Probability and Statistics

Part-7

MA2103 - 2023

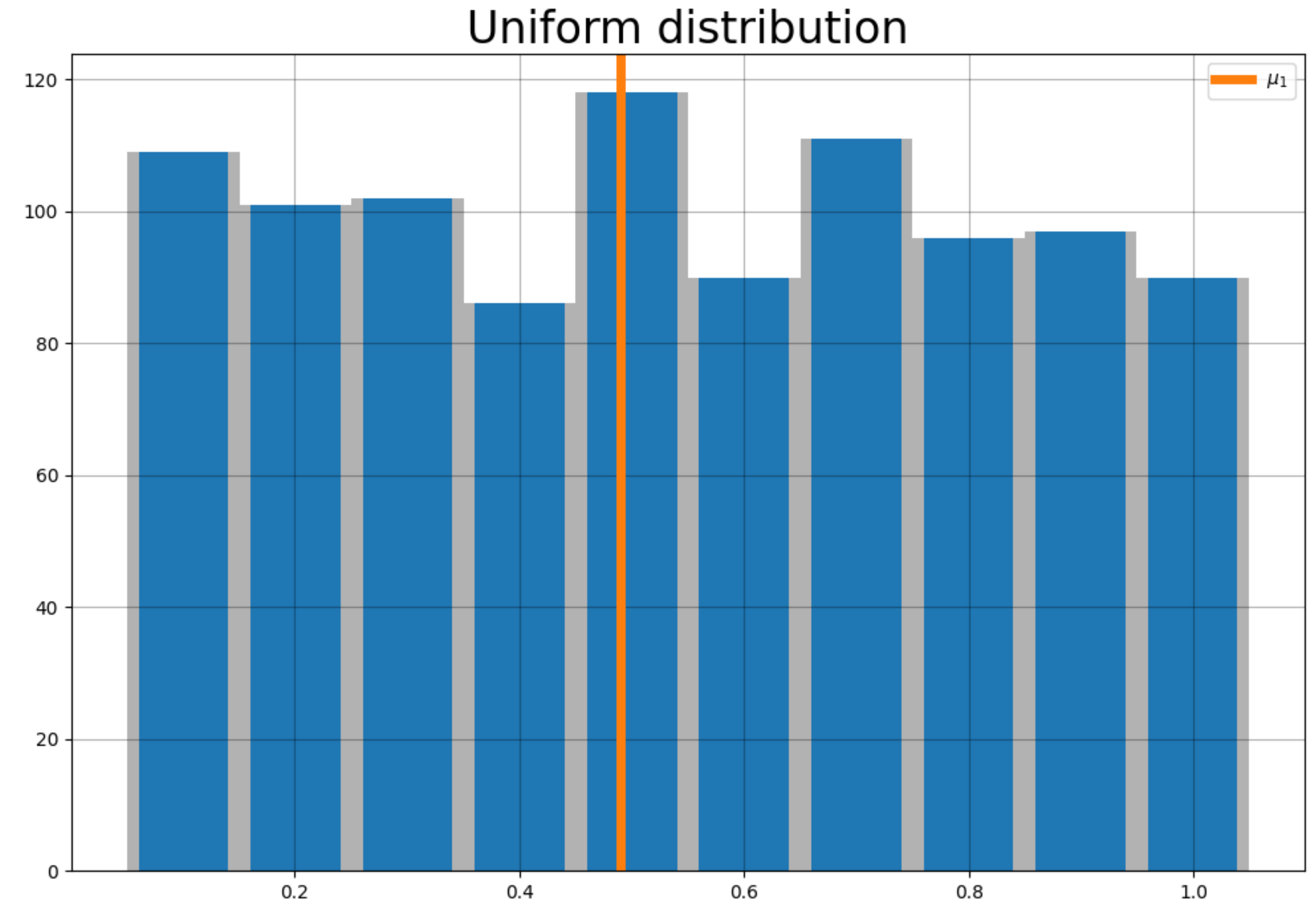
Rajesh Kumble Nayak

Means of uniform random number

