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BATCH : B10

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Mathematics - 2 Tutorial Sheet-1

Q.1. (i) $(D^2-2D+2)y = 0$

for CF : Substituting $y = e^{mx}$

$(m^2-2m+2) = 0$

$m = \frac{2 \pm \sqrt{4-8}}{2}$

$m = 1 \pm i$

$\therefore \quad y(x) = e^x (A\cos x + B\sin x)$.

(ii) $(D^4-81)y = 0$

Substituting $y = e^{mx}$

we get, $m^4 - 81 = 0$

$m = \pm 3, \pm 3i$

$y(x) = Ae^{3x} + Be^{-3x} + C\cos 3x + D\sin 3x$

(iii) $(D^3-1)^2 y = 0$

On substituting $y = e^{mx}$

we get, $m^3 - 1 = 0$

$m = 1, 1, \frac{-1 \pm \sqrt{3}i}{2}, \frac{-1 \pm \sqrt{3}i}{2}$

$y(x) = (A+Bx)e^x + e^{-1/2x}\left((C+Dx)\cos\frac{\sqrt{3}x}{2} + (E+Fx)\sin\frac{\sqrt{3}x}{2}\right)$

Q.2. (i) $(D^2-4D+4)y = e^{2x} + x^3 + \cos 2x$

CF : $m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

$m = 2, 2$

$y(x) = (A+Bx)e^{2x}$

PI : $y = \dfrac{1}{(D-2)^2} e^{2x} + \dfrac{1}{(D-2)^2} x^3 + \dfrac{1}{D^2-4D+4} \cos 2x$

$$y = \frac{x e^{2x}}{2(D-2)} + \frac{1}{-\cancel{4}-4D+\cancel{4}}(\cos 2x) + \frac{1}{(D-2)^2} x^3$$

$$y = \frac{x^2 e^{2x}}{2} + \frac{1}{-4D}(\cos 2x) + \frac{1}{(D-2)^2} x^3$$

$$\cancel{y = \frac{x^2 e^{2x}}{4} + \frac{1}{9}(1+2D)^{-1}(\cos 2x)}$$

$$y = \frac{x^2 e^{2x}}{2} - \frac{1}{8}\sin 2x + \frac{1}{4}\left(1 - \frac{D}{2}\right)^{-2} x^3$$

$$y = \frac{x^2 e^{2x}}{2} - \frac{1}{8}\sin 2x + \frac{1}{4}\left[1 + \frac{2 \times D}{2} + \frac{3D^2}{4} + \frac{4D^3}{8} + \dots\right] x^3$$

$$= \frac{x^2 e^{2x}}{2} - \frac{1}{8}\sin 2x + \frac{x^3}{4} + \frac{3x^2}{4} + \frac{9}{8}x + \frac{3}{4}$$

$\therefore$ $y(x) = CF + PI$

$$= (A+Bx)e^{2x} + \frac{x^2 e^{x}}{2} + \frac{x^3}{4} + \frac{3x^2}{4} + \frac{9}{8}x + \frac{3}{4} - \frac{1}{8}\sin 2x$$

Ans

(iii) $(D^2 - 6D + 13)y = 16e^{3x}\sin 4x + 3^x$

CF : $m^2 - 6m + 13 = 0$

$$m = \frac{6 \pm \sqrt{36 - 52}}{2} = 3 \pm 2i$$

$$y = e^{3x}(A\cos 2x + B\sin 2x)$$

PI : $y = \dfrac{16 e^{3x}\sin 4x}{D^2-6D+13} + \dfrac{3^x}{(D^2-6D+13)}$

$$= e^{3x}\frac{16}{(D+3)^2 - 6(D+3) + 13}\sin 4x + \frac{e^{x\log_e 3}}{(D^2-6D+13)}$$

$$= \frac{16e^{3x}}{D^2+4}\sin 4x + \frac{e^{x\log_e 3}}{(\log_e 3)^2 - 6(\log_e 3) + 13}$$

$$= \frac{-16e^{3x}\sin 4x}{12} + \frac{3^x}{(\log_e 3)^2 - 6(\log_e 3) + 13}$$

$$= -\frac{4}{3}e^{3x}\sin 4x + \frac{3^x}{(\log_e 3)^2 - 6(\log_e 3) + 13}$$

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$$y(x) = CF + PI$$
$$= e^{3x}(A\cos 2x + B\sin 2x) - \frac{4}{3}e^{3x}\sin 4x + \frac{3^x}{(\log 3)^2 - 6(\log 3) + 13}$$

