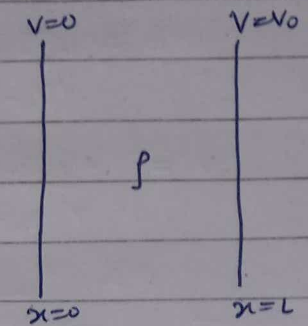
Physics - 2 (15B11PH211)

Tutorial - 3

Q.1. By poisson's eq^n :



$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon_0}$$

$$\frac{\partial v}{\partial x} = -\frac{\rho x}{\varepsilon_0} + A$$

$$V = -\frac{\rho}{\varepsilon_0}\frac{x^2}{2} + Ax + B \quad — ①$$

At $x=0$, $V=0 \Rightarrow B = 0$

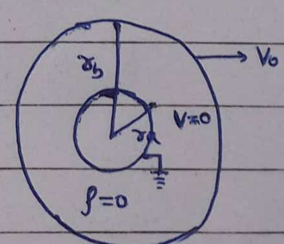
At $x = L$, $V = V_0 \Rightarrow A = \frac{V_0}{L} + \frac{\rho_0 L x}{2\varepsilon_0}$

$$V = \frac{-\rho x^2}{2\varepsilon_0} + \frac{V_0 x}{L} + \frac{\rho_0 L x}{2\varepsilon_0}$$

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{x} = \frac{-\partial}{\partial x}\left(\frac{-\rho x^2}{2\varepsilon_0} + \frac{V_0 x}{L} + \frac{\rho_0 L x}{2\varepsilon_0}\right)\hat{x}$$

$$\vec{E} = \left(\frac{\rho x}{\varepsilon_0} - \frac{V_0}{L} - \frac{\rho L}{2\varepsilon_0}\right)\hat{x}$$

Q.2. Here potential is varying with 'r'
Not with θ & ϕ. so laplace Eq^n



$$\nabla^2 V = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right) = 0$$

$$r^2\frac{\partial v}{\partial r} = A \Rightarrow \frac{\partial v}{\partial r} = \frac{A}{r^2}$$

$$V = -\frac{A}{r} + B \quad — ①$$

At $r = r_a$, $V=0$ $\Rightarrow$ $0 = -\frac{A}{r} + B$ $\rightarrow$ $B = \frac{A}{r_a}$ — (2)

At $r = r_b$, $V = V_0$ $\Rightarrow$ $A = \frac{V_0(r_a - r_b)}{(r_b - r_a)}$ — (3)

$$B = \frac{r_b V_0}{(r_b - r_a)} \quad — (4)$$

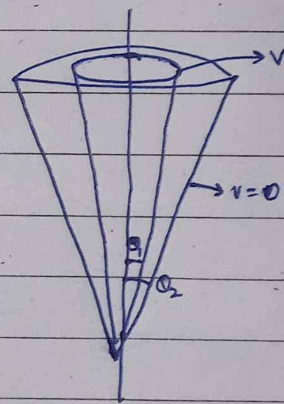
$$V = \frac{-V_0 r_a r_b}{r(r_b - r_a)} + \frac{V_0 r_b}{(r_b - r_a)}$$. This is potential b/w two shell.

$$\vec{E} = -\vec{\nabla} V = -\frac{dV}{dr}\hat{r} = \frac{V_0 r_a r_b}{r_b - r_a}\left(\frac{-1}{r^2}\right)\hat{r}$$

$$\vec{E} = -\frac{V_0 r_a r_b}{r^2(r_b - r_a)}\hat{r} \quad \text{Ans}$$

Direction of $\vec{E}$ is $-\hat{r}$ because of higher to lower potⁿ.

