INTRO TO QUANTUM COMPUTING

Week 21 Lab

GROVER’S SEARCH ALGORITHM

<insert TA name>

<insert date>

PROGRAM FOR TODAY

- Announcements
- Canvas attendance quiz
- Questions from last week
- Pre-lab zoom feedback
- Lab content
- Post-lab zoom feedback

ANNOUNCEMENTS

- This week's homework assignment is to submit your name how you'd like it printed on your certificate for passing this course! PLEASE fill it out!
- There is no other homework - enjoy Spring Break ☺
- **Spring Break:** No lecture or lab for the next TWO weeks! Cheat sheets and review notebook will be uploaded on Canvas.

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CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz (using the password posted below and in the chat).
  - This is lab number:
  - Passcode:

- **Question:** Which topics from this semester would be the most helpful to review? **(Please select all that apply)**

- **This quiz is not graded, but counts for your lab attendance!**

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PRE-LAB ZOOM FEEDBACK

On a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

In lecture this week, Amir described Grover's search algorithm

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QUESTIONS FROM LAST WEEK

What is the role of the oracle?

- The oracle implements a specific function by using quantum gates
  - For Deutsch-Jozsa, it implements a constant or balanced function
  - For Grover's, it implements a function that returns -1 for one state and +1 for all other states
- We're not allowed to peek inside the oracle to find out the function
- We can only ask the oracle questions in the form of sending it inputs, and analyzing the outputs
- The complexity of our algorithm is the number of queries we need to figure out the oracle function

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LEARNING OBJECTIVES FOR LAB 20

- Understanding Grover's algorithm
  - Introduction to the search problem
  - Implementing the algorithm for 2 qubits
  - Circuits for each step of the implementation
  - Coding the algorithm for 2 qubits

- More on oracles*

*Optional content

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INTRO TO GROVER'S ALGORITHM

- Grover's algorithm performs search in an **unstructured** list
  - Database search
  - Optimization problems (searching through possible solutions)
  - Statistics (finding mean, median etc.)

- The complexity of Grover's algorithm is $O(\sqrt{N})$, a quadratic speedup over classical linear search, which is $O(N)$

- **Check:** Approximately how many operations do you need to perform to search a database with 10000 entries using linear search? Using Grover search?

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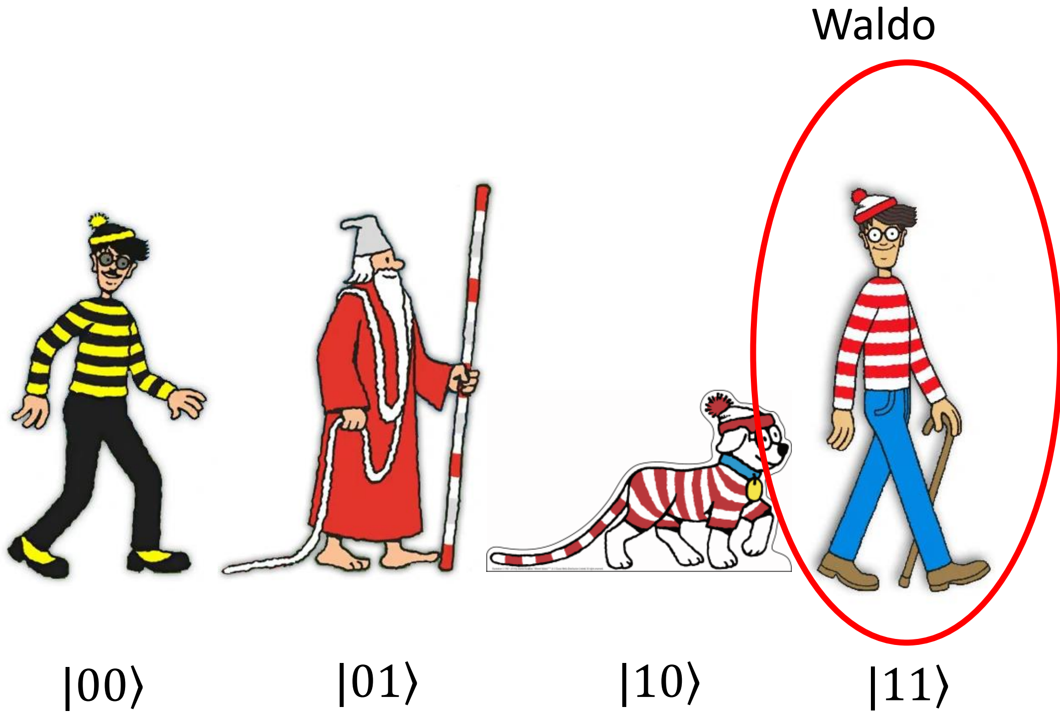
STEPS FOR GROVER’S ALGORITHM

