Worksheet 1: Elementary programming

If you are using Julia or Python, we recommend using a jupyter notebook. In WeLearn, you need to submit this file. Please clearly indicate in the markup cells, the number of the question for which you are writing the program. Also, please remember to add documentation through comments in your program.

You may also use scripts and use REPL to evaluate them. In that case, please keep all your files for a particular worksheet in a folder and you may upload the compressed archive of that folder.

Please feel free to ask for help!

1. (2 points) Write a function `is_prime` that checks whether a given integer (n) is prime. For this, check whether any integer from 2 to $[\sqrt{n}]$ divides n. If it does, then the function should return `false` else it should return `true`.

2. (4 points) Write a function that generates all prime numbers between any two given integers.

3. (4 points) Write a function `f(x, n)` that evaluates the series

$$\exp(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

   upto n^{th} term, for given x and n.

4. (4 points) Write a program that prints the calculated values of `f(x,n)` for $x = 0.1, 0.2, \ldots, 1.0$ for $n = 4, 8, 12, 16, 20$. Your program should print a table where rows correspond to changing x and columns correspond to changing n. The table should be formatted to contain exactly 6 decimal places for each number.

5. (6 points) Write a function that uses the continued fraction

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \ldots}}}}$$

   to evaluate π upto n terms. Also write a function that uses Leibniz formula

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots$$

   to evaluate π upto n terms.

   Write a program that prints the output of both the functions for $n = 1, 2, \ldots, 10$ (as a formatted table with 8 decimal places for each number). Compare their convergence by plotting them in the same plot versus n.