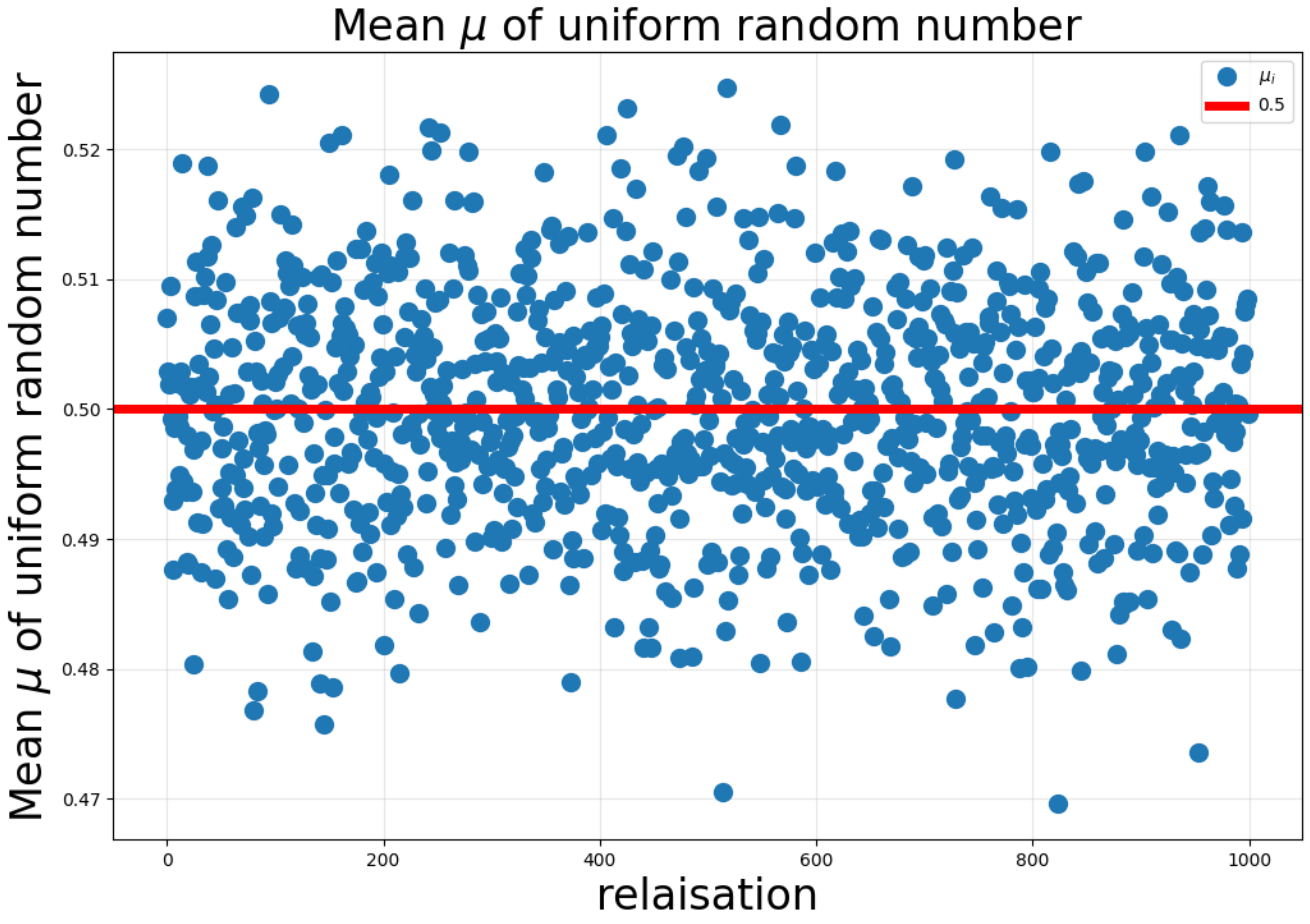
Lets take a realisation of uniform random number between $(0, 1)$ say r_1, let μ_1 be the mean.

