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BATCH : B10

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Physics - 2 (15B11PH211)

Tutorial - 2

Q.1. (a) Gauss law :

→ differential form ⇒ $\nabla . \vec{E} = \frac{\rho}{\varepsilon_0}$

→ Integral form ⇒ $\oint \vec{E} . d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$

(b) charge enclosed divided by the permitivity (ε_0).

(c) zero

(d) $\phi = \oint \vec{E} . d\vec{A} \neq$

$$\nabla . \vec{E} = \frac{\rho}{\varepsilon_0} \quad \because \vec{E} = -\nabla \vec{V}$$

$$\therefore \nabla(-\nabla V) = \frac{\rho}{\varepsilon_0}$$

$$-\nabla^2 V = \frac{\rho}{\varepsilon_0} \text{ Ans}$$

(e) $\vec{E}$ is path independent, $\nabla \times \vec{E} = 0$ & $\vec{E} = -\nabla V$

Q.2. (a) As we move farther and farther away from the plane, more and more charge comes into ~~over~~ 'field of view' (a cone shape extended out from our life), and this compensates for the diminishing effect of any particular piece. Similarly for infinite line charge.

(b) Discontinuous, σ/ϵ_0 ; always continuous.

(c) $V(b) - V(a) = \int_a^b \vec{E}\cdot d\vec{l} = 0 \Rightarrow V(a) = V(b)$

as $\vec{E}$ within or at the surface of conductor is zero.

(d) $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$; $\vec{f} = P = \frac{1}{2\epsilon_0}\sigma^2\hat{n} = \frac{1}{2}\epsilon_0 E^2$

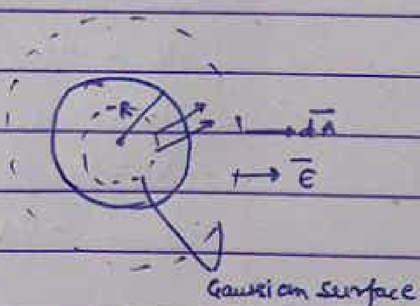
Q.3. $\rho = \epsilon_0 \nabla\cdot\vec{E} = \epsilon_0 \frac{1}{r^2}\frac{d}{dr}(r^2 k r^3) = 5\epsilon_0 k r^2$

$Q_{enc.} = \epsilon_0 \oint_s \vec{E}\cdot d\vec{a} = \epsilon_0 \int_0^{2\pi}\int_0^{\pi} (kR^3\hat{r})\cdot(R^2\sin\theta\, d\theta\, d\phi)\hat{r}$

$= 4\pi\epsilon_0 k R^5$

$Q_{enc.} = \int \rho\, dv = \int_0^R (5\epsilon_0 k r^2)(4\pi r^2 dr) = 4\pi\epsilon_0 k R^5$

Q.4. $r > R$



$\oint_s \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0}\int \rho\, dv$

$\int_0^{2\pi}\int_0^{\pi} |\vec{E}|\, r^2\sin\theta\, d\theta\, d\phi = \frac{1}{\epsilon_0}\int_0^{2\pi}\int_0^{\pi}\int_0^R r^2\sin\theta\, dr\, d\theta\, d\phi$

$|\vec{E}|(\cancel{4\pi} r^2) = \frac{\rho}{3\epsilon_0}(\cancel{4\pi} R^3)$

$\vec{E} = \frac{\rho}{3\epsilon_0}\frac{R^3}{r^2}\hat{r}$ Ans

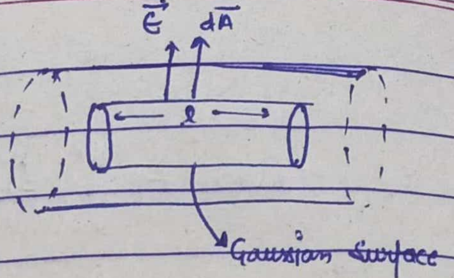
$r = R$: $\vec{E} = \frac{\rho R}{3\epsilon_0}\hat{r}$ Ans

$r < R$: $|\vec{E}|(4\pi r^2) = \frac{\rho(4\pi r^3)}{3\epsilon_0}$

$\vec{E} = \frac{\rho}{3\epsilon_0} r\,\hat{r}$ Ans

Q.5. $r < R$

$$\oint \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\int_0^{l}\int_0^{2\pi}\int_0^{r} \rho\, r\, dr\, d\phi\, dz$$



