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Batch B10

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Mathematics (15B11MA111)

Assignment - 1

Q.1. $f(x,y) = 2x^2 - xy + y^2 + 3x - 4y + 1$, $(-1,1)$, $|x+1| < 0.1$ $|y-1| < 0.1$

$f_x = 4x - y + 3 \rightarrow f_x(-1,1) = -2$ $\qquad f_{xxx}(-1,1) = 0$

$f_y = -x + 2y - 4 \rightarrow f_y(-1,1) = -1$ $\qquad f_{yyy}(-1,1) = 0$

$f_{xx} = 4 \rightarrow f_{xx}(-1,1) = 4$ $\qquad f_{xxy}(-1,1) = 0$

$f_{yy} = 2 \rightarrow f_{yy}(-1,1) = 2$ $\qquad f_{yyx}(-1,1) = 0$

$f_{xy} = -1 \rightarrow f_{xy}(-1,1) = -1$

for Quadratic Polynomial :

$$f(x,y) = f(-1,1) + (x+1)f_x(-1,1) + (y-1)f_y(-1,1) + \frac{1}{2!}\left((x+1)^2 f_{xx}(-1,1) + (y-1)^2 f_{yy}(-1,1) + 2xy f_{xy}(-1,1)\right)$$

$$= -2 + (x+1)(-2) + (y-1)(-1) + \frac{1}{2}\left((x+1)^2 4 + (y-1)^2 2 + 2xy(-1)\right)$$

$$= x - y - 3 + \frac{1}{2}\left(4(x+1)^2 + 2(y-1)^2 - 2xy\right)$$

~~for absolute maximum error :~~

for linear Expression :-

$f(x,y) = \underline{x - y - 3}$ Ans

for absolute maximum error for linear expression :-

$$R_L \le \frac{B}{2}\{|x+1| + |y-1|\}^2$$

$$\le \frac{B}{2}\{0.1 + 0.1\}^2$$

$$\le \frac{B}{2}(0.08) \qquad \because B = \max\{|f_{xx}|, |f_{yy}|, |f_{xy}|\} = 4$$

$$R_L \le 2 \times 0.08 = 0.16$$ Ans

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Q.2. $f(x,y) = \sin(x+2y)$

$f_x = \cos(x+2y) \rightarrow f_x(0,0) = 1$
$f_y = 2\cos(x+2y) \rightarrow f_y(0,0) = 2$
$f_{xx} = -\sin(x+2y) \rightarrow f_{xx}(0,0) = 0$
$f_{yy} = -4\sin(x+2y) \rightarrow f_{yy}(0,0) = 0$
$f_{xy} = -2\sin(x+2y) \rightarrow f_{xy}(0,0) = 0$
$f_{xxx} = -\cos(x+2y) \rightarrow f_{xxx}(0,0) = -1$
$f_{yyy} = -8\cos(x+2y) \rightarrow f_{yyy}(0,0) = -8$
$f_{xxy} = -2\cos(x+2y) \rightarrow f_{xxy}(0,0) = -2$
$f_{yyx} = -4\cos(x+2y) \rightarrow f_{yyx}(0,0) = -4$

Tailor series upto 3rd order :

$$f(x,y) = f(0,0) + xf_x + yf_y + \frac{1}{2!}(x^2 f_{xx} + y^2 f_{yy} + 2xy f_{xy}) + \frac{1}{3!}(x^3 f_{xxx} + y^3 f_{yyy} + 3x^2y f_{xxy} + 3y^2x f_{yyx})$$

$$= 0 + x + 2y + \frac{1}{2}(0) + \frac{1}{6}(-x^3 - 8y^3 - 6x^2y - 12xy^2)$$

$$= x + 2y - \frac{1}{6}(x^3 + 8y^3 - 6x^2y - 12xy^2)$$

Absolute maximum error :

$$R_c \leq \frac{B}{24}\{|x| + |y|\}^4$$

$$\leq \frac{B}{24}\{0.2\}^4$$

$$\leq \frac{B}{24} \times 0.0016$$

$\therefore B = \max\{|f_{xxx}|, |f_{yyy}|, |f_{xxy}|, |f_{yyx}|, |f_{xxyy}|\}$

$$R_c \leq \frac{16}{24} \times 0.0016$$

$\therefore B = 16$

$$R_c \leq 0.001067$$ Ans

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Q.3. $x^2+2xy+z^2$, $2x+y=0$, $x+y+z=1$

$\phi_1(x,y,z) = 2x+y$

$\phi_2(x,y,z) = x+y+z-1$

$F(x,y,z,\lambda) = x^2+2xy+z^2+\lambda(2x+y)+\lambda(x+y+z-1)$

$\frac{\partial F}{\partial x} = 2x+2y+2\lambda+\lambda = 0$, $\frac{\partial F}{\partial y} = 2x+\lambda+\lambda = 0 \rightarrow \lambda = -x$

