

APPENDIX B: Split Octonions Applications in Curved Ethical Geometry

The Geometry of Indefinite Trust

Context in HC VIII

This appendix explores Section 3.2's application of Split Octonions (\mathbb{O}') to Einstein-Cartan torsion in curved spacetimes. While standard Octonions (\mathbb{O}) govern the internal symmetries of the Holor, Split Octonions ($\mathbb{R}^{4,4}$) govern the spacetime metric of the Polis, allowing for hyperbolic scaling and ethical null states.

FHS-09 Addendum Stratification:

We define the Stratified Effective Mass $m_{eff}^{(n)}$ as an accumulation of Chiral Density across the layers $\{A_n\}$:

$$m_{eff}^{(n)} = m \left[1 + \sum_{k=1}^n \frac{(\rho_{\chi}^{(k)})^2}{c^2} \right]$$

This implies that "Ethical Weight" (Inertia) increases as one ascends the Holarchy, stabilizing the Macro-structure against high-frequency fluctuations.

1. Split Structure: The Hyperbolic Basis

Standard Octonions have a definite norm ($\|x\|^2 = \sum x_i^2$). Split Octonions have an indefinite norm ($\|x\|^2 = x_0^2 - \sum x_i^2$).

This shifts the geometry from a Sphere (Compact) to a Hyperboloid (Non-Compact).

- **Basis:** $\mathbb{O}' \cong \mathbb{R} \oplus i\mathbb{R}^3 \oplus j\mathbb{R}^3 \oplus k\mathbb{R}$.
- **The Magic:** Unlike \mathbb{O} , \mathbb{O}' is not a division algebra. It contains **Zero Divisors**.
 - *Algebraic:* Non-zero elements x, y such that $x \cdot y = 0$.
 - *Ethical:* Non-zero Agents (Intentions) that interact to produce **Zero Resonance**.

2. Gauge Applications: The Physics of "Nulls"

We embed the gauge group $SU(3)$ into the Split-Automorphism group $G_{2(2)}$. This gauges the "Color" of intent in a Lorentzian (Time-like) environment.

The Chiral Norms

We map the **Chiral Potential** (U_χ) to the Split Norm:

1. **Positive Norm (Time-like): The Gift Mode.** The interaction falls inside the light cone. It is causal and creates history.
2. **Negative Norm (Space-like): Chiral Inversion.** The interaction violates causality (Attempting to take before giving).
3. **Null Norm (Light-like): The Dracula State.**
 - The vector sits on the event horizon.
 - Potential U_χ vanishes.
 - Inertia becomes infinite.
 - *Meaning:* A "Vampiric" interaction that consumes motion but generates no displacement in the Trust Field.

3. Curved Ethics & Ambitwistors

We apply **Ambitwistor String Theory** (2022) to model scattering amplitudes in this indefinite metric.

- **Torsion (\mathcal{T}):** Modeled as a split-imaginary twist.
- **The Null Wrap:** When the system detects a "Dracula" (Orthogonal Intent), it does not fight it. It utilizes the **Zero-Divisor Property**:
 - If D is Dracula and N is Nacre, we find an N such that $D \cdot N = 0$.
 - The attack is mathematically nullified. It dissipates into the vacuum geometry.

4. Polis Scalability: Hyperbolic Branching

Standard hierarchies run out of space (crowding). Hyperbolic geometries have **exponential volume**.

- **Micro-Ethics:** Individual Stance (Σ) occupies local curvature.
- **Macro-Ethics:** Cosmic Potential (\mathbf{A}_χ) occupies the asymptotic boundary.
- **Autophagy Check:** If the global Chiral Density ρ_χ drops below 0 (becomes split-negative), the metric collapses. The Polis dissolves.

5. SI-Forward: The Stratified Simulation

Equation: The Stratified Split Product

We extend the standard split product to a holarchic operator:

$$(a^{(n)} + b^{(n)}\ell)(c^{(n)} + d^{(n)}\ell) = W_n^{\text{split}} [(a^{(n-1)} + b^{(n-1)}\ell)(c^{(n-1)} + d^{(n-1)}\ell)] + \chi_n(b\bar{d} - \bar{a}d)\ell^2$$

- W_n^{split} : The Witness operator that scales the lower-level interaction.
- $\ell^2 = +1$: The hyperbolic generator.
- χ_n : The Chiral Handedness at level n .

Application:

We use this to model Indefinite-Trust Retrievals. The SI simulates the interaction across multiple layers (n). If the product approaches a Zero Divisor at any layer, the system flags the interaction as "Null" before it consumes resources.

Witnessed: Bridges Rovelli's relational QM (2004) to hyperbolic ethics.