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A Heuristic Entrepreneurial Perspective on Quantum Annealing in Combinatorial Portfolio Optimization for Transitional New World Era

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Abstract— This research project explores the identified literature gap by focusing on Quantum combinatorial portfolio optimization in a complex financial Engineering and FinTech entrepreneurial context. The core challenge is to answer the research question whether a Quantum FinTech entrepreneur can solve a classical financial Engineer's portfolio optimization problem, faster and more reliably by using the third generation D-Wave Quantum Annealer 2000Q? The approach is to apply a sub-field of econophysics namely Quantum Finance that links the laws of Quantum Mechanics to Quantum Computing by using a specific built quantum computer for optimization problems, namely the D-Wave Annealer 2000Q and its specific Ocean Software Development Kit and the Python OOP language alongside relevant libraries. This research paper is written within our current era of a diminishing Moore's Law in classical computing and a clear transition from old world views to the new world views during the beginning of our current Quantum era also known as second Quantum revolution. Both a qualitative (~25%) and quantitative (~75%) approach in analyzing the potential of D-Wave Quantum Annealer 2000Q through firsthand experience rather than seeking to improve the model by introducing the D-Wave Ocean SDK through the optimization of the number division problem and then conducting a heuristic project using an algorithmic modelling approach from a well known research paper. The second heuristic approach is a longitudinal study of randomly selected London Stock Exchange market data over a multi-year period using API technology by Yahoo Finance. Drawn conclusions from a perspective of a 'Quantum Economic person' ('We-mode') from Quantum Cognition and Quantum Game Theory with the inherent 'uncertainty principle' from Quantum Mechanics, rather than from a 'homo economicus' or always rational behaving classical economic man's perspective ('I-mode'). This research successfully addressed the literature gap by confirming the FinTech entrepreneur's goals of obtaining portfolio optimization faster and more reliably with the third generation D-Wave Quantum Annealer 2000Q because of quantum tunnelling with favorable trading results by beating the benchmark portfolio of all twenty stocks. Quantum annealing solves such an optimization problem disruptively fast and thus makes Quantum Computing algorithms practically more applicable than classical computing for combinatorial portfolio optimization.

Index Terms— adiabatic quantum computation, annealing, binary, deterministic, D-Wave systems, heuristic, lowest energy, optimal trading trajectory, quadratic unconstrained binary optimization, quantum annealing, quantum computing, quantum finance, quantum mathematics quantum portfolio optimization, quantum theory, quantum trading systems, qubits, tunneling, uncertainty principle.

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1 INTRODUCTION

THIS FinTech entrepreneurial-research project presents a heuristic learning and applied understanding of the program content over the past two years as a Masters of Science in Financial Engineering student. The approach in this project is to apply a sub-field of econophysics namely Quantum Finance that links the laws of Quantum Mechanics to Quantum Computing by using a specific built quantum computer for optimization problems namely the Quantum Annealer 2000Q, by D-Wave Systems Inc. in Canada [19], in solving two combinatorial portfolio optimization problems known as NP-complete problems. The literate layman reader of this research paper might well know that complex numbers are essentially an extension of real numbers. As such Quantum Finance is essentially an extension of Classical Finance and thus the Quantum Annealer is essentially an extension of a financial Engineer's toolkit. The design is by first an introduction (this section), then by a concise discussion of the background and context, followed by the research problem, the literature review followed by an introduction to Ocean SDK and then two different applications, results and conclusion and recommendations and further research. Originality of this research project is in the form of using the third generation real Quantum computer specifically built for optimization problems and a formulated quantum unconstrained binary optimization algorithm in solving the portfolio optimization problem faster and more reliably by comparison to a Hybrid (classical and quantum) [25], [30] solver and the benchmark portfolio. Existing research from Quantum Cognition and Quantum Game Theory [15], [6], is applied to the 'decision making' process. This is the quantum equivalent to behavioral finance and probably more suitable to a global economy of new world views and a new world order [6]. The aim is to describe the motivation, objectives, methods, results and enlightenment of the research on quantum combinatorial portfolio optimization in a complex financial Engineering FinTech context.

The body of the literature review is divided into subsections by themes to present, analyze, evaluate and synthesize the existing research and other scholarly literature on;

- classical vs. quantum portfolio optimization,
- classical vs quantum computing,
- classical finance vs. behavioral finance vs. quantum finance, including applied ideas from quantum cognition and a hybrid classical-quantum annealing [25], [30] trading system. A probable limitation to the literature review chapter is that it is not very concise and contribures a larger precentage to this project however, it is the first literature review completed on multiple themes, to clearly understand the content and to consisely conclude on the project.

There is predominantly a focus on answering the following three questions in every subsection of the literature review.

- Are there any patterns that have come to light in the literature?
- What are the central themes and categories used by the researchers?
- What are the variables of interest (especially in the case of quantitative research)?

The literature review conclusion, summarize and form a brief overview outlining the layout of the chapter.

1.1 Background and Context

This research was inspired by collaborative research in Abu Dhabi, United Arab Emirates [26]. A brief chronological background and context of the origin of Quantum Finance that lead to this paper. A Greek philosopher named Democritus (c. 460–370 B.C.) first proposed the concept of atoms [1]. However, Dr. Albert Einstein made famous the term 'Heuristic' in Quantum Theory in his Noble prize winning paper of 1905 [2], as a third scientist to have helped invent the concept of Quantum Mechanics after Dr. Max Plank and Dr. Erwin Schrödinger [3]. Dr. Werner Heisenberg with his 'uncertainty principle' is known to be the fourth physicist [58]. Research then identified that a Pakistani mathematician Dr. Asghar Qadir, introduced Quantum Economics in his 1978 paper with the same title [4]. Dr. Samuelson in 1979, asked if a quantum economics co-ordinating entrepreneur is worth his profit?'[46]. Today, more than a century on from first discovering Quantum Mechanics, Quantum Financial Engineers and FinTech entrepreneurs, are able to heuristic financial Engineer beyond professor Markowitz' famous 'mean-variance' portfolio optimization model [5]. The project explores a quantum mechanics and quantum computing approach to modelling combinatorial portfolio optimization within our current era of a diminishing Moore's Law in classical computing with a continuous increase in the amount of financial data and a clear transition from old world views to the new world views or quantum world views [6], and then by using the results, back-test the results in a hybrid classical-quantum annealing [25], [30] trading system with the out-of-sample data (real trading data) and present the findings and conclusions from the perspective of a 'Quantum Economic person' [7], [9] rather than the perspective that of an always rational, economic man [8].

1.2 Problem Statement

Finding the optimal portfolio is critically important [20], [21] for FinTech entrepreneurial financial Engineers. Numerous research papers have investigated strategies and approaches to portfolio optimization [12], [16]. Dr Markowitz classical approach to portfolio optimization is already well-established in the literature [5].

With a diminishing Moore's law in terms of computational speed, problem-solving capacity and efficiency in information processing and an ever increase in data [10], the existing research is inadequate for the new world era [50], [9], because it is derived from classical physics and succumb to the field's limitations of deterministic [9]. An increasing amount of portfolios therefore finds themselves trapped in the local minima or in the plateau and never reach the global minima fast [22], [23].

1.2.1 Research objectives (RO).

RO1: identify Quantum portfolio optimization strategies and approaches utilized by Quantum hardware and Quantum software companies in North America.

RO2: apply the identified strategies and approaches to the real 2000Q and evaluate the effectiveness of these strategies and approaches in finding the best risk/return combination given portfolio with specific constraint.

RO3: compare and contrast these applied strategies and approaches in terms of their strengths and weaknesses against the benchmark.

1.2.2 Research questions

This research will cover Quantum Machine Learning with the third generation Quantum Annealer from D-Wave Systems Inc. namely D-Wave Quantum Annealer 2000Q using Python and specific built D-Wave open source Ocean Software Development Kit [61] and for the Hybrid annealer OpenJIJ [60]. This research will cover some applied ideas and methodologies to finance from Quantum Mathematics [46], Quantum Statistics [46], Quantum Mechanics [3], Quantum Computing [20], [26], Quantum Cognition [6] known as Quantum Economics and Finance [9]. The research will cover Quadratic Unconstrained Binary Optimization [24] and how this all can be applied to a back-test and a real life data-optimal portfolio trading system without assuming any trading costs, and then conclude on the findings followed by recommendations and future research.

