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Math 2605 Project Written Component

Part 1 e)

Part 1 f):

1. By using LU or QR-factorizations, we can solve the problem with forward and backward substitution. This process involves a concrete number of simple algebraic operations directly proportional to the size of the give matrix. On the other hand, calculating the inverse matrix would not be easily be formulated into a procedure that can given to a computer. If one did use such a procedure, it would require many more multiplication and division operations, which would increase error due to the finite precision of floating point storage.
2. Using LU or QR-factorizations does not incur significant error, as seen in the graphs above. As n increases, there is little change in the error of (LU – H)/(QR – H) and (Hx – b), meaning that these factorizations can be used effectively when scaled to higher n values. For (Hx – b), there seems to be an exponential increase in error until n=14, then it drops back down to no growth. The increase in error, which was recorded to be no larger than 1E-12 for (Hx - b), is well worth the decrease in runtime.

Part 2:

Overall, it seems that the Gauss-Seidel method is much more efficient at decoding bitsteams than the Jacobi method. Regardless of the length of the bitstream, the Gauss-Seidel method always takes 2 iterations to reach the solution with the desired precision. However, the Jacobi method takes more iterations to achieve the error tolerance as the length of the initial stream increases. We found that the Jacobi methods takes 11, 53, and 104 iterations to reach the error tolerance when solving for the original stream x using the encoded stream y1. We also found that the Jacobi methods takes about twice as many iterations to solve for x using y1 than y0.

Part 3:

1. The matrix A denotes the proportion of the current population that moves on to the next age group. The first row appears to be the reproduction rate for each age group, which adds to the population. Many of the factors that cause this are physical, such as old age or lack of health care. Also, it is unknown what population this describes, so there could be differences in social structure and availability of clean water.
2. The population tends to increase fairly quickly, starting with 16,474 and going to 17,258 to 22,151. This seems to indicate a logarithmic growth rate with it eventually capping out at some point.
3. The largest eigenvalue of the matrix is .99999999. This means that the population will become stable in the long run, after a period of growth. It appears to converge exactly to one, which would fit the predicted growth pattern.
4. A decrease in the birth rate of the second age group made very little impact on the eigenvalue of the function. The calculated eigenvalue ended up being 1.00000002, which is still very close to one. This means that the population remained very stable even after these changes.