$$\mu_1 = \sum_{i}^{N} x_{1i} \approx 0.5$$

μ_1 is not exactly 0.5



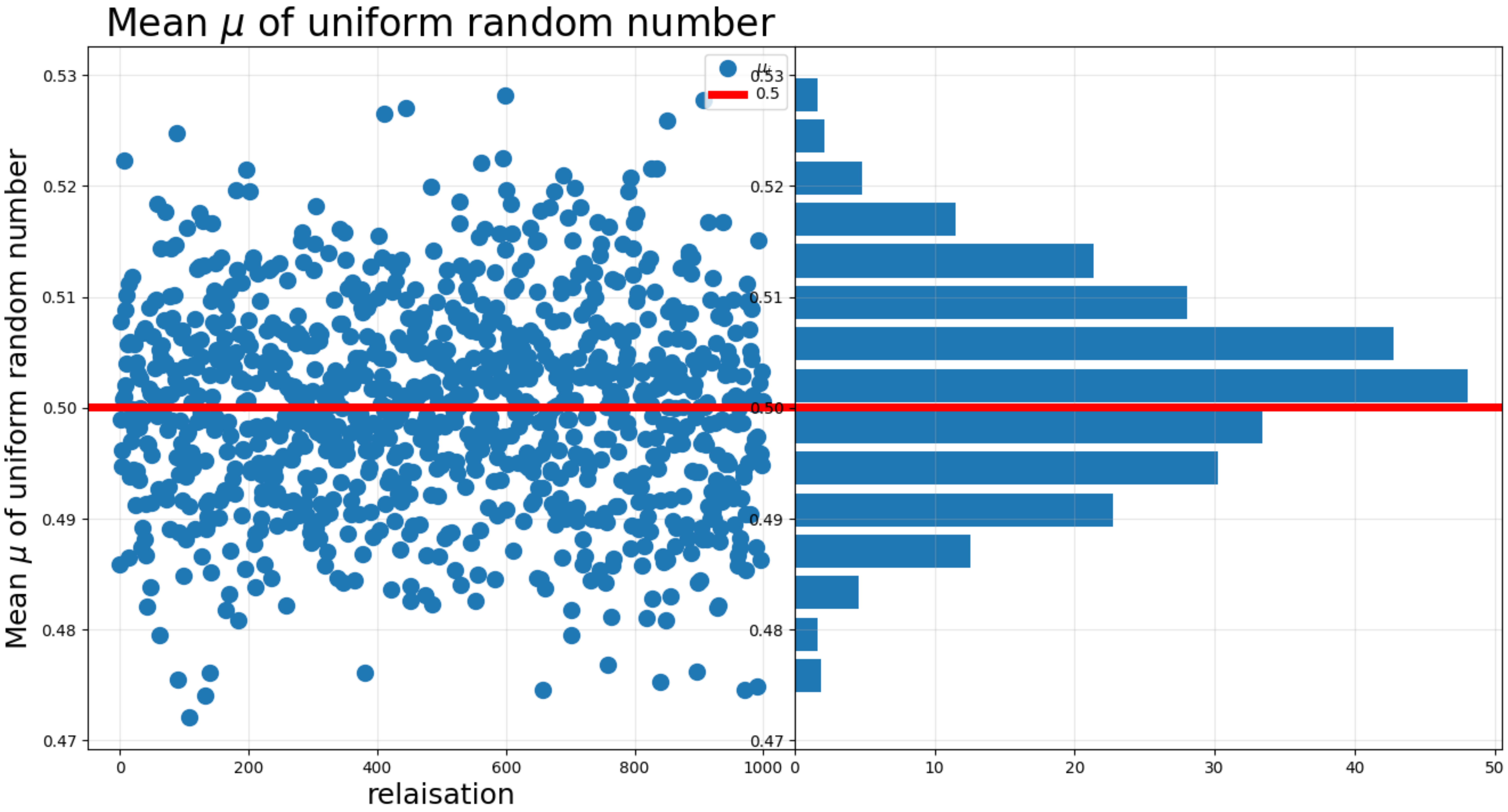
If we generate another realisation random number r_2 with same distribution, μ_2 will be slightly different from the μ_1 and so on.



Means of different realisation, μ_i of a random distribution is a random number!

Means of uniform random number

Means of different realisation, μ_i of a random distribution is a random number!

