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Probability and Applied Statistics Project 2 Formula Sheet

Definition 3.11

This finds Poisson probability distribution of a set of data at specific instances:

$$p(y) = \frac{\lambda^y}{y!}e^{-\lambda}, \quad y = 0, 1, 2, \ldots, \lambda > 0$$

Theorem 3.11

This finds the mean and variance of a random variable with the Poisson distribution:

$$\mu = \mathrm{E(Y)} = \lambda$$

$$\sigma^2 = \mathrm{V(Y)} = \lambda$$

Theorem 3.14

This theorem, known as Tchebysheff's Theorem, is used to determine a lower bound for the probability that the random variable Y of interest falls in an interval $\mu \pm \mathrm{k}\sigma$:

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \;\; or \;\; P(|Y - \mu| \geq k\sigma \leq 1) - \frac{1}{k^2}$$

Definition 4.1

Let Y denote any random variable. The distribution function of Y is denoted as such:

$$\mathrm{F(y) = P(Y \leq y)\ for\ } -\infty < \mathrm{y} < \infty$$

Definition 4.3

The distribution function for a continuous random variable is denoted by F(y) as such:

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

Theorem 4.3

In regard to Y, a random variable, potentially having a density function along with a < b, the probability to find Y is as follows:

$$P(a \leq Y \leq b) = \int_a^b f(y)dy$$

Definition 4.5

The expected value of a continuous random variable Y is:

$$E(Y) = \int_{-\infty}^{\infty} yf(y)dy$$

Theorem 4.4

If g(y) were to be a function of Y, the equation is as follows:

$$E[g(Y)] = \int_{-\infty}^{\infty} g(y)f(y)dy$$

Definition 4.8

A random variable Y is said to have a normal *probability distribution* if the requirements are met and take the form of:

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/(2\sigma^2)}, -\infty < y < \infty.$$

Theorem 4.7

When Y is a normally distributed random variable with the parameters μ and σ, then:

$$\mathrm{E(Y)} = \mu \text{ and } \mathrm{V(Y)} = \sigma^2$$

Definition 4.9

When random variable Y has parameters > 0 and β > 0, the gamma distribution is:

$$\mathrm{f(y)} = \begin{cases} \dfrac{\mathrm{y}^{\alpha\text{-}1} e^{\text{-y}/\beta}}{\beta^{\alpha}\Gamma(\alpha)}, & 0 \leq \mathrm{y} < \infty \\ 0, & \text{elsewhere} \end{cases}$$

where

$$\Gamma(\alpha) = \int_{-\infty}^{\infty} \mathrm{y}^{\alpha\text{-}1}\, e^{\text{-y}}\mathrm{dy}$$

Theorem 4.8

If Y has a gamma distribution with parameters α and β, then:

$$\mu = \mathrm{E(Y)} = \alpha\beta \text{ and } \sigma^2 = \mathrm{V(Y)} = \alpha\beta^2$$

Definition 4.11

When random variable Y has parameter β > 0, the exponential distribution is:

$$\mathrm{f(y)} = \begin{cases} \dfrac{1}{\beta} e^{\text{-y}/\beta}, & 0 \leq \mathrm{y} < \infty \\ 0, & \text{elsewhere} \end{cases}$$

Theorem 4.8

If Y has an exponential distribution with parameter β, then:

$$\mu = \mathrm{E(Y)} = \beta \text{ and } \sigma^2 = \mathrm{V(Y)} = \beta^2$$

Definition 4.12

The following is when a random variable Y is capable of being put through Beta Probability Distribution (indicated by its parameters):

$$f(y) = \begin{cases} \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)}, & 0 \leq y \leq 1, \\ 0, & elsewhere, \end{cases} \quad \text{where } B(\alpha,\beta) = \int_0^1 y^{\alpha-1}(1-y)^{\beta-1}dy = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

Theorem 4.11

If Y is a beta-distributed random variable with parameters $\alpha > 0$ and $\beta > 0$, then:

$$\mu = \mathrm{E(Y)} = \frac{\alpha}{\alpha+\beta} \text{ and } \sigma^2 = \mathrm{V(Y)} = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Theorem 4.13

