

**Portfolio Optimisation   
using Modern Portfolio Theory & Python**

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**Acknowledgements**

**Abstract**

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# Introduction

## Background

This section aims to set the scene of the project, providing context and introducing the problem at hand. An investment is defined as “The Action or process of investing money for profit” (Oxford Dictionary, 2017). There are lots of different options that investors are faced with when it comes to choosing what to invest in. There are stocks, bonds, options, futures and countless more to consider. For this project, the focus is going to be on stocks. A stock is a portion of shares issued by a company (Oxford Dictionary, 2017). When investing into stocks, or anything for that matter, the general goal is going to be that when it comes time to sell, they have risen in value enough that the investor has made a profit. However, making that profit almost never comes free. Each investment typically has an “Expected Return” and a “Risk” aspect associated with it. These can be defined in a number of ways. In the context of this project, “Expected Return” is defined as the anticipated profit or loss on an investment characterised by a historical average. “Risk” is defined as the volatility of an investment’s return, which is effectively the likelihood that an investment’s actual return will be different to the expected return. This is characterised as variance. The best possible investment would be one that had a very high return with no risk attached. Sadly, this is never the case as almost every investment is guaranteed to have some kind of inherent risk (Thangavelu, 2015). To achieve those higher returns, investors have to be prepared to take on the extra risk involved in doing so.

It would be unusual for investors to only invest in one asset at a time Instead, what is created is a portfolio. A portfolio is “a range of investments held by a person or organisation” (Oxford Dictionary, 2017). Due to the insanely large amount of asset options that are available for investors to choose from, this introduces a small part of the problem that this project addresses. How do you decide what assets to invest in? Traditionally when creating portfolios, investors would select assets using anecdotal analysis or statistics of each asset individually. This changed when Harry Markowitz’s paper “Portfolio Selection” was released in March 1952 compiled within The Journal of Finance. Markowitz put forward among others, the idea of looking at how assets move and interact with the portfolio as a whole rather than on an individual basis. Instead of looking at an asset’s individual variance, it instead considers the risk of the portfolio to be determined by the covariance between each asset. In finance, covariance is a measure of how much the returns of two assets move together, contextualising the risk of a specific asset with the rest of the portfolio (Investopedia, 2017). One of Markowitz’s biggest ideas is that investors can potentially decrease the amount of risk they are having to take for the same return through means of diversification (Shipway, 2009). By spreading investments out over a larger number of diverse assets, the risk associated is generally going to be lower than if the investments were made in only one or two assets. This is due to each asset reacting differently to certain scenarios. With a large and diverse portfolio, there are going to be scenarios in which some assets will pay off, and some that won’t (Elton and Gruber, 1997).

The age old question of how to distribute your budget between each of your assets forms the problem that this project aims to provide a solution for. Modern Portfolio Theory (MPT) is a combination of Markowitz’s “Portfolio Selection” theory and William Sharpe’s later contributions in 1964 with his Capital Asset Pricing Model (CAPM). It is an investment framework that provides guidance for the creation of portfolios based on the maximisation of expected returns for a given risk (Mangram, 2013). Markowitz said that for every level of risk, there exists what he described as an “efficient portfolio”. This being the portfolio that provides the maximum return for that specific amount of risk. The Mean-Variance analysis techniques featured within Modern Portfolio Theory provides insight into mathematical ways of identifying and calculating efficient portfolios through distribution of budget, optimising the trade-off between risk and return. Despite its reputation, original forms of Markowitz’s optimisation model aren’t extensively used for larger portfolios due to the computational complexity involved in optimising them (Konno and Yamazaki, 1991). Along with the larger advancements of computing into the financial world, optimisations like these are becoming easier to do, due to strides in both software and hardware. It is this area in which the project will take place; using mean-variance analysis to craft and optimise select portfolios to achieve the desired goals be it; minimum variance, maximum return or somewhere in between.

## Aims & Objectives

### Aim & Rationale

The aim and rationale of this project is to create a proprietary asset allocation system that demonstrates the effectiveness of Modern Portfolio Theory in creating investment strategies. It should be able to retrieve relevant and up-to date stock data providing visualisation and descriptive analysis to the user. It will allow the user to craft a portfolio using their choice of stocks from the available selection. Using concepts from Modern Portfolio Theory such as Mean-Variance analysis, this portfolio can then be optimised in order to achieve the user’s desired goals. The statistics of the portfolio’s potential performance will be displayed by the system so the user then decide whether to proceed with that portfolio as their investment strategy.

### Objectives

The objectives of this project are to:

* Use research to acquire and develop a greater understanding of Modern Portfolio Theory to help pinpoint requirements and guide implementation.
* Find and consider various options for a reliable online source of relevant financial data.
* Implement solutions using Python to achieve the following:
  + A retrieval system to fetch up-to-date price data for a specified list of stocks.
  + Perform statistical analysis on the data to provide the user with descriptive summaries of each stock as well as graphic visualisation.
  + Allow the user to craft custom portfolios manually choosing from the available selection.
  + Perform mean-variance optimisation to achieve the user’s desired portfolio goal.
  + Provide the user with summarising statistics of their optimised portfolio.
* Create a full graphical user interface to contain and situate the functionality of the solutions that have been created to form an interactive desktop application.

# Literature Review

This chapter illustrates and explains some of the underlying concepts of this project through the review of relevant academic work. Description and analysis of their work is performed, furthered by presenting a comparison of their findings with work of a similar nature.

## Modern Portfolio Theory

### Background

The original founder of Modern Portfolio Theory (MPT): Harry Markowitz had his paper “Portfolio Selection” first published in the Journal of Finance, March 1952. His work and theories provided completely new and revolutionary insight into many areas of finance, totally changing the traditional methods of managing investments, awarding him a Nobel Prize in Economic Sciences in 1990. (Elton and Gruber, 1997). He characterised and formulated the problem of financial portfolio selection as a more than just a consideration of return, but as a combination of both; risk and return, suggesting that there is an unavoidable trade-off to be made. MPT assumes there to be a positive relationship between risk and return for each investment, meaning that to obtain a higher expected return, there has to be a higher risk involved. This introduced the concept of mean-variance analysis into portfolio theory, called the Markowitz Model. The Markowitz model can also be called the mean-variance model because it takes the expected return as a mean and uses variance or standard deviation as a measure of risk. It explains that for every level of given risk in a portfolio, there is what Markowitz describes as an “efficient portfolio”, aka one portfolio that provides the greatest return for that level of risk. This laid the foundation for other theories such as the “Capital Asset Price Model” which extended the analysis of determining what assets should be included into a portfolio (Elton and Gruber, 1997).

### The Markowitz Model

The Markowitz

### Model Inputs

A common way to calculate inputs for the Markowitz model is to look at historical data as a means to estimate values for sample mean for expected return and variance for risk. There is contention about the accuracy and effectiveness of using historical data as inputs for the mean-variance optimisation model. Fabozzi et al (2002) discuss the effectiveness of using historical data as an input, stressing that there is a strong level of importance in assessing the economic conditions when using that period to estimate return and variance. It goes on to suggest that there are reasons for the use of historical data, including arguments about the length of the histories, as well as the political and financial state of that market playing a major role in assessing the viability of the historical data. They argue that only after an economy has a long record of good performance and stability before should they be considered for use as an estimation. To contrast this point, Michaud (1989) created a similar implementation of a mean-variance optimiser using historical data. He theorised that the poor performance of the model was due to the use of historical data. Going on to suggest that a trait of the optimiser was causing it to maximise the error contained within the data due to what he believed was because the estimations made ignored what he described as the “inherent multivariate nature” of the problem. Sharpe (1999) argues that a lot of performance measures are justified in their use of historic data in some applications due to the likelihood of predicted relationships that can be extrapolated in theory but not always in practice.

