1. Find the MLE of the following pdf, where the sample size is $n$.

   (1) $f_X(x;\alpha,\beta)=\beta^{-\alpha}\alpha x^{\alpha-1}1_{(0,\beta)}(x)$ where $\alpha>0,\beta>0.$ Also derive the MME in this problem.

   (2)

$$f_X(x;\theta)=\begin{cases}1_{(0,1)}(x) & \theta=0\\ (2\sqrt{x})^{-1}1_{(0,1)}(x) & \theta=1\end{cases}$$

   (3) $f_X(x;\theta)=\frac{\theta}{1-\theta}x^{(2\theta-1)/(1-\theta)}1_{(0,1)}(x)$ where $\theta\in(\frac{1}{2},1)$

2. Suppose $Y_1,\ldots,Y_n$ are a random sample from $N(\beta_0+\beta_1 x_i,\sigma^2)$ $i=1,\ldots,n$ where $x_i$'s are known constants, find a sufficient statistic for $\theta=(\beta_0,\beta_1,\sigma^2)$

3. Suppose $X_1,\ldots,X_n$ are random variables with finite mean and finite variances.

   (1) If $X_1,\ldots,X_n$ are independent with $E(X_i)=\alpha i$ and $Var(X_i)=\sigma^2$ .Find the best **unbiased** estimator for $\alpha$ in the class $\hat\alpha=\sum_{i=1}^n c_iX_i$

   (2) If $X_1,\ldots,X_n$ are identically distributed with mean $\mu$ and variance $\sigma^2$ but are correlated with correlation coefficient $\rho$. Find the best unbiased estimator for $\mu$ in the class

$$\{\hat\mu=\sum_{i=1}^n c_iX_i : c_i\in\mathbb{R},\sum_{i=1}^n c_i=1\}$$

4. Suppose $X_1,\ldots,X_n$ are a random sample from a Weibull distribution

$$f_X(x)=\frac{\alpha}{\beta}x^{\alpha-1}\exp(-x^\alpha/\beta)1_{(0,+\infty)}(x)$$

   where $\alpha>0$, $\beta>0$. Suppose $\alpha$ is known and $\beta$ is unknown. Find an **unbiased** estimator for $\beta$ such that it can reach the Cramer-Rao lower bound. Furthermore, show that the estimator is consistent to $\beta$

5. Find the generalized likelihood ratio of $H_0: a=1$ versus $H_1: a\neq 1$ from the $N(\theta,a\theta^2)$ family where $\theta>0$ is an unknown parameter.

6. Let $X_1,\ldots,X_n$ be a random sample from $U(\theta,\theta+1)$. And we test $H_0:\theta=0$ versus $H_1:\theta>0$ we reject $H_0$ if $X_{(n)}\geq 1$ or $X_{(1)}\geq k$ where $k$ is a constant.

   (1) Find $k$ so that the test has a significance level $\alpha$.

   (2) Derive the power curve of the test in (1).

7. Suppose $X_1,\ldots,X_n$ are random samples from

$$f(x,\theta)=e^{-(x-\theta)}1_{[\theta,+\infty)}$$

   find a $(1-\alpha)100\%$ confidence interval for $\theta$ based on the estimator $X_{(1)}$ .