Tutorial 03 - 28/08/2024

Problem 1: Transformation to elliptical coordinate system is given $\{x, y\} \longrightarrow \{\mu, \nu\}$

$$x = a \cosh\mu \cos\nu$$

$$y = a \sinh\mu \sin\nu$$

find the expression for K.E and obtain the EOM for a free particle:

$$\left.\begin{aligned} x &= a \cosh\mu \cos\nu \\ y &= a \sinh\mu \sin\nu \end{aligned}\right] \longrightarrow (1)$$

$$\dot{x} = a \sinh\mu \cos\nu\, \dot{\mu} - a \cosh\mu \sin\nu\, \dot{\nu}$$

$$\dot{y} = a \cosh\mu \sin\nu\, \dot{\mu} + a \sinh\mu \cos\nu\, \dot{\nu}$$

$$\begin{aligned} \dot{x}^2 + \dot{y}^2 = {} & a^2 \sinh^2\mu \cos^2\nu\, \dot{\mu}^2 + a^2 \cosh^2\mu \sin^2\nu\, \dot{\nu}^2 \\ & \cancel{- 2a^2 \sinh\mu \cosh\mu \cos\nu \sin\nu\, \dot{\mu}\dot{\nu}} \\ & a^2 \cosh^2\mu \sin^2\nu\, \dot{\mu}^2 + a^2 \sinh^2\mu \cos^2\nu\, \dot{\nu}^2 \\ & \cancel{+ 2a^2 \cosh\mu \sinh\mu \sin\nu \cos\nu\, \dot{\mu}\dot{\nu}} \end{aligned}$$

$$\begin{aligned} = {} & a^2 \dot{\mu}^2 \left(\sinh^2\mu \cos^2\nu + \cosh^2\mu \sin^2\nu\right) \\ & + a^2 \dot{\nu}^2 \left(\cosh^2\mu \sin^2\nu + \sinh^2\mu \cos^2\nu\right) \end{aligned}$$

$$\dot{x}^2 + \dot{y}^2 = a^2 \left(\cosh^2\mu \sin^2\nu + \sinh^2\mu \cos^2\nu\right)\left(\dot{\mu}^2 + \dot{\nu}^2\right)$$

now $\cosh^2\mu \sin^2\nu + \sinh^2\mu \cos^2\nu$

$$= \cosh^2\mu \sin^2\nu + \sinh^2\mu - \sinh^2\mu \sin^2\nu$$

$$(\cosh^2\mu - \sinh^2\mu)\sin^2\nu + \sinh^2\mu$$

$$\sin^2\nu + \sinh^2\mu$$

also sames as $(\cosh^2\mu - \cos^2\nu)$

$$T = \frac{1}{2} m \left(\sin^2\nu + \sinh^2\mu\right)\left(\dot{\mu}^2 + \dot{\nu}^2\right)$$

The equation of motion for free particle is

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\mu}}\right) - \frac{\partial T}{\partial \mu} = 0 \ ; \ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\nu}}\right) - \frac{\partial T}{\partial \nu} = 0$$

$$\frac{\partial T}{\partial \dot{\mu}} = m\left(\sin^2\nu + \sinh^2\mu\right)\dot{\mu}$$

$$\frac{\partial T}{\partial \mu} = m \sinh\mu \cosh\mu \left(\dot{\mu}^2 + \dot{\nu}^2\right)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\mu}}\right) = m\left(2\sin\nu\cos\nu\,\dot{\nu} + 2\sinh\mu\cosh\mu\,\dot{\mu}\right)\dot{\mu} + m\left(\sin^2\nu + \sinh^2\mu\right)\ddot{\mu}$$

eqn of motion

$$m\left(\sin^2\nu + \sinh^2\mu\right)\ddot{\mu} + m\left(2\sin\nu\cos\nu\,\dot{\nu}\dot{\mu} + 2\sinh\mu\cosh\mu\,\dot{\mu}^2\right) - m\sinh\mu\cosh\mu\left(\dot{\mu}^2 + \dot{\nu}^2\right)$$

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similarly

$$\frac{\partial L}{\partial \dot{\nu}} = m\left(\sin^2\nu + \sinh^2\mu\right)\dot{\nu}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\nu}}\right) = m\left(2\sin\nu\cos\nu\,\dot{\nu} + 2\sinh\mu\cosh\mu\,\dot{\mu}\right)\dot{\nu} + m\left(\sin^2\nu + \sinh^2\mu\right)\ddot{\nu}$$

$$\frac{\partial L}{\partial \nu} = m\sin\nu\cos\nu\left(\dot{\mu}^2 + \dot{\nu}^2\right)$$

