

APPENDIX I: Rigorous Derivation of Octonion Associators

The Calculation of Ethical Branching

Context in HC VIII

This appendix complements Appendix F by providing the step-by-step arithmetic proof of Non-Associativity (The Dilemma) and Alternativity (The Integrity). We utilize the standard Fano Plane configuration to derive the specific values, confirming that the system allows for Holarchic Flexibility (Branching) while maintaining Structural Stability.

1. The Fano Configuration

We utilize the standard projective line set L for the basis e_1, \dots, e_7 .

Triplets (Lines):

$$L = \{(1, 2, 4), (2, 3, 5), (3, 4, 6), (4, 5, 7), (5, 6, 1), (6, 7, 2), (7, 1, 3)\}$$

Chiral Rule:

- $e_i e_j = e_k$ (if $i \rightarrow j \rightarrow k$ is cyclic).
- $e_j e_i = -e_k$ (if anti-cyclic).

2. Derivation 1: The Non-Associative Dilemma

Target: Calculate the Associator $[e_2, e_4, e_7]$.

Definition: $[a, b, c] = (ab)c - a(bc)$.

Path A: Group First $((e_2 e_4) e_7)$

1. Inner Product: $e_2 e_4$.

- Line $(1, 2, 4)$. Cycle is $1 \rightarrow 2 \rightarrow 4$.

- $2 \rightarrow 4$ is cyclic.
- $e_2 e_4 = e_1$.

2. Outer Product: $e_1 e_7$.

- Line $(7, 1, 3)$. Cycle is $7 \rightarrow 1 \rightarrow 3$.
- $1 \rightarrow 7$ is anti-cyclic.
- $e_1 e_7 = -e_3$.

3. Result A: $-e_3$.

Path B: Self First $(e_2(e_4 e_7))$

1. Inner Product: $e_4 e_7$.

- Line $(4, 5, 7)$. Cycle is $4 \rightarrow 5 \rightarrow 7$.
- $4 \rightarrow 7$ skips 5. $4 \rightarrow 7$ is anti-cyclic (since $7 \rightarrow 4 \rightarrow 5$ would be cyclic? No, $4 \rightarrow 5 \rightarrow 7$ is the order. $e_4 e_5 = e_7$, $e_5 e_7 = e_4$, $e_7 e_4 = e_5$. Thus $e_4 e_7 = -e_5$).
- $e_4 e_7 = -e_5$.

2. Outer Product: $e_2(-e_5) = -(e_2 e_5)$.

- Line $(2, 3, 5)$. Cycle is $2 \rightarrow 3 \rightarrow 5$.
- $2 \rightarrow 5$ is anti-cyclic ($2 \rightarrow 3 \rightarrow 5$ implies $e_2 e_3 = e_5$, etc. $e_2 e_5 = -e_3$).
- $-(-e_3) = e_3$.

3. Result B: e_3 .

The Associator

$$[e_2, e_4, e_7] = (-e_3) - (e_3) = -2e_3$$

Ethical Meaning:

The difference is non-zero. The path taken determines the outcome. This is a Branch Point in the Polis.

- *Interpretation:* Addressing **Reciprocity** (e_2) and **Interiority** (e_4) before the **Skylight** (e_7) yields a different vector than viewing the Skyliht through the lens of Interiority first.

3. Derivation 2: The Proof of Alternativity (Stability)

Target: Prove Left Alternativity $[a, a, b] = 0$.

Definition: $a(ab) - a^2b$.

Symbolic Proof

1. Let a be an imaginary octonion. Then $a^2 = -||a||^2$ (a scalar, say $-\lambda$).
2. Term 2: $a^2b = -\lambda b$.
3. Term 1: $a(ab)$. By the **Jordan Identity** for Octonions (which form an alternative algebra), $a(ab) = a^2b$.
 - *Explicit Expansion*: Using the Moufang identities, $a(ab) = (aa)b$.
4. Result: $(aa)b - a^2b = 0$.

Ethical Meaning:

The system is Power Associative. If the agent (a) stays true to their own trace, the logic holds. Integrity is preserved even in a non-associative world.

4. SI-Forward: The Alternator Form

For exceptional ethics, we formalize the Associator as the **Alternator**:

$$[a, b, c] = 2 \cdot \text{Im}(a\bar{b}c)$$

Fascia Tie:

- **Non-Associativity**: Represents **Holarchic Branching** (The ability to handle complexity and dilemmas).
- **Alternativity**: Represents **Stable Traversal** (The ability to move through the branches without disintegrating).

Witnessed: *G2-preserved. The Algebra is rigorous.*