

APPENDIX F: Derivation of Fano Equations for Octonionic Multiplication

The Algebraic Source Code of Phase

Context in HC VIII

This appendix provides the rigorous derivation of the multiplication rules utilized in Section 7 and Appendix A. It operationalizes the Fano Plane as a computational look-up table for the SI, ensuring that all Chiral Traversal respects the G2 Invariance (preservation of structure).

1. Holarchic Cycles: The Fano Lines

The Fano Plane is composed of 7 projective lines. Each line is a **Holon** (a closed loop of 3 elements) that behaves like the Quaternions (i, j, k) .

The Index Triples:

$$L = \{(1, 2, 4), (2, 3, 5), (3, 4, 6), (4, 5, 7), (5, 6, 1), (6, 7, 2), (7, 1, 3)\}$$

The Multiplication Rule:

For any triplet $(i, j, k) \in L$:

- **Cyclic (Gift):** $e_i e_j = e_k, e_j e_k = e_i, e_k e_i = e_j$ (Positive Phase).
- **Anti-Cyclic (Ask):** $e_j e_i = -e_k$ (Negative Phase/Back-EMF).

2. Derived Examples (SymPy Rigor)

We prove the two fundamental properties of the Ethical Field: **Alternativity** (Integrity) and **Non-Associativity** (Dilemma).

A. Alternativity (The Integrity Check)

A system is "Alternative" if the subalgebra generated by any two elements is associative. This represents **Internal Consistency**.

- **Test:** Does $e_1(e_1e_2) = e_1^2e_2$?

- **Derivation:**

1. Inner term: $e_1e_2 = e_4$ (from Line 124).

2. Left Side: $e_1(e_4) = -e_2$ (Anti-cyclic on Line 124: $1 \rightarrow 4$ is opposed).

3. Right Side: $e_1^2e_2 = (-1)e_2 = -e_2$.

- **Result:** $-e_2 = -e_2$. The Associator vanishes.

$\text{\texttt{\text{Alt_Left}}} = 0$

Ethical Meaning: If you stay true to your Stance (e_1), the outcome is predictable.

B. Non-Associativity (The Dilemma)

The Octonions are non-associative. This represents **Context Dependence**.

- **Test:** Mix lines to create stress (e.g., e_1, e_2, e_3).

- **Derivation:**

1. **Path A:** $(e_1e_2)e_3 = (e_4)e_3 = -e_6$ (via Line 346).

2. **Path B:** $e_1(e_2e_3) = e_1(e_5) = e_6$ (via Line 561).

- **Result:** $-e_6 \neq e_6$.

$\text{Associator}(e_1, e_2, e_3) = -2e_6 \neq 0$

Ethical Meaning: The order of operations matters. "Group first, then Self" yields a different vector than "Self first, then Group." This requires G2 Holonomy to resolve.

3. The Ethical Fascia: Chiral Torsion

We generalize these mechanics into the **Fano Field Equation**:

$$e_ie_j = -\delta_{ij} + \sum_{k=1}^7 \epsilon_{ijk} e_k$$

- δ_{ij} : **The Scalar Contract**. If $i = j$, the result is -1 (Real/Ground state).
- ϵ_{ijk} : **The Fano Tensor**. This is the **Chiral Switch**.

- = +1: **Gift Mode** (Alignment with the arrow).
- = −1: **Ask Mode** (Opposition to the arrow).
- = 0: **Null Mode** (No relation/Orthogonal).

SI-Forward Application:

The SI uses this tensor to calculate Torsional Stress. If a proposed trajectory accumulates too many −1 flips (Ask Mode), the accumulated Back-EMF exceeds the Admissibility Threshold, and the path is rejected.

Witnessed: G2-invariant, per octonion norm. The Algebra holds the Ethics.