Himanshu Dixit

B64178

Batch - B10

DATE / /
PAGE

Bridge Course (21B19GE111)

Assignment Unit 2

Q.1.

$$x^2+y^2+z^2=6$$
$$x^2-y^2+2z^2=2$$
$$2x^2+y^2-z^2=3$$

$$\longrightarrow \begin{bmatrix}1&1&1\\1&-1&2\\2&1&-1\end{bmatrix}\begin{bmatrix}x^2\\y^2\\z^2\end{bmatrix}=\begin{bmatrix}6\\2\\3\end{bmatrix}$$

let A X B

$\therefore\ X = A^{-1}B$

$|A| = 1(1-2)-1(-1-4)+(1+2) = -1+5+3 = 7 \neq 0$

i.e A^{-1} exist.

$$M = \begin{bmatrix}-1&-5&3\\-2&-3&-1\\3&1&-2\end{bmatrix} \rightarrow C = \begin{bmatrix}-1&5&3\\2&-3&1\\3&-1&-2\end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix}-1&2&3\\5&-3&-1\\3&1&-2\end{bmatrix}$$

$\therefore\ A^{-1} = \frac{1}{|A|}\text{adj } A$

$$A^{-1} = \frac{1}{7}\begin{bmatrix}-1&2&3\\5&-3&-1\\3&1&-2\end{bmatrix}$$

$$X = \frac{1}{7}\begin{bmatrix}-1&2&3\\5&-3&-1\\3&1&-2\end{bmatrix}\begin{bmatrix}6\\2\\3\end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix}7\\21\\14\end{bmatrix} = \begin{bmatrix}1\\3\\2\end{bmatrix} = \begin{bmatrix}x^2\\y^2\\z^2\end{bmatrix}$$

on comparing : $x = \pm 1$, $y = \pm\sqrt{3}$, $z = \pm\sqrt{2}$

Q.2.

$$\frac{1}{x}+\frac{2}{y}+\frac{4}{z}=1$$
$$\frac{2}{x}+\frac{3}{y}+\frac{8}{z}=0$$
$$-\frac{1}{x}+\frac{9}{y}+\frac{10}{z}=5$$

$$\longrightarrow \begin{bmatrix}1&2&4\\2&3&8\\-1&9&10\end{bmatrix}\begin{bmatrix}1/x\\1/y\\1/z\end{bmatrix}=\begin{bmatrix}1\\0\\5\end{bmatrix}$$

let A X = B

$\therefore\ X = A^{-1}B$

$|A| = 1(30-72)-2(20+8)+4(18+3) = -42-56+84 = -14$

DATE / /
PAGE

i.e A^{-1} exist. $m = \begin{bmatrix} -42 & 28 & 21 \\ -16 & 14 & 11 \\ 4 & 0 & -1 \end{bmatrix} \rightarrow C = \begin{bmatrix} -42 & -28 & -21 \\ 16 & 14 & -11 \\ 4 & 0 & -1 \end{bmatrix}$

$\therefore Adj A = C^T = \begin{bmatrix} -42 & 16 & 4 \\ -28 & 14 & 0 \\ -21 & -11 & -1 \end{bmatrix}$ $\quad \therefore A^{-1} = \frac{1}{|A|} adj A$

$A^{-1} = \frac{1}{-14} \begin{bmatrix} -42 & 16 & 4 \\ -28 & 14 & 0 \\ -21 & -11 & -1 \end{bmatrix}$

$X = \frac{1}{-14} \begin{bmatrix} -42 & 16 & 4 \\ -28 & 14 & 0 \\ -21 & -11 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$

$= \frac{1}{-14} \begin{bmatrix} -22 \\ -28 \\ -26 \end{bmatrix} := \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}$ $\quad \therefore$ on comparing

$x = \frac{7}{11}, \; y = \frac{1}{2}, \; z = \frac{7}{13}$ Ans

Q.3. $a_1x + b_1y + c_1z = 0$
$a_2x - b_2y + c_2z = 0$
$a_3x + b_3y - c_3z = 0$

$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & -b_2 & c_2 \\ a_3 & b_3 & -c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

let $\quad A \quad X \quad = B$

for trivial Solⁿ $|A| \neq 0$ — ①

$a_1x + b_1y + c_1z = 3$
$a_2x - b_2y + c_2z = 7$
$a_3x + b_3y - c_3z = 11$

$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & -b_2 & c_2 \\ a_3 & b_3 & -c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 11 \end{bmatrix}$

let $\cdot \; A \quad X \quad = C$

$|A| \neq 0 \quad \therefore$ It has unique solution $\rightarrow X = A^{-1}C$
(A^{-1} exist)

Q.4. $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

(i) $(adj A)^{-1} = adj(A^{-1}) \longrightarrow$ L.H.S : $M_A = \begin{bmatrix} 14 & 9 & -1 \\ 9 & 4 & -1 \\ -1 & -1 & -1 \end{bmatrix} \rightarrow Cof_A = \begin{bmatrix} 14 & -9 & -1 \\ -9 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}$

$adj A = (Cof_A)^T = \begin{bmatrix} 14 & -9 & -1 \\ -9 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}$ let $adj A = C$

$|C| = 14(-5) + 9(9+1) - 1(-5) = -70 + 90 + 5 = 25 \neq 0 \quad C^{-1}$ Exist

$M_C = \begin{bmatrix} -5 & 10 & -5 \\ 10 & -15 & 5 \\ -5 & 5 & -25 \end{bmatrix} \longrightarrow C_C = \begin{bmatrix} -5 & -10 & -5 \\ -10 & -15 & -5 \\ -5 & -5 & -25 \end{bmatrix}$ $\quad \therefore C^{-1} = \frac{1}{|C|} adj C$

