YDSE

① $\phi = \frac{2\pi}{\lambda} x$

② $y = \frac{x D}{d}$, $x = n\lambda$ for max.
$x = (2n+1)\frac{\lambda}{2}$ for min.

③ $\beta = \frac{\lambda D}{d}$

④ Visibility, $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

⑤ $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$
$A^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos\phi$

⑥ if a thin filf is placed in on path ~~say~~ S_1P
effective path $= S_1P + (\mu - 1)t$
$\Rightarrow x = \frac{yd}{D} - (\mu-1)t = n\lambda$ (for max.)
$y_0 = \frac{D}{d}(\mu-1)t$ (shifted)

⑦ Angular width. $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$

Fresnel Biprisim

1. $\delta = (\mu - 1)\alpha$
2. $D = a + b$
3. $d = 2\delta a$
4. $\beta = \frac{\lambda D}{d}$
5. If a biconvex is placed b/w prism and eyepiece

   $D > 4f$

   $\frac{I}{O} = \frac{h_i}{h_o} = \frac{v}{u}$ $\boxed{d = \sqrt{d_1 d_2}}$ d_1, d_2 are image heigh of slit

6. if base angles are different $\alpha \to \alpha_1 + \alpha_2$
7. if film is placed $\Delta = (\mu_1 - 1)t_1 - (\mu_2 - 1)t_2$ = shift

---

Principle of Reversibility

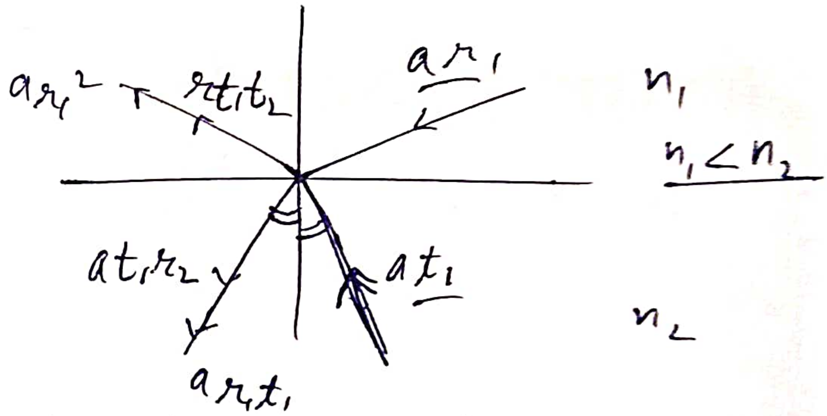
r = ref. coff.

t = transmission coff.

$\boxed{t_1 t_2 = 1 - r_1^2}$

$\boxed{r_2 = -r_1}$

Stock's law



There is a phase change(π) if light is reflected from denser medium not in case of rarer.

$$a_r = \frac{n_1 - n_2}{n_1 + n_2} a_i \qquad a_t = \frac{2n_1}{n_1 + n_2} a_i$$

3

LLOYD's MIRROR

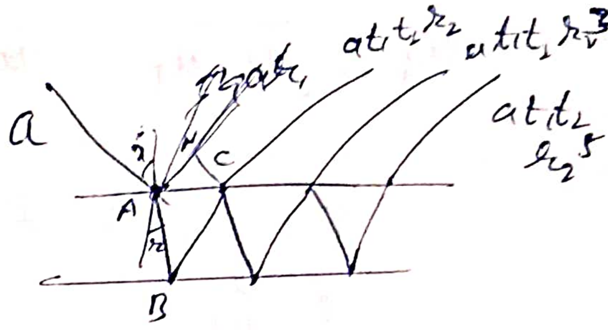
1. $x = n\lambda$ (destructive)
2. $x = (2n+1)\frac{\lambda}{2}$ (~~des~~ constructive)

Thin film

$a t_1 t_2 r_2 + a t_1 t_2 r_2^3 \ldots$

$= a r_2$

$= -a r_1$



$\Delta = \Delta x = n_2(AB+BC) - n_1 AN$

$\boxed{= 2n_2 t \cos r}$

$\Delta x = n\lambda \Rightarrow$ destructive

$\Delta x = (2n+1)\frac{\lambda}{2} =$ const.

In case of reflection

Transmission

$\Delta x = n\lambda \Rightarrow$ constructive

Excessively thin film

$t << \lambda$

$\Delta = 2\mu t \cos r - \frac{\lambda}{2} = n\lambda$ [constr]

$\Rightarrow \Delta \approx \frac{\lambda}{2} \Rightarrow \phi \approx 90° \Rightarrow$ dark film

---

Excessively thick.

