INTRO TO QUANTUM COMPUTING

LECTURE #14

# The Qubit & Bloch Sphere

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# ANNOUNCEMENTS

# TODAY'S LECTURE

➔ Use the Bloch Sphere to conceptualize the state of a qubit

➔ View quantum gates as actions on the Bloch sphere

# What is a qubit?

- Building block of quantum computers
- A two-level system
- Can be in a superposition of two values

# Qubit Reviewed

**Superposition:** a qubit can be $|0\rangle$ and $|1\rangle$ at the same time!

This is how we show it: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$|0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

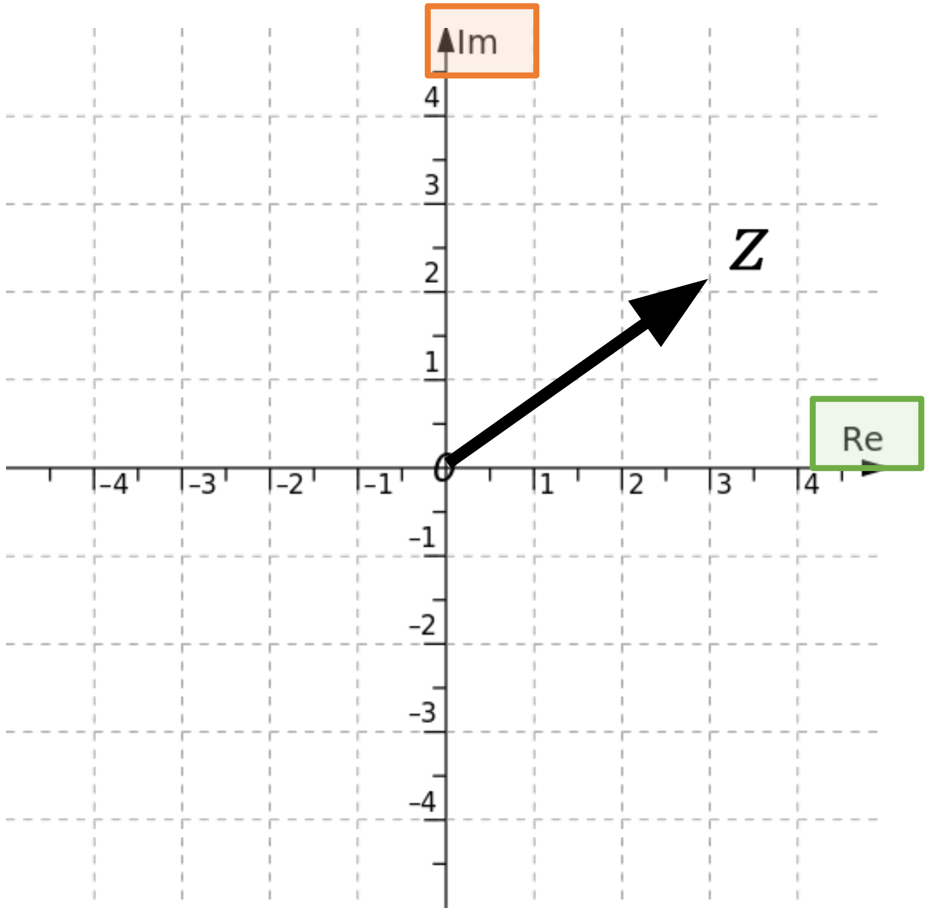
# Complex Numbers Review

A complex number consists of both a ***real*** and ***imaginary*** component.

