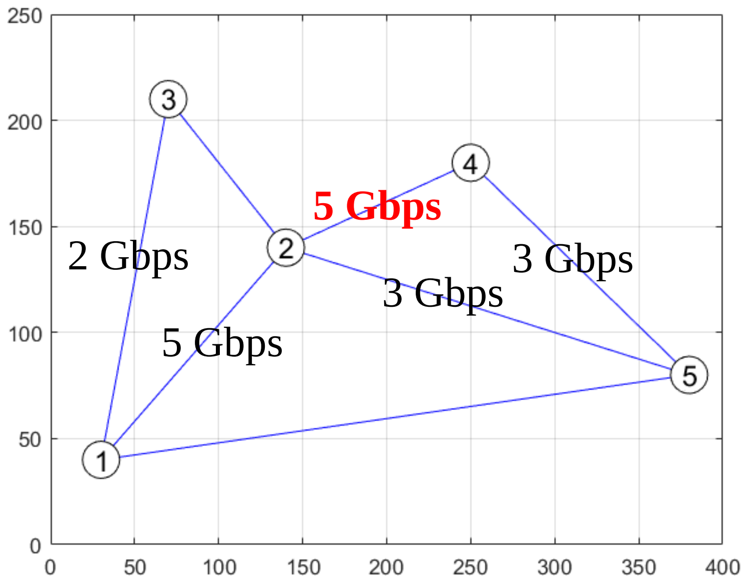
- **Path 4 = 3 1 2 5**
- Path 5 = 3 2 1 5
- Path 6 = 3 1 2 4 5

Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

Traffic flows:

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Flow 3:
- **Path 1 = 2 4**
- **Path 2 = 2 5 4**
- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

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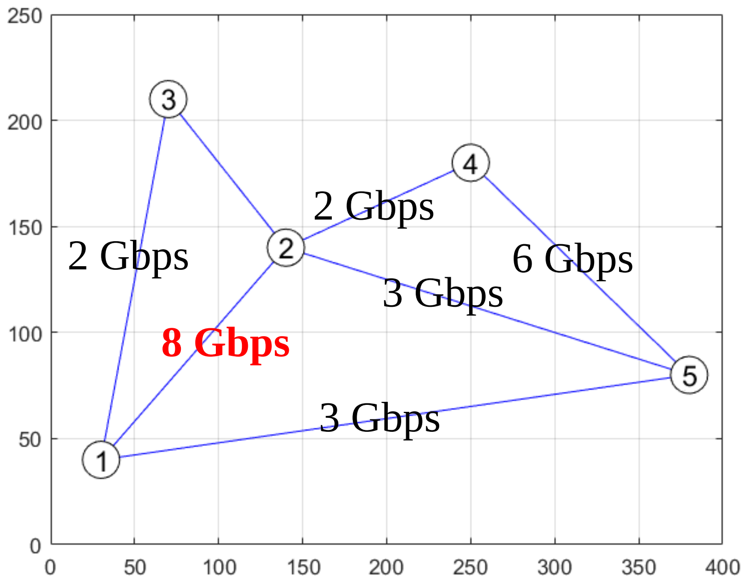
- Path 5 = 3 2 1 5
- Path 6 = 3 1 2 4 5

Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

Traffic flows:

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- Path 1 = 2 4
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Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

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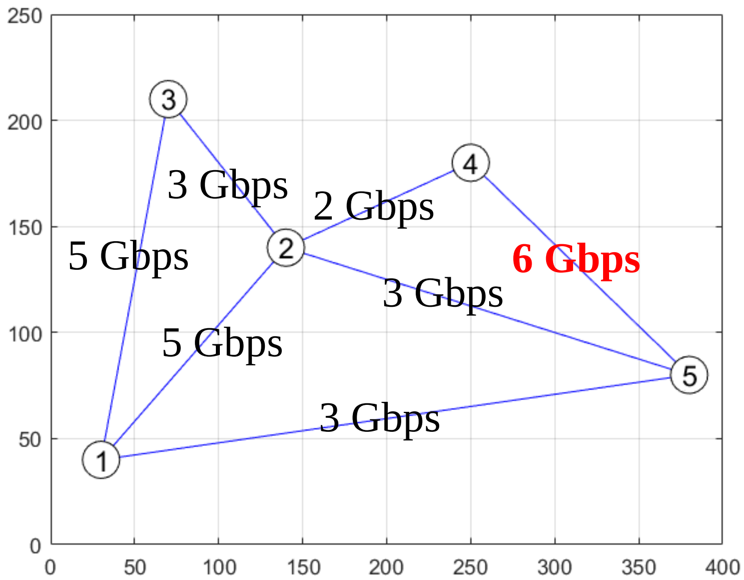
Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

