HOMEWORK 14

THE QUBIT AND THE BLOCH SPHERE

1. A **qubit** is best described as

   a) a quantum harmonic oscillator.

   b) a quantum system with exactly two discrete eigenstates.

   c) a classical system with two opposite states.

   d) a quantum system with infinite eigenstates.

2. Qubit states are represented geometrically as

   a) curves in the x-y plane.

   b) surfaces in a 3D space.

   c) vectors of varying magnitude.

   d) vectors that point to the surface of a sphere.

3. True or False: every point on the Bloch Sphere represents a **valid, normalized qubit state**.

   a) True

   b) False

4. Which of the following operators is used to perform a **rotation about the y-axis** on a qubit state on the Bloch Sphere.

   (a) σ_x

   (b) σ_y

   (c) σ_z

   (d) Hadamard Gate

5. Which of the following operators is used to perform a **rotation about the z-axis** on a qubit state on the Bloch Sphere.

   (a) σ_x

   (b) σ_y

   (c) σ_z

   (d) Hadamard Gate

Questions 6-10 are in relation to the following quantum state:

$$|\phi\rangle = e^{i\frac{\pi}{3}}\left(\frac{1}{\sqrt{3}}|+\rangle + e^{i\frac{\pi}{6}}\sqrt{\frac{2}{3}}|-\rangle\right)$$

Answer **Questions 6-10** by correctly identifying the various properties of the state $|\phi\rangle$.

6. What is/are the **basis state(s)**?

   (a) $e^{i\frac{\pi}{3}}$

   (b) $e^{i\frac{\pi}{6}}$

   (c) $\frac{1}{\sqrt{3}}$ and $\sqrt{\frac{2}{3}}$

   (d) $|+\rangle$ and $|-\rangle$

   (e) None of the above

7. What is/are the **probability amplitude(s)**?

   (a) $e^{i\frac{\pi}{3}}$

   (b) $e^{i\frac{\pi}{6}}$

   (c) $\frac{1}{\sqrt{3}}$ and $\sqrt{\frac{2}{3}}$

   (d) $|+\rangle$ and $|-\rangle$

   (e) None of the above

8. What is/are the **global phase(s)**?

   (a) $e^{i\frac{\pi}{3}}$

   (b) $e^{i\frac{\pi}{6}}$

   (c) $\frac{1}{\sqrt{3}}$ and $\sqrt{\frac{2}{3}}$

   (d) $|+\rangle$ and $|-\rangle$

   (e) None of the above

9. What is/are the **relative phase(s)**?

   (a) $e^{i\frac{\pi}{3}}$

   (b) $e^{i\frac{\pi}{6}}$

   (c) $\frac{1}{\sqrt{3}}$ and $\sqrt{\frac{2}{3}}$

   (d) $|+\rangle$ and $|-\rangle$

   (e) None of the above

10. What is/are the **eigenvalue(s)**?

    (a) $e^{i\frac{\pi}{3}}$

    (b) $e^{i\frac{\pi}{6}}$

    (c) $\frac{1}{\sqrt{3}}$ and $\sqrt{\frac{2}{3}}$

    (d) $|+\rangle$ and $|-\rangle$

    (e) None of the above

11. Which of the following properties does **not** have any influence on the outcome of measurements of quantum states?

    a) Relative Phase

    b) Probability Amplitude

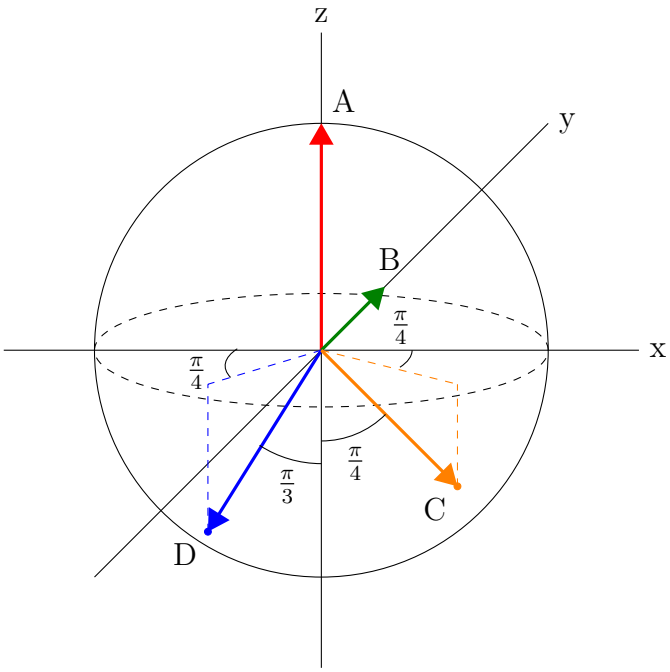
    c) Global Phase

    d) Eigenvalues of operators

In lecture we learned that the state of a qubit in a quantum computer is given by a two dimensional vector of the form

$$|\psi\rangle = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\phi}\sin\left(\frac{\theta}{2}\right) \end{pmatrix}$$

Questions 12-15 relates to the following Bloch sphere with vectors that represent various qubit states.



Hint: remember that the angle θ is measured from the **positive z-axis**. Some vectors in the diagram may not be labeled in this way.

12. What is the qubit state represented by vector **A**?

    a) $|0\rangle$

    b) $\cos\left(\frac{\pi}{3}\right)|0\rangle + e^{-i\frac{3\pi}{4}}\sin\left(\frac{\pi}{3}\right)|1\rangle$

    c) $\frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$

    d) $\cos\left(\frac{3\pi}{8}\right)|0\rangle + e^{-i\frac{\pi}{4}}\sin\left(\frac{3\pi}{8}\right)|1\rangle$

13. What is the qubit state represented by vector **B**?

    a) $|0\rangle$

    b) $\cos\left(\frac{\pi}{3}\right)|0\rangle + e^{-i\frac{3\pi}{4}}\sin\left(\frac{\pi}{3}\right)|1\rangle$

    c) $\frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$

    d) $\cos\left(\frac{3\pi}{8}\right)|0\rangle + e^{-i\frac{\pi}{4}}\sin\left(\frac{3\pi}{8}\right)|1\rangle$

14. What is the qubit state represented by vector **C**?

    a) $|0\rangle$

    b) $\cos\left(\frac{\pi}{3}\right)|0\rangle + e^{-i\frac{3\pi}{4}}\sin\left(\frac{\pi}{3}\right)|1\rangle$

    c) $\frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$

    d) $\cos\left(\frac{3\pi}{8}\right)|0\rangle + e^{-i\frac{\pi}{4}}\sin\left(\frac{3\pi}{8}\right)|1\rangle$

15. What is the quantum state represented by vector **D**

    a) $|0\rangle$

    b) $\cos\left(\frac{\pi}{3}\right)|0\rangle + e^{-i\frac{3\pi}{4}}\sin\left(\frac{\pi}{3}\right)|1\rangle$

    c) $\frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle)$

    d) $\cos\left(\frac{3\pi}{8}\right)|0\rangle + e^{-i\frac{\pi}{4}}\sin\left(\frac{3\pi}{8}\right)|1\rangle$