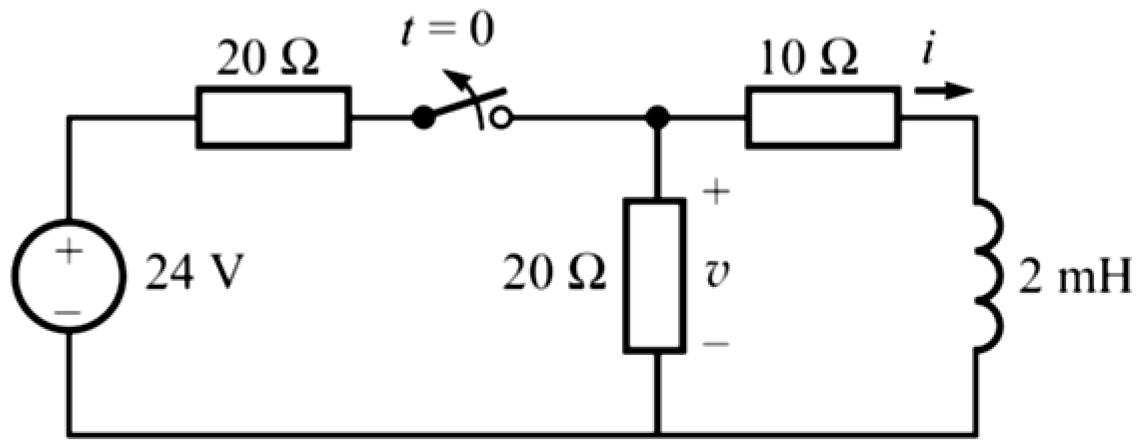
ELECTRICAL SCIENCE-II (15B11EC211)

UNIT 1
LECTURE 2

Example1



The circuit has been in the condition shown for a long time. The switch is opened at $t = 0$.

(i) Determine the current $i(0+) = I_0$.

(ii) Find v_R across 20-Ω resistor at the instant just after the switch is opened.

(iii) Find v_L across the inductor immediately after the switch is opened.

Solution : (*i*) Under steady-state condition, the voltage drop across an inductor is zero and it behaves as a short-circuit. The equivalent resistance faced by the 24-V source is

$$R_{\text{eq}} = 20\,\Omega + (20\,\Omega \,||\, 10\,\Omega) = 26.67\,\Omega$$

The current supplied by the 24-V source,

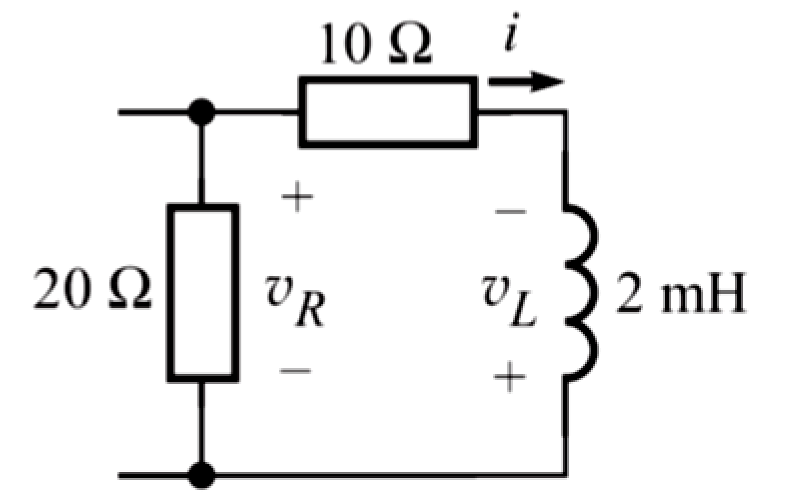
$$I = \frac{V}{R_{\text{eq}}} = \frac{24}{26.67} = 0.9 \text{ A}$$

By current division,

$$I_L = 0.9 \times \frac{20}{20+10} = 0.6 \text{ A}$$

Immediately after the switch is opened, the current through the inductor remains the same. Hence,

$$i(0^+) = I_0 = \mathbf{0.6\ A}$$



The circuit after the switch is opened ($t > 0$).

