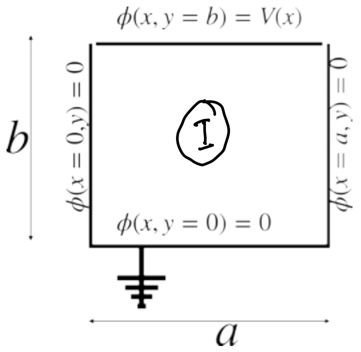
Tutorial – 02 16/01/2025 8:00 AM

Problem 1 : find electric potential inside (region (I)) for the boundary condition given in the figure

$$\nabla^2\phi = 0$$

with $\phi(x, y=0) = 0$; $\phi(x, y=b) = V(x)$

$$\phi(x=0, y) = 0 = \phi(x=a, y)$$



Sol^n : We need to solve $\nabla^2\phi = 0$

The Cartesian Coordinate is most suitable for this problem

The Laplacian in Cartesian system is given by

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \quad \text{——— (1)}$$

We make an ansatz that $\boxed{\phi(x,y) = \phi_x(x)\,\phi_y(y)}$ ——— (2)

Substituting (2) in (1) and simplifying, we get.

$$\frac{d^2\phi_x(x)}{dx^2} + \alpha^2\phi_x(x) = 0 \quad \& \quad \frac{d^2\phi_y}{dy^2} - \alpha^2\phi_y = 0$$

we propose sol^n

$$\phi_x = A_n \sin\frac{n\pi}{a}x$$

for $n = 1, 2, 3 \ldots\ldots$

with this we get $\alpha = \alpha_n = \frac{n\pi}{a}$

for ϕ_y we propose

$$\phi_y = B_n' \sinh\frac{n\pi}{a}y$$

with these we get

$$\phi = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{a} \sinh\frac{n\pi y}{a}$$

which satisfy boundaray condition at

$$\phi(x, y=0) = 0 \; ; \; \phi(x=0, y) = 0 = \phi(x=a, y)$$

We need to satisfy final condition, $\phi(x, y=b) = V(x)$

$$\phi(x, y=b) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi x}{a}\right) \sinh\left(\frac{n\pi b}{a}\right)$$

$$\|$$

$$V(x) = \sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi b}{a}\right) \sin\left(\frac{n\pi x}{a}\right) \quad \text{——— } (3)$$

C_n can be determined from Fourier components of $V(x)$

If $f(x)$ is $\mathbb{L}^2$ function in the interval $[0, a]$ then

it can be expressed as,

$$f(x) = a_0 + \sum_{n=1} a_n \cos\frac{n\pi x}{a} + \sum_{n=1} b_n \sin\frac{n\pi x}{a}$$

$$a_0 = \frac{1}{a}\int_0^a f(x)\,dx$$

$$a_n = \frac{2}{a}\int_0^a f(x)\cos\frac{n\pi x}{a}\,dx$$

$$b_n = \frac{2}{a}\int_0^a f(x)\sin\frac{n\pi x}{a}\,dx$$

multiply eq (3) by $\sin\frac{m\pi x}{a}$ on both side and integrate

from $0 \rightarrow a$

$$\frac{a}{2}\int_0^a \sin\frac{m x\pi}{a} V(x)\,dx = \frac{a}{2}\int \sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi b}{a}\right) \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right)$$

$$b_n = \frac{a}{2} C_n \sinh\left(\frac{n\pi b}{a}\right)$$

$$C_n = \frac{2}{a} b_n \frac{1}{\sinh\left(\frac{n\pi b}{a}\right)}$$

The Sol^n is

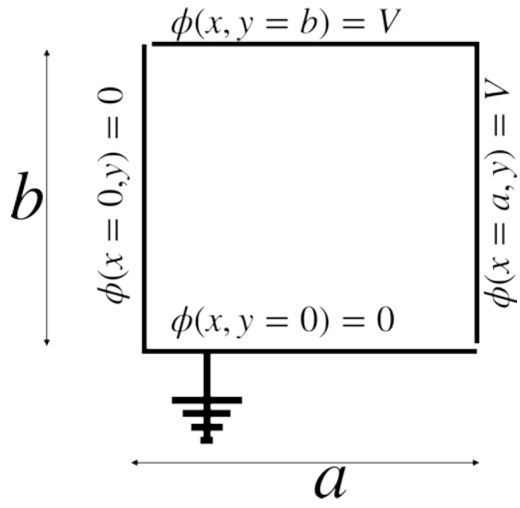
$$\phi(x, y) = \sum_{n=1}^{\infty} \frac{2 b_n}{a} \frac{1}{\sinh\left(\frac{n\pi b}{a}\right)} \sin\left(\frac{n\pi x}{a}\right) \sinh\left(\frac{n\pi y}{a}\right)$$

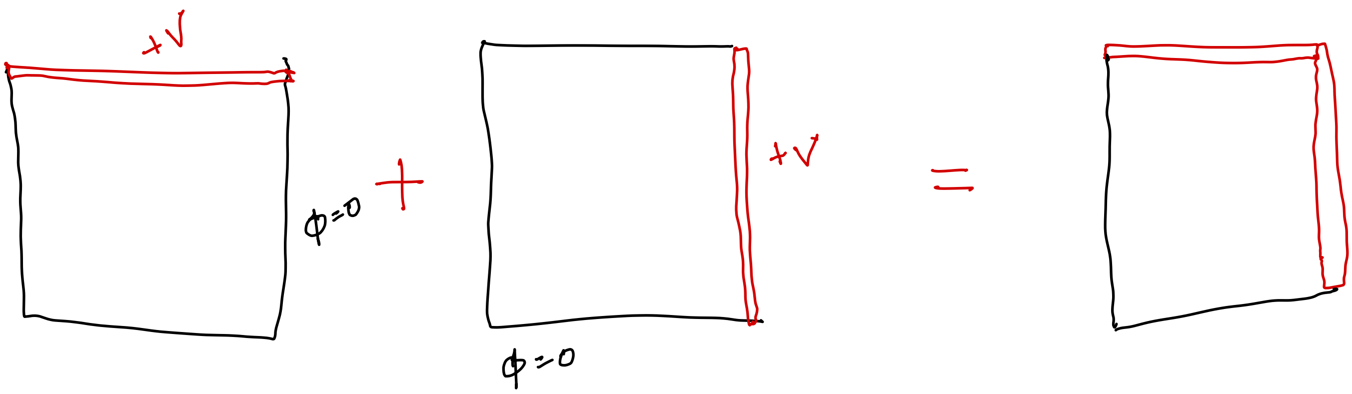
Problem 2: find electric potential inside (region (I)) for the boundary condition given in the figure

$$\nabla^2 \phi = 0$$

with $\phi(x, y=0) = 0$; $\phi(x, y=b) = V$

$\phi(x=0, y) = 0$ $\phi(x=a, y) = V$





Sol ⊔ , Sol ⊏ and add the sol^n algebra need to done!