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Probability I

MA2202

Assignment 2

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Exercise 1. (10 Points)
There are n boxes numbered $1, 2, \ldots, n$, among which the rth box contains $r-1$ white cubes and $n-r$ red cubes. Suppose, we choose a box at random and we remove two cubes from it, one after another, without replacement.

(a) Find the probability of the second cube being red.
(b) Find the probability of the second cube being red, given that the first cube is red.

Exercise 2. (10 Points)
Let $(\Omega, \mathcal{E}, P)$ be a probability space and let $A_1, A_2, \ldots, A_n \in \mathcal{E}$ with $P(A_1 \cap \cdots \cap A_n) \neq 0$. Show that $P(A_1 \cap \cdots \cap A_n)$ is equal to

$$P(A_1)\, P(A_2 \,|\, A_1)\, P(A_3, |A_2 \cap A_1) \cdots P(A_n \,|\, A_{n-1} \cap \cdots \cap A_1).$$

Exercise 3. (10 Points)
Let $(\Omega, \mathcal{E}, P)$ be a probability space and let $A_1, A_2, \ldots, \in \mathcal{E}$ be pairwise mutually exclusive. Let $A = \cup_{n=1}^{\infty} A_n$ and let $B \in \mathcal{E}$ with $P(B) \neq 0$. Show that

$$P(A \,|\, B) = \sum_{n=1}^{\infty} P(A_n \,|\, B).$$

Exercise 4. (9+3+3 points)
We are familiar with the famous Monty Hall problem. Now suppose, instead of 3 doors, there are n doors, only one among which has a prize behind it.

(a) Find the probability of winning upon switching given that Monty opens k doors. Will switching benefit you?
(b) Find the probability of winning upon switching given that Monty opens maximum number of doors. Will switching benefit you?
(c) Find the probability of winning upon switching given that Monty opens no doors. Will switching benefit you?

Exercise 5. (10 points)
Dropping two points uniformly at random on $[0, 1]$, the unit interval is divided into three segments. Find the probability that the three segments obtained in this way form a triangle.

Maximum score: 50 points. Please mention your name, roll no. and **group** in your answersheet. Please submit your answersheet by 11:59 p.m. on **Frbruary 1, 2024** in the DMS mailbox for MA2202, which is designated with your group name. Submit only your final answersheet! **Multiple answersheets or corrections to the original answersheet will not be accepted, irrespective of the time of their submission.**