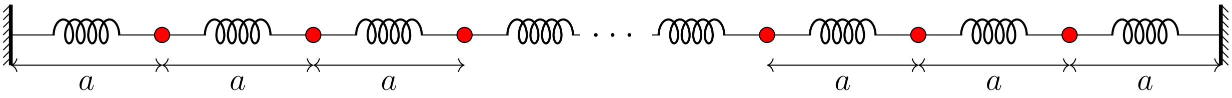
Total marks: 40

1. Consider a beaded string of N beads each of mass m (approximated as a long chain of spring-mass system as shown in the figure). The beads are uniformly placed on the string and the string has a uniform tension T. The horizontal distance between any two beads in equilibrium is a. The unstretched lengths of the springs are negligible. [14]



   (a) Find the equation of the motion of n^{th} bead for the longitudinal mode of vibration. [2]

   (b) Assuming normal mode vibration, find the normal mode frequency ω_m for m^{th} mode. [2]

   (c) Find the amplitudes of the beads in m^{th} mode ($A_n^{(m)}$ using the notation used in the class). [2]

   (d) Plot the dispersion relation ω versus k. [2]

   (e) Check whether we have $\omega_{N+2} = \omega_N$? [2]

   (f) Check whether we have $A_n^{(N+2)} = A_n^{(N)}$? [2]

   (g) Qualitatively plot $A_n^{(1)}$ and $A_n^{(N)}$ for all n. [2]

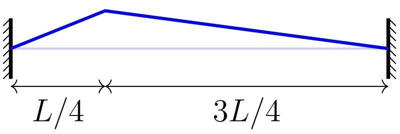
2. Consider two pendulums, a and b, with the same string length L, but with different bob masses, M_a and M_b. They are coupled by a spring of spring constant K which is attached to the bobs. Assuming small angle oscillations, [8]

   (a) Find the equations of motion using angles of the pendulums (w.r.t. the vertical) as dynamical variables. [3]

   (b) Find the normal modes and the normal frequencies. [2+2]

   (c) For $M_a = M_b = M$, does this reduce to the case considered in the class? [1]

3. Consider the following string, with the given configuration [8]



   (a) Find the Fourier representation of the string. You should use the sine representation for the string (not the full representation). [7]

   (b) Show that normal modes having nodes at $L/4$ are absent. [1]

   (c) Check numerically that your solution matches with the given shape. You may submit your codes.

4. Consider the following pattern. Find the Fourier representation of this pattern. Use the complete representation (using sine and cosine). Also, check numerically that your solution matches with the given shape. You may submit your codes. [10]