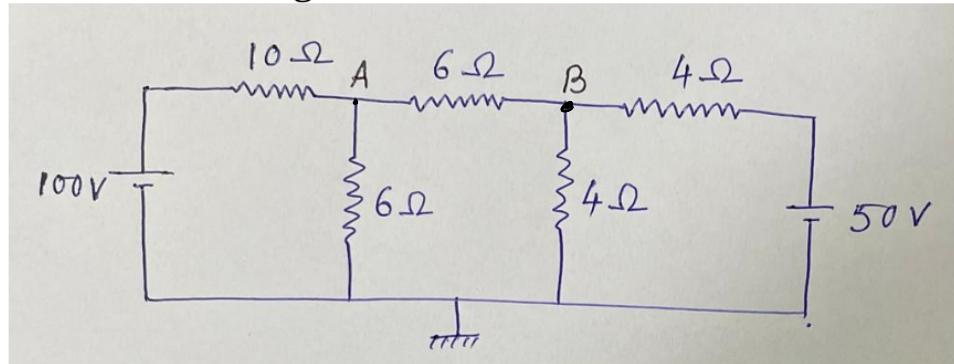


1.

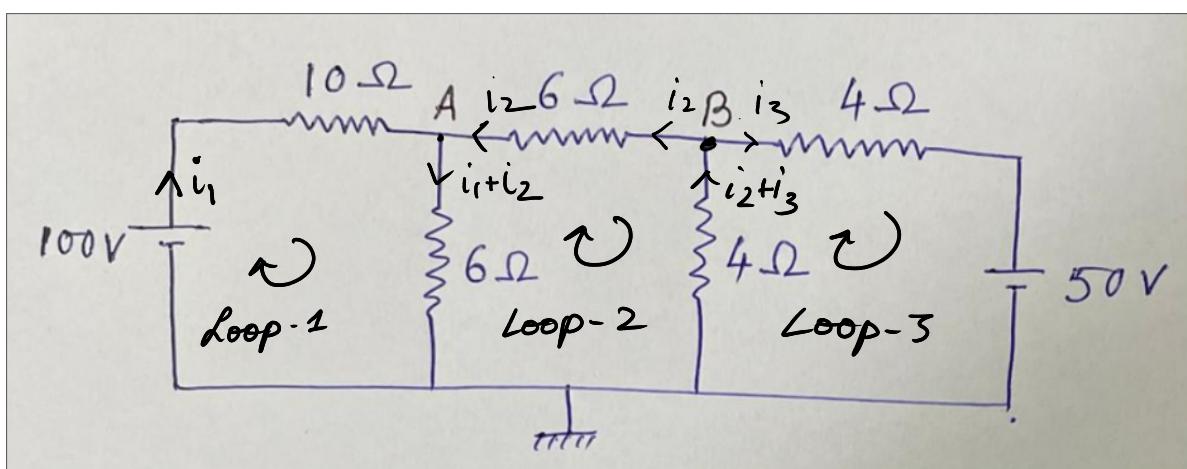
1. Consider the following circuit:



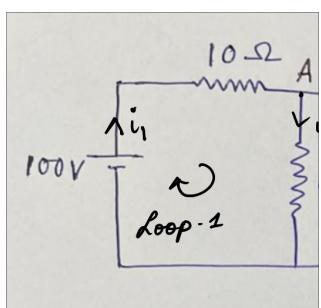
Calculate the voltage at point A and B with respect to the ground.

[10]

Ans:-



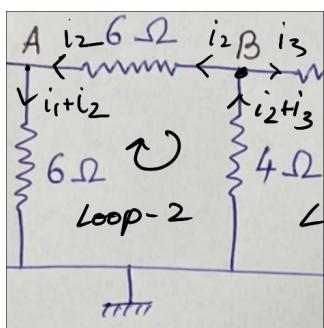
Loop-1



$$+100 - 10i_1 - 6(i_1 + i_2) = 0$$

$$\Rightarrow 10i_1 + 6(i_1 + i_2) = 10 \Rightarrow \underline{16i_1 + 6i_2 = 10}$$

