PH3101, Spring 2024 **Mid-Term Exam** DPS, IISER Kolkata

Instructor: Dr. Rajesh Kumble Nayak
3:00 PM, 25 September, 2024.
Duration 90 Min.

- Read each question carefully.
- Answer all questions.
- Ensure all parts of the answer for a given question are in the same place.
- Write neatly, and do all rough work on the answer sheet.
- Do not write on the question paper.
- Keep answers concise and to the point.

Good luck!

Q - 1(20 Marks)
In a generalised coordinate system $\{\sigma, \tau\}$ the Lagrangian is given by,

$$\mathcal{L} = \frac{m}{2}\left(\frac{1}{\sigma^2+\tau^2}\right)\left[\dot{\sigma}^2+\dot{\tau}^2\right]$$

1. Find the Hamiltonian for the system
2. Find the Hamiltonian Equation of motion.

Q - 2(20 Marks)
In a generalised coordinate system $\{q_1, q_2\}$ the Lagrangian is given by,

$$\mathcal{L} \,=\, \dot{q_1}^2 \,+\, \dot{q_2}^2 \,+\, k_1\, {q_1}^2 \,+\, k_2\, \dot{q}_1 \dot{q}_2\,,$$

1. Find the generalised momenta $\{\, p_1,\, p_2\,\}$.
2. Express the generalised velocities $\{\dot{q}_1, \dot{q}_2\}$ in terms of generalised momenta $\{\, p_1,\, p_2\,\}$.
3. Find the Hamiltonian for the system. It is not necessary to simplify the Hamiltonian to its simplest form.

Q - 3(5 Marks)
For a Hamiltonian system, the Hamiltonian $H\left(q_i, p_i, t\right)$ satisfies the Hamiltonian equations of motion. Show that if $\frac{\partial H}{\partial t} \,=\, 0$, then $\frac{dH}{dt} \,=\, 0$ or H is constant of motion.

Q - 4(5 Marks)
Lagrangian for a system is given by

$$\mathcal{L} \;=\; \frac{m}{2}\left(\dot{x}^2+\dot{y}^2\right) + q\left(\dot{x}A_x + \dot{y}A_y\right) - q\Phi(x,y)$$

Here, $\vec{A} = (A_x,\, A_y)$. Find the Hamiltonian.