$t >> \lambda$

$\Delta = n\lambda \Rightarrow$ constructive

$\Delta = 2\mu t \cos r$

Anti reflective coating

μ_{air}

μ_f

μ_g

$\mu_{air} < \mu_f < \mu_g$

$\Rightarrow \Delta x = (2n+1)\frac{\lambda}{2}$ for minima, for $\perp$ incidence, $r=0$

$2\mu_f t = \frac{\lambda}{2} \Rightarrow \boxed{t = \frac{\lambda}{4\mu_f}}$

Ex. MgF_2 (1.38)

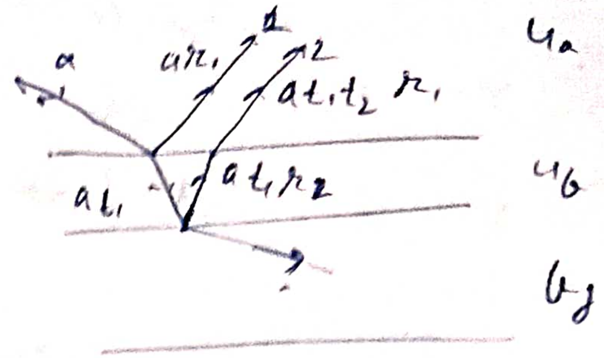
(4)

For complete destructive interference of 1 & 2

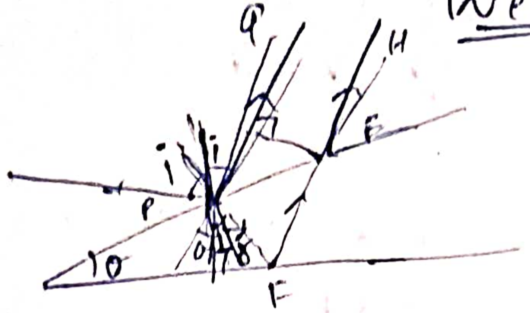
$a r_1 = a t_1 t_1' r_1$

solve to get

$\mu_f = \sqrt{\mu_a \mu_g} = \sqrt{\mu_g}$



Wedge shaped films



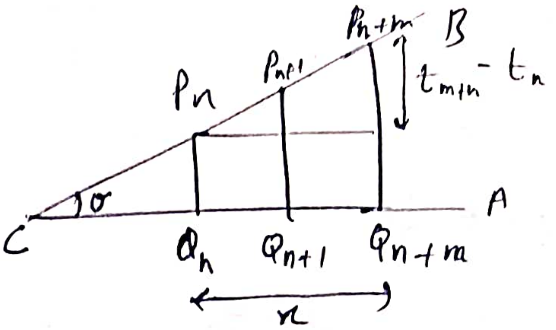
$\Delta x = \mu(PF + FE) - PK$

$= 2\mu t \cos(r+\theta)$

In real $\theta \approx 0 \Rightarrow \Delta x = 2\mu t \cos(r)$

$\Delta x = (2n+1)\frac{\lambda}{2}$ = Constructive

$\Delta x = n\lambda \Rightarrow$ destructive.



$r = 0$ for ⊥ incidence

$P_nQ_n = t_n$. let n^{th} is bright fringe.

$2\mu \, P_nQ_n = (2n+1)\frac{\lambda}{2}$

$2\mu \, P_{n+1}Q_{n+1} = (2n+3)\frac{\lambda}{2}$

$\Rightarrow \boxed{(P_{n+1}Q_{n+1} - P_nQ_n) = \frac{\lambda}{2\mu}}$

$\Rightarrow$ next bright fringe will appear at thickness change

$\Delta t = \frac{\lambda}{2\mu}$

Similarly for $(n+m)^{th}$ bright fringe $(P_{n+m}Q_{n+m} - P_nQ_n) = \frac{m\lambda}{2\mu}$

$\boxed{t_{m+n} - t_n = \frac{m\lambda}{2\mu}}$

For $(n+m)^{th}$ bright fringe

$$t_{m+n} - t_n = \frac{m\lambda}{2\mu}$$

for small θ, $\theta = \tan\theta = \frac{t_{m+n} - t_n}{x} = \frac{m\lambda}{2\mu x}$

$$\Rightarrow \frac{x}{m} = \boxed{\frac{\lambda}{2\mu\theta} = \beta}$$

<u>Note</u> Fringe at apex is dark.

If we derive it for air film we can easily get

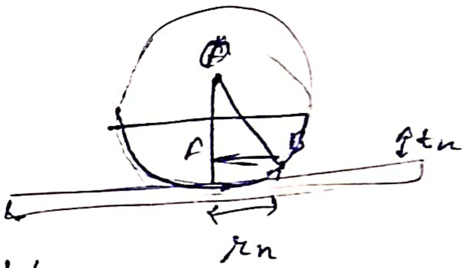
$$t_{m+n} - t_n = \frac{m\lambda}{2}$$

next bright fringe at ~~at~~ $\Delta t = \frac{\lambda}{2}$

for $\boxed{\beta = \frac{\lambda}{2\theta}}$

Reflectivity $= r^2$, ~~r = refl.~~ coff.