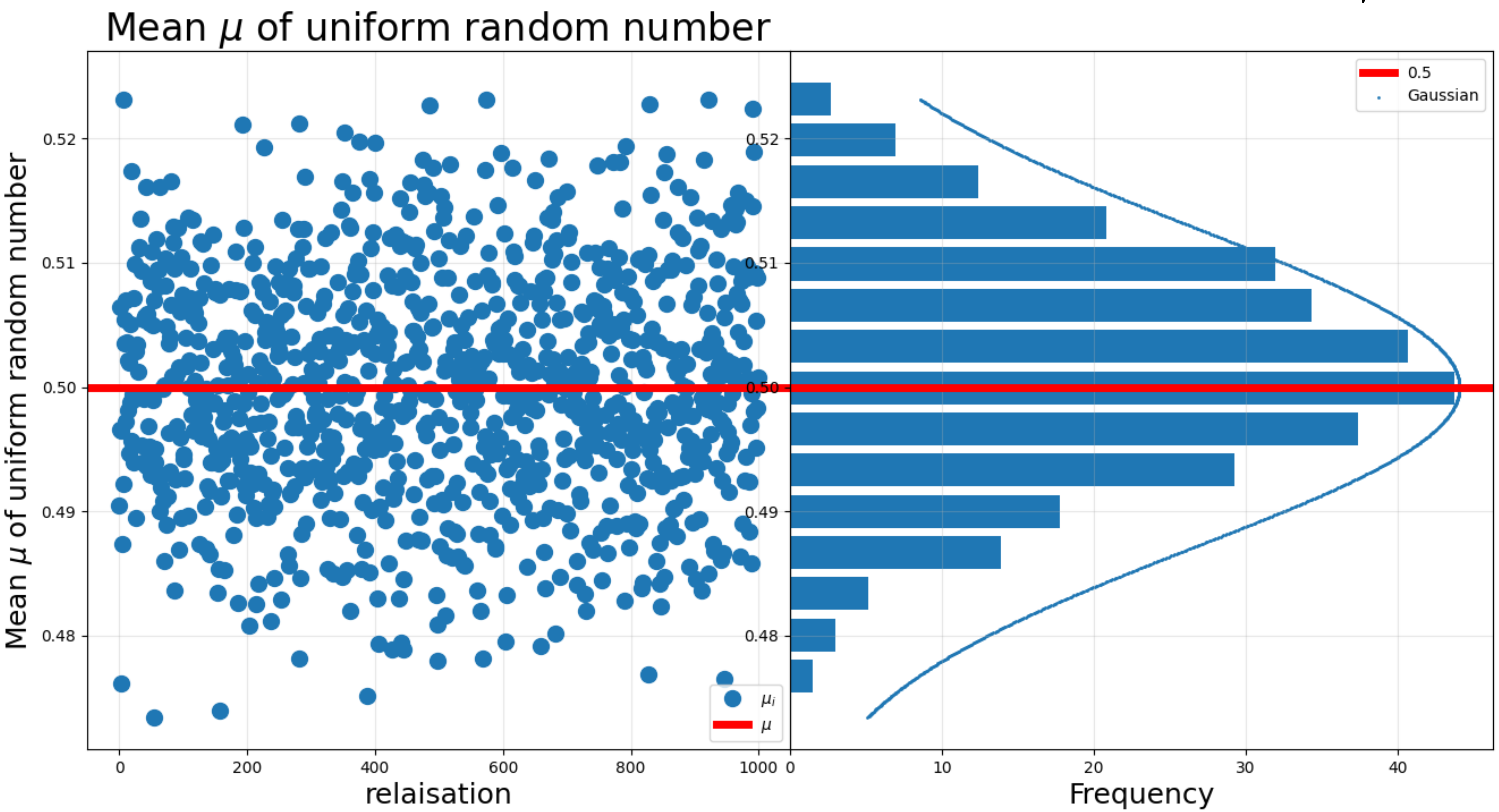


Means of uniform random number

Means of different realisation, μ_i of a random distribution is a random number!

The histogram fits well with Gaussian distribution!

$$p(x) = \frac{1}{\sigma\sqrt{2pi}} e^{-\left(\frac{x-\mu}{2\sigma}\right)^2}$$



Is there a connection between uniform and normal distribution ?

