Solution: Tutorial 8 (Physics 1) Odd-2021

① $n_f = 2, n_i = 3, \frac{1}{\lambda} = R_H\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] = \frac{5R_H}{36}$

$\Rightarrow \lambda = 656\ nm$ (visible) and $\nu = 4.57 \times 10^{14}\ Hz$

For shortest wavelength, $n_i = \infty$ so $\frac{1}{\lambda} = \frac{R_H}{4}$

$\Rightarrow \lambda_{short} = 364.6\ nm$ (UV)

whereas for longest wavelength $n_i = 3$, $\lambda_{long} = 656\ nm$

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② Since, total number of elements in the periodic table is equal to total number of electrons combined in all shells, for $n = 6$

$$\text{Total elements} = 2n^2 + 2(n-1)^2 + 2(n-2)^2 + \cdots 2(1)^2$$
$$= 2[6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2]$$
$$= 182$$

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③ $L = \sqrt{l(l+1)}\,\hbar$ & $L_z = m_l\hbar$, here $m_l = 0$ till $\pm l$

$L_{z_{max.}} = l\hbar$ so % change $= \frac{L - L_{z_{max.}}}{L}$ %

$$= \frac{\sqrt{l(l+1)}\,\hbar - l\hbar}{\sqrt{l(l+1)}\,\hbar} = 1 - \frac{l}{\sqrt{l(l+1)}}$$

$l = 1$ for p-orbital so % change $= 29\%$.
Similarly for d-orbital ($l = 2$) and f-orbital ($l = 3$) % changes are 18% and 13% respectively

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(Contd.→)

④ ORBITAL

$L = \sqrt{l(l+1)}\,\hbar$, $L_z = L\cos\theta = m_l \hbar$ and $m_l = -1, 0, 1$

$$\cos\theta = \frac{m_l}{\sqrt{l(l+1)}} = -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$$ Hence, $\theta = 135°, 90°, 45°$

SPIN

$S = \sqrt{s(s+1)}\,\hbar$, $S_z = S\cos\theta = m_s \hbar$ and $m_s = -\frac{1}{2}, \frac{1}{2}$

$$\cos\theta = \frac{m_s}{\sqrt{s(s+1)}} = -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$ Hence, $\theta = 44.7°, 125.2°$

TOTAL

$J = \sqrt{j(j+1)}\,\hbar$, $J_z = J\cos\theta = m_j \hbar$ and $m_j = m_l \pm m_s$

$\therefore m_j = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$

$$\cos\theta = \frac{m_j}{\sqrt{j(j+1)}} = -\frac{3}{\sqrt{15}}, -\frac{1}{\sqrt{15}}, \frac{1}{\sqrt{15}}, \frac{3}{\sqrt{15}}$$

Hence, $\theta = 140.7°, 104.9°, 75°, 39.2°$

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⑤ For a $2\,^2P$ state, $l = 1$ (for P orbital) and multiplicity here $M = 2 = 2S+1 \Rightarrow S = \frac{1}{2}$

Since $j = l \pm s$ so $j = 1 \pm 1/2 = \frac{1}{2}, \frac{3}{2}$

so values of $j = \frac{5}{2}, \frac{7}{2}$ not possible and therefore states $2\,^2P_{5/2}$ and $2\,^2P_{7/2}$ can not exist.

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⑥ For $n=2$, $l = 0, 1$. Also $M = 2S+1 = 1$ (singlet), $M = 3$ (Triplet)

For excited He, 1 electron in $n=2$ & 1 electron in $n=1$, so possible configurations Ⓐ $1s^1 2s^1$, Ⓑ $1s^1 2p^1$

For Ⓐ: $1s^1 2s^1$, for both electrons: $l_1 = 0$, $l_2 = 0$, hence, $L = 0$ & $s_1 = 1/2$, $s_2 = 1/2$, so $S = 0, 1$,

$\because J = |L-S|$ to $|L+S|$ and $M = 2S+1$

(i) $L=0$, $S=0$, $J=0$, $M=1$, designation: $2\,^1S_0$ (Singlet)

(ii) $L=0$, $S=1$, $J=1$, $M=3$, designation: $2\,^3S_1$ (Triplet)

For Ⓑ: $1s^1 2p^1$, for both electrons: $l_1 = 0$, $l_2 = 1$, $L = 1$, $s_1 = 1/2$, $s_2 = 1/2$ so $S = 0, 1$, $J = |L-S|$ to $|L+S|$ and $M = 2S+1$

(i) $L=1$, $S=0$, $J=1$, $M=1$, designation: $2\,^1P_1$ (Singlet)

(ii) $L=1$, $S=1$, $J=0,1,2$, $M=3$, designation: $2\,^3P_0, 2\,^3P_1, 2\,^3P_2$ (Triplet)

(7) For H-atom, here $l = 2, s = 1/2$ so $j = \frac{3}{2}, \frac{5}{2}$

Since, $J = \sqrt{j(j+1)}\,\hbar$ hence $J = \sqrt{15}\,\hbar/2$, $J = \sqrt{35}\,\hbar/2$

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(8) Here, $\lambda_1 = 589$ nm, $\lambda_2 = 589.6$ nm, $E = \frac{hc}{\lambda}$

So, $\Delta E = hc\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = 2\mu_B B$, $\Rightarrow B = 18.5$ T

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(9) Given: $B = 0.3$ T, $\lambda = 450$ nm $\because \nu = \frac{c}{\lambda}$ so $d\lambda = -\frac{\lambda^2}{c}d\nu$

$\therefore \Delta\nu = \frac{eB}{4\pi m} = 4.196 \times 10^9$ Hz Hence, $\Delta\lambda = 2.83 \times 10^{-12}$ m

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(10) Given: $B = 5$T, $\Delta\lambda = 500$ nm

$\because \frac{e}{m} = \frac{4\pi c \Delta\lambda}{B\lambda^2}$ putting values; $\frac{e}{m} = 9.74 \times 10^{-10}$ C/kg

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End of Solution