Furthering the discussion of effective inputs for the Markowitz model, Konno and Yamazaki (1991) question the use of standard deviation of historical returns as a measure of risk. They note that an investor’s perception of risk is not symmetrical around the mean of the returns. Arguing that investors aren’t as concerned when there is large positive deviation from the mean compared to small negative deviations and going further to say that market returns are generally not distributed symmetrically. Ways to classify these return fluctuations have been devised through the use of a newer estimator called “Downside Risk”, which is the estimation of an asset’s potential to decline in value (Investopedia, 2017). Rockafellar and Uryasev (2000) assess the use of alternatives to standard deviation in optimisation practices, recommending the use of more intricate methods to characterise risk, such as Value-at-Risk (VaR), or even Conditional Value-at-Risk (CVaR). Both of which are based on the use of standard deviation of normal distributions and contain undesirable attributes described as a “lack of subadditivity and convexity” making optimisations more computationally difficult as well as an increase in implementation technicality.

### Diversification

The concept of diversification is one of the most important parts of Markowitz’s work. Even today, the concept of diversification is commonly used in portfolio management, as it such an intuitive appeal as well as an empirical backing. Diversification is the process of spreading investments over a wide range of investment types and areas in an attempt to reduce unsystematic risk (Markowitz, 1952). There are two types of risk that the financial world. There is systematic and unsystematic risk, both of which are a component of the overall risk that is present within assets and portfolios. Systematic risk can be defined as the risk that is inherent to an entire market, meaning that it is something that all assets that are traded within that market are subject to. An example of Systematic Risk would be the effects applied on the whole market by events such as The Great Recession, not the event itself. Events that have these affects are what is known as Systemic Risks (Investopedia, 2017). Unsystematic Risk is the part of an investment’s risk that is specific to that investment, influenced by events attributable to the investment itself or the sub-group it is in. Not the entire market system like Systematic Risk (Investopedia, 2017).

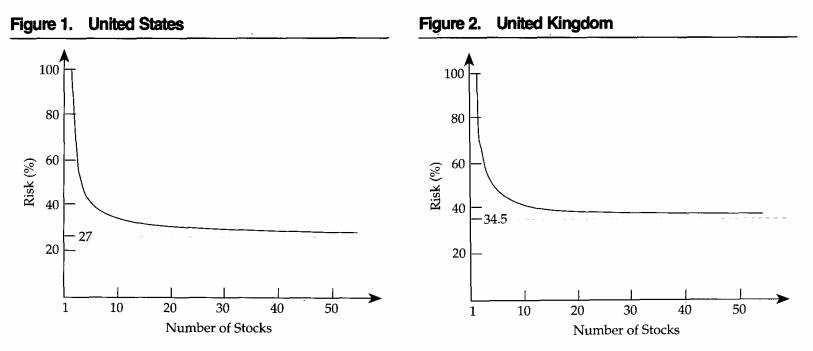
The effectiveness of Diversification at reducing unsystematic risk is generally undisputed. Solnik (1995) analysed the effects of diversification on markets outside of the US finding that it was almost just as effective in European markets. However, he noted that the proportionality of the number of assets compared to the amount risk reduced was not consistent and it was impossible to completely eliminate risk after a certain point because of what can be attributed to the assets moving together. He suggested that it is possible to reduce risk past this certain point through further diversification extending the holding of assets across multiple markets rather than just one. 

Figure 1 - The effects of Diversification in different markets (Solnik, 1995)

Raffestin (2014) builds on the summary provided by Solnik (1995), investigating the effectiveness of Diversification at reducing the impact caused by systemic risks, theorising that Diversification can make investors safer individually but can actually increase the effects caused by a systemic risk due to common asset holdings between each investor. He described the connections made by these common asset holdings as “endogenous covariance” that can actually propagate the effects of systemic risk through the system. He found again, that investors can minimise their risk by spreading wealth across not only a greater amount of assets, but more distant assets, potentially increases the overall stability of a market. This further reinforces the importance of strong Diversification in a portfolio.

### Portfolio Performance Evaluation

There are a number of different metrics to measure the overall performance of a portfolio. One of the oldest and most influential performance metrics is the Sharpe Ratio developed by William Sharpe in 1966 (Sharpe, 1994). It calculates the overall reward to risk ratio of an asset or portfolio through the use of a benchmark, typically described as a “risk-free asset”. The choice of risk-free asset weighs in heavily to the effectiveness of the Sharpe Ratio. It’s debated in practice whether there are assets that are totally risk free. However, in theory index funds and other assets such as government bonds are typical candidates when it comes to selecting a risk free asset.

Lediot and Wolf (2008) test the overall performance and robustness of the Sharpe Ratio as a performance metric, finding that the use of it in fairly stable conditions can actually be quite effective. However, they state that as the Sharpe Ratio generally requires the use of historical data as an input, it can produce misleading conclusions. Primarily occurring when there are tails heavier than the normal distribution of the data, noting that it shouldn’t really be used because these tails are common within financial returns. Christie (2005) largely tends to agree with Lediot and Wolf’s (2008) conclusion suggesting that the biggest limitation of the Sharpe Ratio is that its inputs; expected return and risk are measured with error, leading the Sharpe Ratio itself to contain error.

# Methodology

Every project will usually have some kind of project management and software development methodology that it will follow. For very small projects, methodologies aren’t really much of a concern, as they generally don’t last long enough or aren’t intense enough to utilise the full benefit of following an established software development methodology. However, as the project grows in size, it becomes more and more important to be following a methodology as it can make time and cost more foreseeable, as well as boosting efficiency by sticking to a thought-out schedule (Awad, 2005). This makes it important to consider a range of methodologies for both project management and software development. Before choosing the most optimal methodology, relevant analysis would need to done for this particular project.

All of the software development methodologies out there each have their own strengths and weaknesses that can become more or less apparent depending on the type of project. One of the most influential concepts that affects the viability of a software development methodology is the initial state of the software’s requirements (Balaji and Murugaiyan, 2012). The way the requirements were set out at the beginning was very uncertain. The artefact being produced from the project was always going to be making use of various concepts within Modern Portfolio Theory (MPT). As MPT is a relatively old theory, all concepts featured within it are already laid out and are not going to change. This makes it seem possible to create a full list of requirements from what had been researched, and proceeding with a traditional sequentially structured methodology such as Waterfall. However, due to the sheer amount of concepts situated within MPT, it is extremely hard at the beginning of the project to fully gauge how many of these concepts were realistically going to be implemented, and in general, how far the artefact could go. The project supervisor acted not far from what could be considered a type of client to the project. The direction and requirements that were dictated by it were often the result of in-depth discussion with the project supervisor. This set a slight feature-driven-development tone to the project as the outcomes of these discussions provided both general direction and priority. This uncertainty alone would rule out the use of a sequential methodology like Waterfall. It tends to benefit from having very well defined requirements, as well as a strong time and cost forecast which would be unable to accommodate this type of project (Balaji and Murugaiyan, 2012).

Going off the exclusion of rigid methodologies such as Waterfall, the logical choice is something a lot more flexible that could adapt to the initial lack of knowledge in the chosen field, making determination of detailed requirements and calculating an accurate time plan for the project impractical. This lent itself to choosing a more adaptable methodology that could account for these unknown details, providing a framework for both software development and project management. In terms of project management, some of the more intricate and established methodologies such PRINCE2 seemed a little too process heavy and intensive for a solo project. Something such as PRINCE2 is more suitable and effective for a larger team, where allocating people and resources is more of a complex problem. It would be unnecessary and perhaps detrimental to try incorporate this large of a methodology for a small project like this (Matos and Lopes, 2013).

