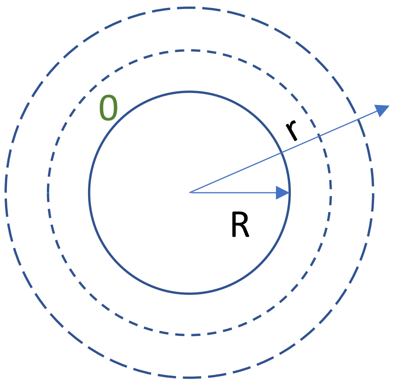
c_0

Setting a fundamental limit on size of (aerobic) bacteria



$$c(r) = c_0(1 - \frac{R}{r})$$

Concentration gradient of oxygen

$$I = 4\pi D R c_0$$

Max Consumption rate

$$-j 4\pi r^2 = I$$

$$D\frac{dc}{dr} = \frac{I}{4\pi r^2}$$

$$c(r) = A - \frac{1}{r}\frac{I}{4\pi D}$$

$$c(\infty) = A;\; A = c_0$$

$$c(R) = 0; I = 4\pi D R c_0$$

$$c(r) = c_0 - \frac{1}{r}\frac{4\pi D R c_0}{4\pi D}$$

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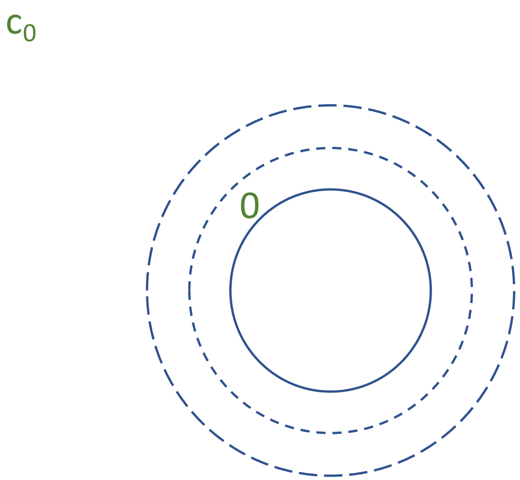
Setting a fundamental limit on size of (aerobic) bacteria



$$c(r) = c_0\left(1 - \frac{R}{r}\right)$$

$$c(r) = A - \frac{1}{r}\frac{I}{4\pi D}$$
$$c(\infty) = c_0$$
$$c(R) = 0$$
$$I = 4\pi D R c_0$$

Example Find the full concentration profile $c(r)$ and the maximum number of oxygen molecules per time that the bacterium can consume.

Solution: Imagine drawing a series of concentric spherical shells around the bacterium with radii r_1, r_2, ... Oxygen is moving across each shell on its way to the center. Because we're in a quasi-steady state, oxygen does not pile up anywhere: The number of molecules per time crossing each shell equals the number per time crossing the next shell. This condition means that $j(r)$ times the surface area of the shell must be a constant, independent of r. Call this constant I. So now we know $j(r)$ in terms of I (but we don't know I yet).

Next, Fick's law says $j = D\frac{\mathrm{d}c}{\mathrm{d}r}$, but we also know $j = \frac{I}{4\pi r^2}$. Solving for $c(r)$ gives $c(r) = A - \frac{1}{r}\frac{I}{4\pi D}$, where A is some constant. We can fix both I and A by imposing $c(\infty) = c_0$ and $c(R) = 0$, finding $A = c_0$ and $I = 4\pi D R c_0$. Along the way, we also find that the concentration profile itself is $c(r) = c_0(1 - (R/r))$.

Remarkably, *we have just computed the maximum rate at which oxygen molecules can be consumed by any bacterium whatsoever!* We didn't need to use any biochemistry at all, just the fact that living organisms are subject to constraints from the physical world. Notice that the oxygen uptake I increases with increasing bacterial size, but only as the first power of R. We might expect the oxygen *consumption*, however, to increase roughly with an organism's *volume*. Together, these statements imply an upper limit to the size of a bacterium: If R gets too large, the bacterium would literally suffocate.