(iii) $(D^2+1)y = \operatorname{cosec} x$

CF : $m^2+1 = 0$

$m = \pm i$

$y = A\cos x + B\sin x$

PI : $y = \frac{1}{(D^2+1)} \operatorname{cosec} x$

$$= \frac{1}{(D+i)(D-i)} \operatorname{cosec} x$$

$$= \frac{1}{2i}\left[\frac{1}{(D-i)} - \frac{1}{(D+i)}\right]\operatorname{cosec} x$$

$$\frac{1}{(D-i)}\operatorname{cosec} x = e^{ix}\int e^{-ix}\operatorname{cosec} x\, dx$$

$$= e^{ix}\int(\cos x - i\sin x)\operatorname{cosec} x\, dx$$

$$= e^{ix}\int(\cot x - i)\,dx = e^{ix}\left[\log|\sin x| - ix\right] + c$$

Similarly, $\frac{1}{(D+i)}\operatorname{cosec} x = e^{-ix}\left[\log(\sin x) + ix\right] + c$

$$y(x) = CF + PI$$
$$= (A\cos x + B\sin x) + \frac{1}{2i}\left[e^{ix}(\log\sin x - ix) - e^{-ix}(\log(\sin x) + ix)\right]$$

Q.3. (i) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

let $x = e^z$ $\qquad D_1 = \frac{d}{dz}$

$(D_1(D_1-1) + D_1 + 1)y = 0$

$(D_1^2 + 1)y = 0$

CF: $m^2+1 = 0$

$m = \pm i$

$y(x) = A\cos z + B\sin z$

PI: $y = 0$

$\therefore$ $y(x) = CF + PI$

$= A\cos(\log x) + B\sin(\log x)$ Ans

(ii) $x^2y'' + 4xy' + 2y = 0$

By cauchy euler method :

$(D_1(D_1-1) + 4D_1 + 2)y = 0$

$(D_1^2 + 3D_1 + 2)y = 0$

$y(x) = ?$ $\quad m^2+3m+2 = 0$

$(m+2)(m+1) = 0$

$m = -2,\ m = -1$

$y(x) = Ae^{-2z} + Be^{-z}$

$y(x) = \frac{A}{x^2} + \frac{B}{x}$ Ans

(iii) $x^2y'' - 5xy' + 9y = 0$

By cauchy euler method : $\quad x = e^z$

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$$(D_1(D_1-1) - 5D_1 + 9)y = 0$$

$$(D_1^2 - 6D_1 + 9)^2 y = 0$$

Characterstic Equation : $m^2 - 6m + 9 = 0$

$(m-3)^2 = 0$

$m = 3, 3.$

$$y(x) = Ae^{3z} + Bx e^{3z}$$

$$y(x) = (A + Bx) x^3 \text{ Ay}$$

(iv) $x^3y''' + 3x^2y'' + xy' + y = \sin(\log x) + x$

By cauchy euler Eqn : $x = e^z$

$$(D_1(D_1-1)(D_1-2) + 3(D_1)(D_1-1) + D_1 + 1)y = \sin z + e^z$$

$$(D_1(D_1^2 - 3D_1 + 2) + 3(D_1^2 - D_1) + D_1 + 1)y = \sin z + e^z$$

$$(D_1^3 - \cancel{3D_1^2} + \cancel{2D_1} + \cancel{3D_1^2} - \cancel{3D_1} + \cancel{D_1} + 1)y = \sin z + e^z$$

CF : Characterstic eqn : $m^3 + 1 = 0$

$m = -1, -\omega, -\omega^2$

$$m = -1, \frac{+1-\sqrt{3}i}{2}, \frac{1+\sqrt{3}i}{2}$$

$$\therefore y(x) = Ae^z + e^{+1/2 z}\left(A\cos\frac{\sqrt{3}}{2}z + B\sin\frac{\sqrt{3}}{2}z\right)$$

$$y(x) = Ax + x^{1/2}\left(A\cos\left(\frac{\sqrt{3}}{2}\log x\right) + B\sin\left(\frac{\sqrt{3}}{2}\log x\right)\right)$$

PI : $y = \frac{1}{(D_1^3+1)}\sin z + \frac{1}{(D_1^3+1)}e^z$

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$$y(x) = \frac{1}{(1-D_1)} \sin z + \frac{1}{2} e^z$$

$$= \frac{(1+D_1)}{1-D_1^2} \sin z + \frac{1}{2} e^z$$

$$= (1 + D_1 + D_1^2 + D_1^3 + \dots) + \frac{1}{2} e^z$$

$$= (\sin z + \cos z - \sin z - \cos x + \sin z + \cos z + \dots) + \frac{1}{2} e^z$$

$$= \sin z + \cos z + \frac{1}{2} e^z$$

$$= \sin(\log x) + \cos(\log x) + x$$

$$= \frac{(1+D_1)}{2} \sin z + \frac{1}{2} e^z$$

$$= \frac{\sin z + \cos z}{2} + \frac{e^z}{2}$$

$$= \frac{\sin(\log x) + \cos(\log x) + x}{2}$$

$$y(x) = CF + PI$$

$$= Ax + x^{1/2}\left(A\cos\left(\frac{\sqrt{3}}{2}\log x\right) + B\sin\left(\frac{\sqrt{3}}{2}\log x\right)\right) + \frac{\sin(\log x) + \cos(\log x) + x}{2} \quad \text{Ans}$$