1. Suppose X_1, X_2, X_3 are independent exponential random variables with parameter $\lambda_1, \lambda_2, \lambda_3$, respectively. Let $X = X_1 + X_2, Y = X_2 + X_3$ Find the joint distribution of X and Y.

2. Suppose the distribution of Y conditional on $X = x$ is $N(x, x^2)$ and that marginal distribution of X is $U(0,1)$ Find $Cov(X,Y)$ and show that $\frac{Y}{X}$ and X are independent.

3. Suppose $X_1, X_2, \cdots, X_n$ are independent standard normal random variables. Let $Y_1 = \sum_{i=1}^{n} a_i X_i, Y_2 = \sum_{i=1}^{n} b_i X_i$ Show that Y_1 and Y_2 are independent if and only if $\sum_{i=1}^{n} a_i b_i = 0$

4. Suppose X and Y are random variables with finite variances. Show that
    1. X and $Y - E[Y|X]$ are uncorrelated.
    2. $Var(Y - E[Y|X]) = E[Var(Y|X)]$

5. Suppose $X_1, X_2, \cdots$ are i.i.d random variables with the distribution $U(-1,1)$ $S_n = \sum_{i=1}^{n} \arcsin |X_i|$
    1. Solve $P(|S_{1200}| < 1200\pi)$
    2. Find the maximum value of n such that $P(|S_n| \leq 150\pi) \geq 0.9$

6. Suppose random variables X and Y satisfy a bivariate normal distribution $N(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$
    1. Show that X and Y are independent if and only if $\rho = 0$
    2. Show that for all α, β which satisfies $\alpha^2 + \beta^2 \neq 0$ $Z = \alpha X + \beta Y$ is a normal random variable. Find $E[Z], Var(Z)$
    3. If$\mu_x = 0, \mu_y = 0, \sigma_x^2 = 1, \sigma_y^2 = 1$ Show that $E[\max(X,Y)] = \sqrt{\frac{1-\rho}{\pi}}$

7. Suppose $X_1, X_2, \cdots, X_n$ are independent random variables with variance σ_i^2 ,respectively. Find the weights a_i ($\sum_{i=1}^{n} a_i = 1$) to minimize the variance of $\sum_{i=1}^{n} a_i X_i$