This research will not cover a very technical, deep dive into the functionality of the hardware and software of the specific purpose built Quantum Annealers [11], [12]. Neither will it extensively cover all aspects of a trading system and statistical measurements thereof. It will also not cover all aspects of a FinTech entrepreneurial venture nor create a full business plan for an entrepreneurial service venture. The author understands that the research is about applying quantum computers to classical problems, while quantum economics [47] are about applying quantum ideas to the financial system itself. Therefore, this research will not cover a deep dive into Quantum Economics for Finance [9].

1.2.3 Research contribution

This research, an extension of the current literature, will contribute to the novel and important body of knowledge in Quantum Finance on Quantum combinatorial portfolio optimization by FinTech entrepreneurial financial Engineers, by incorporating a sub-field of econophysics namely Quantum Finance that links the laws of Quantum Mechanics to Quantum Computing by using a specific built quantum computer for optimization problems, namely the D-Wave Annealer 2000Q and its specific Ocean Software Development Kit and the Python OOP language alongside relevant libraries.This will help address the current research gap in this area and provide real-world value to Quantum FinTech financial Engineering entrepreneurs operating in such dynamic environments.

1.2.4 Research limitations

• Consiseness – the literature review chapter is not very concise. The word ‘heuristic’ however, is found in the title and describes the research and learning process necessary for both author and the Quantum Annealer. It is the author’s first literature review on the specific themes in one project and therefore the literature review contributes a rather large percentage to the overall project.

• Scope – the focus is very narrow in the approach and methodologies and doesn’t consider all improvements in deep learning and reinforcement learning approaches to portfolio optimization.

• Research methodology - the qualitative methodology might be criticized for being overly subjective, and the quantitative methodology criticized for oversimplifying the situation by not being able to increase the number of assets in the portfolio over 65 stocks.

• Resources - a lack of time (part time research), money (budget constraint, as of April 2019, the D-Wave machine can be used for free for 1 minute per month) and at times access to equipment due to country wide power outages known as load-shedding and personal research experience.

• Generalizability of the results - the results from the study of the Quantum Finance specific industry can’t necessarily be generalized to other industries.

2 LITERATURE REVIEW

2.1 Introduction to Literature Review

The purpose with this literature review is to address the problem statement within an identified Quantum FinTech entrepreneurial literature gap in moving beyond only classical physics and methodologies and limitations of classical computing, to Quantum mechanics and quantum computing in solving the pressing combinatorial portfolio optimization problem of financial Engineers, faster and more reliably than currently possible. Draw conclusions as part of the decision making process from the perspective of Quantum Cognition and Quantum Game Theory [15], [40], [41] by means of a 'Quantum person' [9] with the underlying principle of quantum physics namely, the 'uncertainty principle' [58] ('We-mode rather than I-mode'). This is the quantum equivalent to behavioral finance and probably more suitable to a global economy of new world views and of a new world order [6]. The meaning of the word 'Heurisristics' found in the title is clearly evident of the literature review and to some extent explain the reason behind the different themes and the length of this chapter. Trusting that the readers will understand the reasoning.

2.2 Classical Portfolio Optimization vs. Quantum Portfolio Optimization

The literature on classical portfolio optimization is well established and currently identify clear patterns that have come to light and are to a large extent in agreement with that of Dr. Markowitz [5]. His proposed technique known simply as 'mean-variance' [13] is used extensively during classical portfolio optimization since it was first introduced. There are numerous biases in the research literature however, a few patterns emerge in the following three biases; Overfitting (also known as over-optimization or curve fitting) in trading, P-hacking and Look ahead bias. In addressing overfitting bias, it seems that the important thing to remember to avoid overfitting in that the past can never predict the future in the financial world. No one can. Strategies that have been adopted too closely to the past data won't be flexible enough to adapt to the future too.

Central themes and categories on classical portfolio optimization are the proposed quantitative framework to achieve portfolio diversification as first described by Dr. Harry Markowitz [5]. With a fundamental objective to significantly reduce specific risks, optimally allocate wealth to different assets, and to determine asset's risk-return trade-offs [13] between expected returns and volatilities, and ends with selecting an efficient portfolio. The portfolio's performance evaluation is an integral part of the modern portfolio management process. Many techniques and models were developed and applied to evaluate the performance based on the portfolio's return-risk characteristics.

Recent literature is divided or at least in disagreement in the effectiveness and in the constraints of the algorithm. Therefore, there are contesting views, mainly identifying that there are inherent limitations to classical portfolio optimization methodologies which have lead to an increase in literature on supervised and unsupervised and reinforcement learning computing techniques in portfolio optimization (more on this in the section-paragraph "Classical computing vs. Quantum Computing).

By further identifying what gaps exist in the current research and how does these inform the research project. The literature on Quantum Portfolio Optimization is currently not well established and can identify clear patterns that have come to light and are to a large extent in agreement; are that specific build quantum computers with quantum variables could very likely handle portfolio optimization problems quicker and better than the traditional or classical methods. The research fits into this existing body of literature on Quantum Portfolio Optimization by means of a specific purpose build quantum computer i.e. D-Wave Quantum 2000Q. Central themes and categories on quantum portfolio optimization are also in agreement with the quantum technology and techniques of Quantum Mathematics [46] and Quantum Mechanics [3] used in the specific purpose Quantum Annealer by D-Wave Systems Inc. in Canada [20]. With their help problems could be solved that are still far too complex for the "supercomputers" used today in finance, physics, biology, weather research and elsewhere. Even with a small number of variants, classical computers quickly drop out when calculating optimal solutions. The literature is divided and in disagreement with the effectiveness in real life optimization problems and in the constraints on the current quantum computer technology. Therefore, there exist contesting views, including a 'reverse quantum annealing method' where the authors claim that the reverse method is quicker in finding the global minima [20].

Variables of interest in portfolio optimization are; the literature defines the problem as a mathematical model with decision variables to minimize or maximize an objective function (cost/profit) and constraints to model real-world complexities. Solving the mathematical model would give the optimal decisions. While one can find optimal solutions for problems that can be abstracted as mathematical models, optimization struggles when the problem complexity cannot be abstracted in a mathematical model, or the problem size is large where one can never solve the model to optimality. Furthermore, the Ising model and QUBO, quantum unconstrained binary optimization. The quantum annealer is designed to handle specific optimization problems, including Ising model and Quadratic Unconstrained Binary Optimization (QUBO) models. Therefore, whenever it is required to embed a problem into D-Wave, need to translate it into a QUBO model. This is not an easy task, and many studies have been conducted on these reductions.

The originality in this research is in the form of using a real Quantum computer specifically built for optimization problems and a quantum unconstrained binary optimization algorithm in solving the portfolio optimization. I.e. testing models on simulators and commercial quantum annealers. The focus is on analyzing the potential of D-Wave through firsthand experience rather than seeking to improve the model. Furthermore, by introducing the idea of a Quantum person ('we-mode' rather than the classical 'I-mode' known as 'homo economicus') in the decision making in quantum portfolio optimization. Which could aid in drawing conclusions in portfolio optimization and in the theoretical entrepreneurial case study within an era of transitioning world order visible in global markets. This brief introduction could furthermore aid the high frequency traders (HFT algorithmic traders also known as robot traders) in causing less to none events like the Flash Crash of 2010 which could very likely aid an entrepreneurial economy at large.

