Home work 04

Step 1 :

$$\int_{-1}^{5}\int_{+1}^{5} F(x,y)\,dx\,dy$$

Where $f(x,y) = xy + 3$

$$\int_{-1}^{5}\int_{+1}^{5} (xy+3)\,dx\,dy$$

⇒ we take the antiderivative of the x-plane or directions

$$= \int_{-1}^{5}\left[\frac{x^2}{2}\cdot y + 3x\right]_{1}^{5} dy$$

$$= \int_{-1}^{5}\left(\left(\frac{(5)^2\cdot y}{2} + 3(5)\right) - \left(\frac{1^2\cdot y}{2} + 3(1)\right)\right) dy$$

$$= \int_{-1}^{5}\left(\frac{25y}{2} + 15 - \frac{1y}{2} - 3\right)dy = \int_{-1}^{5}\left(\frac{25y}{2} - \frac{y}{2} + 15 - 3\right)dy = \int_{-1}^{5}\frac{24y}{2} + 12$$

$$\therefore \int_{-1}^{5}(12y+12)\,dy = 12\int_{-1}^{5}(y+1)\,dy$$

⇒ Integrate or take the antiderivative of the y-plane or direction

$$= 12\left[\frac{y^2}{2} + y\right]_{-1}^{5} = 12\left(\left(\frac{5^2}{2} + 5\right) - \left(\frac{(-1)^2}{2} - 1\right)\right) = 12\left(\frac{25}{2} + 5 - \frac{1}{2} + 1\right)$$

$$= 12\left(\frac{25-1}{2} + 6\right) = 12\left(\frac{24}{2} + 6\right) = 12(12+6) = 12\times 18$$

$$= 216$$

Therefore, $\int_{-1}^{5}\int_{+1}^{5}(xy+3)\,dx\,dy = 216$

Homework 06 (#1)

Step 2: Solve

$$\int_{-1}^{5}\int_{+1}^{5} F(x,y)\,dx\,dy$$

using Simpson's Rule

we have

$$\int_{y_{min}}^{y_{max}}\int_{x_{min}}^{x_{max}} F(x,y)\,dx\,dy = \frac{hk}{9}\Bigg[F(x_1,y_1) + F(x_P,y_1) + F(x_1,y_Q) + F(x_P,y_Q) + \sum_{\substack{j=2\\ j=\text{even}}}^{Q-1} 4\left[F(x_1,y_j) + F(x_P,y_j)\right] + \sum_{\substack{j=3\\ j=\text{odd}}}^{Q-2} 2\left[F(x_1,y_j) + F(x_P,y_j)\right] +$$

$$\sum_{\substack{i=2\\ i=\text{even}}}^{P-1} 4\left[F(x_i,y_1) + F(x_i,y_Q)\right] + \sum_{\substack{i=3\\ i=\text{odd}}}^{P-2} 2\left[F(x_i,y_1) + F(x_i,y_Q)\right] +$$

$$\sum_{\substack{j=3\\ j=\text{odd}}}^{Q-2}\sum_{\substack{i=2\\ i=\text{even}}}^{P-1} 8F(x_i,y_j) + \sum_{\substack{j=2\\ j=\text{even}}}^{Q-1}\sum_{\substack{i=3\\ i=\text{odd}}}^{P-2} 8F(x_i,j) +$$

$$\sum_{\substack{j=2\\ j=\text{even}}}^{Q-1}\sum_{\substack{i=2\\ i=\text{even}}}^{P-1} 16F(x_i,y_j) + \sum_{\substack{j=3\\ j=\text{odd}}}^{Q-2}\sum_{\substack{i=3\\ i=\text{odd}}}^{P-2} 4F(x_i,y_j)\Bigg]$$

For Ease of computing and structuring, I'll solve these in segments.

Where P is the number of integration points (x) = 5

" Q " " " " " " (y) = 7

" h " " interval size for x-direction = 4

" k " " " " " y-direction 6

$$\int_{-1}^{5}\int_{+1}^{5} f(x,y)\,dx\,dy = \int_{-1}^{5}\int_{+1}^{5} \cancel{x}\,(+3 = ??$$

~~FE~~ x: Integration Points = 1, 2, 3, 4, 5

y: Integration points -1, 0, 1, 2, 3, 4, 5

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Solving or Computing by Part (#2)

(i)

$\Rightarrow \frac{hk}{9}$; $h = \frac{b_x - a_x}{n_x} = \frac{5-1}{4} = \frac{4}{4} = 1$

$k = \frac{b_y - a_y}{n_y} = \frac{5-(-1)}{6} = \frac{6}{6} = 1$

$\Rightarrow \frac{hk}{9} = \frac{1 \cdot 1}{9} = \frac{1}{9}$

$$\boxed{\begin{array}{l} P = 5 \\ Q = 7 \end{array}}$$

(ii)

