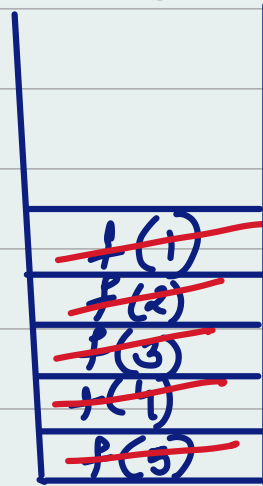


Q) $N = 5$, Print all the numbers from N till 1 : 5, 4, 3, 2, 1

$f(5) \rightarrow f(4) \rightarrow f(3) \rightarrow f(2) \rightarrow f(1)$
5 4 3 2 1

fun (n)
 prints (n)
 fun (n-1)

base condition \rightarrow if $n == 0$



return;

```
{ psvm ( ) {  
    fun (5);  
}
```

```
static void fun (int n) {  
    if (n == 0) {  
        return;  
    }
```

```
    System.out.println(n);  
    fun (n-1);  
}
```

```
}
```

so for printing 1 to 5 using recursion calls fun (n: 6)

& if (n == 6)
return;

also fun^c would be fun(n+1) ←

Q) factorial of a number:

$$N = 5 \quad \text{Ans} = 5! = 5 \times 4 \times 3 \times 2 \times 1 \\ = 120$$

$$\begin{aligned} \text{Fact}(5) &= 5 * \text{Fact}(4) \\ &= 5 * 4 * \text{fact}(3) \\ &\quad \swarrow 3 * \text{fact}(2) \\ &\quad \quad \swarrow 2 * \text{fact}(1) \end{aligned}$$

$$F(N) = N * F(N-1)$$

Trace:

$$F(5) = 5 * F(5-1) = 120$$

$$F(5) = 5 * F(4) \quad \begin{matrix} 2 & 4 \\ \nearrow & \nearrow \\ & 6 \end{matrix}$$

$$F(4) = 4 * F(4-1)$$

$$F(3) = 3 * F(3-1) \quad \begin{matrix} 2 \\ \nearrow \end{matrix}$$

$$F(2) = 2 * F(2-1) \quad \begin{matrix} 2 \\ \nearrow \end{matrix}$$

Base Case $F(1) = 1$ ←

$$F(1) = 1$$

Return;

Code:

```
psvm ( ) {  
    int ans = 5;  
    system.out.println(factorial(ans));  
}  
  
static int factorial(int n) {  
    if (n <= 1) {  
        return 1;  
    }  
    return n * factor(n-1);  
}
```

Q) Sum of digits

$$N = 1342$$
$$\text{remainder} = N \% 10$$

$$N = N/10$$

$$\text{ans} = 1 + 3 + 4 + 2 = 10$$

1 + sum of digits of (342)

$$\begin{array}{r} 134\textcircled{2} \\ \hline 10 \end{array} = 2 \quad \begin{array}{r} 3 + F(42) \\ \hline 10 \end{array} \rightarrow \begin{array}{r} 13\textcircled{4} \\ \hline 10 \end{array} = 4 \rightarrow \begin{array}{r} 13 \\ \hline 10 \end{array} = 3 \rightarrow \begin{array}{r} 1 \\ \hline 10 \end{array} = 1$$

$$F(N) = F(N/10) + (N \% 10)$$

$$F(N) = F(N/10) + \text{remainder}$$

F (1342)

Complexity: $O(\log(N))$

return $2 + F(134)$

return $(4 + F(13))$

Code in mac

$n --$ vs $-- n$ // $n = n - 1$

$n --$ first pass
for $n = 5$

↓
subtract first
than pass
for $n = 5$

Q:] Reverse a number using recursion.

$N = 1$ to 5 return 5 to 1 .

OR $N = 1825$ to 5281

↓
 $1825 \rightarrow 5 + f(182)$

take the last
element put
the first.

↓
 $2 + f(18)$

↓
 $8 + f(1) \rightarrow$ return
base condition

$F(N) = (N \% 10)$

$F(N/10)$

1) $\text{fun}(n)$ { 2 void
 if ($n == 0$) {
 return;
 }
 remainder = $n \% 10$;
 sum = $\text{sum} * 10 + \text{rem}$;
 fun($n / 10$);

1825
 $5 + f(182)$
 $(5 * 10 + 2) + f(18)$
 $52 + f(18)$
 $521 + f(8)$

2) $N = 12345$
 $5 * 10000 + 1234$
 $5 * 10^4 + f(1234)$

$F(N, \text{arguments}) =$
 rem, $\text{arguments} - 1$
 $(N \% 10) * 10 +$
 $F(N / 10, \text{arguments} - 1)$

$\rightarrow 4 * 10^3 + f(123)$

$\rightarrow 3 * 100 + f(12)$

$\rightarrow 2 * 10 + f(1)$ return

Here helper function is created.

In code line
 return remainder * (long)(Math.pow(10, digits-1))
 + helper($n / 10$, digits-1);

8) Palindrome or not.

$N = 12321$

start ✓ se ✓ end
1 2 3 2 1

How can we solve this through recursion
code in the max through recursion.

9) Count number of zeroes in a number.

$N = 30204$

Ans = 2

① If we take count inside argument

$F(N, 0) \rightarrow$ If digit = 0

$F(N/10, C+1)$

Ex:
 $\frac{100}{10} = \frac{10}{10} = 1$
 $0+1 \Rightarrow 1$

gmp to count

else

$F(N/10, C)$

$\frac{30204}{10} = \frac{3020}{10}$
 $1+1 = 2$

$F(30204, 0)$

$= C+1 = 0+1$
 $= 1$
 $\frac{302}{10} = 30$

$F(3020, 0)$

$\frac{30}{10} = 3$

$F(30, 1)$

$1+1 = 2$

$F(30, 1) \rightarrow 2$
 $F(3, 2) \rightarrow 2$
 $F(0, 2) \rightarrow \text{return ans}$

```

public class Count_0 {
    psvm() {
        sout (count (30210));
    }

```

```

    static int count (int n) {
        return helper (n, 0);
    }

```

// special patterns, how to pass a value to above calls.

```

static int helper (int n, int count) {
    if (n == 0) {
        return count;
    }
    int remainder = n % 10;
    if (remainder == 0) {
        return helper (n/10, count+1);
    }
}

```

return helper($n/10$, count),

* special example to return same value to above function calls

8) Given an integer num, return the number of steps to reduce it to zero

In one step, the current number is even, you have to divide it by 2, otherwise subtract one from it

Input num = 14

output = 6

- 1) 14 is even, divide by 2 obtain 7.
- 2) $7 - 1 = 6 \rightarrow$ second step.
- 3) $6 / 2 = 3 \rightarrow$ 3rd step
- 4) $3 - 1 = 2 \rightarrow$ 4th step
- 5) $2 / 2 = 1 \rightarrow$ 5th step
- 6) $1 - 1 = 0 \rightarrow$ 6th step
stop //

helper function/ We need to pass these values in recursion calls, that is why we need to put it in the argument (helper function)

code:

```
class Solution {  
    public int numberOfSteps (int num) {  
        return helper(num, 0);  
    }  
    static int helper (int num, int steps) {  
        if (num == 0) {  
            return steps;  
        }  
        if (num % 2 == 0) {  
            return helper(num/2, steps+1);  
        }  
        return helper(num-1, steps+1);  
    }  
}
```