An Agile methodology would be most fitting for a project of this specification. It provides multiple opportunities to assess the overall direction the project is heading in, which accommodates the uncertainty of this project’s final goal. The iterative and incremental nature of Agile makes it extremely appealing as there are regular intervals in which new requirements can be gathered and work done previously can be built on and improved (Agile Methodology, 2008). While Agile itself is not a methodology, there are methodologies that use Agile principles. One of the most popular ones is SCRUM (SCRUM, 2017). One of the problems with using SCRUM, is that it is so heavily focused on team communication and feedback, that a lot of its benefits are wasted on a one-man project. For this project, the chosen methodology is Agile in nature, primarily combining aspects of Extreme Programming (XP) and Feature-Driven Development (FDD). XP accommodates the heavy software development aspect of the project as well as the presence of changing requirements and FDD relates to the prioritising of features to include in the final release of the artefact.

## Project Management

### Time Planning

Time planning is something that is essential to any project’s success. However, since the project follows an Agile methodology it is difficult and often useless to try and plan every part of the project accordingly. The project is prone to changing requirements and deadlines throughout, making the process of accurately planning out each stage on a weekly basis a troublesome task. A Gantt chart is a commonly used time planning tool that provides good visualisation of how time is going to be spent over the course of the project (Investopedia, 2017). As the project follows an agile methodology, a hypothetical scenario of how the project is likely going to be structured has been generated. With the use of iterations, the generated Gantt chart details rough estimations of each iteration’s length and contents.

<Gantt Chart Here>

### Risk Management

As the project contained multiple software based components, it was important to consider some of the different risks that were involved. An essence of project management that could be utilised to summarise and handle this risk is what is called a Risk Matrix. A Risk Matrix can be defined as a structured approach to identifying risk, defining which risks are most dangerous to the project, what impact they might have, and how they can be dealt with (Garvey and Lansdowne, 1998).

A Risk Matrix has been formulated to provide evaluation and insight into potential risks that may be present within the project. <Talk about what is included in Risk Matrix>

<Risk Matrix Here>

### Development Tools

This section delves into the main tools and applications that were used throughout the whole process of creating the project’s final artefact. It explains what the tools were used for, how they were chosen and if applicable, what were some possible alternatives. Tools that were only used for more specific areas of project development are discussed during the development section of the report.

#### Programming Language

One of the most important tools utilised in a software development project is the programming language that it is created in. There are programming languages that are much more suited to specific tasks than others and it’s helpful to pick a suitable one so as to not make things purposely difficult or inefficient. When deciding what programming language is going to be used, it is important to classify what kind of problems are needing to be solved using language. A brief overview of the tasks in this project involve a series of data analysis and handling techniques including fairly comprehensive mathematical procedures. Finally, the creation of a graphical user interface (GUI) is also necessary, which really requires some kind of object-oriented principles.

Based on the data handling and mathematical function requirements alone, some potential candidate languages would be Python (Python Software Foundation, 2017) and R (R Core Team). R is one of the most popular languages used in data science and provides a large majority of the utilities needed for the project. The R environment contains an integrated suite of facilities for data manipulation, calculation and graphical display (R Core Team, 2017). However, R unfortunately does not contain the object oriented features that would be necessary to create a GUI. Python on the other hand, was not initially designed as a data analysis tool. However, due to its general applicability, several high quality modules have been developed to handle and manipulate data, providing similar functionality to R. This has made Python extremely popular in the Data Science world and a real competitor to R (Muenchen, 2017). This makes Python the ideal choice for this project due to its versatility as it can perform everything necessary to assure its completion. There are quite a few different distributions of Python, each coming with their own pre-packaged modules. Since this project is fairly heavy on mathematical and data handling processes, Anaconda (Continuum Analytics, 2017) was the chosen distribution as it already contained a lot of the necessary Python modules with it, making it the logical choice.



Figure 2 - Graph showing the popularity of languages found in Data Science job listings. (Muenchen, 2017)

#### Integrated Development Environments (IDE)

IDEs can be fairly important when it comes to developing software. They package most of the fundamental tools that a developer is going to need into a single graphical user interface. This streamlines the process of writing code as it provides many quality of life improvements that can often be overlooked in terms of importance. Spyder (Spyder Developer Community, 2017) is a cross-platform IDE that is used for a lot of Python projects involving data processing and mathematics. Some of its main features that were utilised during the development process were things such as its integrated console. It has an interactive shell called IPython that provides support for data visualisation and also contains a flexible interpreter providing a quick and built-in way to run the program in development, making debugging and general program execution a lot faster. It provided instant, on-screen console output pertaining to the program as well as detailing bugs were they to occur.

Code completion also provides a huge boost to productivity, especially within object oriented projects. It helps by providing suggestions of relevant attributes pertaining to that specific class or object, as well as displaying information about that function’s parameters. This can become extremely useful when the project contains multiple instances of a complex class by making it easier to see how that class is set up.

A minor but important feature none the less, is syntax highlighting. Code can get difficult to read and understand when the size of the project begins to ramp up. Spyder contains settings that allow full customisation of syntax colours, providing easy ways to differentiate between various elements within code, such as functions, classes and data types. This greatly increases the overall clarity of the code by making its structure much more visually defined, making it easier to find errors stemming from specific areas of the code.

#### Version Control

With any project it is necessary to have some kind of centralised storage for all work created that it accessible from anywhere. Some storage systems are a lot more advanced and contain more effective forms of Version Control. A Version Control System (VCS) is a system that records all changes made to a set of files that are located within that storage system. It provides date and time specifications, as well as exact character changes that provides the user with the ability keep track of changes and recall older versions if necessary (Chacon and Straub, 2014). The benefits of using a VCS become more noticeable and apparent when doing a larger and more complex project. It becomes even more useful in a project such as this because of its iterative nature and use of Agile methodologies.

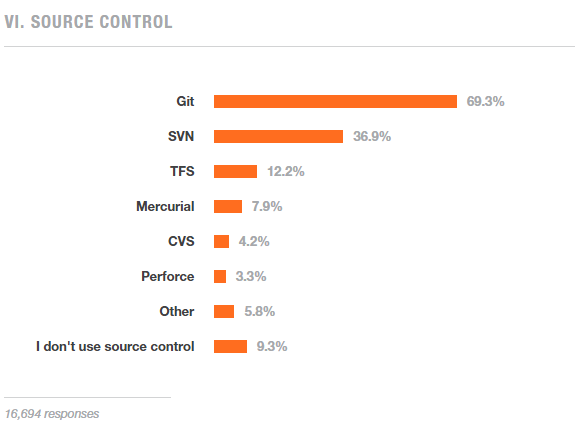


Figure 3 - Stackoverflow's developer survey - Most popular Version Control (Stackoverflow, 2015)

In 2015, Stackoverflow ran a survey for developers asking them a series of questions. One of the questions was their chosen VCS. Figure 2 shows the results from that survey which had almost seventeen thousand responses. The results show that roughly seventy percent of those developers used some version of Git (Hamano, 2017). This provided a solid indicator of which VCS is most popular and likely the most effective.

Git and an extension of that being GitHub (GitHub, 2017) was used for this project. University students get free access to a series of private online GitHub storage repositories that are ideal for a project like this as they have practically unlimited storage space. GitHub was used extensively throughout the project from the very beginning. Project work was often not done in the same place and usually on different systems. The work stored in the repository ranged from code used in development to diagrams and word documents as it was logical to keep everything in one place. The centralised nature of a system like GitHub provided ways to pull up-to-date versions of the project to any system and then push any changes that were made during that session back to the online repository. This made working on the project very flexible as it was extremely easy to access the repository from almost any location should it have been necessary. In conjunction with the push & pull system that GitHub incorporates, the amount of information that is available pertaining to each change made is extensive. This can help keep track of progress made during the project, as well as retaining old solutions that may have otherwise been discarded should they prove useful again.

