Total marks: 25

1. Consider a one-dimensional oscillator of unit mass and of natural frequency $\omega_\circ$ (natural frequency: frequency of undamped oscillation). The oscillator experiences a velocity dependent damping with a damping coefficient γ. [8]

   Initially the oscillator was at rest $x(0) = \dot{x}(0) = 0$. The oscillator is subjected to a force given by $F_\circ \cos \omega t$.

   (a) Find the complete solution as a function of time. [6]

   (b) Check the case when the driving frequency is equal to the natural frequency. [2]

2. Consider how to solve the steady-state motion of a forced oscillator if the driving force is of the form $F = F_\circ \sin \omega t$ instead of $F_\circ \cos \omega t$. [5]

3. A block of mass m is connected to a spring, the other end of which is fixed. There is also a viscous damping mechanism. The following observations have been made on this system: [12]

   1. If the block is pushed horizontally with a force equal to mg, the static compression of the spring is equal to h.
   2. The viscous resistive force is equal to mg if the block moves with a certain known speed u.

   (a) For this complete system (including both spring and damper) write the differential equation governing horizontal oscillations of the mass in terms of m, g, h, and u. [2]
   Answer the following for the case that $u = 3\sqrt{gh}$:

   (b) What is the angular frequency of the damped oscillations? [1]

   (c) After what time, expressed as a multiple of $\sqrt{h/g}$, is the energy down by a factor $1/e$? [1]

   (d) What is the quality factor Q of this oscillator? [2]

   (e) This oscillator, initially in its rest position, is suddenly set into motion at $t = 0$ by a bullet of negligible mass but nonnegligible momentum traveling in the positive x direction. Find the value of the phase angle ϕ in the equation $x = Ae^{-\alpha t/2} \cos(\omega t - \phi)$ that describes the subsequent motion, and sketch x versus t for the first few cycles. [3]

   (f) If the oscillator is driven with a force $mg \cos \omega t$, where $\omega = \sqrt{2g/h}$, what is the amplitude of the steady-state response? [3]