INTRO TO QUANTUM COMPUTING

Week 14 Lab

THE BLOCH SPHERE

Corbin McElhanney

February 9, 2021

PROGRAM FOR TODAY

- Logistics
- Canvas attendance quiz
- Pre-lab zoom feedback
- Questions from last week
- Lab content
- Post-lab zoom feedback

LOGISTICS

- Student assistant office hours
  - Every **Friday 8 am-2 pm EST, and Sunday 10 am-12 pm EST (UTC-5)**
  - Student Assistants are available to review lab and lecture materials, walk through homework problems, or answer any other content-related questions you might have at the end of each week
  - Zoom link available on Canvas
- Friday homework review sessions
  - Every **Friday 4-5 p.m. EST (UTC-5)**
  - Review, ask questions, work through weekly homework problems with an instructor
  - Zoom links available on Canvas
  - Recordings will be made available if you cannot attend live
- Create your IBM Quantum Experience accounts (instructions on Canvas)
  - We will be using QE in lab next week!

Canvas attendance quiz passcode: 4758

CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz.

Lab Number: 1 | Quiz Password: 4758

- **This quiz not graded, but counts for your lab attendance!**

PRE-LAB ZOOM FEEDBACK

On a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

QUESTIONS FROM PAST WEEK

What is phase? → Global phase ← time doesn't change
→ Relative phase ← time changes

$e^{i\alpha}\left(\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle\right)$

$=$

$\frac{1}{\sqrt{2}}|0\rangle+e^{i\alpha}\frac{1}{\sqrt{2}}|1\rangle$ ←

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LEARNING OBJECTIVES FOR LAB 14

- Locating states on the Bloch sphere
  - Latitude and longitude
  - Bloch sphere and Stern Gerlach experiment

quantum state

- Visualizing the application of gates on states
  - Rotation operators
  - Rotating qubit states on the Bloch sphere

quantum gates

- Comparing modes on a piano and a violin*

*Optional content

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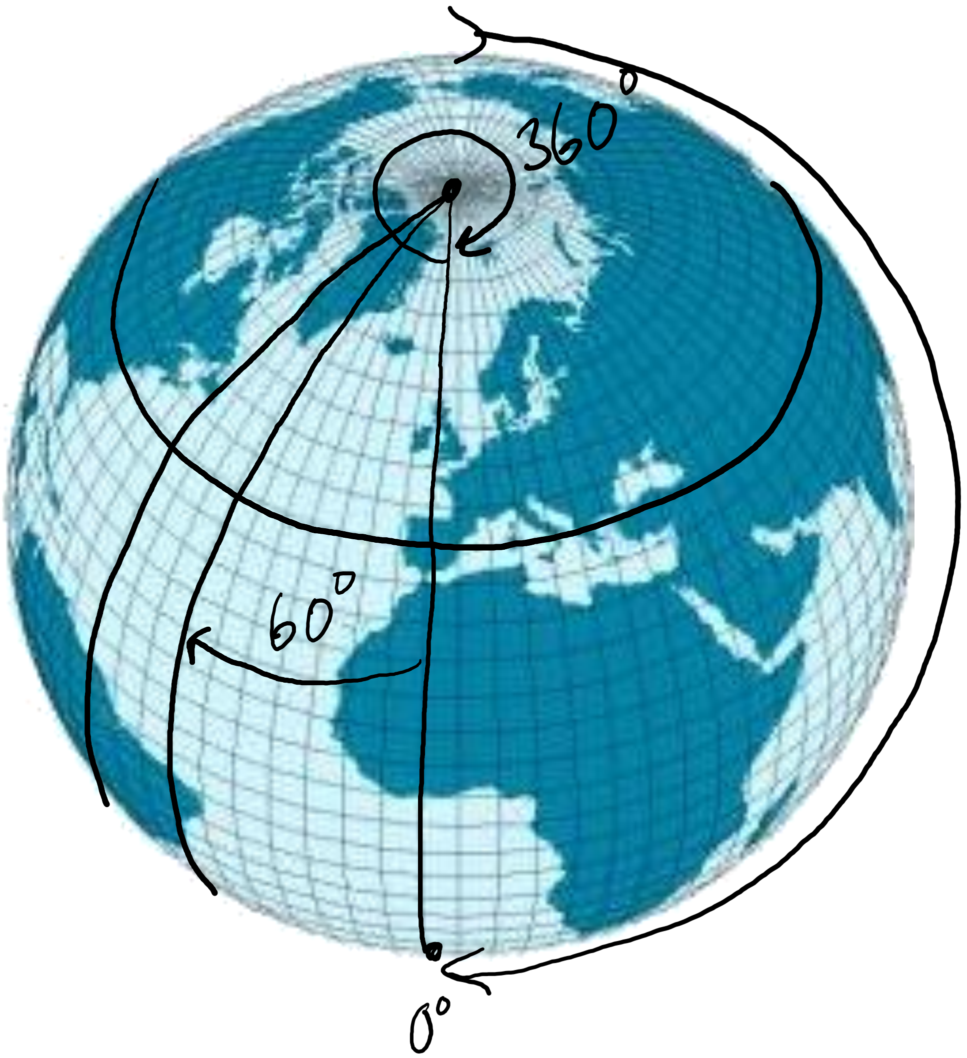
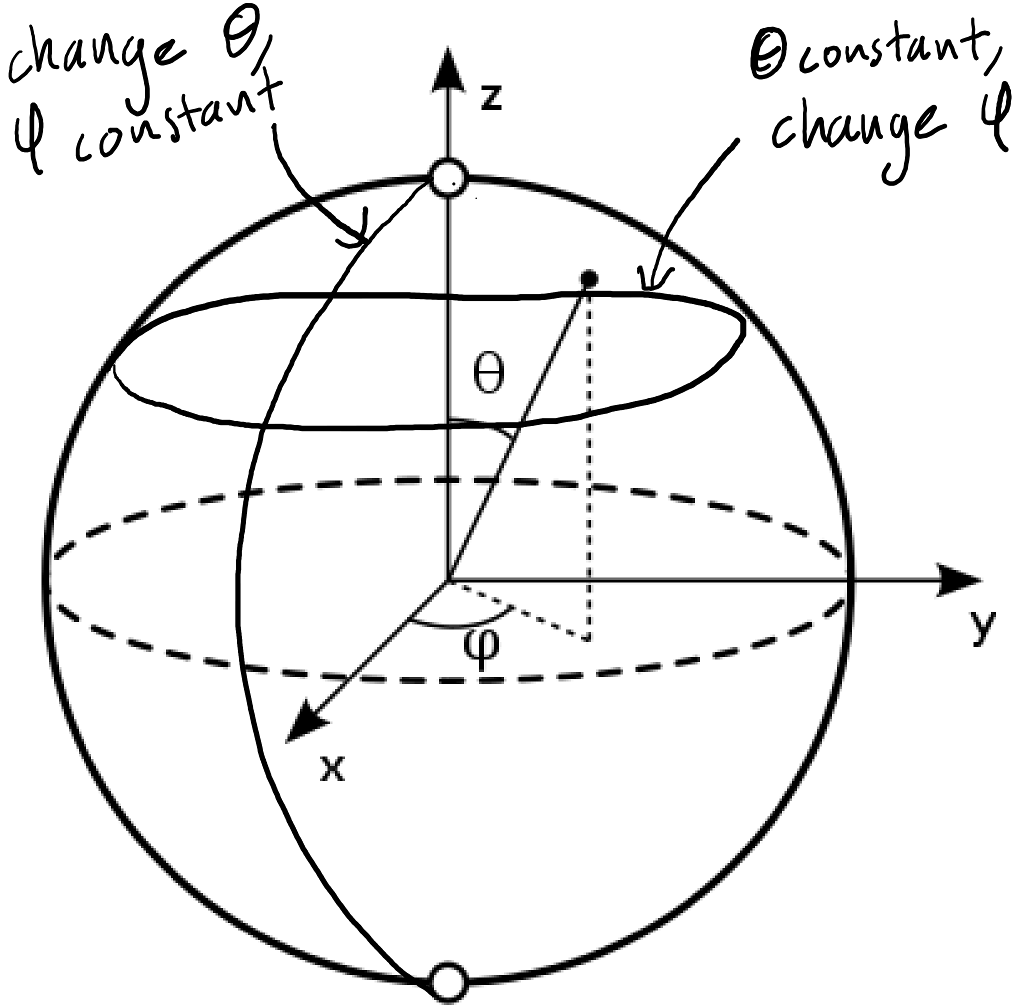
LOCATING A PLACE ON EARTH



- How would you tell me where your hometown is?
- Latitude and longitude! Two angles

LOCATING POINTS ON A SPHERE



$$\theta \in [0, \pi]$$

$$\varphi \in [0, 2\pi)$$

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CHARACTERISTICS OF BLOCH SPHERE

- The Bloch sphere is a visual tool to show single qubit states
- All single qubit states exist on the 2-D surface of the sphere
- The angle θ gives us the proportions of $|0\rangle$ and $|1\rangle$
- The angle φ gives us the relative phase of the qubit
- The Bloch sphere can also be used to visualize qubit transformations

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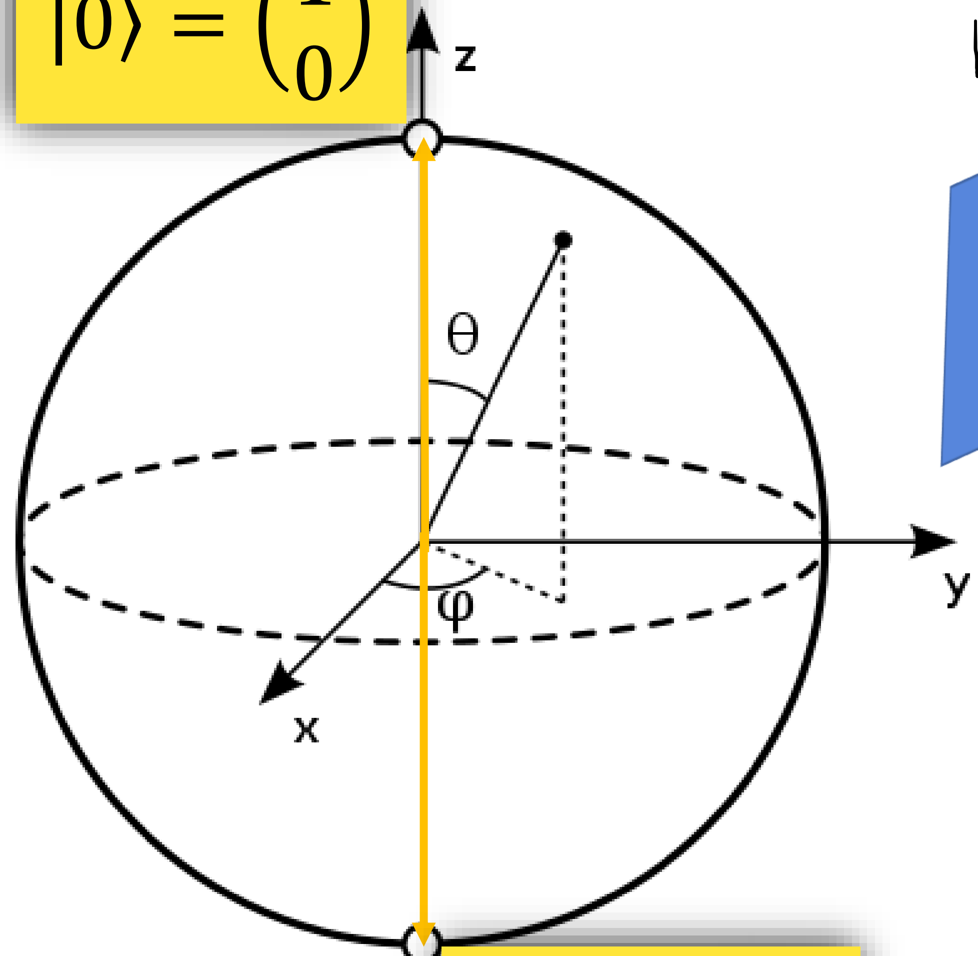
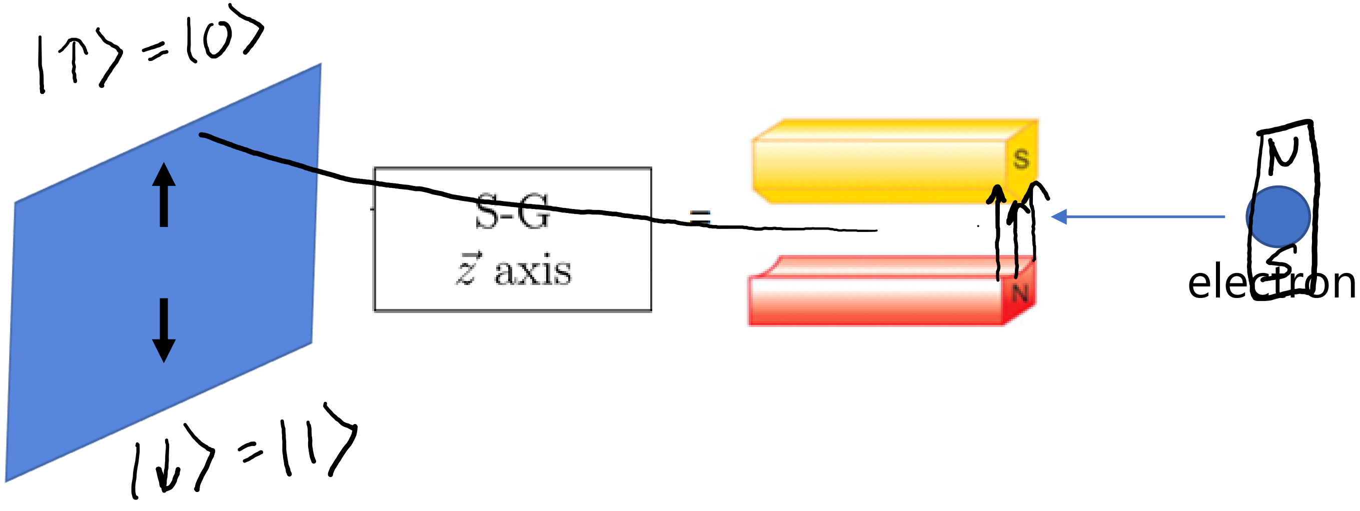
LOCATING POINTS ON A SPHERE

