

Mathematics (15B11MAIII)

9.1.
$$f(x,y) = 2x^2 - xy + y^2 + 3x - 4y + 1$$
, (-1,1), $|x+1| < 0.1$ | $|y-1| < 0.1$

$$f_{x} = 4x - y + 3 \rightarrow f_{x}(-1,1) = -2 \qquad f_{xxx}(-1,1) = 0$$

$$f_{y} = -x + 2y - 4 \rightarrow f_{y}(-1,1) = -1 \qquad f_{yyy}(-1,1) = 0$$

$$f_{xx} = 4 \rightarrow f_{xx}(-1,1) = 4 \qquad f_{xxy}(-1,1) = 0$$

$$f_{yy} = 2 \rightarrow f_{yy}(-1,1) = 2 \qquad f_{yyx}(-1,1) = 0$$

$$f_{xy} = -1 \rightarrow f_{xy}(-1,1) = 1$$

$$f(x_1y) = f(-1,1) + (x+1)f_x(-1,1) + (y-1)f_y(-1,1) + 1 (x+1)^2 f_{xx}(-1,1) + 2xyf_{xy}$$

$$= -2 + (x+1)(-2) + (y-1)(-1) + 1 (x+1)^2 4 + (y-1)^2 2 + 2xy(-1)$$

=
$$\chi - y - 3 + \frac{1}{2} \left(4(x+1)^2 + 2(y-1)^2 - 2xy \right)$$

for absolute maximum error:

for linear expression:
$$f(x,y) = x-y-3 \qquad A_y$$

for absolute maximum ex rox for linear expression:
RL & B { | x+1|2+ |y-1|}^2

Q-2-
$$f(x,y) = \sin(x+2y)$$

$$f_{xx} = cos(x+2y) \rightarrow f_{x}(0,0) = 1$$

$$f_{y} = 2cos(x+2y) \rightarrow f_{y}(0,0) = 2$$

$$f_{xx} = -sin(x+2y) \rightarrow f_{xx}(0,0) = 0$$

$$f_{yy} = -4sin(x+2y) \rightarrow f_{yy}(0,0) = 0$$

$$f_{xy} = -2sin(x+2y) \rightarrow f_{xy}(0,0) = 0$$

$$f_{xxx} = -cos(x+2y) \rightarrow f_{xxx}(0,0) = -1$$

$$f_{xxy} = -9cos(x+2y) \rightarrow f_{xxx}(0,0) = -8$$

$$f_{xxy} = -2cos(x+2y) \rightarrow f_{xxy}(0,0) = -2$$

$$f_{xyx} = -4cos(x+2y) \rightarrow f_{xxx}(0,0) = -2$$

$$f_{xyx} = -4cos(x+2y) \rightarrow f_{xxx}(0,0) = -4$$

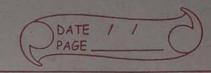
Tailon Series upto 3rd order:

$$f(x,y) = f(0,0) + xfx + yfy + \frac{1}{2!} (x^2fxx + y^2fy + 2xy fxy) + \frac{1}{3!} (x^3fxxy + y^3fyxy + 3x^3fxxy + y^2fyx + 2xy^2) + \frac{1}{3!} (x^3fxxy + y^3fyxy + y^3fxy) + \frac{1}{3!} (x^3fxxy + y^3fxyy + y^3fxy + y^3fxyy + y^3fxyy$$

Absolute maximum essor

60 B = max { | fxxx |, | fxxx |, | fxxx | , | fxxx | }

Re < 0.001067 00



$$F(x,y,z,\lambda) = x^2 + 2xy + z^2 + \lambda(2x+y) + \lambda(x+y+z-1)$$

$$\frac{\partial F}{\partial x} = 2x + 2y + 2\lambda + \lambda = 0 \quad , \quad \frac{\partial F}{\partial y} = 2x + \lambda + \lambda = 0 \quad \rightarrow \lambda = -\infty$$

$$\lambda = -2(x + y)$$

$$\frac{\partial F}{\partial z} = 2z + \lambda = 0 \rightarrow \lambda = -2z$$

$$2(x+y) = x = 2z \xrightarrow{\text{fut}} (x+y)+z = 1$$

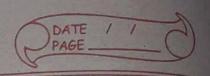
$$\frac{3x+x}{2}=1 \rightarrow x=\frac{1}{2}$$