NEWTON RING



$t_n = r_n^2/2R = \frac{r_n^2}{2R}$ (by pythagores's)

for bright fringe $2\mu t\cos r = (2n+1)\lambda/2$

$$2t = 2(n+1)\frac{\lambda}{2}$$

$$2\frac{r_n^2}{2R} = (2n+1)\lambda/2$$

$$\Rightarrow \boxed{r_n = \sqrt{\frac{(2n+1)\lambda R}{2}}}$$

for dark $\boxed{r_n = \sqrt{n\lambda R}}$

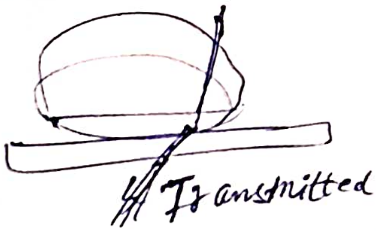
Center is dark

for dark $\quad r_n^2 = n\lambda R$

$D_n^2 = 4n\lambda R$

$D^2_{n+m} = 4(n+m)\lambda R$

$$\boxed{\lambda = \frac{D^2_{n+m} - D_n^2}{4mR}}$$



Bright $\quad 2\mu t \cos r = n\lambda$

Dark $\quad 2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

air as thin film, $r \approx 0$, $\mu = 1$

$2t = n\lambda \qquad 2t = (2n+1)\frac{\lambda}{2}$

but $t = \frac{r_n^2}{2R}$

bright $\quad 2 \cdot \frac{r^2}{2R} = n\lambda \Rightarrow r\sqrt{n\lambda R}$

dark $\quad r = \sqrt{\frac{(2n+1)\lambda R}{2}}$

Centre is bright.

$$\boxed{\lambda = \frac{D^2_{n+m} - D_n^2}{4mR}}$$

$D_n^2 = 4n\lambda R$ for dark

$D_{n+m}^2 = 4(n+m)\lambda R$

$$\boxed{D_{n+m}^2 - D_n^2 = 4m\lambda R} \quad \text{— (1)}$$

$\Rightarrow 2\mu t_n = n\lambda \Rightarrow$ dark

$t_n = \dfrac{r_n'^2}{2R} \Rightarrow \dfrac{2\mu r_n'^2}{2R} = n\lambda \Rightarrow r_n'^2 = \dfrac{n\lambda R}{\mu}$

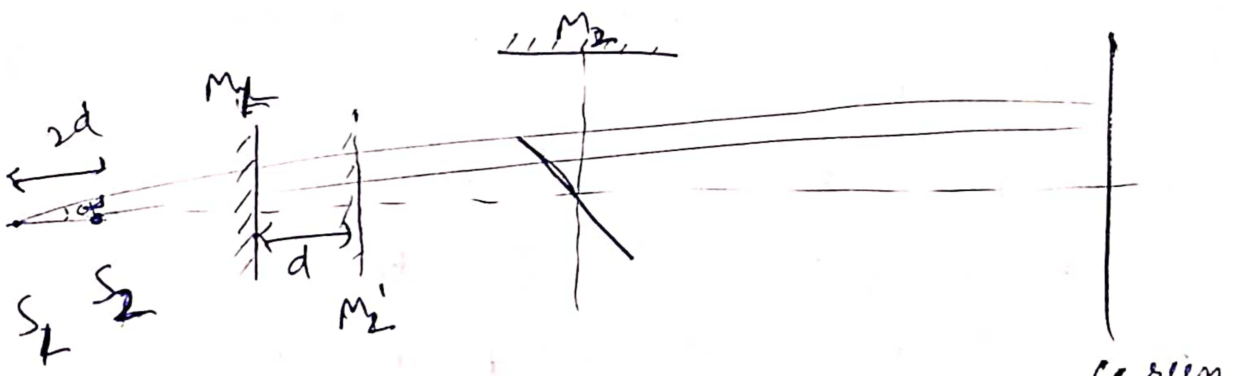
$D_n'^2 = \dfrac{4n\lambda R}{\mu}$

$\Rightarrow D_{n+m}'^2 - D_n'^2 = \dfrac{4m\lambda R}{\mu}$ — (2)

$$\boxed{\mu = \frac{4m\lambda R}{D_{n+m}'^2 - D_n'^2}}$$

$$\Rightarrow \boxed{\mu = \frac{D_{n+m}^2 - D_n^2}{D_{n+m}'^2 - D_n'^2}}$$

Michelson Interferometer



$\Delta x = 2d\cos\theta$

8

$2d\cos\theta = n\lambda \Rightarrow$ destructive ~~[illegible]~~

$2d\cos\theta = \left(n+\frac{1}{2}\right)\lambda \Rightarrow$ Constructive

---

$\Rightarrow$ If by moving M_1 ~~&~~ moves d_0 toward M_2 collapses N fringes we have:-

[$d\downarrow \Rightarrow$ fringes collapse at centre]

$$2d = m\lambda$$

$$2(d-d_0) = (m-N)\lambda \qquad [\theta=0 \text{ looking at centre}]$$

$$m\lambda - 2d_0 = m\lambda - N\lambda$$

$$\boxed{\lambda = \frac{2d_0}{N}}$$

If source have 2 wave length λ_1 & λ_2 and interferometer is set at ~~[illegible]~~ initially $\Rightarrow$ both overlap at $\theta=0$ / $d\approx 0$.

