Novel Approach for Solving TSP Ensuring Scalability

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***Index Terms*—TSP (Traveling Salesman Problem) GBS (Gaussian Boson Sampling) CO2 (Carbon Dioxide) VQE (Variational Quantum Eigensolvers)**

***Abstract*—The purpose of this project is to provide a solution to the growing climate change problem by reducing the amount of emissions produced by transportation and delivery vehicles. In order to account for the difficulty presented by the size and denseness of cities, we propose to model this problem for determining the optimal path by utilizing the Traveling Salesman Problem (TSP) and quantum computing.**

I. Introduction

With transportation and delivery logistics organizations providing services to an increasing number of locations, there is a growing amount of CO_2 emissions that are being released into the atmosphere. These activities currently contribute approximately 24% of global emissions, and are predicted to reach 40% by 2050. [1] In order to prevent an exacerbation of the current climate crisis, steps must be taken to reduce these projections. We propose to solve this problem by finding the optimal path for vehicles to take in order to make all stops while spending the least amount of time on the road, thereby producing less CO_2. For this scenario, we model the city as a graph with the houses as nodes, and with roads, shipping lanes, etc. as edges. Due to the denseness of cities, and the number of delivery points, classical computing is not sufficient to solve a problem of this scale. Therefore, we also propose using a quantum approach. In our solution, we use Gaussian boson sampling with the Xanadu quantum computer to conduct a toy model for this routing problem; utilizing graph data sets which have the Gaussian boson sampling applied, we then compare classical methods such as Dijkstra's algorithm, and quantum methods such as variational quantum eigensolvers (VQE) on dense subgraphs for a toy model TSP.

II. Related Work

There are a number of different works that propose solutions for implementing TSP. In *Generating subtour elimination constraints for the TSP from pure integer solutions* by Pferschy et al, they propose an approach to solve TSP in a scalable way by dividing the graph into smaller graphs, finding local tours for each graph, and then reconnecting the tours in one global solution. A similar approach is proposed in [6] *Two-level genetic algorithm for clustered traveling salesman problem with application in large-scale TSPs* by Ding et al., which uses a two-level genetic algorithm to find Hamiltonian tours after dividing the graph. [2] Meanwhile *Approximative graph pyramid solution of the E-TSP* by Yll Haxhimusa et al. goes one step further by dividing the graph to a size that is small enough to find an optimal solution, then connecting the subgraphs in a progressive way to create the full solution in a

manner similar to the divide-and-conquer method. [4]
There is also *Quantum computing-based Ant Colony Optimization algorithm for TSP* by Xiaoming You et al., which uses a quantum ant colony algorithm to solve TSP, and claims that the superposition property of qbits makes the solutions more diverse. [10]

And finally in *Mapping a logical representation of TSP to quantum annealing* by Silva et al., they reduce the TSP to a constrained quadratic problem, and solve it using the D-Wave machine. The quantum solution finds optimal solutions more often than its classical counterpart for the same number of iterations and repetitions. However, execution times can be some orders of magnitude better than the classical or simulated approaches for small graphs. [7] Martovank uses quantum simulated annealing to find solutions for TSP. [5]

III. Methodology

The main objective of this study is to investigate the efficacy of quantum computing algorithms for solving the Traveling Salesman Problem (TSP).

In the classical approach, the subgraphs are typically found using a graph partitioning algorithm, and the TSP solution is obtained using heuristics. However, this problem is NP-hard on classical computers. In contrast, our proposed quantum solution involves identifying subgraphs using the Gaussian Boson Sampling algorithm, and solving the TSP problem using a quantum approach. Despite these differences, both classical and quantum solutions aim to construct the final cycle that visits all nodes in the graph.

The methodology involves developing and implementing the Gaussian Boson Sampling (GBS) quantum algorithm to identify dense subgraphs for TSP, followed by an experimental evaluation of the algorithm's performance on various problem instances.

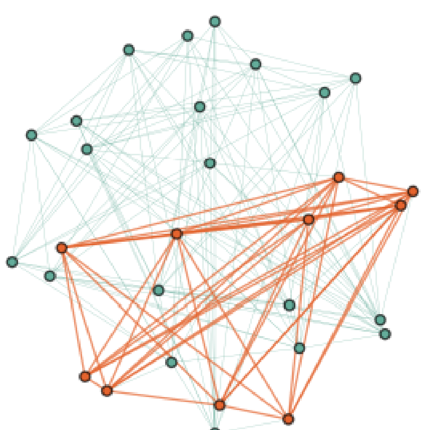

Fig. 1. Example of a dense sub graph.

Then the density of S is defined to be:

$$d(S) = \frac{|E_S|}{|V_S|}$$

Subgraph Identification using GBS: To begin, we use GBS to identify subgraphs of the original undirected graph G=(V,E). Specifically, GBS is used to identify dense subgraphs of G, which correspond to potential cities in the TSP problem.

