* Prime Numbers: 2, 3, 5, 7, 13, . . .

```
for (i=2, i<N, i++){
        if ( N%i ){
                Not a prime
        }
}return Prime;
```

Another example

1 × 36
2 × 18
3 × 12
4 × 9
6 × 6
9 × 4
12 × 3
18 × 2
36 × 1

→ 36

→ Hence only make checks for numbers ≤ sqrt(N)

Complexity O($\sqrt{N}$)

This is repeated hence, ignore

Q] N = 40, [Numbers that are less than equal to 40 which are prime numbers]

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 multiple

2 , 3 , ~~4~~ , 5 , ~~6~~ , 7 , ~~8~~ , ~~9~~ , 10 ,
11 , 12 , 13 , 14 , 15 , 16 , 17 , 18 , 19 ,
20 , 21 , 22 , 23 , 24 , 25 , 26 , 27 , 28 ,
29 , 30 , 31 , 32 , 33 , 34 , 35 , 36 , 37 ,
38 , 39 , 40

O → False (All those multiple of prime number will automatically get cancelled)
X → True

So the program will be that all the multiples of i*2 = j will automatically be not prime.

* code:

```
class ArrayPrime
  psvm {
   int n = 100;
  boolean [] prime = new boolean [n+1]
 // If n should be included than size
   should be (n+1) → upto 40 not 39.
 Sieve (n, prime)
}
static void sieve( int n, boolean [] primes){
   for (int i=2 , i*i<=n , i++ ){
```

```
if ( !primes[i] ){   (2+2+2+...
                       3+3+3+3.)
for (int j=2 ; j = i*2 ; j+=i){
      prime [j] = true;
   } }
  }
}
for (int i=2 ; i<=n; i++){
   if (!primes[i]) {
   System.out.println (i+"  ");
 }
}
```

Time complexity analysis:-

first loop → $\frac{n}{2} + \frac{n}{3} + \frac{n}{5} + \frac{n}{7} + \cdots \frac{n}{p}$

p → highest prime number less than n

$$n\underbrace{\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\cdots\right)}_{\text{for primes}}$$ Harmonic Progression

$\log(\log N)$

Total Time complexity: O (N * log(log N))

8) Finding square root of a number: Ex: 36

mid

0 18 36

↓

is 18 * 18 more than 36
No, its not check into left side for end = mid-1

if (mid * mid > n)
    end = mid - 1
else
    start = mid+1

Ex: 40

sqrt (40) = 6.32,

6 → Int value with above way

.32 → but how can we get the decimal value.

[This is for 0.1]

root = 6 [is 6*6 <= 40, yes it is then increment the decimal value by 1] similarly upto 6.3

= 6.2

Ans → = 6.3

= ~~6.4 * 6.4~~

reverse back

The same thing for 0.01, by taking a loop

```
double increment = 0.1;
for(int i=0; i<=n; i++){
    while (root* root <= n){
        root = root+ increment,
    }
    increment =
    increments/10;
```

Time complexity ≈ log(N)

Code: in the mac.

Q) Newton Raphson Method :-

$$\text{root} = \frac{\left(X + \frac{N}{X}\right)}{2}$$

↙ actual root of n

x = sqrt that you have assumed

Why the formula works :-

$$\sqrt{N} = \frac{x + \frac{N}{x}}{2}$$

If guess is correct then,

$$\sqrt{N} = \frac{\sqrt{N} + \frac{\cancel{\sqrt{N}} \cdot \sqrt{N}}{\cancel{\sqrt{N}}}}{2}$$

$$\boxed{\cancel{2}\sqrt{N} = \cancel{2}\sqrt{N}} \; //$$

$$\text{root} = \sqrt{X + \frac{N}{N}}$$

You will find your ans when error < 1.

error = $| root - X |$ | if(Math.abs(root-x)<1)
break;

① Assign x to N itself ⇒ double x = n;

②

③ update the value of x = root

Complexity: $O((\log N) F(N))$

$F(N)$ = cost of calculating $\frac{f(x)}{f'(x)}$ with some N digit precision.

Code in the mac.

Q) Factors of a number:

what are all the numbers that divide

Ex: 20? ⇒ 1, 2, 4, 5, 10, 20

Run a loop for 1 to 20   Brute force

Time complexity upto N times O(N)

Space complexity constant

```
psvm(  ){
```

```
    factors(20),
}
static void factors1(int n){
    for (int i=1; i<=n ; i++){
        if (n% i ==0){
            Sout(i+ " ");
        }
    }
}
```

i=1, Is 20 divisible by 1; yes.

i=2, Is 20 divisible by 2; yes;

i=4 " " yes

i=5 " " yes

20%1 ↓

$20 * 1 = 20$

$20 \% 2 \Rightarrow 10 * 2$

$4 * 5 = 20$

$5 \times 4 = 20$

$\rightarrow 2 * \underbrace{10 \text{ or } 10*2}_{\text{repeated}}$

only check till sqrt of(n)

$O(\sqrt{n})$

Code in Mac

* Properties of Modulo (%)

- $(a+b)\%m = [(a\%m)+(b\%m)]\%m$
- $(a-b)\%m = [(a\%m)-(b\%m)]\%m$
- $(a*b)\%m = [(a\%m)*(b\%m)]\%m$
- $(a/b)\%m = [(a\%m)*(b^{-1}\%m)]\%m$

$b^{-1}\%m \Rightarrow$ multiplicative modulo inverse (MMI)

Ex: $(6*y)\%7 = 1$

Here $y = 6$, because of multiplicative modulo inverse.

