Homework #3

#1

#1

Integration Point	x-value	function value f(x)
x_1 (lower bound)	0.0	0.0000
x_2	0.25	0.0158
x_3	0.50	0.1499
x_4	0.75	0.9624
x_5 (upper bound)	1.0	8.0000

Solve $\int_0^1 x^3(1+x^4)^3\,dx$ using Simpson's rule

$$\int_a^b F(x)dx \approx \frac{h}{3}\left[F(x_1)+F(x_p)+2\sum_{i=3}^{p-2}F(x_i)+4\sum_{i=2}^{p-1}F(x_i)\right]$$

(odd nodes only; even nodes only)

where $h=\frac{b-a}{p-1}$ where $b=1$, $a=0$; $p=5$ = no of intervals

$x_p=1$, $x_1=0$

$$h=\frac{1-0}{4}=\frac{1}{4}$$

$$\therefore \int_0^1 x^3(1+x^4)^3\,dx = \frac{1}{12}\left[F(x_1)+F(x_5)+2(F(x_3))+4(F(x_2)+F(x_4))\right.$$

From the table,

$F(x_1)=0$; $F(x_5)=8$; $F(x_3)=0.1499$; $F(x_4)=0.9624$

$$\therefore \int_0^1 x^3(1+x^4)^3\,dx = \frac{1}{12}\left[0+8+2(0.1499)+4(0.0158+0.9624)\right]$$

$$= \frac{1}{12}(12.2126) = 1.017717 \approx 1.0177$$

using simpson's rule $\int_0^1 x^3(1+x^4)^3\,dx = 1.0177$

#2

Calculating the Error:

$$\text{Error\%} = \frac{|0.9375 - 1.0177|}{0.9375} \times 100.0$$

$$= \frac{|-0.0802|}{0.9375} \times 100.0$$

$$= \frac{0.080021 \times 100}{0.9375} = \frac{8.021}{0.9375}$$

$$\text{Error\%} = 8.5557\%$$