Q.4.
$$u = x(1-8^2)^{-1/2}$$
, $v = y(1-8^2)^{-1/2}$, $u = z(1-8^2)^{-1/2}$

$$\frac{\partial x}{\partial w} \frac{\partial y}{\partial w} \frac{\partial z}{\partial w} = \frac{\partial x}{\partial w} \frac{\partial y}{\partial w} \frac{\partial z}{\partial w} = \frac{\partial x}{\partial w} \frac{\partial y}{\partial w} \frac{\partial z}{\partial w} = \frac{\partial x}{\partial w} \frac{\partial y}{\partial w} \frac{\partial z}{\partial w} = \frac{\partial x}{\partial w} \frac{\partial y}{\partial w} \frac{\partial z}{\partial w} = \frac{\partial x}{\partial w} \frac{\partial y}{\partial w} \frac{\partial z}{\partial w} = \frac{\partial x}{\partial w} \frac{\partial z}{\partial w} \frac{\partial z}{\partial w} = \frac{\partial z}{\partial w$$

$$\frac{\partial u}{\partial x} = \frac{1}{\sqrt{1 - x^2 - y^2 - z^2}}, \quad V = \frac{1}{\sqrt{1 - x^2 - y^2 - z^2}}, \quad W = \frac{Z}{\sqrt{1 - x^2 - y^2 - z^2}}$$

$$\frac{\partial y}{\partial x} = \sqrt{1-x^2-y^2-z^2} + x(-x) - \frac{1-y^2-z^2}{(1-x^2-y^2-z^2)} - \frac{1-y^2-z^2}{(1-x^2)^{3/2}}$$



$$\frac{\partial v}{\partial x} = \frac{-y - 2x}{2(1 - x^2)^3} = \frac{x_1 y}{(1 - x^2)^{3/2}} = 0$$

$$\frac{\partial \omega}{\partial x} = \frac{zx}{(1-x^2)^{3/2}} - 3$$

$$\frac{\partial(u_1v_1\omega)}{\partial(x_1y_1z)} = \frac{1-y^2-z^2}{(1-x^2)^{3/2}} \frac{xy}{(1-x^2)^{3/2}} \frac{xz}{(1-x^2)^{3/2}} = \frac{1-y^2-z^2}{(1-x^2)^{3/2}} \frac{xz}{(1-x^2)^{3/2}} \frac{1-y^2-z^2}{(1-x^2)^{3/2}} \frac{xz}{(1-x^2)^{3/2}} = \frac{1-y^2-z^2}{(1-x^2)^{3/2}} \frac{xz}{(1-x^2)^{3/2}} \frac{xz}{(1-x^2)^{3/2}} \frac{xz}{(1-x^2)^{3/2}} = \frac{1-x^2-z^2}{(1-x^2)^{3/2}} \frac{xz}{(1-x^2)^{3/2}} \frac{xz}{(1-x^2)^{3/2}} = \frac{1-x^2-z^2}{(1-x^2)^{3/2}} \frac{xz}{(1-x^2)^{3/2}} \frac{xz}{(1-x^2)^{3/2}} = \frac{1-x^2-z^2}{(1-x^2)^{3/2}} = \frac{1-x^2-z^2}{(1-x^2)$$

$$= \frac{1}{(1-x^2)^{9/2}} \begin{vmatrix} 1-y^2-z^2 & y^2 & y^2 \\ y^2 & 1-x^2-z^2 & y^2 \end{vmatrix} = \frac{1}{(1-x^2)^{9/2}} \begin{vmatrix} 1 & 1 & 1 \\ y^2 & (1-x^2-z^2) & y^2 \\ z^2 & z^2 & 1-x^2-y^2 \end{vmatrix} = \frac{1}{2^2} \frac{1}$$

$$= \frac{1}{(1-y^2)^{9/2}} \frac{1}{y^2} \frac{0}{(1-x^2-y^2-z^2)} = \frac{(1-y^2)^2}{(1-y^2-y^2-z^2)}$$