- Like the Deutsch-Jozsa algorithm, Grover’s algorithm uses **superposition** and **interference** to speed up search

- Here are the steps in Grover’s algorithm:
  1. Apply H gate to all qubits – **superposition**
  2. Apply **oracle** to all qubits, and apply Grover’s **diffusion operator** – **interference**
  3. Measure all qubits

LET'S APPLY THESE STEPS!

- We're going to apply these steps to search for Waldo among 4 characters
- Let's assume that Waldo is 4th in the list of characters
- How many qubits do we need to represent these 4 characters?
- With 2 qubits, we only need to go through the steps once. With more qubits (larger databases), we would repeat step 2 until the probability of measuring the right result is maximized.



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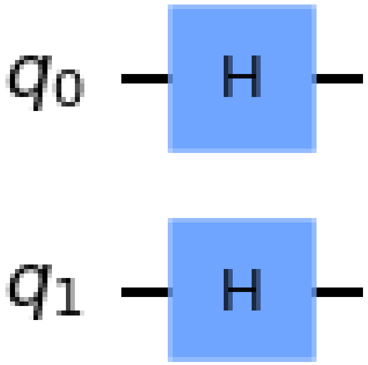
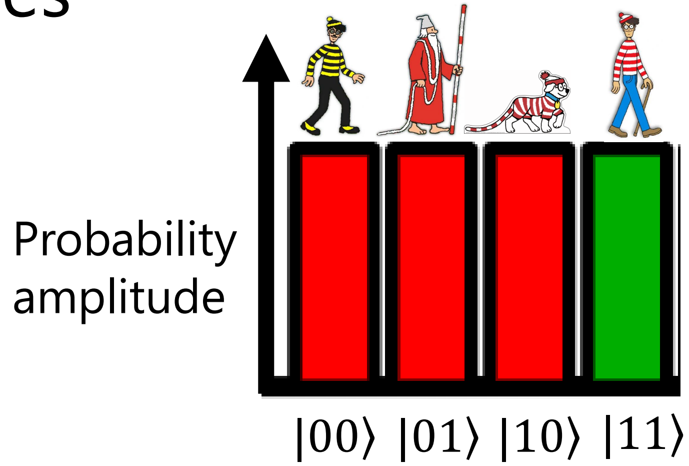
STEP 1: APPLYING H GATE

- In this step, we apply an H gate to both qubits



- By applying the H gates, we are starting off with equal probability amplitude in all possible states – we don't know where Waldo is ☺



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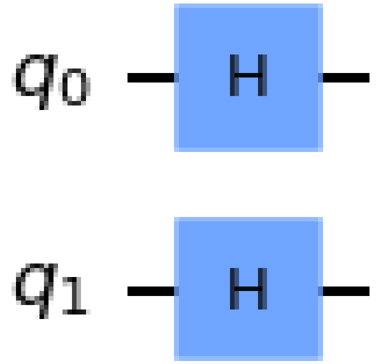
STEP 1: APPLYING H GATE

- **Question**: Write the statevector for the state $|q_1 q_0\rangle$ after an H gate is applied to both qubits (hint: use tensor products)



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WHAT IS OUR GOAL HERE?

