Q2: Clustering

Q-2

- We know that k-means assign each data point to a unique cluster by calculating distance to the cluster center. This is kind of hard assignment of clusters.

- On other hand, Gaussian mixture clustering performs soft assignment of each data point to the cluster using probabilities.

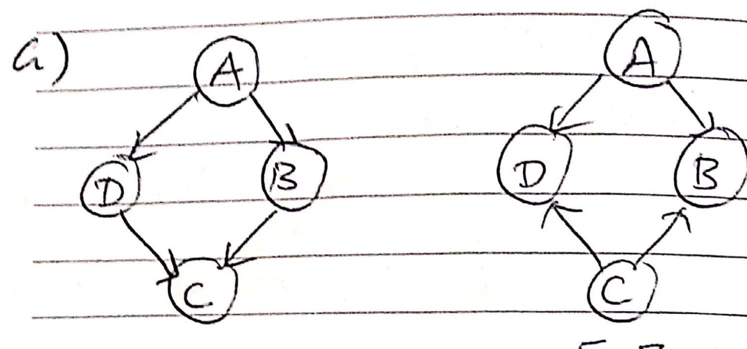
- Hence, if Gaussian mixture has large variance, then points on the edges between clusters may have different assigned clusters even if cluster centers are identical in both methods.

- So, yes, few points from different clusters in k-means solution can be assigned to same cluster in Gaussian mixture solution.

Q3: Bayesian Networks

Q-3: Bayesian Networks:

a)



[1] [2]

→ $A \perp C \mid B, D$

⇒ $A \perp C \mid S$, where $S = \{B, D\}$

- $\boxed{\text{For Network 1:}}$

via D - When we look at the path from A to C, we notice that arrows meet head-to-tail at D and D is in set S.

via B - Also, at node B as well, arrows ~~are~~ meet head-to-tail and B is in set S.

- Hence, from rule 1 of d-separation

algorithm, we can say that for Network 1, $A \perp C \mid B, D$ holds true.

For Network 2:

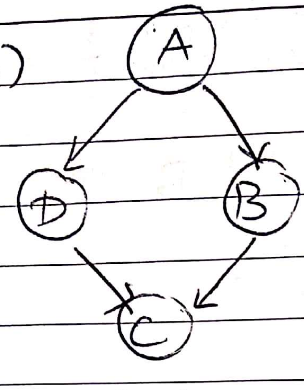
via D — Looking at path from A to C via D, We can notice that arrows meet head-to-head at D, But D is in the set S.

- hence, rule-2 holds false

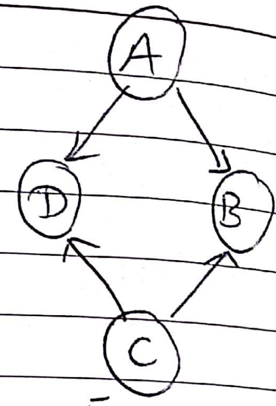
via B — For B as well, arrows are meeting head-to-head at B, But B is in the set S.

→ Hence, rule-2 of d-separation algorithm doesn't satisfy.

→ So, we can say that for Network 2 $A \perp C \mid B, D$ holds false b)



[1]



[2]

→ $B \perp D \mid A, C$

⇒ $B \perp D \mid R$, where $R = \{A, C\}$

For Network I :

via A — When we look at the path from B to D via A, we can notice that arrows meet tail-to-tail at A and nod A is in set R.

— It satisfies 1st sule of d-separation.

via C — Now, if we go via C, then arrows meet head-to-head at C, But C is in the set R.

— hence, 2nd sule of d-separation fails.

⇒ As, one of the rules of d-separation satisfied, we can say that

$B \perp D \mid A, C$ holds true

→ $\boxed{\text{For Network 2}}$:

$\boxed{\text{via } A}$ - We can notice that arrows meet tail-to-tail at node A and A is in set R.

$\boxed{\text{via } C}$ - Also, for node C, arrows meet tail-to-tail and C is in set R.

- Hence, from the 1st rule of d-separation, we can say that

$B \perp D \mid A, C$ holds true for network-2 as well.