is it a lucky coincidence or *La mano de Dios*

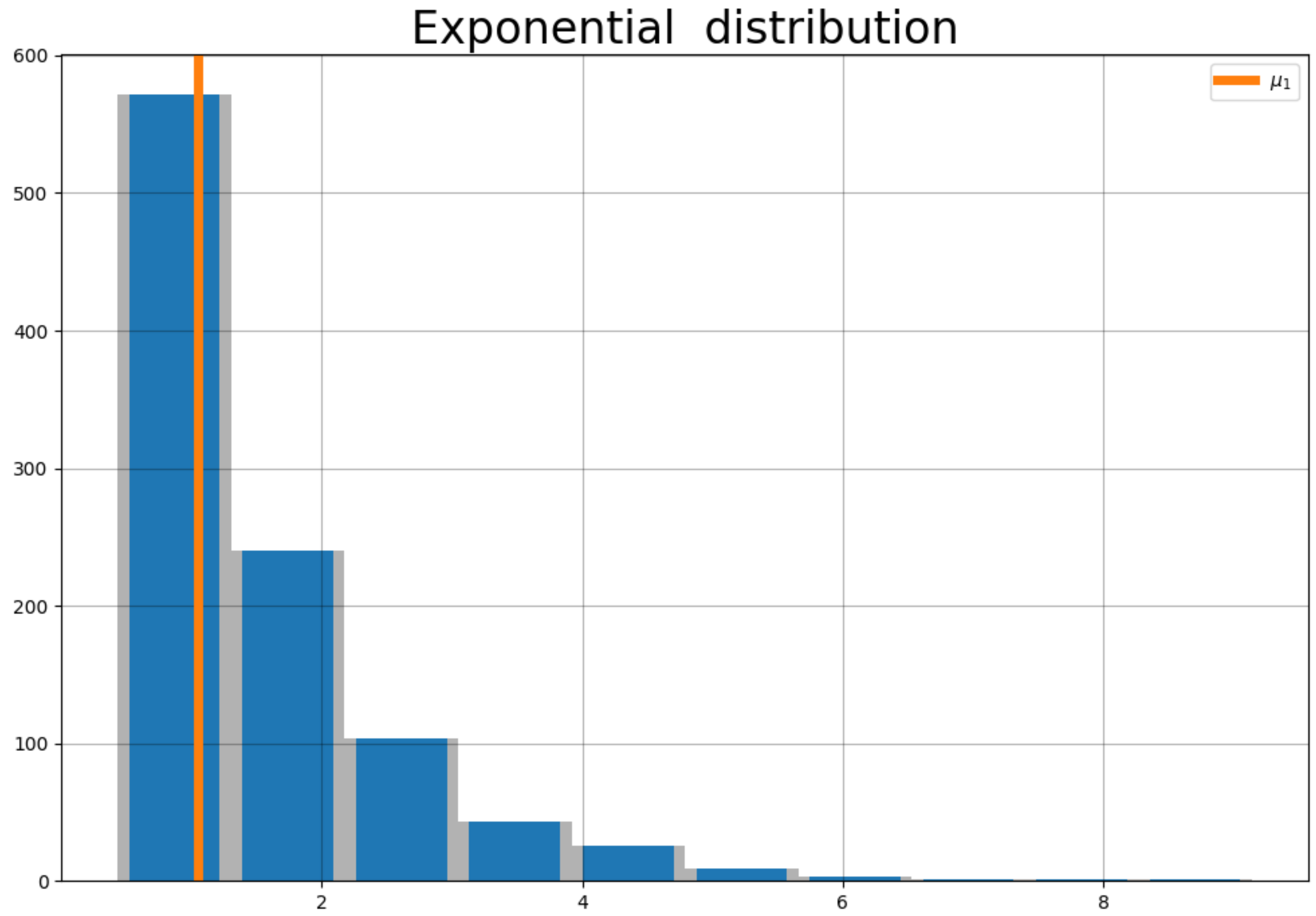
We will have to test with more distribution!