$\Rightarrow f(x_1, y_1) + f(x_5, y_1) + f(x_1, y_7) + f(x_5, y_7)$

$= \cancel{4} + (\cancel{2}) + 8 + 28 = 36$

(iii)

$\Rightarrow \sum_{\substack{j=2 \\ j=\text{even}}}^{7-1} 4[F(x_1, y_j) + F(x_5, y_j)] = \sum_{\substack{j=2 \\ j=\text{even}}}^{6} 4[F(x_1, y_j) + f(x_5, y_j)]$

$= 4[(F(x_1, y_2) + F(x_5, y_2)) + (F(x_1, y_4) + F(x_5, y_4)) + (F(x_1, y_6) + (x_5, y_6))]$

$= 4[(3+3) + (5+13) + (7+23)] = 4(54) = 216$

(iv)

$\Rightarrow \sum_{\substack{j=3 \\ j=\text{odd}}}^{7-2} 2[F(x_1, y_j) + F(x_5, y_j)] = \sum_{\substack{j=3 \\ j=\text{odd}}}^{5} 2[F(x_1, y_j) + f(x_5, y_j)]$

$2[(F(x_1, y_3) + F(x_5, y_3)) + (F(x_1, y_5) + F(x_5, y_5))]$

$2[(\cancel{4} + \cancel{2}8 + 6 + 18)] = 2(36) = \underline{72}$

(v)

$\Rightarrow \sum_{\substack{i=2 \\ i=\text{even}}}^{5-1} 4[F(x_i, y_1) + F(x_i, y_7)] = \sum_{\substack{i=2 \\ i=\text{even}}}^{4} 4[F(x_i, y_1) + f(x_i, y_7)]$

$= 4[(F(x_2, y_1) + F(x_2, y_7)) + (F(x_4, y_1) + F(x_4, y_7))]$

$= 4[(1+13) + (-1+23)] = 4(36) = \underline{144}$

Vi

$$\sum_{\substack{i=3\\ i=odd}}^{P-2} 2[F(x_i,y_1)+F(x_i,y_7)] = \sum_{\substack{i=3\\ i=odd}}^{3} 2[F(x_i,y_1)+F(x_i,y_7)] \quad (\#3)$$

$$= 2[F(x_3,y_1)+F(x_3,y_7)] = 2[0+18] = 2(18) = 36$$

Vii

$$\sum_{j=3}^{Q-2} \sum_{i=2}^{P-1} 8F(x_i,y_j) = \sum_{\substack{j=3\\ j=odd}}^{5} \sum_{\substack{i=2\\ i=even}}^{4} 8F(x_i,y_j)$$

$$= 8[F(x_2,y_3)+F(x_4,y_3)+F(x_2,y_5)+F(x_4,y_5)] = 8(5+7+9+15) = \overset{8(36)}{\underline{288}}$$

Viii

$$\sum_{\substack{j=2\\ j=even}}^{7-1} \sum_{\substack{i=3\\ i=odd}}^{5-2} 8F(x_i,y_j) = \sum_{\substack{j=2\\ j=even}}^{6} \sum_{\substack{i=3\\ i=odd}}^{3} 8F(x_i,y_j)$$

$$= 8[F(x_3,y_2)+F(x_3,y_4)+F(x_3,y_6) = 8[3+9+15] = 8(27) = \underline{216}$$

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$$\sum_{\substack{j=2\\ j=even}}^{7-1} \sum_{\substack{i=2\\ i=even}}^{5-1} 16F(x_i,y_j) = \sum_{\substack{j=2\\ j=even}}^{6} \sum_{\substack{i=2\\ i=even}}^{4} 16F(x_i,y_j)$$

$$= 16[F(x_2,y_2)+F(x_4,y_2)+F(x_2,y_4)+F(x_4,y_4)+F(x_2,y_6)+F(x_4,y_6)]$$

$$= 16(3+3+7+11+11+19) = 16(54) = \underline{864}$$

(x)

$$\sum_{\substack{j=3\\ j=odd}}^{7-2} \sum_{\substack{i=3\\ i=odd}}^{5-2} 4F(x_i,y_j) = \sum_{\substack{j=3\\ j=odd}}^{5} \sum_{\substack{i=3\\ i=odd}}^{3} 4F(x_i,y_j)$$

$$= 4[F(x_3,y_3)+F(x_3,y_5)] = 4[6+12] = 4(18) = \underline{72}$$

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Therefore,

$$\int_{-1}^{5}\int_{+1}^{5} f(x,y)\,dx\,dy = \int_{-1}^{5}\int_{+1}^{5} (xy+3)\,dx\,dy$$

Putting it all together!

$$\int_{-1}^{5}\int_{+1}^{5} (xy+3)\,dx\,dy = \frac{1}{9}\left(36+216+72+144+36+288+216+864+72\right)$$

$$= \frac{1}{9}(1944)$$

$$= \underline{\underline{216}}$$