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B64178

DATE / PAGE ①

Mathematics T2 Examination - Phase-2 (odd 2021)

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BATCH : B10

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SUB. NAME : Mathematics - 1

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Q.1. — $1/84$

Q.2. — $\pi/112$

Q.3. — 36

Q.4. — $-\frac{1}{2}(e^2+1)$

Q.5. — $4/7$

Q.6. — $x^2+y^2 = 10$

Q.7. — 45

Q.8. — $1024/15$

Q.9. — $1502/105\, a^4$

Q.10. — $ye^x + x\sin y + c$

Himanshu Dixit

B64178

DATE / /
PAGE 2

Q.1. $\int_0^1 x^3(1-x^2)^5\,dx \qquad x^2 = t \qquad x\,dx = \frac{dt}{2}$

$\int_0^1 t(1-t)^5 \frac{dt}{2} = \frac{1}{2}\int_0^1 t^{2-1}(1-t)^{6-1}dt = \frac{1}{2}B(2,6)$

$= \frac{1}{2}B(2,6) = \frac{1}{2}\frac{\Gamma 2\,\Gamma 6}{\Gamma 8} = \frac{1}{2}\frac{5!}{7!} = \frac{1}{2\times 42}$

$= \frac{1}{84}$ Ans

Q.2. $\left(\frac{x}{2}\right)^1 + \left(\frac{y}{2}\right)^1 + \left(\frac{z}{2}\right)^1 = 1$

$a = b = c = 2, \quad p = q = r = 1, \quad \alpha = \frac{7}{2}, \beta = \frac{5}{2}, \gamma = 2$

$\iiint_T x^{5/2} y^{3/2} z^1\,dx\,dy\,dz = ?$

$= \frac{a^\alpha b^\beta c^\gamma}{pqr} \frac{\Gamma\alpha/p\;\Gamma\beta/q\;\Gamma\gamma/r}{\Gamma\left(\alpha/p + \beta/q + \gamma/r + 1\right)}$

$= \frac{2^{7/2}2^{5/2}2^2}{1}\frac{\Gamma 7/2\;\Gamma 5/2\;\Gamma 2}{\Gamma 9} = 2^8 \frac{5/2\times 3/2\times 1/2\sqrt{\pi}\times 3/2\times 1/2\sqrt{\pi}}{8!}$

$= \frac{2^8}{8!}\frac{45}{2^5}\pi = \frac{2^3\times 45}{8!}\pi = \frac{\pi}{112}$ Ans

Q.3 $\int_C F.dr = ?$ $\qquad \frac{x-1}{2} = \frac{y-2}{4} = \frac{z-2}{4}$ (line segment from (1,2,2) to (3,6,6))

$\frac{x-1}{\cancel{2}} = \frac{y-2}{\cancel{4}\,2} \qquad \frac{x-1}{2} = \frac{z-2}{4}$

$y = 2x - \cancel{2} + \cancel{2} \qquad dz = 2dx$

$dy = 2dx$

Himanshu Dixit

B64178

DATE
PAGE 3

$$\int_1^3 x\,dx + 2x2x\,dx + 2x2x\,dx$$

$$\frac{x^2}{2} + 2x^2 + 2x^2 \Big]_1^3 = \frac{9}{2} + 18 + 18 - \frac{1}{2} - 2 - 2$$

$$4 + 16 + 16 = 36 \text{ Ans}$$

Q.4. $\int_1^e \int_1^{\log y} \int_1^{e^x} dz\,dx\,dy$

$$\int_{y=1}^{e} \int_{x=1}^{\log y} \int_{z=1}^{e^x} dz\,dx\,dy = \int_{y\,1}^{e} \int_{x\,1}^{\log y} e^x - 1\,dx\,dy$$

$$\int_1^e e^x - x \Big]_1^{\log y} dy = \int_1^e y - \log y - e + 1\,dy$$

$$\frac{y^2}{2} - (e+1)y \Big]_1^e - \int_1^e \log y\,dy \qquad \left[\int \log x\,dx = x\log x - x \right]$$

$$\frac{e^2}{2} - (e+1)e - \left(\frac{1}{2} - (e+1)\right) - y\log y - y \Big]_1^e$$

$$\frac{e^2}{2} - e(e+1) - \frac{1}{2} + (e+1) - (\cancel{e} - \cancel{e} - 0 + 1)$$

$$\frac{e^2}{2} - \frac{1}{2} + (\cancel{1} - e^2) - \cancel{1} = -\frac{e^2}{2} - \frac{1}{2} = -\frac{1}{2}(e^2 + 1) \text{ Ans}$$