But if we move M_1 away/toward plate by $d \Rightarrow$

if $\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = \frac{1}{2}+n \Rightarrow$ fringes disappear ($\lambda_1 \approx \lambda_2$)

if $\frac{2d}{\lambda_1} - \frac{2d}{\lambda_2} = 1+n \Rightarrow$ " appear.

as if $2d\cos\theta' = m\lambda_1$ & $\Rightarrow$ $2d\cos\theta' = \left(m+\frac{1}{2}\right)\lambda_2$

maxima of λ_2 & minima of $\lambda_1 \Rightarrow$ fade

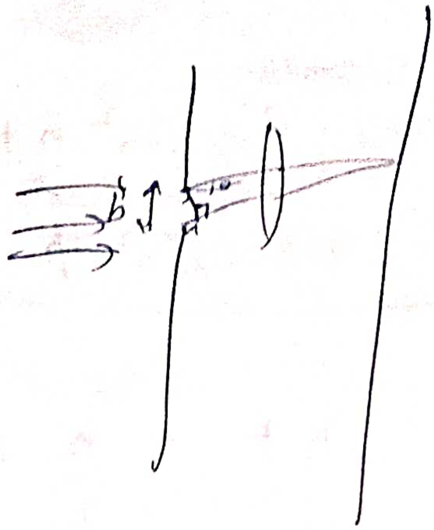
Instead of discrete λ if we have $\lambda \to \lambda + \Delta\lambda$ all then no interference if

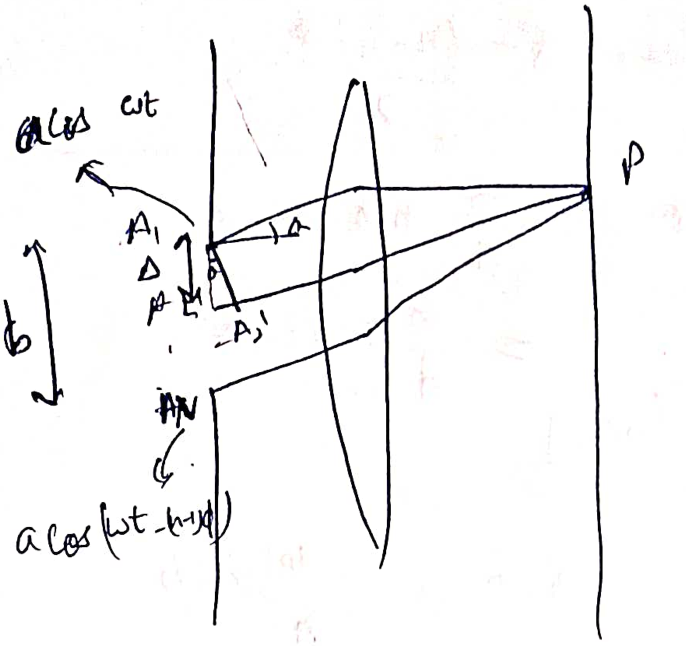
$$\frac{2d}{\lambda} - \frac{2d}{\lambda+\Delta\lambda} \geq \frac{1}{2} \qquad \left(\lambda + \Delta\lambda \approx \lambda \text{ if } \lambda_i \approx \lambda_f\right)$$

$$\Rightarrow \boxed{2d \geq \frac{\lambda^2}{\Delta\lambda}}$$

(9) - Single Slit

Fraunhofer





$$b = (n-1)\Delta$$

when $\Delta \to 0$ $\quad n\Delta \to b$

$$\underline{A_2 A_2' = \Delta \sin\theta} \Rightarrow \phi = \left(\frac{2\pi}{\lambda}\right)\Delta \sin\theta$$

Resultant field at P $\Rightarrow$

$$E = a\left[\cos\omega t + \cos(\omega t - \phi) + \cdots \cos(\omega t - (n-1)\phi)\right]$$

$$= a\,\frac{\sin n\phi/2}{\sin \frac{\phi}{2}} \cos\left(\omega t - (n-1)\frac{\phi}{2}\right) = R\cos(\omega t - \delta)$$

$$\Rightarrow \boxed{R = \frac{a\sin(n\phi/2)}{\sin(\phi/2)}} \quad \& \quad \delta = \frac{(n-1)\phi}{2}$$

$$\boxed{R\cos\delta = \frac{a\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}} \cos\frac{(n-1)\phi}{2}}$$

$$\boxed{R\sin\delta = \frac{a\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}} \sin\frac{(n-1)\phi}{2}}$$

$$\boxed{R^2 = \frac{a^2 \sin^2 \frac{n\phi}{2}}{\sin^2 \phi/2}}$$

$$\tan\delta = \tan\frac{(n-1)\phi}{2}$$

$$\Rightarrow \delta = \frac{(n-1)\phi}{2}$$

$$\frac{n\phi}{2} = \frac{n}{2}\frac{2\pi}{\lambda}\Delta \sin\theta = \frac{\pi}{\lambda} n\Delta \sin\theta$$

$$= \frac{\pi}{\lambda} b \sin\theta$$

$$\Rightarrow \boxed{\frac{n\phi}{2} = \frac{\pi}{\lambda} b\sin\theta} = \beta \text{ (say)}$$

$$\Rightarrow R = \frac{a\sin\beta}{\sin\beta/n}$$

if $n \to \infty \Rightarrow \sin(\beta/n) = \beta/n$

$$\Rightarrow R = \frac{an\sin\beta}{\beta} = \frac{A\sin\beta}{\beta}$$

$(A = na)$ say.

