The Deutsch-Josza algorithm

- The DJ algorithm allows us to determine if the function is constant or balanced with just one check, provided we express the function as an **oracle**.
- We know that we want our quantum states to be modified by the application of the function, so that we can figure out if the function is **constant or balanced**. Therefore, we will have to express the **function in the form of an operator**, or a gate.
- Also, just like all other quantum operators, the gate we make for the function will have to be reversible. An **oracle** is exactly this reversible gate form of the function we want to check.

In [1]:
```
from random import getrandbits
from qiskit import *
import numpy as np
```

STEPS

Here are the steps of the DJ algorithm for a n-bit function:

- Initiate a quantum circuit of n+1 qubits
- Apply an X gate to the last qubit (for a phase kickback)
- Apply Hadamard transform to all the qubits
- Apply the oracle to the qubits
- Apply H gates to all the qubits
- Measure $q0$. If the result is 0, the oracle is constant. If it is 1, the oracle is balanced.

Notice that we only invoked the oracle (i.e. applied the function) once in this algorithm.

Defining randomized function

In [2]:
```
def get_function(n,out = False):
    '''Get the function for the 2^n inputs'''
    inputs = 2**(n)
    bit = getrandbits(1)
    func = []

    # is bit is 0 -> constant
    if bit == 0:
        if out:
            print("Constant function selected!")
        one = getrandbits(1)
        if one is True:
            func = [1 for _ in range(inputs)]
        else:
            func = [0 for _ in range(inputs)]
        func.insert(0,'constant')
    # is bit is 1 -> balanced
    else:
        if(out):
            print("Balanced function selected!")
        func.append('balanced')
        per = np.random.permutation(inputs)
        for k in per:
            # at even you get 0, at odd you get 1
            # since even number of elements, always equal
            # zeroes and ones
            func.append(k%2)

    return func
```

Defining oracle for n bits

- Basically just apply an MCX gate whenever f(x) is 1 for an input combination and nothing when it is zero

In [8]:
```
def get_oracle(function):
    qubits = int(np.log2(len(function)-1))
    qc = QuantumCircuit(qubits+1, qubits,name='Oracle')

    # now see what type of function is it
    if(function[0] == 'balanced'):
        function = function[1:]

        # wherever it is 0, don't do anyhing
        for i, k in enumerate(function):
            if k == 1:
                # first get the binary representation of i
                b = bin(i)[2:]
                b = b.zfill(qubits)[:qubits]
                # apply X gates
                qc.barrier()
                for j, h in enumerate(b):
                    if(h == '1'):
                        qc.x(qubits - j -1)

                # apply mcx pertaining to the fact that
                # bit has to be flipped
                qc.mcx([i for i in range(qubits)], qubits)

                # reverse the operation of X application
                for j, h in enumerate(b):
                    if(h == '1'):
                        qc.x(qubits - j - 1)
                qc.barrier()
            else:
                pass
    else:

        function = function[1:]
        if(sum(function) == 0): # nothing to be done
            pass
        else:                   # all the qubits need to be inverted
            qc.x(qubits)
    return qc
```

