PH3102 Quantum Mechanics Assignment 1

Instructor: Dr. Siddhartha Lal Autumn Semester, 2024

Start Date: August 6, 2024 Submission Deadline: August 13, 2024 .
Submit your answers to the Tutor at the start of the tutorial.

Q1. Mathematical Preliminaries [5 marks]

- (a) Solve the integrations (i) $\int_0^\infty xe^{-\alpha x^2}\,dx$, (ii) $\int_0^\infty x^3e^{-\alpha x^2}\,dx$, (iii) $\int_0^\infty x^5e^{-\alpha x^2}\,dx$
- (b) Calculate $\nabla \cdot \vec{r}$, $\nabla \times \vec{r}$
- (c) If $\begin{vmatrix} y^2+3y & y-1 & y+3 \\ y+1 & -2y & y-4 \\ y-3 & y+4 & 3y \end{vmatrix} = Ay^4 + By^3 + Cy^2 + Dy + E$, calculate the value of A, E.

Q2. Time Average [5 marks]

Imagine you are standing on the roof of the Research Complex (RC) building at a height H above the ground. You now drop a stone from there with an initial velocity 0. Your friend photographs the fall of the stone, and measures the distance of the falling stone every picosecond (i.e., at extremely small time intervals). What is the average of all these distances?
Hint: Ignore air resistance during the free fall, and take time to be a continuous variable. Compute the probability dt/T for a photo being clicked in an interval dt of the stone's fall through a distance lying in the interval x and $x + dx$ from the top of the building, and T being the total fall time through a distance H, as a function of x, dx and H. Now use this probability distribution to compute the average displacement.

Q3. Step Potential [10 marks]

Consider the step potential in one spatial dimension given by

$$V(x) = \begin{cases} 0 & \text{for } x < 0, \\ V_0 & \text{for } x \geq 0\ . \end{cases}$$

(a) Write the Schrödinger equation for this potential.
(b) Solve the Schrödinger equation for the cases (i) $E > V_0$ and (ii) $E < V_0$. Comment on the physically allowed states from your solutions.
(c) Compute the analytic expressions for the reflection (R) and transmission (T) coefficients for both cases (i) $E > V_0$ and (ii) $E < V_0$. Do you find any differences between classical and quantum mechanics in these cases? If so, what are they?
(d) Calculate the analytic expression for the probability density ($P(x)$) of the wavefunction $x \geq 0$ for the case $E < V_0$. Evaluate the expression for the cases $P(x)|_{x\to 0}$ and $P(x)|_{x\to\infty}$, and comment on the results you obtain.