Solutions to Sheet 4

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Exercise 1

From the given points the centered datamatrix

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 3.5 & 0 & 0.5 \\ 2.5 & 2 & 1.5 \\ -2.5 & -2 & -1.5 \\ -3.5 & 0 & -0.5 \end{bmatrix}$$

can be created. The covariance matrix

$$C = A^T A = \begin{bmatrix} 37 & 10 & 11 \\ 10 & 8 & 6 \\ 11 & 6 & 5 \end{bmatrix}$$

is derived from the datamatrix A. From the characterisitc polynominal of C $-\lambda^3 + 50\lambda^2 - 264\lambda$ the eigenvalues

$$\lambda_1 = 44, \lambda_2 = 6, \lambda_3 = 0$$

can be calculated. Using the corresponding eigenvectors

$$v_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} -1 \\ -4 \\ 7 \end{pmatrix}$$

the PCA-transformation

$$U = \begin{bmatrix} 3\sqrt{11} & -\sqrt{6} & -\sqrt{66} \\ \sqrt{11} & 2\sqrt{6} & -4\sqrt{66} \\ \sqrt{11} & \sqrt{6} & 7\sqrt{66} \end{bmatrix}$$

of A can be calculated. Therefore the first and second principal components of A are

$$\mathbb{P}_1 = \text{span}\left(\begin{pmatrix} 3\sqrt{11} \\ \sqrt{11} \\ \sqrt{11} \end{pmatrix}\right), \mathbb{P}_2 = \text{span}\left(\begin{pmatrix} -\sqrt{6} \\ 2\sqrt{6} \\ \sqrt{6} \end{pmatrix}\right)$$

. That leads to the best fit subspaces

$$\mathbb{U}_1 = \mathbb{P}_1, \mathbb{U}_2 = \text{span}\left(\sqrt{11}\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, \sqrt{6}\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}\right)$$