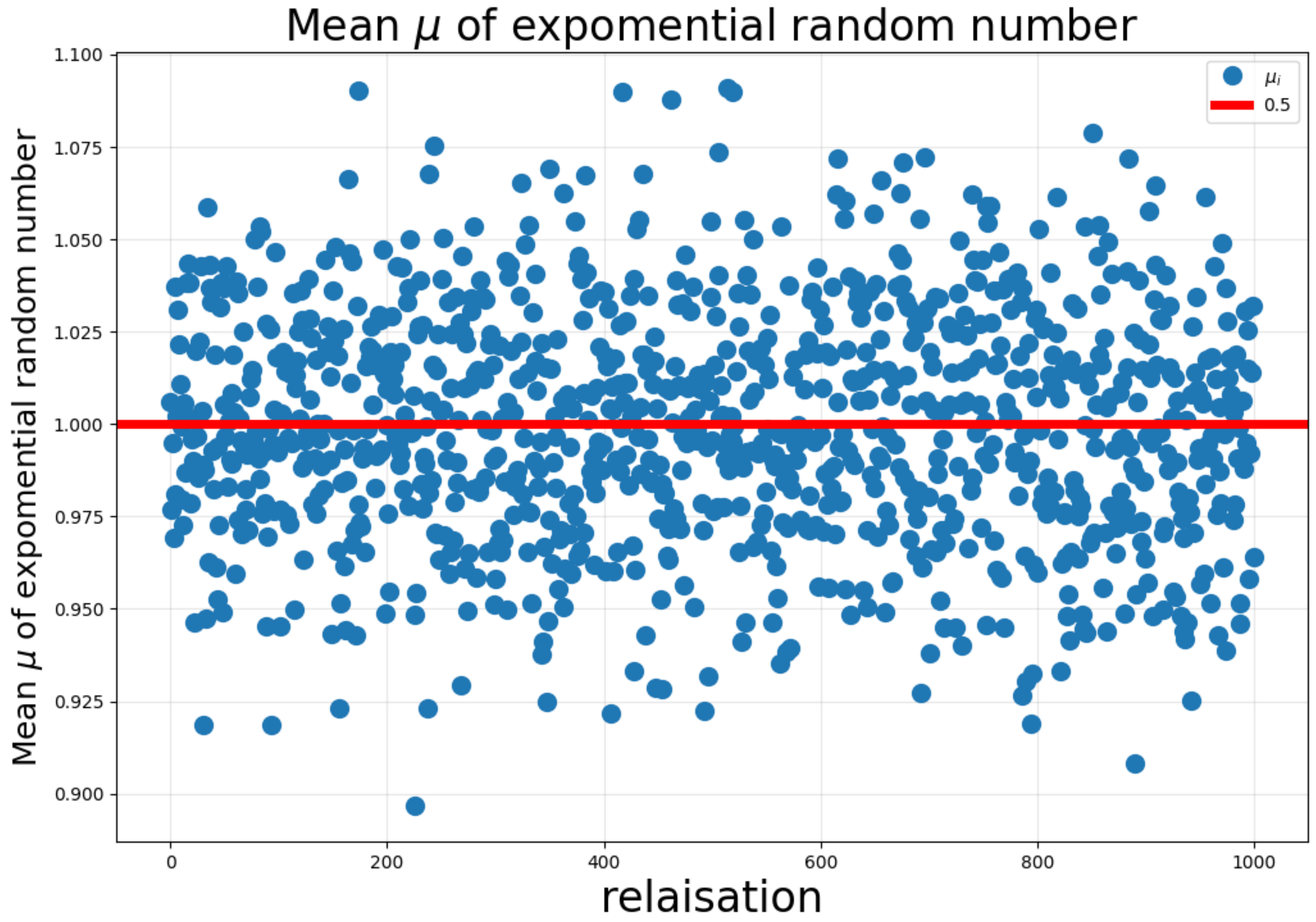
Exponential distribution

Let's try with exponential distribution given by $p(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

The mean μ can be computed using $\mu = \int_{-\infty}^{\infty} x\, p(x) dx = \frac{1}{\lambda}$

Variance of a distribution is given by $\sigma^2 = \int_{-\infty}^{+\infty} (x-\mu)^2\, p(x)\, dx = \frac{1}{\lambda^2}$