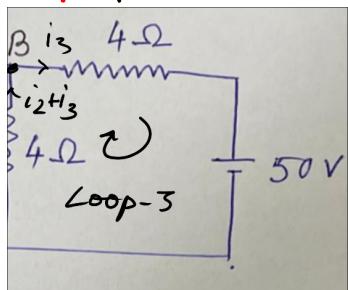
Loop-2



$$6(i_1 + i_2) + 6i_2 + 4(i_2 + i_3) = 0$$

$$\Rightarrow 6i_1 + 16i_2 + 4i_3 = 0$$

Loop-3



$$\begin{aligned}
 -4(i_2 + i_3) - 4i_3 - 50 &= 0 \\
 \Rightarrow 4(i_2 + i_3) + 4i_3 &= -50 \\
 \Rightarrow 4i_2 + 8i_3 &= -50
 \end{aligned}$$

$$16i_1 + 6i_2 + 0 = 100$$

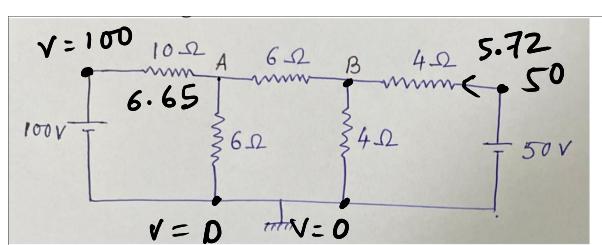
$$6i_1 + 16i_2 + 4i_3 = 0$$

$$0 + 4i_2 + 8i_3 = -50$$

$$\Rightarrow \begin{pmatrix} 16 & 6 & 0 \\ 6 & 16 & 4 \\ 0 & 4 & 8 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 100 \\ 0 \\ -50 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \frac{1}{188} \begin{pmatrix} 50 \\ -8 \\ -43 \end{pmatrix}.$$

$$i_1 = 6.65 \text{ A}, \quad i_2 = -1.064 \text{ A}, \quad i_3 = -5.72 \text{ A}$$



$$\begin{aligned}
 V_A &= (100 - 10 \cdot 6.65) \text{ V} \\
 &= (100 - 66.5) \text{ V} \\
 &= 33.5 \text{ V}
 \end{aligned}$$

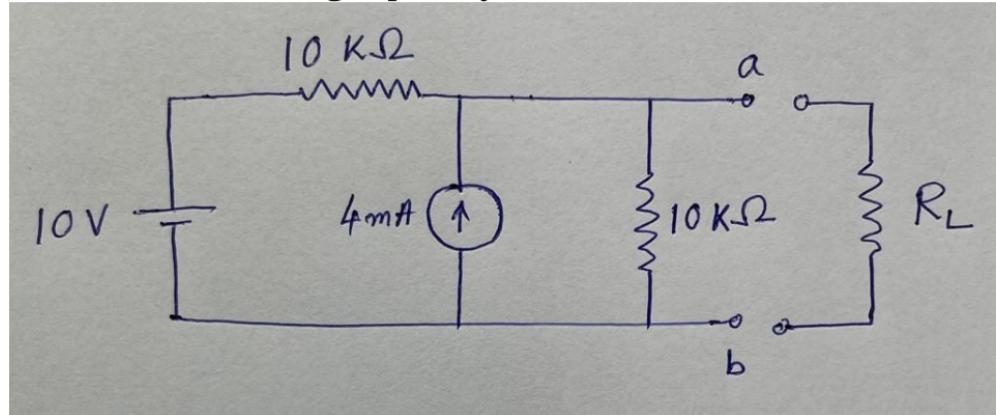
$$V_B = (50 - 4 \cdot 5.72) \text{ V} = (50 - 22.88) \text{ V} = 27.12 \text{ V}$$

