PH3102 Quantum Mechanics   Assignment 2

Instructor: Dr. Siddhartha Lal   Autumn Semester, 2024

Start Date: August 13, 2024   Submission Deadline: August 20, 2024 .

Submit your answers to the Tutor at the start of the tutorial.

Q1. Concept of a operator. [5 marks]

Consider $\hat{O}$ to be an operator defined by

$$\hat{O} = |\phi\rangle \langle\psi| ,$$

where $|\phi\rangle$ and $|\psi\rangle$ are two vectors of the state space.
(a) Give the condition for $\hat{O}$ to be Hermitian.
(b) Calculate $\hat{O}^2$. State the condition for which $\hat{O}$ can be a valid projection operator?
(c) Show that $\hat{O}$ can always be written in the form of $\hat{O} = \lambda P_1 P_2$, where λ is a constant to be determined and P_1 and P_2 are projection operators corresponding to the vectors $|\phi\rangle$ and $|\psi\rangle$ respectively.

Q2. Characteristics of real wavefunction. [5 marks]

Consider a real-valued wavefunction $\psi(x)$.
(a) For this $\psi(x)$, show that the expectation value of momentum given by $\langle\hat{p}\rangle$ is zero.
(b) Now show that if $\psi(x)$ has a mean momentum given by $\langle\hat{p}\rangle$, $e^{\frac{ip_0 x}{\hbar}}\psi(x)$ has mean momentum $\langle\hat{p}\rangle + p_0$.
Use the Dirac "bra-ket" notation to carry out the computations.

Q3. Coherent States. [5 marks]

For the simple harmonic oscillator with the time independent wavefunctions $\psi_n(x)$ satisfying

$$H\psi_n(x) = \hbar\omega(n + \frac{1}{2})\psi_n(x) \ ,$$

consider the superposition at time $t = 0$

$$\psi(x, t = 0) = \sum_{n=0}^{\infty} c_n \psi_n(x) \ . \tag{1}$$

(a) How should the coefficients be chosen so that $\psi(x, 0)$ is an eigenstate of the lowering operator $\hat{a}$ with eigenvalue α (a given complex number), i.e.,

$$\hat{a}\psi(x, 0) = \alpha\psi(x, 0) \ ?$$

(b) Using the expression for $\hat{a}$, find the explicit form of the wavefunction at $\psi(x, 0)$. Ensure that $\psi(x, 0)$ is correctly normalised.
Note that eigenstates of $\hat{a}$ are referred to as "coherent states".