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Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

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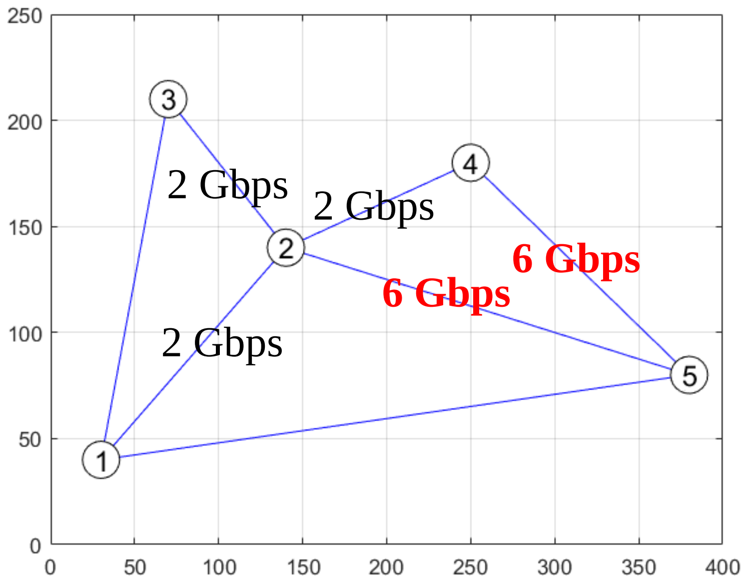
Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

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Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

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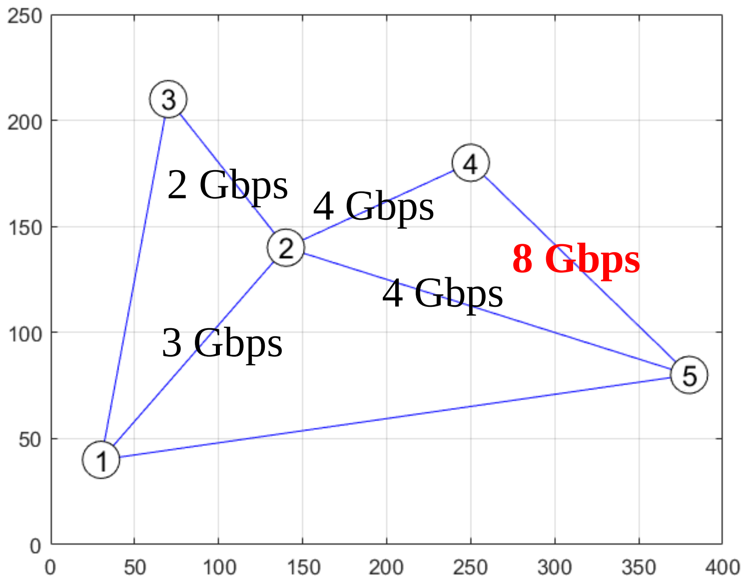
Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

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Worst link load: **5 Gbps**

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Flow 1: Path 4 →Path1

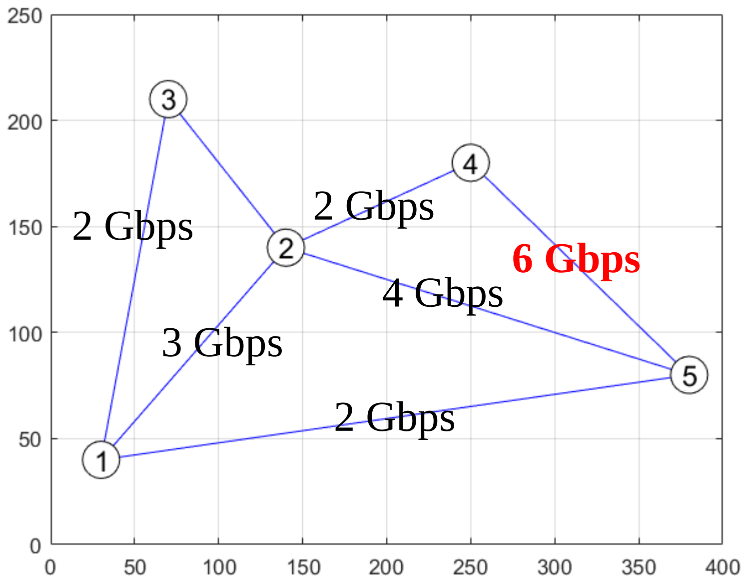
Flow 1:
- Path 1 = 1 2 4
- Path 2 = 1 3 2 4
- Path 3 = 1 5 4
- **Path 4 = 1 2 5 4**
- Path 5 = 1 3 2 5 4
- Path 6 = 1 5 2 4

