11. August 2023

Linear Algebra I (MA2102)

Assignment 1

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Exercise 1. (5+5 points)
(*i*) Explain why a system of homogeneous linear equations with more variables than equations always has a nontrivial solution, whereas a system of such equations with more equations than variables may not have any nontrivial solution.
(*ii*) Show that a matrix with more columns than rows (resp. more rows than columns) does not have a left (resp. right) inverse.

Exercise 2. (10 points) Compute the determinant of the following matrix:

$$\begin{pmatrix} 2 & -2 & & & & & \\ -1 & 5 & -2 & & & 0 & \\ & -2 & 5 & -2 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & -2 & 5 & -2 & \\ & 0 & & & -2 & 5 & -1 \\ & & & & & -2 & 2 \end{pmatrix}_{n\times n}$$

Exercise 3. (10 points) Let λ be an eigenvalue of an $n \times n$ real matrix A. Show that there exists a positive integer $k \leq n$ such that

$$|\lambda - a_{kk}| \leq \sum_{j=1,\, j\neq k}^{n} |a_{jk}|.$$

Exercise 4. (10 points) Show that an $n \times n$ real matrix is invertivle if and only if its columns span $\mathbb{R}^n$.

Exercise 5. (5+10 points) (*i*) Let V be the set of all real numbers. Define the binary operation "addition" on V by

$$x \boxplus y = \text{ the maximum of } x \text{ and } y$$

for all $x, y \in V$ and define an operation of "scalar multiplication" by

$$\alpha \boxdot x = \alpha x$$

for all $\alpha \in \mathbb{R}$ and $x \in V$. Is V is a vector space over $\mathbb{R}$ under the above operations? Justify your answer!

(*ii*) Let V be the set of all positive real numbers. Define the binary operation "addition" on V by

$$x \boxplus y = xy$$

for all $x, y \in V$. Define an operation of "scalar multiplication" by

$$\alpha \boxdot x = x^{\alpha}$$

for all $\alpha \in \mathbb{R}$ and $x \in V$. Show that V is a vector space over $\mathbb{R}$ under the above operations. Provide a basis of V.

Maximum score: 50 points. You may answer as many questions as you wish. If the sum of your total score exceeds 49, you shall get the maximum score. Please mention your name, roll no. and **group** in your answersheet. Please submit your answersheet by 8:00 a.m. on 17th August, 2023 in the DMS mailbox for MA2102, which is designated with your group name.