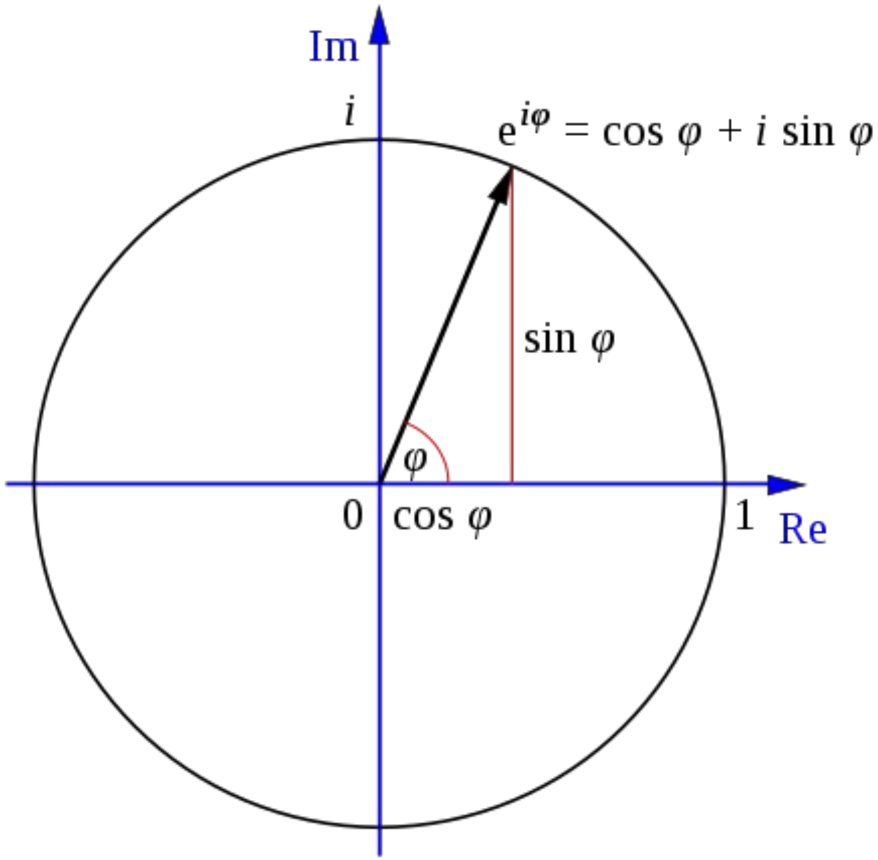
$$z = a + i\, b$$

$$z^* = a - i\, b$$

$$|z|^2 = z \cdot z^* = z^* \cdot z$$



# EULER'S FORMULA & COMPLEX EXPONENTIALS




Source: Wikipedia (CC)

**Euler's formula:**

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

Polar representation of complex numbers!

$$z = x + iy = |z|(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

$$r = |z| = \sqrt{x^2 + y^2}$$

(vector radius)

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

(vector angle)