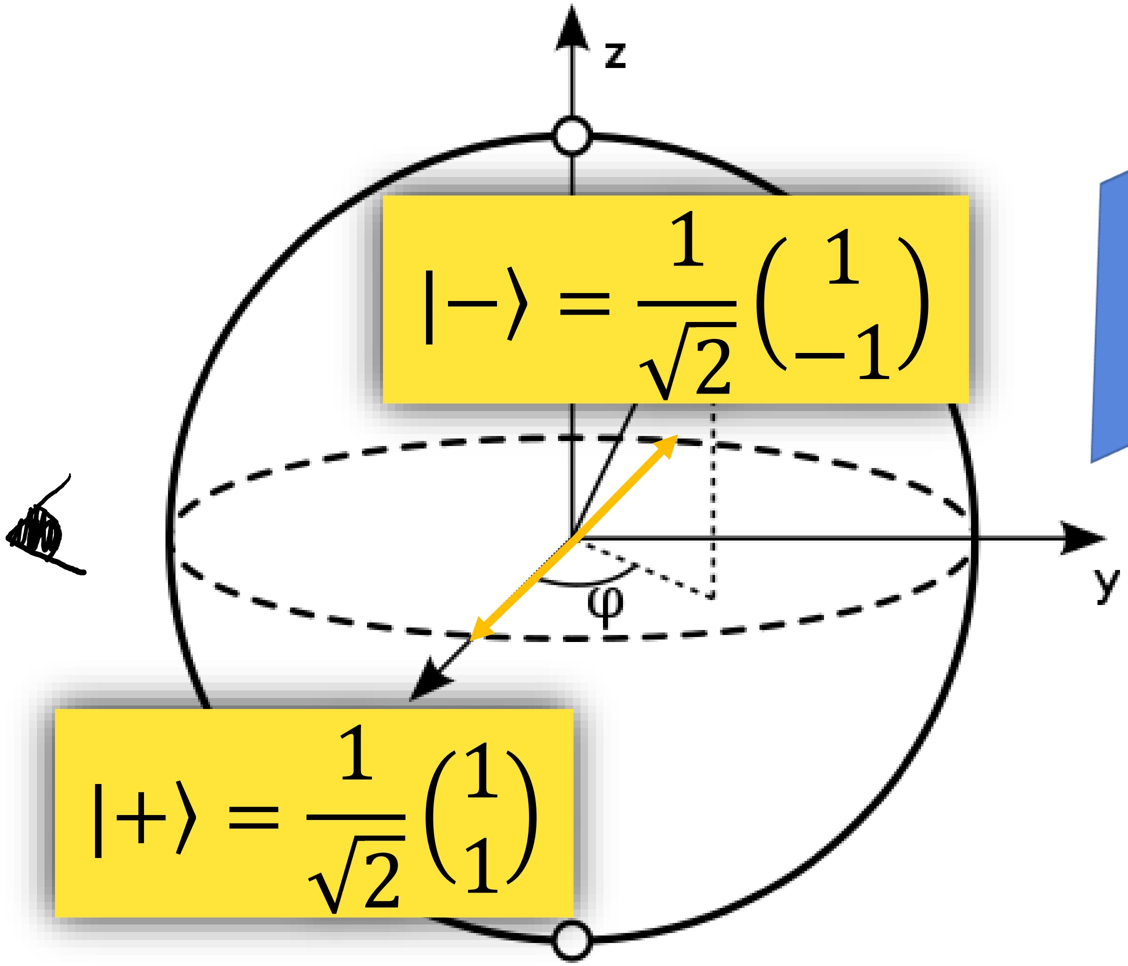
$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

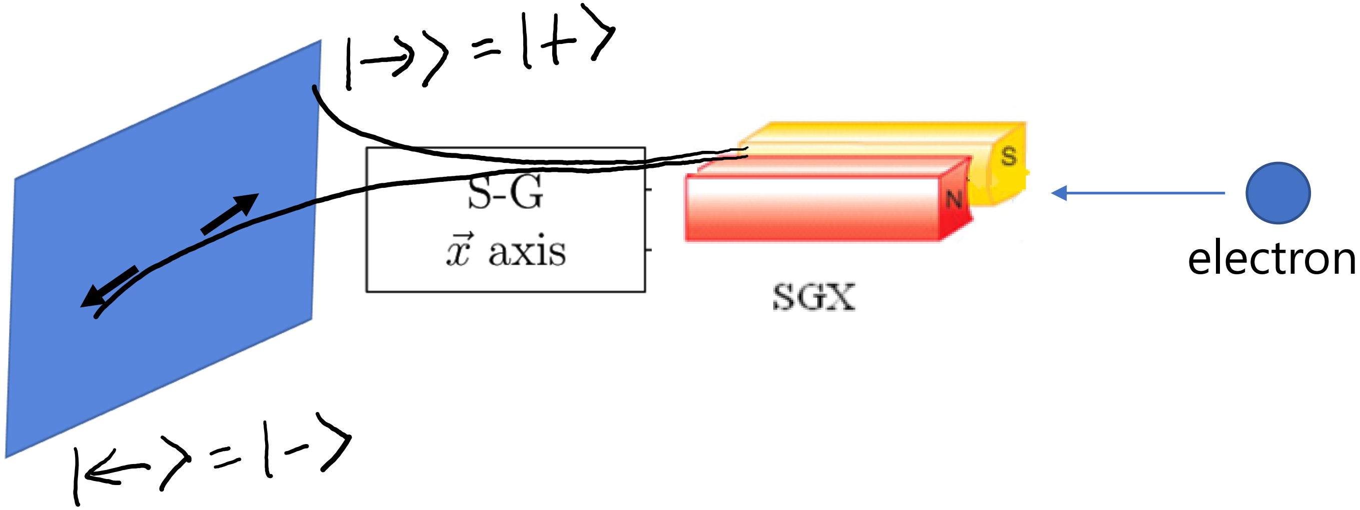


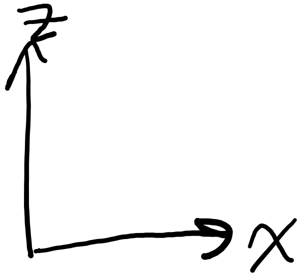
$|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$



LOCATING POINTS ON A SPHERE







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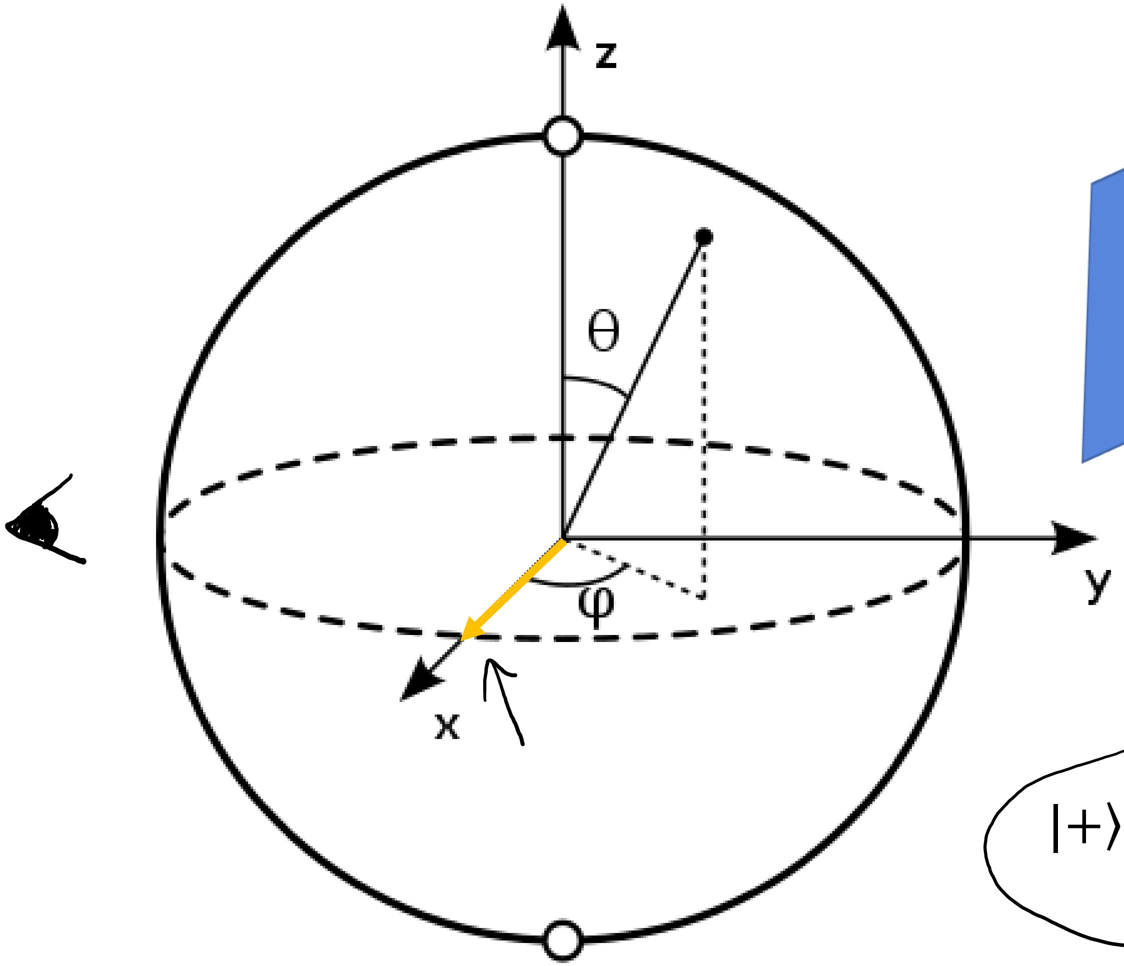
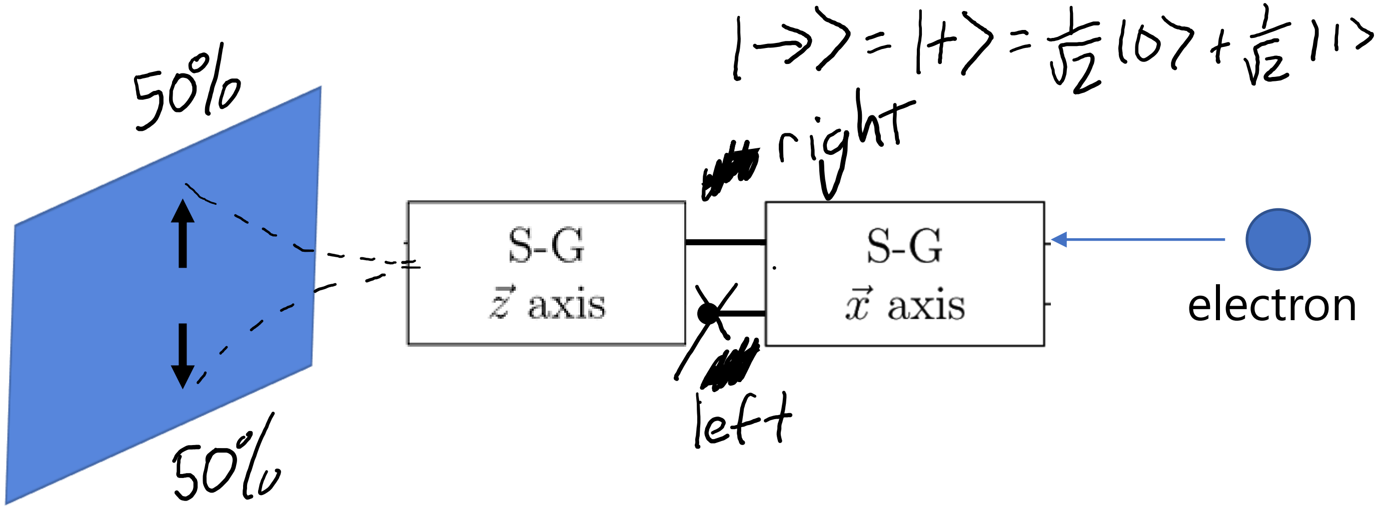
LOCATING POINTS ON A SPHERE





$$|+\rangle = |\rightarrow\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$$

