Laplace Transformation formulae

$$L(f(t)) = \int_0^\infty e^{-st} f(t)\,dt = F(s)$$

(i) $L(1) = \frac{1}{s}$

(ii) $L(e^{at}) = \frac{1}{s-a}$

(iii) $L(t) = \frac{1}{s^2}$

(iv) $L(t^n) = \frac{n!}{s^{n+1}}$ = (n is positive integer)

$= \frac{\lceil n+1}{s^{n+1}}$

(v) $L(\sin at) = \frac{a}{s^2+a^2}$

(vi) $L(\cos at) = \frac{s}{s^2+a^2}$

(vii) $L(\sinh at) = \frac{a}{s^2-a^2}$

(viii) $L(\cosh at) = \frac{s}{s^2-a^2}$

First shifting theorem

If $L(f(t)) = F(s) \Rightarrow L(e^{at} f(t)) = F(s-a)$

change of scale property

$L(f(t)) = F(s)$ then $L(F(at)) = \frac{1}{a} F\left(\frac{s}{a}\right)$

Laplace of multiplication by t^n

$L(t^n f(t)) = (-1)^n \frac{d^n}{ds^n} F(s)$

where $F(s) = L(f(t))$

Laplace of division by t

$$L\left(\frac{f(t)}{t}\right) = \int_s^\infty F(s)\,ds$$ where $F(s) = L(f(t))$

Laplace of Integral

If $L(f(t)) = F(s)$ then

$$L\left(\int_0^t F(t)\,dt\right) = \frac{1}{s}F(s)$$

Laplace transform of derivatives

$$L(f(t)) = F(s)$$

$$L(f'(t)) = sL(F(t)) - f(0)$$

$$L(f''(t)) = s^2L(F(t)) - sf(0) - f'(0)$$

$$L(f^n(t)) = s^nL(F(t)) - s^{n-1}f(0) - s^{n-2}f'(0)$$
$$- - - - - - f^{n-1}(0).$$