# What is phase?

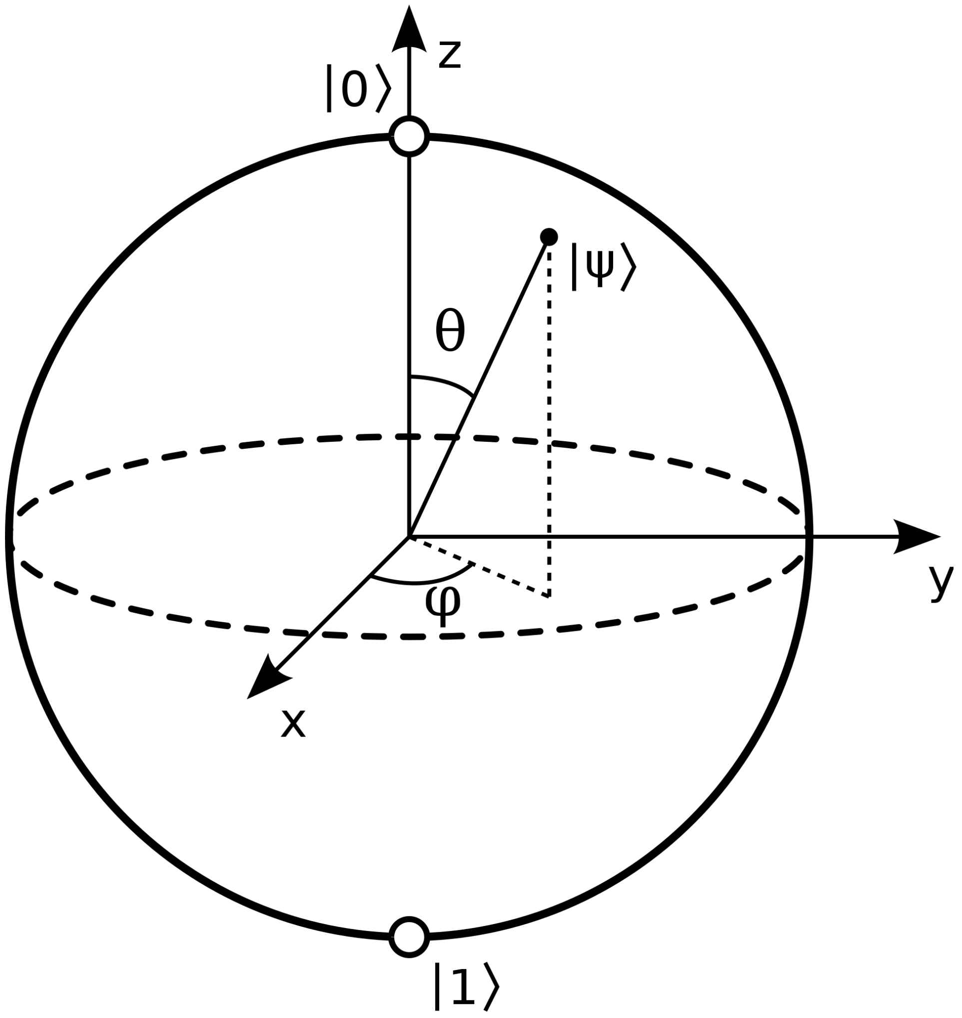
$$\boxed{e^{i\varphi}}$$

In quantum mechanics global phase has **NO *physical significance*** and **NO *effect*** therefore can be ignored!!

$$e^{i\varphi}\,|\psi\rangle \rightarrow |\psi\rangle$$

# Bloch Sphere

The ***state*** of a qubit can be represented as a point on the Bloch Sphere



# Qubit

Back to our qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

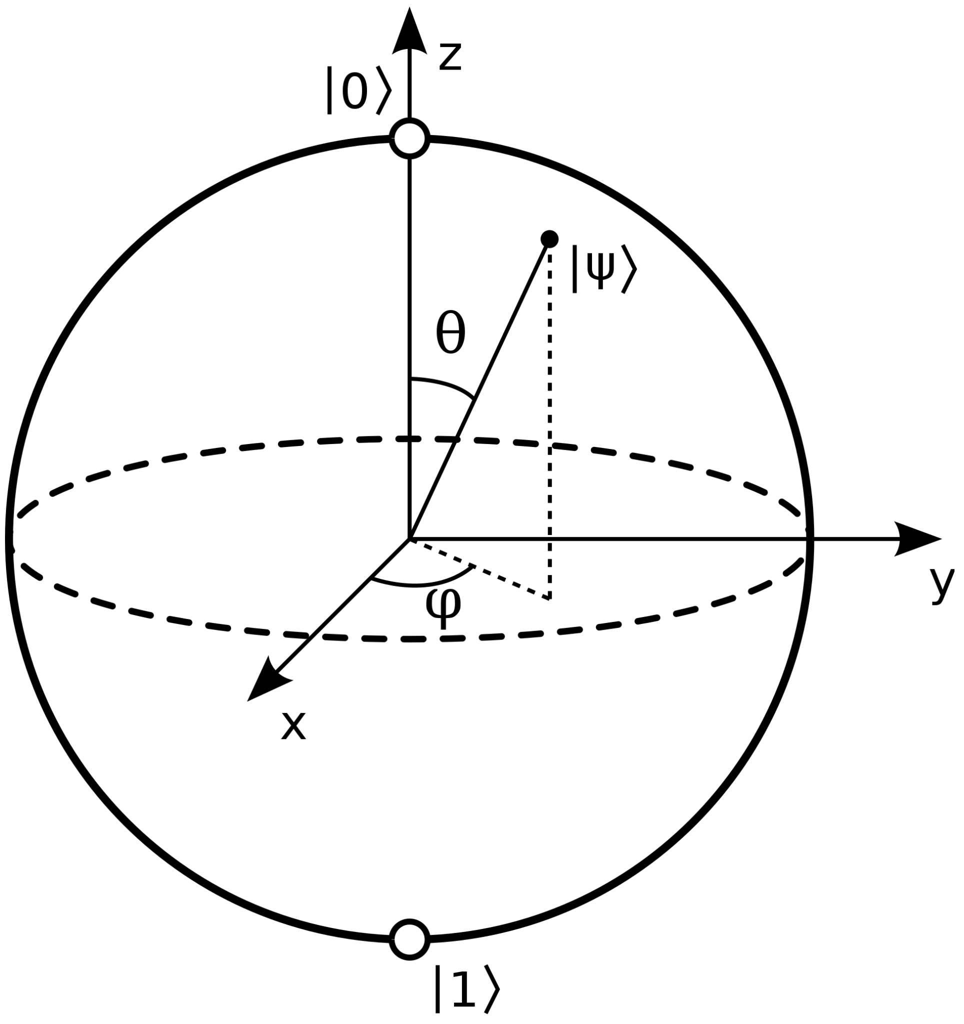
# Bloch Sphere

$|\psi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)\, e^{i\varphi}|1\rangle$

