# HOMEWORK 4

## VECTORS AND MATRICES

1. Compute the following dot product:

   $$\begin{pmatrix} 5 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

   a) 46
   b) 52
   c) $\begin{pmatrix} 30 \\ 16 \end{pmatrix}$
   d) $\begin{pmatrix} 40 \\ 12 \end{pmatrix}$

2. Compute the following dot product:

   $$\begin{pmatrix} 3 \\ 10 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

   (a) $\begin{pmatrix} 3 \\ 70 \end{pmatrix}$
   (b) $\begin{pmatrix} 21 \\ 10 \end{pmatrix}$
   (c) 21
   (d) 73

3. Compute the following dot product:

$$\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}$$

a) 11

b) 8

c) $\begin{pmatrix} 2 \\ -6 \\ 12 \end{pmatrix}$

d) $\begin{pmatrix} 3 \\ -4 \\ 4 \end{pmatrix}$

4. State whether or not the following vectors are orthogonal:

$$\begin{pmatrix} 1 \\ 5 \end{pmatrix} \text{ and } \begin{pmatrix} -5 \\ 1 \end{pmatrix}$$

a) True

b) False

5. State whether or not the following vectors are orthogonal:

$$\begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$$

a) True

b) False

**Please read before attempting problems 6 and 7.**

Linear combinations are very important in many fields of mathematics and physics. In quantum mechanics, superposition states are represented mathematically as linear combinations of other quantum states. The next few problems may be quite new, so please take a look at this worked example to help with them.

Consider the vectors:

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

A linear combination of these vectors means creating a new vector in the following way

$$\vec{v} = x\vec{a} + y\vec{b}$$

for constants $x$ and $y$.

**Example:** $\begin{pmatrix} 30 \\ 5 \end{pmatrix}$

In order to solve this we must find coefficients $x$ and $y$ such that:

$$\begin{pmatrix} 30 \\ 5 \end{pmatrix} = x\vec{a} + y\vec{b}$$
$$\begin{pmatrix} 30 \\ 5 \end{pmatrix} = x\begin{pmatrix} 1 \\ 2 \end{pmatrix} + y\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$
$$\begin{pmatrix} 30 \\ 5 \end{pmatrix} = \begin{pmatrix} x \\ 2x \end{pmatrix} + \begin{pmatrix} 3y \\ y \end{pmatrix}$$

Which results in the system:

$$\begin{aligned} 30 &= x + 3y \\ 5 &= 2x + y \end{aligned} \tag{4.1}$$

Solving this gives the result that

$$\vec{v} = -3\vec{a} + 11\vec{b}$$

6. Given basis vectors

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Find the coefficients ($x$ and $y$) in the linear combination that yields the following vector:

$$\begin{pmatrix} 5 \\ 5 \end{pmatrix} = x\vec{a} + y\vec{b}$$

a) $x = 4, y = -2$
b) $x = 3, y = 4$
c) $x = 2, y = 1$
d) $x = -5, y = -1$

7. Given basis vectors

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Find the coefficients ($x$ and $y$) in the linear combination that yields the following vector:

$$\begin{pmatrix} -7 \\ 16 \end{pmatrix} = x\vec{a} + y\vec{b}$$

a) $x = 11, y = -6$
b) $x = -7, y = 20$
c) $x = 13, y = 2$
d) $x = 5, y = -5$

8. Evaluate the expression

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

a) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$
b) $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$
c) $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$
d) $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$

9. Evaluate the expression

$$\begin{pmatrix} 1 & 4 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

a) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

b) $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$

c) $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

d) $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$

10. Evaluate the expression

$$\begin{pmatrix} -2 & 6 \\ 9 & 4 \end{pmatrix} \begin{pmatrix} 3 & 3 \\ -5 & 1 \end{pmatrix}$$

a) $\begin{pmatrix} -6 & 18 \\ -45 & 4 \end{pmatrix}$

b) $\begin{pmatrix} 4 & 9 \\ 2 & -10 \end{pmatrix}$

c) $\begin{pmatrix} -36 & 0 \\ 7 & 31 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 9 \\ 4 & 5 \end{pmatrix}$