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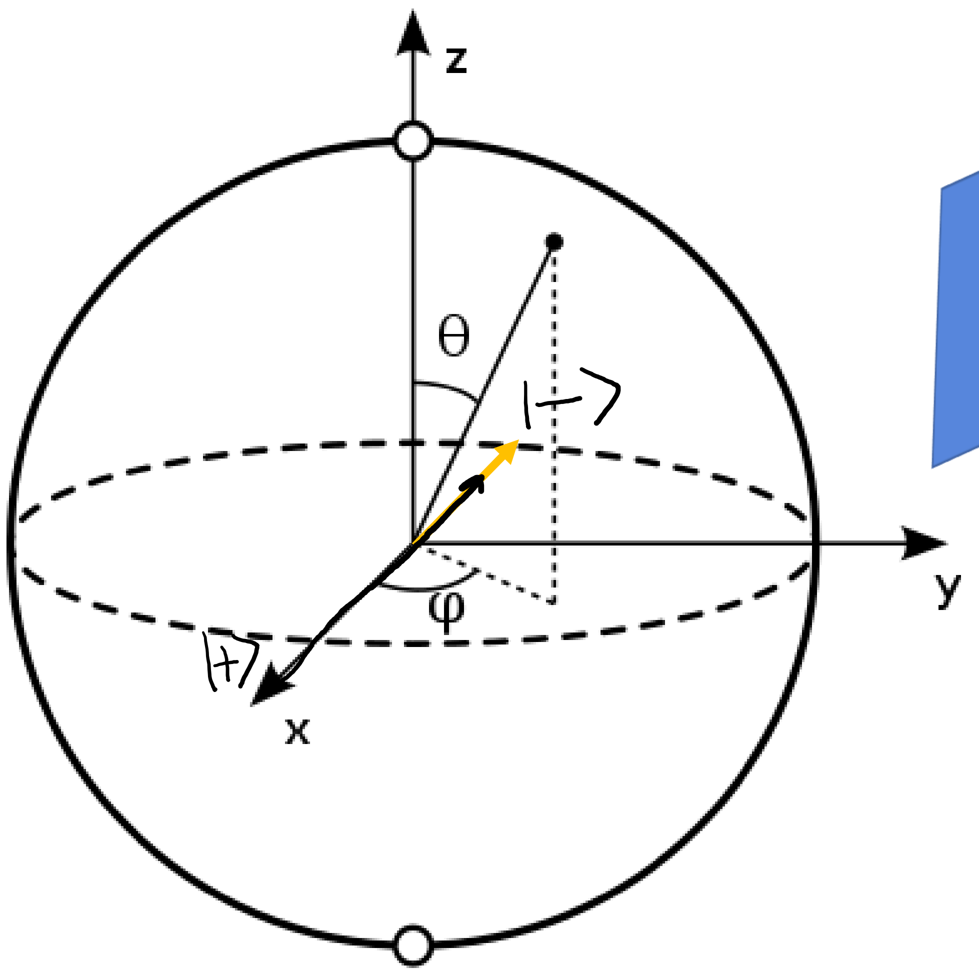
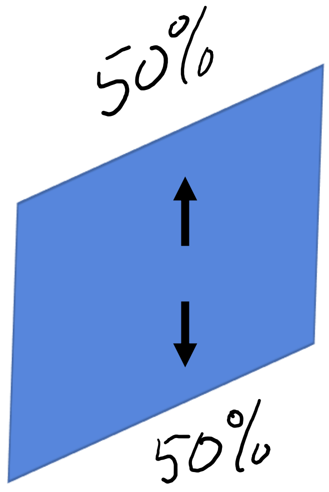
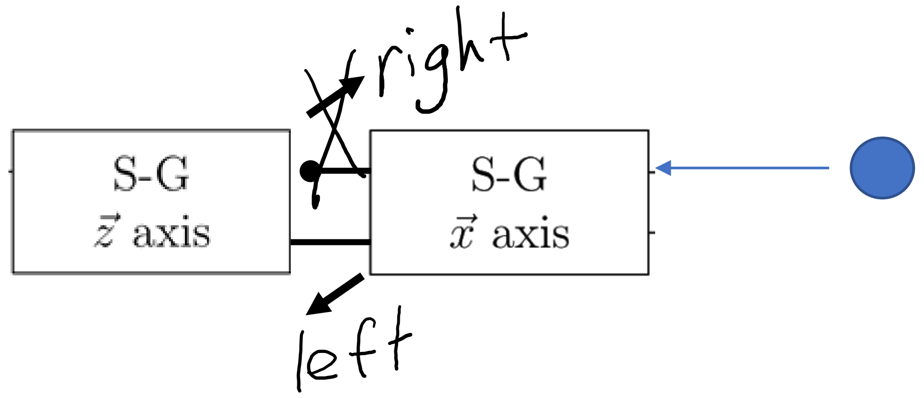
LOCATING POINTS ON A SPHERE







$$|-\rangle = |\leftarrow\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

θ : proportion of $|0\rangle$ and $|1\rangle$ in your state

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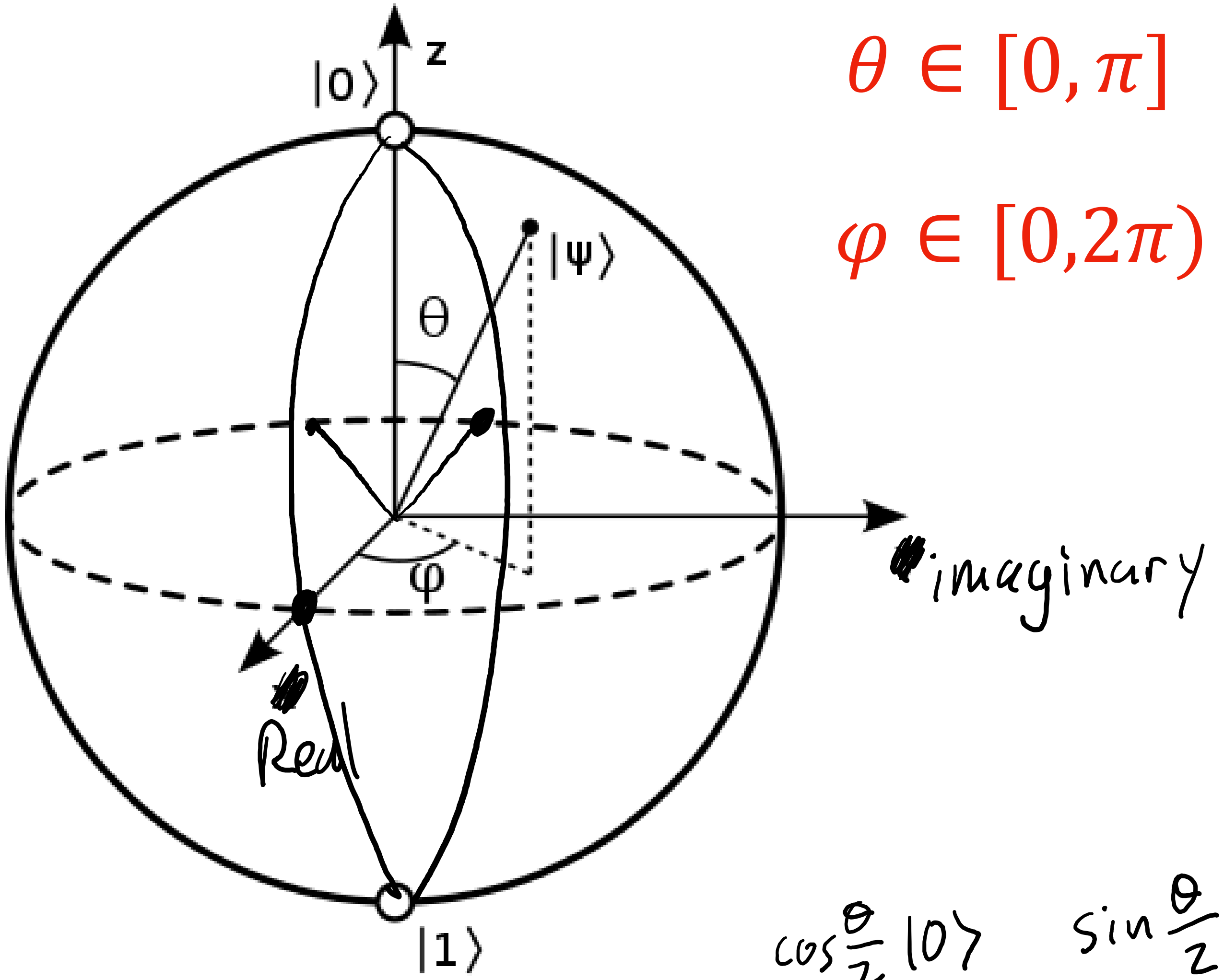
THE BLOCH SPHERE

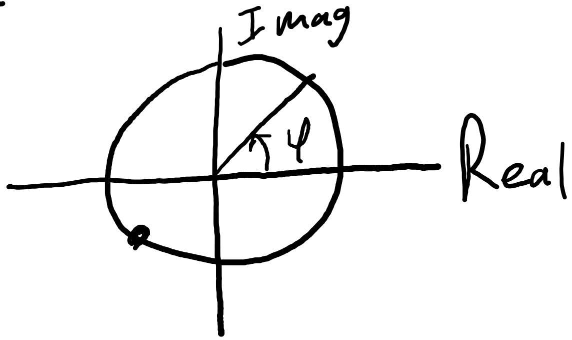


$\theta \in [0, \pi]$

$\varphi \in [0, 2\pi)$

$$\psi(\theta, \varphi) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\varphi} \sin\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$e^{i\varphi} = \cos\varphi + i\sin\varphi$



$\cos\frac{\theta}{2}|0\rangle$ $\sin\frac{\theta}{2}|1\rangle$

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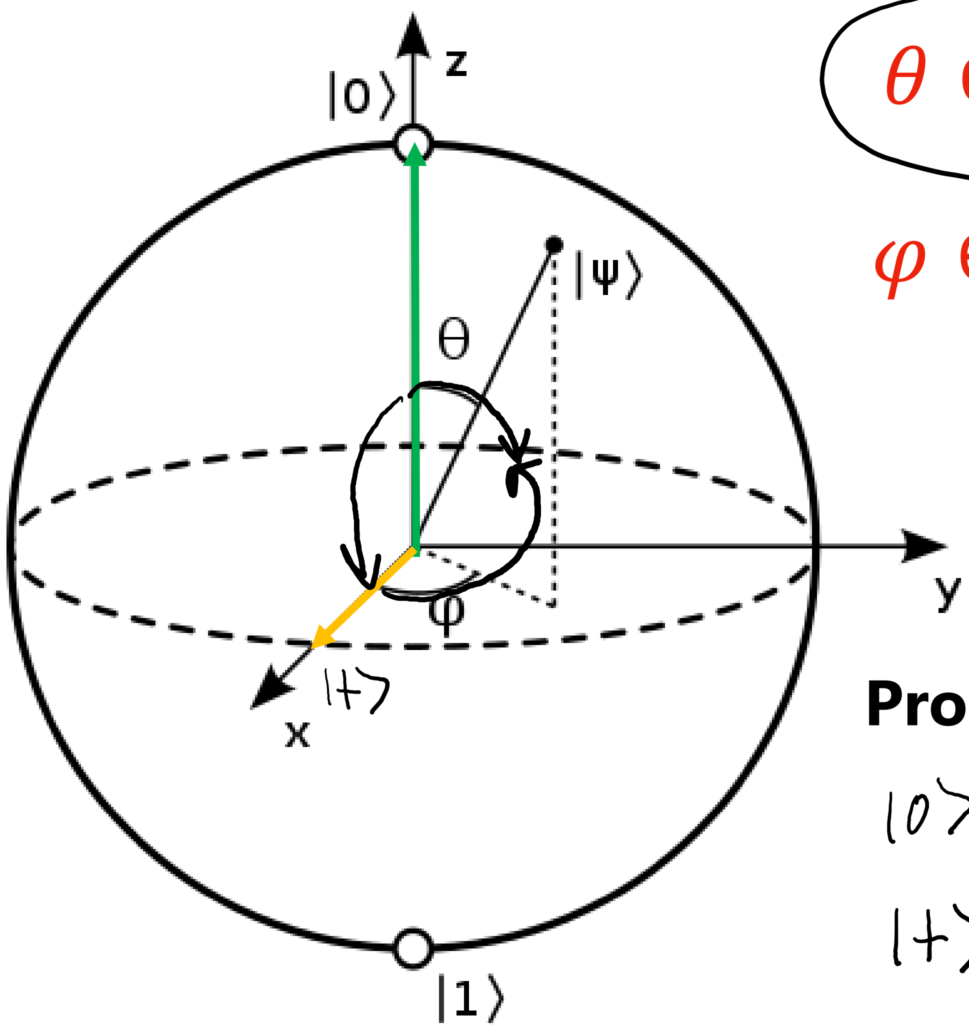
THE BLOCH SPHERE



$\theta \in [0, \pi]$

$\varphi \in [0, 2\pi)$

$= \underbrace{\cos\frac{\theta}{2}}\,|0\rangle + \underbrace{e^{i\varphi}}\sin\frac{\theta}{2}\,|1\rangle$

$$\psi(\theta, \varphi) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\varphi}\sin\left(\frac{\theta}{2}\right) \end{pmatrix}$$

Problem: What are θ and φ for $|0\rangle$ and $|+\rangle$?

$|0\rangle$: $\theta = 0$, φ anything

$|+\rangle$: $\theta = \frac{\pi}{2}$, $\varphi = 0$

$\begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$ $\cos^{-1}\left(\frac{3}{5}\right) =$

$=$

QUESTIONS?

Questions on content so far?

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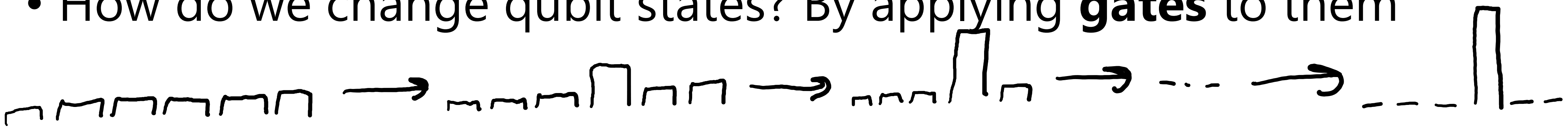
QUANTUM GATES

- So far, we've seen single qubit states and visualized them on the Bloch sphere
- In a quantum computer, we want to **change** qubit states
  - **Example:** Searching for a name in a list
  - Initially we don't know where the name is, so each entry in the list is equally likely to be the name we're searching for. The state of the qubits is an equal superposition
  - At the end, we want the state of the qubits to be just the name we are searching for. The qubit state has to be **changed** to perform this search.

