



CHALLENGE COMPLETED

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Sub-Superdense Coding

200 points

Backstory

Zenda and Reece have worked hard to implement classical computation reversibly. "*This is all rather boring though,*" says Trine. "*Let's introduce entanglement and have some fun!*" Trine tells them that the next [Laws of Infodynamics](#) will constrain how much classical information can be sent using entangled quantum resources.

Sending information with entangled states

Entanglement is a valuable resource that can be used to send information, as we will explore in this challenge. Superdense coding, for instance, uses Bell pairs to send two classical bits with a single qubit.

Trine has misplaced her usual Bell states, and instead provides Zenda with 2 qubits and Reece with 1 qubit. They are entangled forming the following state:

$$|\Psi\rangle = \frac{|00\rangle_Z|0\rangle_R + |11\rangle_Z|1\rangle_R}{\sqrt{2}}.$$

Zenda wants to send Reece 3 bits of information (i, j, k) , where each bit can take the value 0 or 1. To do so, she will apply an **encoding gate** on her two qubits and then send them to Bob.

Reece, who now has the three qubits and knows the strategy Zenda uses to encode, performs a decoding gate that will generate the state $|i, j, k\rangle$. Let's look at the following drawing to understand it.

Your goal will be to devise a coding and decoding strategy such that Reece can decode Zenda's bits, as shown in the figure above.

The strategy to build the encoding gate is akin to [superdense coding](#) for two bits of information, where all the Bell basis states are produced by acting only on one of the qubits in an entangled pair. Here, we want to produce all the GHZ basis states, which are analogous to the Bell basis for three qubits. They are given by

$$|GHZ\rangle_{ijk} = \frac{1}{\sqrt{2}}\left(|0jk\rangle + (-1)^i|1\bar{j}\bar{k}\rangle\right),$$

where $\bar{j} = 1 - j$ and $\bar{k} = 1 - k$. However, we should do this by acting only on the first two qubits (i.e. Zenda's share) of the entangled state $|\Psi\rangle$ that Trine provided. The decoding gate is also inspired by superdense coding, so reviewing the topic will help you a lot in this challenge!

► Laws of Infodynamics Part II: The Second and Third Laws

Challenge code

You simply have to complete two quantum functions:

- `encode`: quantum function that will define an operator to be applied only on Zenda's qubits. This function will depend on the bits (i, j, k) to be encoded.
- `decode`: quantum function that defines the operator that Reece will use to retrieve the bits that Zenda sent. In this case, the operator does not have any information about Zenda's bits, so the same operators will always be applied regardless of the state that Zenda sends Reece.

Output

In this challenge, we will not judge your solution using public or private test cases. Instead, we will check that, for all combinations of i, j , and k , the entire encoding and decoding circuit behaves as expected.