Where we are after step 1:

- After step 1 (applying the H gates), our statevector is $|q_1 q_0\rangle = \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$
- The equal superposition means that we don't know which of the 4 characters Waldo is, yet

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WHAT IS OUR GOAL HERE?

Where we want to end of the algorithm:

- After our search is done, we'd like our statevector to be 1 where Waldo is, and 0 everywhere else.
- We would like the probability to be 1 at the position Waldo is in, and zero everywhere else. This is **amplitude amplification**.
- So, if Waldo is in the 4th position, we'd like $|q_1 q_0\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ at the end of the algorithm

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STEP 2: APPLYING ORACLE

- Remember the oracle!
- The oracle is a fancy gate that gives us answers to queries
- What do we want the oracle to do?

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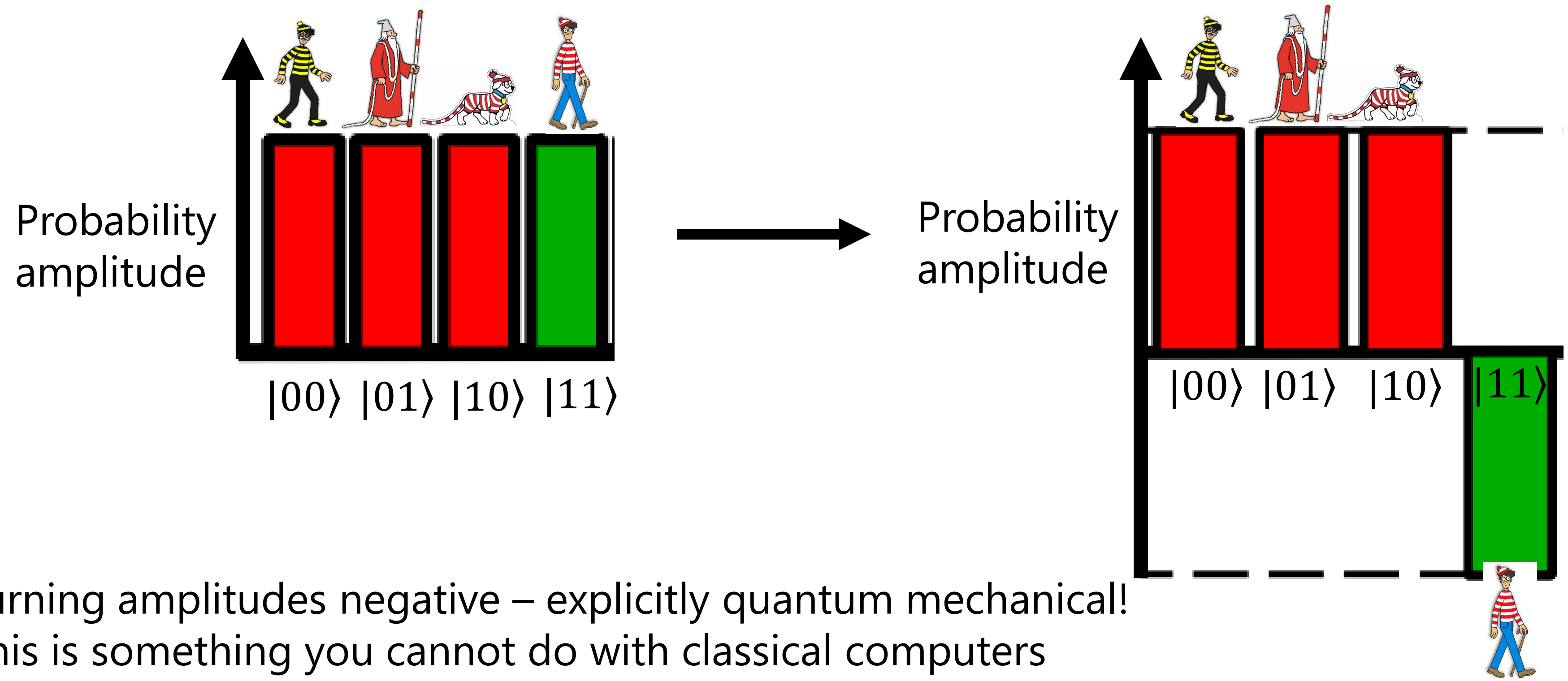
STEP 2: APPLYING ORACLE