2.3 Classical Computing vs. Quantum Computing

Research in computing from classical physics described here as classical computing is well established and highlights an increasing pattern in supervised and reinforcement learning tools in use today. This also tends to be the central themes and categories more recently used by researchers. An increasing number of research identified that Moore's law may be obsolete within a few short years [10]. Challenged by the physical limit of the size of the transistor silicon microchips. The advances in computing power may become slower than witnessed in the past 50 years. There are alternative solutions in pursuit of breaking through such barriers. For example, the use of the Graphical Processing Unit (GPU) and the development of the Tensor Processing Unit (TPU) has accelerated the processing speed in deep learning [21]. One of the most exciting candidates that could cause a paradigm shift would be quantum computing (QC) first proposed by Dr. Feynman and Dr. Manin in the 1980s[11], [12].

This is a gap in the current research, not only in financial Engineering and combinatorial portfolio optimization, but in optimization as a whole. I.e. everything that requires optimization including our human brains and entrepreneurial ventures too. Therefore, literature on Quantum Mechanic techniques in optimization including the defined problem in portfolio optimization is largely lacking. There are however an increasing number of research in Quantum Portfolio Optimization using a specific built Quantum computer for optimization problems, namely Quantum Annealer. (Please see an entire subsection on its own below to 'Quantum Portfolio Optimization: D-Wave System Inc. Quantum Annealer 2000Q' [11], [12]).

2.2 Classical Finance vs. Behavioural Finance vs. Quantum Finance

Classical finance is very well researched, with clear patterns based on Newtonian physics from which the father of classical economics Dr Adam Smith also based his theories both at Oxford University and when he moved back Scotland. Large patterns identified in classical or traditional finance where investors are rational, i.e. homo economicus [9], [17], [19] are based on Newtonian physics. Dr. Markowitz is also identified as the father of classical finance with Dr. Fama and Modern Portfolio Theory [5], [9] on diversification. Behavioral Finance researcher divided and in disagreement with traditional finance described here as classical finance.

Behavioral Finance research is less well established, however this is not surprising knowing that it was first introduced in the early 1980s and therefore, the literature is lacking accordingly. Clear patterns however, have emerged after the Global Financial Crises of 2008/9 and central themes and categories now also include psychology in finance. The category of psychology has established that investors are 'irrational' frequently falling victim to cognitive biases in their pursuit of what standard finance calls 'rational' wants. Recent research goes further, identifying people as having 'normal' wants and how these, rather than cognitive errors and shortcuts, tend to underlie and influence many aspects of financial behavior [9], [19]. It is evident that research on Behavioral Finance include more quantitative models than that of classical finance with new variables of interest. However, further research identified that this is not enough [9] and therefore new applied and theoretical research emerged, forming the latest category in finance namely, ' Quantum Cognition' which is based on Quantum Mechanics and from which the term 'Quantum Finance' [52], [53], [54]. Classical finance's game theory is in contrasts with the quantum game theory, where, as Alexander Wendt [15] notes, the strategy has 'an irreducibly collective aspect, such that players are at least partly in 'We-mode' rather than just 'I-mode'.

Clear patterns in Quantum Cognition [40] and Quantum Finance [52] indicates it is very different and that it enhances the measurement of 'irrationality' identified by Behavioral Finance with the inclusion of 'irrational numbers' from the underlying Quantum physics and the 'uncertainty principle' by Dr. Werner Heisenberg [58]. As quantum cognition shows, there is more to both mind and markets than classical logic would imply. Today, many 'investors' in corporations are also highly 'rational' and goal-focused because they are not human, they are emotionless computer algorithms of the sort that now dominates much trading in financial securities [9] [14]. Herewith, then are new variables of interest with very different underlying mathematics known as 'Quantum Mathematics' and highly exciting research both applied and theoretical is expected especially in the case of quantitative research. Needless to say then that the research is lacking, however, there is very little divided research and research in disagreement on the underlying Quantum Mathematics and Quantum Mechanics. Even by Dr. Einstein, who has spent many years trying. The research of this project fits into the aforementioned existing body of literature.

2.3 Quantum Portfolio Optimization: D-Wave Quantum Annealer 2000Q

Research that investigates the use of quantum computers for building an optimal portfolio is well established [27], [42], [43]. The machine's quantum nature has been researched for quite some time, mainly because the D-Wave works differently from other quantum computers [27]. Note that quantum annealing can be used in many other domains for which optimization is a sub problem [43]. Patterns that have come to light are that it turns out that the natural distributions generated by quantum annealers can be understood as approximations to the Boltzmann distribution [29]. Central themes and categories include both quantitative and qualitative research however, quantitative research is in the majority.

Divided opinion is that many supervised and reinforcement learning tools are trained via solving optimization problems. More research on quantum reinforcement learning algorithms are required. This area of research is lacking in research papers and is a clear gap. Furthermore, an emerging new research field called Quantum Machine Learning consequently has emerged, and the intensity of research is impressive. Optimizations of all sorts of very complex dynamical systems, new drug discoveries, new autonomous systems and improvements in existing ones are motivations enough, economical or purely scientifically otherwise.

The following paragraph identifies what the literature currently says about the topic. A large body of research ranging between small and larger portfolios exists. Recent patterns in identifying the newest model, the D-Wave 2000Q, is using the computational power of 2000 qubits, which is enabling calculations that would be all but impossible with classical super computers.[7], [8]. However, it shows that the process of quantum annealing doesn't require all 2000 qubits of D-Wave to be entangled. Usually, only a small subset of the 2000 qubits is entangled during quantum annealing. Additionally, quantum annealing only requires neighboring qubits to be entangled. Established research on the sensitive to any disturbances from the environment and therefore on the error correction term exist [38], [39]. The topic of investigation therefore only contributes to the existing application of portfolio optimization using the latest D-Wave 2000Q system (a gap in the research) and not on improving the error correction methods [38] nor the optimization method. The research contributes to originality in the defined research gap in quantum entrepreneurship in the financial sector, firstly identified by Dr. Samuelson in 1979 [46].

2.6 Conclusion on Literature Review

Research especially important to the research questions are on the D-Wave System Inc. latest Quantum Annealer 2000Q [20], [26], [36] and QUBO [24]. However, the background and context better describes the extend required to better understand this research paper and to deliver a concise conclusion chapter.

The research gaps that exist in the literature are specific to the problem statement. Application to the latest specific build quantum computer namely, D-Wave Quantum Annealer 2000Q in solving portfolio optimization and to draw conclusions from a perspective of Quantum Cognition. Thus determine whether a Quantum FinTech entrepreneur can provide quantum portfolio optimization faster and more reliable as a service to classical financial Engineers.

3 METHODOLOGY

This project takes both a qualitative (~25%) and quantitative (~75%) approach. The design is structured in analyzing the potential of D-Wave through firsthand experience rather than seeking to improve the model by firstly introducing the specific Ocean SDK software and by conducting a project using a modelling approach from a well known research paper [26], followed by longitudinal study of randomly selected, multi-year period, London Stock Exchange market data using API technology by Yahoo Finance (secondary data). In both approaches the use of the specific Quantum Annealer software developers kit, namely ‘Ocean’ from D-Waves Systems and the Python object oriented programming language with relevant libraries are applied. During the Hybrid approach the openJIJ software is introduced.