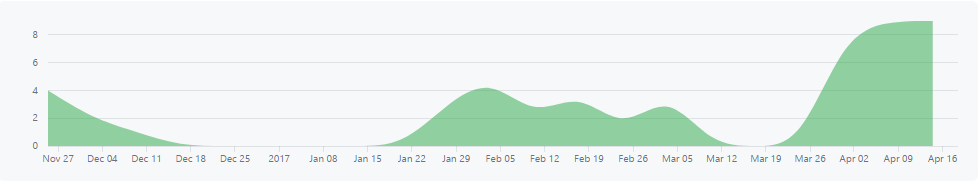


Figure 4 - Evidenced Use of GitHub throughout project: Commit distribution

#### Qt Designer

Creating the visual design and layout for Graphical User Interfaces (GUI) through pure coding practices can become an extremely time consuming process unless the exact look is solidified from the outset and not many changes are made. For projects where the visuals of the GUI aren’t the main aspect of the development, many developers will opt to use other software to accelerate and streamline the creation process. Qt (The Qt Company, 2017) itself is an application framework that runs on several different software and hardware platforms without requiring much code adaptation. Qt Designer is one of those applications that use this framework. It provides a toolkit containing useful features such as prebuilt widgets and layout structures to help users facilitate their application functionality in a timely manner (Blanchette and Summerfield, 2006). When it came designing and creating the layout of the GUI for this project, Qt Designer was used to handle almost all of the design tasks. It was extremely useful in the creation of the GUI wrapper as it saved a lot of time by automatically generating code for the layout designed within Qt Designer. Meaning that all the visuals of the application were instantly setup and all that needed to be done were tasks such as connecting inputs to specific functions and handling the outputs.

# Implementation

This section details the full development process for the artefact created during this project. It does not follow the traditional Software Development Life Cycle (SDLC) due to the Agile iterative methodology that has been adapted to this project. The development in this project is done over multiple iterations that repeat the typical stages found in the SDLC. The figure below roughly outlines the overall process used in the development of this artefact. Requirements were gathered before each iteration to figure out what was needing to be done. Suitable design was then generated based on those requirements which would then be implemented. Relevant testing to a varying degree would be performed at the end of each iteration to ensure that the result of that iteration was working as intended before starting the next. Some of the later iterations rely on and utilise functionality produced in the earlier iterations which makes it imperative that the dependencies found within the earlier iterations are fully complete. More in-depth testing will be done for the completion of the final artefact, once everything has been brought together. The project supervisor provided significant input when it came to gathering requirements and assessing the output from each iteration to ensure that it was time to begin the next iteration.

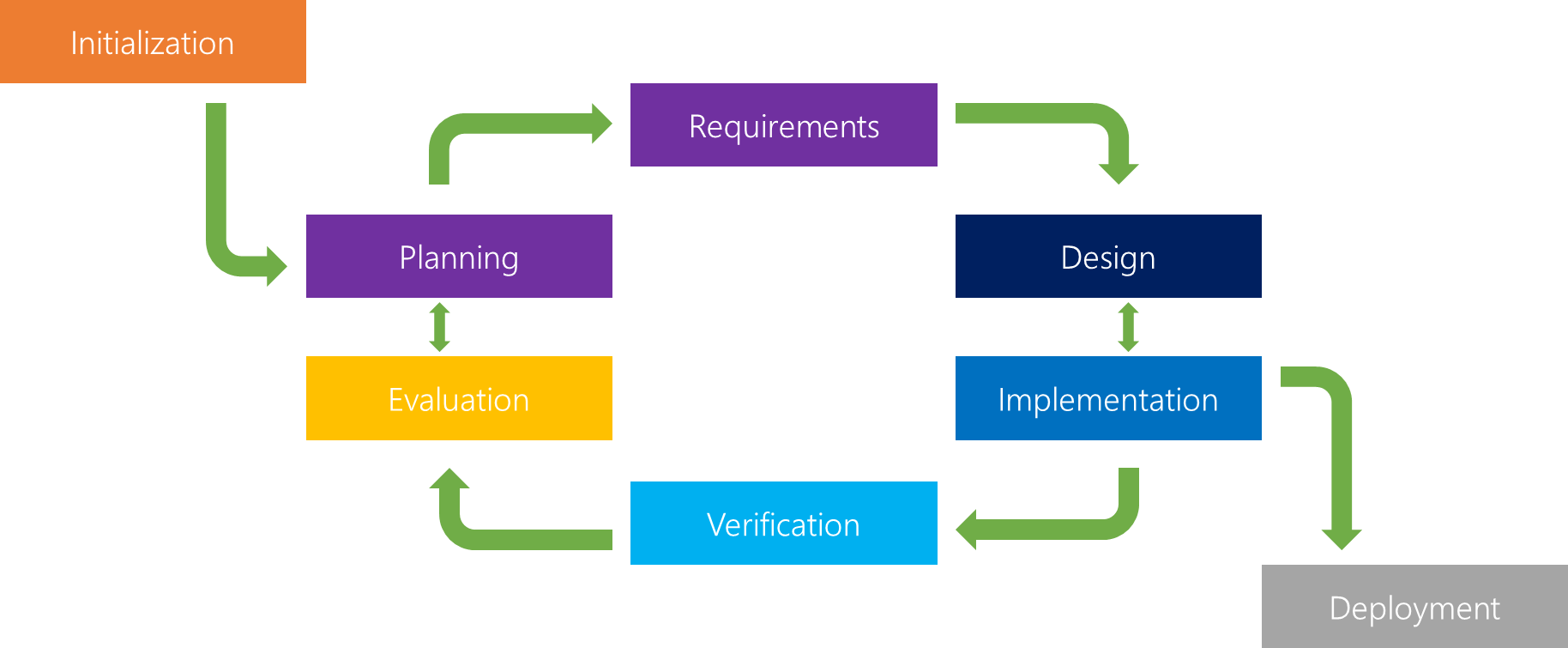


Figure 5 - Iterative version of SDLC. (Powell-Morse, 2016)

### Requirements

The project artefact as described in the Aims & Objectives section of the report is a portfolio management system that can provide descriptive statistics and visualisation of an asset’s performance as well as provide optimised asset allocation suggestions for a portfolio. Upon meeting the supervisor of the project, possible directions and solutions were discussed on what kind of analysis was to be done, as well as possible models that could be implemented as a means of portfolio optimisation. As a primary direction of the project, the Markowitz model as detailed by Harry Markowitz, was suggested as a model to start with, which could then be built and expanded on later if time prohibited it. This made logical sense as the Markowitz model is seen as one the most intuitive models as well as being considered the foundation of portfolio theory as it’s known today. Following the details of the Markowitz model is ideal as a starting solution to the project aim as it provides a means of asset analysis as well as portfolio optimisation techniques. There are various areas of the Markowitz model to implement which have been separated into steps and divided between development iterations. Each step generally requires the completion of the previous step in order to be implemented. Requirements that are specific to the implementation of certain step are discussed and explained in greater detail at the beginning of that iteration. The overall development focuses on the implementation of these stages of the Markowitz model to achieve the project aim.

## Retrieving and Handling Data

As per the requirements of the project, the acquisition of relevant, up-to-date financial data is of utmost importance. After discussion with the project supervisor and further research into previous implementations of the Markowitz model, it was found that historical data pertaining to an asset’s price is often used as a means to generate estimations for input values that are used for analysis and optimisation later on in the Markowitz Model. Furthermore, historical data is often used by investors to gather descriptive statistics that can provide guidance and indication as to what stocks are good investments. As the project specifically focuses on the use of stocks as the chosen asset class, appropriate stock price data will need to be acquired and stored appropriately.

