Comp251 Assignment 4

Ahmed Hegazi
260725697.

Q2

(i) $T(n) = 25\,T\left(\frac{n}{5}\right) + n.$

$a = 25 \quad b = 5 \quad k = \log_5(25).$

$n^{(\log_5 25 - \log_5(25) - 1)} = n.$

$F(n) = (n^{k-\epsilon})$

$\epsilon = \log_5(25) - 1.$ (case 1)

Then $T(n) = \Theta(n^k) = \Theta(n^{\log_5 25}) = \Theta(n^2)$

(ii) $T(n) = 2T\left(\frac{n}{3}\right) + n\log n.$

$a = 2 \quad b = 3 \quad k = \log_3(2)$

$F(n) = n\log n = \log(n) \times n^{\log_3(2) + (1 - \log_3(2))}$

For big(n)

$F(n) = \alpha \cdot n^{k+\epsilon}$ First condition ✓

Second condition. $2F\left(\frac{n}{3}\right) = \frac{2n}{3}\log\left(\frac{n}{3}\right) \leq c(n\log(n))$

For $c = \frac{5}{6}$.

So $T(n) = \Theta(n\log n)$

(Case 3)

$(iii)\ T(n) = T(\frac{3}{4}n) + 1$ Case 2

$a = 1 \cdot b = \frac{3}{4} \cdot k = \log_{(\frac{3}{4})}(1) = 0$

$F(n) = 1 = n^k \log^0(n)$

$f(n) = \theta(n \log(n))$

then $T(n) = \theta(n \log(n))$

(iv) $T(n) = 7 \cdot T(\frac{n}{3}) + n^3$ Case 3

$a = 7 \quad b = 3 \cdot k = \log_3(7)$

$f(n) = n^3 = \Omega(n \log_3(7) \cdot (3 - \log_3(7)))$

also; $7(\frac{n}{3})^3 = \frac{7}{9} n^3 \leq \frac{15}{18} n^3$

then $T(n) = \theta(n^3)$

Q2 $T_A(n) = 7\,T(\frac{n}{2}) + n^2$

$a = 7, \; b = 2, \; k = \log_2(7)$

$F(n) = n^2 = n^{\log_2(7)} \cdot (\log_2(7) - 2)$

then $f(n) = \theta(n^{\log_2(7)} - \log_2(7) - 2))$

then $T(n) = \theta(n^{\log_2(7)})$

$T_B(n) = \alpha\, T_B(\frac{n}{4}) + n^2 \quad a = \alpha \quad b = 4$

$F(n) = n^2$

$$f(n) = n^2 \cdot n^{\log_4(\alpha - \log_4(\alpha) - 2)}$$

$$f(n) = n^{k+\epsilon} \qquad \epsilon = \log_4((\alpha) - 2)$$

$\epsilon > 0$ then $\log_4(\alpha) - 2 > 0$.

$\log_4(\alpha) > 2 \Rightarrow \alpha > 16$.

Then if $\alpha > 16$.

$$f(n) = \theta(n^{(\log_4(\alpha) - \log_4(\alpha) - 2)})$$

$$T(n) = \theta(n^{\log_4(\alpha)}).$$

$$\Rightarrow T_B(n) < T_A(n).$$

$$n^{\log_4(\alpha)} < n^{\log_2(7)}.$$

$$\alpha < 4^{(n^{\log_2(7)})}.$$

$$\alpha <= 49.$$

$$\boxed{\text{Max } \alpha = 48}$$

(v) $T(n) = T(\frac{n}{2}) + n(2-\cos(n))$

$a=1 \quad b=2 \quad k = \log_2(1) = 0$

$f(n) = n(2-\cos n) \geq n$

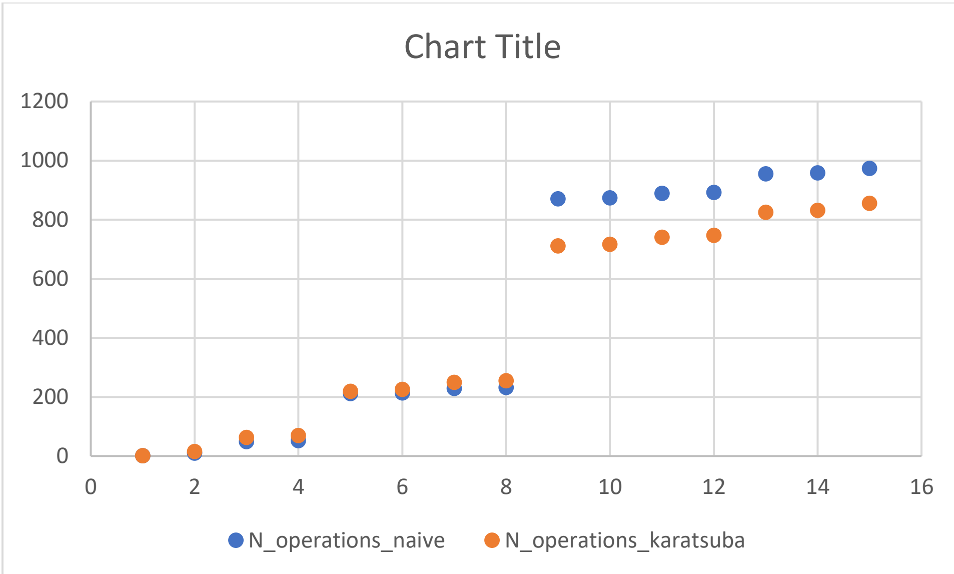
$n = n^{k+1}$

$f(n) = \Omega(n^{k+1}) \quad \epsilon = 1$

$= \frac{n}{2}(2-\cos(\frac{n}{2})) \geq \frac{3}{4}(2-\cos(n))$

$T(n) = \Theta(n(2-\cos(n)))$

Case 3



As the results show the naïve algorithm has a smaller time complexity for smaller values since the number of brute force additions subtractions and multiplications is smaller than the number that is performed by the Karatsuba algorithm. but as the size of the numbers increase the number of recursions done by Karatsuba is smaller than that of the naïve algorithm and so Karatsuba algorithm has a smaller time finding the answer. In conclusion the Karatsuba algorithm performs better than the naïve algorithm for sufficiently large size (n).