Grover's Search

- How do we change qubit states? By applying **gates** to them



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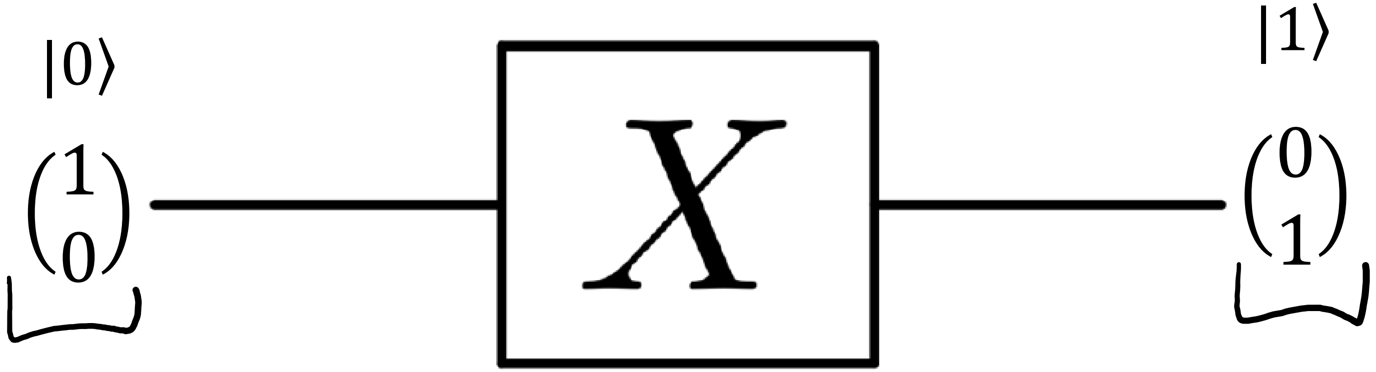
APPLYING GATES TO STATES



$$\underbrace{\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}}_{\text{X-gate matrix}}\underbrace{\begin{pmatrix}1\\0\end{pmatrix}}_{|0\rangle} = \underbrace{\begin{pmatrix}0\\1\end{pmatrix}}_{|1\rangle}$$

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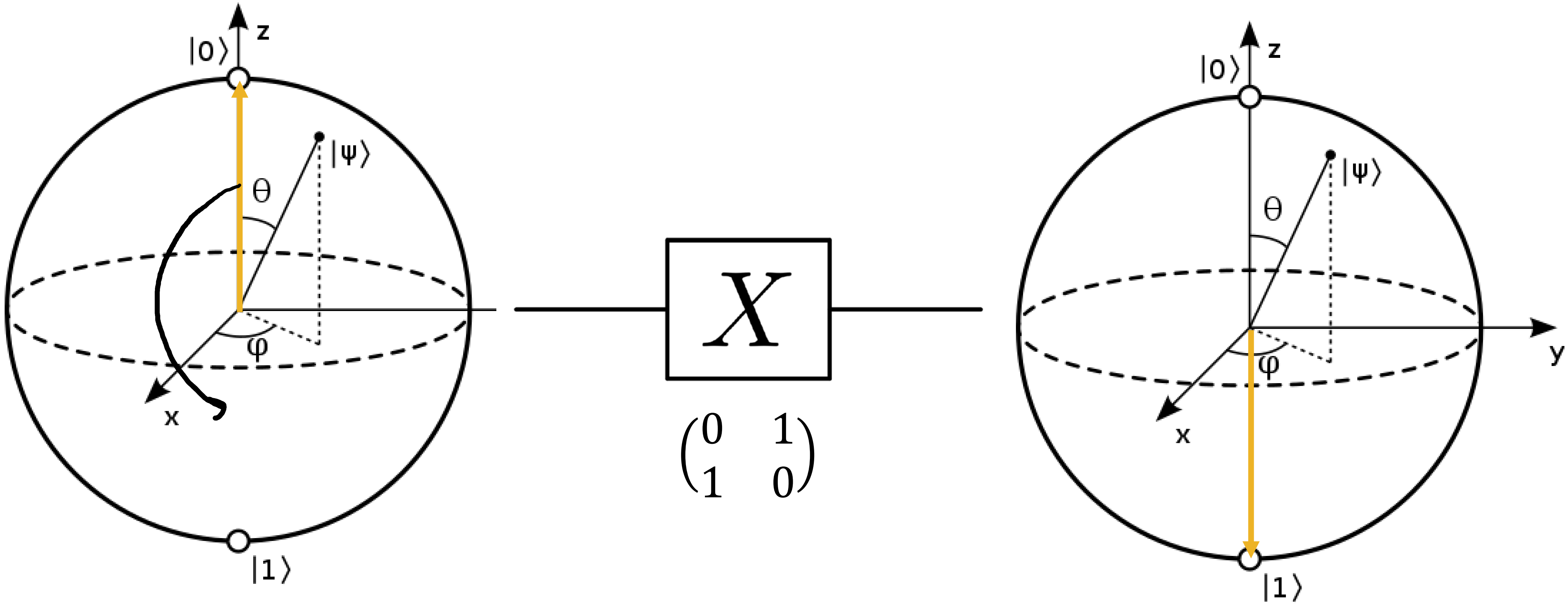
APPLYING THE X GATE



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

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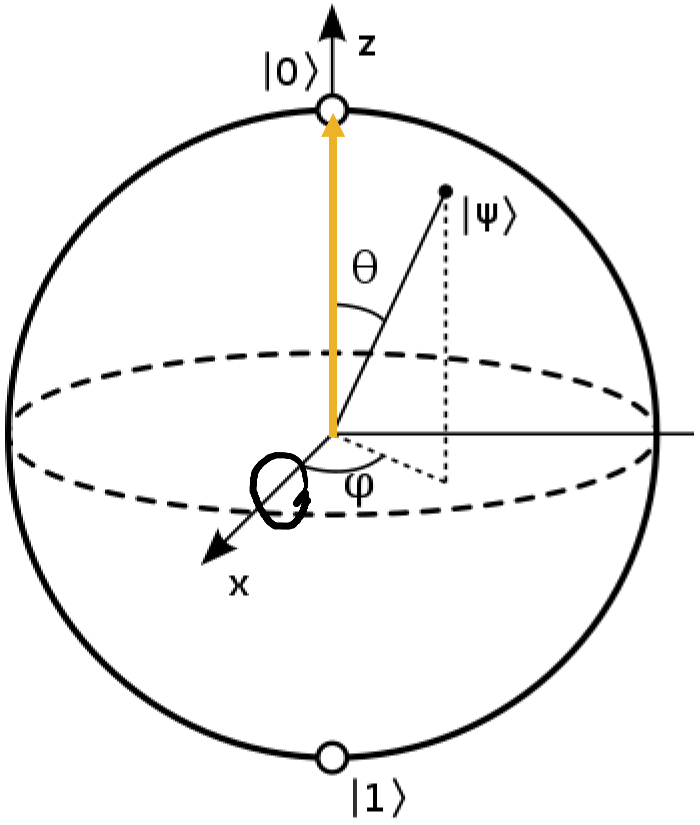
GENERAL X-ROTATION GATE

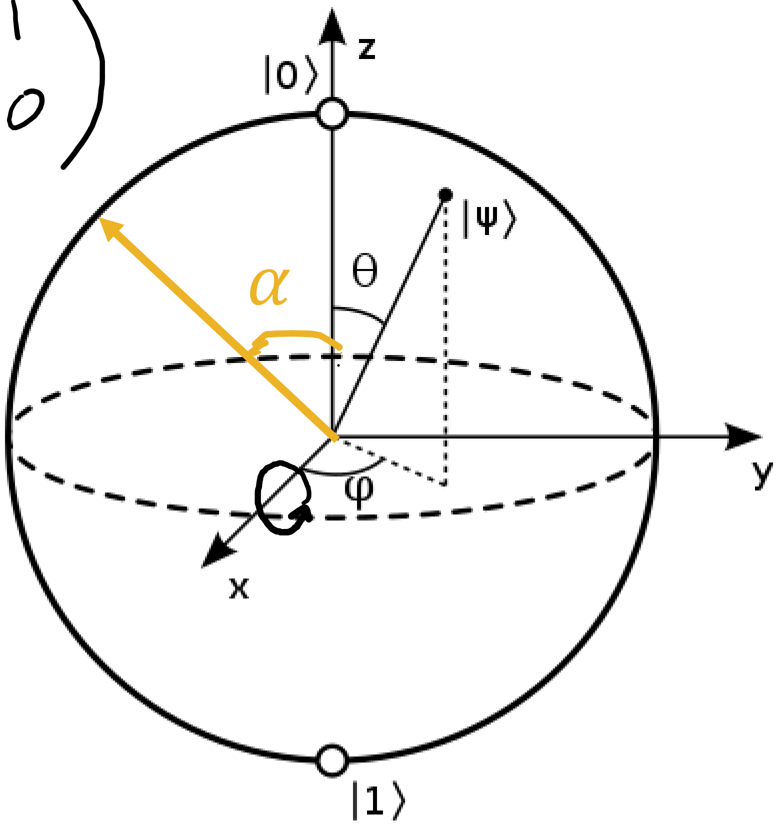


$$\begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} = -i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

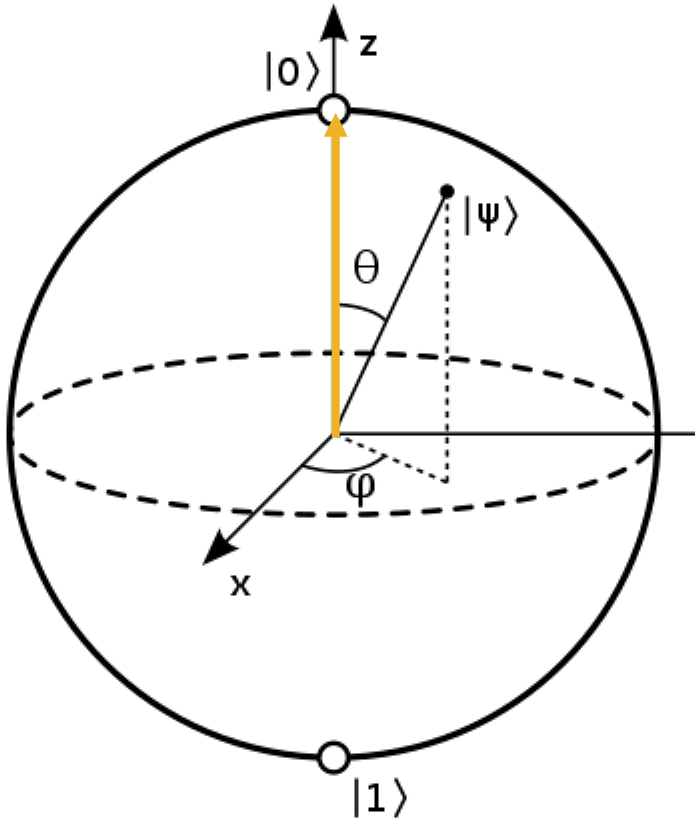
$R_X(\pi)$ X

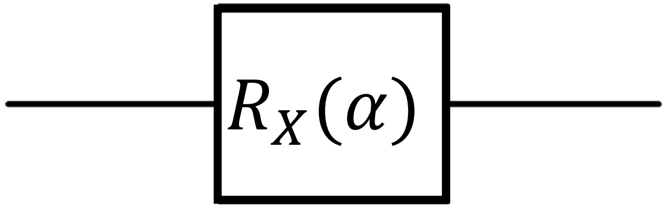
$R_X(\alpha)$

$$\begin{pmatrix} \cos\frac{\alpha}{2} & -i\sin\frac{\alpha}{2} \\ -i\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{pmatrix}$$

