Problem 1: Optional Problem

Error Mitigation Code

An automated irrigation system uses a complex algorithm that takes in environmental data (temperature, humidity, sunlight, etc.) and uses it to determine whether the system should be off (output bit 0) or on (output bit 1). However, the system is quite faulty, and is prone to errors. 10% of the time it encounters a bit flip error, where the output bit shows 0 when it should be 1 or vice versa.

a) If the algorithm is executed once a day, what is the expected value for the number of errors in the system over the course of one year?

To counter these errors, engineers have implemented a 3 bit error code in the system. The code works by encoding one logical bit using 3 physical bits. This means that a logical 0, written as $\bar{0}$, is encoded as 000. **Any combination of physical bits that contain 2 or more 0's, is still recognized as** $\bar{0}$. Similarly, a logical 1, $\bar{1}$, is encoded as 111, and **any string with two or more 1's is interpreted as** $\bar{1}$. The irrigation system is now only on if it reads $\bar{1}$ and off if it reads $\bar{0}$.

b) For each of the possible combinations of 3 physical bits, state whether they would be interpreted as $\bar{0}$ or $\bar{1}$:

Physical Bits	Logical Bit
000	$\bar{0}$
001	$\bar{0}$
010	
011	
100	
101	
110	$\bar{1}$
111	$\bar{1}$

c) How many simultaneous bit flip errors must occur in the system to cause $\bar{0}$ (initially 000) to be incorrectly interpreted as $\bar{1}$?

d) Given the 10% error rate for the physical bits, what is the probability of encountering an error where $\bar{0}$ is flipped to $\bar{1}$?

Need to verify the last answer though...

Problem 2: Optional Challenge Problem

Quantum Tunneling

One of the strangest phenomena in quantum mechanics is quantum tunneling. Quantum tunneling is also a feature of the wave-like nature of quantum objects. It is the idea that there is sometimes a small but significant probability that quantum particles may overcome energy barriers.

Consider a quantum particle confined to a box that has a very large width L. The particle moves between the walls of the box, but each time it makes contact with the walls, there is a small probability p that the particle will tunnel through the wall. Fig.1 shows a sketch of the setup.

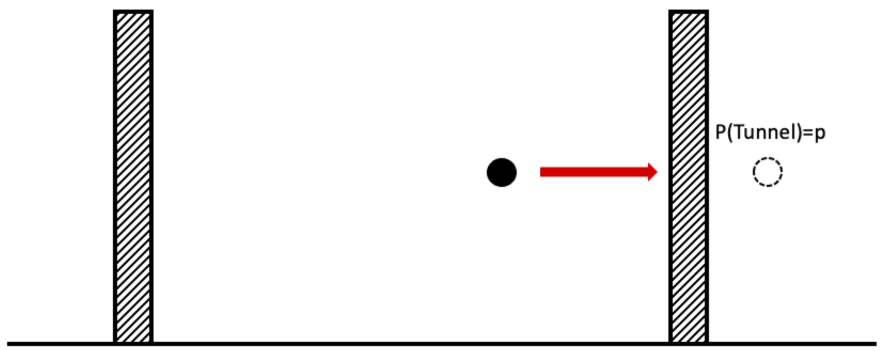

Figure 1: Sketch of a particle trapped in a box. The probability of the particle tunneling through the walls of the box is p

a) What is the probability that the particle tunnels through the wall on the 1st collision?

b) What is the probability that the particle does not tunnel through the wall after the 1st collision?

c) What is the probability that the particle tunnels through the wall on the 2nd collision?

d) What is the probability distribution $P(n)$, that describes the probability of the particle tunneling through the wall on the nth collision? (Note: This probability distribution is known as a *geometric distribution*.)

e) Show that this probability distribution is normalized by verifying that:

$$\sum_{n=1}^{\infty} P(n) = 1$$

The following identity for a geometric series may be useful: $\sum_{m=0}^{\infty} r^m = \frac{1}{1-r}$

The expected number of collisions before the particle escapes $< n >$ is given by the expected value of our probability distribution:

$$< n >= \sum_{n=1}^{\infty} nP(n)$$

It can be shown that for this type of distribution $< n >$ takes the following form:

$$< n >= \frac{1}{p} \tag{1}$$

f) If there is a 0.1% probability of the particle tunneling in each collision, what will be the average number of collisions before it escapes?

Extra Challenge: Try to derive Eq 1. **Warning**: this is very difficult to do. We encourage students who are interested in the derivation to try on their own, then use the provided walk-through of the derivation if necessary.