1. Analytical: HOMEWORK #2

$$\int_0^1 x^3(1+x^4)^3\,dx$$

→ Expand the bracket to integrate each power of x

$$=\int_0^1 x^3(1+x^4)(1+x^4)^2\,dx$$

$$=\int_0^1 x^3(1+x^4)(1+2x^4+x^8)\,dx$$

$$=\int_0^1 (x^3+x^7)(1+2x^4+x^8)\,dx$$

$$=\int_0^1 \left[x^3(1+2x^4+x^8)+x^7(1+2x^4+x^8)\right]dx$$

$$=\int_0^1 (x^3+2x^7+x^{11}+x^7+2x^{11}+x^{15})\,dx$$

$$=\int_0^1 (x^3+3x^7+3x^{11}+x^{15})\,dx$$

→ Take the integral of each power of x

$$=\left[\frac{x^4}{4}+\frac{3x^8}{8}+\frac{3x^{12}}{\cancel{12}\,4}+\frac{x^{16}}{16}\right]_0^1 = \left[\frac{4x^4+6x^8+4x^{12}+x^{16}}{16}\right]_0^1$$

$$=\left(\frac{4(1)^4+6(1)^8+4(1)^{12}+(1)^{16}}{16}\right)-\left(\frac{4(0)^4+6(0)^8+4(0)^{12}+(0)^{16}}{16}\right)$$

$$=\frac{4+6+4+1}{16}=15/16=0.9375$$

2. Setup:

$$\int_0^1 f(x)\,dx = \int_0^1 x^3(1+x^4)^3\,dx$$

$$f(x) = x^3(1+x^4)^3$$

if we are to divide $0 \rightarrow 1$ into four (4) equal parts we'll have to get quarter points

$\Rightarrow$ 0.0, 0.25, 0.5, 0.75, 1.0

thus,

$$F(0.0) = 0^3(1+0.0^4)^3 = 0.0000$$
$$F(0.25) = 0.25^3(1+0.25^4)^3 = 0.0158$$
$$F(0.5) = 0.5^3(1+0.5^4)^3 = 0.1499$$
$$F(0.75) = 0.75^3(1+0.75^4)^3 = 0.9624$$
$$F(1.00) = 1.0^3(1+1.0^4)^3 = 8.0000$$

Node	x Value	function value $f(x)$
x_1 (lower bound)	0.00	0.0000
x_2	0.25	0.0158
x_3	0.50	0.1499
x_4	0.75	0.9624
x_5 (upper bound)	1.00	8.0000

3) Mechanical: Rectangle Rule (3)

Recall, repeated Rectangle Rule

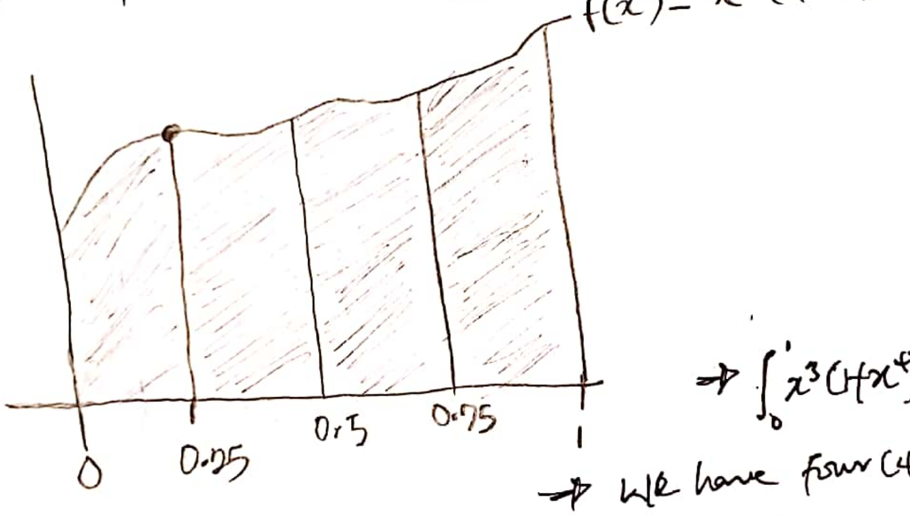
$$\int_a^b F(x)\,dx \approx h\sum_{k=1}^{N} F(x_k)$$