$\lambda = \frac{-2(x+y)}{3}$

$\frac{\partial F}{\partial z} = 2z+\lambda = 0 \rightarrow \lambda = -2z$

$\therefore \frac{2(x+y)}{3} = x = 2z \xrightarrow{\text{put}} (x+y)+z = 1$

$\frac{3x}{2}+\frac{x}{2} = 1 \rightarrow x = 1/2$

$\left[x = 1/2,\ y = 1/4,\ z = 1/4 \right]$ Ans

$x^2+2xy+z^2 = \frac{1}{4}+\cancel{2}\times\frac{1}{\cancel{2}}\times\frac{1}{4}+\frac{1}{16} = \frac{1}{2}+\frac{1}{16} = \frac{9}{16}$ Ans

Q.4. $u = x(1-r^2)^{-1/2}$, $v = y(1-r^2)^{-1/2}$, $w = z(1-r^2)^{-1/2}$

$r^2 = x^2+y^2+z^2$

L.H.S : $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} \cancel{(1-r^2)^{-1/2}} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$

$\cancel{\frac{\partial u}{\partial x} = x\left(\frac{-1}{2}\right)}$ $u = \frac{x}{\sqrt{1-x^2-y^2-z^2}}$, $v = \frac{y}{\sqrt{1-x^2-y^2-z^2}}$, $w = \frac{z}{\sqrt{1-x^2-y^2-z^2}}$

$\cancel{1-u^2-v^2-w^2}$

$\frac{\partial u}{\partial x} = \frac{\sqrt{1-x^2-y^2-z^2} - \frac{x(-x)}{\sqrt{1-x^2-y^2-z^2}}}{(1-x^2-y^2-z^2)} = \frac{1-y^2-z^2}{(1-r^2)^{3/2}}$ — (1)

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$$\frac{\partial v}{\partial x} = \frac{-y\cdot -2x}{2(\sqrt{1-r^2})^3} = \frac{xy}{(1-r^2)^{3/2}} \quad \text{—— ②}$$

$$\frac{\partial w}{\partial x} = \frac{zx}{(1-r^2)^{3/2}} \quad \text{—— ③}$$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{1-y^2-z^2}{(1-r^2)^{3/2}} & \frac{xy}{(1-r^2)^{3/2}} & \frac{xz}{(1-r^2)^{3/2}} \\ \frac{xy}{(1-r^2)^{3/2}} & \frac{1-x^2-z^2}{(1-r^2)^{3/2}} & \frac{yz}{(1-r^2)^{3/2}} \\ \frac{xz}{(1-r^2)^{3/2}} & \frac{yz}{(1-r^2)^{3/2}} & \frac{1-x^2-y^2}{(1-r^2)^{3/2}} \end{vmatrix} = \frac{1}{(1-r^2)^{9/2}} \begin{vmatrix} 1-y^2-z^2 & xy & xz \\ xy & 1-x^2-z^2 & yz \\ xz & yz & 1-x^2-y^2 \end{vmatrix}$$

$$= \frac{1}{(1-r^2)^{9/2}} \begin{vmatrix} 1-y^2-z^2 & x^2 & x^2 \\ y^2 & 1-x^2-z^2 & y^2 \\ z^2 & z^2 & 1-x^2-y^2 \end{vmatrix} = \frac{1}{(1-r^2)^{9/2}} \begin{vmatrix} 1 & 1 & 1 \\ y^2 & (1-x^2-z^2) & y^2 \\ z^2 & z^2 & 1-x^2-y^2 \end{vmatrix}$$

$$= \frac{1}{(1-r^2)^{9/2}} \begin{vmatrix} 1 & 0 & 0 \\ y^2 & (1-x^2-y^2-z^2) & 0 \\ z^2 & 0 & (1-x^2-y^2-z^2) \end{vmatrix} = \frac{(1-r^2)^2}{(1-r^2)^{9/2}}$$

$$= (1-r^2)^{2-9/2} = (1-r^2)^{-5/2} \quad \text{R.H.S} \quad \text{; hence Proved}$$

Q.5. (i) box is open at the top :

$$S\cdot A = S = lb + 2(bh + lh)$$

$$S = xy + 2(xz + yz)$$

$$v = lbh = xyz$$

$$f(x,y,z,\lambda) = xyz + \lambda(xy + 2xz + 2yz - S)$$

$$\frac{\partial f}{\partial x} = yz + \lambda y + 2\lambda z \quad , \quad \frac{\partial f}{\partial y} = xz + \lambda x + 2\lambda z$$

$$\frac{\partial f}{\partial z} = xy + 2\lambda x + 2\lambda y$$

$$\frac{\partial f}{\partial x} = 0 \longrightarrow \lambda = \frac{-yz}{y+2z} \quad , \quad \frac{\partial f}{\partial y} = 0 \rightarrow \lambda = \frac{-xz}{x+2z}$$

$$\frac{\partial f}{\partial z} = 0 \longrightarrow \lambda = \frac{-xy}{2x+2y}$$

