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Mathematics - II (15B11MA211)

Tutorial Sheet - 4

Q.1. $\sum (-1)^{n-1} \frac{1}{n^p}$

(a) for convergence :- use leibinize Test
(i) $U_{n+1} < U_n$ is True for in series
(ii) $\lim_{n\to\infty} U_n = \lim_{n\to\infty} \frac{(-1)^\infty}{n^p} = 0$ ✓

Hence, lebinize test is satisfies series in convergence $(p \geq 0)$

(b) for absolute ~~Conver~~ convergen :-
$|U_n|$ is convergence
$|U_n|$ is $\frac{1}{n^p}$ and according to p-series test

$p > 1$, $|U_n|$ is convergent. hence, Absolute convergent when $p > 1$ ✓

Q.2. $\frac{2}{1^2} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \dots$

Series is Alternating series & $U_n = (-1)^{n+1} \frac{(n+1)}{n^2}$

According leibinize Test ① This is Alternating decreasing series ✓
② $\lim_{n\to\infty} (-1)^{n+1} \left(\frac{n+1}{n^2}\right) = 0$ ✓

hence, series is convergence as leibinze Test satisfies.

Now, check for $|U_n| = \frac{n+1}{n^2}$ Let $V_n = \frac{1}{n} \geq U_n$

Acc. to D. Alambart Test :-

$\lim_{n\to\infty} \left|\frac{U_n}{V_n}\right| = \lim_{n\to\infty} \left| \frac{n+1/n^2}{1/n} \right| = 1$; ~~Test fails~~ unique & exist.

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V_n is divergence as $p=1$ so $|U_n|$ is divergence

$\therefore$ Series is condionally convergence.

Q.3. $1-2x+3x^2-4x^3+\ldots\ldots$

This is Power series !

$U_n = nx^{n-1}$

$U_{n+1} = (n+1)x^n$

Acc. to D. Alambert ratio test :

$$\lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)x^n}{(n)x^{n-1}}\right| = |x|$$

if $|x|<1 \to$ series is Convergent

if $|x| \geq 1 \to$ series is divergent !

Q.4. $x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots$

$$U_n = \frac{\cancel{(2n-1)}\, x^{2n-1}}{2n-1}$$

$$U_{n+1} = \frac{x^{2n+1}}{2n+1}$$

Acc. to ratio test :

$$\lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right| = \lim_{n\to\infty}\left|\frac{x^{2n+1}}{2n+1} \times \frac{2n-1}{x^{2n-1}}\right|$$

$$= \lim_{n\to\infty}\left|x^2\left(\frac{2-1/n}{2+1/n}\right)\right| = x^2$$

$x^2 < 1 \to$ series convergence in $-1 < x < 1$

$x^2 = 1$ test fail , $x^2 > 1 \to$ series divergence

$x = \pm 1$

$x = 1$ $\quad\quad$ $x = -1$

$U_n = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$ $\quad\quad$ $U_n = -\left(1 - \frac{1}{3} + \frac{1}{5} - \ldots\right)$

Both satisfies leibinze Test hence, series is convergent in $x = \pm 1$ $\therefore$ convergence in $-1 \leq x \leq 1$

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Q.5. $1 + a\cos x + a^2\cos 2x + a^3\cos 3x + \dots + a^n\cos nx + \dots$

$U_n = a^n \cos nx$

$|U_n| = |a^n \cos nx| = |a^n|$

$a^n \cos nx \le |a^n \cos nx| \le |a^n|$

By Geometric Progression Test : $r = |a|$

if $r < 1$ GP is Convergence and By M-Test.

U_n is uniform convergence

Q.6. $\sum \frac{n(x+2)^n}{3^{n+1}} = \sum a_n z^n$ let.

$\therefore$ To find radius of convergence :

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left(\frac{n+1}{3^{n+2}} \times \frac{3^{n+1}}{n}\right) = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)\frac{1}{3} = \frac{1}{3}$$

$\frac{1}{3} = \frac{1}{R}$ $\therefore$ $R = 3$ Ans

Region of Convergence is : $|z| \le R$

$|x+2| \le 3$

$-3 \le x+2 \le 3$

$-5 \le x \le 1$ Ans