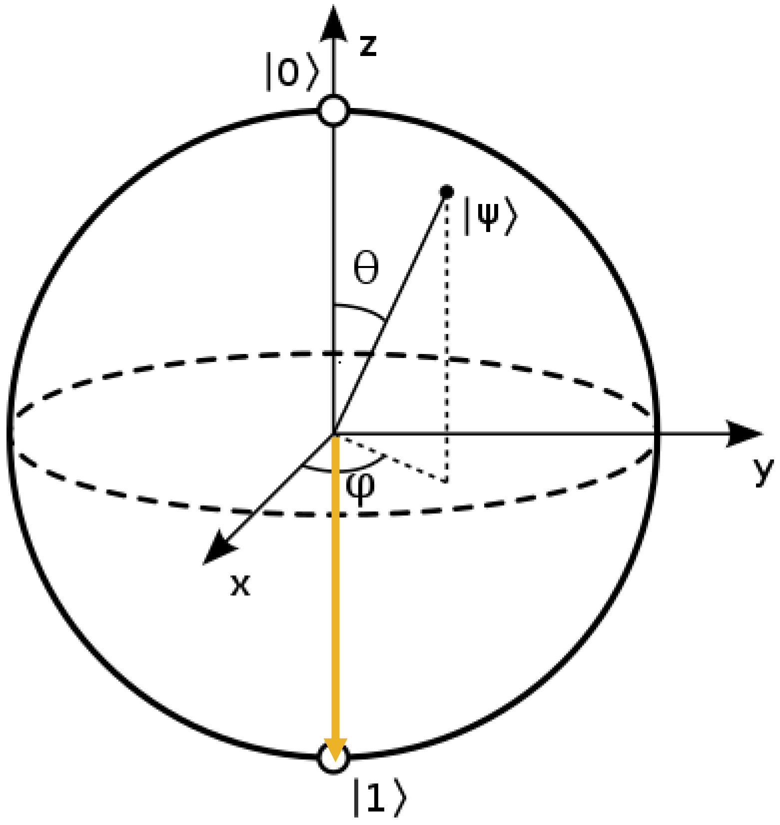


GENERAL X-ROTATION GATE





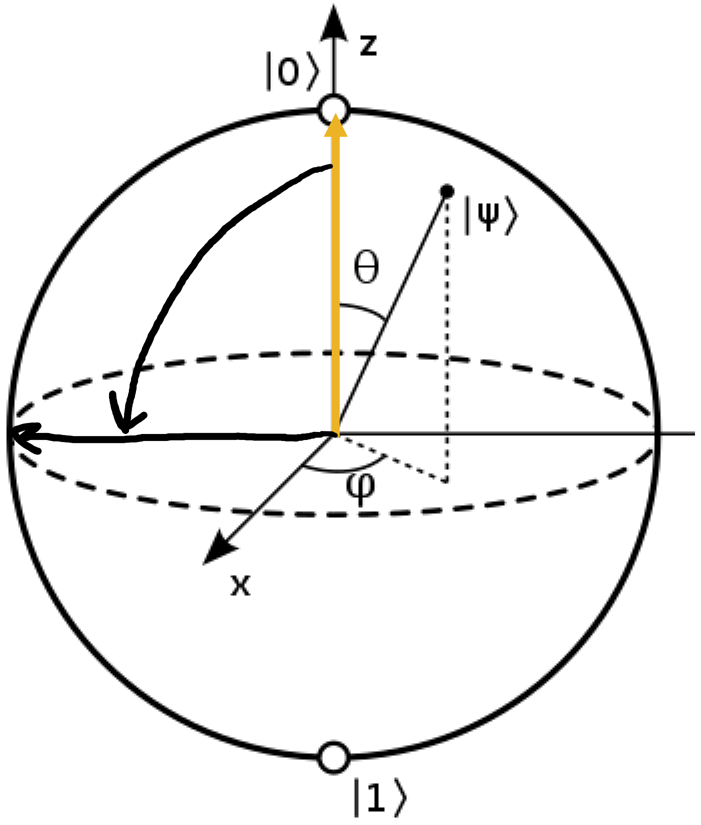
$$\begin{pmatrix} \cos\frac{\alpha}{2} & -i\sin\frac{\alpha}{2} \\ -i\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{pmatrix}$$



Problem 1: Taking Santa to see penguins

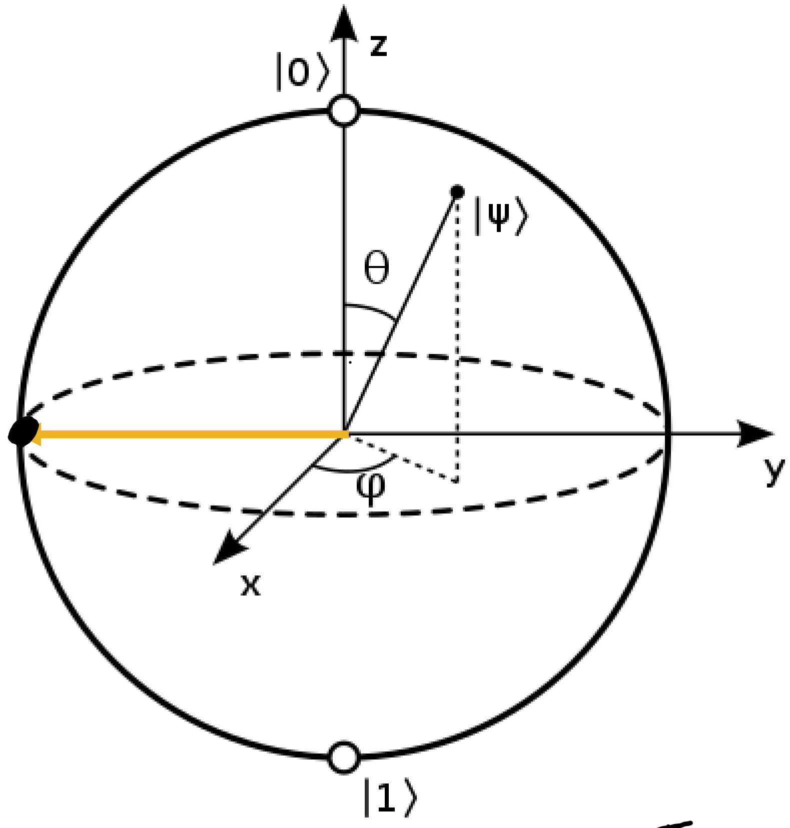
What is α, if we apply the R_X gate to $|0\rangle$ and we want the final state to be $|1\rangle$? $\alpha = 180^\circ = \pi$

GENERAL X-ROTATION GATE



$R_X(\alpha)$

$$\begin{pmatrix} \cos\frac{\alpha}{2} & -i\sin\frac{\alpha}{2} \\ -i\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{pmatrix}$$

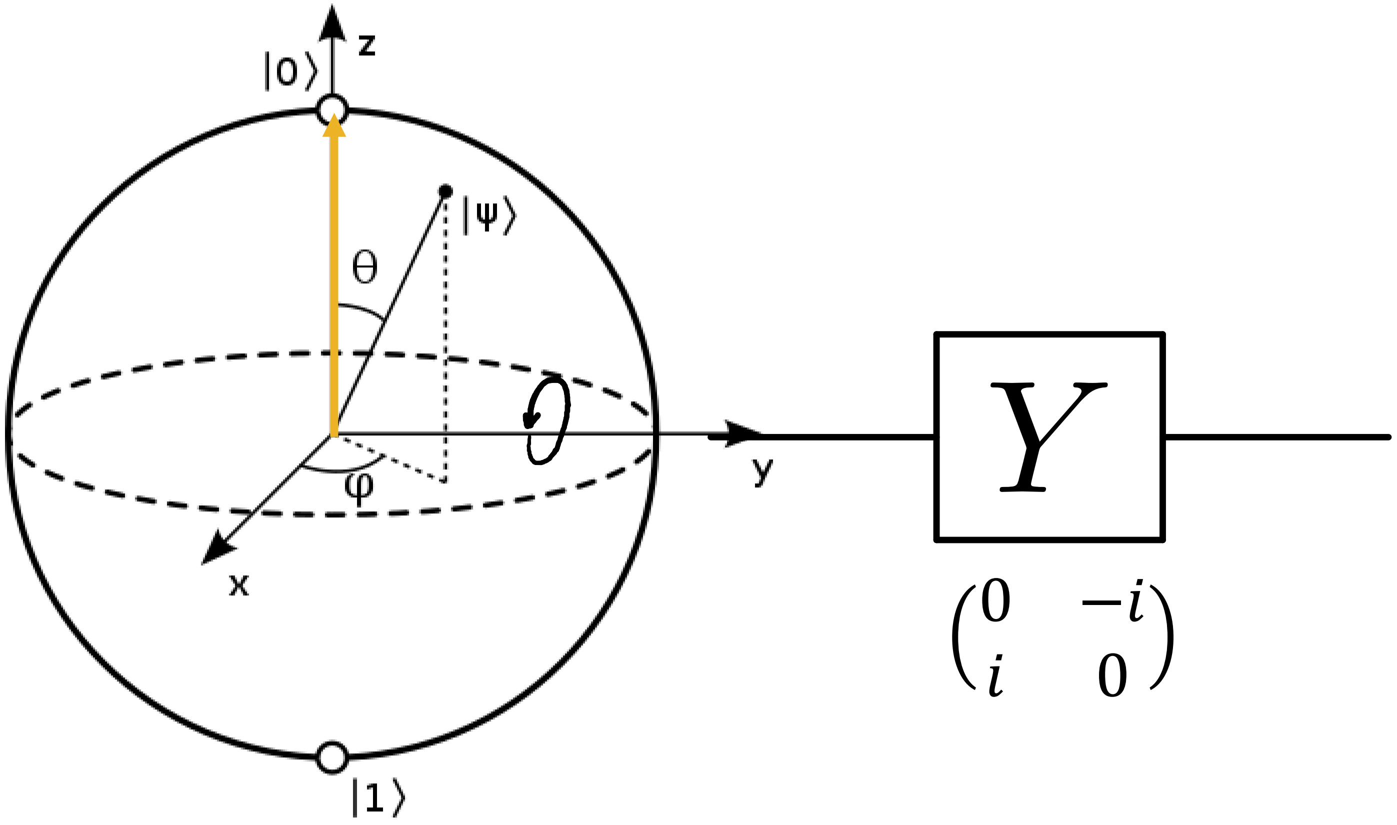


$\alpha = 90^\circ = \frac{\pi}{2}$

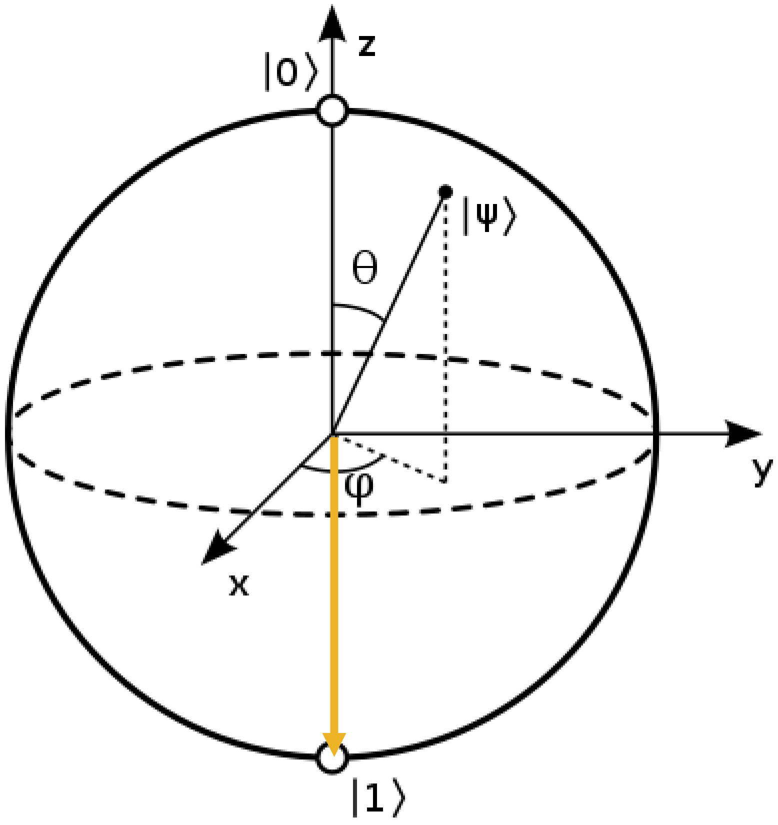
Problem 2: Taking Santa to the beach

What is α, if we apply the R_X gate to $|0\rangle$ and we want the final state to be $\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$?

APPLYING THE Y GATE



$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$



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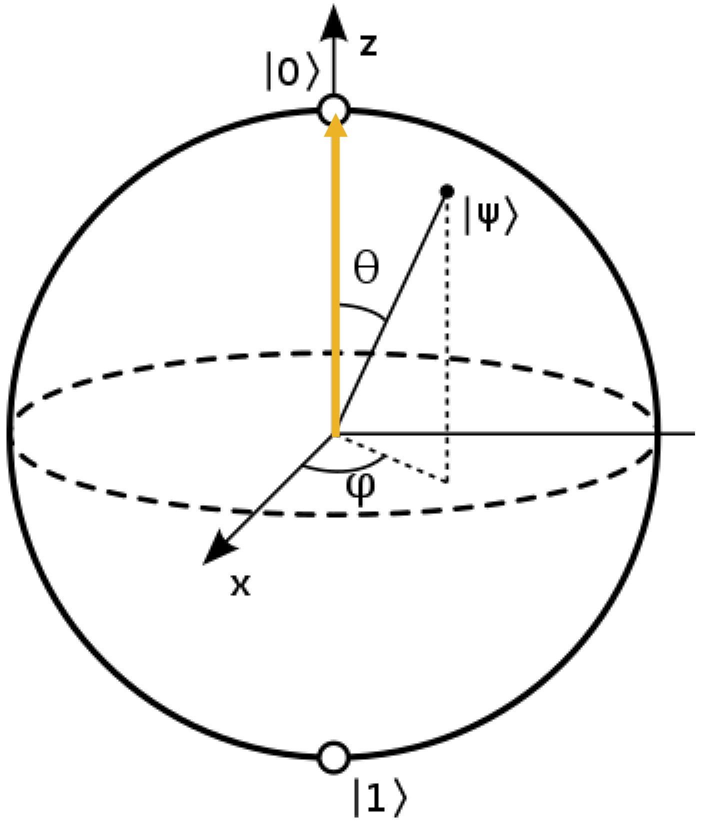
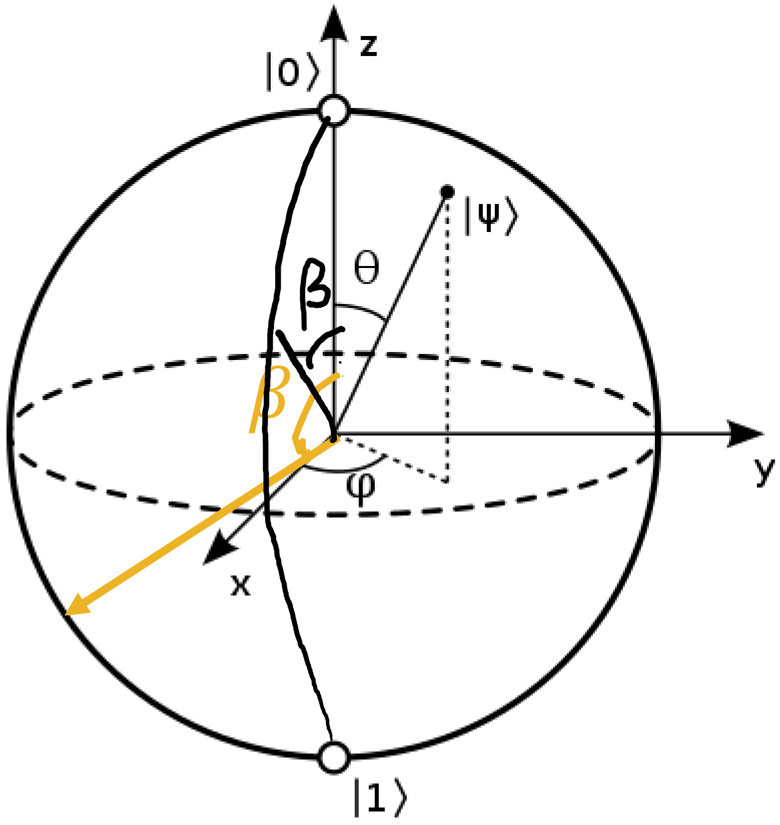
GENERAL Y-ROTATION GATE