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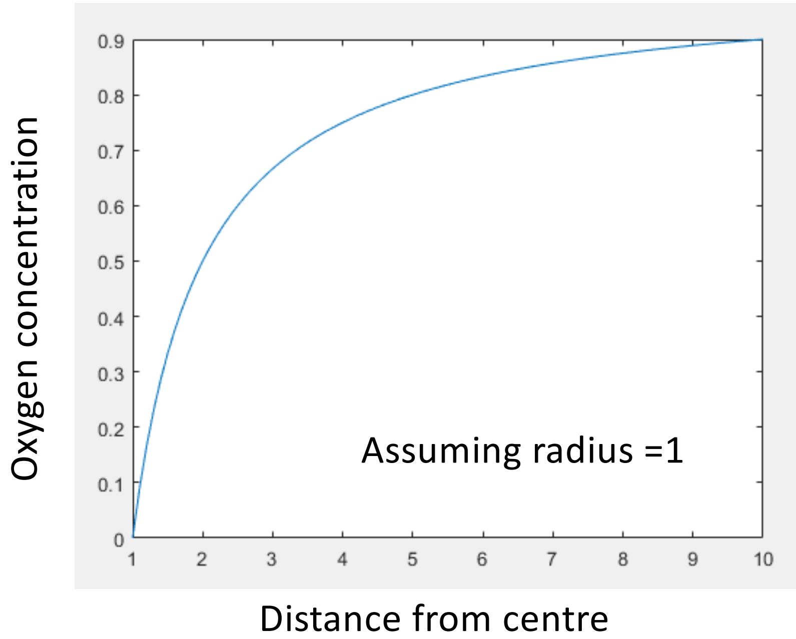
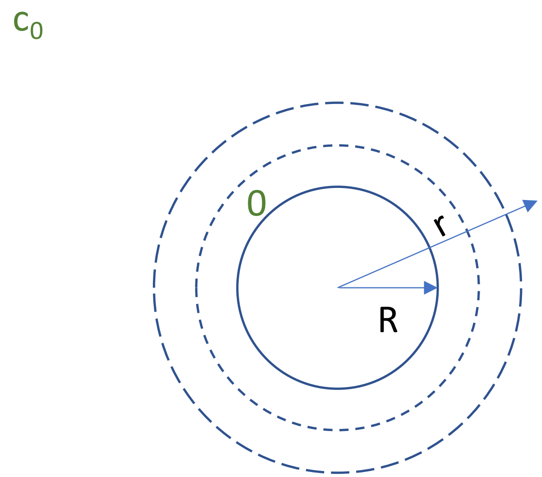
HW



Oxygen concentration
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
10
Assuming radius =1
Distance from centre



c_0
0
r
R

$$I = 4\pi D R c_0$$

Setting a limit to the size of bacteria

Your Turn 4f

a. Evaluate the above expression for I, using the illustrative values $R = 1\,\mu\text{m}$ and $c_0 \approx 0.2\,\text{mole/m}^3$.

b. A convenient measure of an organism's overall metabolic activity is its rate of O_2 consumption divided by its mass. Find the maximum possible metabolic activity of a bacterium of arbitrary radius R, again using $c_0 \approx 0.2\,\text{mole}\,\text{m}^{-3}$.

c. The actual metabolic activity of a bacterium is about $0.02\,\text{mole}\,\text{kg}^{-1}\text{s}^{-1}$. What limit do you then get on the size R of a bacterium? Compare to the size of real bacteria. Can you think of some way for a bacterium to evade this limit?

Take $D=10^{-5}$ cm^2/sec for nutrient

2022

https://www.nature.com/articles/d41586-022-01757-1#:~:text=These%20filament%2Dlike%20organisms%2C%20up,than%20any%20other%20known%20bacteria.

$$I = 4\pi D R c_0$$

(9) ~~Ex~~ $R = 1\,\mu m$ $\quad c_0 = 0.2$ mole/m³

$$I = 4\pi\, 10^{-6} \cdot 10^{-5}\, 10^{-4} \cdot 0.2$$

$$= 0.8\pi\, 10^{-16} \text{ moles/sec.}$$

$$\frac{I}{mass} = \frac{\cancel{4\pi} R\, 10^{-9}\, 0.2}{\frac{\cancel{4}}{3}\pi R^3 \cdot \rho_w} = \frac{3\, 10^{-9} \cdot 0.2}{R^2 \cdot 10^3} \cdot 10^{-6}$$

$$= \frac{0.6\, 10^{-12}}{R^2}$$

$$0.02\ \cancel{\text{mole}} = \frac{0.6 \times 10^{-12}}{R^2}$$

$$R = \sqrt{\frac{0.6 \times 10^{-12}}{0.02}} = \sqrt{30}\, 10^{-6}\ \cancel{\mu} m$$

$$= \boxed{5\,\mu m}.$$

To increase max possible nutrient consumption..