$$V_A = 33.5 \text{ V}$$

$$V_B = 27.12 \text{ V}$$

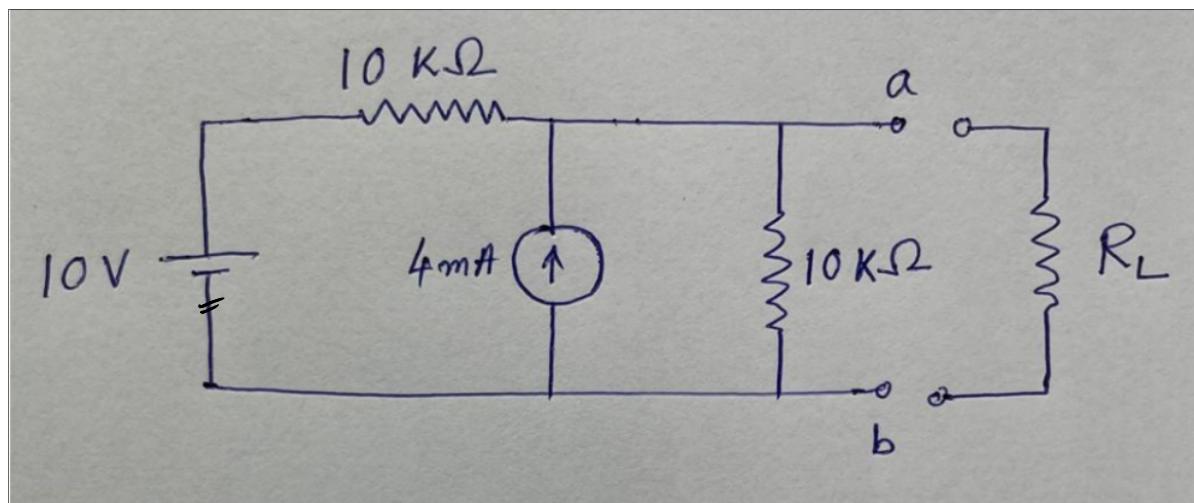
27

2. Consider the following 1-port system:

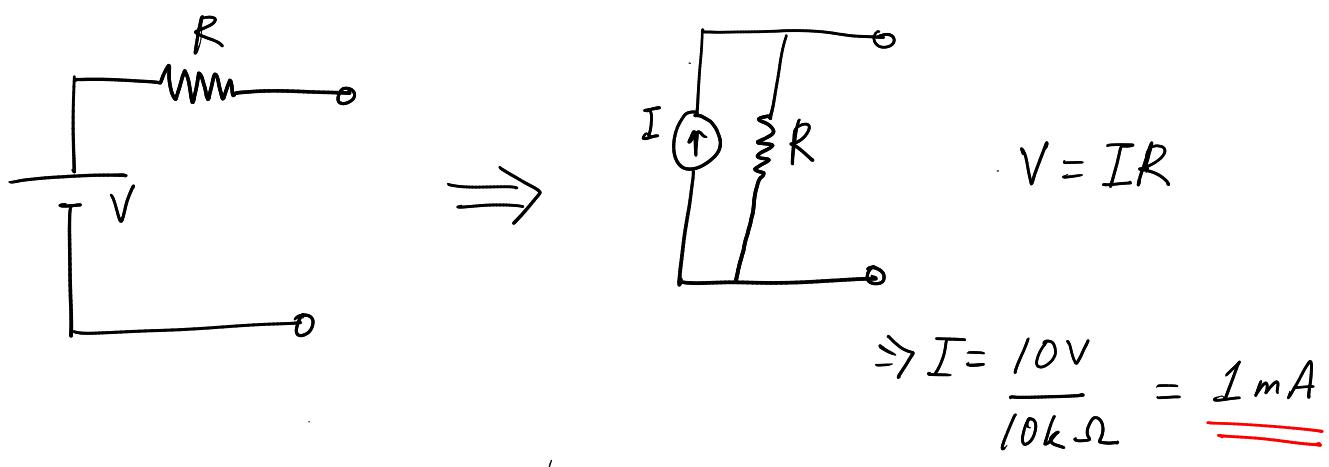


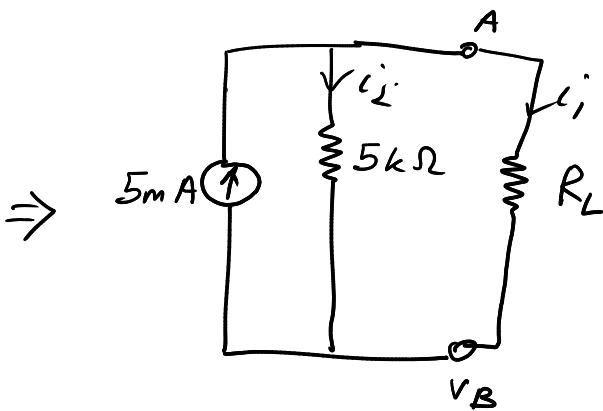
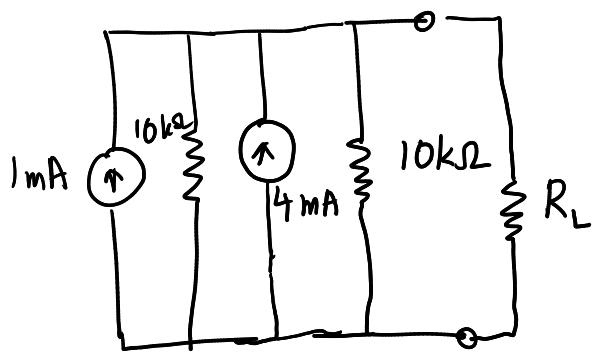
Attach load resistors $50\ \Omega$, $500\ \Omega$, $5\ k\Omega$, $50\ k\Omega$ and $500\ k\Omega$ to the port and calculate the resulting voltage across the load along with the current and power. [5+5+5]