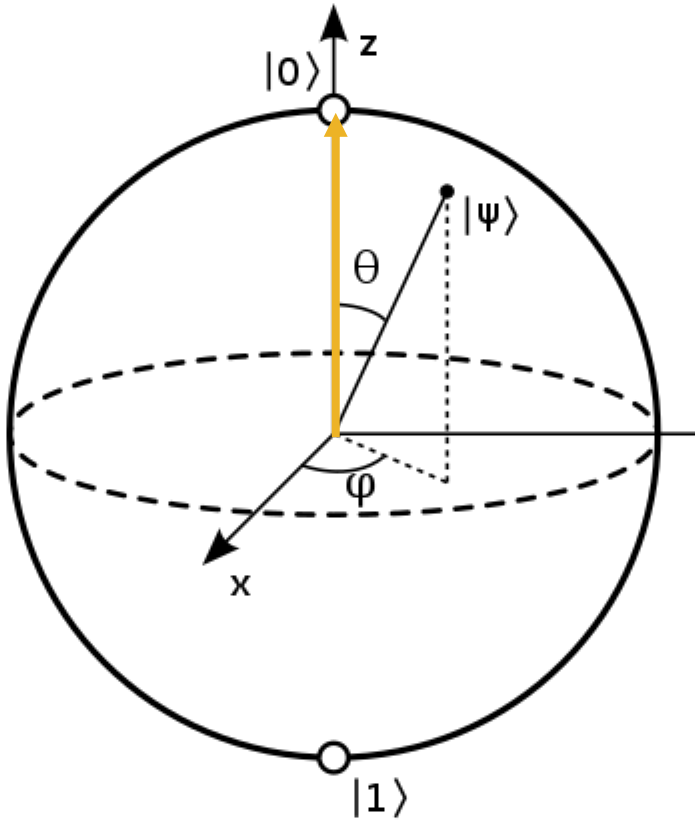


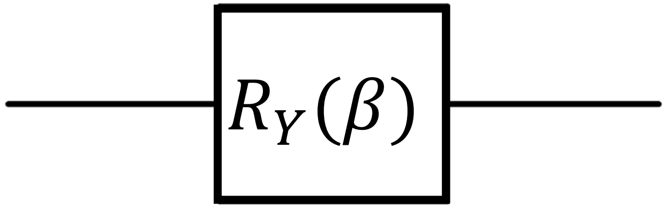
$$R_Y(\beta)$$

$$\begin{pmatrix} \cos\frac{\beta}{2} & -\sin\frac{\beta}{2} \\ \sin\frac{\beta}{2} & \cos\frac{\beta}{2} \end{pmatrix}$$

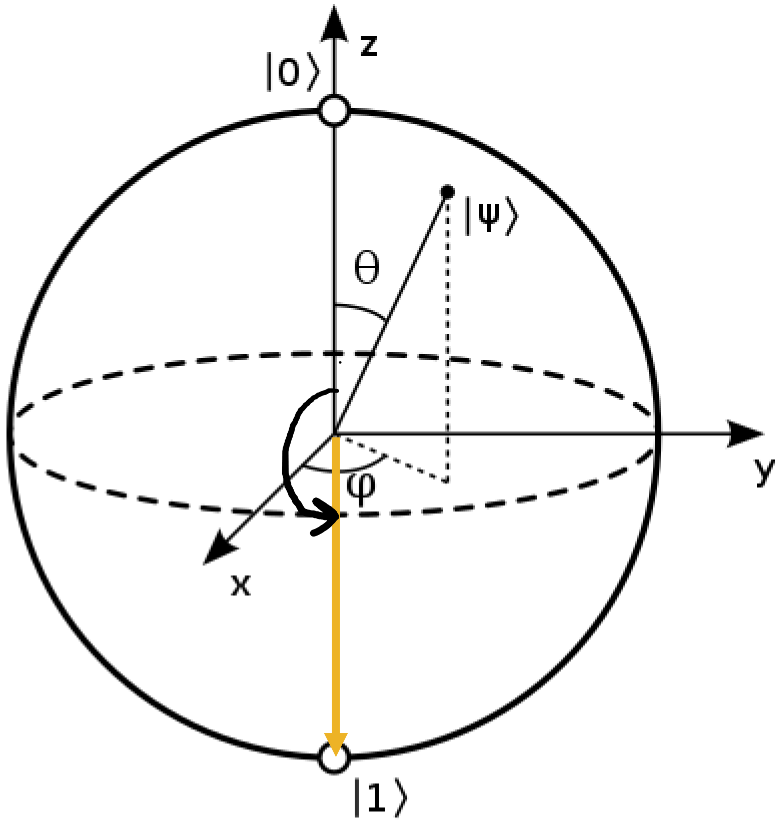


GENERAL Y-ROTATION GATE





$$\begin{pmatrix} \cos\frac{\beta}{2} & -\sin\frac{\beta}{2} \\ \sin\frac{\beta}{2} & \cos\frac{\beta}{2} \end{pmatrix}$$



Problem 3: What is β, if we apply the R_Y gate to $|0\rangle$ and we want the final state to be $|1\rangle$?

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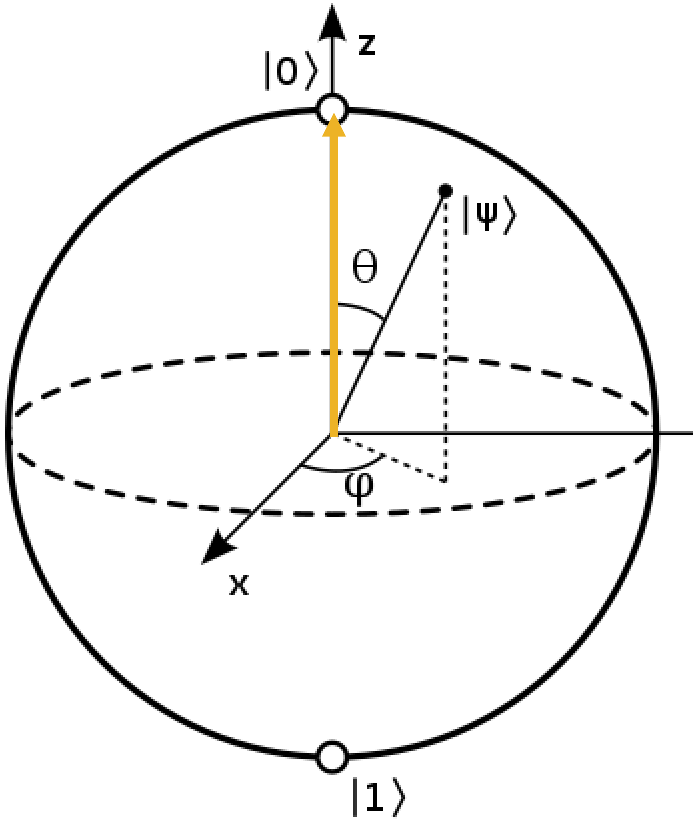
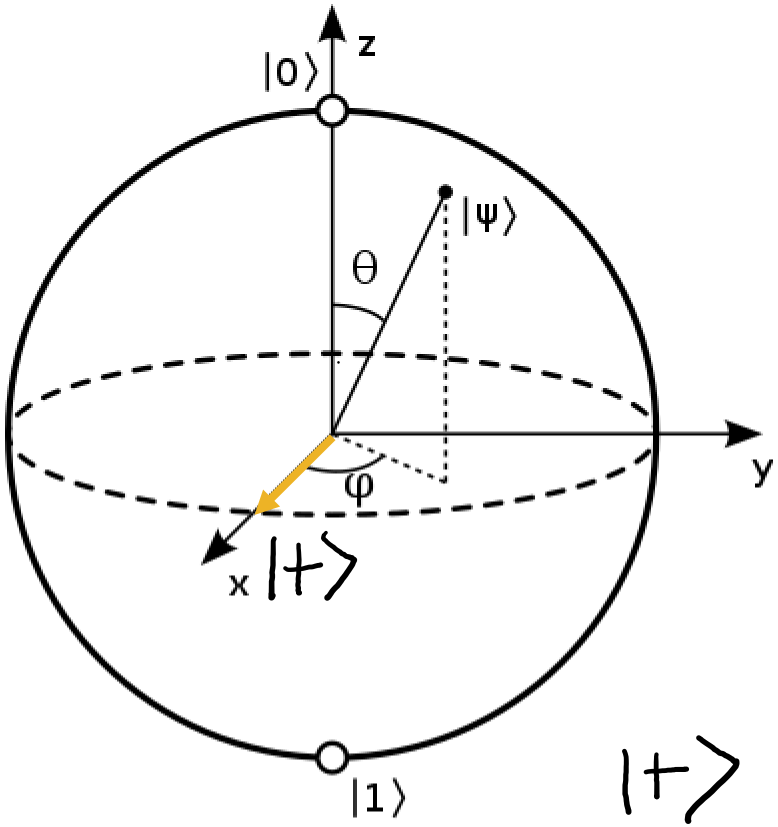
GENERAL Y-ROTATION GATE



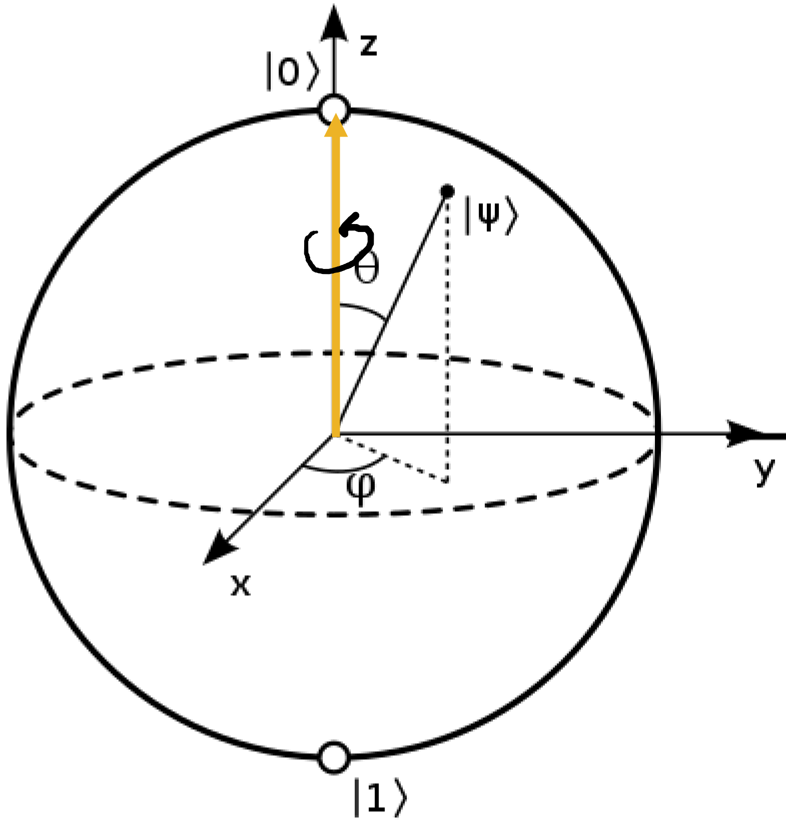
$$R_Y(\beta)$$

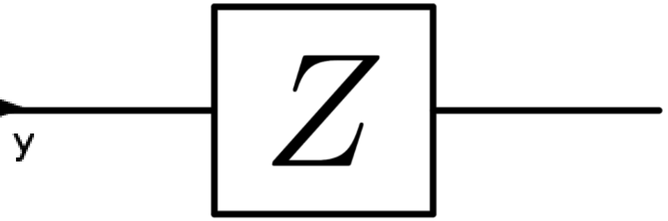
$$\begin{pmatrix} \cos\frac{\beta}{2} & -\sin\frac{\beta}{2} \\ \sin\frac{\beta}{2} & \cos\frac{\beta}{2} \end{pmatrix}$$



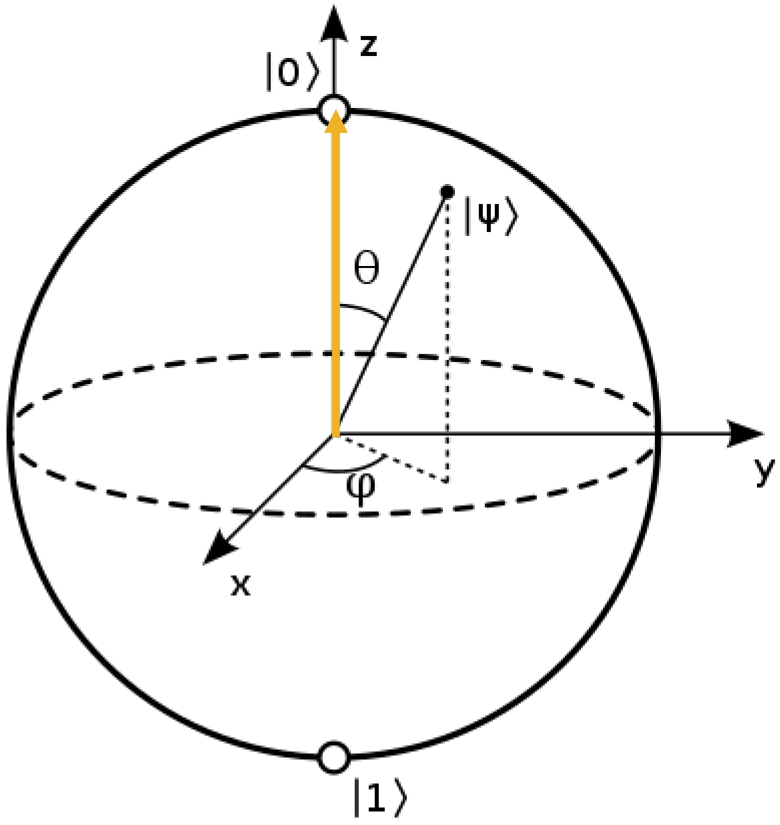
Problem 4: What is β, if we apply the R_Y gate to $|0\rangle$ and we want the final state to be $\overbrace{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)}^{|+\rangle}$?