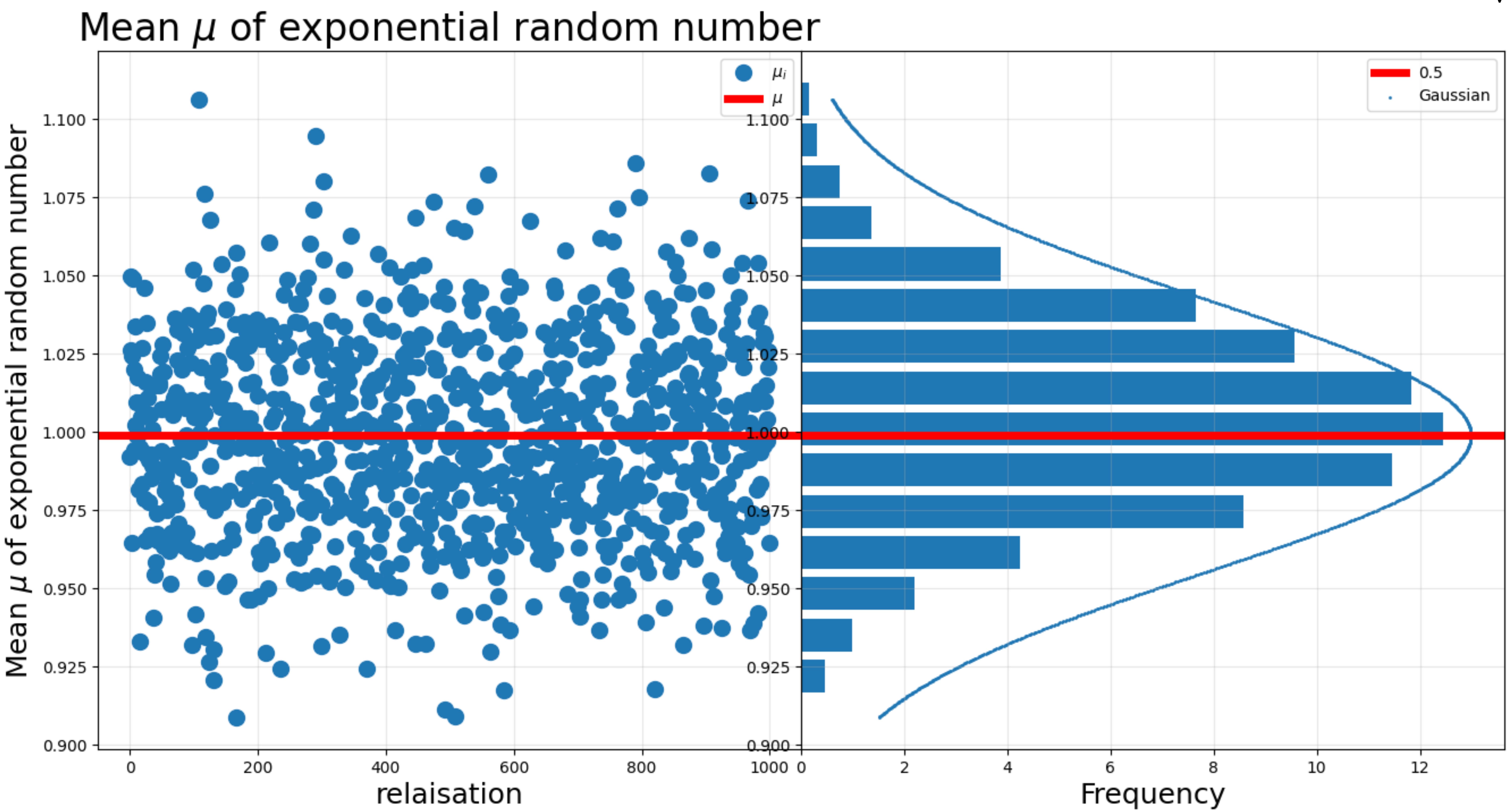


Means of Exponential random number

Means of different realisation, μ_i of a random distribution is a random number!

The histogram fits well with Gaussian distribution!

$$p(x) = \frac{1}{\sigma\sqrt{2pi}} e^{-\left(\frac{x-\mu}{2\sigma}\right)^2}$$



This is no lucky coincidence !

and indeed a golden hand!

This is true for the most of the cases!

Mean of realisations of random number generated from a distribution is Gaussian!

This is called Central limit theorem in statistics.