DATE / /
PAGE

$$C^{-1} = \frac{1}{25}\begin{bmatrix} -5 & -10 & -5 \\ -10 & -15 & -5 \\ -5 & -5 & -25 \end{bmatrix} \rightarrow (\mathrm{Adj}\, A)^{-1} = \frac{-1}{5}\begin{bmatrix} 1 & 2 & 1 \\ +2 & 3 & 1 \\ 1 & 1 & +5 \end{bmatrix}$$

R.H.S : $\mathrm{adj}\,(A^{-1}) = \mathrm{adj}\left(\frac{1}{|A|}\mathrm{adj}\,A'\right) = \mathrm{adj}\left[\frac{1}{(14-18-1)}\begin{bmatrix} 14 & -9 & -1 \\ -9 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}\right]$

$$\mathrm{adj}\begin{bmatrix} -14/5 & 9/5 & 1/5 \\ 9/5 & -4/5 & -1/5 \\ 1/5 & -1/5 & +1/5 \end{bmatrix} = \begin{bmatrix} -1/5 & -2/5 & -1/5 \\ -2/5 & -3/5 & -1/5 \\ -1/5 & -1/5 & -1 \end{bmatrix} = \frac{-1}{5}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & +5 \end{bmatrix}$$

hence, L.H.S = R.H.S

(ii) from above :

$|A| = -5 \neq 0$ i.e A^{-1} exist

$\mathrm{adj}\,A = \begin{bmatrix} 14 & -9 & -1 \\ -9 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}$ hence, $A^{-1} = X = \frac{-1}{5}\begin{bmatrix} 14 & -9 & -1 \\ -9 & 4 & 1 \\ -1 & 1 & -1 \end{bmatrix}$

from above : $\mathrm{adj}\, x = \mathrm{adj}\,(A^{-1}) = \frac{-1}{5}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & +5 \end{bmatrix}$

$$|x| = \frac{-14}{5}\left(\frac{+4}{25}+\frac{1}{25}\right) + \frac{9}{5}\left(\frac{-9}{25}-\frac{1}{25}\right) + \frac{1}{5}\left(\frac{9}{25}-\frac{4}{25}\right)$$

$$= \frac{-14}{25} + \frac{18}{25} + \frac{1}{25} = \frac{1}{5}$$

$$(A^{-1})^{-1} = x^{-1} = \frac{1}{|x|}\mathrm{adj}\, x = 5 \times \frac{1}{5}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = A$$

; hence Proved

Q.5 $$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} \xrightarrow{R_1 \to R_1+R_2+R_3} = 2\begin{vmatrix} x+y & x+y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = 2(x+y)\begin{vmatrix} 1 & 1 & 1 \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

$$= 2(x+y)\begin{vmatrix} 1 & 0 & 0 \\ y & x & x-y \\ x+y & -y & -x \end{vmatrix} = 2(x+y)\left[-x^2+xy-y^2\right] = -2(x^3+y^3)$$

$C_2 \to C_2 - C_1$
$C_3 \to C_3 - C_1$

(Expand along 1st Row)

DATE / /
PAGE

Q.6. $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix} = \begin{vmatrix} 1 & x & y \\ 0 & y & 0 \\ 0 & 0 & x \end{vmatrix} = 1(xy-0) = xy$ Ans

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

(Expand along 1st coloumn)

Q.7. (i) $\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$

L.H.S

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta-\alpha & (\beta-\alpha)(\beta+\alpha) & -(\beta-\alpha) \\ \gamma-\alpha & (\gamma-\alpha)(\gamma+\alpha) & -(\gamma-\alpha) \end{vmatrix} = (\beta-\alpha)(\gamma-\alpha) \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ 1 & \beta+\alpha & -1 \\ 1 & \gamma+\alpha & -1 \end{vmatrix}$

$C_1 \to C_1 + C_3$

$= (\beta-\alpha)(\gamma-\alpha) \begin{vmatrix} \alpha+\beta+\gamma & \alpha^2 & \beta+\gamma \\ 0 & \beta+\alpha & -1 \\ 0 & \gamma+\alpha & -1 \end{vmatrix} = (\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha)(-\beta-\alpha+\gamma+\alpha)$

(expand along C_1)

$= (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$ R.H.S

L.H.S = R.H.S ; hence Proved

(ii) $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$

L.H.S : By using Property $\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & py^3 \\ z & z^2 & pz^3 \end{vmatrix}$

$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + (pxyz) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (1+pxyz) \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$= (1+pxyz) \begin{vmatrix} x & x^2 & 1 \\ y-x & (y-x)(y+x) & 0 \\ z-x & (z-x)(z+x) & 0 \end{vmatrix} = (1+pxyz)(y-x)(z-x) \begin{vmatrix} x & x^2 & 1 \\ 1 & y+x & 0 \\ 1 & z+x & 0 \end{vmatrix}$

$= (1+pxyz)(y-x)(z-x)[z+\cancel{x}-y-\cancel{x}]$ (expand along 3rd coloumn)

$= (1+pxyz)(x-y)(y-z)(z-x)$ Ans R.H.S

L.H.S = R.H.S ; hence Proved

(iii) $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

L.H.S : $C_1 \to C_1+C_2+C_3$ $\begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$

$R_2 \to R_2-R_1$
$R_3 \to R_3-R_1$ $\begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix}(a+b+c) = (a+b+c)\left[(2b+a)(2c+a)-(a-b)(a-c)\right]$

(expand along C1)

$= (a+b+c)\left[4bc+2ab+2ac+\cancel{a^2}-\cancel{a^2}+ac+ab-bc\right]$

$= 3(a+b+c)(ab+bc+ac)$ R.H.S

L.H.S = R.H.S ; Hence Proved