Q.5. $f(x,y,z) = x^3 - xy^2 - z^2$

$$\nabla f = \left(\hat{i}\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right)(x^3 - xy^2 - z)$$

$$= (3x^2 - y^2)\hat{i} - 2xy\,\hat{j} - \hat{k}$$

$$\nabla f_{(1,1,0)} = 2\hat{i} - 2\hat{j} - \hat{k}$$

$$\text{unit vector of } V = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{4+9+36}} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$$

Himanshu Dixit

B64178

DATE
PAGE (5)

Q.10 $\bar{F}(x,y) = (ye^x + \sin y)\,\hat{i} + (e^x + x\cos y)\,\hat{j}$

$$\text{Curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ye^x + \sin y & e^x + x\cos y & 0 \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(\cancel{e^x} + \cancel{\cos y} - \cancel{e^x} - \cancel{\cos y})$$

$$= 0$$

hence, curl$(\bar{F}) = 0$ i.e. $\bar{F}$ is conservative $\vec{F}$ Ans

$\vec{F} = \nabla \bar{F}$ (on comparing)

$(ye^x + \sin y) = \frac{\partial F}{\partial x}$ — ①

$\frac{\partial F}{\partial y} = e^x - x\cos y$ — ②

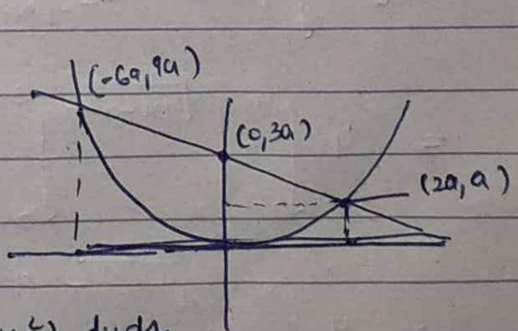
$F = ye^x + x\sin y + g(y,z)$

$\frac{dF}{dy} = e^x + x\cos y + \frac{\partial g}{\partial y}$ — ③

from ② & ③ : $\frac{\partial g}{\partial y} = 0$ $\therefore$ $g(y) =$ Constant.

$$\boxed{\bar{F} = ye^x + x\sin y + C}$$ Ans

Q.9. $\int_0^{2a}\int_{\frac{x^2}{4a}}^{3a-x} (x^2+y^2)\,dy\,dx$



$$\int_{y=a}^{3a}\int_0^{3a-y} (x^2+y^2)\,dy\,dx + \int_0^a\int_0^{2\sqrt{ay}} (x^2+y^2)\,dy\,dx$$

$y = 3a - x$

$x^2 = 4ay$

$(x+6a)(x-2a) = 0$

$x^2 = 4a(3a-x)$

$x^2 + 4ax - 12a^2 = 0$

Himanshu Dixit

B64178

$$\int_0^{3a} \frac{x^3}{3} + y^2x \Big]_0^{3a-y} dy + \int_0^{a} \frac{x^3}{3} + y^2x \Big]_0^{2\sqrt{ay}} dy$$

$$\int_0^{3a} \frac{(3a-y)^3}{3} + (3a-y)y^2\, dy + \int_0^{a} \frac{8a^{3/2}y^{3/2}}{3} + 2a^{1/2}y^{5/2}\, dy$$

$$-\frac{(3a-y)^4}{12} + \frac{\cancel{3}ay^3}{\cancel{3}} - \frac{y^4}{4}\Big]_0^{3a} + \frac{8}{3}a^{3/2}\frac{y^{5/2}}{5/2} + 2a^{1/2}\frac{y^{7/2}}{7/2}\Big]_0^{a}$$

$$27a^4 - \frac{81a^4}{4} + \frac{81a^4}{12} + \frac{16}{15}a^4 + \frac{4}{7}a^4$$

$$\frac{27}{2}$$

$$\left(\frac{136080 - 102060 + 34020 + 5376 + 2880}{5040}\right)a^4$$

$$\frac{76296}{5040}a^4 = \frac{3179}{210}a^4$$

Q.8.

