Himanshu Dixit

B64178

Batch B10

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Bridge Course (21B19GE111)

Assignment Unit 1

Q.1. (i) $A \cap B = \{3,5,7,9,11\} \cap \{7,9,11,13\} = \{7,9,11\}$ Ans

(ii) $A \cap C \cap D = \{3,5,7,9,11\} \cap \{11,13,15\} \cap \{15,17\} = \phi$ Ans

(iii) $B \cap C = \{7,9,11,13\} \cap \{11,13,15\} = \{11,13\}$ Ans

(iv) $A \cap (B \cup C) = \{3,5,7,9,11\} \cap (\{7,9,11,13\} \cup \{11,13,15\})$
$= \{3,5,7,9,11\} \cap \{7,9,11,13,15\}$
$= \{7,9,11\}$ Ans

(v) $(A \cup D) \cap (B \cup C) = \{3,5,7,9,11,15,17\} \cap \{7,9,11,13,15\}$
$= \{7,9,11,15\}$ Ans

Q.2. (i) $(A \cup B)' = A' \cap B'$

L.H.S : $(\{2,4,6,8\} \cup \{2,3,5,7\})' = \{2,3,4,5,6,7,8\}' = \{1,9\}$

R.H.S : $\{2,4,6,8\}' \cap \{2,3,5,7\}' = \{1,3,5,7,9\} \cap \{1,4,6,8,9\} = \{1,9\}$

L.H.S = R.H.S ; hence Proved

(ii) $(A \cap B)' = A' \cup B'$

L.H.S : $(\{2,4,6,8\} \cap \{2,3,5,7\})' = \{2\}' = \{1,3,4,5,6,7,8,9\}$

R.H.S : $\{2,4,6,8\}' \cup \{2,3,5,7\}' = \{1,3,5,7,9\} \cup \{1,4,6,8,9\} = \{1,3,4,5,6,7,8,9\}$

L.H.S = R.H.S ; hence Proved.

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Q.3. $f: R \to R$ $\quad f(x) = [x]$

for one-one : let $x_1 = 1.2$ & $x_2 = 1.5$

$f(x_1) = 1$ & $f(x_2) = 1$

$\therefore$ $x_1 \neq x_2$ but $f(x_1) = f(x_2)$ hence, f'(n) is not one-one.

for Onto : every element of codomain did not have pre-image in domain such as 1.1, 19.5 etc.

hence, f'(n) is not onto.

Q.4. (i) $f(x) = |x|$, $g(x) = |5x-2|$

$fog(x) = ||5x-2|| = |5x-2|$ Ans, $gof(x) = |5|x|-2|$ Ans

(ii) $f(x) = 8x^3$, $g(x) = x^{1/3}$

$fog = fog(x) = f(g(x)) = 8(g(x))^3 = 8(x^{1/3})^3 = 8x$ Ans

$gof = gof(x) = g(f(x)) = (f(x))^{1/3} = (8x^3)^{1/3} = 2x$ Ans

Q.5. $f: R^+ \to [4,\infty)$ $\quad f(x) = x^2+4$

for one-one : let x_1 & x_2 be two R^+ no. such that $f(x_1) = f(x_2)$

$x_1^2 + \cancel{4} = x_2^2 + \cancel{4}$

$\therefore$ $x_1 = x_2$ $\quad [x_1 + x_2 \neq 0]$

hence, the function is one-one

for onto : $y = x^2+4 \in$ co-domain.

$\therefore$ $x^2 = y-4$

$x = \sqrt{y-4} \in$ domain.

hence, every element of co-domain has its pre-image in domain, i.e. the function is onto.

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If the function is one-one & onto then the function is invertible.

$$f^{-1}(x) = \sqrt{x-4} \quad \text{Ans}$$

Q.6. (a) $\sin^{-1}(-1/2)$ $\qquad [-\pi/2, \pi/2]$

$-\sin^{-1}(1/2) = -\frac{\pi}{6}$ Ans

(b) $\cos^{-1}\frac{\sqrt{3}}{2}$ $\qquad [0, \pi]$

$= \frac{\pi}{6}$ Ans

(c) $\operatorname{cosec}^{-1}(2)$ $\qquad [-\pi/2, \pi/2] - \{0\}$

$= \frac{\pi}{6}$ Ans

Q.7. $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x \quad ; \quad \frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

L.H.S : let $x = \sin\theta \in [-1/\sqrt{2}, 1/\sqrt{2}]$

$= \sin^{-1}(2\sin\theta\cos\theta) = \sin^{-1}(\sin 2\theta) \qquad \theta \in [-\pi/4, \pi/4]$

$= 2\theta = 2\sin^{-1}x$ R.H.S

hence, L.H.S = R.H.S ✓