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Tutorial sheet - 8 (PHYSICS)

(Q1) Balmer series, $R = 1.097 \times 10^7 m^{-1}$

Shortest wavelength $\rightarrow \frac{1}{\lambda_s} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right)$.

$= 364.6nm$

longest wavelength $= 656.3nm$

(Q2) 32 elements

(Q3) P state

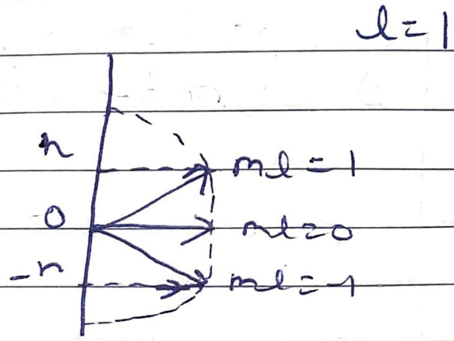
$l = 1$

$m_l = 0, \pm 1$

$L = \sqrt{l(l+1)}\hbar = \sqrt{2}\hbar$

$L_z(max) = \hbar$

Percentage difference $= (\sqrt{2} - 1)/\sqrt{2} = 29.3\%$



d state

$l = 2$

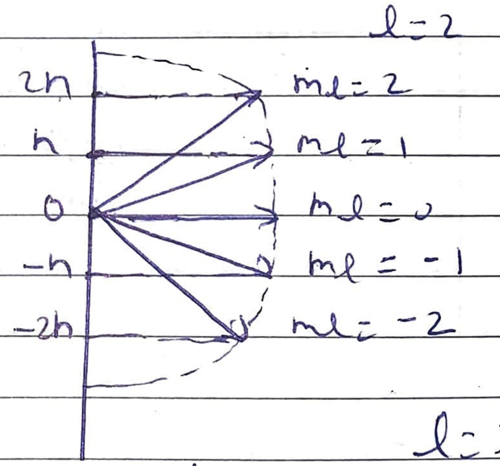
$m_l = 0, \pm 1, \pm 2$

$L = \sqrt{l(l+1)}\hbar = \sqrt{6}\hbar$

$l_z(max) = 2\hbar$

percentage difference

$= (\sqrt{6} - 2)/\sqrt{6} = 18.4\%$



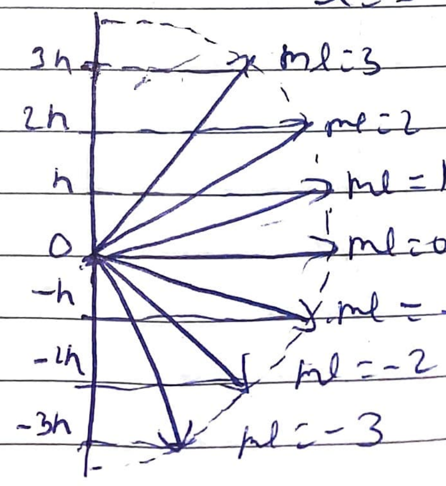
f state

$l = 3$

$m_l = 0, \pm 1, \pm 2, \pm 3$

$L_z(max) = 3\hbar$

percentage difference $= (\sqrt{12} - 3)/\sqrt{12} = 13.4\%$



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Orbital angular momentum

(Q4) $|\vec{L}| = \hbar\sqrt{l(l+1)}$

with its projection on the z axis is

$L_z = m_l \hbar$

Spin angular momentum

$|\vec{S}| = \hbar\sqrt{s(s+1)}$

with its projection on the z axis

$S_z = m_s \hbar$

Total angular momentum

$|\vec{J}| = \hbar\sqrt{j(j+1)}$

with its projection on the z-axis

$J_z = m_j \hbar$

(Q5) They are impossible to exist because the electrons ~~too~~ cannot stably occur in their orbitals