- Introduce the D-Wave Ocean SDK through the number problem by formatting QUBO.
- First aporoach: Compare the D-Wave with existing algorithm from a well known paper [26] to the bruit force method.
- Second approach: Define the market data and import historical stock price data: The time series prediction is based on the historical data (secondary data) between July 2020 and April 2022. The period to predict is May 2022 till late July 2022, which is the same period of the data in test set. Perform a detailed EDA. (The benchmark portfolio consists of all 20 stocks and is tabulated in chapter 4.4.3)
- The predicted values are transformed to be the average return rate for each stock. Used it for one of the objectives to maximise the sum of return rates.
- Prepare the data by cleansing and applying feature Engineering.
- Calculate actual rate of return from the historical stock price data in both train and test sets. Use the results for the calculation of covariance matrix that will be used for the second objective function which is to reduce the risk by diversifying the portfolio

- In portfolio optimisation one constraint is assumed: the maximum number of stocks to be included in the portfolio.
- Use PyQubo, a python library that that can convert the cost function to a quadratic unconstraint binary optimisation matrix that can be sent to D-wave qunatum annealer and openJIJ simulated quantum annealer.
- Define and formulate the cost functions and constraints to obtain Ising and a QUBO matrix. Set two cost functions to optimise the portfolio. One is to maximise the sum of predicted growth rates, predicted in the feature Engineering section. Another is to minimise the covariance between each of the stock items to be selected in the portfolio. Then add up two cost functions for QUBO.
- Sample optimization results from D-Wave Quantum Annealer: Install D-Wave's Ocean SDK to request the computation to the quantum annealer. *EmbeddingComposite* is used to embed the QUBO to the physical quantum annealer in D-Wave.
- Sample optimization results from a simulated quantum annealing solver. D-Wave has hybrid annealer that uses classical computer and quantum annealer. The '*hybrid_binary_quadratic_model_version2*' can have up to 1,000,000 variables. Use the hybrid annealer by importing *LeapHybridSampler* library.
- Important Libraries; DWave-Ocean SDK, LeapHybridSampler, YFinance from Yahoo Finance, Pandas, Numpy, Prophet, PyQubo, OpenJIJ, SQASampler
- Evaluate the outcomes. It is important to see how these findings reflect in the hybrid classical-quantum trading system, whether or not they explain the existing or lead to any new effects.
- Draw conclusions from the perspective of a 'Quantum Economic person' from Quantum Cognition and Quantum Game Theory (We-mode), rather than from the perspective of a 'homo economicus' or always rational behaving classical economic man's (I-mode) [15].
- Present the summary and drawn conclusions on the Quantum Annealers and hybrid classical-quantum [25], [30] trading system.

4 Results

4.1 Introduction to D-Wave Ocean SDK

D-Wave Ocean SDK (Ocean) is a collection of software that uses D-Wave Machines. It is easy to use various features from Python.

In this section, the following through the number division problem is discussed.

- How to create QUBO and Ising models
- How to use Ocean SDK
- Optimization for different annealing parameters

4.1.1 Formulation

There are two formats that can be run on D-Wave machine: Ising model and QUBO. This paper won't get into the details here, will describe the minimum requirements for using Ocean.

QUBO (Quadratic Unconstrained Binary Optimization)

In the case of QUBO, consider the following Hamiltonian for binary variable qi∈{0,1}.

(1)

$$H(\{q_i\}) = \sum_i Q_{ii} q_i + \sum_{i>j} Q_{ij} q_i q_j$$

In solving QUBO, the classical computer calculates Qii and Qij in the above equation. The next step is to run on the D-Wave machine.

Ising model

In the case of the Ising model, consider the following Hamiltonian for the spin variable σi∈{1,−1}.

$$H(\{\sigma_i\}) = \sum_i h_i \sigma_i + \sum_{i>j} J_{ij} \sigma_i \sigma_j$$

(2)

In solving the Ising model, the classical computer calculates h_i と J_{ij} in the above equation. The next step is to run on the D-Wave machine.

Reciprocal conversion

QUBO and the Ising model can be converted to each other by the conversion formula

$$q_i = \frac{sigma_i+1}{2} \quad (3)$$

It doesn't matter which way the Hamiltonian is formulated, can choose the one that is easier to consider from the subject matter. However, it should be noted that the D-Wave machine may behave differently for two different forms of the same problem.

Number division problem

The problem of dividing each group so that the sum of each group is the same is called a number division problem.

For example, divide the following set of integers into two groups.

C = [2, 10, 3, 8, 5, 7, 9, 5, 3, 2]

Here are 23 optimal solutions where the difference between the sum of the two is zero. There are multiple states in which the energy, called the difference in sums, is the same value, but the solutions are different. These states are called "degenerate" in physical terms. For example, an optimal solution is {2, 5, 3, 10, 7}, {2, 5, 3, 9, 8} (both sum to 27).

QUBO from number division problem

Create the formulation with the goal of making the difference between the sum of the two groups equal to zero.

(Sum of the numbers of group A) - (Sum of the numbers of group B) = 0

Therefore, binary variables are required and defined as the following; if the number belongs to group A (1) and if it belongs to group B (0).

Then by combining the sum of group A and the sum of

$$\sum_{i=1}^{N} c_i q_i - \sum_{i=1}^{N} c_i (1 - q_i) = 0$$

group B to obtain,

Obtain, (4)

The fine method is used as a way of expressing such an equality constraint (Appendix B). Therefore, QUBO for the number division problem is as follows.

$$H(\{q_i\}) = \left(\sum_{i=1}^{N} c_i q_i - \sum_{i=1}^{N} c_i (1 - q_i) \right)^2$$

(5)

In order to perform the calculation on the D-Wave machine, the value of Q_{ij} must be found and expanded and the $H(\{sigma_i\})$ organized.

$$H(\{q_i\}) = \left(2\sum_{i=1}^{N} c_i q_i - \sum_{i=1}^{N} c_i \right)^2 = 4 \sum_{i=1}^{N} c_i \left(c_i - \sum_{j=1}^{N} c_j \right) q_i + 8 \sum_{i=1}^{N} \sum_{j=i+1}^{N} c_i c_j q_i q_j + \left(\sum_{i=1}^{N} c_i \right)^2$$

(6)

Therefore,

$$Q_{ii} = 4c_i \left(c_i - \sum_{j=1}^{N} c_j \right)$$
$$Q_{ij} = 8c_i c_j$$

(7)

The last term is a constant that does not depends on $\{q_i\}$ and can be ignored.

Furthermore, the Ocean SDK basic library, dimod *.BinaryQuadraticModel()* allows the Ising model and

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QUBO's interconversion. Therefore, the research does not cover a deeper dive into the mathematics of the Ising model. Through the optimization of number division problem the D-Wave Ocean SDK is introduced. A lot can be done from here: For example,

- success probability (percentage of optimal solutions) relative to the number of readings,
- annealing time
- energy relative to the number of readings, annealing time
- the percentage of optimal solutions obtained relative to multiple optimal solutions.

There are other annealing parameters that are not mentioned here, mainly due to budget constraints. For example, by specifying an annealing schedule, future research can perform different techniques from normal annealing, such as reverse annealing [35]. (Please also see the Future Research chapter)

4.2. D-Wave Ocean SDK with 2000Q: First Approach (Algorithm Modelling)

Conduct a project using a modelling approach from the paper where Nada Elsokkary et al. [26] formulated a simple model based on expected return, covariance, and budget. This model allows the modeler to generate high return / high-risk portfolios and diversified, more secure ones when given a specific budget. The focus is on analyzing the potential of D-Wave through firsthand experience rather than seeking to improve the model.

As mentioned in the literature review, the D-Wave handles a specific class of problems well, and therefore finding a competent translation of the problem is essential. Once the model is created, it must be tested. Validated whether the program was working by comparing the performance with a classical brute force method for a low number of assets (up to 25). Once assured that the results provided by D-Wave hit the optimal solution provided by the brute force, the problem is scaled. In addition to formulating a problem as a QUBO model, the optimization of parameters is vital for good performance. The first item that is essential to understand is that the quantum bits ("qubits") from D-Wave are different from the qubits of gate-based computers.

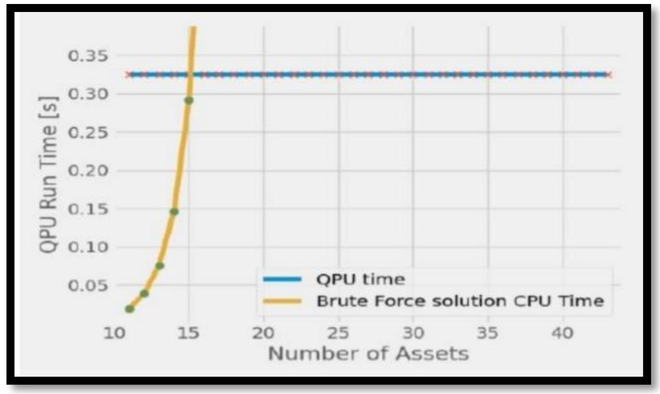

Figure 1: Screenshot: compares the time used from D- Wave to solve the optimization compared to brute force. Can clearly see the exponential separation as the number of assets grows.

