

NEGATIVE LOG LOSS FUNCTION OR CROSS ENTROPY

$$H_p(q) = -\frac{1}{N} \sum_{i=1}^N y_i \cdot \log(p(y_i)) + (1 - y_i) \cdot \log(1 - p(y_i))$$

Binary Cross-Entropy / Log Loss

**DON'T BE AFRAID, IT IS VERY SIMPLE.
JUST BE SYSTEMATIC.**

Well let's take an example.

Let us suppose we want to classify **dogs and cats**. Let us define **1 as cat** and **0 as dogs**.

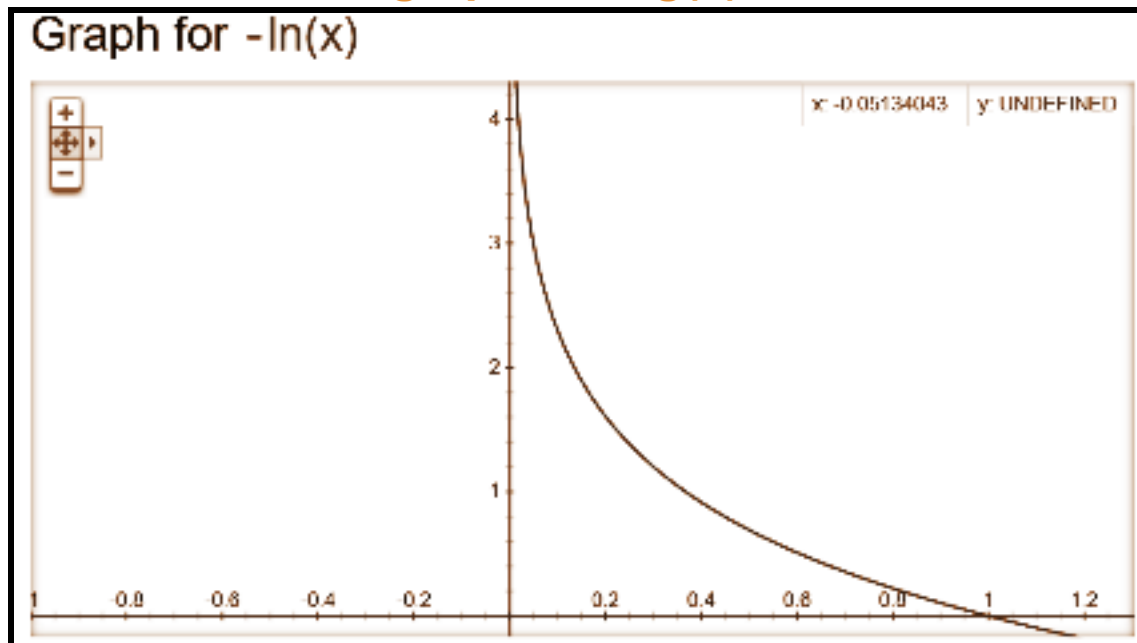
Now we define P_i as the probability to classify a data point as a CAT.

Let us say P is $[0.9, 0.1, 0.2, 0.8]$. This means for the data points 1 and 4 we are 90% sure it is a cat and 80% sure it is a cat respectively.

For the data points 2 and 3, we say that we are 10% sure it is a cat or 90% sure it is a dog and similarly that we are 80% sure it is a dog.

Basis of our loss function is that - The lesser it is, the better it is. Let us analyse why the negative sign.

The below is the graph of $-\log(x)$ with base e.



Note that **we are using probabilities. They lie in range $[0,1]$.**

Now see that if we have a binary data point ,namely data point 1 here, its y value is the value **1** and the **$p(y)$ value is 0.9**. This makes the graph **tend towards 0** and thus **A LOWER VALUE**.

This means a higher probability leads to a lower loss.

This is what we wanted. **The above was in**

fact the explanation of this term->

$$y_i \cdot \log(p(y_i))$$

with of course a **negative sign**.

Now, if our binary data point is **0** then the above term would be **deactivated as y is zero**. But this means that the second term of

sum which is $(1 - y_i) \cdot \log(1 - p(y_i))$ gets **activated**.

Now, $1 - y_i$ becomes 1 and $(1 - p(y_i))$ becomes actually the probability of classifying the data point as 0. Again, it is in line with our objective of defining a loss function which was the higher the probability, the lower the loss. Ofcourse, the negative sign has to be included here also.

Now, if we take the average of ALL THESE LOSSES, WE FINALLY GET THE NEGATIVE LOG LIKELIHOOD LOSS.

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Binary Cross-Entropy / Log Loss

Neat! Not so confusing now.