- What do we want the oracle to do? Remember the lecture



Turning amplitudes negative – explicitly quantum mechanical!
This is something you cannot do with classical computers

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STEP 2: APPLYING ORACLE

- We want the oracle to flip the sign of the state we are searching for, while leaving other states unchanged

- Can you think of a gate that changes the sign of one state, while leaving other states unchanged?

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STEP 2: APPLYING ORACLE

- **The oracle matrix:**

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Z gate

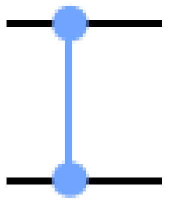
Turning amplitudes negative – explicitly quantum mechanical!
This is something you cannot do with classical computers

STEP 2: APPLYING ORACLE

- **Question:** How can we implement this qubit as a gate?

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

This is a controlled Z gate!



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STEP 2: APPLYING ORACLE

- **Question:** What is the result of applying the oracle matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

on the statevector at the end of step 1?

Hint: Remember matrix-vector multiplication from first semester

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STEP 2: APPLYING ORACLE

- **Question:** What is the result of applying the oracle matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

on the statevector at the end of step 1?

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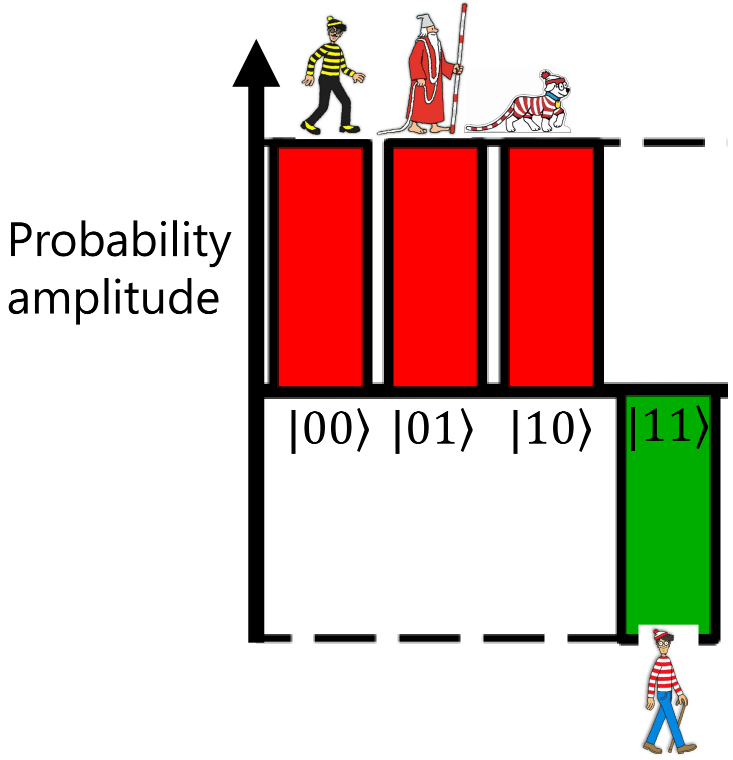
STEP 2: APPLYING ORACLE

- Yay! We made the state of Waldo different – we flipped the sign (phase) of Waldo’s state

- Now that Waldo stands out, we are in a position to find him!