Ans



We convert the battery to a current source and a resistor. Essentially, we go from a Thvenin circuit to a Norton Equivalent.





$$i_1 = \frac{5k\Omega}{(5k + R_L)\Omega} \cdot 5mA$$

$$i_2 = \frac{R_L}{(5k + R_L)} \cdot 5mA$$

$$V_{AB} = i_1 R_L = \frac{25 R_L}{(5000 + R_L)} \quad V$$

$$i_{AB} = i_1 = \frac{25}{(5000 + R_L)}$$

$$P_{AB} = \frac{625 R_L}{(5000 + R_L)} W$$

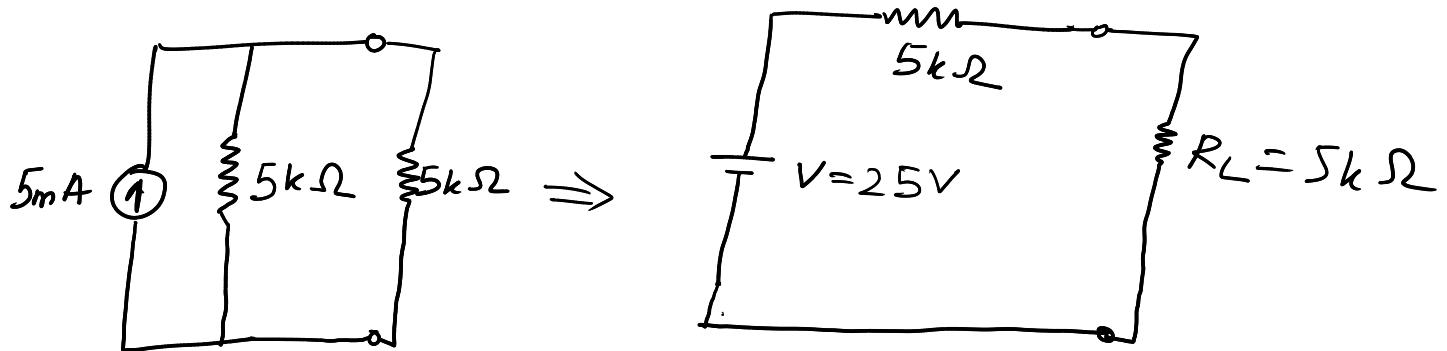
R_L	V_{AB}	i_{AB}	P_{AB}
1. 50Ω	0.25 V	4.95 mA	1.23 W
2. 500Ω	2.27 V	4.55 mA	10.33 W
3. $5k\Omega$	12.5 V	2.5 mA	31.25 W
4. $50k\Omega$	22.73 V	0.45 mA	10.33 W
5. $500k\Omega$	24.75 V	0.05 mA	1.23 W

3>

3. Considering the circuit as in the problem 2, construct the Thevenin equivalent circuit. Then calculate the current, voltage and power across a load resistance of $5 k\Omega$ that is connected in the equivalent circuit.