APPLYING THE Z GATE





$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



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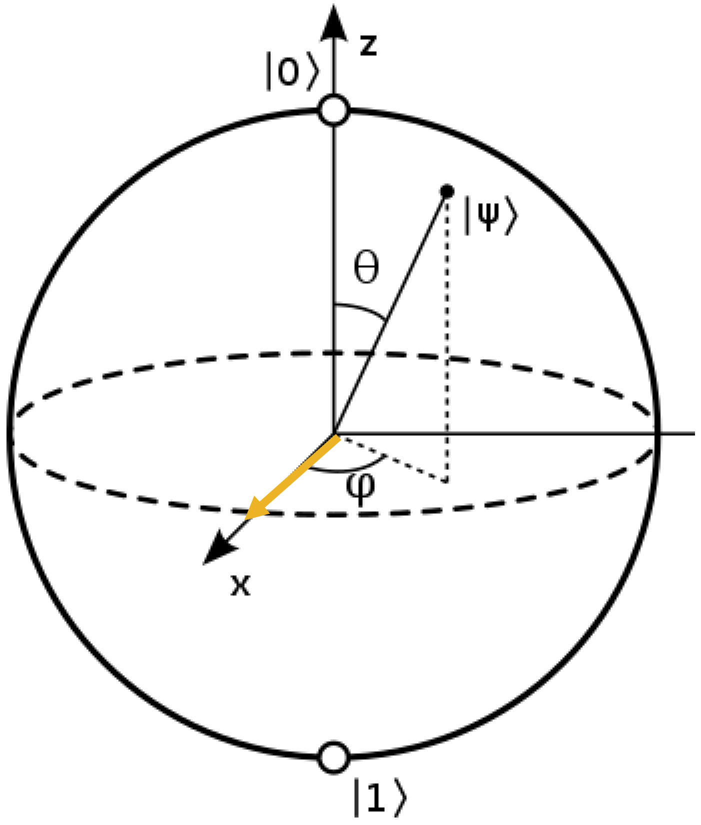
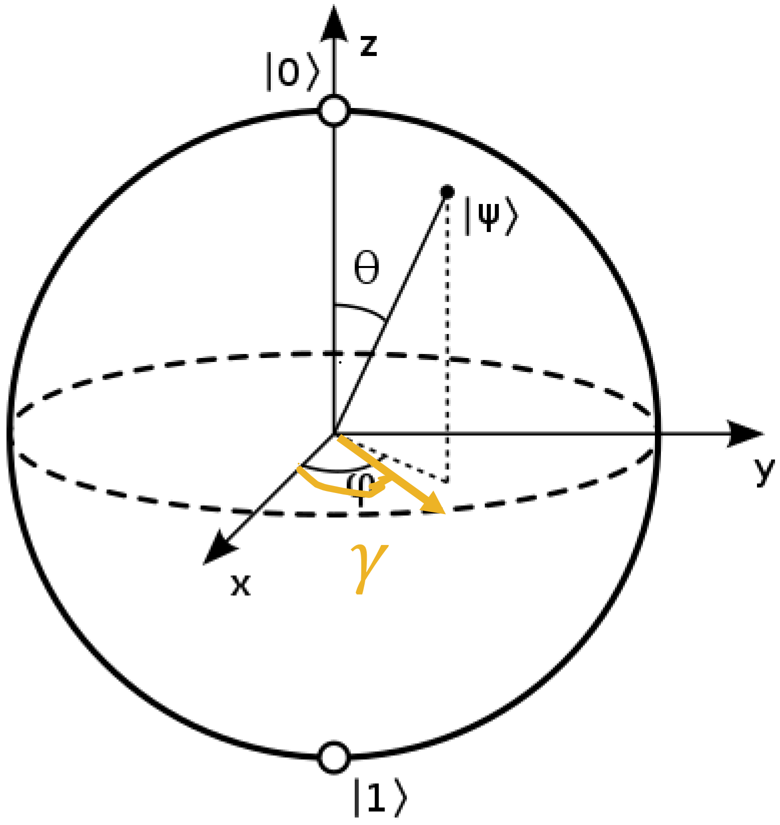
GENERAL Z-ROTATION GATE



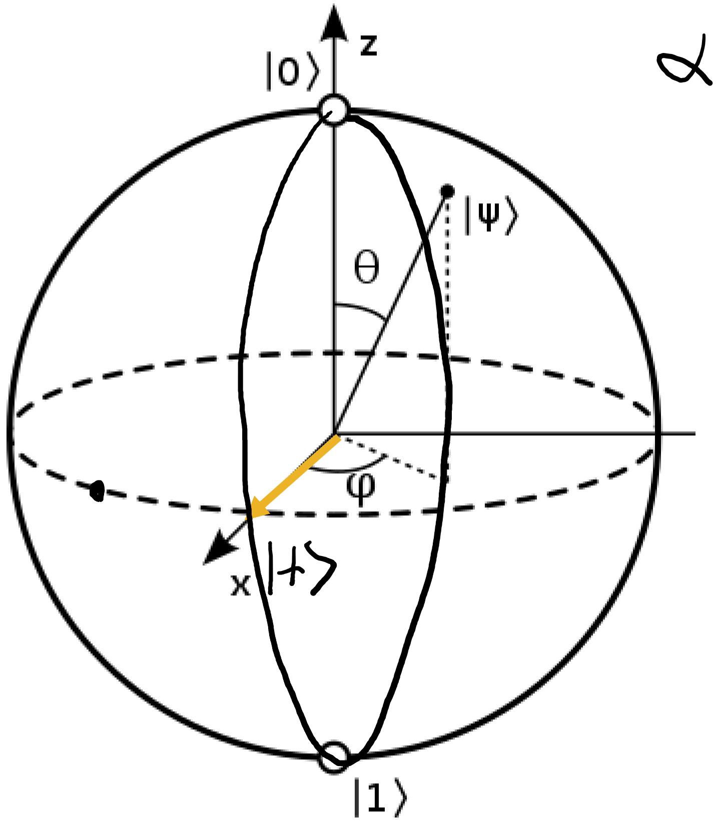
$R_z(\gamma)$

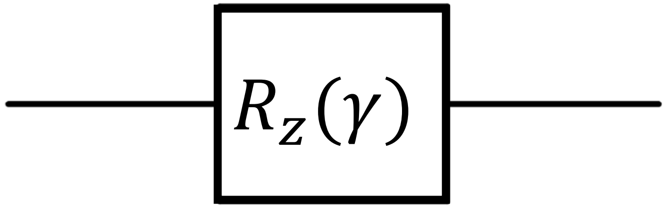
$$\begin{pmatrix} e^{-i\gamma/2} & 0 \\ 0 & e^{i\gamma/2} \end{pmatrix}$$



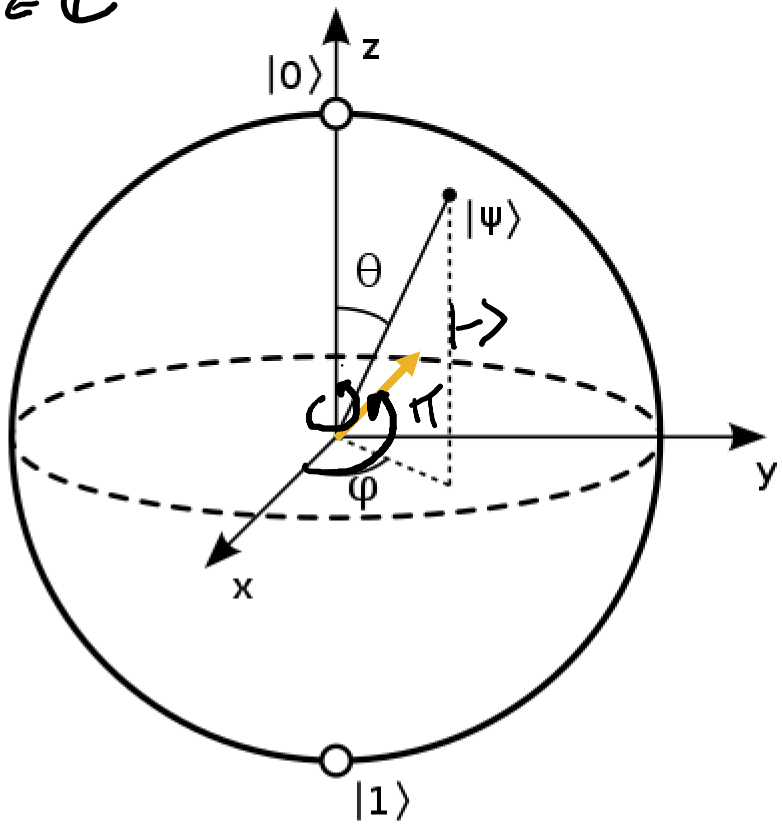
GENERAL Z-ROTATION GATE

$\alpha|0\rangle + \beta|1\rangle, \ \alpha \in \mathbb{R}, \ \beta \in \mathbb{C}$





$$\begin{pmatrix} e^{-i\gamma/2} & 0 \\ 0 & e^{i\gamma/2} \end{pmatrix}$$



Problem 5: What is γ, if we apply the R_Z gate to $|+\rangle$ and we want the final state to be $|-\rangle$?

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KEY TAKEAWAYS

- The Bloch sphere is used to represent single qubit states
- Applying a gate to a single qubit is equivalent to rotating the qubit on the Bloch sphere
- The rotation gates along x, y, and z-axes generalize the X, Y, and Z gates.
  - By combining R_X, R_Y, and R_Z gates, we can make any arbitrary qubit transformation

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COMING UP …

- **Next week: IBM Quantum Experience**
  - Applying gates to qubits
  - Combining gates
  - Visualizing the result of applying gates on qubits
  - Creating circuits for superposition, entanglement, etc.

- **In two weeks: Qiskit**
  - Writing code to define qubits states and gates
  - Applying gates to qubits
  - Developing code to implement quantum algorithms

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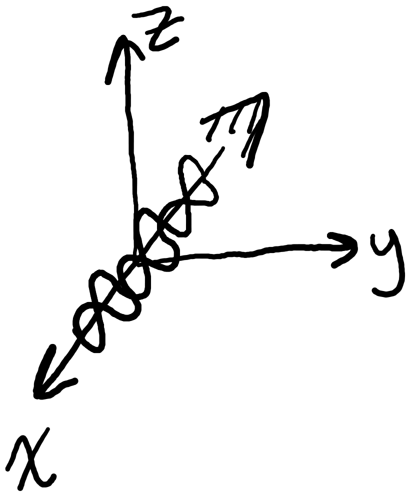
FURTHER READING AND RESOURCES

- https://www.youtube.com/watch?v=MBnnXbOM5S4 – 3Blue1Brown video on the uncertainty principle, as it applies to ripples

- https://www.youtube.com/watch?v=vUVkS1XZVCc – Lecture on the Bloch sphere by Prof. Umesh Vazirani

- https://www.scottaaronson.com/blog/?p=4021 – Blog post by Prof. Scott Aaronson on why quantum amplitudes use complex numbers

QUESTIONS?

