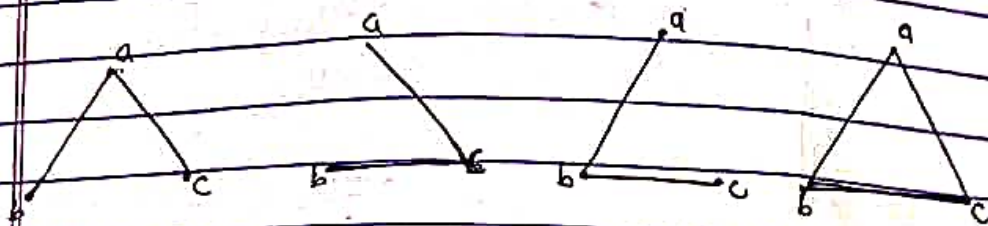
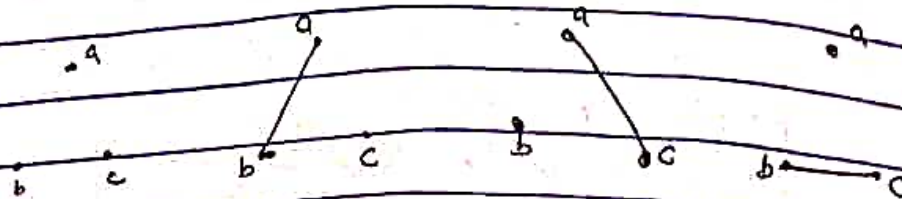
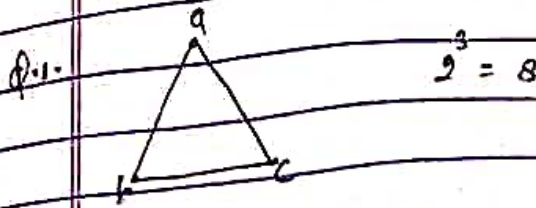


Tutorial Graph Theory (TECS)

- Q.2.
- (a) yes $\{V_1 = \{a, b, c, d\}, V_2 = \{e\}\}$
- (b) yes $\{V_1 = \{b, d, e\}, V_2 = \{a, c\}\}$
- (c) No
- (d) yes $\{V_1 = \{a, b, d, e\}, V_2 = \{c, f\}\}$
- (e) No

- Q.3.
- | | | |
|-----|--------------------------------------|--|
| (a) | Hamiltonian Path \rightarrow ABCDE | Hamiltonian Circuit \rightarrow ABCEDA |
| (b) | n \rightarrow EABCD | n \rightarrow X |
| (c) | n \rightarrow X | n \rightarrow X |
| (d) | n \rightarrow ABcDEFgHI | n \rightarrow ABCDEFgHIAI |

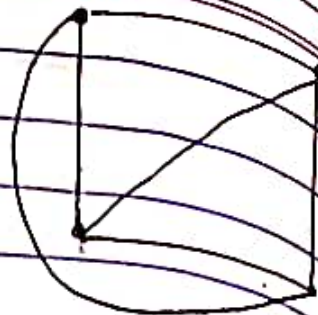
- Q.4.
- | | | |
|-----|------------------------------------|---------------------------------------|
| (a) | Euler Path \rightarrow BCDBAD | Euler Circuit \rightarrow X |
| (b) | Euler Path \rightarrow BCDFBEDAB | Euler Circuit \rightarrow BCDFBEDAB |
| (c) | n \rightarrow X | n \rightarrow X |
| (d) | n \rightarrow BACEDCB | n \rightarrow BACEDCB |

Q.5.

(a)



(b)



(c)

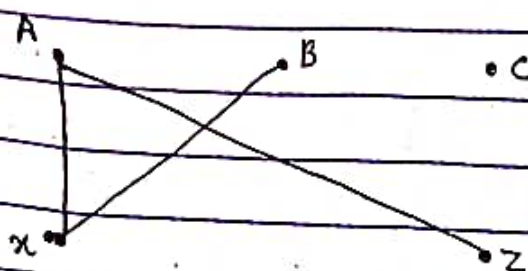
Non-Planar

(d)

Non-Planar

Q.6. (a) $AXYC$, $AXBYC$

(b)



(c)

 $XYCX$

(d)



(e)

$A \rightarrow \{V_1 = (A, Z), V_2 = (X, B, Y, C)\}$
 $X \rightarrow \{V_1 = (A, X, Z), V_2 = (B, Y, C)\}$
 $Y \rightarrow \{V_1 = (Y, C), V_2 = (A, X, B, Z)\}$

(f)

 AX, XY

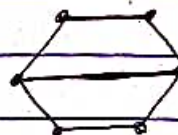
- Q.7. (a) yes
(b) yes

Q.8. The colors need to be assigned to the animals such that no two adjacent nodes have the same color. The minimum numbers of colors required to color the animals is then the number of different habitats needed (as animal with the same color can be placed together in a habitat).

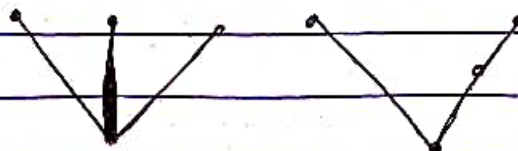
- Q.9. (a) 3
(b) 2
(c) 2
(d) 3

Q.10. C4 & C5 are the only committees that do not share a common member. Hence we require a different colour for every committee except for these two (which may be coloured the same).
So, $6 - 1 = 5$ meetings.

Q.11. Yes, for example, both graphs below contain 6 vertices, 7 edges & have degree (2, 2, 2, 2, 3, 3)



Q.12. 1: for example:



2. This is not possible if we require the graphs to be connected. If not, we could take C_8 as one graph & two copies of C_4 as the other.
3. Not possible. If you have a graph with 5 vertices all of degree 4, then every vertex must be adjacent to every other vertex. This is the graph K_5 .
4. This is not possible. In fact, there is not even one graph with this property (such a graph could have $5 \cdot \frac{3}{2} = 7.5$ edges)

Q.14. If we draw a graph with each letter representing a vertex, and each edge connecting two letters that were consecutive in the alphabet, we would have a graph containing two vertices of degree 1 (A and Z) & the remaining 24 vertices all of degree 2 (for example, DD would be adjacent to both CC & EE). By Brook's theorem, this graph has chromatic number at most 2, as that is the maximal degree in the graph & the graph is not a complete graph or odd cycle. Thus only two boxes are needed.