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STEP 2: APPLYING GROVER'S DIFFUSION OPERATOR

- At the end of step 2, we have $|q_1 q_0\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$

- At the end, we'd like to have $|q_1 q_0\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

- How can we make this transformation? We want a gate that will take the first vector and give us the second

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STEP 2: APPLYING GROVER'S DIFFUSION OPERATOR

- We apply Grover's diffusion operator

$$\frac{1}{2}\begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

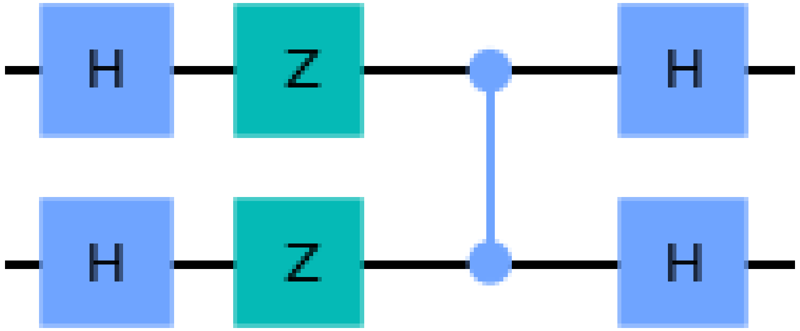
STEP 2: APPLYING GROVER'S DIFFUSION OPERATOR

- **Question** : Find the result of applying the Grover operator on the state after step 2:

$$\frac{1}{2}\begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$$

STEP 2: APPLYING GROVER'S DIFFUSION OPERATOR

- How do we implement Grover's operator as a matrix?
- Complicated! This is where the math helps
- **Answer:**



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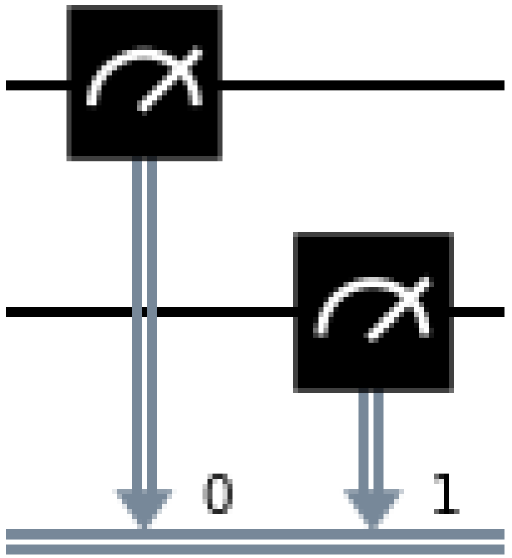
STEP 3: MEASUREMENT

- We measure our qubits to find the final state

- **Check:** If Waldo was found successfully, what should the final state be?



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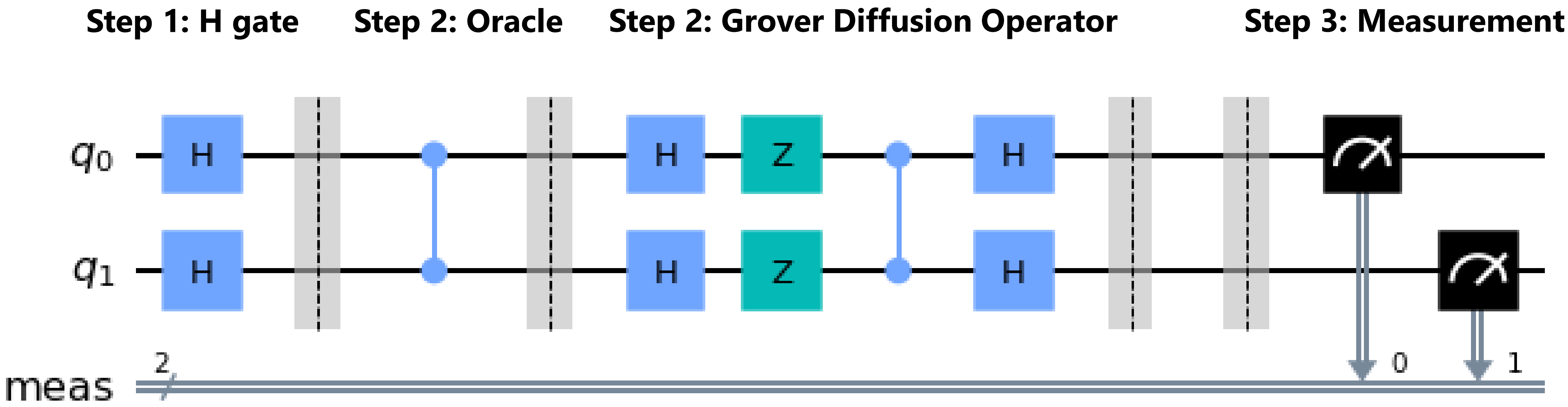
THE FULL GROVER CIRCUIT