### Design

During the design phase different solutions were considered about how best to approach the process of retrieving data. The initial idea was that the data retrieval and storage process was going to be ran in conjunction and separated to the final portfolio analysis tool. As the data had to be kept up-to-date to ensure relevancy, the idea of automatic retrieval on a daily basis seemed appealing because new stock pricing data becomes available every day and it seemed relatively hands-off for the user once it is set up. This reinforced the idea of saving the daily retrieved data to a database hosted on a cloud server. Local storage would not suffice as it lacks accessibility for other systems as well as the continuous need for the user’s system to be available so data can be stored. The crux of continuous availability also applied to the process of running the automatic retrieval program periodically. It made sense to host the automatic retrieval system on a cloud server as well, so it could be constantly active. The use of a database in conjunction with the ability to pull directly from the data source also acts as a backup for if one of them were to become unavailable. FIGURE X shows a conceptual design of the automatic retrieval system in relation to the online financial data source and the created database. Considerations would have to be made about services were available for cloud hosting, for both: the retrieval system and the database.

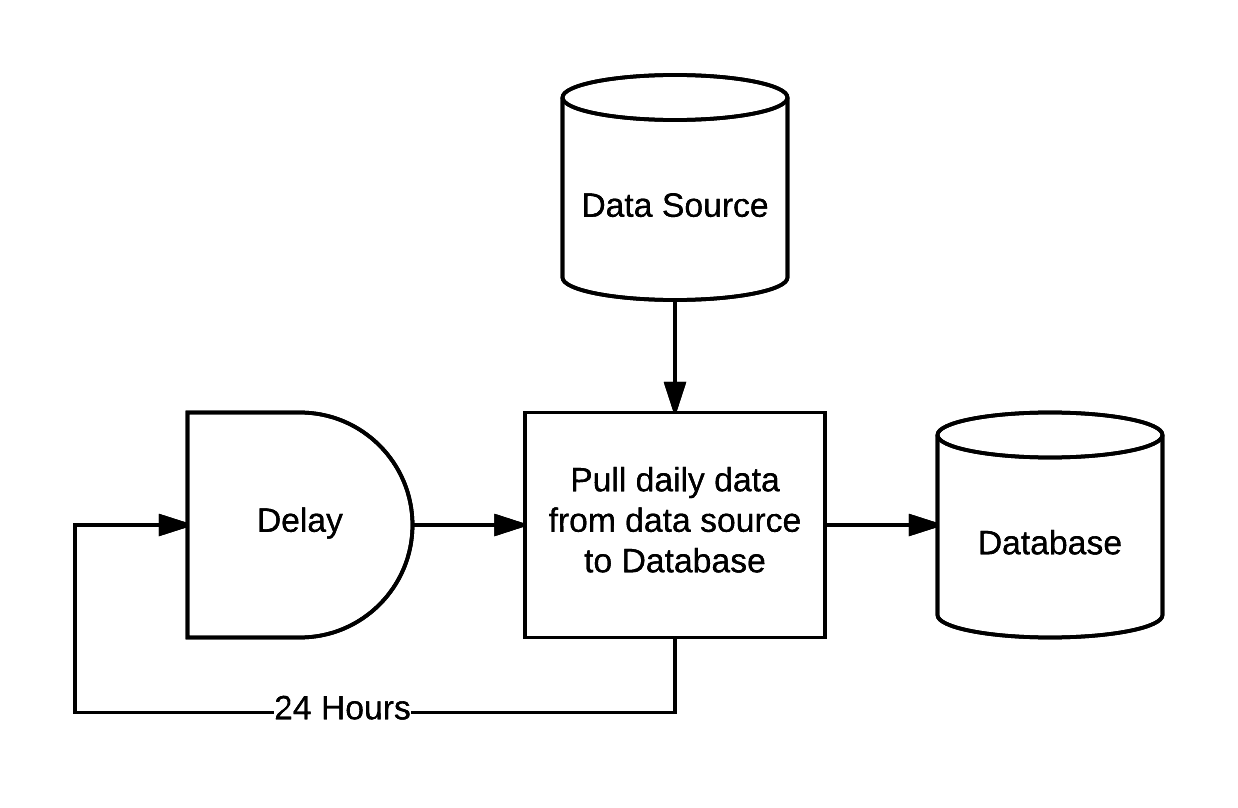


Figure 6 - Flowchart detailing process of Data Retrieval

### Implementation

Going into the implementation phase, implementation goals can be generated for this iteration based off of the initial requirements of the project and the ideas and conclusions arrived at during the design phase. The implementation goals for this particular iteration can be summarised as:

* Find a suitable online source for daily stock pricing data
* Create a database to store the data and find a suitable cloud service to host it.
* Create a Python script that can be run every day to retrieve the pricing data for that particular day and store it in the database, as well as a service to host it.
* Create separate script to then be able to access that data for future use in the artefact.

#### Quandl

When looking for what to use as a source of stock pricing data, there are several different things to consider. The most important things are the legitimacy and accuracy of the data, especially when applied in practice. During development, there isn’t as much of an issue due to their being no real world consequences of using inaccurate data when testing the application. However, it makes sense to ensure the quality of the data at the beginning anyway, even before deployment. Some of the most commonly used online sources of data for individuals and small companies are websites such as Yahoo! Finance (Yahoo!, 2017) or Quandl (Quandl, 2017). Both of these provide free access to a huge amount of different datasets that are updated every day for almost every different asset class. Both websites allow users to easily retrieve data from the desired dataset using the website’s own API. Quandl generally stood out more compared to Yahoo! Finance due to the detailed documentation and instructions they provided for using their API, as well as direct code examples of how to implement it using a number of different languages, most important of which was Python. Quandl have their own Python module that can be easily downloaded and imported, giving access to the functions necessary for making the API calls. The API call function takes several parameters to specify details about the call: name of the dataset, ticker for the stock and start/end date. An example demonstrating the simplicity of calling the Quandl API and storing the data in a variable using Python as per the Quandl Documentation (Quandl, 2017):

data = quandl.get\_table('WIKI/PRICES')

#### MySQL Database

The data storage solution that was chosen for this project involved creating a MySQL database that would be hosted on a cloud server. MySQL is the world’s most popular open-source relational database management system (MySQL, 2017). There are several different services that can be used to create databases, but MySQL seemed the most appropriate due to having previous experience using it, its prestige and the wide range of services that provided cloud hosted instances of it. One of the most popular cloud based services providers is Amazon Web Services (Amazon, 2017). They provide various cloud-based services including instances of MySQL databases and virtual machines making them an obvious choice for hosting both the database and the automatic retrieval system (Butler, 2014). They have the Amazon RDS service for relational databases and the Amazon EC2 service for instances of virtual machines. Both of these services come under Amazon Web Services’ (AWS) free tier providing a certain amount of usage per month, making the use of AWS ideal for a project of this size.

Once both of the AWS cloud instances had been established, the database would then have to be configured so it contained the correct tables for storing the relevant data. Instead of using dozens of SQL queries to set it up, MySQL have their own visual database design tool called MySQL Workbench (Oracle Corporation, 2017) which combines the design, development and management aspects of creating the database into one IDE. This was used throughout the process of creating the database as it provided a centralised way for to be accomplished. This made it a lot easier to create and configure essential parts of the database such as the schema and tables including the handling of connection permissions. It also provided a means of visualising the data stored within the tables making it much easier to test and ensure that the data was being retrieved and handled properly.

#### Retrieval System

Once the database had been set up and a suitable data source had been selected, the next step was to create the Python script that would pull data from the data source and then push it into the database. There are many different pieces of information that can be retrieved pertaining to each stock, but there is only one which is going to be of use to the project, that is the “Closing” price. A stock’s closing price is the final price that it ends on after a day of trading, giving the most recent price valuation until trading begins again the next day. However, closing price as it stands is not particularly useful for analysis due to certain corporate actions such as stock splits, dividends and rights offerings that can appear as bizarre changes in price (Investopedia, 2017). Fortunately, Quandl also provides what is called the “Adjusted” closing price which is the same as the closing price except that the price is amended to include the effects of any corporate actions that may have taken place. It is the main type of historical stock data that is used to calculate historical returns which are used for analysis and optimisations such as those in the Markowitz model (Investopedia, 2017). Initially a list of 15 or so stocks were picked out that were mainly to do with large tech firms. However, after research into the Markowitz model and the effectiveness of diversification, this was changed to include roughly 25+ different stocks. Solnik’s (1995) research into diversification provided a good visualisation of how effective diversification can be. Visualised in FIGURE X, it roughly shows that for US stock markets in which this project takes place, the bulk effect of diversification caps out at around 20 or so stocks. Instead of just increasing the number of stocks, the selection was also made more varied by including stocks of companies outside the tech sector to further increase the potential effectiveness of diversification.

