Q4, 5, 6

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4.

a.

False
$2x^4 \leq c(x^3 + 3x + 2)$
$2x \leq c + c(\frac{3}{x^2} + \frac{2}{x^3})$
$2x - c(\frac{3}{x^2} + \frac{2}{x^3}) \leq c$
as x $\to \infty$, left side $\to \infty$ but right side is constant
$2x^4$ grows faster than $x^3 + 3x + 2$

b.

True
$4x^3 + 2x^2 \times \log x + 1 < 4x^3 + 2x^3 + x^3 < 7x^3$
(as $logx < x$, $2x^2 \times \log x < 2x^3$)
hence: $4x^3 + 2x^2 \times \log x + 1 < cx^3$ when $c = 7$ and $x > 5$

c.

False
$3x^2 + 7x + 1$ is $\omega(x \log x)$ if $x \log x$ is $o(3x^2 + 7x + 1)$
and x^2 grows faster than $x \log x$

d.

True
$x^2 + 4x$ is $\Omega(x \log x)$ if $x \log x$ is $O(x^2 + 4x)$
$x \log x$ grows more slowly than x^2 therefore True when C =1 and k = 10

e.

False
$f(x) + g(x)$ is not $\Omega(f(x) \times g(x))$

5.

a.

$log_3 9 = 2$
1st option $T(n)$ is $\Theta(n^2 \log n)$

b.

$\log_2 4 = 2$
2nd option $T(n)$ is $\Theta(n^2)$

c.

Master theorem cannot be applied as a is not a constant.

d.

$log_3 3 = 1$
3^{rd} option
$3(\frac{n}{3}) \leq cn$
True for $c > 3$
$T(n) = \Theta(n)$

e.

cannot be applied as $a < 1$

6.

b.
Selectionsort does not have a best and worst case, it always checks every number, the merge function is the important one, it is worst case when the lists contain alternate numbers
e.g.
8,6,4,2
7,5,3,1
so for $n = 8$, the worst case would be 8,6,4,2,7,5,3,1
and for $n = 16$ the worst case would be 16,12,8,4,14,10,6,2,15,11,7,3,13,9,5,1 because after each merge, the next lists being merged will still have alternating numbers.