Flow 2:
- Path 1 = 1 5
- Path 2 = 1 2 5
- **Path 3 = 1 2 4 5**
- Path 4 = 1 3 2 5
- Path 5 = 1 3 2 4 5

Flow 4:
- Path 1 = 3 2 5
- **Path 2 = 3 2 4 5**
- Path 3 = 3 1 5
- **Path 4 = 3 1 2 5**
- Path 5 = 3 2 1 5
- Path 6 = 3 1 2 4 5

Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 3:
- Path 1 = 2 4
- **Path 2 = 2 5 4**
- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

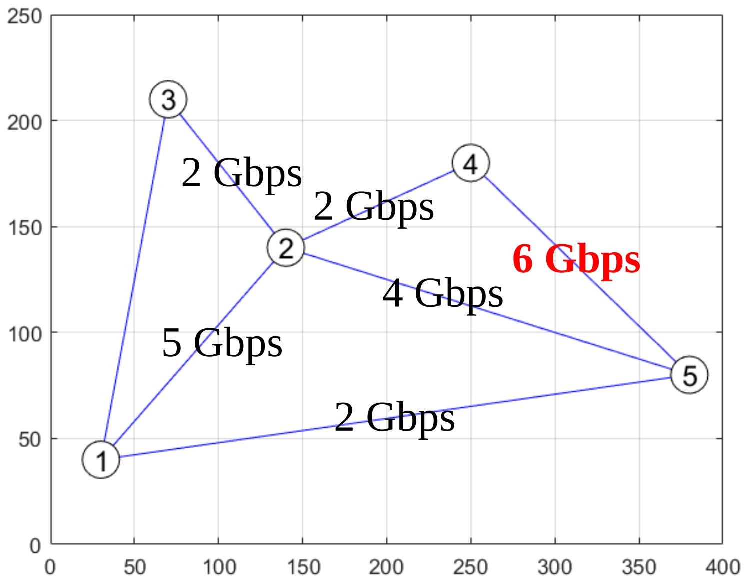
Flow 1:
- Path 1 = 1 2 4
- Path 2 = 1 3 2 4
- Path 3 = 1 5 4
- **Path 4 = 1 2 5 4**
- Path 5 = 1 3 2 5 4
- Path 6 = 1 5 2 4

Flow 2:
- Path 1 = 1 5
- Path 2 = 1 2 5
- **Path 3 = 1 2 4 5**
- Path 4 = 1 3 2 5
- Path 5 = 1 3 2 4 5

Flow 4:
- Path 1 = 3 2 5
- Path 2 = 3 2 4 5
- **Path 3 = 3 1 5**
- **Path 4 = 3 1 2 5**
- Path 5 = 3 2 1 5
- Path 6 = 3 1 2 4 5

Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 3:
- Path 1 = 2 4
- **Path 2 = 2 5 4**
- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Worst link load: **5 Gbps**

Best move so far:
Flow 1: Path 4 →Path1

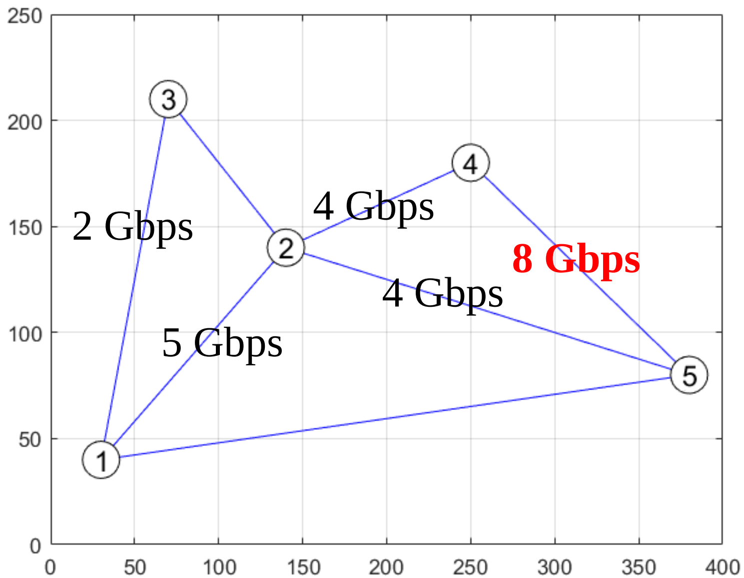
Flow 1:
- Path 1 = 1 2 4
- Path 2 = 1 3 2 4
- Path 3 = 1 5 4
- **Path 4 = 1 2 5 4**
- Path 5 = 1 3 2 5 4
- Path 6 = 1 5 2 4

