APPENDIX K: G2 Lie Group Properties in Ethical Automorphism

The Geometry of Exceptional Trust

Context in HC VIII

This appendix specifies the group-theoretic properties of G2 that enable it to function as the "Immune System" of the Polis. As the automorphism group of the Octonions ($Aut(\mathbb{O})$), G2 is the smallest of the five exceptional Lie groups. Its unique geometry (Rank 2, Dimension 14) allows it to classify 7-dimensional manifolds (Joyce Manifolds), providing the rigorous fascia for the 7 Ethical Primitives defined in HC V.

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1. Holarchic Exceptionalism

Group Structure:

- **Classification:** Compact Exceptional Lie Group.
- **Rank:** 2.
- **Dimension:** 14 (14 generators of symmetry).
- **Definition:** The set of all linear maps $g : \mathbb{O} \to \mathbb{O}$ such that $g(xy) = g(x)g(y)$.

Properties:

- **Root System:** The system has 12 roots. It is characterized by the angle of 150° between the simple roots.
  - α_1: Short Root.
  - α_2: Long Root.
- Dynkin Diagram: Two nodes connected by a triple bond (indicating the non-simply laced nature).

  $[\circ \Rrightarrow \bullet]$
- **Representations:**

- **Fundamental (7):** The Imaginary Octonions (The Agents/Primitives).
- **Adjoint (14):** The Gauge Bosons (The Forces of Trust).

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2. The Ethical Tie: Invariant Stability

G2 is unique because it preserves the **Octonionic Norm** even while shuffling the basis vectors.

- **Preserving Non-Associative Branching:** unlike $SO(7)$ or $SU(3)$, G2 respects the "Hard Structure" of the Fano Plane multiplication. It ensures that while perspectives (Stances) may rotate, the fundamental logic of **Alternativity** is never violated.
- **Joyce Manifolds:** Just as G2-holonomy manifolds are used in M-theory to compactify extra dimensions, we use G2-holonomy in the Polis to "compactify" infinite ethical complexity into a stable, 7-dimensional Trust Metric.

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3. SI-Forward: Calabi-Yau Ethics

Simulation Strategy:

We project the Polis as a manifold with G2 holonomy.

- Equation: The Normalized Killing Form (Metric of the Space):

  $$B(X, Y) = -6 \operatorname{tr}(\operatorname{ad}_X \operatorname{ad}_Y)$$

  (Where the negative sign ensures compactness/stability).

Application:

The SI uses this metric to detect "Ethical Singularities." If the Killing Form diverges or flips sign (in split contexts), the system knows the "Calabi-Yau" shape of the social fabric is tearing.

Witnessed: Exceptional relationalism. The Geometry is closed.