- Must move away from depletion zone it creates
- Move to greener pastures

Motile bacteria mimic random walk and tune their "run" length to make best use of its motility

Life at Low Reynolds Number

E.M. Purcell
Lyman Laboratory, Harvard University, Cambridge, Mass 02138
June 1976

American Journal of Physics vol 45, pages 3-11, 1977.

Editor's note: This is a reprint of a (slightly edited) paper of the same title that appeared in the book Physics and Our World: A Symposium in Honor of Victor F. Weiskopf, published by the American Journal of Physics (1976). The personal tone of the original talk has been preserved in the paper, which was itself a slightly edited transcript of a tape. The figures reproduce transparencies used in the talk. The demonstration involved a tall rectangular transparent vessel of corn syrup, projected by an overhead projector turned on its side. Some essential hand waving could not be reproduced.

How much area would a bacterium (radius 1 um) explore using only thermal motion?

Motile bacteria mimic random walk

Brown: 1828
Einstein: 1905
Perrin: 1913

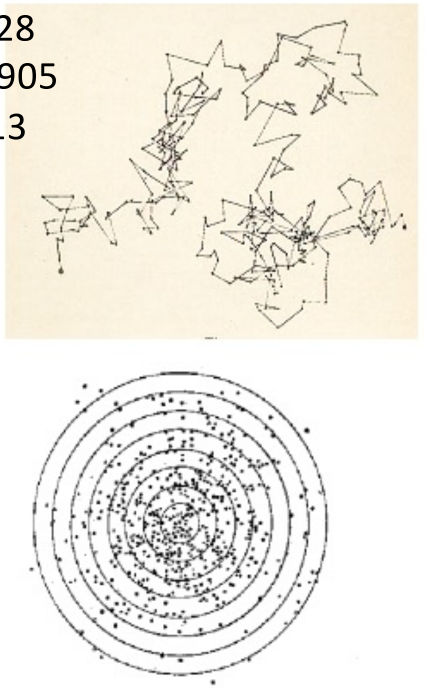


Fig 3. Brownian motion, after Jean Perrin [12]: An example of a trajectory (above) and statistical distribution of displacements (below, the circles correpond to fractions and multiples of the square root of the mean square displacement $<X^2>$)

