Q4, 5, 6

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4.

a.

False
$2x^4 \leq c(x^3 + 3x + 2)$
$2x \leq c + c(\frac{3}{x^2} + \frac{2}{x^3})$
$2x - c(\frac{3}{x^2} + \frac{2}{x^3}) \leq c$
as x $\to \infty$, left side $\to \infty$ but right side is constant
$2x^4$ grows faster than $x^3 + 3x + 2$

b.

True
$4x^3 + 2x^2 \times \log x + 1 < 4x^3 + 2x^3 + x^3 < 7x^3$
(as $logx < x$, $2x^2 \times \log x < 2x^3$)
hence: $4x^3 + 2x^2 \times \log x + 1 < cx^3$ when $c = 7$ and $x > 5$

c.

False
$3x^2 + 7x + 1$ is $\omega(x \log x)$ if $x \log x$ is $o(3x^2 + 7x + 1)$
and x^2 grows faster than $x \log x$

d.

True
$x^2 + 4x$ is $\Omega(x \log x)$ if $x \log x$ is $O(x^2 + 4x)$
$x \log x$ grows more slowly than x^2 therefore True when C =1 and k = 10

e.

False
$f(x) + g(x)$ is not $\Omega(f(x) \times g(x))$

5.

a.

$log_3 9 = 2$
$T(n)$ is $\Theta(n^2 \log n)$

b.

$\log_2 4 = 2$
$T(n)$ is $\Theta(n^2)$

c.

Master theorem cannot be applied as a is not a constant.

d.