Classtest 1 of 3

Instructor: Dr. Rajesh Kumble Nayak
6:00 PM, 1 September 2023.
Duration: 50 Minutes.

- Answer all the question.
- All question carry equal marks.
- No calculators are allowed!

Q - 1: Plot the function

$$f(x) = \frac{1}{1-x} + \frac{1}{1+x},$$

In an appropriate domain.

$$f(x) = \frac{1}{1-x} + \frac{1}{1+x} = \frac{1+\cancel{x}+1-\cancel{x}}{1-x^2} = \frac{2}{1-x^2}$$

$$f(x) = \frac{2}{1-x^2} \;;\; = \frac{2}{(1-x)(1+x)}$$

domain $(-\infty, \infty]$

singular point $x = \pm 1$

at $\pm\infty \to 0$; at $0 \to 2$

at ± 1 ; let us first look

at $x_0 = -1-\epsilon$

$$f(x) = \frac{2}{(1-x)(1+x)} \qquad (1-x) = \cancel{1} - (-\cancel{1} - \epsilon) = 2+\epsilon$$

$$(1+x) = \cancel{1} - \cancel{1} - \epsilon = -\epsilon$$

$$f(x) \longrightarrow -\infty$$

as $\epsilon \longrightarrow 0$ at -1

Sides to at -1

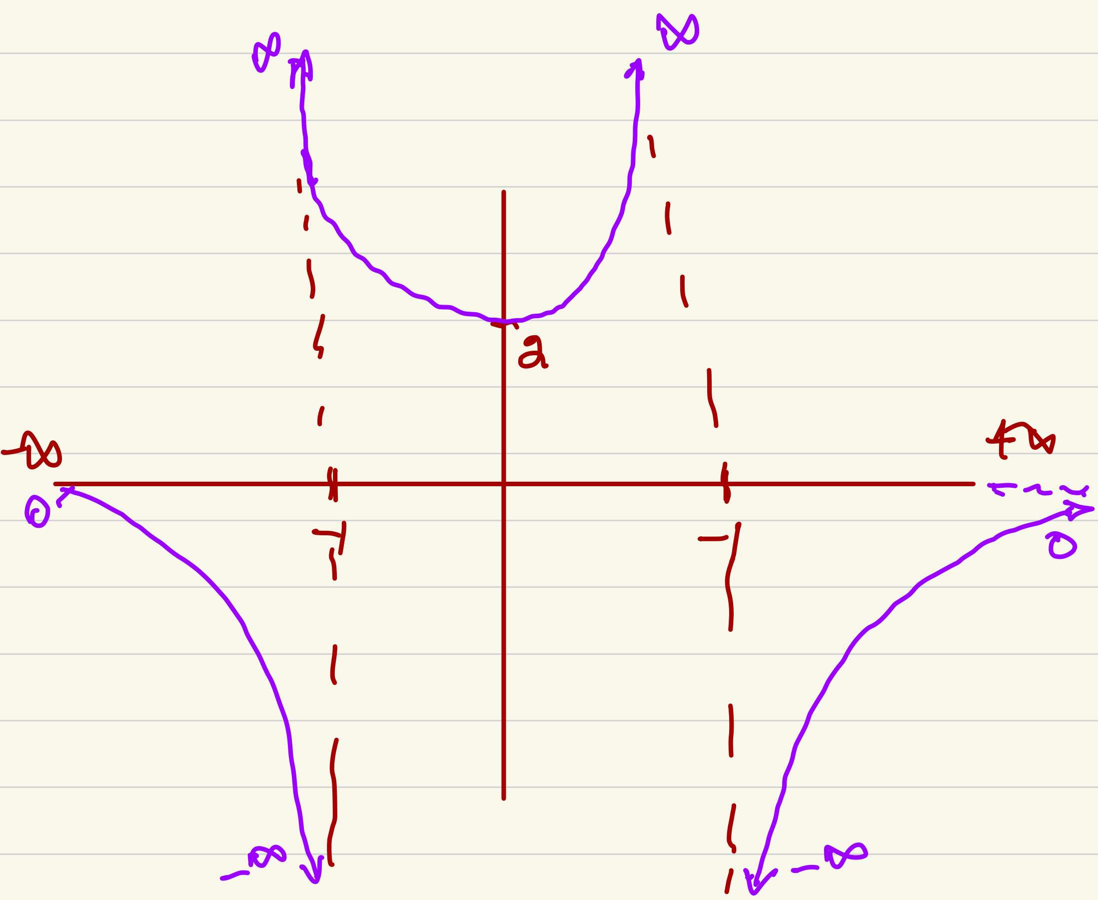