Q.3. potential is constant with r & ϕ only variable with θ. Now laplace Eqⁿ $\nabla^2 V = 0$



$$\frac{d}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right) = 0$$

$$V = A\ln\left(\tan\frac{\theta}{2}\right) + B \quad — (1)$$

$\theta = \theta_2$, $V=0$ $\Rightarrow$ $B = -A\ln\left(\tan\frac{\theta_2}{2}\right)$ — (2)

$\theta = \theta_1$, $V = V_1$ $\Rightarrow$ $A = \frac{V_1}{\ln\left(\frac{\tan\frac{\theta_1}{2}}{\tan\frac{\theta_2}{2}}\right)}$ — (3)

$$B = \frac{-V_1}{\ln\left(\frac{\tan\frac{\theta_1}{2}}{\tan\frac{\theta_2}{2}}\right)}\ln\left(\tan\frac{\theta_2}{2}\right) \quad — (4)$$

$$V = V_1\frac{\ln(\tan\theta/2) - \ln(\tan\theta_2/2)}{\ln(\tan\theta_1/2) - \ln(\tan\theta_2/2)}$$. This is the pot. b/w the cones.

$$\vec{E} = -\vec{\nabla} V$$

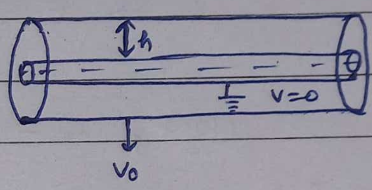
$$= -\frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta}$$

$$\vec{E} = -\frac{1}{r}\left(\frac{\frac{V_1}{\tan\theta/2}\sec^2\theta/2\ \frac{1}{2}}{\ln\left(\tan\frac{\theta_1}{2}\right) - \ln\left(\tan\frac{\theta_2}{2}\right)}\right)\hat{\theta}$$

$$= \frac{-\frac{1}{r}\frac{V_1}{2}\frac{\cos\theta/2}{\sin\theta/2}\frac{1}{\cos\theta/2}}{\ln\left(\tan\frac{\theta_1}{2}\right) - \ln\left(\tan\frac{\theta_2}{2}\right)}\hat{\theta}$$

$$\vec{E} = \frac{-V_1}{r\sin\theta\left(\ln(\tan\theta_1/2) - \ln(\tan\theta_2/2)\right)}\hat{\theta} \quad Ans$$

Q.4. (i) Pot^n is variable with s so laplace eq^n



$$\nabla^2 V = 0$$

$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) = 0$$

$$V = A\ln(s) + B \quad \text{—} ①$$

At $s = a$, $V = 0 \Rightarrow B = -A\ln a$

$s = b$, $V = V_0 \Rightarrow A = V_0/\ln(b/a)$ — ②

so $B = -V_0 A/\ln(b/a)$ — ③

$$V = \frac{V_0\ln(s/a)}{\ln(b/a)} \quad \text{—} ④ \quad Ans$$

(ii) $\vec{E} = -\nabla V = -\frac{dV}{ds}\hat{s} = \frac{-V_0\left(\frac{1}{s/a}\right)\frac{1}{a}\hat{s}}{\ln(b/a)}$

$$\vec{E} = \frac{-V_0}{s\ln(b/a)}\hat{s} \quad Ans$$

Direction of $\vec{E} \longrightarrow -\hat{s}$ (higher pot. to lower pot.)

(iii) $C = Q/V$

V = pot^n diff. $= V_0 - 0 \Rightarrow V = V_0$

To find Q, find the charge density of (+)ve conductor (i.e cylinder of rod b)

for inner surface of outer cylinder :

$$\left.\frac{dV_{above}}{ds}\right|_{s=b} - \left.\frac{dV_{below}}{ds}\right|_{s=b} = \frac{-\sigma_b}{\varepsilon_0}$$

$\because$ $V_{above} = V_0$ & $\frac{dV_0}{ds} = 0$

$$\sigma_b = \frac{\varepsilon_0 V_0}{b\ln(b/a)}$$

This is the charge density of outer cylinder. it is uniform

charge $Q_b = \int \sigma_b \, da = \frac{\varepsilon_0 V_0}{b\ln(b/a)} \cdot 2\pi b l$.