$$\boxed{I = R^2 = I_0 \frac{\sin^2\beta}{\beta^2}}$$

$\boxed{I_0 \text{ is intensity at } \theta = 0}$

$$R = \frac{A\sin\beta}{\beta} = \frac{A}{\beta}\left[\beta - \frac{\beta^3}{\lfloor 3} + \frac{\beta^5}{\lfloor 5} - \frac{\beta^7}{\lfloor 7} \cdots\cdots\right]$$

$$= A\left[1 - \frac{\beta^2}{\lfloor 3} + \frac{\beta^4}{\lfloor 5} - \cdots\cdots\right]$$

$$R_{max} = A \Rightarrow \beta = 0 \Rightarrow \frac{\pi b \sin\theta}{\lambda} = 0$$

$$\Rightarrow \cancel{b\sin\theta} = \boxed{\theta = 0}$$

OR

$$I_{max} = I_0 \Rightarrow \left(\frac{\sin\beta}{\beta}\right)^2 = 1 \Rightarrow \beta \to 0$$

$$\Rightarrow \theta \to 0$$

$I_{min} = 0 \Rightarrow I_0 \frac{\sin^2\beta}{\beta^2} = 0$ ~~$\Rightarrow \beta = \pm m\lambda$~~

~~but~~ $\beta \neq 0$

$\Rightarrow \beta = m\pi \, , \boxed{\beta \neq 0}$

$\Rightarrow \boxed{b\sin\theta = m\lambda}$ Minima condition

---

$I = I_0 \frac{\sin^2\beta}{\beta^2} \Rightarrow \frac{dI}{d\beta} = I_0 \left[\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3} \right] = 0$