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(Q6) The lowest excited state of helium is represented by the electron configuration 1S2S. There are four possible state from 1S2S configuration: a singlet state

$$\Psi^{+}(1,2) = \frac{1}{\sqrt{2}}\left(\psi_{1s}(1)\,\psi_{2s}(2) + \psi_{2s}(1)\,\psi_{1s}(2)\right)\sigma_{0,0}(1,2)$$

and three triplet states

$$\Psi^{-}(1,2) = \frac{1}{\sqrt{2}}\left(\psi_{1s}(1)\,\psi_{2s}(2) - \psi_{2s}(1)\,\psi_{1s}(2)\right)$$

$$\begin{cases} \sigma_{1,1}(1,2) \\ \sigma_{1,0}(1,2) \\ \sigma_{1,-1}(1,2) \end{cases}$$

Using Hamiltonian in equation we can compute the approximate energies

$$E^{\pm} = \iint \Psi^{\pm}(1,2)\, H\, \Psi^{\pm}(1,2)\, d\tau_1\, d\tau_2$$

The reduced form

$$E^{\pm} = I(1s) + I(2s) + J(1s,2s) \pm K(1s,2s)$$

In one electron integral →

$$I(a) = \int \psi_a(r)\left\{-\frac{1}{2}\nabla^2 - \frac{Z}{r}\right\}\psi_a$$

The coulomb integrals

$$J(a,b) = \iint \psi_a(r_1)^2\, \frac{1}{r_{12}}\, \psi_b(r_2)^2\, d\tau_1$$

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The exchange integrals →

$$\iint \psi_a(r_1)\psi_b(r_1)\frac{1}{r_{12}}\psi_a(r_2)\psi_b(r_2)\,d\tau_1$$

The coulomb integral represent replusive potential energy. The exchange integral arises because of exchange symmetry requirement of the wavefunction. Both J and K can be shown to be positive quantities. Therefore the lower sign represents the state of lower energy making the triplet state of the configuration 1s2s lower in energy than the singlet state.

(Q7) $\frac{h}{2\lambda}$

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(Q8) $\Delta E = \frac{2eh}{2m} B$

$= \frac{eh}{2\pi m} B$

$B = \frac{2\pi m \Delta E}{eh}$

The difference in energy levels corresponds to the difference in wavelengths for the two transitions

$$\nu = \frac{c}{\lambda}$$

$$\Delta\nu = \frac{c}{\lambda^2}\Delta\lambda \text{ (ignoring -ve sign)}$$

$$\Delta E = h\Delta\nu$$

$$= \frac{hc\Delta\lambda}{\lambda^2}$$

$$B = \frac{2\pi m}{eh} \frac{hc\Delta\lambda}{\lambda^2}$$

$$= \frac{2\pi mc\Delta\lambda}{e\lambda^2}$$

$$= \frac{2\pi(9.1095\times10^{-31})(3\times10^{8})(0.6\times10^{-9})}{(1.602\times10^{-19})(5.89\times10^{-7})^2}$$

$$= 18.61$$

The effective magnetic field due to the atom's nucleus

(Q9) $\Delta\lambda = \frac{4\mu B\lambda^2}{hc} = \frac{9.27\times10^{-24}\times0.3\times(450)^2\times(10^{-9})^2}{6.63\times10^{-34}\times3\times10^{8}}$

$$= 2.83\times10^{-12}\ m$$

(Q10) The wavelength seperation is given by

$\Delta\lambda = eB\lambda^2/4\pi mc$

$\Delta\lambda = \cdot 0323$ ~~nm~~ $= 3.23\times10^{-11}$ m

$\lambda = 500\times10^{-9} = 5\times10^{-7}$

$B = 5$ Tesla

$c = 3\times10^{8}$ m/s

$$3.23\times10^{-11} = \frac{e\times1\times(5\times10^{-7})^2}{4\times 3.14\times m\times 3\times10^{8}}$$

$$\frac{3.23\times10^{-11}\times4\times3.14\times3\times10^{8}}{5\times10^{-7}\times5\times10^{-7}} = \frac{e}{m}$$

$$4.868\times\frac{10^{-11}\times10^{8}}{10^{-14}} = \frac{e}{m}$$

$4.868\times10^{11} = e/m$