$$Q_b = \frac{\varepsilon_0 V_0 2\pi l}{\ln(b/a)}$$

Now, $C = \frac{Q_b}{V}$ $\Rightarrow$ $C = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$

In general, capacitance per unit length

$$c' = \frac{C}{l} = \frac{2\pi\varepsilon_0}{\ln(b/a)}$$

Q.5. $\vec{K} = \pi(0.3\hat{x} + 0.4\hat{y})$, $\omega = 1000\pi$

$$\vec{B} = \frac{\vec{K}\times\vec{E}}{\omega} = \frac{1}{\omega}(0.3\hat{x} + 0.4\hat{y}) \times E_0 \cos(\pi(0.3x + 0.4y - 1000t))\hat{k}$$

$$\bar{B} = 10^{-4} E_0 \cos[\pi(0.3x + 0.4y - 1000t)](4\hat{i} - 3\hat{j})$$

Q.6. $D = \varepsilon E = \frac{\varepsilon_0 V}{d}$, $i_d = \frac{\partial D}{\partial t} = \frac{\varepsilon_0}{d}\frac{dV}{dt}$

hence, $i_d = J_d . S = \frac{\varepsilon S}{d}\frac{dV}{dt} = \frac{C dV}{dL}$

which is the same as the conduction current, given by

$$i_c = \frac{dQ}{dt} = \frac{S\, d\rho_s}{dt} = \frac{S\, dD}{dt} = \frac{\varepsilon S\, dE}{dt} = \frac{\varepsilon S}{d}\frac{dV}{dt} = \frac{C dV}{dt}$$

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$$I_d = \frac{2\times10^{-9}}{36\pi} \times \frac{5\times10^{-4}}{3\times10^{-3}} \cdot 10^3 \times 50\cos10^3 t$$

$$= 147.4\cos10^3 t \ nA \ \text{Ans}$$

Q.7. $4x+3y=0 \rightarrow \frac{x}{3}+\frac{y}{4}=0$

$$\vec{B} = -\frac{E_0}{c}\cos(qx-qct)\,\hat{y}$$

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{1}{\mu_0}\left(E_0 \times \frac{E_0}{c}\cos^2\theta\right)\hat{x}$$

$$\langle\vec{S}\rangle = \frac{E_0^2}{2\mu_0 c}\,\hat{x}$$

$$I = \langle\vec{S}\rangle\,\hat{n} = \frac{E_0^2}{2\mu_0 c}\cos(90-\alpha) = \frac{E_0^2}{2\mu_0 c}\sin\alpha = \frac{2c\varepsilon_0 E_0^2}{5}$$

$\therefore \tan\alpha = 4/3 \rightarrow \sin\alpha = 4/5$

$$I \approx 0.4c\varepsilon_0 E_0^2 \approx \frac{1}{2}c\varepsilon_0 E_0^2 \ \text{Ans}$$

Q.8. let charge per unit length be λ, hence $I=\lambda u$ in z-direction.

The magnetic field at a distance r is $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$

The Electric " " " " " is $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$

$$= \frac{I}{2\pi\varepsilon_0 u r}\hat{r}$$

Hence, Poynting vector $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{I^2}{4\pi^2\varepsilon_0 u r}\hat{z}$

Q.9 when Electromagnetic wave is reflected by mirror, momentum transfered to the mirror per unit area per second is twice the momentum of light striking the mirror per unit area per second.

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i.e. $\frac{dP}{dt} = \frac{2 \times \text{Power}}{c} = \frac{2 \times 10 \times 10^{-3}}{3 \times 10^{8}} = 6.6 \times 10^{-11}\ kg\ m/s^2$

The force exerted on the reflecting mirror is

$F = \frac{dP}{dt} = 6.6 \times 10^{-11}\ N$ Ans.