Let us write $x = -1+\epsilon$

$$1-x = 1-(-1+\epsilon) = 2-\epsilon$$

$$1+x = \cancel{1} - \cancel{1} + \epsilon = \epsilon$$

$$f(x) = \frac{1}{(2-\epsilon)(\epsilon)} \longrightarrow +\infty$$

as $x \longrightarrow -1$ from the



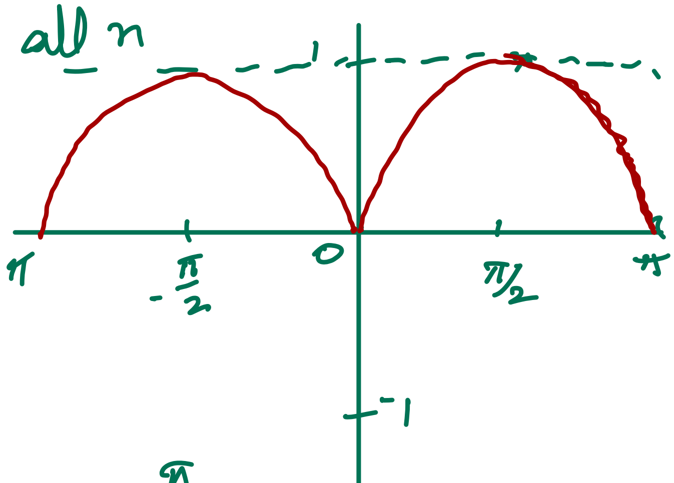
Q - 2: Find the Fourier series expansion of the function

$$f(x) = \begin{cases} \sin x & x > 0 \\ -\sin x & x < 0 \end{cases},$$

Make plot of function $f(x)$, first two individual terms and the partial sum of two terms.

the function is even function around zero

So $b_n = 0$ for all n



Let us find a_0

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} -\sin x\,dx + \int_{0}^{\pi} \sin x\,dx$$

$$= \frac{2}{2\pi}\int_{0}^{\pi} \sin x\,dx = \frac{4}{2\pi} = \frac{2}{\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} \sin x \cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} \sin x \cos nx\,dx$$

