HOMEWORK 12

QUANTUM MECHANICS 3

In this week we learned about the postulates of quantum mechanics. These postulates are mathematical statements about the way quantum mechanics works that we assume are true. Extrapolating on the postulates leads us to many of the predictions of quantum mechanics which we observe in experiments. First we will go over what these postulates tell us about the behavior of quantum states, then examine some of the implications.

For **Questions 1-6**, select the statement which best completes the given postulate of quantum mechanics.

1. The first postulate establishes how quantum states can be represented mathematically. It states that

   a) a quantum state can be represented as a number on the number line.

   b) a quantum state can be represented as a ket $|\psi\rangle$ in a Hilbert space.

   c) a quantum state can be represented as a spherical surface.

   d) a quantum state $|\psi\rangle$ can be represented as a two dimensional vector $\begin{pmatrix} x \\ y \end{pmatrix}$.

2. The second postulate of quantum mechanics establishes how **observable quantities** are represented in quantum mechanics. In quantum mechanics, all **observable quantities** are

   a) Hermitian operators.

   b) vectors.

   c) complex numbers.

   d) real numbers.

3. Measurements in quantum mechanics are probabilistic. The third postulate of quantum mechanics establishes what values are possible for the measurement of a given observable. Upon measuring an observable represented by the operator $\hat{A}$, the possible outcomes of the measurement are given by

   (a) the **elements** of the matrix $\hat{A}$.

   (b) the **inverse** of $\hat{A}$.

   (c) the **transpose** of $\hat{A}$

   (d) the **eigenvalues** of $\hat{A}$.

4. The fourth postulate of quantum mechanics tells us the probability of measuring a given value for an operator $\hat{A}$. The postulate states that the probability of measuring a state $|\psi\rangle$ in the eigenstate of $\hat{A}$ $|a_1\rangle$ is obtained by

   (a) taking the **inner product** of $|\psi\rangle$ and $|a_1\rangle$.

   (b) applying $\hat{A}$ to state $|a_1\rangle$.

   (c) applying $\hat{A}$ to state $|\psi\rangle$.

   (d) taking the **outer product** of $|\psi\rangle$ and $|a_1\rangle$.

5. The fifth postulate of quantum mechanics establishes how measurement affects a quantum state. After measurement of an observable $\hat{A}$

   (a) the quantum state collapses to a **random** quantum state.

   (b) the quantum state collapses to an **eigenstate** of the observable $\hat{A}$.

   (c) the quantum state disappears.

   (d) the quantum state collapses to a **column vector** contained in $\hat{A}$.

6. The sixth postulate of quantum mechanics states how quantum states change in time. The evolution of a quantum state in time is

   (a) determined by the measurement of an **observable** $\hat{A}$ on the state.

   (b) constant throughout time.

   (c) determined by the action of a **unitary operator** $\hat{U}(t)$ that preserves the normalization of the state.

   (d) the same for all quantum states.

Questions 7-14 refer to the following scenario.

Consider an experiment which measures an observable represented by the operator $\hat{O}$ (this could be position, momentum, energy, spin, etc.). $\hat{O}$ has 4 eigenvalues: α, β, γ and δ. These correspond to 4 eigenstates: $|A\rangle, |B\rangle, |C\rangle$ and $|D\rangle$ in the following way:

$$\hat{O}|A\rangle = \alpha|A\rangle \qquad \hat{O}|B\rangle = \beta|B\rangle \qquad \hat{O}|C\rangle = \gamma|C\rangle \qquad \hat{O}|D\rangle = \delta|D\rangle$$

Note that these states are all **orthogonal** to each other. That is,

$$\langle A|B\rangle = 0, \langle A|C\rangle = 0, \langle A|D\rangle = 0, \langle B|C\rangle = 0, \langle B|D\rangle = 0, \langle C|D\rangle = 0$$

7. If observable $\hat{O}$ is measured, in general how many possible values could the measurement take?

   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) There are infinitely many possibilities.

8. Which of the following are possible outcomes of the measurement of observable $\hat{O}$? (Select all that apply)

   a) α
   b) β
   c) γ
   d) δ
   e) $\alpha + \beta$
   f) $\alpha + \beta + \gamma + \delta$
   g) $2\alpha - 3\beta$

For **Questions 9-14**, we will now consider the measurement of a specific state:

$$|\psi\rangle = \frac{1}{\sqrt{6}}|A\rangle + \frac{1}{\sqrt{3}}|B\rangle + \frac{1}{\sqrt{2}}|D\rangle$$

9. Upon measurement of $\hat{O}$ for $|\psi\rangle$, what is the probability that the value α is measured?
   a) 0
   b) $\frac{1}{2}$
   c) $\frac{1}{6}$
   d) $\frac{1}{3}$

10. Upon measurement of $\hat{O}$ for $|\psi\rangle$, what is the probability that the value β is measured?
    a) 0
    b) $\frac{1}{2}$
    c) $\frac{1}{6}$
    d) $\frac{1}{3}$

11. Upon measurement of $\hat{O}$ for $|\psi\rangle$, what is the probability that the value γ is measured?
    a) 0
    b) $\frac{1}{2}$
    c) $\frac{1}{6}$
    d) $\frac{1}{3}$

12. Upon measurement of $\hat{O}$ for $|\psi\rangle$, what is the probability that the value δ is measured?
    a) 0
    b) $\frac{1}{2}$
    c) $\frac{1}{6}$
    d) $\frac{1}{3}$

13. If we perform a measurement of $\hat{O}$ on $|\psi\rangle$ and the value we obtain for the measurement is δ, which of the following gives the state of $|\psi\rangle$ **after this measurement**? (Recall the postulate on wavefunction collapse)

a) $|\psi\rangle = \frac{1}{\sqrt{6}}|A\rangle + \frac{1}{\sqrt{3}}|B\rangle + \frac{1}{\sqrt{2}}|D\rangle$

b) $|\psi\rangle = |B\rangle$

c) $|\psi\rangle = \frac{1}{\sqrt{2}}|A\rangle + \frac{1}{\sqrt{2}}|B\rangle$

d) $|\psi\rangle = |D\rangle$

14. If we perform a measurement of $\hat{O}$ on $|\psi\rangle$ and the value we obtain for the measurement is α, which of the following gives the state of $|\psi\rangle$ **after this measurement**? (Recall the postulate on wavefunction collapse)

a) $|\psi\rangle = \frac{1}{\sqrt{6}}|A\rangle + \frac{1}{\sqrt{3}}|B\rangle + \frac{1}{\sqrt{2}}|D\rangle$

b) $|\psi\rangle = |A\rangle$

c) $|\psi\rangle = \frac{1}{\sqrt{2}}|B\rangle + \frac{1}{\sqrt{2}}|D\rangle$

d) $|\psi\rangle = |C\rangle$