$\Rightarrow \sin\beta[\beta - \tan\beta]$

$\Rightarrow \boxed{\beta = \tan\beta \text{ (for } I_{max})}$

$\sin\beta = 0 \rightarrow$ $\beta = 0 \Rightarrow$ Central maxi

$\beta = m\pi \Rightarrow$ minima

$\Rightarrow \boxed{b\sin\theta = n\lambda}$

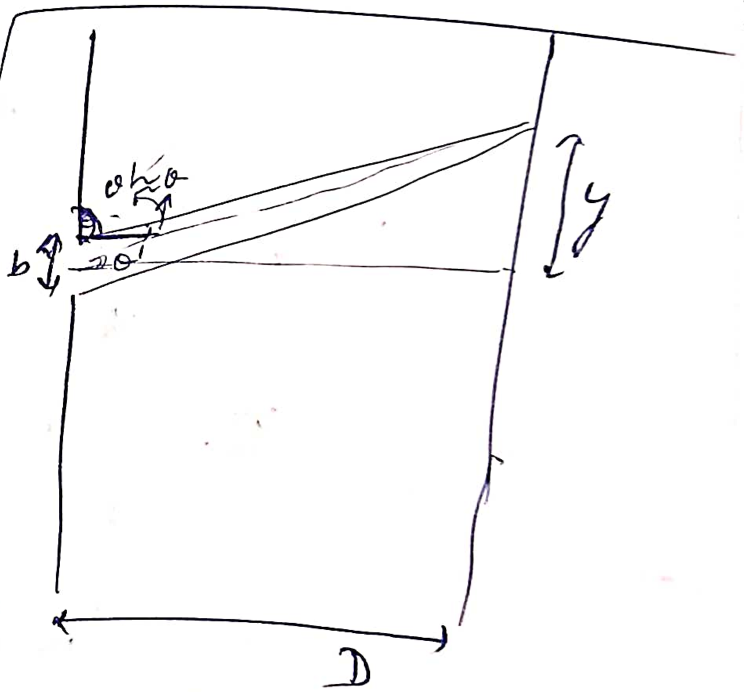
$\Rightarrow \beta = 1.43\pi$

$\beta = 2.46\pi$

- - - -

$\beta = \tan\beta$

$\boxed{b\sin\theta = (n+\frac{1}{2})\lambda}$



$\tan\theta = y/D \approx \theta \approx \sin\theta$

$\sin\theta = y/D$

1st minima is

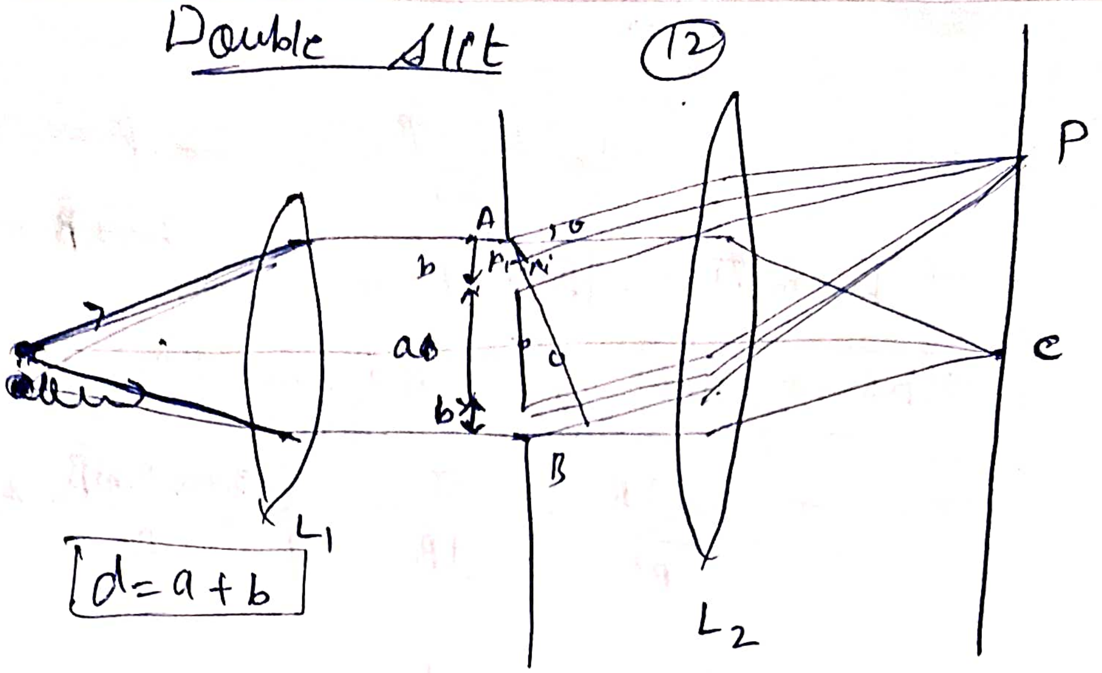
$b\sin\theta = \lambda$

$\sin\theta = \frac{\lambda}{b}$

$\frac{\lambda}{b} = \frac{y}{D} \Rightarrow y = \frac{\lambda D}{b}$

$\boxed{\text{slight width} = \frac{2\lambda D}{b}}$

Double slit (12)



$\boxed{d = a + b}$

By slit 1 at P $\quad E_1 = \frac{A \sin\beta}{\beta} \cos(\omega t - \beta)$

—— 2 —— $\quad E_2 = \frac{A \sin\beta}{\beta} \cos(\omega t - \beta - \phi_1)$

$$\phi_1 = \frac{2\pi}{\lambda} d \sin\theta$$

$$E = E_1 + E_2 = 2A \frac{\sin\beta}{\beta} \cos\gamma \cos\left(\omega t - \beta - \frac{1}{2}\phi_1\right)$$

$$\gamma = \frac{\phi_1}{2} = \frac{\pi}{\lambda} d \sin\theta$$

$$\boxed{I = 4I_0 \frac{\sin^2\beta}{\beta^2} \cos^2\gamma}$$

If $b \approx 0$

$\Rightarrow \frac{\sin^2\beta}{\beta} \approx$ Constant

$I = 0 \Rightarrow \beta = n\pi$ OR $\gamma = (2n+1)\frac{\pi}{2}$

$\Rightarrow b \sin\theta = m\lambda$

& $d \sin\theta = \left(n + \frac{1}{2}\right)\lambda$

$= (2n+1)\frac{\lambda}{2}$

13

Maxima Occur When ~~$\gamma = 0, \pi, 2\pi \ldots$ & $\beta = (2n+1)\frac{\pi}{2}$~~

~~$\Rightarrow d\sin\theta = 0\lambda, 2\lambda, \ldots$ & $b\sin\theta = (2n+1)\frac{\lambda}{2}$~~

~~$b\sin\theta = \lambda, 2\lambda, 3\lambda \ldots$ missing~~

$\beta = (2n+1)\frac{\pi}{2} \Rightarrow b\sin\theta = (2n+1)\frac{\pi}{2}$

& $\gamma = 2n\pi \Rightarrow d\sin\theta = n\lambda$

Missing

direction of [diffraction minima = interference maxima]

$d.\sin\theta = b\sin\theta$

$$\Rightarrow \boxed{\frac{d}{b} = \frac{m}{n}}$$

01

Polarisation

Basic facts:- (a) Interference & diffraction $\Rightarrow$ light is a wave.

(b) Max-well conjuctured light is e-m wave. ($\vec{E} \perp \vec{B}$ $\perp$ to direction of propagation of light.)

(c) interference cant tell light is longitudinal or transverse. Polarisation solve this issue and tell us that light is transverse (As logitudinal waves cant be polarised)

Classification of Polarised light

Light (EM Wave)

- Polarised
  - linearly/plane (PP) $\updownarrow$
  - Circularly (CP) (tip of $\vec{E}$ describe circle)
    - left (LCP)
    - right (RCP)
  - Elliptically (EP)
    - left (LEP)
    - Right (REP)
- Unpolarised

2.) (15)

Problem in solving in Light (polarised) classification.

(i) (a) A line through origin ⇒ $y = mx$

(b) Circle ⇒ $x^2 + y^2 = a^2$

(c) Ellipse ⇒ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(ii) If $\vec{E}$ is given make components $\vec{E}_x$ & $\vec{E}_y$

let $x = |\vec{E}_x|$ $y = |\vec{E}_y|$ and then check which of the three equations are satisfied by x, y.

Note ① If phase diff b/w $\vec{E}_x$ & $\vec{E}_y$ is $\pm n\pi$ then it is linearly polarised.

② Convert $\vec{E}_y$ & $\vec{E}_x$ both to sin or cos.

③ If ϕ (phase diff) $= \pm \frac{n\pi}{2}$ & $|\vec{E}_x| = |\vec{E}_y|$ ⇒ Circular Polarised

④ Else it is elliptical polarised.