North pole: $|0\rangle$

South pole: $|1\rangle$

Classical bits are only on the North pole or the South pole



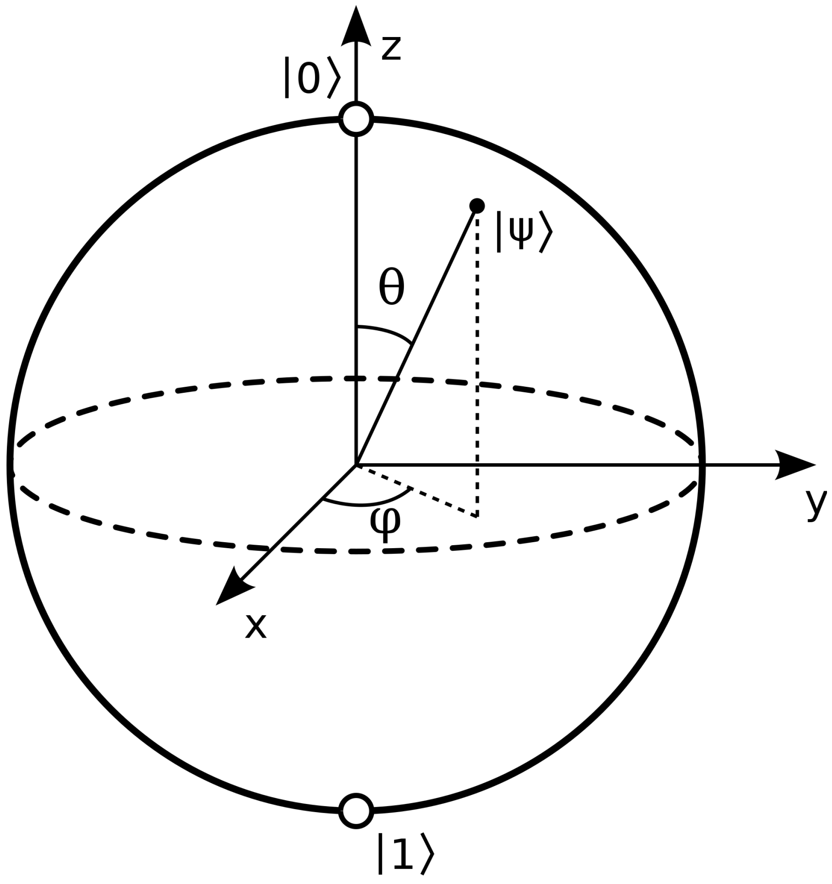
# Bloch Sphere

$|\psi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)\, e^{i\varphi}|1\rangle$

What is the state of the qubit based on the position on the Bloch sphere?

$\theta=0$:

$\theta=\pi$:



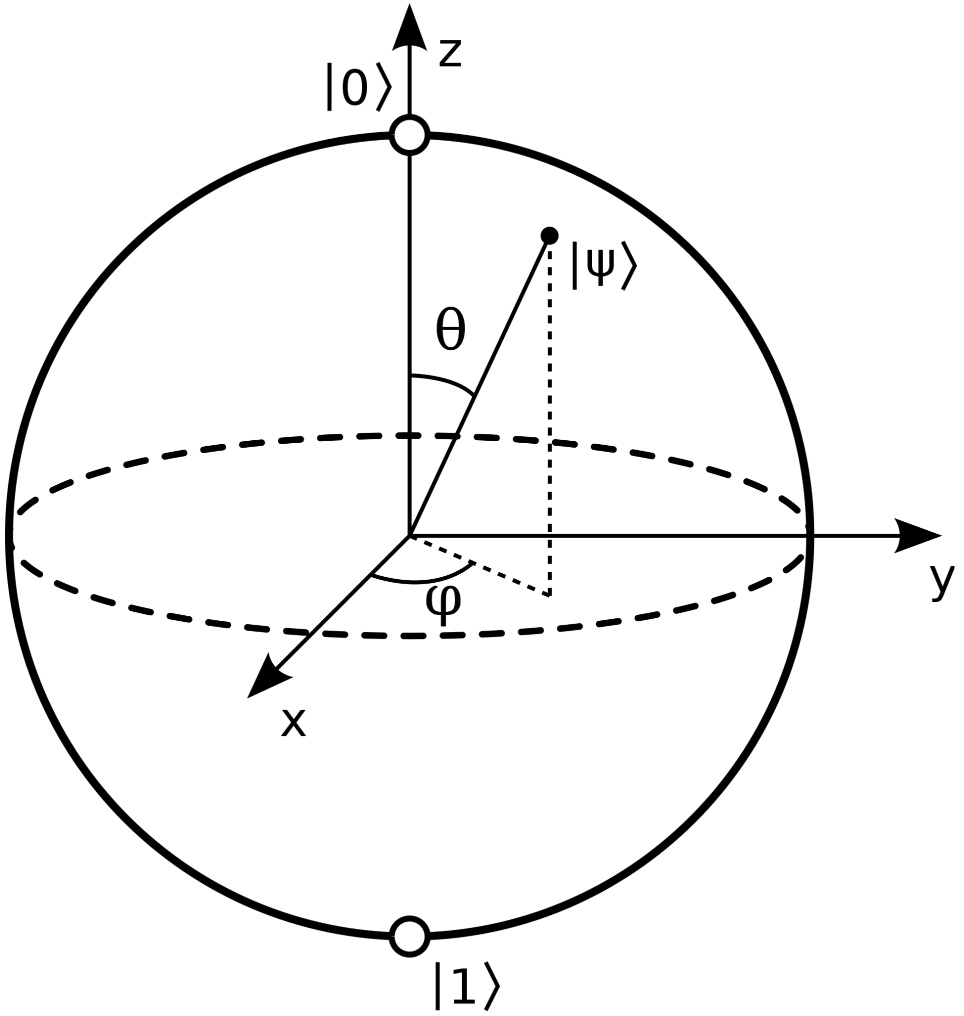
# Bloch Sphere

$|\psi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)\, e^{i\varphi}|1\rangle$

More examples:

$\theta = \pi/2$ , $\varphi = 0$:

$\theta = \pi/2$, $\varphi = \pi$ :



# 10 MIN BREAK!

# Quantum gates

- Maps one the quantum state to another quantum state
- Moves a point on the Bloch Sphere to another point
- We can represent them with matrices

# Important Quantum Gates

Pauli gates:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Pauli-X operator (X)

$$\sigma_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$$

Pauli-Y operator (Y)

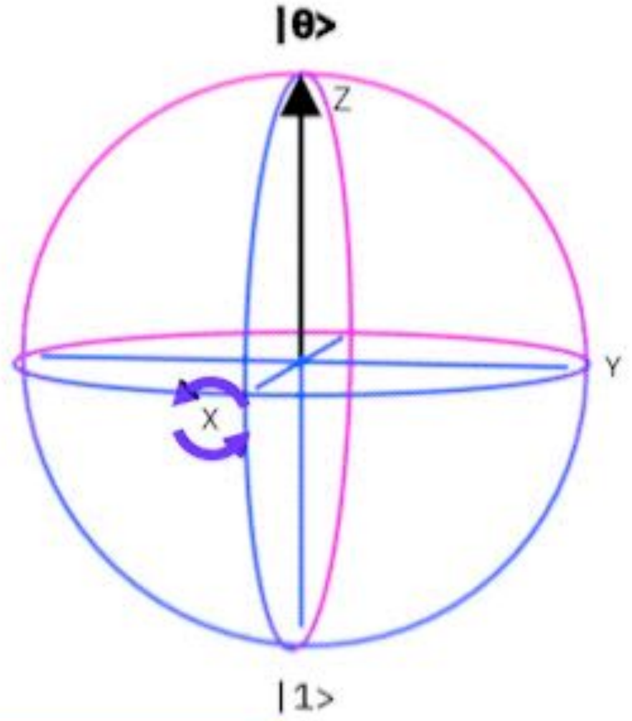
$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

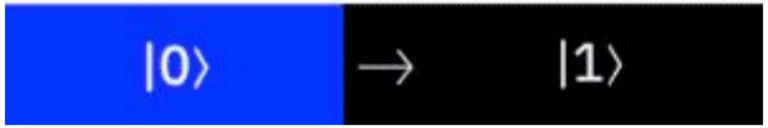
Pauli-Z operator (Z)

