Homework 04

#1

1) Analytical

a) $\int_0^1 x\sqrt{1-x^2}\,dx$

using u-substitution approach

where $u = 1-x^2$

$\frac{du}{dx} = -2x$; $\Rightarrow du = -2x\,dx$

N.B We can rewrite our Integral as

$= \int_0^1 -\frac{1}{2} \cdot -2x\sqrt{1-x^2}\,dx$ $\left\{ \text{since } \frac{-1}{2} \cdot -2 = 1 \right\}$

$= \frac{-1}{2}\int -2x\,dx\sqrt{1-x^2}\,dx$ (We can take the Constant out of the Integral)

from the above Integral, we see that,

$u = 1-x^2$ & $du = -2x\,dx$

$\therefore \frac{-1}{2}\int du \cdot \sqrt{u} \approx -\frac{1}{2}\int u^{1/2}\,du$

$\int_0^1 x\sqrt{1-x^2}\,dx = \frac{-1}{2}\left(\frac{u^{3/2}}{3/2}\right)\Big|_0^1 = \frac{-1}{2}\left(\frac{u^{3/2}}{1} \times \frac{2}{3}\right)\Big|_0^1$

$= \frac{-1}{2}\left(\frac{2u^{3/2}}{3}\right)\Big|_0^1 = -\left(\frac{u^{3/2}}{3}\right)\Big|_0^1$ substitute $1-x^2$ for u

$= -\left(\frac{(1-x^2)^{3/2}}{3}\right)\Big|_0^1$

$= -\left(\frac{(1-(1^2))^{3/2}}{3}\right) - \left[-\frac{(1-0^2)^{3/2}}{3}\right]$

$= -\left(\frac{(1-1)^{3/2}}{3}\right) - \left(-\frac{1^{3/2}}{3}\right) = -\frac{0}{3} + \frac{1}{3} = 1/3$

$\therefore \int_0^1 x\sqrt{1-x^2}\,dx = 1/3$

(b) $\int_1^2 2x e^{x^2} dx$

#2

- still using u-substitution approach

$u = x^2$

$\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$

$\therefore$ we can rewrite our integral as

$\int_1^2 2x\,dx\, e^{x^2}$

recall that $du = 2x\,dx$ and $u = x^2$

$\int_1^2 e^u du = e^u + c \Big|_1^2$

substituting for u and computing, we'll have,

$e^{2^2} - e^{1^2} = e^4 - e^1 = 51.8798682046852 \approx 51.8799$

$\therefore \int_1^2 2x e^{x^2} dx = \underline{51.8799}$

(c) $\int_0^1 F(x)\,dx$ where $\begin{cases} f(x) = 0.0 & \text{for } x \leq 0.5 \\ f(x) = 1.0 & \text{for } x > 0.5 \end{cases}$

$= \int_0^{0.5} 0\,dx + \int_{0.5}^1 1\,dx$

$= 0 + x\Big|_{0.5}^1$

$= 1 - 0.5 = 0.5$

$\therefore$ 0.5

#3

d) $\int_3^5 x\,dx = \left[\frac{x^{1+1}}{1+1} + c\right]_3^5$

$$= \frac{x^2}{2} + c \Big|_3^5$$

$$= \frac{5^2}{2} - \frac{3^2}{2} = \frac{25-9}{2} = \frac{16}{2} = 8.0$$

$$\therefore \int_3^5 x\,dx = 8.0$$

I feel the reason why

$$\int_0^1 F(x)\,dx \quad \text{where} \begin{cases} F(x) = 0.0 & \text{when } x \le 0.5 \\ F(x) = 1.0 & \text{when } x > 0.5 \end{cases}$$

is because of the numbers of integration point that is needed to estimate.

The structure of the above formula is

$$\int_0^1 F(x)\,dx = \int_0^{0.5} 0\,dx + \int_{0.5}^1 1\,dx$$

We'll have to ~~integrate~~ consider the two integral when using the numerical method!

(ii) $\int_3^5 x\,dx$

The reason it has 4 interval is because the simpson's rule is an accurate method in evaluating polynomial feature which x is a degree 1 polynomial. Hence, if it takes 2 interval to integrate with some errors, the 4th interval will correct the error! ~~at~~ leading to a perfect answer.

(iii) To correct the ~~many~~ interval to 4 intervals with the integral

$$\int_0^1 F(x)\,dx \quad \text{where} \begin{cases} f(x) = 0.0 & \text{when } x \le 0.5 \\ f(x) = 1.0 & \text{when } x > 0.5 \end{cases}$$

We should get rid of the first condition and assume that $F(x) = x$ with lowerbound $= 0.0$ and upper bound $= 1.0$

Such that,

$$\int_0^1 x\,dx = \underline{\underline{0.5}}$$

And if we iterate using my code, we will get 4 intervals accurately giving us this answer.