Solution of Tutorial 4 (2021) 15B11PH111

① The grating equation $(a+b)\sin\theta = \pm n\lambda$ with $n = 1, 2, 3 \cdots$

$(a+b)$ = grating element

$\lambda = 5000 Å = 5000 \times 10^{-10}$ m , $(a+b) = \frac{2.54}{2620}$ cm $= 9.69 \times 10^{-6}$ m

$$n_{max} = \frac{(a+b)}{\lambda}\sin\theta_{max} = \frac{a+b}{\lambda} = 19.38 \approx 19$$

maximum no. of orders visible in the spectrum.

② $(a+b)\sin\theta = n\lambda$

or, $\frac{10^{-2}}{4000}\sin\theta = 3 \times 680 \times 10^{-9} = 2.04 \times 10^{-6}$

$\Rightarrow \sin\theta = 0.816$ or, $\theta = \sin^{-1}(0.816) = 54.68°$

③ Resolving power of grating $\rightarrow \frac{\lambda}{d\lambda} = nN$

$\lambda = 5890 Å$, $\lambda + d\lambda = 5896 Å$, $d\lambda = 6 Å$, $n = 2$

$$N = \frac{\lambda}{n\, d\lambda} = \frac{5890}{2 \times 6} = \frac{5890}{12} = 490.8 \approx 491 \text{ line,}$$

for $n = 3$, $N = \frac{5890}{18} = 327.22 \approx 327$

④ Since yellow line of specific order(n) is superimposed with blue line of next higher order (n+1), so

$$(a+b)\sin\theta = \pm n \times 6000 \quad \text{—(i)}$$
$$(a+b)\sin\theta = \pm(n+1)\, 4800 \quad \text{—(ii)}$$

Comparing (i) and (ii) we get

$n \times 6000 = (n+1) \times 4800$

$\Rightarrow n = 4$. Substituting n in (i) we get

$(a+b)\sin\theta = 4 \times 6000$

or, $(a+b) \times \frac{3}{4} = 24000 \Rightarrow (a+b) = 32000 \times 10^{-10}$ m $= 3.2 \times 10^{-6}$ m

⑤ Suppose, nth order of spectral line of λ_1 coinsides with mth order of spectral line of λ_2

$$(a+b)\sin\theta = n\lambda_1$$
$$(a+b)\sin\theta = m\lambda_2$$

Let $\lambda_1 = 6000 A^\circ$, $n = 5$

so, $5 \times 6000 = m \times \lambda_2 \Rightarrow \lambda_2 = \frac{30000}{m}$

then
$\lambda_2 = 7500 A^\circ$ for $m = 4$
$\lambda_2 = 6000 A^\circ$ " $m = 5$
$\lambda_2 = 5000 A^\circ$ " $m = 6$
$\lambda_2 = 4285.7 A^\circ$ " $m = 7$
$\lambda_2 = 3750 A^\circ$ " $m = 8$

So the spectral lines within $4000 A^\circ - 7000 A^\circ$ will be.
$4285.7 A^\circ$ and $5000 A^\circ$.

(6) The limit of resolution of microscope for λ is

$$d = \frac{1.22\lambda}{2\mu \sin\alpha}$$

$\lambda = 5461 A^\circ = 5.461 \times 10^{-7}$ m, $d = 4 \times 10^{-7}$ m

Numerical aperture $NA = \mu \sin\alpha = \frac{1.22\lambda}{2d} = 0.833$

(7) $\sin\theta_c = \frac{n_2}{n_1} = \frac{1}{1.5}$
$\theta_c = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8^\circ$

(8) $n_1 = 1$, $n_2 = 1.52$
Brewster angle $\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.52}{1}\right) = 56.7^\circ$
Refraction angle $\theta_R = 90^\circ - 56.7^\circ = 33.4^\circ$

(9) For air-water interface.
$\tan\theta_p = \frac{n_{water}}{n_{air}} = \frac{1.33}{1.00} \rightarrow \theta_p = 53.1^\circ$

From Snell's law
$n_{air} \sin\theta_p = n_{water} \sin\theta_2 \Rightarrow \sin\theta_2 = \frac{1}{1.33} \sin 53.1^\circ$
$\Rightarrow \theta_2 = \sin^{-1}\left(\frac{\sin 53.1^\circ}{1.33}\right) = 36.9^\circ$

For water-glass interface.
$\tan\theta_p = \tan\theta_3 = \frac{n_{glass}}{n_{water}} = \frac{1.50}{1.33}$
$\Rightarrow \theta_3 = 48.4^\circ$

so Angle between surface, $\theta = \theta_3 - \theta_2 = 11.5^\circ$

(10) $I = I_0 \cos^2\theta \rightarrow$ Malus law

$I_1 = \frac{I_0}{2}$, $I_2 = \frac{I_0}{2}\cos^2\theta$

$I_3 = \frac{I_0}{2}\cos^2\theta \times \cos^2(90^\circ - \theta)$

$= \frac{I_0}{2}\cos^2\theta \times \sin^2\theta = \frac{I_0}{8}\sin^2 2\theta$

(11) $I = I_{max}\cos^2\theta$

$\Rightarrow \theta = \cos^{-1}\sqrt{\frac{I}{I_{max}}}$

(a) $\frac{I}{I_{max}} = \frac{1}{3} \Rightarrow \theta_1 = \cos^{-1}\sqrt{\frac{1}{3}} = 54.7^\circ$

(b) $\frac{I}{I_{max}} = \frac{1}{5} \Rightarrow \theta_2 = \cos^{-1}\sqrt{\frac{1}{5}} = 63.4^\circ$

(c) $\frac{I}{I_{max}} = \frac{1}{10} \Rightarrow \theta_3 = \cos^{-1}\sqrt{\frac{1}{10}} = 71.5^\circ$