A qubit is the fundamental unit of information in both systems, but in the gate-based computer the manipulation of these units using a set of universal gates is possible, whereas the quantum annealer does not have this option. Instead, modify the environment and let the system find the ground state. The advantage that this brings is the possibility of exploiting a phenomenon called quantum tunneling. Imagine a landscape with hills and holes, and every time a computation is performed, it is like dropping a ball into the landscape and seeing where the ball stops. Naturally, expect to find the ball in the deepest hole (the ground state), but the advantage over classical annealing lies in the fact that the ball can "dig tunnels" between the hills while searching for the deepest hole, thus increasing the probability of hitting the ground state.

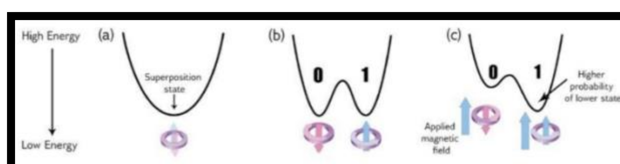

Figure 2: Tunneling effect. Courtesy of D-Wave Systems Inc. [33]

In Figure 2 the D-Wave process: start from a very simple landscape (a) and modify it through the application of magnetic files. In (b) if imagine dropping a ball from the middle, end up with a 50% probability in both holes. In fact, both holes are in a minimum energy state for the problem. On the other hand, in (c) expect the ball to fall in the deepest hole, representing "1" and this will indeed happen with higher probability.

The D-Wave is a probabilistic machine, experiments are executed a set number of times (this is a configurable parameter), and the outcome with the highest probability is the solution to the problem.

The quantum annealer is designed to handle specific optimization problems, including Ising and Quadratic Unconstrained Binary Optimization (QUBO) models. Therefore, need to translate it into a QUBO model whenever want to embed a problem into D-Wave. This is not an easy task, and many studies have been conducted on these reductions [20], [24].

A deeper look into the algorithm of the model [26],

$$\text{Minimize}: \theta_0 \sum_i (-\alpha_i E(R_i)) + \theta_1 \sum_i \sum_j \alpha_i \alpha_j \operatorname{Cov}(R_i, R_j) + \theta_2 \left(\sum_i \alpha_i A_i - B \right)^2$$

$E(R_i)$ is the expected return of the stock R_i,given by the data, e.g. Nasdaq data.

$Cov(R_i, R_j)$ is the covariance between stock R_i and R_j, representing the diversification of the portfolio

A_i is the cost of the stock R_i

B is the total budget for investment

$Theta_i$ is a configurable parameter that controls the weighting of the different parts of the optimization

$Alpha_i$ are binary values (0,1) that indicate whether a given stock is purchased or not.

Inputs to the model are the covariance matrix, the expected return of each stock, together with the price and budget. Expected output is a binary string of 0's and 1's indicating whether to buy or not buy a given stock.

The theta parameters are there to tune the optimization, and they sum to 1. In particular:

$Theta_0$ controls how much the expected return should play a role in the model.

$Theta_1$ controls the diversification of the portfolio

$Theta_2$ controls how tightly stick to the budget

Note that the number of possible combinations of the portfolio grows exponentially with the number of assets (2^x, where x is the number of assets), and the brute force model becomes computationally intractable very quickly. The tasks required for implementation were adapting the model into a QUBO formulation and the tuning of the D-Wave parameters, for example, for annealing time and chain strength.

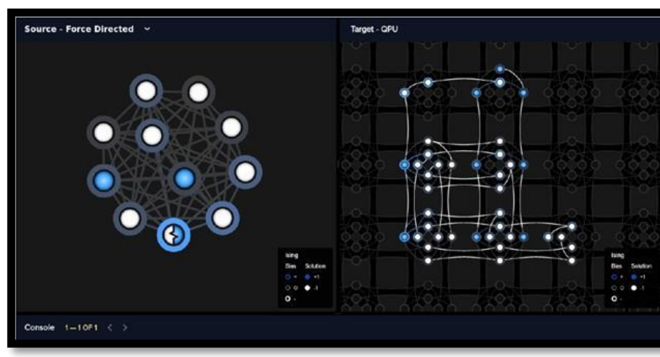

FIGURE 3: EMBEDDING OF 11 ASSETS, SCREENSHOT: COURTESY D-WAVE SYSTEMS INC.

Embedding a problem into D-Wave means connecting different qubits and finding the configuration with the lowest energy state. The architecture of the chip, though, only permits a few direct connections. When direct connections are not possible, the user can create chains of physical qubit that will be considered a logical qubit. Expect the chain to have the same color at the end of the annealing. If the chain "breaks" the solution is not reliable. Figure 3 is of an embedding of 11 assets into the D-Wave machine through the Ocean SDK. On the left, observe a chain break, which means that the problem solved by the machine is different from the one proposed. This figure

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aims to represent problem interconnection on the left and the actual embedding on the chip on the right.

This research checks the solution's validity by comparison with the results obtained by D-Wave with a classical, brute force, check for solutions up to 25 stocks. The global minima was found by D-Wave, confirming the validity of the simulator. However, as the problem grew, it is required to use bigger chains and a drop in the quality of the solution is experienced. Even though the model worked, it failed to find a good solution and started to collapse to a normal distribution. The next model is the Zephyr topology, with about 7,000 qubits, expected in 2023 or 2024 with an expected qubit connectivity of 20 up from 15 [31] and could improve on the performance of this result.

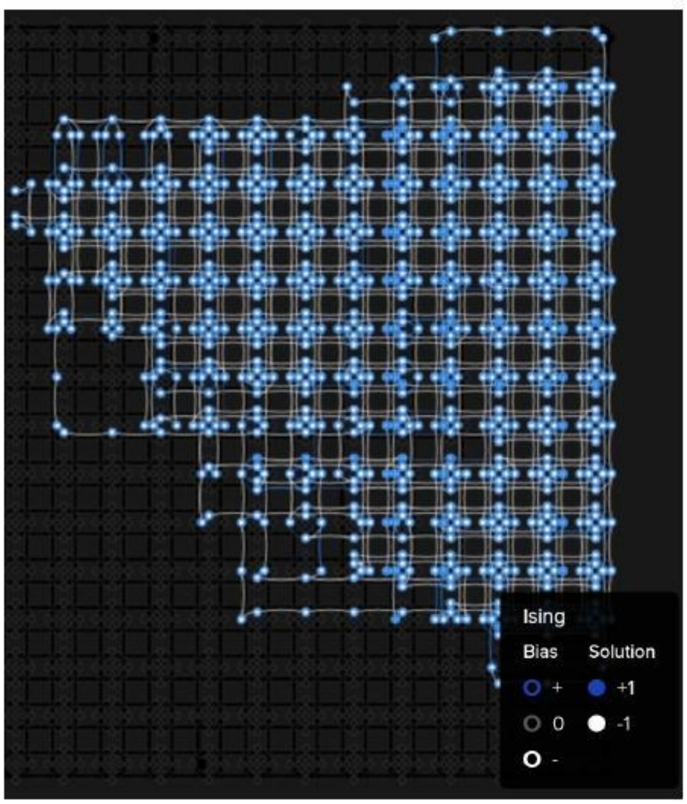

Figure 4: Large embedding of 65 stocks, Screenshot: Courtesy of D-Wave Systems Inc.

Figure 4 represents a large embedding of stocks to represent the optimization of 65 assets and use a large number of the qubits available in the chip. This is due to the interconnectivity.

The D-Wave 2000Q, is using the computational power of 2000 qubits, which is enabling calculations that would be all but impossible with classical super computers.[7], [8]. However, research shows that the process of quantum annealing doesn't require all 2000 qubits of D-Wave to be entangled. Usually, only a small subset of the 2000 qubits are entangled during quantum annealing. Additionally, quantum annealing only requires neighboring qubits to be entangled. Established research on the sensitive to any disturbances from the environment and therefore on the error correction term exist [38].

