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Math 2605 Project Written Component

Part 1 e)

Part 1 f)

i) By using LU or QR-factorizations, we can solve the problem with forward and backward substitution. This process involves a concrete number of simple algebraic operations directly proportional to the size of the give matrix. On the other hand, calculating the inverse matrix would not be easily be formulated into a procedure that can given to a computer. If one did create such a procedure, it would require many more multiplication and division operations, which would increase error due to the finite precision of floating point storage.

ii) Using LU or QR-factorizations does not incur significant error, as seen in the graphs above. As n increases, there is little change in the error of (LU – H)/(QR – H) and (Hx – b), meaning that these factorizations can be used effectively when scaled to higher n values. The increase in error, which was recorded to be no larger than 1E-12 for (Hx - b), is well worth the decrease in runtime.

Part 3:

1. The matrix A denotes the proportion of the current population that moves on to the next age group. The first row appears to be the reproduction rate for each age group, which adds to the population. Many of the factors that cause this are physical, such as old age or lack of health care. Also, it is unknown what population this describes, so there could be differences in social structure and availability of clean water.
2. The population tends to increase fairly quickly, starting with 16,474 and going to 17,258 to 22,151. This seems to indicate a logarithmic growth rate with it eventually capping out at some point. The population seems to shift too much in this data set to find a correlation in population change, although for the most part it appears to be going down slowly.

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|  |  |  |  |  |  |  |  |  |  | Total | %Change |
| 2010 | 6.35 | 1.47 | 1.615 | 1.62 | 1.89 | 1.76 | 1.36 | .924 | .36 | 17.349 | 22.2 |
| 2020 | 5.1875 | 4.445 | 1.2495 | 1.4535 | 1.458 | 1.6632 | 1.408 | 1.0472 | .3696 | 18.2815 | 5.37 |
| 2030 | 8.1624 | 3.63125 | 3.77825 | 1.12455 | 1.30815 | 1.28304 | 1.33056 | 1.08416 | .41888 | 22.12124 | 21.0 |
| 2040 | 9.656485 | 5.71368 | 3.0865625 | 3.400425 | 1.012095 | 1.151172 | 1.026432 | 1.0245312 | .433664 | 26.5050467 | 19.8 |
| 2050 | 13.41322675 | 6.7595395 | 4.856628 | 2.77790625 | 3.0603825 | .8906436 | .9209376 | .79035264 | .40981248 | 33.87942932 | 27.8 |

1. The largest eigenvalue of the matrix is .99999999. This means that the population will become stable in the long run, after a period of growth. It appears to converge exactly to one, which would fit the predicted growth pattern.
2. A decrease in the birth rate of the second age group made a little impact on the eigenvalue of the function. The calculated eigenvalue ended up being 1.00000002, which is still very close to one. While the increase was slightly higher in this one, the population remained fairly stable even after these changes. It will be unable to reach the point where it will grow out of control.

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|  |  |  |  |  |  |  |  |  |  | Total |
| 2030 | 5.4954 | 3.63125 | 3.77825 | 1.12455 | 1.30815 | 1.28304 | 1.33056 | 1.08416 | .41888 | 19.45424 |
| 2040 | 7.477735 | 3.84678 | 3.0865625 | 3.400425 | 1.012095 | 1.151172 | 1.026432 | 1.0245312 | .433664 | 22.4593967 |
| 2050 | 8.86487875 | 5.2344145 | 3.269763 | 2.77790625 | 3.0603825 | .8906436 | .9209376 | .79035264 | .40981248 | 33.87942932 |