1. (15')If $0 < P(B) < 1$,show that A and B are independent if and only if

   (1)$P(A|B) = P(A|B^c)$

   (2)$P(A|B) + P(A^c|B^c) = 1$

2. (15')There are n coins in the box. When flipped the ith coin will turn up heads with probability p_i which satisfies $\frac{1}{2} < p_1 < p_2 < \cdots < p_n < 1$. A coin is randomly selected from the box and is then repeatedly flipped. If there are exactly m heads in the first $2m$ flips, show that the probability of the first coin is picked is greater than $\frac{1}{n}$.

3. (15') Suppose$X \sim U(0, \pi)$ Show that the expected value of random variable $Y = \tan X$ does not exist.

4. (10')Suppose $X \sim Geometry(\frac{1}{3})$ Find the PMF of random variable $Y = X/(X+1)$

5. (15')Show that if $X \sim Beta(\alpha, \beta)$ then

$$E[g(x)(\beta - (\alpha - 1)\frac{(1-X)}{X})] = E[(1-X)g'(X)]$$

6. (15')Suppose $x_i \sim U(-0.5, 0.5), S_n = \sum_{i=1}^{n} X_i$

   (1)Solve $P(|S_{1200}| < 15)$

   (2 Find the maximum value of n such that $P(|S_n| \leq 10) \geq 0.9$

7. (15')The pdf of r.v X is

$$f_X(x) = \begin{cases} Ae^{-(x-\mu)^2/2\sigma_1^2} & x \leq \mu \\ Ae^{-(x-\mu)^2/2\sigma_2^2} & x > \mu \end{cases}$$

   (1)Find the value of A

   (2)Solve $Var(X)$