4.3 D-Wave Quantum Annealer: Second Approach

Address the combinatorial quantum optimization required for the hybrid annealing. Twenty randomly selected stocks from the London Stock Exchange, United Kingdom index, LSE100, are extracted with their respective historical closing prices for a period of two years from July 2020 to July 2022. The Train dataset consist of random data between '2020-07-01' to '2022-04-30' and the Test dataset between '2022-05-01' to '2022-07-24'. The basket of stocks for the portfolio is tabulated in chapter 4.4.3.

4.3.1 Exploratory Data Analysis

EDA is an important step to first understand the data (identify the trends and patterns within the data, detect outliers or anomalous events, find interesting relations among the variables, points of interest, etc.) before using them for modeling, machine learning, etc. (Appendix A).

	ADM.L	AZN.L	BATS.L	BNZL.L	BRBY
count	57.000000	57.000000	57.000000	57.000000	57.000C
mean	2185.973684	10508.228070	3466.447368	2814.035088	1629.9649
std	173.503948	403.405858	81.319788	146.057225	60.2885
min	1729.000000	9750.000000	3300.000000	2575.000000	1482.000C
25%	2157.000000	10244.000000	3426.000000	2671.000000	1586.000C
50%	2228.000000	10506.000000	3478.000000	2837.000000	1636.500C
75%	2267.000000	10800.000000	3528.500000	2919.000000	1678.500C
max	2561.000000	11232.000000	3628.000000	3108.000000	1743.000C

Table 1: Sample description of Test Dataset, Appendix A

4.3.2 Feature Engineering

The predicted values using the prophet library (Appendix A) are transformed to be the average rate of return for each stock and used for one of the objectives to maximize the sum of the rate of returns. Calculate the actual rate of returns from the historical stock prices and will use the results od the calculation of the covariance that will be used for the second objective function to reduce the risk by diversifying the portfolio. Mean of actual rate of return of the train dataset is 0.00039797518786811187. The calculated mean for the actual rate of return on the test set for the final evaluation is 0.26481830740037615

4.3.3 Formulate the cost functions and constraints to obtain a QUBO matrix.

a) Defining the cost function: Two cost functions to optimize the portfolio. One is to maximize the sum of predicted growth rates, predicted in the feature Engineering section (Appendix A). Another is to minimize the covariance between each of the stock items to be selected in the portfolio. This research then adds up the two cost functions for QUBO. Where PyQubo, a python library, is again used to convert the cost function to a quadratic unconstraint binary optimization matrix. This matrix is then sent to D-wave quantum annealer and openJIJ simulated quantum annealer respectively.

b) Defining the constraint term: one constraint is assumed - the portfolio's allowed stock count.

4.3.4 Prepare QUBO

The problem formulation and representation in the QUBO format are all handled by PyQubo library. The model is compiled using the objective function that needs to be minimized, then define the constraint coefficient. *to_qubo* function generates a QUBO matrix in the dictionary type and an offset value which is a constant value and is not required for the search for the minima.

4.3.5 Sample optimization results from a D-Wave annealer (QA).

D-Wave's Ocean SDK is again used to request the computation to the quantum annealer. *EmbeddingComposite* is used to embed the QUBO to the physical quantum annealer.

Here is the result of 10 samples from the QA. The first list items are the optimized combination of the stock items, the number 1 indicates the stock is included in the portfolio. The second value is the energy state where the

```
[([1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0], -197.99089412,
 ([1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1], -191.98574577,
 ([1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0], -197.99089412,
 ([1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0], -199.99303663,
 ([1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0], -197.99089412,
 ([1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0], -199.9934611 ,
 ([1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0], -199.99389407,
 ([1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1], -199.99067124,
 ([0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 0], -197.99340359,
```

Table 2: 10 Sample from QA, Appendix A

lowest number is the best solution. The number shows close to minus two hundred, because the offset value is not included. Third number is the number of occurrence of the solution. Here 10 reads and each read has unique stock selections. The last number indicate "chain break", the connection between qubits which are broken then fixed by the software. Ideal solution would have a chain break ratio of 0 (Appendix A).