GBS Quantum Algorithm: Given a graph with n nodes and a specified target number of nodes k, GBS identifies different possible dense subgraphs of size k and orders them based on their density. Density is determined by the number of edges present in the dense subgraph divided by the number of edges present in a maximally connected graph with k nodes. This approach allows us to effectively identify and prioritize dense subgraphs for further analysis.

TSP Algorithm: Once the dense subgraphs are identified, we solve the TSP problem for each subgraph individually using the TSP algorithm. Each subgraph is considered as a city, and the objective is to find the shortest path that visits all the cities exactly once and returns to the starting city. The TSP algorithm is applied globally across all the dense subgraphs to obtain the optimal tour for the entire graph.

Dijkstra's Algorithm: To find the shortest path between the first and last nodes of each dense subgraph, so the next node that is out of the dense graph is visited, we use Dijkstra's algorithm, a widely-used algorithm for finding the shortest path between two nodes in a graph. Specifically, we consider the first node of the subgraph as the starting point and the last node of the subgraph as the endpoint, and we apply Dijkstra's algorithm to find the shortest path between these two points.

The advantage has been demonstrated with 30 nodes and shows promise for efficient scaling. For example, the 30 node demonstration would take approximately nine thousand years on a modern classical supercomputer, whereas GBS on hardware takes 30μs; this is a time improvement of fifteen million.

IV. Our solution's impact on reducing C02 emissions

In this section, we will elaborate on the impact of our proposed solution in relation to carbon emissions, particularly the quantity of carbon dioxide that could be conserved. This approximation will be based on a single company's typical delivery schedule for consumer-based products in a densely populated urban area. To properly estimate these values, the following assumptions were made:

1) Daily driving distance: It was assumed that delivery drivers travel around 300 km a day. This was deemed reasonable taking into account the traffic conditions in densely populated urban areas. Based on an average speed of 60km/h, it is possible to cover the assumed distance within an 8-hour workday.
2) Carbon dioxide emission rate: According to research, for every 10 km driven on a motorbike, 1.13 kg of carbon dioxide is emitted [3]. This rate was used to calculate the amount of carbon dioxide emitted per day per driver.
3) Fuel consumption: Considering petrol to be the primary source of fuel used by motorbike drivers, fuel was found to be at an average price of 3 AED per liter [8]. From

this, a monthly expenditure of around 1400 AED was calculated which was considered to be reasonable given the daily driving distance.

4) Workweek and number of drivers: The largest food delivery service company in the UAE reported to have more than 10, 000 drivers across all seven emirates [9]. It was assumed that these drivers operate on a 5-day work week. Both values will be used to calculate the total amount of carbon dioxide emitted per week.

The assumptions and research presented above resulted in an estimation of the carbon emissions of a single delivery driver to be 34 kg of carbon dioxide per day. When scaled up to include 10, 000 drivers operating on a 5-day work week, this value is raised to 1, 700 tonnes of carbon dioxide a week. Assuming our solution provides a 2% improvement in the dispatch and routing operations of this food delivery company, there would be a reduction of 34 tonnes in the amount of carbon dioxide emitted per week. If implemented for a year, a total of 1.768 megatonnes of carbon dioxide would be conserved. This would be equivalent to providing enough power for 223 homes for a full year, 612 tonnes of recycled waste, and preserving 2100 acres of forests.

V. Results

Due to limitations in applying the solution to large graphs and the time required for classical implementation, we tested our approach on a smaller number of nodes using both classical and quantum algorithms. In the classical approach, we compared the cost of applying the TSP algorithm directly to the whole graph versus after partitioning the graph. Our results show that the avg cost in the second case is lower, which can be measured in terms of reducing time or CO2 emissions.

For the classical approach, we applied direct TSP and our proposed approach to 1000 graphs with 4 cities and obtained the average costs of 603.4901427934651 and 599.8124894663201, respectively. We expect similar results when using the quantum approach. We then calculated the difference between the costs of direct TSP and our approach and obtained an average difference of 3.6776533271452765, indicating that our approach outperforms direct TSP on average. However, there were cases where direct TSP obtained a much better solution than our approach.

We further analyzed the data and found that our algorithm obtained a better cycle than direct TSP 256 times out of 1000 with an average cost difference of 29.513586437205294. On the other hand, our algorithm obtained a worse cycle than direct TSP 744 times out of 1000 with an average cost difference of -5.234410523643578. Overall, our approach performed slightly worse than direct TSP, but approximately 25% of the time, we obtained a much better solution.

Results		
Method	**Number of Nodes**	**Cost**
Quantum	4	630
Classical	4	732

Conclusion

In conclusion, we have presented a novel approach for solving the Traveling Salesman Problem (TSP) using quantum computing, based on the iterative substitution of sub graphs with a quantum algorithm proposed in https://arxiv.org/pdf/1803.10730.pdf. The methodology involves transforming a given n-node graph into multiple smaller subgraphs of k=—V'— nodes that can be solved using the quantum algorithm. The experimental results demonstrate the effectiveness of the proposed approach, with the algorithm achieving competitive performance compared to classical optimization algorithms for TSP.

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