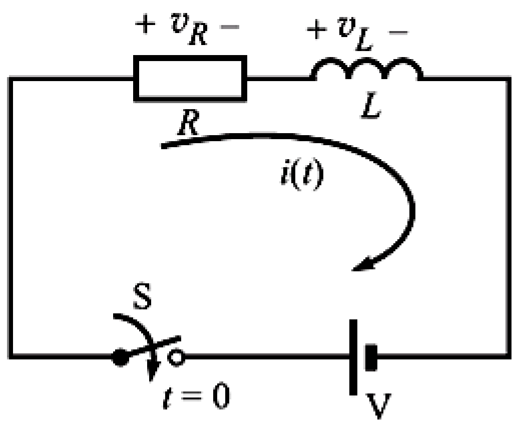
(ii) The voltage across the 20-Ω resistor,

$$v_R = (-I_0)R = -0.6 \times 20 = \mathbf{-12\ V}$$

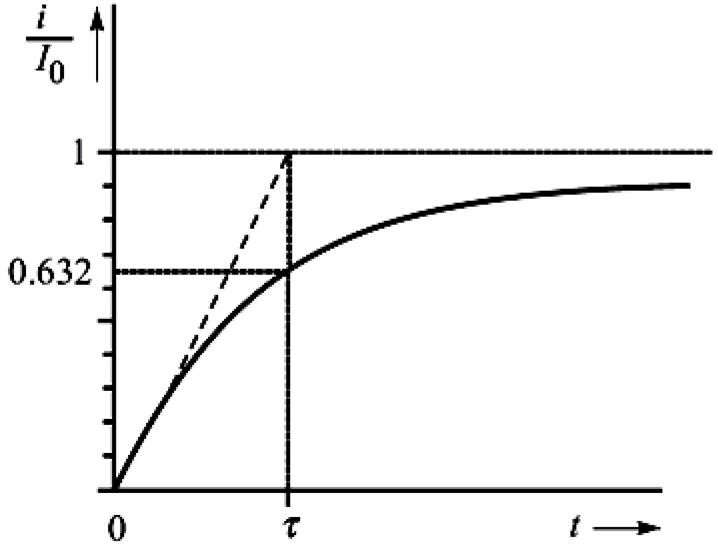
(iii)

$$e = L\left.\frac{di}{dt}\right|_{t=0} = L\times\left(I_0\times\frac{R}{L}\right) = I_0R = 0.6\times(20+10) = \mathbf{18\ V}$$

Growth of Current in Series *RL* Circuit



(*a*) The circuit.



(*b*) The response.

- Since the current in an inductor cannot change by a finite amount in zero time, we must have $i(0+) = 0$.
- After $t = 0$, the current slowly increases and approaches its steady state value $I_0 = V/R$.

The response to this circuit for $t > 0$ can be found as

$$i(t) = I_0(1 - e^{-t/\tau})$$

The value of $i(t)/I_0$ at $t = \tau$,

$$\frac{i(\tau)}{I_0} = (1 - e^{-1}) = (1 - 0.368)$$

or $$i(\tau) = 0.632 I_0$$

- Thus, in one time constant the response rises to 63.2 % of its final value.
- It takes about five time constants for the current to grow to its final steady state value.

Rate of Growth of Current

The initial rate of growth of current is given by the slope of the curve at the origin.

$$\left.\frac{di}{dt}\right|_{t=0} = -\left(-\frac{I_0}{\tau}\right)e^{-t/\tau}\Big|_{t=0} = \frac{I_0}{\tau} = \frac{V}{R}\frac{R}{L} = \frac{V}{L}$$

- Thus, the smaller the value of *L*, the faster the current rises to its final value.

Example 2

A coil having an inductance of 14 H and a resistance of 10 Ω is connected to a dc voltage source of 140 V, through a switch.

- (*a*) Calculate the value of current in the circuit at an instant 0.4 s after the switch has been closed.
- (*b*) Once the current reaches its final steady state value, how much time it would take the current to drop to 8 A after the switch is opened ?