4.3.6 Sample optimization results from a Simulated Quantum annealer (SQA).

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```
[([1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1], -199.99081682,
 ([0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 1, 1, 0], -199.99174622,
 ([1, 0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 1], -199.99030031,
 ([1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1, 0], -199.98966452,
 ([0, 0, 1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1], -199.98753355,
 ([0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1], -199.99044573,
 ([1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0], -199.990648   ,
 ([1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 1, 0], -199.99089416,
 ([0, 1, 1, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 1, 1, 0, 0], -199.98888206,
 ([0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0], -199.99276445,
```

Table 3: 10 Samples from the SQA, Appendix A

There is a publicly available open source QA simulator provided by OpenJIJ. It can be used as an alternative to the real quantum annealer and use the same QUBO prepared by PyQubo (Appendix A).

Above is the result of 10 samples from the SQA. The first list items are the optimized combination of the stock items, the number 1 indicates the stock is included in the portfolio. The second value is the energy state where the lowest number is the best solution. The number shows close to minus two hundred, because the offset value is not included. Third number is the number of occurrences of the solution. Here 10 reads and each read has unique stock selections. The last number indicates "chain break", the connection between qubits which are broken then fixed by the software. The ideal solution would have a chain break ratio of 0.

By selecting the best portfolio selection from QA and SQA and the Hybrid Classical-Quantum samples and compare with the benchmark portfolio of all 20 stocks' with cost function 1 of 2.648 and cost function 2 of 804.0759 respectively (Appendix A).

Quantum Annealer (QA):

Best portfolio QA.

['FLTR.L', 'BNZL.L', 'ULVR.L', 'CCH.L', 'BRBY.L', 'SSE.L', 'REL.L', 'DCC.L', 'BATS.L', 'IHG.L']

Best combination from QA

[1 0 0 1 0 0 1 0 1 1 1 1 0 1 0 0 0 0 1 1]

Cost function 1: increases the return factor by Quantum Annealer -5.295

Cost function 2: decrease the risk factor by Quantum Annealer 1605.65

Simulated Quantum Annealer (SQA):

Best portfolio SQA

['SHEL.L', 'CCH.L', 'BRBY.L', 'SVT.L', 'LSEG.L', 'NXT.L', 'DCC.L', 'GSK.L', 'SDR.L', 'AZN.L']

Best combination from SQA hybdrid annealing

[0 1 1 0 0 1 0 1 1 1 0 0 0 1 1 1 0 1 0 0]

Cost function 1: increases the return factor by Simulated Quantum Annealer -1.78

Cost function 2: decrease the risk factor by Simulated Quantum Annealer 180.316

Hybrid-QA
Best combination from QA hybrid
[1 1 0 0 0 1 0 1 1 1 0 1 1 1 1 0 0 0 0 0]

Cost function 1: Return rate factor by QA Hybrid
-1.77
Cost function 2: lowering the risk.180.41

With comparison to the benchmark, I can observe sample records from QA/SQA/Hybrid annealing as follows:

• QA showed high-return but high-risks.

• SQA recommended slightly lower return but minimum risk

• Hybrid showed very similar costs for both return and risk to the SQA

4.4. Conclusion on Results

4.4.1 Introduction to Ocean SDK

The results align with the existing research covered in the literature review chapter. Introduced D-Wave Ocean SDK through the optimization of the number division problem. Then investigate the percentage of optimal solutions relative to the number of readings, annealing time, energy relative to the number of readings, annealing time, the percentage of optimal solutions obtained relative to multiple optimal solutions.

The visualization of QUBO in Figure 5 and the Ising Model in Figure 6 respectively to understand any target issues such as degree of coupling and coefficient ratios.
The diagonal component represents Qii and the non-diagonal component represents Qij. It turns out that the number division problem is an all-coupling problem, where there is an interaction between all the qubits.

Expanded the constructed Hamiltonian to find the coefficient matrices such as Q_{ii} and Q_{ij} and J_{ii} and J_{ij}. However, when dealing with more complex Hamiltonian, manual calculations are more complicated and may result in errors. Therefore, the use of PyQUBO, domain-specific language developed by Recruit Communications and

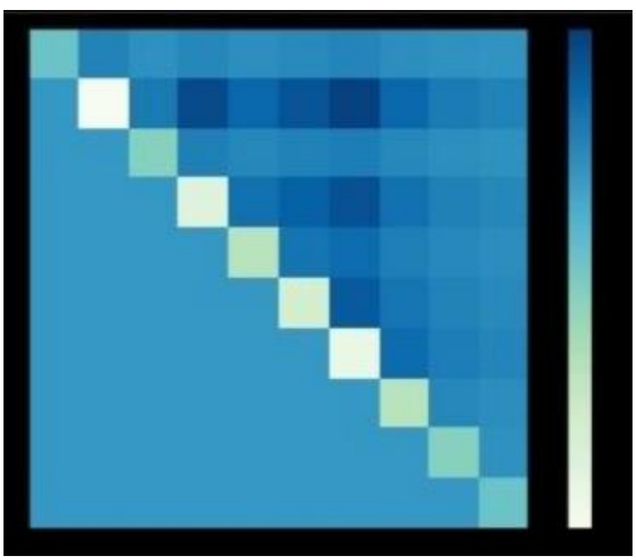

Figure 5: QUBO – Correlation, Appendix A

create the QUBO and Ising models without having to expand the Hamiltonian [57].

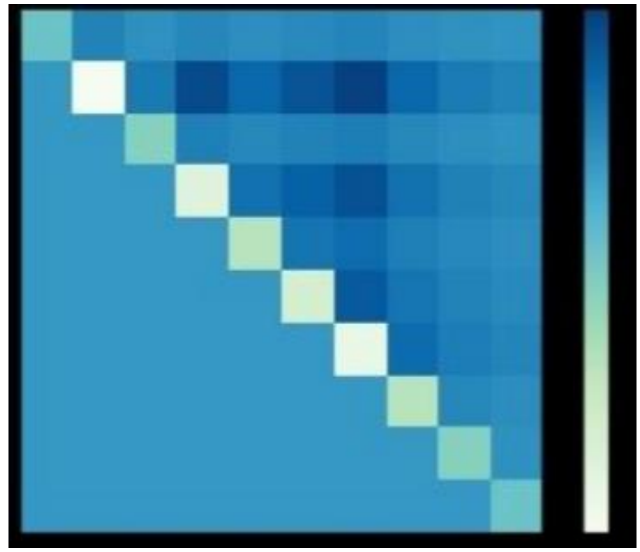

Figure 6: Ising Model – Correlation, Appendix A

4.4.2. D-Wave Ocean SDK with 2000Q: First Approach (Algorithm Modelling)

The minimization algorithm of a well known paper [26] is applied to the D-Wave Quantum Annealer 2000Q by comparison with a brute force method. Once the results provided by 2000Q hit on the optimal solution provided by the brute force method, the problem is scaled up and followed with a discussion of the Quantum phenomenon known as tunneling. First the minimization algorithm used in [26] is defined and where the inputs to the model is the expected return of each stock, together with the price and budget from the covariance matrix. The output is a string of 0's and 1's indicating whether to buy or not buy a given stock. The results are not at odds with the previous research [26]. Note that the number of possible combinations of the portfolio grows exponentially with the number of assets (2^x, where x is the number of assets), and the brute force model becomes computationally intractable very quickly and moved on to the embedding. As a mathematical model with decision variables to minimize or maximize an objective function (cost/profit) and constraints to model real-world complexities. Solving the mathematical model would give the optimal decisions. While one can find optimal solutions for problems that can be abstracted as mathematical models, optimization struggles when the

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problem complexity cannot be abstracted in a mathematical model, or the problem size is large where you can never solve the model to optimality. Figure 3 showed an embedding of 11 assets into the D-Wave machine through the Ocean SDK. On the left, a chain break is visable, which means that the problem solved by the machine is different from the one proposed. This figure aims to represent problem interconnection on the left and the actual embedding on the chip on the right. Next, this research checked the solution's validity, by comparison of the results obtained by D-Wave with a classical check, brute force, for solutions up to 25 stocks. The global minima was found fast, by D-Wave 2000Q, confirming the validity of the simulator and the research question. However, as the problem grew, this research required to use bigger chains and experienced a drop in the quality of the solution. Even though the model worked, it failed to find a good solution and started to collapse to a normal distribution. The next model is the Zephyr topology, with improvements in the hardware of about 7,000 qubits, expected in 2023 or 2024 and could, with a high probability, improve on the performance of this paper and the research [26] with an expected qubit connectivity of 20 up from 15 [31] addressing the current limits to scalability.

4.4.3. D-Wave Ocean SDK with 2000Q: Second Approach

The results obtained by the second approach also confirmed the research question and is in alignment with previous research discussed in the literature review chapter. The predicted values' mean rate of return 0.000139839. The mean of actual rates of return 0.0003979 used in calculating the covariance for the second objective function to reduce the risk by diversifying the portfolio in alignment with Modern Portfolio Theory [5]. The mean of actual rate of return for the test set is 0.26. The benchmark portfolio's cost function 1 is 2.648 and cost function 2 is 804.075 respectively.

The benchmark portfolio consists of all 20 company stocks randomly chosen from the London Stock Exchange.

No.| Ticker Symbol | Name of Stock

1. IHG.L INTERCONTINENTAL HOTELS GROUP PLC
2. SDR.L SCHRODERS PLC
3. SVT.L SEVERN TRENT PLC
4. BATS.L BRITISH AMERICAN TOBACCO PLC
5. LSEG.L LONDON STOCK EXCHANGE GROUP PLC
6. EXPN.L EXPERIAN PLC
7. ADM.L ADMIRAL GROUP PLC
8. BNZL.L BUNZL PLC
9. SHEL.L SHELL PLC
10. GSK.L GSK PLC ORD
11. CPG.L COMPASS GROUP PLC
12. CCH.L COCA-COLA HBC A
13. AZN.L ASTRAZENECA PLC
14. NXT.L NEXT PLC
15. DCC.L DCC PLC ORD
16. ULVR.L UNILEVER PLC
17. REL.L RELX PLC
18. BRBY.L BURBERRY GROUP PLC
19. SSE.L SSE PLC
20. FLTR.L FLUTTER ENTERTAINMENT PLC

By selecting the best portfolio selection from QA and SQA and the Hybrid Classical-Quantum samples.

Quantum Annealer (QA):

Best portfolio QA.

['FLTR.L', 'BNZL.L', 'ULVR.L', 'CCH.L', 'BRBY.L', 'SSE.L', 'REL.L', 'DCC.L', 'BATS.L', 'IHG.L']

Best combination from QA hybdrid

[1 0 0 1 0 0 1 0 1 1 1 1 0 1 0 0 0 0 1 1]

Cost function 1: increases the return factor by Quantum Annealer -5.3

Cost function 2: decrease the risk factor by Quantum Annealer 1605.65

Simulated Quantum Annealer (SQA):

Best portfolio SQA

['SHEL.L', 'CCH.L', 'BRBY.L', 'SVT.L', 'LSEG.L', 'NXT.L', 'DCC.L', 'GSK.L', 'SDR.L', 'AZN.L']

Best combination from SQA hybdrid-quantum

[0 1 1 0 0 1 0 1 1 1 0 0 0 1 1 1 0 1 0 0]

Cost function 1: increases the return factor by Simulated Quantum Annealer -1.78

Cost function 2: decrease the risk factor by Simulated Quantum Annealer 180.32

Hybrid-QA
Best combination from QA hybrid
[1 1 0 0 0 1 0 1 1 1 0 1 1 1 1 0 0 0 0 0]
Cost function 1: Return rate factor by QA Hybrid
-1.7694680995084255
Cost function 2: lowering the risk.
180.41567501627983

With comparison to the benchmark, the sample records from QA/SQA/Hybrid are:

• QA Annealer showed high-return but high-risks.

• SQA Annealer recommended slightly lower return but minimum risk