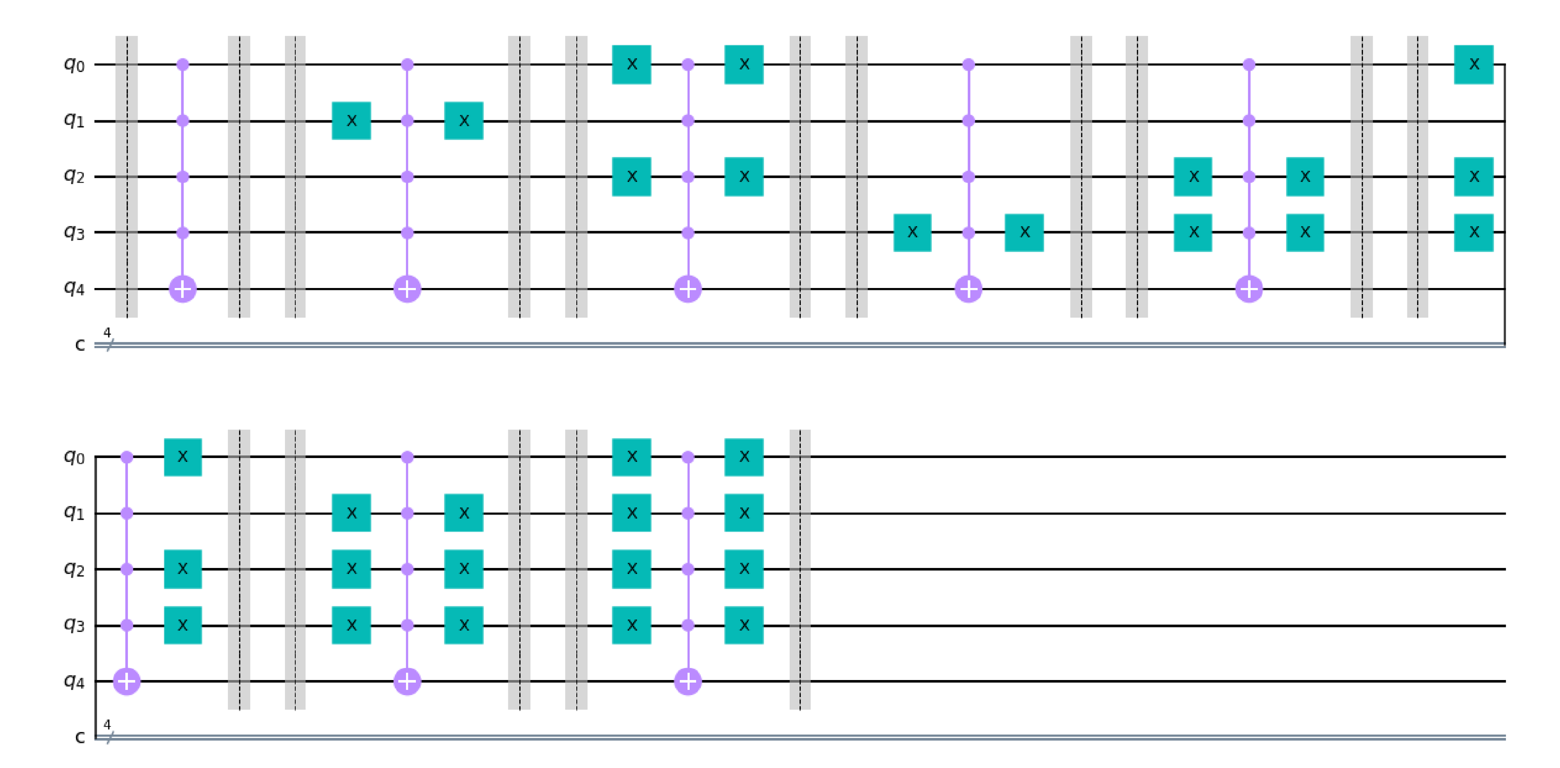
Basis for correct identification

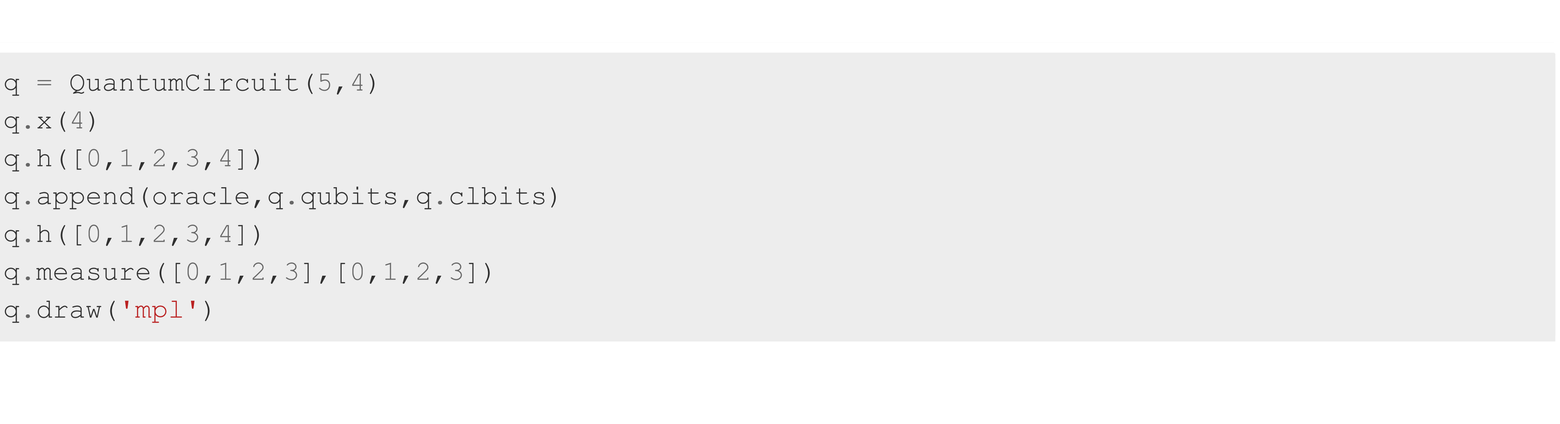
- If the function is balanced, then the **proabability of measuring the state |00...0>** cancels out and we would have atleast one 1 in the measurement of the query register.
- If the function is constant, then the **probability of the measuring the state |00...0>** is 100% and thus we would have the state |*00...0*> with a probability of 1.