The process of retrieving data was done using two similar but separate Python scripts. The first script would retrieve the bulk of the pricing data from a desired starting point. A year’s worth of data was chosen for this project. This would act as an initialisation script that would ideally only need to be run one time upon setting up the database. The initialisation script loops through the list of selected stock names, calling the Quandl API for each one retrieving all the data available for that stock. The adjusted closing price column of each stock is renamed to the name of the stock and then appended to an empty container. A Pandas dataframe is used to store and handle all of the data, as it is relatively easy to use while having high performance and a bunch of analytical functions built into it, including one that pushes it to an SQL database (Pandas, 2016). Once the adjusted close price of each stock had been appended this dataframe was pushed to the database that had been setup which would now contain the daily adjusted closing price of each stock in a time series format.

Figure 7 - Code example of retrieving and compiling each stock's adjusted close price using Quandl’s API

allData **=** pd**.**DataFrame**([])**

**for** stock **in** availableStockList**:**

fullHolder **=** pd**.**DataFrame**(**quandl**.**get**(**"WIKI/"**+**stock**,** trim\_start **=** start**,** authtoken **=** token**))**

closeHolder **=** pd**.**DataFrame**(**fullHolder**[**'Adj. Close'**])**

closeHolder**.**columns **=** **[**stock**]**

**if** allData**.**empty**:**

allData **=** closeHolder

**else:**

allData **=** allData**.**join**(**closeHolder**,** how**=**'outer'**)**

The second Python script was created to automate the retrieval of new data on a daily basis, instead of retrieving a year’s worth of data every time. It is set up the same way as the first script that was created, except that it would only retrieve the data for the day that it has been executed on, instead of the whole time period. It then appends the data for that day to the end of the MySQL database. To automate this process, the virtual machine server that was set up using Amazon Web Services was configured to contain a virtual Linux system that came with Python already. The reason for choosing a Linux system is that Linux comes with a utility called Cron that allows the user to schedule commands or scripts to be run automatically at a certain time and date (HostGator, 2017). The Python script that was created to be run automatically was then uploaded onto the virtual server, and then using Cron, was scheduled to be run every day at 11:30pm. To do this, a Cron job was setup using the Crontab interface which can be accessed through the Linux terminal. Cron jobs take time and date parameters followed by the action or script that you want it to run at this time. 23:30pm was chosen because financial data sources like Quandl can take some time to update the current day’s data, so a late evening time was chosen to account for the possibility that it may take longer. Once that had been set up, the database will be automatically updated every day with each stock’s adjusted closing price.



Figure 8 – Setting up the Cron job.

### Testing

The retrieval system acts as a separate system to the asset allocation system, so separate testing was done for this on it’s own. As the future parts of the artefact require historical stock pricing data, the most important outcome of this iteration that must be achieved; is a system that retrieves adjusted close price data in the correct format which can then be used for later iterations. The functionality that needs to be verified can be split into two separate components:

* Ensuring that data is retrieved from Quandl for each stock and that it is in the desired format.
* Confirming that the database is being populated with data.

As the testing required for these two parts of the system is a fairly simple matter of verification, a simple black-box test was performed to check that the results from system match the expected results. APPENDIX FIGURE is a table created to facilitate the results of the black-box testing, providing description of the function being tested, the expected result and the actual result.

## Stock Analysis and Visualisation

Now that pricing data has been collected pertaining to each stock in the list, the next step is to provide descriptive statistics for both the user, and to use later on as an input for the Markowitz model. This will form the beginning of what will eventually become the full Python application. Some of the main statistics of a stock that an investor would look to assess its performance are its expected return and the risk associated with it. A common metric used to describe a stock’s expected return is the mean of its periodic returns. This project is using one year’s worth of data which is a relatively small amount, so a stocks returns are going to be recorded daily. By assuming they are normally distributed, the risk of an asset can also be derived using daily returns. This is done by calculating the standard deviation or variance of the returns. In this iteration, any visual output pertaining to individual stocks will also be done here.

### Design

Due to the objectives of this iteration being fairly linear, there wasn’t that much pre-implementation design that was done. As this iteration focuses on analysing, calculating and displaying statistics of stocks individually, it made sense to apply some of Python’s object-oriented capabilities and treat each stock as its own object through the creation of a Stock class. This would provide the ability to centralise every piece of information that is relevant to that particular stock. This is useful for a number of reasons: it makes the storage and handling of each piece of data for every stock a lot less overwhelming and messy, while making them a lot easier to access and call. The design idea for the stock class can be seen in FIGURE X through the use of a class diagram. It shows the initial conceptual idea of what attributes and functions would contained within that class.

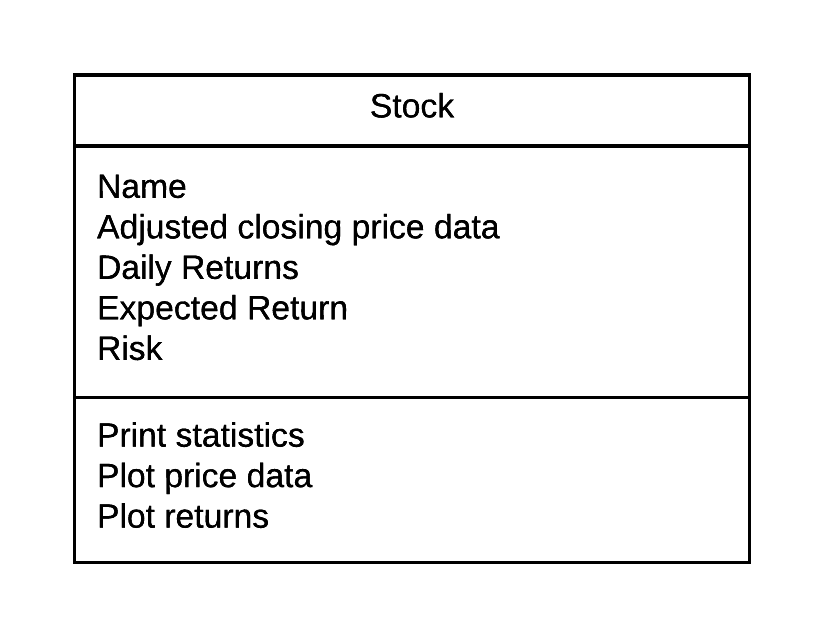


Figure 9 - Conceptual stock class design

### Implementation

* Create a stock class to consolidate all data and methods associated with individual stocks.
* Calculate and store daily returns of each stock using its adjusted closing price.
* Calculate estimations for a stock’s risk and expected return by calculating the mean and variance of the daily returns.
* Provide appropriate visualisation of data and statistics.

Upon running the application, all the adjusted closing price data is retrieved. The constructor for the stock class that has been created requires a valid name only. The list of stock names is then iterated through and a Stock object is created for each of them. When they are initialised they are given attributes for the name of the stock and the appropriate closing price data that goes with it. This data is then used to calculate other pieces of information which form the remaining attributes of the object.

#### Daily Returns

Using the pricing data that was retrieved, a series of historical returns can now calculated for each stock. The rate of return of an asset can be described as the percentage change in value between two time periods. As the data that has been retrieved in this project is the price valuation of the stock at the end of each trading day, the rate of return is used to describe the change in stock price between each trading day. In this project, a stock’s rate of return denoted by is the current day’s price minus the previous day’s price divided by the previous day’s price. This gives the percentage change in price over time period .