[2+1+1+1]

Ans. We have the Norton equivalent in Problem 2. We can just take the Norton equivalent and convert it to a Thevenin equivalent circuit.



$$V = 5mA \times 5k\Omega = 25V$$

$$R_{eff} = 10k\Omega$$

$$i_{AB} = V_{AB} / R_{eff} = 25V / 10k\Omega = 2.5mA$$

$$\begin{aligned} P_{AB} &= i^2 R_L = (2.5 \times 10^{-3})^2 \times 5 \times 1000 \\ &= 6.25 \times 5 \times 10^{-3} W \\ &= 31.25 mW \end{aligned}$$

$$V_{AB} = i_{AB} R_L = 2.5mA \times 5k\Omega = 12.5V$$

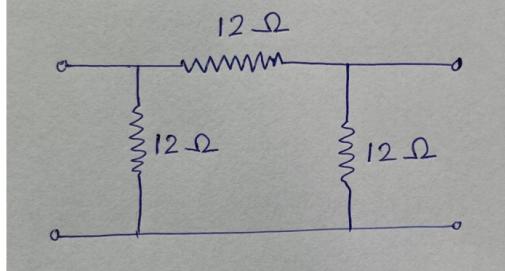
$$i_{AB} = 2.5mA$$

$$V_{AB} = 12.5V$$

$$P_{AB} = 31.25 mW$$

4.

4. Calculate the z-, y- and h-parameters for the following circuit:



[4+4+4]

$Z \rightarrow y, h$
parameters?

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

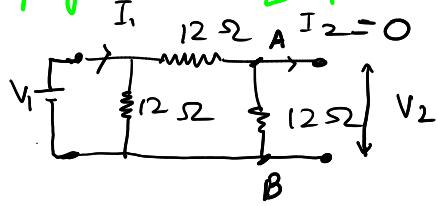
$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

Keeping $I_2=0$ [Open output]



$$\begin{aligned} R_{eq} &= (12+12) \Omega \parallel 12 \Omega \\ &= 24 \Omega \parallel 12 \Omega \\ &= \frac{24 \times 12}{24+12} \Omega = \boxed{8 \Omega} \end{aligned}$$

Keeping $I_2=0$

$$V_1 = 8I_1 \text{ [Ohm's Law]}$$

$$\Rightarrow Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = 8 \Omega$$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

$$V_2 = 12 I_{AB}$$

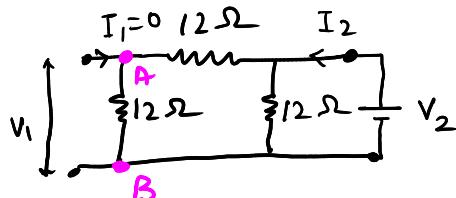
$$I_{AB} = \frac{12}{12+24} \times I_1 = \frac{12}{36} I_1 = I_1 / 3$$

$$= 4 I_1$$

$$\Rightarrow \frac{V_2}{I_1} \Big|_{I_2=0} = 4 \Omega = Z_{21}$$

$$Z_{11} = 8 \Omega \quad Z_{21} = 4 \Omega$$

Keeping $I_1=0$ [Open input]



$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

$$\Rightarrow Z_{22} = 8 \Omega$$

We have

$$V_2 = 8 I_2$$

$$V_1 = 12 \Omega I_{AB} \quad I_{AB} = \frac{12}{12+24} I_2 = \frac{I_2}{3}$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = 4 \Omega$$

$$Z_{11} = 8 \Omega \quad Z_{12} = 4 \Omega \quad Z_{21} = 4 \Omega \quad Z_{22} = 8 \Omega$$

$$Z = \begin{bmatrix} 8 & 4 \\ 4 & 8 \end{bmatrix}$$

Calculating h

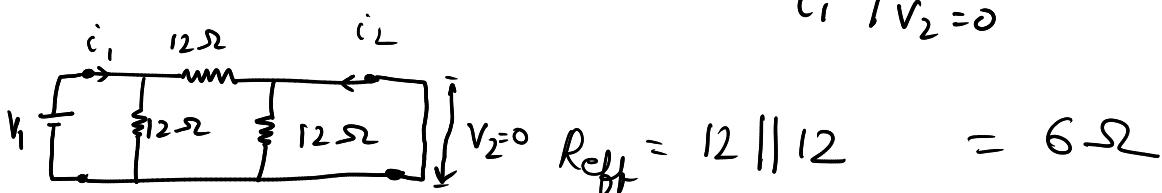
$$h = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

$$h_{11} = \left. \frac{V_1}{i_1} \right|_{V_2=0} \Omega$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{i_1=0}$$