Let Y be a random variable with the finite mean μ and variance σ^2. Then, for any $k > 0$:

$$\mathrm{P}(|\mathrm{Y} - \mu| < \mathrm{k}\sigma) \geq 1 - \frac{1}{\mathrm{k}^2} \text{ or } \mathrm{P}(|\mathrm{Y} - \mu| \geq \mathrm{k}\sigma) \leq \frac{1}{\mathrm{k}^2}$$

Definition 5.1

Let Y_1 and Y_2 be discrete random variables. The *joint* (or bivariate) *probability function* for Y_1 and Y_2 is given by:

$$\mathrm{p}(y_1, y_2) = \mathrm{P}(Y_1 = y_1, Y_2 = y_2), \qquad -\infty < y_1 < \infty, -\infty < y_2 < \infty$$

Definition 5.2

For any random variables Y_1 and Y_2, the joint (bivariate) distribution function $F(y_1, y_2)$ is:

$$\mathrm{F}(y_1, y_2) = \mathrm{P}(Y_1 \leq y_1, Y_2 \leq y_2), \qquad -\infty < y_1 < \infty, -\infty < y_2 < \infty$$

Definition 5.3

Y_1 and Y_2 are continuous random variables with joint distribution function $F(y_1, y_2)$**.** If there exists a nonnegative function $f(y_1, y_2)$ and Y_1 and Y_2 are said to be *jointly continuous random variables***,** we get the *joint probability density function* which is shown as:

$$F(y_1, y_2) = \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f(t_1, t_2) dt_2 dt_1$$

Definition 5.4

Y_1 and Y_2 are jointly discrete random variables with probability function $p(y_1, y_2)$**.** Then the *marginal probability functions* of Y_1 and Y_2, respectively, are given by:

$$p_1(y_1) = \sum_{\text{all } y_2} p(y_1, y_2) \text{ and } p_2(y_2) = \sum_{\text{all } y_1} p(y_1, y_2)$$

Also, Y_1 and Y_2 are jointly continuous random variables with joint density function $f(y_1, y_2)$**.** Then the *marginal density functions* of Y_1 and Y_2, respectively, are given by:

$$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_2 \text{ and } f_2(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_1$$

Definition 5.5

Y_1 and Y_2 are jointly discrete random variables with joint probability function $p(y_1, y_2)$ and marginal probability functions $p_1(y_1)$ and $p_2(y_2)$, respectively, then the *conditional discrete probability function* of Y_1 given Y_2 is:

$$p(y_1 \mid y_2) = P(Y_1 = y_1 \mid Y_2 = y_2) = \frac{P(Y_1 = y_1, Y_2 = y_2)}{P(Y_2 = y_2)} = \frac{p(\boldsymbol{y_1}, \boldsymbol{y_2})}{p_2(y_2)} \text{provided } p_2(\boldsymbol{y_2}) > 0$$

Definition 5.6

If Y_1 and Y_2 are jointly continuous random variables with joint density function $f(y_1, y_2)$, then the *conditional distribution function* of Y_1 given $Y_2 = y_2$ is:

$$F(y_1 \mid y_2) = P(Y_1 \leq y_1 \mid Y_2 = y_2)$$

Definition 5.7

Let Y_1 and Y_2 be jointly continuous random variables with joint density $f(y_1, y_2)$ and marginal densities $f_1(y_1)$ and $f_2(y_2)$, respectively. For any y_2 such that $f_2(y_2) > 0$, the conditional density of Y_1 given $Y_2 = y_2$ is given by:

$$f(y_1 \mid y_2) = \frac{f(\boldsymbol{y_1}, \boldsymbol{y_2})}{f_2(y_2)}$$

and, for any y_1 such that $f_1(y_1) > 0$, the conditional density of Y_2 given $Y_1 = y_1$ is given by:

$$f(y_2 \mid y_1) = \frac{f(\boldsymbol{y_1}, \boldsymbol{y_2})}{f_1(y_1)}$$

Definition 5.8

For distribution function making use of joint distribution function, Y_1 and Y_2 are said to be independent if and only if:

$$F(y_1, y_2) = F_1(y_1)F_2(y_2)$$

for every pair of real numbers (y_1, y_2). If Y_1 and Y_2 are not independent, they are said to be dependent.