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Physics - 2 (15B11PH211)

Tutorial - 1

Q.1. $f(x,y,z) = x^2 + y^3 + z^4 \qquad (2,-3,-4)$

$$\nabla f = \left(\hat{i}\frac{d}{dx} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{dz}\right) f = 2x\hat{i} + 3y^2\hat{j} + 4z^3\hat{k}$$

$$\nabla f \Big]_{(2,-3,-4)} = 4\hat{i} + 27\hat{j} - 256\hat{k} \quad \text{Ans}$$

Q.2. $A = y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k} \qquad \text{divergence}(A)_{(1,2,3)} = ?$

$$\text{div}(A) = \frac{d}{\partial x}y^2 + \frac{d}{dy}(2xy + z^2) + \frac{\partial}{\partial z}(2yz)$$

$$= 2x + 2y$$

$$\text{div}(A) \Big]_{(1,2,3)} = 2 + 4 = 6 \quad \text{Ans}$$

Q.3. $\vec{A} = yz\hat{i} + xz\hat{j} + xy\hat{k}$

$$\text{Curl}(\bar{A}) = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & xz & xy \end{vmatrix} = \hat{i}(x - x) - \hat{j}(y - y) + \hat{k}(z - z)$$

$$= 0 \quad \text{Ans}$$

Q.4. $\phi = 4x^2 + 3y^2 - 9z^2$

$$\vec{E} = -\nabla\phi = -(8x\hat{i} + 6y\hat{j} - 18z\hat{k})$$

$$\vec{E} \Big]_{(3,4,5)} = -(24\hat{i} + 24\hat{j} - 90\hat{k}) \quad \text{Ans}$$

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Q.5. $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$ $\rho = 30x^2y$.

&

$-1 \le y \le 0$?

$$Q = \int \rho\, dz = \int_0^1\int_0^1\int_0^1 30x^2y \; dx\,dy\,dz$$

$$= 30 \int_0^1\int_0^1 x^2y \; z\Big]_0^1 dx\,dy = 30\int_0^1 x^2 \frac{y^2}{2}\Big]_0^1 dx$$

$$= 15\int_0^1 x^2\,dx = \frac{15}{3} x^3\Big]_0^1 = 5C \text{ Ans}$$

$$Q = \int_{-1}^{0}\int_0^1\int_0^1 30x^2y \; dz\,dx\,dy = 30\int_{-1}^{0}\int_0^1 x^2y \; dx\,dy$$

$$= 30\int_{-1}^{0}\left[\frac{x^3}{3}\right]_0^1 y\,dy = 10\int_{-1}^{0} y\,dy$$

$$= -5C \text{ Ans}$$

Q.6. $\rho(r \le a) = \rho_0 \frac{r}{a}$

$$Q = \int \rho\, dz = \int_{r=0}^{a}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \left(\rho_0 \frac{r}{a}\right) r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= \pi \rho_0 a^3 \text{ Ans}$$

Q.7. $Q_1 = 30nC$, $Q_2 = 150nC$, $Q_3 = -70nC$

$$\phi = \frac{Q_{net}\Big]_{encl.}}{\varepsilon_0} = \frac{110\, nC}{\varepsilon_0} = \frac{110 \times 10^{-9}\, C}{\varepsilon_0}$$

$$= 1.243 \times 10^4\, Nm^2/C^2$$

Q.8. lines enter = 5000

lines leave = 3000

ϕ = no. of lines passing normally through the surface.

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$\phi = 5000 - 3000 = 2000$

$q_{encl.} = \phi \varepsilon_0$

$= 2000 \times 8.85 \times 10^{-12} = 1.77 \times 10^{-8}$ Coloumb Ans

Q.9 $\vec{S} = 100\hat{k}$, $\vec{E} = 8\hat{i} + 4\hat{j} + \hat{k}$

$\phi = \vec{E}.\vec{S} = (8\hat{i} + 4\hat{j} + \hat{k}) . 100\hat{k}$

$= 100$ units Ans

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Physics Tutorial 1

Assignment - 1 :

Gauss Theorem - The Electric flux passing through any closed surface is $1/\varepsilon_0$ times, the total charge enclosed inside it.

Mathematically, $\phi = \frac{q}{\varepsilon_0}$

Proof :-

let a charge 'q' be situated at a point O. within a closed surface 's'. Point 'p' is situated on the closed surface at a distance r from O. the intensity of $\vec{E}$ at point p will be :

$$\vec{E} = \frac{kq}{r^2} \quad — (1)$$

Electric flux passing through area 'ds' enclosing point 'p'.

$$d\phi = \vec{E} \cdot d\vec{s}$$

$$d\phi = |\vec{E}||d\vec{s}| \cos\theta$$

flux passing through the whole surface 's'

$$\iint_S d\phi = \iint_S \vec{E} \cdot d\vec{s} \cos\theta \quad — (2)$$

from (1) & (2) :

$$\phi = \iint_S \frac{kq}{r^2} ds \cos\theta$$

$$\phi = kq \iint_S \frac{ds \cos\theta}{r^2}$$

$$\phi = kq\,\omega$$

here 'ω' $\rightarrow$ Solid Angle subtended by the closed surface 's' at O is 4π thus.

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

$\boxed{\phi = 0}$ when charge is outside the body $\therefore q = 0$