Tutorial sheet - 6 - Solution

Physics - 1 (15B11PH111) ODD 2021

Q.1- $S = \frac{10\theta_1}{l_1 c_1} = \frac{10\theta_2}{l_2 c_2} \Rightarrow \frac{20\times 10}{10\times 0.05} = \frac{35\times 10}{l_2 \times 0.1} \Rightarrow l_2 = 8.75\ cm$ Ans

Q2. $m_1 = 10$ gram, $m_2 = 15$ gram, $l_1 = l_2 = 20\ cm$, $V = 100\ cm^3$

$$\theta = \theta_1 + \theta_2 \Rightarrow S_1 l_1 \frac{m_1}{V} + S_2 l_2 \frac{m_1}{V}$$

$$7^\circ = 66.54 \times 2 \times \frac{10}{100} + S_2 \times 2 \times \frac{15}{100}$$

$$S_2 = \frac{100}{2\times 15}(7 - 6.654\times 2) = -21.02\ deg\ cm^3/g\text{-}dm$$

Ans

Q.3- Since the mixture of the two solution is in volume ratio 1:2, therefore the ratio of the length of the two solution in the mixture of length 30 cm will also be 1:2.

So length l_1 occupied by first ~~sol~~ solution $= 30\times\frac{1}{3} = 10\ cm$

" l_2 " " second solution $= 30 \times \frac{2}{3} = 20\ cm$

Again rotation $\propto$ length

for first solution

$$\frac{+42}{\theta_1} = \frac{20}{10}$$

$$\theta_1 = +21^\circ$$

for second solution

$$\frac{-27}{\theta_2} = \frac{30}{20}$$

$$\theta_2 = -18^\circ$$

Optical rotation of the mixture $= +21^\circ - 18^\circ = +3^\circ$ (Dextro)

Q.4- According to G.N transformation relation

$$x' = x - vt \ (i), \quad y' = y \ (ii), \quad z' = z \ (iii), \quad t' = t \ (iv)$$

velocity $u_x' = \frac{dx'}{dt'}$, $u_x = \frac{dx}{dt}$, $u_y' = \frac{dy'}{dt}$, $u_y = \frac{dy}{dt}$, $u_z' = \frac{dz'}{dt'}$, $u_z = \frac{dz}{dt}$

from eq^n. (i), (ii) & (iii)

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v \Rightarrow u_x' = u_x - v; \quad u_y' = u_y; \quad u_z' = u_z \quad (v)$$

Since $u_x' \neq u_x \Rightarrow$ velocity is variant.

for ac. diff eqⁿ (V) w.r.to t

$$\frac{du_x'}{dt'} = \frac{du_x}{dt} - 0 \quad \text{as } v \text{ is constant}$$

$$a_x' = a_x \quad \text{Similarly } a_y' = a_y,\ a_z' = a_z$$

this shows acceleration is invariant under G.T. transformation

Q.5 - Circular lamina (in motion along x or y axis) appear as an ellipse to a stationary observer in frame S. Consider d is the diameter of the circle and motion is along x axis - the diameter d get contracted along x axis such that $d_x = d\sqrt{1-\frac{v^2}{c^2}}$

Now the area of ellipse thus formed is

$$A_e = \pi \times \frac{d}{2} \times \frac{d}{2}\sqrt{1-\frac{v^2}{c^2}} = \frac{\pi d^2}{4}\sqrt{1-\frac{v^2}{c^2}}$$

for observer of frame S, the area of circular lamina is

$$A_c = \pi d^2/4$$

According to question

$$A_e = A_c/2$$

$$\frac{\pi d^2}{4}\sqrt{1-\frac{v^2}{c^2}} = \left(\frac{\pi d^2}{4}\right)/2 \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2} \Rightarrow v^2 = \frac{3}{4}c^2$$

$$v = \frac{\sqrt{3}}{2}c = 2.6 \times 10^8 \text{ m/s.}$$

Q.6. $v = 0.8c$, Component of the length along the direction of motion & $\perp$er to the direction of motion is respectively

$$L_x = L_0 \cos 60^\circ = \frac{L_0}{2} \quad \& \quad L_y = L_0 \sin 60^\circ = \sqrt{3}\frac{L_0}{2}$$

length contraction occurs along the direction of motion

$$L_x' = \frac{L_0}{2}\sqrt{1-\left(\frac{0.8c}{c}\right)^2} = 0.3L_0.$$

So new Length of the rod $L' = \sqrt{L_x'^2 + L_y^2}$ as no change in $\perp$er direction of motion.

$$L' = \sqrt{(0.3L_0)^2 + \left(\frac{\sqrt{3}L_0}{2}\right)^2} = 0.917\,L_0$$

$$\text{length contraction } (\%) = \frac{1-0.917}{1} \times 100 = 8.3\%$$

Ans

Q.7. As clock loses 1 min in 1 hour. So there will be 59 min in each hour.

$\Delta t_0 = 59$ min , $\Delta t = 60$ min

$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow 1-\frac{v^2}{c^2} = \frac{59^2}{60^2} \Rightarrow v = 5.45\times10^{7}$ m/s.

Q.8. At rest - $t_{half} = 17.8\times10^{-9}$ s. , $v = 0.8c$

$t'_{half} = \frac{t_{half}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{17.8\times10^{-9}}{\sqrt{1-\left(\frac{0.8c}{c}\right)^2}} = 2.97\times10^{-8}$ s.

Ans.

Q.9. for round trip. $\Delta t = 2\left(\frac{L_0}{v} - \frac{L_0'}{v}\right)$

$\Rightarrow \Delta t = 2\frac{L_0}{v}\left(1-\left(\sqrt{1-\frac{v^2}{c^2}}\right)\right) = \frac{2\times4c}{0.9c}\left(1-\left(\sqrt{1-\left(\frac{0.9c}{c}\right)^2}\right)\right)$

$= 5$ years.

Ans.