Solution :

The time constant, $\tau = L/R = 14/10 = 1.4$ s

(a) The final steady state value of the current,

$$I_0 = \frac{V}{R} = \frac{140}{10} = 14 \text{ A}$$

The value of current at t = 0.4 s is given by

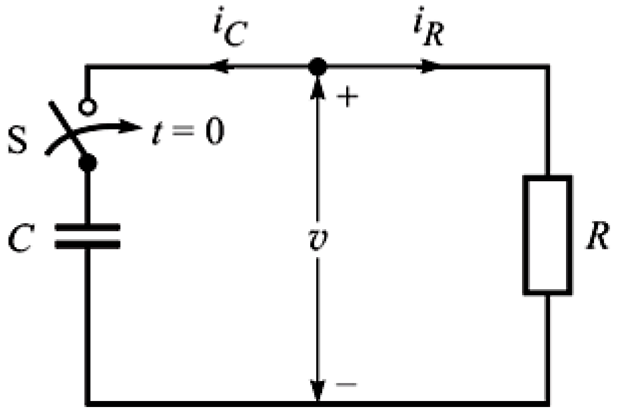
$$i = I_0(1 - e^{-t/\tau}) = 14(1 - e^{-0.4/1.4}) = \mathbf{3.479\ A}$$

(b) For decaying current,

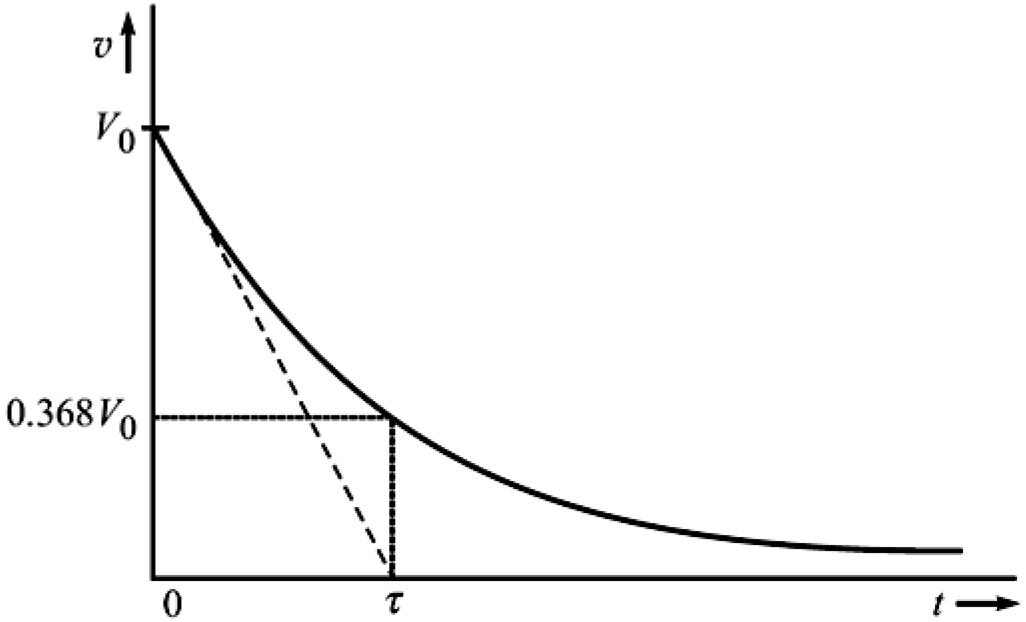
$$i(t) = I_0 e^{-t/\tau} \quad \text{or} \quad 8 = 14e^{-t/1.4} \quad \text{or} \quad e^{-t/1.4} = 0.5714$$

$$-\frac{t}{1.4} = \ln 0.5714 = -0.5596 \qquad \Rightarrow \qquad t = \mathbf{0.7834\ s}$$

The Simple *RC* Circuit (Discharging of a Capacitor)



(a) The circuit.



(b) The voltage response.

The total current leaving the node at the top of the circuit diagram must be zero. Therefore,

$$i_C + i_R = 0 \qquad \text{or} \qquad C\frac{dv}{dt} + \frac{v}{R} = 0 \qquad \text{or} \qquad \frac{dv}{dt} + \frac{v}{CR} = 0$$

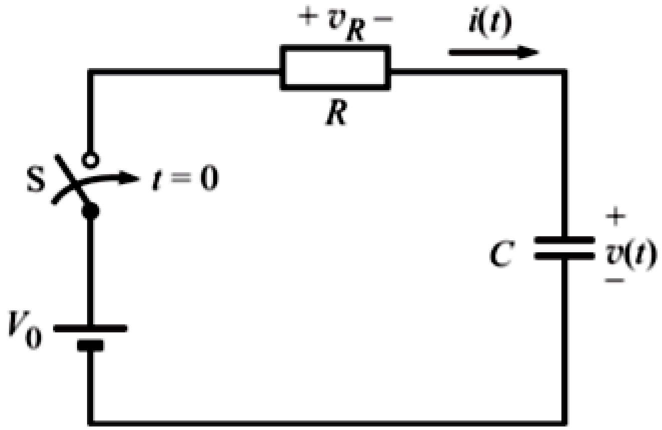
The solution of this equation is

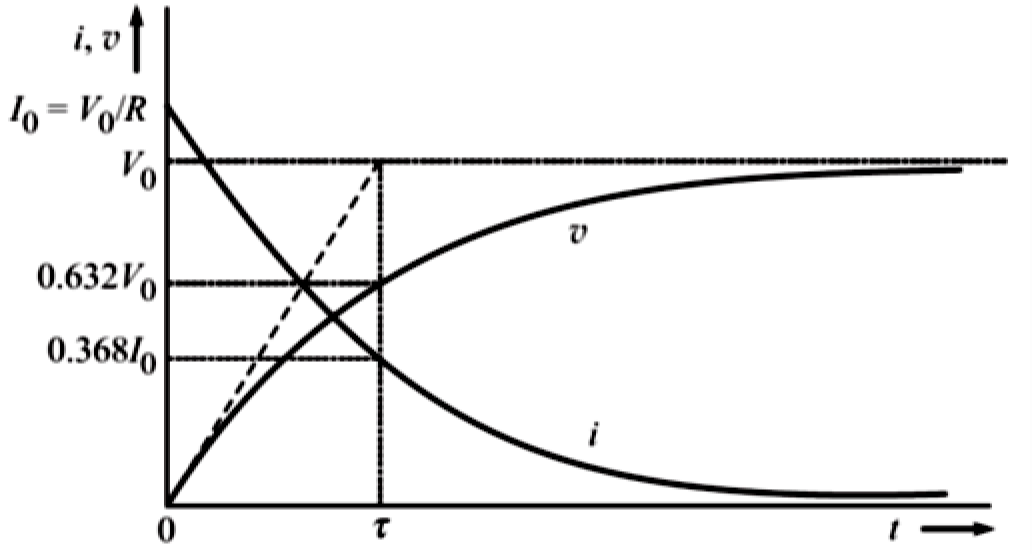
$$v = V_0 e^{-t/RC} = V_0 e^{-t/\tau}$$

The time constant, $\tau = RC$

As per definition of the time constant τ, the slope of the curve at $t = 0$ is given as $-V_0/\tau$.

Charging of a Capacitor





(a) The circuit. (b) The voltage and current response.

- Since the voltage across a capacitor cannot change instantaneously, we must have $v(0^+) = v(0^-) = 0$

- It means that initially the capacitor behaves as a short-circuit. The value of the initial current in the circuit,

$$i(0^+) = I_0 = \frac{V_0}{R}$$

$$v(t \to \infty) = V_0 \qquad \text{and} \qquad i(t \to \infty) = 0$$

The response of the circuit can be determined as

$$v = V_0(1 - e^{-t/\tau})$$

References

1. R.C.Dorf and James A. Svoboda, “Introduction to Electric Circuits”,9thed, John Wiley & Sons, 2013.
2. Charles K. Alexander, Matthew N.O. Sadiku,“FundamentalsofElectricCircuits”, 6th Edition,Tata McGrawHill,2019.