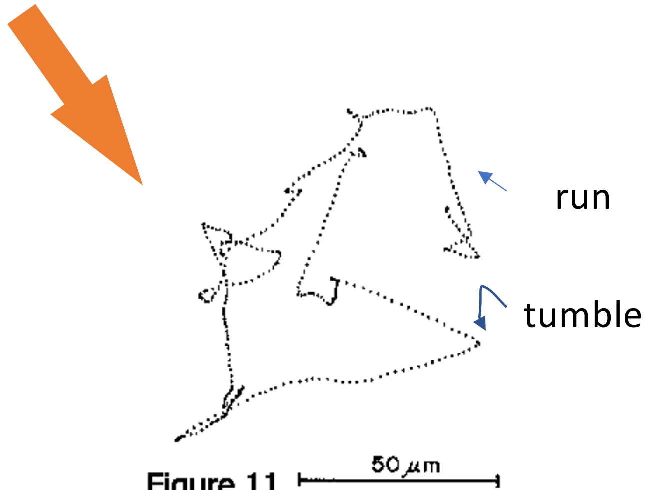


Figure 11

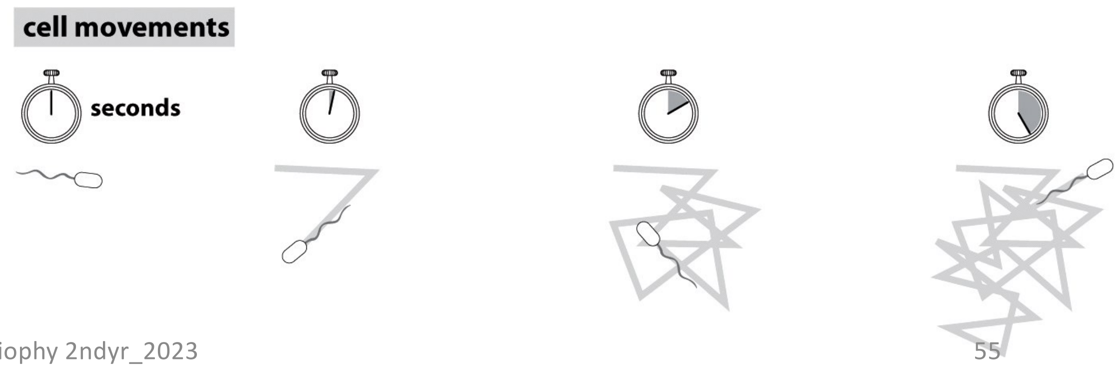


Figure 3.2d Physical Biology of the Cell (© Garland Science 2009)

swimming

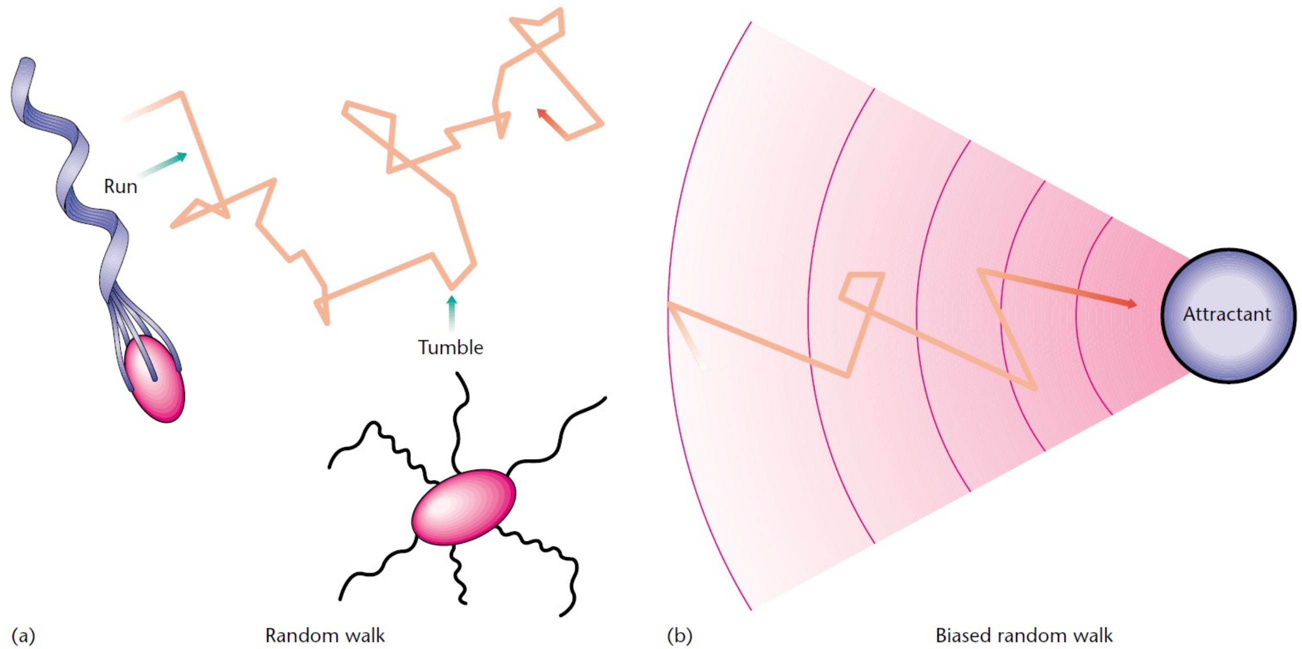
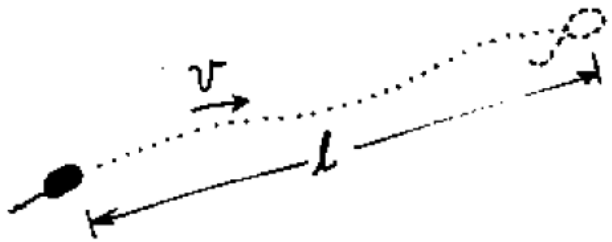


Figure 3 Swimming behaviour of *Escherichia coli* cells. (a) nonstimulated conditions; (b) stimulated conditions.