=
$$(1-8^2)^{2-9/2}$$
 = $(1-8^2)^{-5/2}$ R.H.S; hence Proved

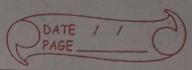
$$S = xy + 2(xz + yz)$$

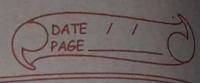
$$\frac{\partial f}{\partial x} = \frac{1}{2} + \frac{1}{2} +$$

$$\frac{\partial f}{\partial z} = xy + 2\lambda x + 2\lambda y$$

$$\frac{\partial f}{\partial x} = 0 \rightarrow \lambda = -yz$$
, $\frac{\partial f}{\partial y} = 0 \rightarrow \lambda = -xz$

$$\frac{\partial f}{\partial z} = 0 \longrightarrow 1 = -xy$$





$$\frac{\partial u}{\partial x} = \frac{1}{8x^{2}} \frac{1}{6uv(u-v)} \frac{1}{-3u} \frac{1}{3u^{2}} \frac{1}{3x^{2}}$$

$$\frac{\partial u}{\partial x} = \frac{1}{6uv(u-v)} \frac{1}{2v - 9v^{2}y^{2}}, \frac{\partial v}{\partial x} = \frac{1}{6uv(u-v)} \frac{1}{9u^{2}x^{2} - 2u}$$

$$\frac{\partial u}{\partial x} = \frac{1}{6uv(u-v)} \frac{1}{2v - 9v^{2}y^{2}}, \frac{\partial v}{\partial y} = \frac{1}{6uv(u-v)} \frac{1}{9v^{2}x^{2} - 2v}$$

$$\frac{\partial u}{\partial y} = \frac{1}{6uv(u-v)} \frac{1}{2v - 9v^{2}y^{2}}, \frac{\partial v}{\partial y} = \frac{1}{6uv(u-v)} \frac{1}{6uv(u-v)}$$

$$\frac{\partial u}{\partial y} = \frac{1}{6uv(u-v)} \frac{1}{6uv(u-v)} \frac{1}{6uv(u-v)}$$

$$\frac{\partial u}{\partial y} = \frac{1}{4v - 9v^{2}x^{2}} \frac{1}{2v^{2}x^{2} - 2u}$$

$$\frac{\partial u}{\partial y} = \frac{1}{4v - 9v^{2}x^{2}} \frac{1}{2v^{2}x^{2} - 2u}$$

$$= \frac{1}{6uv(u-v)^{2}} \frac{1}{8u^{2}v^{2} - 2uv - 8u^{2}v^{2}x^{2}y^{2} + 18uv^{2}x^{2} - 1}{8uv^{2}x^{2} - 4vv - 8u^{2}v^{2}x^{2}y^{2} + 18uv^{2}y^{2}}$$

$$= \frac{1}{36uv(u-v)^{2}} \frac{1}{8u^{2}v^{2} - 2vv - 8u^{2}v^{2}x^{2}y^{2} + 18uv^{2}y^{2}}$$

$$= \frac{1}{36uv(u-v)^{2}} \frac{1}{18u(y^{2}x^{2}) + 8uv^{2}(x^{2} - y^{2})}$$

$$= \frac{1}{36uv(u-v)^{2}} \frac{1}{18u(y^{2}x^{2}) + 8uv^{2}(x^{2} - y^{2})}$$

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$$= \frac{1}{36uv(u-v)^{2}} \frac{1}{18u(v^{2}x^{2}) + 8uv^{2}(v^{2})}$$

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