Himanshu Dixit

B64178

B10

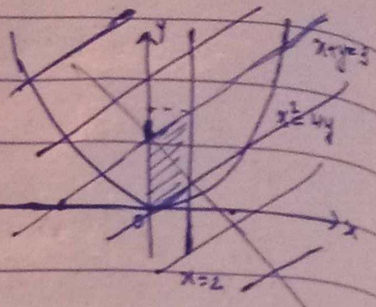
DATE / /
PAGE

Maths (Assignment Integrals)

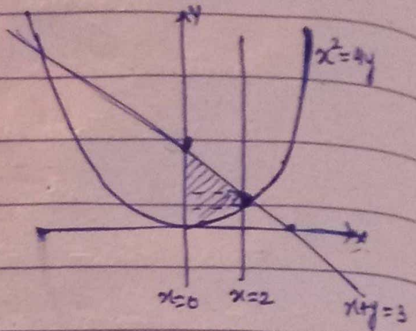
Q.1. $$\int_0^2 \int_{x^2/4}^{3-x} xy \, dx \, dy$$

$y = 3-x \qquad x = 2$

$y = \frac{x^2}{4} \qquad x = 0$



$$\int_{y=1}^{y=3} \int_{x=0}^{x=3-y} xy \, dx \, dy + \int_{y=0}^{y=1} \int_{x=0}^{x=2\sqrt{y}} xy \, dx \, dy$$



$$\int_1^3 y \left[\frac{x^2}{2}\right]_0^{3-y} dy + \int_0^1 y \left[\frac{x^2}{2}\right]_0^{2\sqrt{y}} dy$$

$$\int_1^3 \frac{y(3-y)^2}{2} dy + \int_0^1 2y^2 \, dy$$

$$\frac{1}{2}\int_1^3 9y + y^3 - 6y^2 \, dy + \left[\frac{2}{3}y^3\right]_0^1$$

$$\frac{1}{2}\left[\frac{9y^2}{2} + \frac{y^4}{4} - \frac{\cancel{6}^{2}y^3}{\cancel{3}}\right]_1^3 + \frac{2}{3} = \frac{1}{2}\left(\frac{81}{2} + \frac{81}{4} - 54\right) + \frac{2}{3} - \frac{1}{2}\left(\frac{9}{2} + \frac{1}{4} - 2\right)$$

$$\frac{1}{2}\left(\frac{81 \times 3}{4} - 54\right) - \frac{1}{2}\left(\frac{19}{4} - 2\right) + \frac{2}{3}$$

$$\frac{1}{2}\left[\frac{27}{4}\right] - \frac{1}{2}\left[\frac{11}{4}\right] + \frac{2}{3} = \frac{\cancel{16}^{2}}{\cancel{8}} + \frac{2}{3} = \frac{8}{3}$$

Q.2. $$\int_{y=0}^{\infty} \underbrace{\int_{x=0}^{\infty} e^{-xy} \sin nx \, dx \, dy}_{\text{let}} = I$$

$$I = \int_x^{\infty} \cancel{\sin nx \frac{e^{-xy}}{-y}}^{0} - \int_0^{\infty} n \cos nx \frac{e^{-xy}}{-y} dx$$

$$I = -\left[n \cos nx \cdot \frac{e^{-xy}}{y^2}\right]_0^{\infty} + \int_0^{\infty} -n^2 \sin nx \frac{e^{-xy}}{y^2} dx$$

$$I = \frac{n}{y^2} - \frac{n^2}{y^2} I \rightarrow I\left(1 + \frac{n^2}{y^2}\right) = \frac{n}{y^2}$$

$$I = \frac{n}{n^2 + y^2}$$

$$\int_0^\infty \frac{n}{n^2+y^2}\,dy \qquad \text{(By using double I-late)}$$

$$\cancel{\frac{n}{n}}\left[\tan^{-1}\frac{y}{n}\right]_0^\infty = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

without changing order :

$$\int_{y=0}^{\infty}\int_{x=0}^{\infty} e^{-xy}\sin nx\,dx\,dy = \int_0^\infty I\,dy = \int_0^\infty \frac{n}{n^2+y^2}\,dy = \frac{\pi}{2} \quad \text{--- (1)}$$

By changing order :