Testing for n = 4 qubits

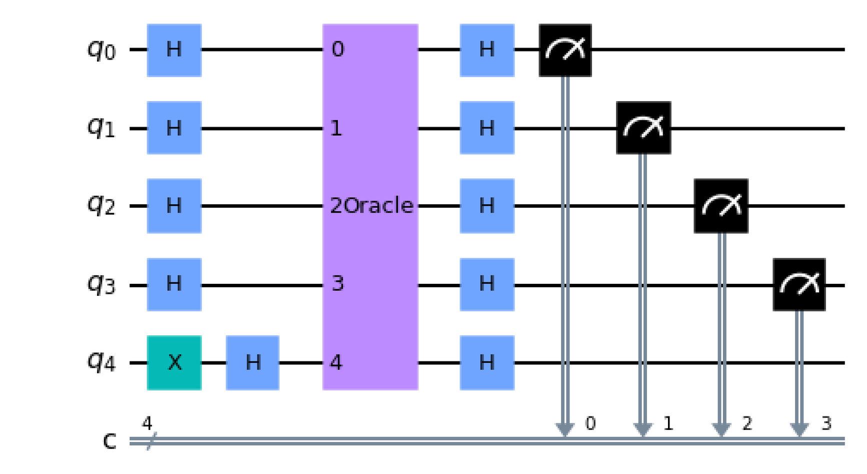
In [9]:
```
func = get_function(4,True)
oracle = get_oracle(func)
print("Function selected :",func)
display(oracle.draw('mpl'))
```

```
Balanced function selected!
Function selected : ['balanced', 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 1, 1, 1]
```



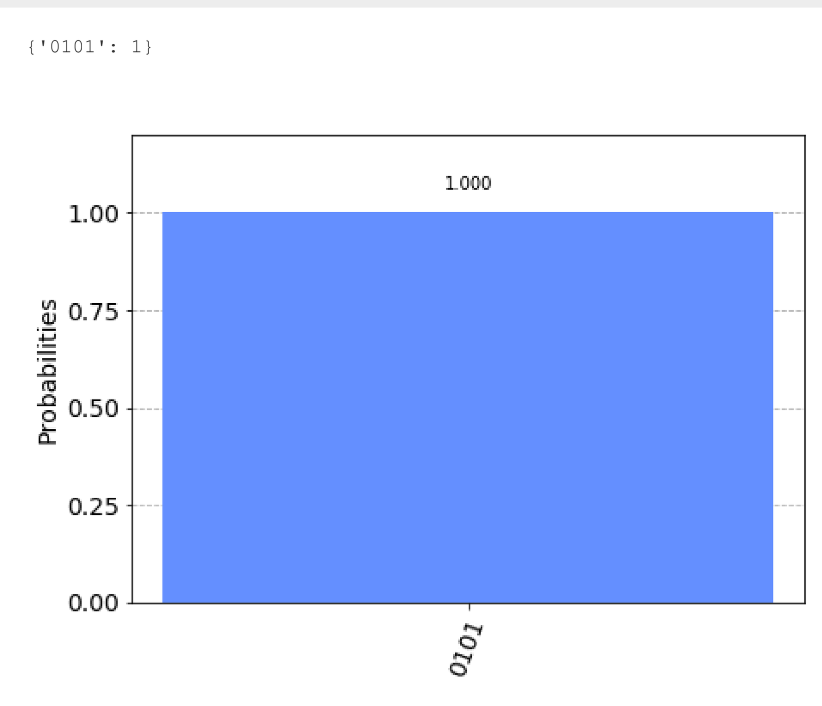


In [10]:
```
q = QuantumCircuit(5,4)
q.x(4)
q.h([0,1,2,3,4])
q.append(oracle,q.qubits,q.clbits)
q.h([0,1,2,3,4])
q.measure([0,1,2,3],[0,1,2,3])
q.draw('mpl')
```