Dispersion

diff. λ ⇒ diff $\angle$ of deviation in prism.

diff colours have diff. λ & μ is also diff for diff wavelength and have different angle of refraction (r)

as $\sin r = \frac{\sin i}{\mu}$

as $v = \nu\lambda$

$$ {}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\nu\lambda_1}{\nu\lambda_2} = \frac{\lambda_1}{\lambda_2} $$

if 1 = air ⇒ ${}_1\mu_2 = {}_{air}\mu_2 = \frac{c}{v_2} = \frac{c}{\nu\lambda_2}$

$$ \Rightarrow \mu = \frac{c}{\nu\lambda_2} \quad (\text{fxn of } \lambda) $$

Doubly refractive material / birefrigence are some material in ~~to~~ which a monochromatic ray split into 2 one is ordinary another is extra-ordinary. with different refrative (μ_o) (μ_e) indices.

optical path of o = $\mu_o t$

" " " $e = \mu_e t$ t = thickness

$\Delta x = (\mu_o - \mu_e) t$

If Δx = quater(half) of incident light/wave then we obtain quater (half) wave plate.

$(\mu_o - \mu_e) t = \frac{\lambda}{4}$ → defines a (QWP)

" " " $= \frac{\lambda}{2}$ → " " half WP

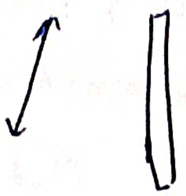
Δx of λ ≈ $\Delta\phi$ of 2π so $\lambda/4 \approx \pi/2$

in linearly PL Phase diff b/w E_x & E_y is 0 or π and in circular PL it is $\pi/2$ thus if we place a proper QWP can transform a CPL into LPL/PPL and vice-versa.

(17) Find nature of polarisation of given light source.

Case 1

analyser



PPL

Rotate analyser for 1 cycle (slowly)
You will get max. Intensity at 2 positions. when axis of analyser is ∥ to plane of polarisation. And we obtain 0 intensity when they are ⊥.

(2)

CPL

36°

No change in output intensity.

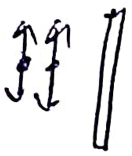
(3)

EPL

36°

2 Clear Maxima & Minima but Intensity at minima is $\neq 0$.

(4) Unpolarised



36°

We will get $I = I_0/2$ intensity output but In this case also intensity is constant as in CPL now what can we do?

(5) So we put a QWP in b/w

360°

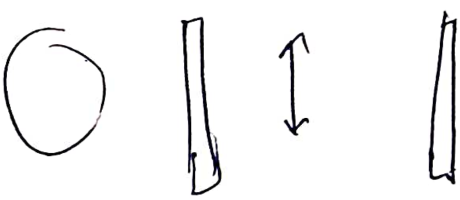
QWP ↓ analyser

unpolarised

↓

Constant Intensity.

Conclusion ⟹ unpolarised light was used.

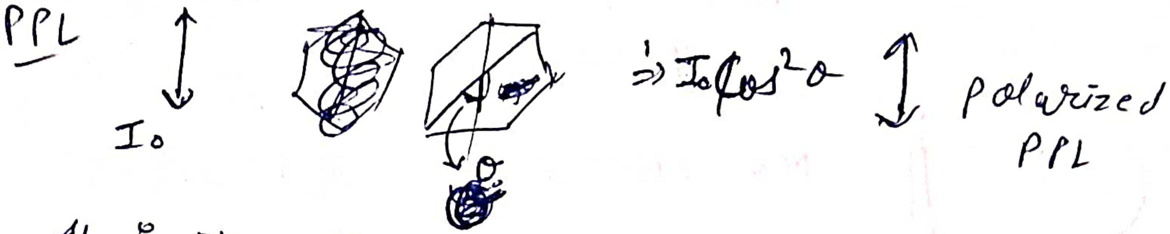


2 maxima & minima ⟹ Circular PL used.

Malus law

If PPL is passed through analyser whose axis makes θ angle with line of polarisation of incident light having intensity I_0 then output is

$$I = I_0 \cos^2\theta$$



<u>Note</u> If incident light is unpolarised the intensity gets half independent of angle θ.

19 Optical Activity

Change of ∠of polarisation of PPL after passing through a solution ~~A~~ by θ° is called optical activity ~~of solution~~.
And solution is optically active.

$\theta \propto$ no. of molecules it interacts per unit volume
$\propto$ length of tube it passes through.

$\theta \propto cl$

$\Rightarrow \boxed{\theta = Slc}$ S = constant called specific rotation

---

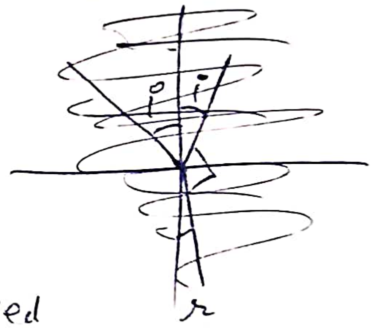
If solution contain mixture of 2 solutions then.

