PH2202 Thermal Physics
Fall Semester - 2024
Indian Institute of Science Education and Research, Kolkata
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Homework: 8 Submission Date: 23/04/2024

The hand written solutions must be submitted at the start of the tutorial.

1. Consider a system in thermal equilibrium with temperature T and with two energy states with energy $E_1 = 0$ and $E_2 = \epsilon$.

   (a) Write down the partition function Z_1 of a *single* particle.

   (b) Write down the partition function Z_2 for *two distinguishable* particles. How Z_2 is related to Z_1?

   (c) Write down the partition function Z_3 for *two indistinguishable* particles. Note that the relation in part (b) does not hold between Z_3 and Z_1. Which configurations (microstates) are spoiling the relation?

2. Consider a quantum mechanical one-dimensional simple harmonic oscillator that is in thermal equlibrium at temperature T. Its energy is given by $E_n = (n + \frac{1}{2})\hbar\omega$, where n is the quantum number of the oscillator with $n = 0, 1, 2, .., +\infty$, and ω is its angular frequency.

   (a) Write down the partition function Z for the system, and simplify it by doing the sum.

   (b) Considering $e^{-\beta E_n}$ is the probability for the particle being in the state of energy E_n, show that $\langle E \rangle = -\frac{\partial}{\partial \beta}\ (ln\ Z)$. Here $\beta = 1/(kT)$, k being the Boltzman constant.

   (b) Calculate the average energy using the above formula.

   (c) Evaluate the average energy in the limit of $\hbar\omega/kT \ll 1$ from the expression in (b). You should recover some familiar result, and justify the result.

   (d) Consider now the opposite limit $\hbar\omega/kT \gg 1$, and find average energy from the expression in (b). Justify the result from your understanding of quantum mechanics for $T \to 0$.