- We've made our full Grover's circuit for 2 qubits!



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TIME TO CODE!

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KEY TAKEAWAYS

- Grover's search algorithm has a complexity of $O(\sqrt{N})$, and provides a quadratic speedup over linear search, which has a complexity of $O(N)$

- Grover's algorithm consists of 3 steps:
  - Put all qubits in a superposition state by applying H gates
  - Apply oracle and diffusion operator repeatedly to amplify the amplitude of the 'right' state
  - Measure

- Grover's algorithm uses the quantum mechanical 'resources' of superposition as well as the ability to change the phase of probability amplitudes to produce interference

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FURTHER READING AND RESOURCES

- Qiskit textbook page on Grover's algorithm
- Prof. Scott Aaronson's notes on oracles
- Using black boxes in quantum computing
- Lecture on quantum algorithm design

POST-LAB ZOOM FEEDBACK

After this lab, on a scale of 1 to 5, how would you rate your understanding of this week's content?

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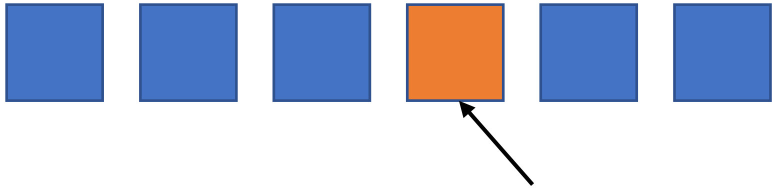
OPTIONAL CONTENT

MORE ON THE ORACLE

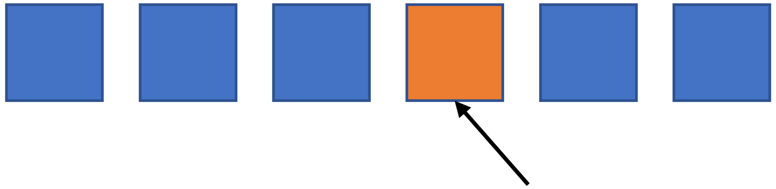
How does an oracle help us in quantum algorithms?

Reframing how we think about solving problems

“Regular” approach		**Oracle approach**
I have a list of N entries. How do I find where the entry I’m looking for is?	→	I have an oracle that “marks” one entry in a list of N entries. How do I figure out which entry it marks?



entry I’m looking for



entry the oracle marks

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MORE ON THE ORACLE

Who designs the oracle?

- Right now, we don’t worry about who made the oracle and how
- We take the oracle as given, and focus on how to figure out which state it marks

- Both for Deutsch-Jozsa and Grover’s, we will discuss which gates implement the oracle if we know which state the oracle marks

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MORE ON THE ORACLE

How does the oracle know where to place the – sign?

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

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MORE ON THE ORACLE

Open questions

- How do we design the oracle, without knowing which function it implements? This is required for a practical implementation

- We want the state we are looking for to behave differently compared to all the other states – needle in a haystack

- Oracle design is an active research area!

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