Flow 2:
- Path 1 = 1 5
- Path 2 = 1 2 5
- **Path 3 = 1 2 4 5**
- Path 4 = 1 3 2 5
- Path 5 = 1 3 2 4 5

Flow 4:
- Path 1 = 3 2 5
- Path 2 = 3 2 4 5
- Path 3 = 3 1 5
- **Path 4 = 3 1 2 5**
- **Path 5 = 3 2 1 5**
- Path 6 = 3 1 2 4 5

Minimizing the worst link load
Hill climbing: step 1

Example network:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 3:
- Path 1 = 2 4
- **Path 2 = 2 5 4**
- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Worst link load: **5 Gbps**

Best move:
Flow 1: Path 4 →Path1

Flow 1:
- Path 1 = 1 2 4
- Path 2 = 1 3 2 4
- Path 3 = 1 5 4
- **Path 4 = 1 2 5 4**
- Path 5 = 1 3 2 5 4
- Path 6 = 1 5 2 4

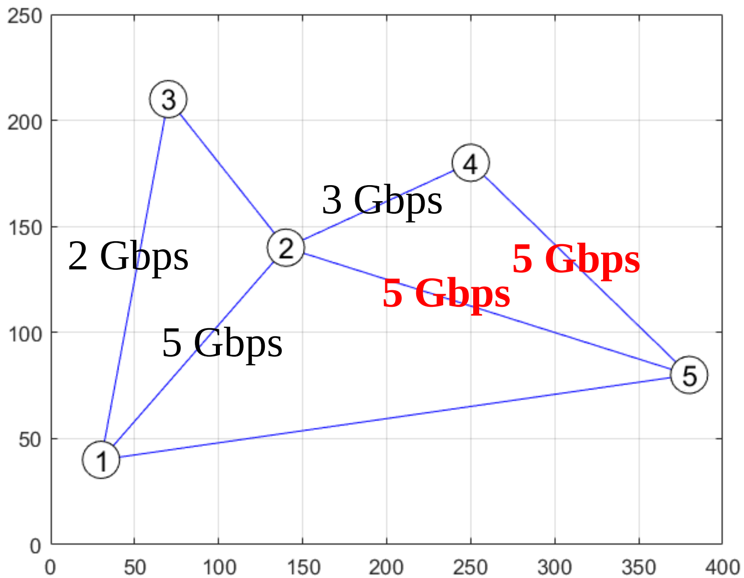
Flow 2:
- Path 1 = 1 5
- Path 2 = 1 2 5
- **Path 3 = 1 2 4 5**
- Path 4 = 1 3 2 5
- Path 5 = 1 3 2 4 5

Flow 4:
- Path 1 = 3 2 5
- Path 2 = 3 2 4 5
- Path 3 = 3 1 5
- **Path 4 = 3 1 2 5**
- Path 5 = 3 2 1 5
- **Path 6 = 3 1 2 4 5**

Last neighbour solution

Minimizing the worst link load
Hill climbing: end of step 1

Example network:



All links with 10 Gbps of capacity

Traffic flows:

t	o_t	d_t	b_t (Gbps)	$\underline{b}_t$ (Gbps)
1	1	4	1.0	1.0
2	1	5	2.0	2.0
3	2	4	3.0	3.0
4	3	5	2.0	2.0

Flow 1:
- **Path 1 = 1 2 4**
- Path 2 = 1 3 2 4
- Path 3 = 1 5 4
- Path 4 = 1 2 5 4
- Path 5 = 1 3 2 5 4
- Path 6 = 1 5 2 4

Flow 2:
- Path 1 = 1 5
- Path 2 = 1 2 5
- **Path 3 = 1 2 4 5**
- Path 4 = 1 3 2 5
- Path 5 = 1 3 2 4 5

Flow 3:
- Path 1 = 2 4
- **Path 2 = 2 5 4**
- Path 3 = 2 1 5 4
- Path 4 = 2 3 1 5 4