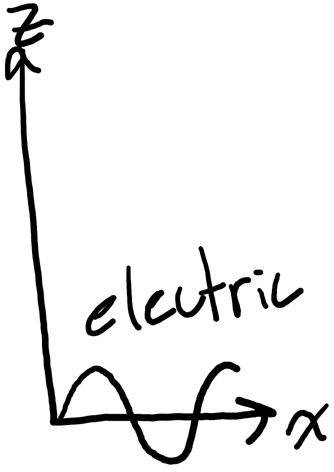


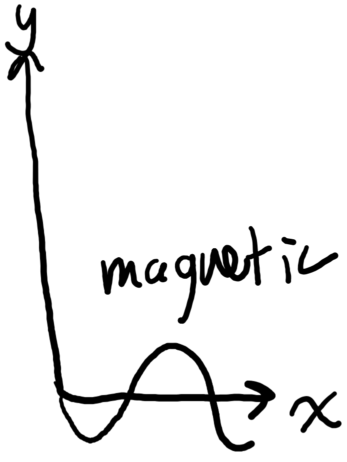
$|V\rangle = |0\rangle$

$|H\rangle = |1\rangle$

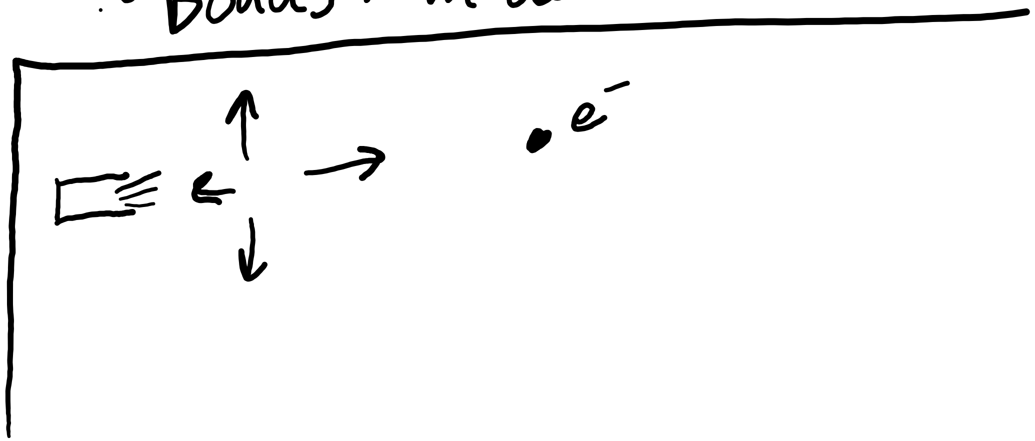
- ✓ Physical phase
- ✓ Grover search
- ✓ Mixed states: $|0\rangle$, 50% ; $|+\rangle$, 50%

Questions on content so far?





- ✓ Bonus: modes



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POST-LAB ZOOM FEEDBACK

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- 5 – The content was easy for me/I already knew all of the content

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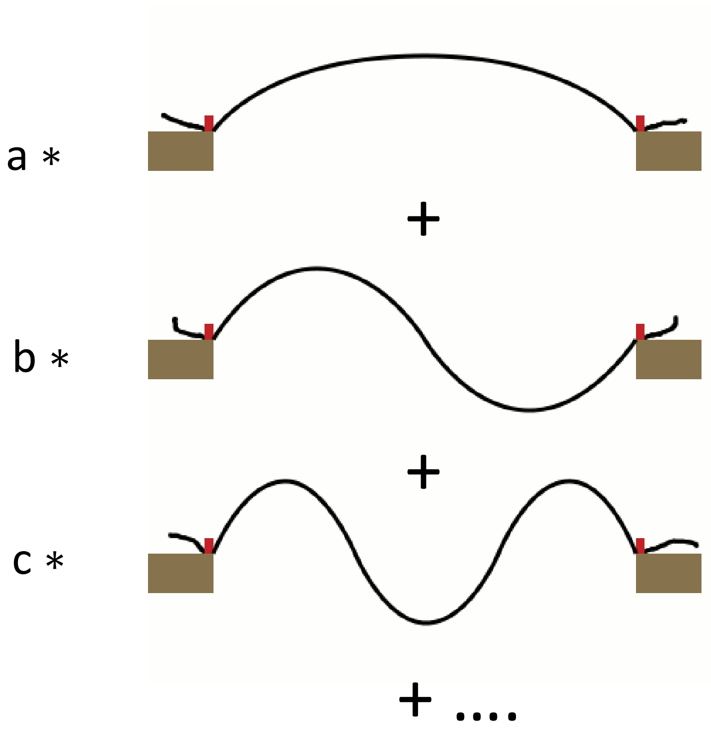
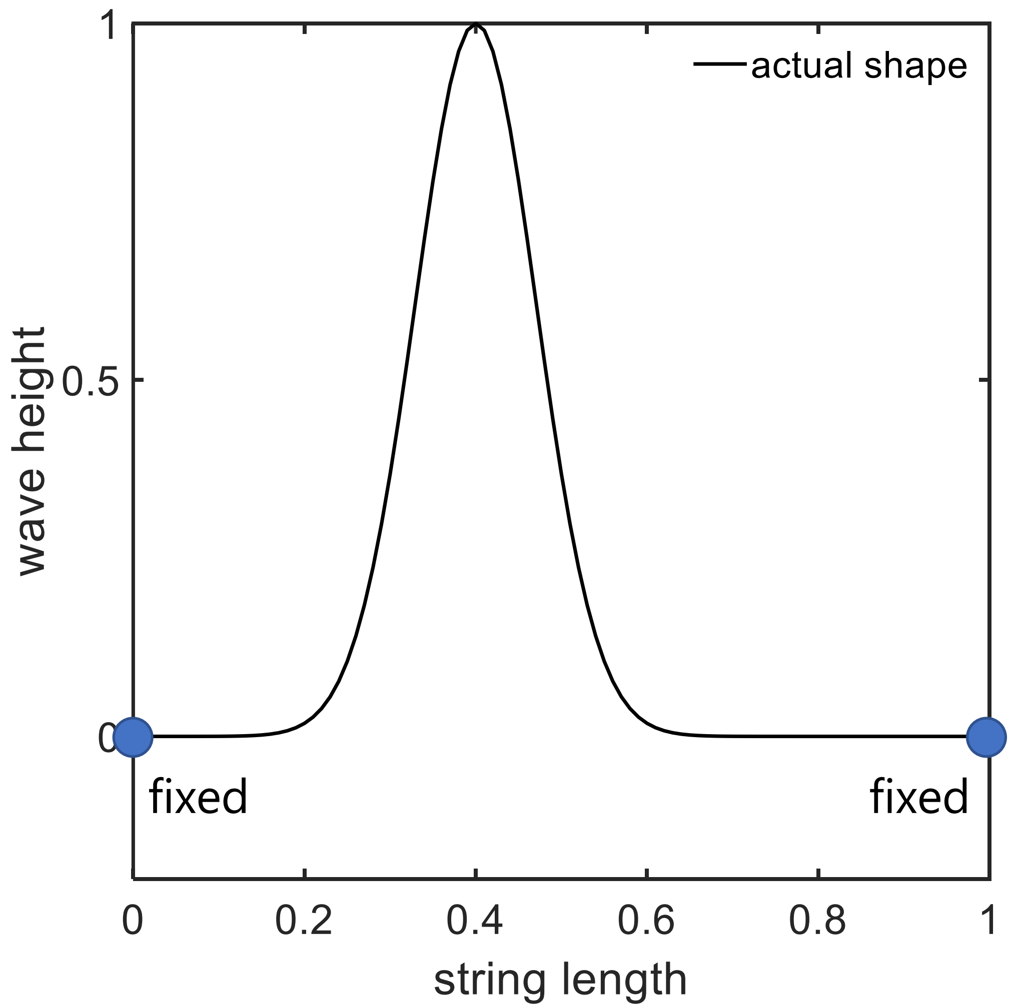
OPTIONAL CONTENT

DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS

The ripple can be "reconstructed" by adding up the modes

actual shape of string =





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THE VIOLIN VS THE PIANO

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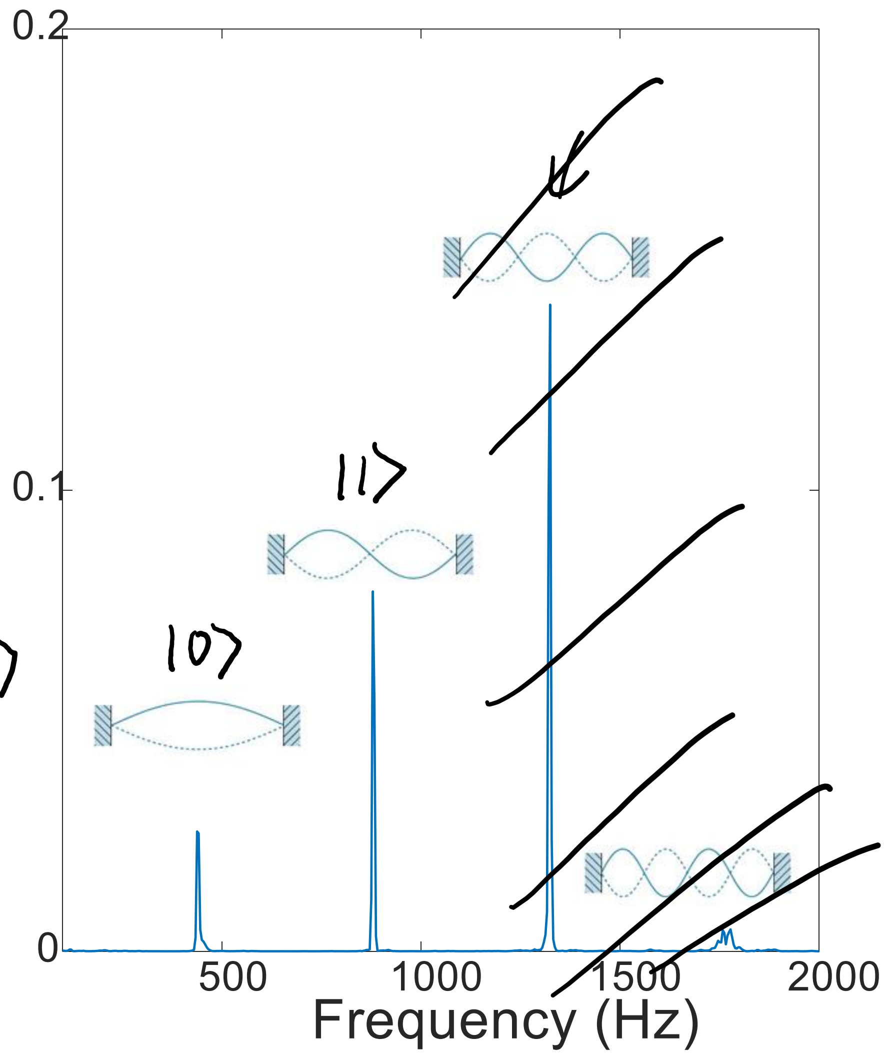
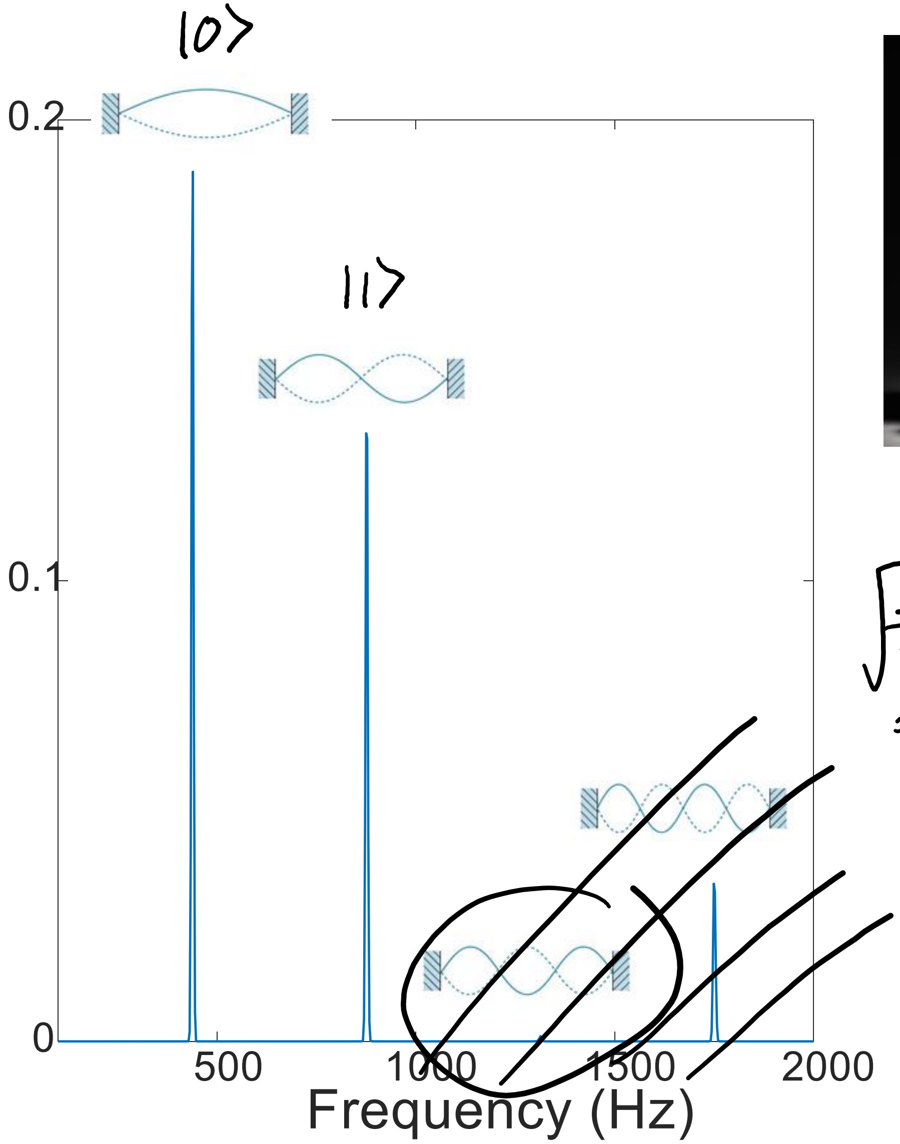
VIOLIN VS PIANO MODES

$$\sqrt{\tfrac{1}{4}}|0\rangle + \sqrt{\tfrac{3}{4}}|1\rangle$$





$$\sqrt{\tfrac{3}{5}}|0\rangle + \sqrt{\tfrac{2}{5}}|1\rangle$$

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CONTRIBUTION OF MODES TO WAVEFUNCTIONS

- The same note played on the violin and piano is different, because the relative contributions (amplitudes) of the different modes (eigenstates) are different

- By changing the relative contribution of the eigenstates in a qubit, we can change the qubit state (wavefuction)

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