$$h_{21} = \left. \frac{i_2}{i_1} \right|_{V_2=0}$$

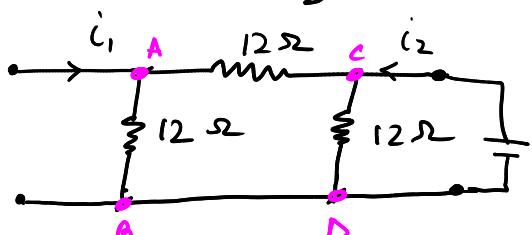
$$h_{22} = \left. \frac{i_2}{V_2} \right|_{i_1=0} S$$



$$V_1 = 6i_1 \Rightarrow h_{11} = 6 \Omega$$

$$i_2 = -\frac{12}{(12+12)} i_1 \Rightarrow \frac{i_2}{i_1} = -\frac{1}{2}$$

$$h_{21} = -\frac{1}{2}$$



$$R_{eff} = 8 \Omega$$

$$V_2 = 8i_2 \Rightarrow \frac{i_2}{V_2} = \frac{1}{8} S$$

$$h_{22} = \frac{1}{8} S$$

$$V_2 = 8 i_2 \quad i_{AB} = \frac{12}{(12+24)} i_2 = \frac{i_2}{3}$$

$$V_1 = 12 i_{AB} = 12 \frac{i_2}{3} = 4i_2 = \frac{8}{2} i_2 = \frac{V_2}{3}$$

$$\frac{V_2}{V_1} = \frac{1}{2} \Rightarrow h_{12} = \frac{1}{2}$$

$$h_{11} = 6\Omega \quad h_{21} = -\frac{1}{2} \quad h_{12} = \frac{1}{2} \quad h_{22} = \frac{1}{8}\text{S}$$

$$h = \begin{pmatrix} 6\Omega & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{8}\text{S} \end{pmatrix}$$

Calculating γ

we know

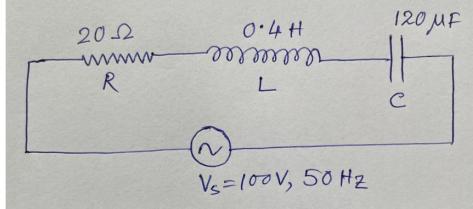
$$Y = Z^{-1}$$

$$= \begin{pmatrix} 8 & 4 \\ 4 & 8 \end{pmatrix}^{-1} = \frac{1}{48} \begin{pmatrix} 8 & -4 \\ -4 & 8 \end{pmatrix}$$

$$Y = \begin{pmatrix} \frac{1}{6} & -\frac{1}{12} \\ -\frac{1}{12} & \frac{1}{6} \end{pmatrix}$$

5. >

5. Consider the following RLC circuit:



Calculate the total circuit impedance, the circuit current, phase angle and draw the voltage phasor diagram. [2+2+2+2]

$$f = 50 \text{ Hz} \quad \omega = 2\pi f \\ = 100\pi$$

$$X_L = \omega L = 40\pi \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 120 \times 10^{-6}} \\ = \frac{1000}{314\pi} 250 = \frac{250}{3\pi}$$

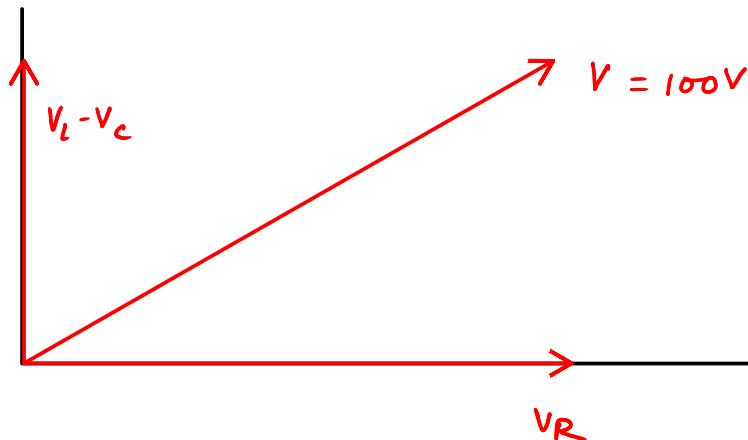
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2} \\ = \sqrt{400 + \left(40\pi - \frac{250}{3\pi}\right)^2} \\ = 101.35 \Omega$$