# Important Quantum Gates

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Pauli-X operator (X)



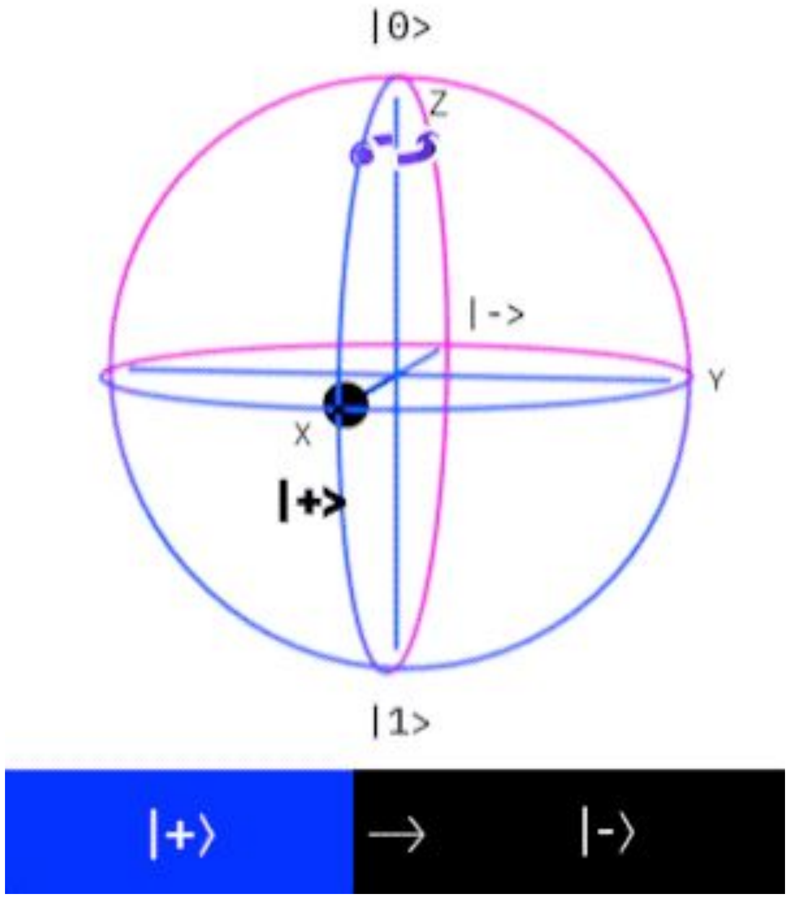




Rotates the state around the ***X*** axis by 180 degrees

# Important Quantum Gates

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli-Z operator (Z)

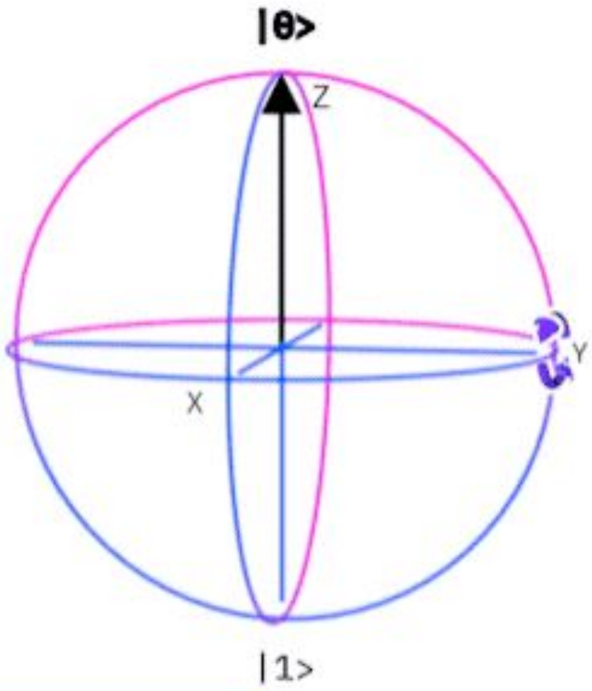


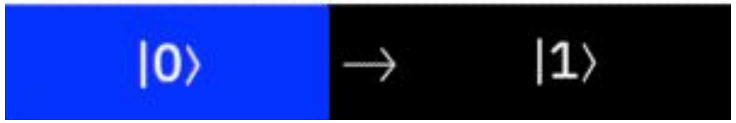

Rotates the state around the ***Z*** axis by 180 degrees

# Important Quantum Gates

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Pauli-Y operator (Y)