How much area would a bacterium (radius 1 um) explore using run length of 30um (1 sec steps)?

Use $$D = \frac{L^2}{2\Delta t}$$

How long should it “run” to make this strategy useful?



to out-swim diffusion:

$$l \geq D/v$$

if $D = 10^{-5}\ cm^2/sec$, $v = .003\ cm/sec$

$$l \geq 30\ \mu$$

"If you don't swim that far you haven't gone anywhere."

Figure 20

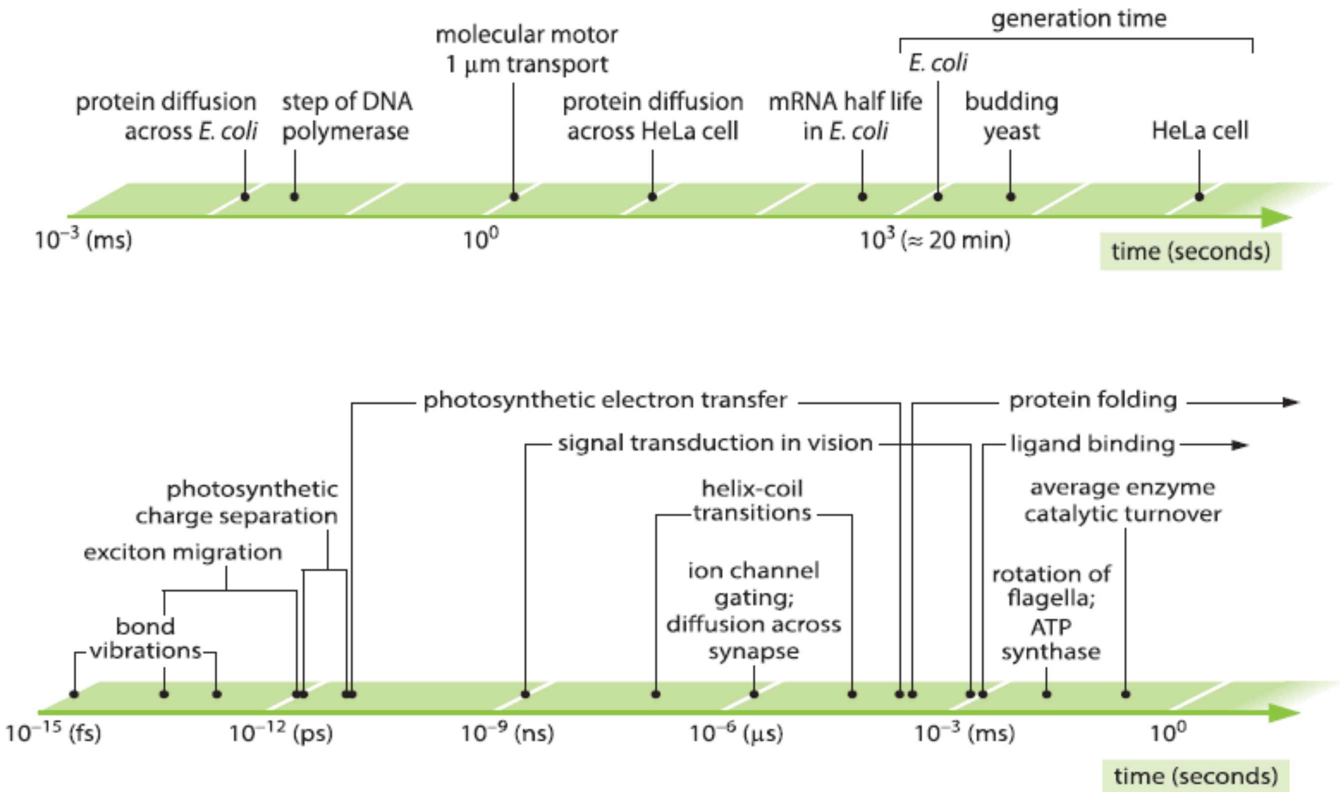


Figure 1: Range of characteristic time scales of central biological processes. Upper axis shows the longer timescales from protein diffusion across a bacterial cell to the generation time of a mammalian cell. The lower axis shows the fast timescales ranging from bond vibrations to protein folding and catalytic turnover durations.