$$I = \frac{V_s}{|Z|} = \frac{100}{101.35} A = 0.988 A$$

$$\text{Phase angle } (\phi) = \tan^{-1} \left(\frac{X_L - X_C}{R} \right) = 78.59^\circ$$

$$V_L = I X_L = 124.25 V \quad V_C = I X_C = 26.22 V$$

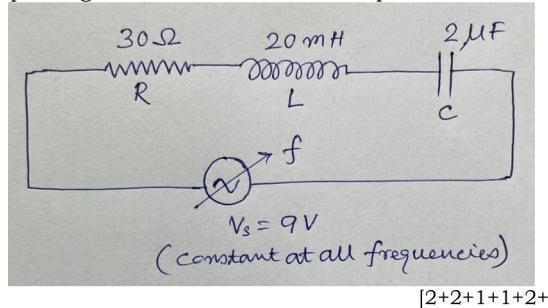
$$V_R = I R = 19.77 V$$



$Z = 101.35 \Omega$	$I = 0.988 A$	$\phi = 78.59^\circ$
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6.

6. Consider the following series resonance network and calculate the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance, the quality factor and the bandwidth of the circuit. Also sketch the corresponding current waveform for all frequencies.



$$f = \frac{\omega}{2\pi} = \frac{10^4}{2} = 5 \times 10^3 \text{ rad s}^{-1}$$

$$\Rightarrow \omega = \frac{10^4}{2}$$

$$= 5 \times 10^3 \text{ rad s}^{-1}$$

At resonance

$$\omega L = \frac{1}{\omega C}$$

$$\Rightarrow \omega^2 = \frac{1}{LC}$$

$$\Rightarrow \omega^2 = \frac{1}{2\pi \times 10^{-3} \times 2 \times 10^{-6}} = \frac{10^8}{4}$$

$$\text{At resonance} \Rightarrow |Z| = R \Rightarrow |Z| = 30 \Omega$$

$$I = \frac{V}{30} = 0.3 \text{ A}$$

$$V_L = I X_L = 0.3 \omega_R L = 0.3 \times 5 \times 10^3 \times 20 \times 10^{-3} = 30 \text{ V}$$

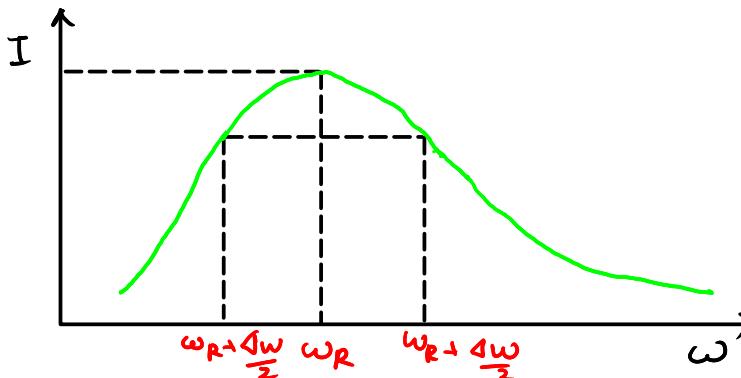
$$\text{At resonance} \quad \omega L = \frac{1}{\omega C}$$

$$\Rightarrow X_L = X_C \Rightarrow I X_L = I X_C \Rightarrow V_L = V_C$$

$$\Rightarrow V_C = 30 \text{ V}$$

$$\text{Quality Factor} \Rightarrow Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{10}{3}$$

$$\text{Bandwidth} \Rightarrow \Delta\omega = \frac{R}{L} = \frac{30}{2\pi \times 10^{-3}} = 1.5 \times 10^3 \text{ rad s}^{-1}$$



$$\omega_R = 5 \times 10^3 \text{ rad s}^{-1}$$

$$I = 0.3 \text{ A}$$

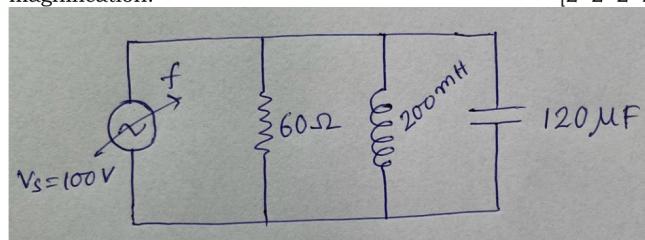
$$V_L = V_C = 30 \text{ V}$$

$$Q = 10/3$$

$$\Delta\omega = 1.5 \times 10^3 \text{ rad s}^{-1}$$

7.

