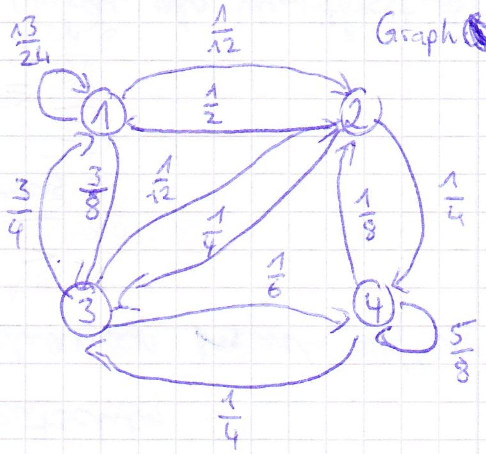
Exercise 2

In this case we have a connected Markov chain because in the given directed graph G_Q there is a path from i to j for all $i,j \in [n]$. So the Lemma 6.4 holds $(\pi_i q_{ij} = \pi_j q_{ji})$

The transition Matrix is:

Graph G_Q:



$$Q = \begin{pmatrix} \frac{13}{24} & \frac{1}{12} & \frac{3}{8} & 0 \\ \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{3}{4} & \frac{1}{12} & 0 & \frac{1}{6} \\ 0 & \frac{1}{8} & \frac{1}{4} & \frac{5}{8} \end{pmatrix}$$

Argument (1): The transition matrix Q is a stochastic matrix, that is, all entries are non-negative and all row sums are 1

Entry q_{24}: argument (1): $x = 1-\frac{1}{2}-\frac{1}{4}=\frac{1}{4} \Rightarrow q_{24}=\frac{1}{4}$

Entry q_{32}: $\pi_3 q_{32} = \pi_2 q_{23} \Leftrightarrow \frac{1}{4} \cdot q_{32} = \frac{1}{12}\cdot\frac{1}{4} \Leftrightarrow q_{32}=\frac{1}{12}$

Entry q_{34}: argument (1): $q_{34} = 1-\frac{3}{4}-\frac{1}{12} \Leftrightarrow q_{34}=\frac{1}{6}$

Entry q_{42}: $\pi_4 q_{42} = \pi_2 q_{24} \Leftrightarrow \frac{1}{6} q_{42} = \frac{1}{12}\cdot\frac{1}{4} \Leftrightarrow q_{42}=\frac{1}{8}$

Entry q_{43}: $\pi_4 q_{43} = \pi_3 q_{34} \Leftrightarrow \frac{1}{6}\cdot q_{43} = \frac{1}{4}\cdot\frac{1}{6} \Leftrightarrow q_{43}=\frac{1}{4}$

Entry q_{44}: argument (1): $q_{44}= 1-\frac{1}{8}-\frac{1}{4} \Leftrightarrow q_{44}=\frac{5}{8}$

Entry q_{13}: $\pi_1 q_{13} = \pi_3 q_{31} \Leftrightarrow \frac{1}{2} \cdot q_{13} = \frac{1}{4}\cdot\frac{3}{4} \Leftrightarrow q_{13}=\frac{3}{8}$

Entry q_{12}: $\pi_1 q_{12} = \pi_2 q_{21} \Leftrightarrow \frac{1}{2} q_{12} = \frac{1}{12}\cdot\frac{1}{2} \Leftrightarrow q_{12}=\frac{1}{12}$

Entry q_{11}: argument (1): $q_{11} = 1-0-\frac{3}{8}-\frac{1}{12} \Leftrightarrow q_{11}=\frac{13}{24}$

With Theorem 6.2 ~~we can say that~~ and Lemma 6.4 we can say that this solution is unique