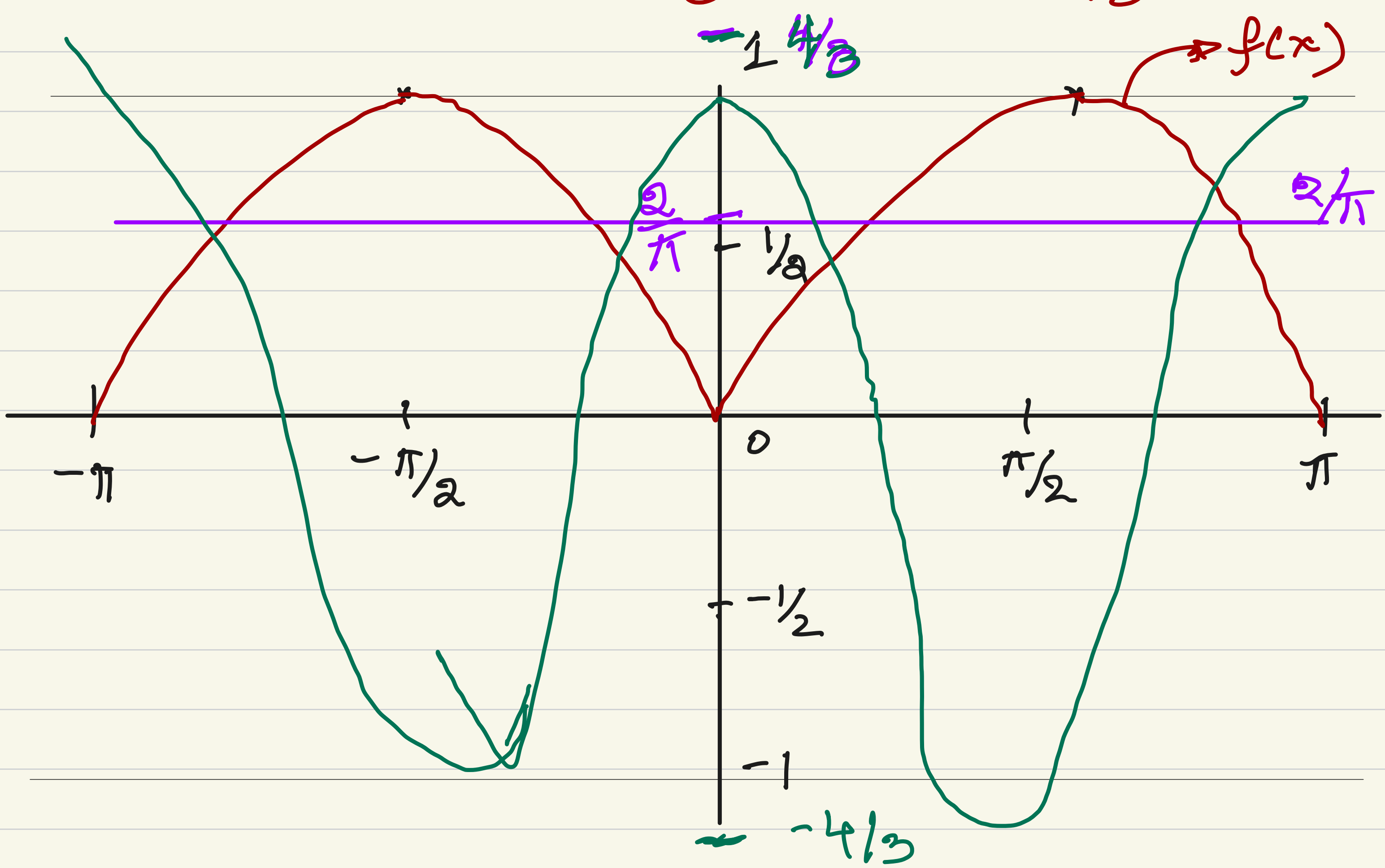
$$a_n = \frac{1 + \cos n\pi}{1-n^2} + \frac{1+\cos n\pi}{1-n^2}$$

$$= \frac{2(1+\cos n\pi)}{1-n^2}$$

$$= 0 \text{ for odd } n$$

$$= \frac{4}{1-n^2} \text{ for even}$$

$$f(x) = \frac{2}{\pi} - \frac{4}{3}\cos 2x - \frac{4}{15}\cos 4x$$



Q - 3: The Dirac delta function $\delta(x)$ is an improper function with the property,

$$\int_a^b f(x)\delta(x-x_0)dx = \begin{cases} f(x_0) & \text{if} a < x_0 < b \\ 0 & \text{0therwise} \end{cases}$$

Assuming $\delta(x - x_0)$ is a $\mathbb{L}^2$ function provide a Fourier Series Expansion in the interval $[-L, L]$ for some $+ve$ L, and $x_0 \in [-L, L]$.

Let $f(x) = \delta(x - x_0)$

$$a_0 = \frac{1}{2\pi}\int f(x)\,dx$$

$$= \frac{1}{2\pi}\int_{-L}^{L} f(x)\,dx = \frac{1}{2\pi}\ ?$$

$$a_0 = \frac{1}{2\pi}$$

$$a_n = \frac{1}{\pi}\int_{-L}^{L} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-L}^{L} \delta(x - x_0)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\cdot\cos n x_0$$

$$b_n = \frac{1}{\pi}\int_{-L}^{L} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-L}^{L} \delta(x-x_0)\sin nx\,dx$$

$$= \frac{1}{\pi}\sin nx_0)$$

$$a_0 = \frac{1}{2\pi}$$

$$a_n = \frac{1}{\pi}\cos nx_0$$

$$b_n = \frac{1}{\pi}\sin nx_0$$

$$\delta(x-x_0) = \frac{1}{2\pi} + \frac{1}{\pi}\cos nx_0\cos nx$$

$$+ \frac{1}{\pi}\sin nx_0\sin nx$$