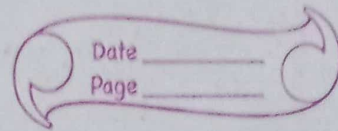


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## Mathematics - II (15B11MA211)

### Tutorial sheet - 4

Q.1.  $\sum (-1)^{n+1} \frac{1}{n^p}$

(a) for convergence :- use Leibniz Test

(i)  $U_{n+1} < U_n$  is True for in series

(ii)  $\lim_{n \rightarrow \infty} U_n = \lim_{n \rightarrow \infty} \frac{(-1)^{n+1}}{n^p} = 0 \quad \checkmark$

Hence, Leibniz test is satisfied Series in Convergence ( $p > 0$ )

(b) for absolute Convergence :-

$|U_n|$  is convergence

$|U_n|$  is  $\frac{1}{n^p}$  and according to p-series test

$p > 1$ ,  $|U_n|$  is Convergent. Hence, Absolute convergent when  $p > 1 \quad \checkmark$

Q.2.  $\frac{2}{1^2} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \dots$

Series is Alternating Series &  $U_n = \frac{(-1)^{n+1} (n+1)}{n^2}$

According Leibniz Test (1) This is Alternating decreasing series  $\checkmark$

(2)  $\lim_{n \rightarrow \infty} \frac{(-1)^{n+1} (n+1)}{n^2} = 0 \quad \checkmark$

Hence, Series is Convergence as Leibniz Test satisfies.

Now, check for  $|U_n| = \frac{n+1}{n^2}$  Let  $V_n = \frac{1}{n} \geq U_n$

Acc. to D-Alembert Test :-

$$\lim_{n \rightarrow \infty} \left| \frac{U_n}{V_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{n+1}{n^2}}{\frac{1}{n}} \right| = 1; \text{ Test fails } \text{uniquely \& exist.}$$



$\forall n$  is divergence as  $p=1$  so  $|u_n|$  is divergence  
 $\therefore$  Series is conditionally convergence.

Q.3.  $1 - 2x + 3x^2 - 4x^3 + \dots$

This is Power Series :

$$U_n = nx^{n-1}$$

$$U_{n+1} = (n+1)x^n$$

Acc. to D. Ambert ratio test :

$$\lim_{n \rightarrow \infty} \left| \frac{U_{n+1}}{U_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)x^n}{nx^{n-1}} \right| = |x|$$

if  $|x| < 1 \rightarrow$  Series is Convergent

if  $|x| \geq 1 \rightarrow$  Series is divergent

Q.4.  $x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$

$$U_n = \frac{x^{2n-1}}{2n-1}$$

$$U_{n+1} = \frac{x^{2n+1}}{2n+1}$$

Acc. to ratio test :

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{U_{n+1}}{U_n} \right| &= \lim_{n \rightarrow \infty} \left| \frac{x^{2n+1}}{2n+1} \cdot \frac{2n-1}{x^{2n-1}} \right| \\ &= \lim_{n \rightarrow \infty} \left| x^2 \left( \frac{2 - 1/n}{2 + 1/n} \right) \right| = x^2 \end{aligned}$$

$x^2 < 1 \rightarrow$  Series convergence in  $-1 < x < 1$

$x^2 = 1$  test fail ,  $x^2 > 1 \rightarrow$  series divergence

$$x = \pm 1$$

$$x = 1$$

$$U_n = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

$$x = -1$$

$$U_n = -\left(1 - \frac{1}{3} + \frac{1}{5} - \dots\right)$$

Both Satisfies Leibniz Test hence, series is Convergent

In  $x = \pm 1$   $\therefore$  convergence in  $-1 \leq x \leq 1$



Q.5.  $1 + a \cos x + a^2 \cos 2x + a^3 \cos 3x + \dots + a^n \cos nx + \dots$

$$U_n = a^n \cos nx$$

$$|U_n| = |a^n \cos nx| = |a^n|$$

$$a^n \cos nx \leq |a^n \cos nx| \leq |a^n|$$

By Geometric Progression Test:  $r = |a|$   
if  $r < 1$  GP is convergence and by M-Test.  
 $U_n$  is uniform convergence

Q.6.  $\sum \frac{n(x+2)^n}{3^{n+1}} = \sum a_n z^n$  let.

$\therefore$  To find radius of convergence:

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \left( \frac{n+1}{3^{n+2}} \times \frac{3^{n+1}}{n} \right) = \lim_{n \rightarrow \infty} \left( \frac{1+\frac{1}{n}}{3} \right) \frac{1}{3} = \frac{1}{3}$$

$$\frac{1}{3} = \frac{1}{R} \quad \therefore R = 3 \quad \text{Ans}$$

Region of convergence is:  $|z| \leq R$

$$|x+2| \leq 3$$

$$-3 \leq x+2 \leq 3$$

$$-5 \leq x \leq 1 \quad \text{Ans}$$