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$$\frac{x}{2(x+y)} = \frac{z}{y+2z} \qquad\qquad \frac{y}{2(x+y)} = \frac{z}{x+2z}$$

$$xy + \cancel{2xz} = \cancel{2xz} + 2zy \qquad\qquad \cancel{2yz} + xy = 2xz + \cancel{2yz}$$

$$x = 2z \quad -(1) \qquad\qquad y = 2z \quad -(2)$$

$$x = y = 2z$$

$$S = xy + 2(xz+yz) = x^2 + 2\left(\frac{x^2}{2} + \frac{x^2}{2}\right) = 3x^2$$

$$x = \sqrt{\frac{S}{3}}, \quad y = \sqrt{\frac{S}{3}}, \quad z = \frac{1}{2}\sqrt{\frac{S}{3}} \quad \text{Ans}$$

(ii) box is closed :

$$S = 2(xy+yz+xz)$$

$$V = xyz$$

$$F(x,y,z,\lambda) = xyz + \lambda\left[\left(2(xy+yz+xz)\right) - S\right]$$

$$\frac{\partial F}{\partial x} = yz + 2\lambda(y+z) = 0, \quad \frac{\partial F}{\partial y} = xz + 2\lambda(x+z) = 0, \quad \frac{\partial F}{\partial z} = xy + 2\lambda(x+y)$$

$$\lambda = \frac{-yz}{2(y+z)} \qquad \lambda = \frac{-xz}{2(x+z)} \qquad \lambda = \frac{-xy}{2(x+y)}$$

$$\frac{y}{y+z} = \frac{x}{x+z} \qquad\qquad \frac{z}{x+z} = \frac{y}{x+y}$$

$$\cancel{xy} + yz = \cancel{xy} + xz \qquad\qquad xz + \cancel{yz} = xy + \cancel{yz}$$

$$x = y \qquad\qquad y = z$$

$$\therefore \; x = y = z.$$

$$S = 2(x^2+x^2+x^2) = 6x^2 \longrightarrow \quad x = \sqrt{\frac{S}{6}}, \; y = \sqrt{\frac{S}{6}}, \; z = \sqrt{\frac{S}{6}} \quad \text{Ans}$$

Q. 6. $u^3+v^3 = x+y, \quad u^2+v^2 = x^3+y^3$. T.P: $\dfrac{\partial(u,v)}{\partial(x,y)} = \dfrac{y^2-x^2}{2uv(u-v)}$

$$\left.\begin{aligned} 3u^2\frac{\partial u}{\partial x} + 3v^2\frac{\partial v}{\partial x} &= 1 \\ 2u\frac{\partial u}{\partial x} + 2v\frac{\partial v}{\partial x} &= 3x^2 \end{aligned}\right] \longrightarrow \begin{bmatrix} 3u^2 & 3v^2 \\ 2u & 2v \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \end{bmatrix} = \begin{bmatrix} 1 \\ 3x^2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \end{bmatrix} = \begin{bmatrix} 1 \\ \cancel{3x^2} \end{bmatrix} \cdot \frac{1}{6uv(u-v)} \begin{bmatrix} 2v & -3v^2 \\ -2u & 3u^2 \end{bmatrix} \begin{bmatrix} 1 \\ 3x^2 \end{bmatrix}$$

$$\frac{\partial u}{\partial x} = \frac{1}{6uv(u-v)} \left[2v - 9v^2x^2 \right], \quad \frac{\partial v}{\partial x} = \frac{1}{6uv(u-v)} \left(9u^2x^2 - 2u \right)$$

for symmetry :

$$\frac{\partial u}{\partial y} = \frac{1}{6uv(v-v)} (2v - 9v^2y^2), \quad \frac{\partial v}{\partial y} = \frac{1}{6uv(u-v)} (9u^2y^2 - 2v)$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} = \begin{vmatrix} \frac{2v-9v^2x^2}{6uv(u-v)} & \frac{2v-9v^2y^2}{6uv(u-v)} \\ \frac{9u^2x^2-2u}{6uv(u-v)} & \frac{9u^2y^2-2u}{6uv(u-v)} \end{vmatrix}$$

$$= \frac{1}{[6uv(u-v)]^2} \left((2v-9v^2x^2)(9u^2y^2-2u) - (2v-9v^2y^2)(9u^2x^2-2u) \right)$$

$$= \frac{1}{(6uv(u-v))^2} \left[18u^2vy^2 - \cancel{4uv} - 81u^2v^2x^2y^2 + 18uv^2x^2 - (18u^2vx^2 - \cancel{4uv} - 81u^2v^2x^2y^2 + 18uv^2y^2) \right]$$

$$= \frac{1}{36u^2v^2(u-v)^2} \left[(18u^2v(y^2-x^2)) + 81u^2v^2\cancel{xy}\,\cancel{(xy-xy)} + 18uv^2(x^2-y^2) \right]$$

$$= \frac{1}{36uv(u-v)^2} \left[18u(y^2-x^2) + \cancel{81uvxy(x-y)} + 18v(x^2-y^2) \right]$$

$$= \frac{(y^2-x^2)\cancel{(18u-18v)}}{\cancel{36}_{2}uv\cancel{(u-v)}(v-v)} = \frac{y^2-x^2}{2uv(v-v)} \quad \text{R.H.S}$$

L.H.S = R.H.S ; hence Proved