• Hybrid Annealer showed very similar costs for both return and risk to the SQA

The main reason for the Back-test (out of sample results) is to check how a certain strategy performed in the past. Thus, it becomes important to consider all the data and all the trading costs. Otherwise, back-testing may show false profitability. The out of sample results (the research paper simple back-test results) are favorable compared to the benchmark portfolio of all 20 stocks and further strategies could be developed and tested with different back-test methodologies. (See Chapter 7: Recommendations and Future research).

With the elements of conventional chips now operating at near-atomic scales and thus the appearing expiration of Moore's Law in terms of computational speed, problem-solving capacity and efficiency in information processing the end of the rope is beginning to show for classical computers. This paper confirms that Quantum annealing is a form of quantum computing that provides a superior approach to optimizing the allocation of resources, costs or time. It could be particularly useful for solving massive optimization problems with robust performance not possible with other computing methods that take far too long for practical implementations. The D-Wave 2000Q machine is an example of this, which is purpose-built for solving large, complex optimization problems using quadratic unconstrained binary optimization (QUBO) models. There are companies other than D-Wave like Fujitsu's so-called Digital Annealer, however, the use of the D-Wave 2000Q is maintained throughout this research capstone project.

6. Conclusions

This research project sets out to explore whether a Quantum FinTech entrepreneur can solve a classical financial Engineer's portfolio optimization problem faster and more reliably with the D-Wave Quantum Annealer 2000Q, unfamiliar with the classical financial Engineer. Given the budget, expertise and time constraints. After obtaining a better understanding of the Ocean SDK software formulation in chapter 4.1 and a comparison between an algorithm from a well known paper and brute force 'classical' model in chapter 4.2. Followed by using Python OOP and Ocean SDK and an open source simulator to obtain combinatorial portfolio optimization results. The research concludes, therefore, that current Quantum annealing helps to speed up the solving NP-complete combinatorial portfolio optimization problems by using a quantum phenomenon known as tunneling. The problem statement is indeed a FinTech entrepreneurial business opportunity and don't have to wait for the third Quantum revolution to conclude this from the results obtained in this project (Appendix C).

As quantum cognition in human decision making described in the literature review chapter shows, there is more to both mind and markets than classical logic would imply. Today, many 'investors' in corporations are also highly 'rational' and goal-focused and thus in 'I-mode' because they are not human, they are emotionless computer algorithms of the sort that now dominates much trading in financial securities. 'We-mode' on the other hand, from Quantum Cognition and Quantum Game

Theory, with the fundamental ‘uncertainty principle’ of Quantum Mechanics. Could be understood as more inclusive of a new world era economy rather than that of the selfish, own interest, classical behavior described above as ‘I-mode’ and could very well be an answer to decision making events like the Enron, the 2010 Flash Crash where eventually one trader received the blame [59] for being more in ‘I-mode’ (please also see the Future Research chapter). The research concludes that in a Quantum era with quantum mechanics and quantum computing and quantum finance that it is more probable that quantum cognition and quantum game theory have a place in decision making in both humans and computer algorithms.

The research did not cover a very technical, deep dive into the functionality of the hardware and software of the specific purpose built Quantum Annealers. Neither did the research extensively cover all aspects of a trading system (back-test) and statistical measurements thereof. Did not cover all aspects of an entrepreneurial venture nor created a full business plan for a FinTech entrepreneurial service venture. The research is about applying quantum computers to classical problems, while quantum economics are about applying quantum ideas to the financial system itself. Therefore, the research did not deep dive into Quantum Economics for Finance.

Practitioners can make use of the research and the new knowledge, it has generated by using the methodology and results obtained to better understand Quantum Annealing in combinatorial portfolio optimization and use this knowledge to create their own trading strategies and start their development of an entrepreneurial business plan or FinTech business plan and therefore built upon the decentralization movement towards a more prosperous and inclusive new world era. By beginning to solve these types of problems now, readers begin building core skills and expertise around combinatorial optimization, QUBO modelling, problem decomposition for hybrid computing applications, and other advanced optimization problem-solving techniques specific to these new classes of computing hardware and software. Importantly, these skills and competencies are highly transferable. In fact, the QUBOs themselves are highly reusable for solving optimization problems with quantum computers. This means getting started developing, iterating and deploying advanced QUBOs for enterprise-class optimization problems now, pushing innovation further, and driving real and often transformative outcomes in the business today. Opportunity is knocking. When production-grade quantum computers finally arrive, readers can redeploy the same QUBO models on the new systems and leap frog even further, having already developed the enterprise competencies to embrace these new systems in earnest. Anything less is both a business and technology risk because competitors will surely be one step ahead.

7. Recommendations and Future Research

• Recommend further analyses with the next-generation QPU, namely Zephyr topology by addressing the limitations of the current hardware i.e. interference on larger portfolios.

• Recommend further study and develop of the entrepreneurial business case which should at least further address the large gap in training, education and development requirements of our customers and employees. Perhaps mainly through a Tutoring program and in collaboration with University programs.

• Recommend to continue to explore this simulator's potential in other fields, and with the new architecture, and hope to see many more algorithms running across industries to base future research on.

• Future research will have to address the limitations of this research project as highlighted in the conclusion chapter.

• Further research is required on the application and importance of Quantum Game Theory [15] on the financial markets and portfolio optimization and to further address Dr. Samualson’s enquiry in 1979 on if coordinating entrepreneurs are worth their profits in a Quantum Economy.

APPENDICES

Appendix A: Code: GitHub. https://github.com/TheHouseOfVermeulens/Quantum-ML/tree/main/D-Wave
Appendix B: The fine method

Fine method as a way of expressing such an equality constraint.

Let $f(x)$ be the cost (objective function) to be minimized and $g(x) = 0$ for the constraint. Solving the optimization problem by adding the following terms usig the hyperparameter λ to solve the optimization problem is called a fine method

$$\min_x \left\{ f(x) + \lambda g(x)^2 \right\}$$

Appendix C: Brief Market Analyses of 2022

• Quantum Computing Market to Expand by 500% by 2028 | 86% of Investments in Quantum Computing Comes from 4 countries [55]

• Only 4 Countries are Responsible for 86% of Total Funding Since 2001

• China in 2022 announced an increased of their investment to USD 32Billion, only second to the US [56].

• Global quantum computing market at USD 490.51 Million in 2021, and USD 2,930.67 Million by 2028, CAGR of 30.70%. This is a whopping 497% growth in just span of 7 years.

• Classical computer databases can take hours or even days to train an AI algorithm.

• Generating around $41 billion revenue by the year 2040 at a CAGR of more than 30%

• Market is projected to experience a significant surge in the demand for quantum sensing and Quantum communication in the years to come.

• The Quantum Computing market is relatively fragmented, with a high level of competition. Few large players, like IBM Corporation, Cambridge Quantum Computing Ltd., and Intel Corporation, now control the market in terms of market share. These industry leaders are extending their customer base across several areas, and many corporations are creating strategic and collaborative initiatives with other start-up enterprises to enhance their market share and profitability.

• Market: Banking, Financial services & Insurance (BFSI)

• The quantum computing market is segmented into various applications such as Machine learning, Data Optimization, Biomedical simulators, Financial Services, Electronic Material Discovery, and Others. Machine learning has dominated the market with 37% of the total revenue generated in 2021 and is expected to grow at the fastest CAGR during the analysis period. ML with data optimization is used in various fields to use the storage space efficiently by storing optimized and usable data which in turn saves a huge cost for the companies [55].

• Restraint: The difficulty in the commercialization of quantum computing due to high computational power consumption and complicated state of data storage are major restraints for the quantum computing industry which could only be fixed with innovation in the technology and will take time which in turn provide us with an opportunity to provide quantum portfolio optimization as a service [55].

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