EOM:

$$m\left(\sin^2\nu + \sinh^2\mu\right)\ddot{\nu} + m\left(2\sin\nu\cos\nu\,\dot{\nu} + 2\sinh\mu\cosh\mu\,\dot{\mu}\right)\dot{\nu} - m\sin\nu\cos\nu\left(\dot{\mu}^2 + \dot{\nu}^2\right) = 0$$

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② Compute the K.E under the transformation from an inertial frame $\{x, y\} \longrightarrow \{x', y'\}$ given by

$$x = x' \cos\omega t - y' \sin\omega t$$
$$y = x' \sin\omega t + y' \cos\omega t$$
(1)

(*) find the equation of motion in coordinate $\{x', y'\}$ for a free particle.

Ans:
$$\dot{x} = \dot{x}' \cos\omega t - \dot{y}' \sin\omega t - \omega x' \sin\omega t - \omega y' \cos\omega t$$

$$\dot{y} = \dot{x}' \sin\omega t + \dot{y}' \cos\omega t + \omega x' \cos\omega t - \omega y' \sin\omega t$$

This get's complicated

we find the inverse relation write (x', y') interms of (x, y) as,

$$x' = x \cos\omega t + y \sin\omega t$$
$$y' = -x \sin\omega t + y \cos\omega t$$

Not to TAs: Make sure all the steps are properly done

$$\dot{x}' = \dot{x}\cos\omega t + \dot{y}\sin\omega t - \omega x\sin\omega t + \omega y\cos\omega t$$

$$\dot{x}' = \dot{x}\cos\omega t + \dot{y}\sin\omega t + \omega y'$$

or

$$\boxed{\dot{x}' - \omega y' = \dot{x}\cos\omega t + \dot{y}\sin\omega t} \quad (2)$$

similarly : $y' = -x\sin\omega t + y\cos\omega t$

$$\dot{y}' = -\dot{x}\sin\omega t + \dot{y}\cos\omega t - \omega x\cos\omega t - \omega y\sin\omega t$$

$$\dot{y}' = -\dot{x}\sin\omega t + \dot{y}\cos\omega t - \omega x'$$

$$\boxed{\dot{y}' + \omega x' = -\dot{x}\sin\omega t + \dot{y}\cos\omega t} \quad (3)$$

$(2)^2 + (3)^3$ $\quad \dot{x}^2 + \dot{y}^2 = (\dot{x}' - \omega y')^2 + (\dot{y}' + \omega x')^2$

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}m\left[\dot{x}'^2 + \dot{y}'^2 + 2\omega(x'\dot{y}' - y'\dot{x}') + \omega^2(x'^2 + y'^2)\right]$$

for free particle generalized force $Q_j = 0$

The equation motion is given by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}'}\right) - \frac{\partial T}{\partial \dot{x}'} = 0$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}'}\right) - \frac{\partial T}{\partial \dot{y}'} = 0$$

are equation of motion

here $T = \frac{1}{2}m\left[\dot{x}'^2 + \dot{y}'^2 + 2\omega\left(x'\dot{y}' - y'\dot{x}'\right) + \omega^2\left(x'^2 + y'^2\right)\right]$

$$\frac{\partial T}{\partial \dot{x}'} = m\left[\dot{x}' - m\omega y'\right]$$

$$\frac{\partial T}{\partial x'} = m\omega\dot{y}' + \omega^2 x'$$

EOM in x' component

$$m\frac{d}{dt}\dot{x}' - 2m\omega\dot{y}' - \omega^2 x' = 0$$

here $T = \frac{1}{2} m \left[\dot{x}'^2 + \dot{y}'^2 + 2\omega \left(x' \dot{y}' - y' \dot{x}' \right) + \omega^2 \left(x'^2 + y'^2 \right) \right]$

$$\frac{\partial T}{\partial \dot{y}'} = m \left[\dot{y}' + m\omega x' \right]$$

$$\frac{\partial T}{\partial y'} = -m\omega \dot{x}' + \omega^2 y'$$

EOM in y' component

$$m \frac{d}{dt} \dot{y}' + 2m\omega \dot{x}' - \omega^2 y' = 0$$