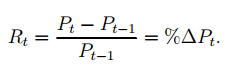


Figure 10 - Rate of Return

The array of daily closing price data is iterated through to calculate the rate of return for each day. Instead of using arithmetic rates of return, what is often used instead is the log returns. Returns are usually assumed to be normally distributed, but when compounding arithmetic returns, it becomes unsymmetrical (Morgan, 2013). To offset this error, log returns are used by taking the natural log of instead. These log returns are stored as an attribute of the stock object in a NumPy array. NumPy is a scientific computing module for Python that contains a more sophisticated array object that can perform a variety of mathematical functions (NumPy, 2017).

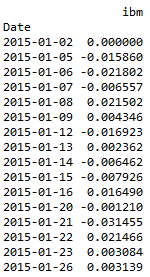


Figure 11 - Generated time series of returns

#### Expected Return and Risk

The expected return and risk of investment into a stock can be calculated using the historical returns of said stock. The expected return of an asset can be calculated by taking the arithmetic mean of the returns. In this case, denotes the expected return of the stock, denotes each of the daily returns and denotes the total number of daily returns there are.

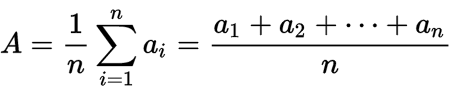


Figure 12 - Expected Return for a stock

The variance or standard deviation of the daily returns is a commonly used metric for describing the risk associated with investment in an asset. The variance of a stock is denoted by with representing each daily return. N denotes the number of daily returns and denotes the arithmetic mean of those returns. To achieve the standard deviation, one just has to square-root the variance.

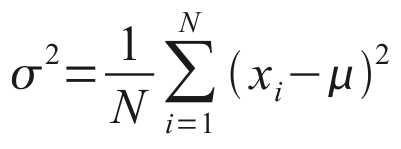


Figure 13 - Risk for a stock

Both of these operations were fairly simple to do through the built-in functionality of NumPy. NumPy contains pre-defined methods for calculating the arithmetic mean, variance and standard deviation. All of these values are then made attributes of the relevant stock object, making them a lot easier to access again for analysis and visualisation.

mean **=** numpy**.**mean**(**self**.**returns**)**

variance **=** numpy**.**var**(**self**.**returns**)**

standardDeviation = numpy**.**std**(**self**.**returns**)**

Figure 14 - Example use of NumPy's built-in functions

#### Visualisation

Visualisation of assets and their properties is an important part of the overall system as it provides a lot of information which can help the user make investment decisions. To create graphical visualisations of each stock’s pricing data and returns, the Matplotlib module was used. Matplotlib is a 2D plotting library which provides easy plotting of data due to its compatibility with a lot of data types such as time-series data which is used here. It has a lot of customisation potential and has good synergy with other modules used such as Pandas, because it handles the Pandas dataframe structure well, allowing multiple columns of data to be plot against the dataframe’s index in one function. It also provides an interactive toolbar which can be used to zoom in on or pan around the canvas in which the data that has been plotted (Droettboom, 2017). This is especially useful in this project as the time period for the data is in days which provides a lot of small points on the graphs which can be hard to see. Visual observations were used during development to verify that the functionality of the graphical visualisations were working correctly. Examples of these graphs are shown below:

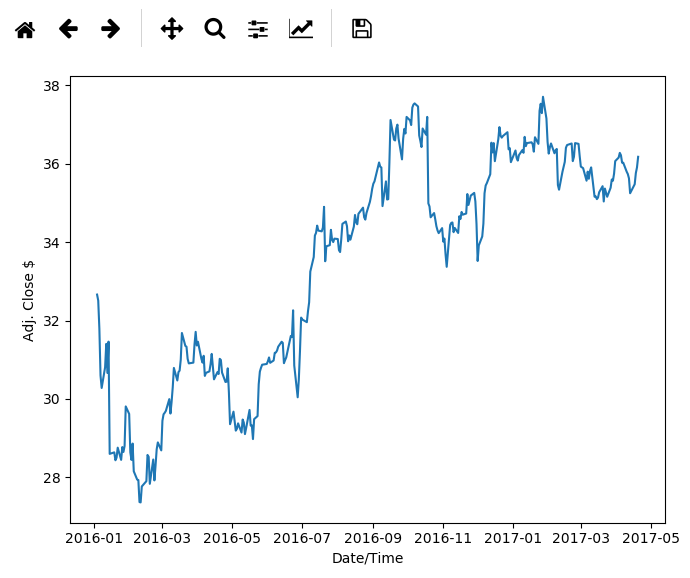


Figure 15 – Example of generated adjusted closing price graph

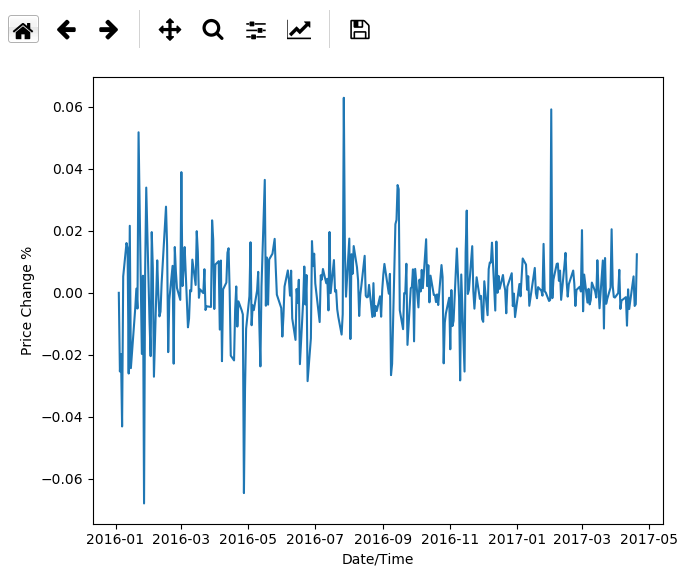


Figure 16 – Example of generated daily returns graph

## Portfolio Creation & Optimisation

This iteration focused on the implementation of the Markowitz portfolio and optimisation model. A portfolio is a grouping of financial assets, or more specifically in this case; stocks. Each portfolio can contain any number of stocks that each have their own weighting. Given a budget that is going to be used to invest, the weighting of an asset in a portfolio corresponds to how much of that budget is going to be invested in that particular stock. The process of choosing the weighting for each asset is called Asset Allocation, which forms the backbone of the problem this project tries to solve using Markowitz’s portfolio theory. Like each asset, each portfolio will have its own expected return and risk that are calculated using methods that incorporate an asset’s weighting as well as individual risk and return values.

### Design

Similar to what was done with each individual stock, creating a new portfolio class meant that every piece of information to do with that specific portfolio would be centralised as an attribute. Each portfolio object could then have its own method for each type of optimisation that could be performed, be it minimum variance, or maximum return. The basis of implementation for calculating each portfolio statistic necessary for performing optimisation is the work laid out by Markowitz himself. Due to the underlying theory being established already, minimal design was needed for this iteration except for the portfolio class structure. The arguments required to initialise the portfolio class were a list of stock names that would be used as the collection of assets that form a portfolio. As each stock object is already initialised when upon retrieval of the stock data, only the name of the stock would need to be passed as an argument to the portfolio object constructor instead of the stock object itself. This makes it so the data needed from each stock object can then be accessed by reference. Each portfolio object would then have its own list of attributes pertaining to relevant statistics that may need to be recalled later. A conceptual example of what the portfolio class could like using a class diagram:



Figure 17 - Portfolio class diagram

### Implementation

* Create a portfolio class to consolidate information and all functions that specifically affect that portfolio.
* Calculate risk and return for a portfolio
* Calculate and plot the Efficient Frontier
* Perform various optimisations of portfolio weights:
  + Minimum Variance
  + Maximum Return
  + Specified Risk Tolerance

To construct a portfolio object, a list of stock names needs to be provided to the constructor. Upon construction, equal weightings which sum to 1 are given to each of stocks in the portfolio. A new dataframe is created to store the compiled returns of each stock included in the portfolio. This will be used to estimate covariance between the assets which is necessary input to the Markowitz optimisation model.