$$z^2 = 4x$$

$$\left.\begin{aligned} z &= \pm 2\sqrt{x} \\ y &= \pm\sqrt{4x-x^2} \\ x &= 0 \text{ to } 4 \end{aligned}\right] \text{ for limits.}$$

$$V = \int_{x=0}^{4}\int_{y=-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}}\int_{-2\sqrt{x}}^{2\sqrt{x}} dz\, dy\, dx$$

$$= 4\int_0^4\int_0^{\sqrt{4x-x^2}}\int_0^{2\sqrt{x}} dz\, dy\, dx$$

$$= 4\int_0^4\int_0^{\sqrt{4x-x^2}} \left[2\sqrt{x}\right] dy\, dx$$

$$= 4\int_0^4 2\sqrt{x}\left(\sqrt{4x-x^2}\right) dx$$

let $x = 4-x$

$$= 4\int_0^4 2\sqrt{4-x}\left(\sqrt{4(4-x)-(4-x)^2}\right) dx.$$

$$= 4\int_0^4 2\sqrt{4-x}\sqrt{x(4-x)}\, dx.$$

$$= 8\int_0^4 \sqrt{4-x}\sqrt{x(4-x)}\, dx..$$

$$= 8\int_0^4 (4-x)\sqrt{x}\, dx = 8\int_0^4 4x^{1/2} - x^{3/2}\, dx$$

$$= 8\left[4\times\frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2}\right]_0^4$$

$$= 8\left[4\times\frac{8}{3/2} - \frac{2^6}{5}\right] = 8\left[\frac{64}{3} - \frac{64}{5}\right]$$

$$= 8\times 64\times\frac{2}{15} = \frac{1024}{15}$$ Ans

Himanshu Dixit

B64178

DATE
PAGE (4)

directional derivative : $(2\hat{i}-2\hat{j}-\hat{k})\left(\frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}\right)$

$$= \frac{4+6-6}{7} = \frac{4}{7} \text{ Ans}$$

Q.6. $f(x,y) = 16-x^2-y^2 \qquad (2\sqrt{2}, \sqrt{2})$

$$f(x,y) = 16-x^2-y^2$$
$$f(2\sqrt{2}, \sqrt{2}) = 16-(2\sqrt{2})^2-(\sqrt{2})^2$$
$$= 16-8-2$$
$$= 6$$

So, the eq. of level curve is :

$$6 = 16-x^2-y^2$$
$$\therefore \boxed{x^2+y^2 = 10} \text{ Ans}$$

Q.7. $\bar{F} = x^2y\,\hat{i} - xy^2\,\hat{j} \qquad r(t) = t\hat{i}+t^2\hat{j} \qquad 0 \le t \le 3$

$$\frac{dr}{dt} = 1+2t$$

$$\vec{F} = t^4\hat{i} - t^6\hat{j} \qquad (x=t,\ y=t^2)$$

$$\int \vec{F}\cdot\frac{d\bar{r}}{dt}\,dt = \int_0^3 (t^4 - t^6)\,dt$$

$$= \left.\frac{t^5}{5} - \frac{t^7}{7}\right]_0^3 = \frac{3^5}{5} - \frac{3^7}{7}$$

$$= \cancel{\frac{3^5}{2} - \frac{81}{2} = \frac{90}{2} = 45 \text{ Ans}}$$

$$= \frac{3^5}{35}\left[7 - 9\times5\right] = -\frac{3^5\times 38}{35} \text{ Ans}$$