$(6*6)\%7 = 1$

$36\%7 = 1$

$$\begin{array}{r} 5 \\ 7\,\overline{)\,36} = 6*6 \\ 35 \\ \hline 1 \end{array}$$

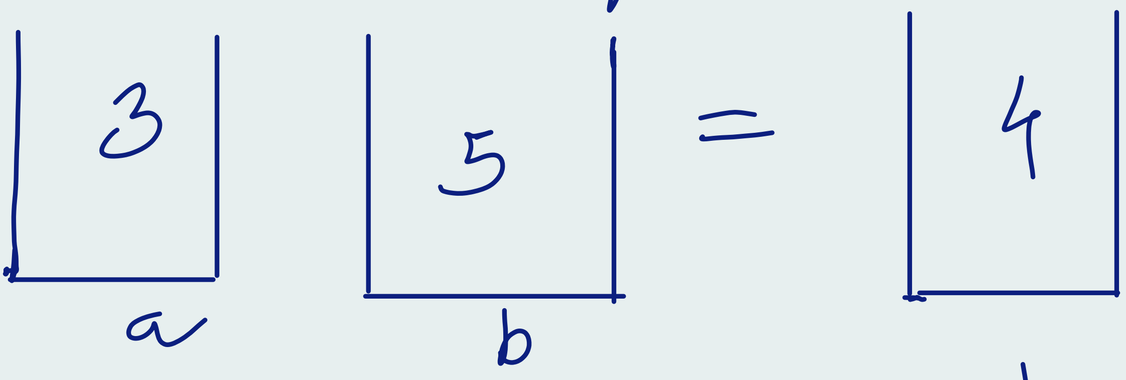
MMI $= b^{-1}\%m$ means that b and m are co-primes → Example. Between 6 & 7 — is exactly only 1 common b/w them and no other numbers are common

It means ~~the~~ 6 & 7 are coprime.

* $(a\%m)\%m = a\%m$
* $(m^x\%m) = 0 \ \forall x \in$ positive Integers

---

* Die-hard Example



$1^{st} \to$ (Intially 2 buckets with $(\overset{a}{0},\overset{b}{0})$ gallons of water) $\to (3, 0) \to (\overset{a}{0}, \overset{b}{3})$

$2^{nd} \to (\overset{a}{0}, \overset{b}{3}) \to$ fill bucket a again $(\overset{\downarrow a}{3}, \overset{b}{3}) \to (\overset{a}{1}, \overset{b}{5})$

Empty 2^{nd} bucket

↳ $(\overset{a}{1}, \overset{b}{0}) \to$ then pass the 1 gallon to b bucket $(0, 1)$

$3^{rd} \to (0, 1) \to (3, 1) \longrightarrow (\overset{a}{0}, \overset{b}{4})$

Ans

Imagine jug a was filled → s_1 times
jug b emptied → s_2 times [removed water]

remainder $= a s^1 - b s^2$
in the b jug

$$\boxed{x = a s^1 + (-b s^2)}$$

$3x + 5y = 4$?

Put x and y as integers what is the minimum (+ve) value, you can have in your equation.

$x = -3,\ y = 2$ [1 is a factor of every number]

$3(-3) + 5(2) = 1$

$3x + 5y = 1$ → minimum value of equation that I can form is 1

→ This is also known as HCF

→ HCF/ of two number
gcd a & b = min +ve value of equation $(ax + by)$ where x and y are integers.

Highest common factor $(4, 18) = \boxed{2}$

$(1, \boxed{2}, 4)$ $(1, \boxed{2}, 3, 6, 9, 18)$

$2 \rightarrow$ Ans Highest common factor

$HCF(3, 9) = \boxed{3}$

$(1, \boxed{3})$ $(1, \boxed{3}, 9)$

$\min(3x + 9y = 3)$

$\Rightarrow 3x + 9y$

$3(x + 3y)$

$\Rightarrow$ put $x = -2$, $y = 1$

$= 3(-2 + 3(1))$

$= 3$

---

a, b

$ax + by = L$ (Litres)

$\boxed{2x + 4y} = 5L$

Note:

whatever HCF you will get, that will come

out as
common.

