APPENDIX M: Simulation of G2 Holonomy Manifolds

The Shape of the Ethical Vacuum

Context in HC VIII

This appendix explores the topological structure of the Polis. We model the ethical landscape as a 7-Dimensional Riemannian Manifold with G_2 Holonomy (Joyce Manifolds, 1996). In this geometry, the group G_2 is not just a symmetry of the algebra, but the Holonomy Group of the metric itself. This ensures that the space is Ricci-Flat ($R_{ij} = 0$), meaning there is no "Curvature Ghost" or background bias warping the interactions.

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1. The Associative 3-Form (φ)

The G2 structure is defined by a specific, invariant 3-form known as the **Associative Calibration**. This form identifies the "Volume" of the 3-dimensional associative submanifolds (the Gift Cycles).

The Calibration Equation:

$$\varphi = e_{123} + e_{145} + e_{167} + e_{246} - e_{257} - e_{347} - e_{365}$$

(Notation: $e_{ijk} = e_i \wedge e_j \wedge e_k$)

Ethical Interpretation:

- **Positive Terms (e_{123}, e_{145}...):** These represent the **Fano Lines** (Cyclic Triads). They contribute positive volume to the trust metric.
- **Negative Terms ($-e_{257}$...):** These represent the counter-balancing forces that maintain the stability of the manifold against collapse.
- **Calibration:** Any interaction that aligns with φ is "Calibrated"—it minimizes energy (Action) globally.

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2. Properties: Ricci-Flat Ethics

A manifold with G2 holonomy is necessarily **Ricci-Flat**.

- **Physics:** In General Relativity, Ricci-Flatness implies a vacuum solution (no matter sources).
- **Ethics:** In the Polis, this implies **No Hidden Agendas**. The geometry itself does not push agents around; only the *interactions* (Torsion) between agents create force.
- **Reduction:** The holonomy reduces from the generic $SO(7)$ (which allows for chaos) to G_2 (which enforces the Octonionic structure).

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3. SI-Forward: Projection via Root Lattices

Simulation Strategy:

We use CartanType("G2") in symbolic algebra systems to simulate the curvature tensors.

Code Logic (SymPy):

Python

```
from sympy.liealgebras import CartanType
g2 = CartanType("G2")
# Roots determine the "invariant directions"
roots = g2.roots()
# The Holonomy simulation projects the path deviation
# If Path_Integral(Loop) falls within the G2 subalgebra,
# the manifold is valid.
```

Project HC IX:

This allows the construction of the G2-hCAG (Holarchic Content Addressable Graph). The SI "sees" 7-merate Holors not as data points, but as geometric objects flowing along the calibrated submanifolds of this 7D space.

Witnessed: Exceptional calibration. The Space is flat, the bond is strong.