Total marks: 20

1. Consider a dispersion relation $\omega = vk$, where the symbols have their usual meanings. We construct a wave packet by choosing sinusoids having a form $\sin(kx - \omega t)$ from $-k_\circ/4$ to $k_\circ/4$, with uniform amplitude for any k. [4]
   (a) Calculate the shape of the wave packet. [3]
   (b) What is the group velocity of this packet? [1]
   (c) [Optional] If possible simulate (calculate and animate) and upload how the packet moves with time.

2. Consider a damped harmonic oscillator whose equation of motion is given by, [6]

$$\ddot{x} + \alpha\dot{x} + \omega_\circ^2 x = 0,$$

   where symbols carry their usual meanings.
   (a) Find the solution of the above equation, for given initial conditions $x(0) = x_\circ$ and $\dot{x}(0) = v_\circ$. [4]
   (b) To find the solution for the critically damped case, check the solution at the limit of vanishing resonance frequency. [2]

3. Consider the following wave equation: [5]

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2},$$

   where, v is a constant speed. Check whether this equation is Lorentz invariant. What if $v = c$, where c is the speed of light in vacuum.

4. Consider a continuous string of length L whose one end is fixed and the other end is free to move (it slide on frictionless rods that pass through massless rings at the end of the string). [5]
   (a) Construct the equation of motion (you start with a beaded string and take the continuum limit). [3]
   (b) Given that the free end is always at the antinode position, find the possible wavelengths. [2]