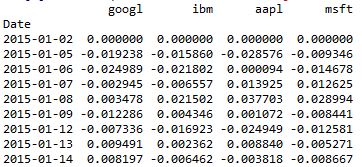


Figure 18 - Compiled returns for stocks in portfolio

#### Expected Return for a Portfolio

The mathematical model of Markowitz’s Portfolio Theory estimates the expected return of portfolio in an intuitive way. The Markowitz model calculates the expected return using the weight and return of each asset in a portfolio. In this case, the asset’s expected return is estimated using arithmetic mean of its historical returns. The expected return of a portfolio is calculated using the following, where and denote the asset’s weight and expected return, respectively.

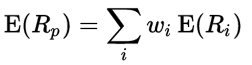


Figure 19 - Expected Return of Portfolio (standard notation)

As an example, the expected return of a portfolio containing two assets, A and B can be described as:



Figure 20 - 2 asset portfolio expected return

However, when the number of assets gets a lot higher, the length of the formula can become excessive. It can be described in a much neater and concise way using matrix notation instead of standard. This also makes it a lot easier to implement using Python as all the asset values can be stored in matrices, allowing the use of matrix functions such as those from NumPy to perform the necessary calculations. For a portfolio that contains three assets; A, B and C, the expected return can be described using the following, where and R are the weight and return of each asset.

Figure 21 - Expected Return of Portfolio (matrix notation)

This is also described as the dot product of the stock weights and the corresponding returns. Upon portfolio creation, the weights and expected return of each asset are already contained within an array, making this procedure simple to do using NumPy’s dot product method.

#### Risk for a Portfolio

If we consider the same three asset portfolio, the variance or risk of that portfolio p can be described by the following, where and Σ denote the weights of each asset and the covariance between each asset. The covariance is a measure of how much the returns of each asset move in tandem with one another (Investopedia, 2017). For example, the covariance of asset A and B is denoted by . Each asset will have its own covariance with every other asset, which make up a covariance matrix.

Figure 22 - Variance of a portfolio

The covariance matrix for each portfolio was calculated using one of NumPy’s methods as seen below, to which the required input argument was a list of returns such as those in PORTFOLIO RETURN FIGURE. The covariance matrix for that particular portfolio was then stored as an attribute of the portfolio object.

self**.**covarianceMatrix **=** numpy**.**cov**(**self**.**returns**)**

Figure 23 - NumPy method for covariance



Figure 24 - 4 asset covariance matrix

#### Efficient Frontier

There are almost a limitless amount of different weighting combinations that sum to 1 that can be given to the assets in the portfolio. Each of these different weightings will produce a different risk and expected return for the portfolio. If you were to plot the risk and expected return for each of these different portfolios, this would make up what is called the “feasible set” or “feasible region”. Markowitz expanded on this idea to say that for each level of risk that is possible for a feasible set, there is what he described as an “efficient portfolio” meaning one that has the highest possible expected return for that specific amount of risk. These efficient portfolios that contain only risky assets form an upper bound called the “efficient frontier”, which are the most efficient portfolios for a given risk level (Elton and Gruber, 1997). Finding the combination of asset weights that form these efficient portfolios forms the optimisation problem.

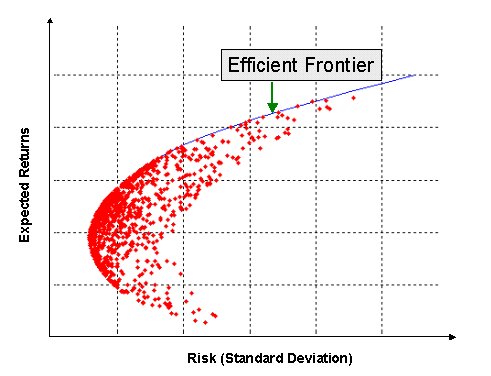


Figure 25 - Efficient Frontier (Finance Train, 2017)

One way of constructing the efficient frontier is to model it as a minimisation problem, with the idea being to minimise the amount of risk for a series of given expected returns. This forms the objective function which is the function that needs to be minimised, in this case the risk or variance of the portfolio.

Any optimisation problem will also have a series of constraints that the optimiser will have to adhere to. The constraints in this scenario are that the asset weights must sum to 1 because we cannot spend more than our total budget and that the expected return of the portfolio must equal one of the given expected returns denoted by μ. For this particular case, shorting of assets isn’t applied so the weight of all assets must be greater than 0.

The whole minimisation problem can be described as:

Minimise:

Subject to:

This is what is described as a quadratic program, which is an optimisation problem that contains a quadratic objective function and linear constraints. To do this, a Python module called CVXOPT was used. It is a software package for convex optimisation that can be used to model minimisation problems such as these. It provides an easy creation process for arguments as well having good performance for solving the problems (Andersen and Vandenberghe, 2016). To use the CVXOPT module, the problem has to be in the correct quadratic form accounting for the use of lagrange multipliers according to the documentation. The inequality constraints are described by and the equality constraints are described by .

Minimise:

Subject to:

When first calculating the efficient frontier using Python, an array of expected return values is produced to act as the return constraint that the risk is going to be minimised for. Roughly 200 of them are generated and distributed appropriately in order to best resemble the portfolios that make up the efficient frontier. The constraints that were stated earlier are transformed into matrix form for input into the optimiser. All 200 of the desired expected returns are iterated through and use as a constraint in the optimiser to return 200 different asset weightings. These 200 portfolios are efficient as they provide the lowest amount of risk for their specified returns.

Figure 26 - Efficient Frontier using CVXOPT

**def** efficientFrontier**(**self**):**

n **=** len**(**self**.**returns**)**

returns **=** numpy**.**asmatrix**(**self**.**returns**)**

N **=** 200

mus **=** **[**10**\*\*(**5.0 **\*** t**/**N **-** 1.0**)** **for** t **in** range**(**N**)]**

P **=** self**.**covarianceMatrix

q **=** cv**.**matrix**(**numpy**.**mean**(**returns**,** axis **=** 1**))**

G **=** **-**cv**.**matrix**(**numpy**.**eye**(**n**))**

h **=** cv**.**matrix**(**0.0**,** **(**n**,**1**))**

A **=** cv**.**matrix**(**1.0**,** **(**1**,**n**))**

b **=** cv**.**matrix**(**1.0**)**

portfolios **=** **[**cv**.**solvers**.**qp**(**mu**\***P**,** **-**q**,** G**,** h**,** A**,** b**)[**'x'**]** **for** mu **in** mus**]**

returns **=** **[**blas**.**dot**(**q**,** x**)** **for** x **in** portfolios**]**

risks **=** **[**numpy**.**sqrt**(**blas**.**dot**(**x**,** P**\***x**))** **for** x **in** portfolios**]**

**return** risks**,** returns

#### Minimum Variance, Maximum Return and Risk Preference

# References

# Appendix

|  |  |  |
| --- | --- | --- |
| Actual Result | The result shows the first three stocks in the dataframe. It contains the date index and the column name as requested meaning that it has passed the test. | This image shows the table featured on the database that was set up, containing all the necessary columns, showing that it is working correctly. |
| Expected Result | A dataframe containing a column of closing price data for each stock with a date index and name of each stock as column title. | The MySQL database should contain the data compiled from the API call. It should have appropriate headings |
| Description | Verification of data being retrieved and also being in the correct format. | Verification of the database being populated appropriately. |