Rotates the state around the ***Y*** axis by 180 degrees

# Rotations on the Bloch Sphere

We're not just limited by 180 degree rotations

For example we can rotate around the x axis by 90 degrees

$$R_x(\pi/2)$$

rotation axis angle

# Rotations on the Bloch Sphere

We can rotate by an angle θ around x or y or z axis:

$$R_x(\theta) = e^{-i\frac{\theta}{2}\sigma_x}$$

$$R_y(\theta) = e^{-i\frac{\theta}{2}\sigma_y}$$

$$R_z(\theta) = e^{-i\frac{\theta}{2}\sigma_z}$$

# Rotations on the Bloch Sphere

Let’s take rotation by an angle θ around the x axis as an example:

$$R_x(\theta) = e^{-i\frac{\theta}{2}\sigma_x} = \cos\frac{\theta}{2}\,I - i\sin\frac{\theta}{2}\,\sigma_x$$

Matrix representation?

# Rotations on the Bloch Sphere

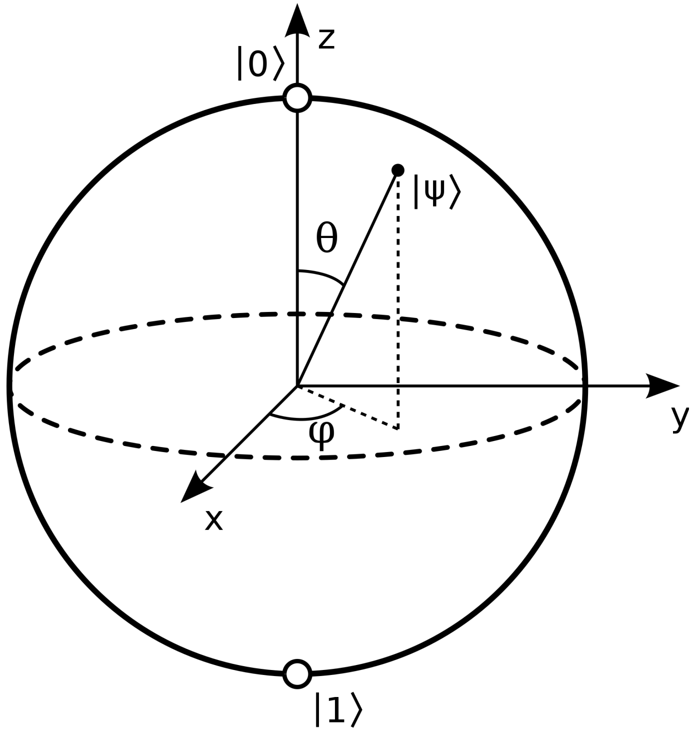
Let’s take rotation by an angle θ around the x axis as an example:

$$R_z(\theta) = e^{-i\frac{\theta}{2}\sigma_z} = \cos\frac{\theta}{2}\, I \; - i\sin\frac{\theta}{2}\,\sigma_z$$

Matrix representation?

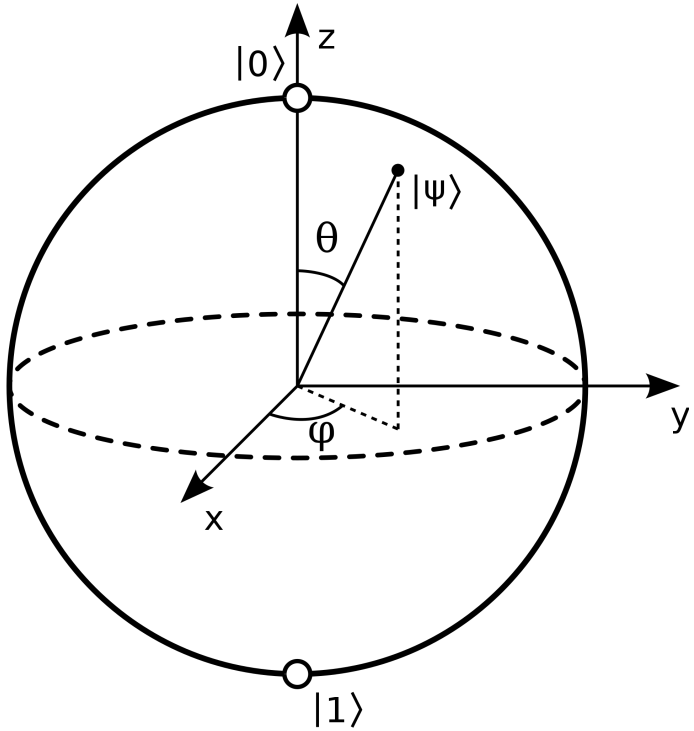
# Rotations on the Bloch Sphere

Examples:



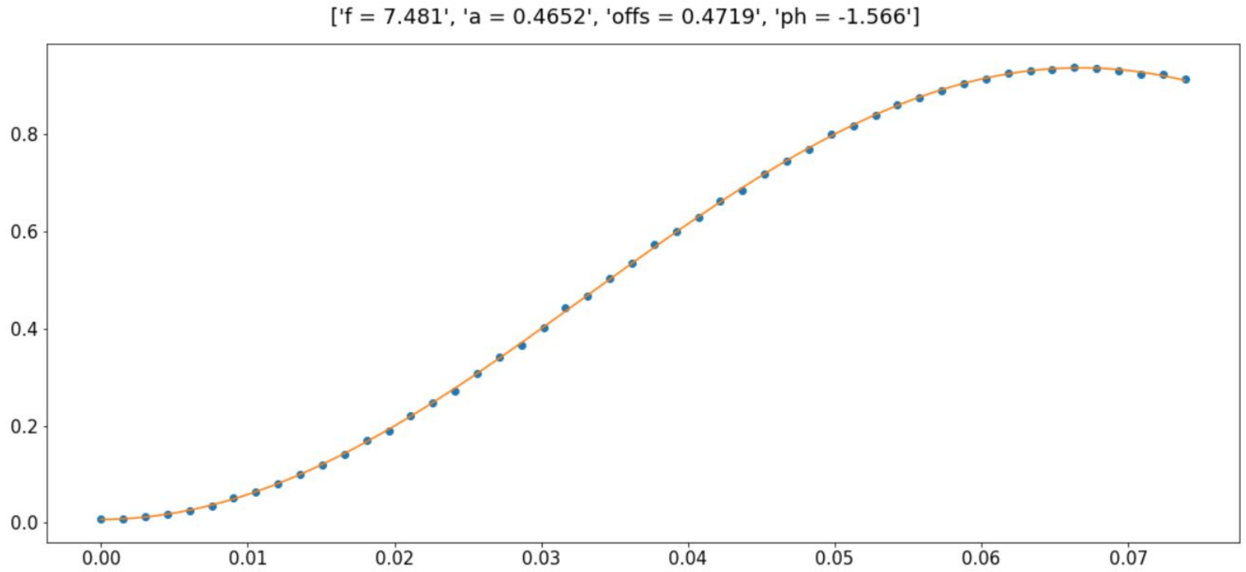
# Rotations on the Bloch Sphere

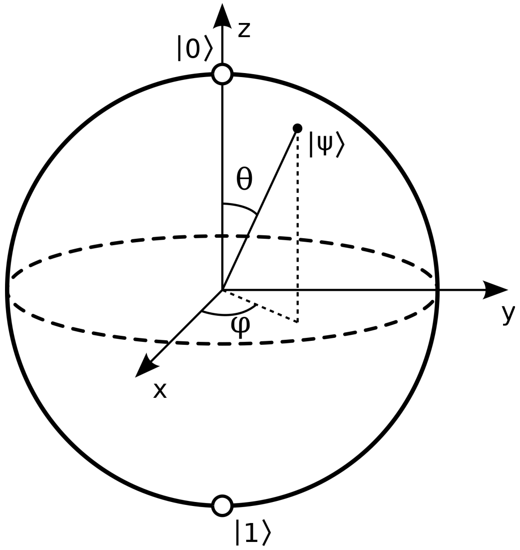
Examples:



# Experimental Implementations

- All quantum gates are engineered using rotations on the Bloch sphere
- In our quantum hardware we can control each transition with some level of accuracy
- Example: Rabi oscillation:



# Quantum Universality

Any quantum computation operation can be made by using a combination of:

**{CNOT, H, S, T}**

# Quantum Universality

What is the S gate?

What is the T gate?