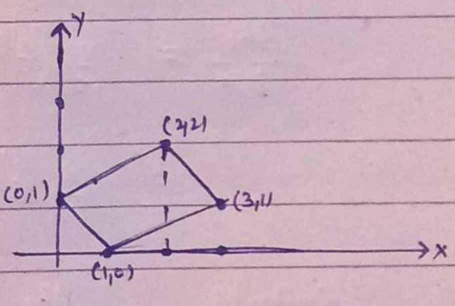
$$\int_{x=0}^{\infty}\int_{y=0}^{\infty} e^{-xy}\sin nx\,dx\,dy = \int_{x=0}^{\infty} \left.\frac{\sin nx\; e^{-xy}}{-x}\right]_0^\infty dx$$

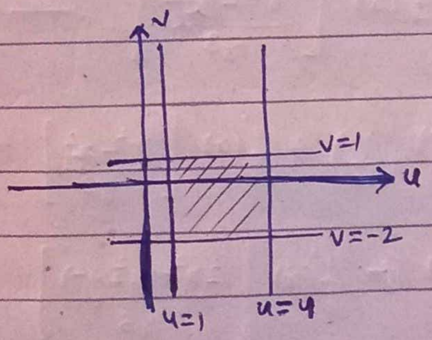
$$= \int_0^\infty \frac{\sin nx}{-x}\left[0-1\right]dx = \int_0^\infty \frac{\sin nx}{x}\,dx \quad \text{--- (2)}$$

from (1) & (2) :

$$\int_0^\infty \frac{\sin nx}{x}\,dx = \frac{\pi}{2}$$; hence Proved

Q.3. $$\iint_R (x+y)^2\,dx\,dy$$





$$y-1 = \frac{0-1}{1-3}(x-3)$$

$$2y-2 = x-3$$

$$x-2y = 1 \quad \text{--- (1)}$$

$$y-2 = \frac{1-2}{3-2}(x-2)$$

$$x+y = 4 \quad \text{--- (2)}$$

$$x+y = 1 \quad \text{--- (3)}$$

$$x-2y = -2 \quad \text{--- (4)}$$

$$\iint_R (x+y)^2\,dx\,dy$$

$$J = \frac{d(x,y)}{d(u,v)}$$

$$d(x,y) = J\,d(u,v)$$

DATE / /
PAGE

$$J = \begin{vmatrix} 2/3 & 1/3 \\ -1/3 & 1/3 \end{vmatrix} = \frac{2}{9} + \frac{1}{9} = \frac{1}{3}$$

$u = x+y$
$v = x-2y$
$v-u = 3y$
$y = \frac{(v-u)}{3}$

$$\partial(x,y) = \frac{1}{3}\,\partial(u,v)$$

$2u+v = 3x$

$x = \frac{2u+v}{3}$

$$\int_{v=-2}^{v=1}\int_{u=1}^{u=4} \frac{u^2}{3}\,du\,dv$$

$$\int_{-2}^{1} \left.\frac{u^3}{9}\right]_1^4 dv = \int_{-2}^{1} \frac{64-1}{9}\,dv = \frac{\overset{7}{\cancel{63}}}{\cancel{9}}\int_{-2}^{1} dv$$

$$= 7\left. v\right]_{-2}^{1} = 7(1+2) = 7\times 3 = 21 \text{ Ans}$$

Q.4. $u = f(r)$ $\quad r^2 = x^2+y^2$ $\quad$ T.P $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$

$$\frac{\partial u}{\partial r} \times \frac{\partial r}{\partial x} = f'(r)\,\frac{x}{r}$$

$r^2 = x^2+y^2$

$\cancel{2}r\frac{\partial r}{\partial x} = \cancel{2}x$

$$\cancel{\frac{\partial^2 u}{\partial r^2}\,\frac{\partial^2 r}{\partial x^2} = f''(r)}$$

$$\cancel{r\frac{\partial^2 r}{\partial x^2} + \left(\frac{\partial r}{\partial x}\right)^2 = 1}$$

$$\frac{\partial u}{\partial x} = \frac{x}{r} f'(r)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{x}{r}\left(f''(r)\times\frac{\partial r}{\partial x}\right) + f'(r)\left[\frac{r - x\frac{\partial r}{\partial x}}{r^2}\right]$$

$$= \frac{x}{r} f''(r)\left(\frac{x}{r}\right) + f'(r)\left[\frac{r^2-x^2}{r^3}\right] \quad \text{— (1)}$$

Similarly, $\frac{\partial^2 u}{\partial y^2} = \frac{y}{r} f''(r)\left(\frac{y}{r}\right) + f'(r)\left[\frac{r^2-y^2}{r^3}\right]$ — (2)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \cancel{\left(\frac{x^2+y^2}{r^2}\right)} f''(r) + f'(r)\left[\frac{2r^2-(x^2+y^2)}{r^3}\right]$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r) \text{ ; hence Proved}$$