$$2\,(x+2y) = 5$$

$$\boxed{x+2y = 2.5} \times$$

$$3x + 6y = 9L$$

$$3\,(x+2y) = 9$$

$$x+2y = 3$$

$$x=1,\ y=2$$

3L and 5l jug, 17 litres of water

$$3x+5y = 17$$

$$1\,(3x+5y) = 17 \,//$$

Euclid's Algorithm:-

$$gcd(a, b) = gcd(rem(b,a), a)$$

This entire formula repeated again & again so we will use recursion

$$gcd(105, 224) = gcd(rem(224, 105), 105)$$

$$= gcd(14, 105)$$

$\downarrow$
~~x~~

$105x + 224y$

$\downarrow$

$14x + 105y$ why subtract?

because the gcd of (105, 224) also divides a linear combination of 105 and 224-

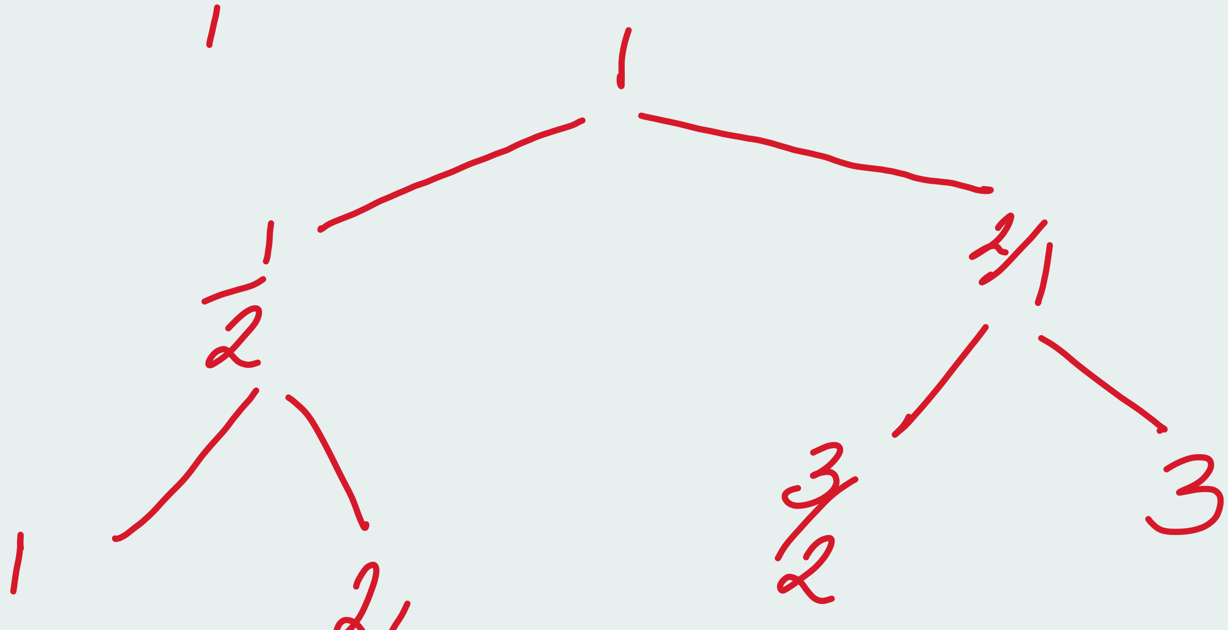
Ex: $224 - 2 \times (105) = 14$ (remainder)

$\text{rem}(12, 24)$

$\downarrow$

$\text{rem}((24, 12), 12) = 12$

$\frac{1}{1}, 2, \overset{3}{4}, 6, \boxed{12}, 24 \qquad 1, 2, \overset{3}{4}, 6, \boxed{12}$



$3 \qquad /3$

LCM:

$$\text{LCM}(a, b)$$

This is equal to that minimum number that are divisible by both A & B

$$\text{lCM}(2, 4) = 4$$
$$\text{lCM}(3, 7) = 21$$

Note: say we have A, B

$$D = \gcd(A, B)$$

$$f = \frac{a}{d}, \quad g = \frac{b}{d}$$

$$\Rightarrow a = fd, \quad b = gd$$

lCM = C, lcm of A & B has to be divisible by both A & B

* Hence $lcM(a, b) = lcM(fd, gd)$

* we know that f and g no other common factor

HCF

$d = 9$

$a = 9, \quad b = 18$

$$f = \frac{a}{d} = 9/9 = \boxed{1}$$

$$g = \frac{b}{d} = \frac{18}{9} = \boxed{2}$$

no other common factors remaining now

* $a = fd \qquad b = gd$

$lcM(f*g*d) \Rightarrow$ This is how above conditions are satisfied.

More info!

$a * b$

$= f*d*g*d$ ↓

d is repeating, so remove it

$lcM = f*g*d$

17, 19 ↓

find a number that is divisible by both 17 & 19.

So, a & b should contain 17 & 19 init.

$$a*b = fd * gd$$

$$= d*(dfg)$$

$$a*b = (HCF)(LCM)$$

Hence formula

$$\boxed{LCM(a,b) = \frac{a* b}{HCF\,(a,b)}}$$