Flow 4:
- Path 1 = 3 2 5
- Path 2 = 3 2 4 5
- Path 3 = 3 1 5
- **Path 4 = 3 1 2 5**
- Path 5 = 3 2 1 5
- Path 6 = 3 1 2 4 5

Worst link load: 5 Gbps

Current solution:
Flow 1 – Path 1
Flow 2 – Path 3
Flow 3 – Path 2
Flow 4 – Path 4

Current solution is updated changing Flow 1 from Path 4 to Path 1

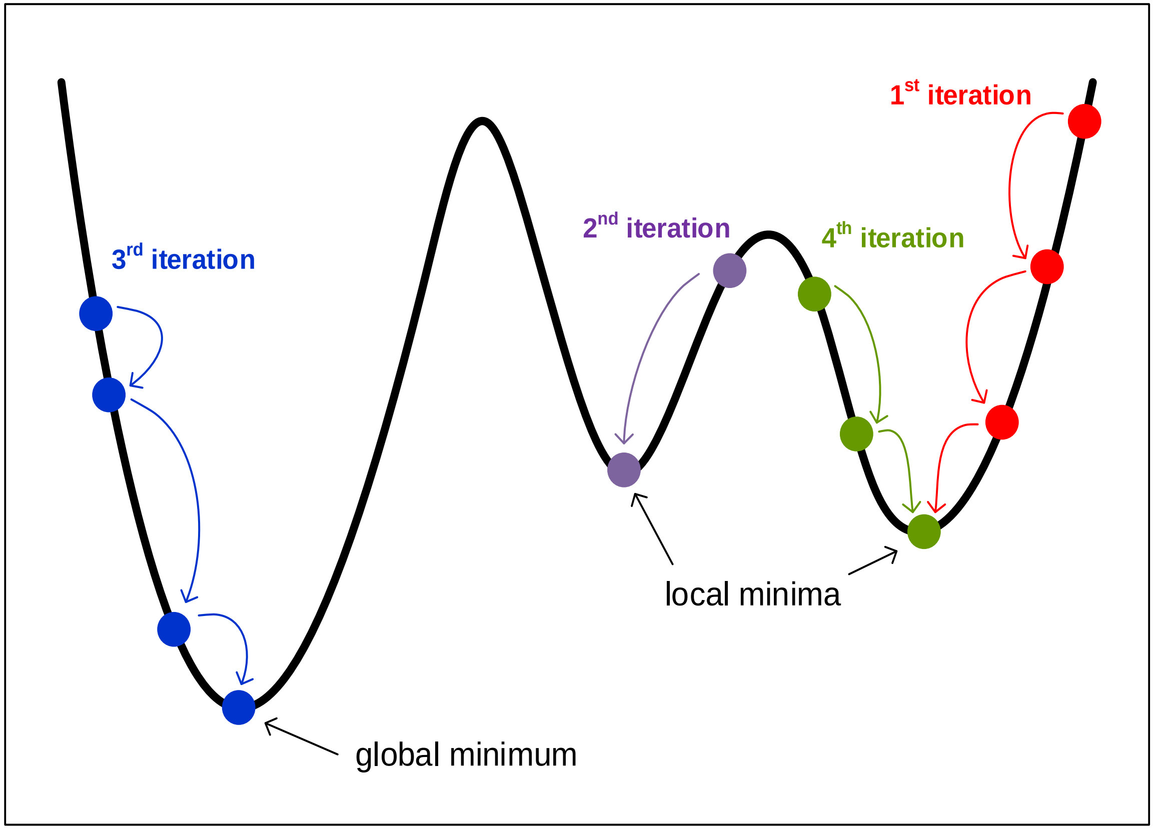
Building a solution from the scratch

+

Getting a better solution from a known solution

Multi Start Hill Climbing Heuristic (I)

- This heuristic combines the two algorithmic strategies:
  1. to build a solution from the scratch
  2. to get a better solution from a known solution



Multi Start Hill Climbing Heuristic (II)

- This heuristic combines the two algorithmic strategies:
  1. to build a solution from the scratch
  2. to get a better solution from a known solution
- In a problem aiming to minimize function $F(x)$, it works as follows:

$f_{best} = +\infty$
repeat
$z = BuildSolution\ ()$
$x = HillClimbing\ (z)$
$f = F(x)$
if $f < f_{best}$ **then**
$x_{best} = x$
$f_{best} = f$
endif
until Stopping Criteria is met

Random strategy or
Greedy Randomized strategy

Hill Climbing starts by the initial solution z and returns solution x

If the aim is to maximize $F(x)$
$f_{best} = -\infty$
$f > f_{best}$