$\theta = \boxed{\theta_1 + \theta_2 = S_1 l c_1 + S_2 l c_2}$ If both have same nature

$\boxed{\theta = |\theta_1 - \theta_2| = |S_1 l c_1 - S_2 l c_2|}$ If both have diff nature

dextro and levo

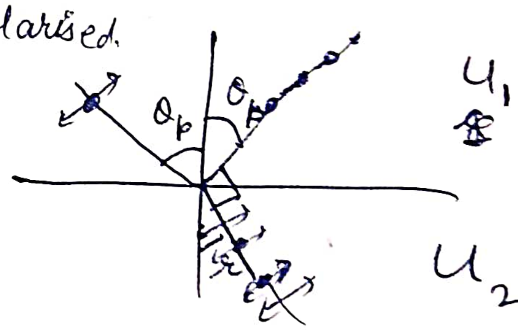
Brewster's law



It states that if $i = \theta_p$ then angle b/w reflected and refracted light is 90° and . If incident light is unpolarised ⇒ reflected will be polarised and refracted will be partially polarised.

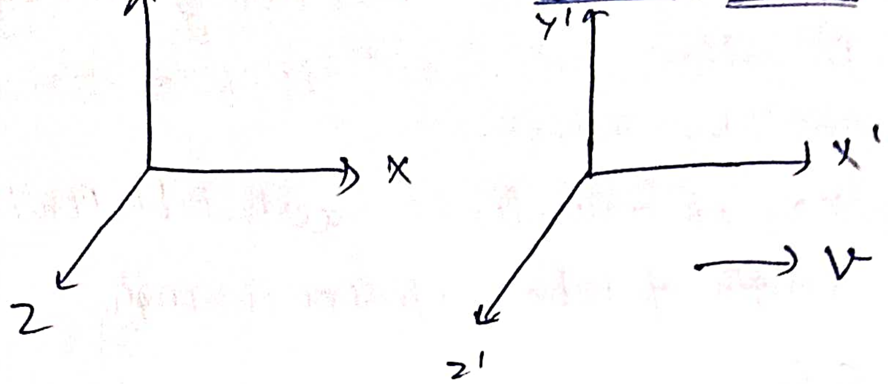
We can easily find that

$\boxed{\theta_p = \tan^{-1} u_2/u_1}$



Relativity

Inertial Frame



Galilian Transformation

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad t' = t$$

$$\Rightarrow V_x' = V_x - v$$

Note:- in all frames velocity of light is same $= c$, whether emitted by moving or stationary source.

Lorentz Transformation

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad y' = y \qquad z' = z \qquad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Inverse

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad y = y' \qquad z = z' \qquad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{K = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Length Contraction

A rod is lying in a moving frame S' along X' of length $= L_0 = X_2' - X_1'$

rod is at rest w.r.t. S'.

$$L_0 = X_2' - X_1' = \frac{X_2 - X_1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}} = KL$$

L is length of ~~that~~ that rod in S frame or rest frame.

clearly $L_0 > L$

Time Dilation

Assume a clock in S' at X'

$t_0 = t_2' - t_1'$ = observed by (observer in S')

The obs. in S will measure it.

$$t = t_2 - t_1 = \frac{t_2' + \frac{vx'}{c^2}}{1/K} - \frac{t_1' + \frac{vx'}{c^2}}{1/K}$$

$$= (t_2' - t_1')K = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$\Rightarrow \quad t > t_0$

$\Rightarrow$ Time ~~some~~ runs slower in S unprime frame / moving frame.

$\Rightarrow$ A stationary clock measures longer time.

Velocity addition

Obs. in S measures,

$$V_x = \frac{dx}{dt}, \quad V_y = \frac{dy}{dt}, \quad V_z = \frac{dz}{dt}$$

Obs. of S' measure it,

$$V_x' = \frac{dx'}{dt'} \qquad V_y' = \frac{dy'}{dt'} \qquad V_z' = \frac{dz'}{dt'} \qquad \underline{y' = y} \quad z' = z$$

$$x' = \frac{x - vt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \Rightarrow dx' = K(dx - v\,dt)$$

$$dt' = K\left(dt - \frac{v\,dx}{c^2}\right)$$

$$\frac{dx'}{dt'} = \boxed{V_x' = \frac{V_x - v}{1 - \frac{vV_x}{c^2}}}$$

Inverse $\boxed{V_x = \frac{V_x' + v}{1 - \frac{vV_x'}{c^2}}}$

similarly

$$V_y' = \frac{V_y\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{vV_x}{c^2}} \qquad V_z' = \frac{V_z\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{vV_z}{c^2}}$$

Relativity of Mass

Mass is more in S' frame. $= m_B$

mass in S $= m_a$

$$\boxed{m_a = m_b\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$m_0 =$ rest mass.

$\boxed{m > m_0}$

Mass & Energy

$$\boxed{T = mc^2 - m_0c^2}$$

$T =$ K.E.

$$\boxed{mc^2 = T + m_0c^2}$$

rest mass energy.

$$P = mv \Rightarrow p^2c^2 = \frac{E^2v^2}{c^2}$$

$$E^2 - p^2c^2 = m_0^2c^4$$

$$E^2 = \frac{m_0^2c^4}{1-\frac{v^2}{c^2}} = m_0^2c^4 + \frac{E^2v^2}{c^2}$$

$$\Rightarrow \boxed{E = \sqrt{m_0^2c^4 + p^2c^2}}$$