7. Considering following parallel resonance network, calculate the resonant frequency, the quality factor, the bandwidth of the circuit, the circuit current at resonance and current magnification. [2+2+2+2]



At resonance,

$$\omega L = \frac{1}{\omega C}$$

$$\Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$Q = R \sqrt{\frac{C}{L}} = 1.47$$

$$\Rightarrow \omega_R = \frac{1}{\sqrt{200 \times 10^{-3} \times 120 \times 10^{-6}}} = 204.12 \text{ rad/s}^{-1}$$

$$\Delta \omega = \frac{1}{RC} = 138.9 \text{ rad s}^{-1}$$

$$I_{max} = \frac{V}{R} = \frac{V}{60} = \frac{100}{60} = 1.67A$$

$$\text{Imagnification} = Q I_{max} = 2.45A$$

$$\omega_R = 204.12 \text{ rad s}^{-1}$$

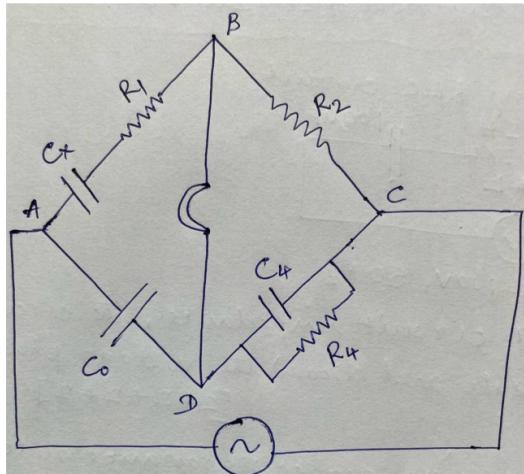
$$Q = 1.47$$

$$\Delta \omega = 138.9 \text{ rad s}^{-1}$$

$$I_{max} = 1.67A$$

$$I_{mag} = 2.45A$$

8.



- (i) For the above AC bridge, derive an expression to determine the capacitance of the unknown capacitor, i.e. C_x and also the series resistance R_1 . [3 + 3]
(ii) If the above bridge is working at a frequency of 2 kHz and the bridge constants at balance are $C_0 = 100 \mu\text{F}$, $C_4 = 50 \mu\text{F}$, $R_2 = 100 \text{k}\Omega$, $R_4 = 50 \text{k}\Omega$, then find the equivalent series circuit of the unknown impedance. [4]

We use the headphones to find the null-point,

$$\frac{Z_{AB}}{Z_{AD}} = \frac{Z_{BC}}{Z_{CD}}$$

$$\Rightarrow \left(R_1 - \frac{j}{\omega C_x} \right) j\omega C_0 \\ = R_2 \left(\frac{1}{R_4} + j\omega C_4 \right)$$

$$\Rightarrow \frac{C_0}{C_x} = \frac{R_2}{R_4} \quad R_1 C_0 = R_2 C_4$$

$$\Rightarrow R_1 = \frac{R_2 C_4}{C_0}$$

$$C_x = \frac{C_0}{R_2} R_4$$

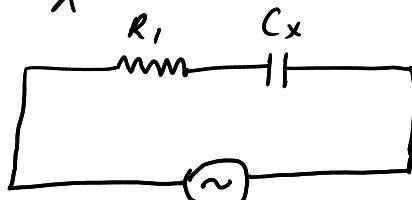
$$(ii) \quad C_x = \frac{100 \mu\text{F}}{100 \text{k}\Omega} 50 \text{k}\Omega = \underline{\underline{50 \mu\text{F}}}$$

$$R_1 = \frac{50 \mu\text{F} \cdot 100 \text{k}\Omega}{100 \mu\text{F}} = \underline{\underline{50 \text{k}\Omega}}$$

$$Z = 50 \times 10^3 - j \left(\frac{1}{2 \times 10^3 \times 50 \times 10^{-6} \times 2 \pi} \right)$$

$$= 50 \times 10^3 - \frac{5}{\pi} j$$

Equivalent series



$$Z = 50 \times 10^3 - j \left(\frac{5}{\pi} \right)$$