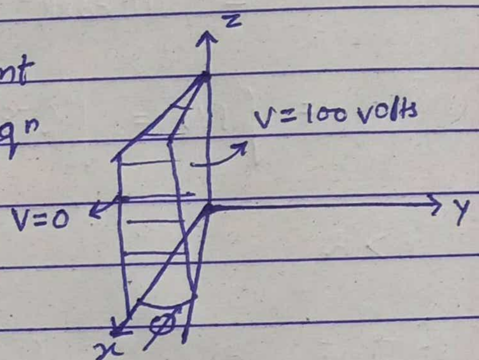
$$|\vec{E}|(2\pi r l) = \frac{\rho}{\varepsilon_0}(\pi r^2 l)$$

$$\vec{E} = \frac{\rho r}{2\varepsilon_0}\,\hat{r} \quad \text{Ans}$$

$r > R$: $$|\vec{E}|(2\pi r l) = \frac{\rho}{\varepsilon_0}(\pi R^2 l)$$

$$\vec{E} = \frac{\rho}{2\varepsilon_0}\frac{R^2}{r}\,\hat{r} \quad \text{Ans}$$

Q.6. Since the potential is constant in x & z, the laplace eq^n



$$\frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} = 0$$

$$V = A\phi + B$$

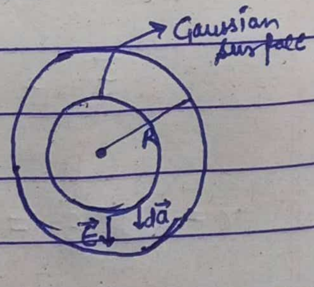
$$B = 0, \quad A = 100/\alpha$$

$$V = 100\,\phi/\alpha \text{ volt}$$

$$\vec{E} = -\nabla V = \frac{-1}{r}\frac{\partial}{\partial\phi}\left(\frac{100\,\phi}{\alpha}\right) = \frac{-100}{r\alpha}\,\hat{\phi} \ \text{(V/m)}$$

Q.7. $r < R$

$$\oint \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\int_0^{\pi}\int_0^{2\pi}\int_0^{r} k r \cdot r^2 \sin\theta\, dr\, d\theta\, d\phi$$



$$|\vec{E}|(4\pi r^2) = \frac{k}{\varepsilon_0}\frac{\cancel{4}\pi r^4}{\cancel{4}}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\pi k r^2\,\hat{r} \quad \text{Ans}$$

$r > R$: $$\vec{E} = \frac{\pi k R^4}{4\pi\varepsilon_0}\frac{1}{r^2}\,\hat{r} \quad \text{Ans}$$

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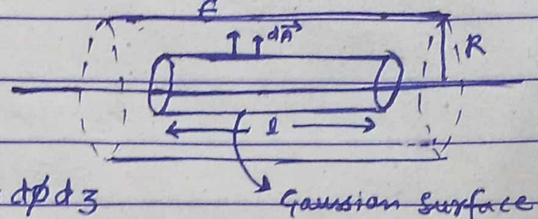
Q.8. $r<R$:



$$\oint \vec{E}\cdot d\vec{A} = \frac{1}{\epsilon_0}\iiint \rho\, dV$$

$$|\vec{E}|(2\pi r l) = \frac{K}{\epsilon_0}\int_0^{l}\int_0^{2\pi}\int_0^{r} r\, r\, dr\, d\phi\, dz$$

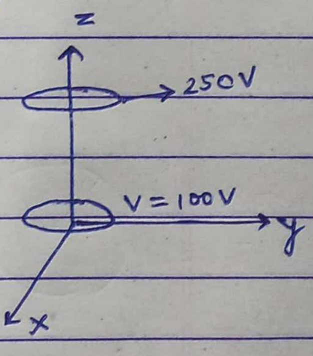
$$|\vec{E}|(2\pi r l) = \frac{K}{\epsilon_0}\,\frac{2\pi r^3 l}{3}$$

$$\vec{E} = \frac{K}{3\epsilon_0} r^2\, \hat{r}$$

$r>R$: $$|\vec{E}|(2\pi r l) = \frac{K}{\epsilon_0}\left(\frac{2\pi R^3 l}{3}\right)$$

$$\vec{E}_{r>R} = \frac{KR^3}{3\epsilon_0}\times\frac{1}{r}\,\hat{r}$$

Q.9. Since the potential is z dependent only the laplace eq^n.



$$\frac{d^2V}{dz^2} = 0 \rightarrow \frac{dV}{dz} = A \quad \text{—(1)}$$

$$V = Az + B$$

from (1) :

$$\frac{dV}{dz} = A = \frac{250-100}{5\times10^{-3}} = 3\times10^{4}\ V/m$$

$$\vec{E} = -\nabla V = \frac{-dV}{dz} = -3\times10^{-4}\,\hat{z}\ V/m$$

$$\vec{D} = \epsilon_0\epsilon_r\vec{E} = -5.84\times10^{-7}\,\hat{z}\ C/m^2$$

Since $\vec{D}$ is const. b/w the plates & $|\vec{D}| = \sigma$ at the conductor surface $\sigma = \pm 5.84\times10^{-7}$ C/m2

(+)ve on upper plate & (-)ve on lower plate.