If you are using a jupyter notebook (recommended), then keep all your programs in a single notebook. A good programming style is to define a function for one task with clearly defined input (arguments) and output. For plots you may use matplotlib (if you are using python) or gnuplot (if you are using c or fortran) or LsqFit module if you are using Julia.

If you are planning to submit separate programs, then please follow the guideline below:

- Keep all files of a worksheet in a single folder.
- Follow a systematic naming convention. You may name the program files as `Q1.py` or `Q1a.py`, `Q1b.py` for question 1 (if you have created multiple files for a single question). The data file should be named as `Q1-data-a.dat` and so on.
- Finally compress the entire folder as a single `.zip` or `.tgz` (using `tar cvfz archive.tgz folder-name/`, and submit the file in WeLearn.

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1. **Cooling a Hot Rod with Fixed-End Temperatures**

   (a) **Physical Setup:**
   A metal rod of length L is held at both ends ($x = 0$ and $x = L$) at a fixed, lower temperature $T_0 = 100$ K. The initial temperature of the rod is uniformly higher, say $T_{\text{init}} = 300\text{K} > T_0$. Let the thermal diffusivity be $\alpha = 10^{-4}$. Thus, you have

   $$u(0,t) = T_0, \quad u(L,t) = T_0, \quad u(x,0) = T_{\text{init}}.$$

   (b) (7 points) **Implementation:**
   Implement a solver that
   - Sets up the tridiagonal system $A\,\mathbf{u}^{n+1} = B\,\mathbf{u}^n$ at each time step.
   - Uses the *Thomas algorithm* for efficient solving.
   - Enforces $u(0,t) = T_0$ and $u(L,t) = T_0$ at every step.

   (c) (3 points) **Plot and Analyze:**
   - Plot the initial temperature distribution ($t = 0$) and the final distribution after some time t_{final}.
   - Observe how the rod cools down to T_0 at both ends.
   - Discuss the long-term solution as $t \to \infty$.

2. **Heating with Different Dirichlet Temperatures at Each End**

   (a) **Physical Setup:**
   Consider the same rod of length L, but now the left end is held at $T_{\text{left}} = 200$ K and the right end at $T_{\text{right}} = 400$ K, where $T_{\text{left}} \neq T_{\text{right}}$. The rod starts from an initial condition $u(x,0) = T_{\text{init}}(x) = 300 + exp(-(x - L/2)^2/2\sigma^2)$ with $\sigma = 0.05$, and the thermal diffusivity is still $\alpha = 10^{-4}$.

   (b) **Dirichlet Boundary Conditions:**
   You have

   $$u(0,t) = T_{\text{left}}, \quad u(L,t) = T_{\text{right}}, \quad u(x,0) = T_{\text{init}}(x).$$

   (c) (7 points) **Numerical Scheme:**
   - Write down the Crank–Nicolson update for interior points $i = 1, \ldots, N-1$.
   - Form the matrices A and B, and describe how the boundary temperatures appear in the right-hand side.
   - Solve repeatedly from t^n to t^{n+1} until some final time t_{final}.

   (d) (3 points) **Results and Steady State:**
   - Plot the temperature profiles at several time steps (e.g. $t_1,\ t_2,\ \ldots$).
   - Show that eventually the solution approaches a *linear* steady state from T_{left} to T_{right}.
   - Investigate the effect of different Δt, Δx, or α on the convergence speed.