We have,

$$\int_0^1 x^3(1+x^4)^3\,dx$$

where $F(x) = x^3(1+x^4)^3$; and $b=1$, $a=0$; and $N=4$

Since, the function is a polynomial function, we'll have something like this



$\Rightarrow \int_0^1 x^3(1+x^4)^3\,dx \approx \frac{b-a}{N}\cdot\sum_{k=1}^{N} F(x_k)$

$\Rightarrow$ We have four (4) element

$$\int_0^1 x^3(1+x^4)^3\,dx \approx \frac{1-0}{4}\sum_{k=1}^{4} x^3(1+x^4)^3 = \frac{1}{4}\left[0^3(1+0^4)^3 + 0.25^3(1+0.25^4)^3 + 0.5^3(1+0.5^4)^3 + 0.75(1+0.75^4)^3\right]$$

$$\approx \frac{1}{4}\left[0 + 0.0158 + 0.1499 + 0.9624\right] = \frac{1}{4}(1.1281)$$

$$= 0.282034\ldots \approx 0.2820$$

$$\text{Error \%} = \frac{|\text{real answer} - \text{numerical answer}|}{\text{real answer}} \times 100.0$$

$\rightarrow$ real answer $= 0.9375$
numerical answer $= 0.2820$

$$\text{Error \%} = \frac{0.9375 - 0.2820}{0.9375}\times 100 = \frac{0.6555}{0.9375}\times 100 = 0.6992\times 100$$

$$= 69.92\% \approx 70\%$$

4) Mechanical: Trapezoid Rule.

From the Repeated Trapezoid Rule with equally spaced nodes/elements: (4)

$$\int_a^b f(x)\,dx \approx h\left(\frac{1}{2}[f(x=a) + f(x=b)] + \sum_{k=2}^{N} f(x_k)\right)$$

$$\int_0^1 x^3(1+x^4)^3\,dx \quad f(x) = x^3(1+x^4)^3$$

where $h = \frac{b-a}{4} = \frac{1-0}{4} = \frac{1}{4}$;

$a = 0$; $b = 1$

* $f(x=a) = f(0) = 0$
* $f(x=b) = f(1) = 1^3(1+1^4)^3 = 8$
* $\sum_{k=2}^{4} f(x_k) = f(x_2) + f(x_3) + f(x_4)$

where $f(x_2) = f(0.25) = 0.0158$
$f(x_3) = f(0.5) = 0.1499$
$f(x_4) = f(0.75) = 0.9624$

putting it all together. We'll have

$$\int_0^1 x^3(1+x^4)^3\,dx \approx \frac{1}{4}\left(\frac{1}{2}(0+8) + 0.0158 + 0.1499 + 0.9624\right)$$

$$\approx \frac{1}{4}\left(\frac{\cancel{8}^{4}}{\cancel{2}}\right) + 0.0158 + 0.1499 + 0.9624)$$

$$\approx = \frac{1}{4}(4 + 1.1281) = 1 + 0.282025 = 1.282025$$

Approximately,

$$\int_0^1 x^3(1+x^4)^3\,dx \approx \underline{1.2820}$$

Calculating The Error in Trapezoid Rule

$$\text{Error \%} = \frac{|\text{real answer} - \text{numerical answer}|}{\text{real answer}} \times 100.0$$

where,
real answer $= 0.9375$
numerical (trapezoid) answer $= 1.2820$

$$\therefore \text{Error \%} = \frac{|0.9375 - 1.2820|}{0.9375} \times 100 = \frac{|-0.3445|}{0.9375} \times 100$$

$$= \frac{0.3445}{0.9375} \times 100 = 0.3675 \times 100 = \underline{36.75\%}$$

The Error is way smaller than the Repeated Rectangle Rule.