In [11]:
```
backend = Aer.get_backend('qasm_simulator')
counts = execute(q,backend = backend,shots=1).result().get_counts()
print(counts)
from qiskit.tools.visualization import plot_histogram
plot_histogram(counts)
```

```
{'0101': 1}
```



- This histogram protrays the fact that since **atleast 1 qubit is non-zero in the final query register**, it shows that the algorithm works and helps us to determine the nature of the function in *a single measurement of the query register**

Testing for n = 4 qubits

In [14]:
```
correct , incorrect = 0,0
for _ in range(1000):
    func = get_function(4,False)
    oracle = get_oracle(func)
    # make the circuit
    q = QuantumCircuit(5,4)
    q.x(4)
    q.h([0,1,2,3,4])
    q.append(oracle,q.qubits,q.clbits)
    q.h([0,1,2,3,4])
    q.measure([0,1,2,3],[0,1,2,3])
#     q.draw('mpl')

    backend = Aer.get_backend('qasm_simulator')
    counts = execute(q,backend = backend,shots=1).result().get_counts()
#     print(counts)
    total = 0
    BIT0 = list(counts.keys())[0]
    for k in BIT0:
        total+= int(k)

    if(total == 0 and func[0] == 'constant'):
        correct +=1
    elif(total != 0 and func[0] == 'balanced'):
        correct +=1
    else:
        incorrect +=1

print("Number of correct identifications :",correct)
print("Number of incorrect identifications :",incorrect)
```

```
Number of correct identifications : 1000
Number of incorrect identifications : 0
```

Testing for n = 5 qubits

In [15]:
```
correct , incorrect = 0,0
for _ in range(1000):
    func = get_function(5,False)
    oracle = get_oracle(func)
    # make the circuit
    q = QuantumCircuit(6,5)
    q.x(5)
    q.h([0,1,2,3,4,5])
    q.append(oracle,q.qubits,q.clbits)
    q.h([0,1,2,3,4,5])
    q.measure([0,1,2,3,4],[0,1,2,3,4])
#     q.draw('mpl')

    backend = Aer.get_backend('qasm_simulator')
    counts = execute(q,backend = backend,shots=1).result().get_counts()
#     print(counts)
    total = 0
    BIT0 = list(counts.keys())[0]
    for k in BIT0:
        total+= int(k)

    if(total == 0 and func[0] == 'constant'):
        correct +=1
    elif(total != 0 and func[0] == 'balanced'):
        correct +=1
    else:
        incorrect +=1

print("Number of correct identifications :",correct)
print("Number of incorrect identifications :",incorrect)
```

```
Number of correct identifications : 1000
Number of incorrect identifications : 0
```

Testing for n = 6 qubits

In [16]:
```
correct , incorrect = 0,0
for _ in range(1000):
    func = get_function(6,False)
    oracle = get_oracle(func)
    # make the circuit
    q = QuantumCircuit(7,6)
    q.x(6)
    q.h([0,1,2,3,4,5,6])
    q.append(oracle,q.qubits,q.clbits)
    q.h([0,1,2,3,4,5,6])
    q.measure([0,1,2,3,4,5],[0,1,2,3,4,5])
#     q.draw('mpl')

    backend = Aer.get_backend('qasm_simulator')
    counts = execute(q,backend = backend,shots=1).result().get_counts()
#     print(counts)
    total = 0
    BIT0 = list(counts.keys())[0]
    for k in BIT0:
        total+= int(k)

    if(total == 0 and func[0] == 'constant'):
        correct +=1
    elif(total != 0 and func[0] == 'balanced'):
        correct +=1
    else:
        incorrect +=1

print("Number of correct identifications :",correct)
print("Number of incorrect identifications :",incorrect)
```

```
Number of correct identifications : 1000
Number of incorrect identifications : 0
```

In [ ]: