Day 13 Ackermann's method

AE353 Spring 2022 Bret1 The eigenvalues of a matrix are the roots of its characteristic polynomial

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \times + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$u = -\begin{bmatrix} 20 & 9 \end{bmatrix} \times$$

$$= 5^{2} + 95 + 20 = (5+4)(5+5) = 51 = -4 = 52 = -5$$

A-BK = [0 1] - [0][20 9]

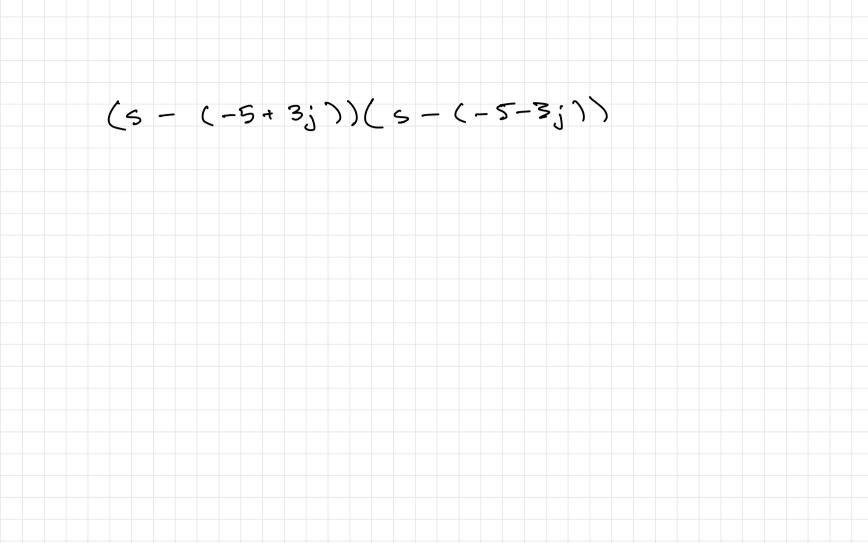
= [0]

Our way to place eigenvalues is to equate coefficients of the characteristic polynomial

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \times + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$
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 $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \times + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} u$

What do we want?

 $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \times + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} u$
 $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \times + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} u$
 $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \times + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} u$
 $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \times + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} u$
 $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\$



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Controllable Canonical Form (CCF)

$$A = \begin{bmatrix} a_1 & \cdots & -a_n \\ T_{(n-1)\times(n-1)} & O_{(n\times 1)\times 1} \end{bmatrix}$$

$$Facts$$

$$det(sI-A) = \begin{bmatrix} s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n \\ 1 & 1 & 1 \end{bmatrix}$$

$$det(sI-A-BK) = \begin{bmatrix} I-a_1-k_1 & \cdots & -a_n-k_n \\ I & 1 & 1 \end{bmatrix}$$

$$det(sI-A-BK) = \begin{bmatrix} s^n + (a_1+k_1)s^{n-1} + \dots + (a_{n-1}+k_{n-1})s + (a_n+k_n) \\ 0 & 1 & 1 \end{bmatrix}$$

Consequence

If you want
$$s^n + r_1 s^{n-1} + \dots + r_{n-1} s + r_n$$

then
$$k_1 = r_1-a_1 + \dots + r_n-a_n$$

Then ... yeasy to find
$$u = -K_{CCF} z$$

solve for V (that's what we need to find K given Kax) How? Accf = VAV Bccf = VB x = Ax+Bu -BCF = VB ACCEBCEF = (VAV) VB = VAB

ACCEBCEF = VAYNAVOB = VAB = VAVZ + VBu ALLE BLEF Acce BCCF = VAB [BCF ACCEBLE ... ACCEBCEF] = V [B AB ... AB] V= Weer W L works as long as is invertible