The Central limit Theorem

whenever a random sample of size N is taken from any distribution with mean μ and variance σ^2, then the sample mean will be approximately a Guassian/ Normal distribution with mean $\bar{\mu}$ and variance $\bar{\sigma}^2$.

$$\bar{\mu} = \frac{1}{M}\sum_{i=1}^{M}\mu_i = \mu$$

$$\bar{\sigma}^2 = \frac{\sigma^2}{N}$$

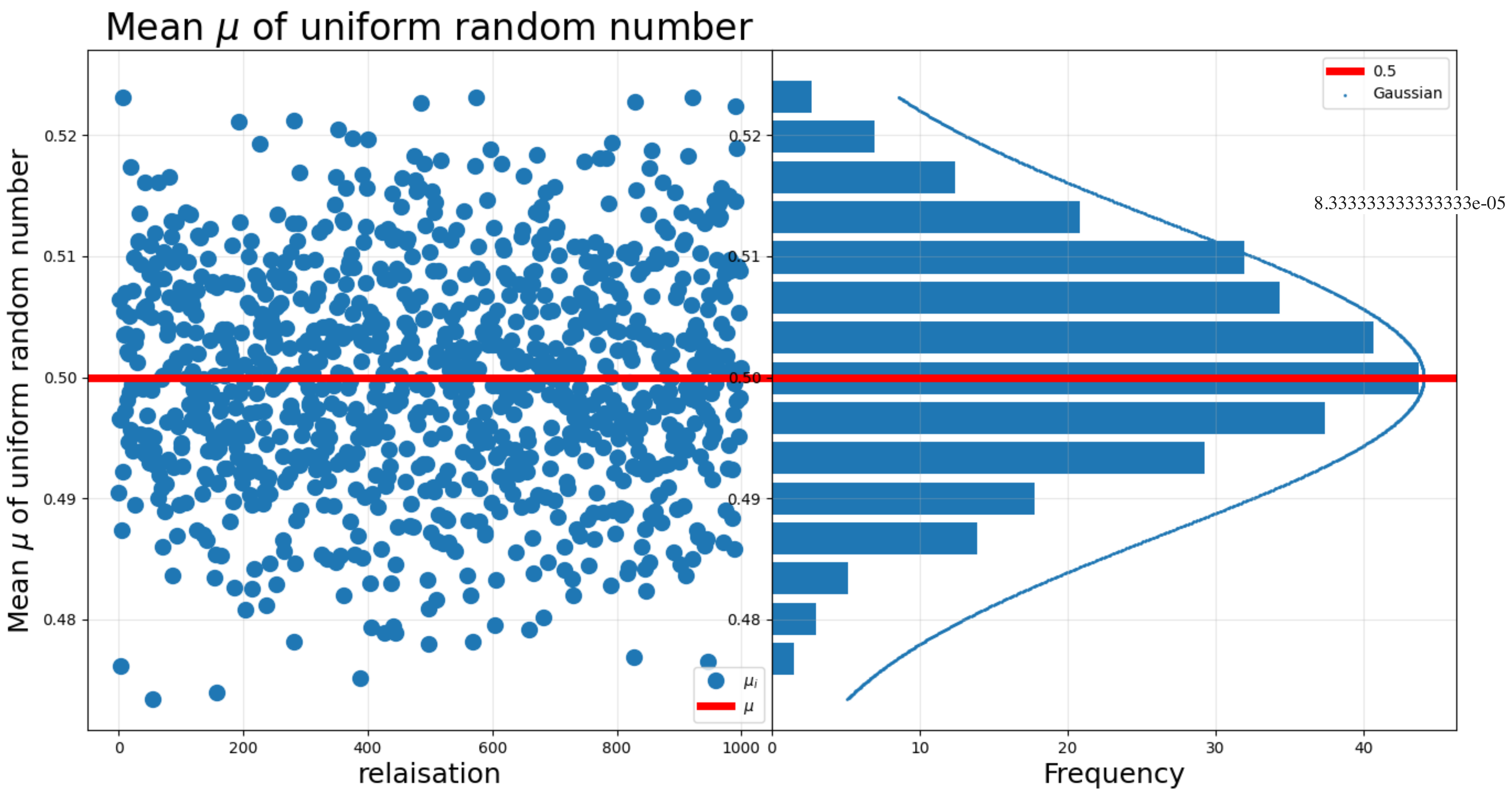
Means of uniform random number

Means of different realisation, μ_i of a random distribution is a random number!

The histogram fits well with Gaussian distribution!

$$p(x) = \frac{1}{\sigma\sqrt{2pi}} e^{-\left(\frac{x-\mu}{2\sigma}\right)^2}$$

We generated uniform random number with $\mu = 0.5$ and $\sigma^2 = \frac{1}{12}$



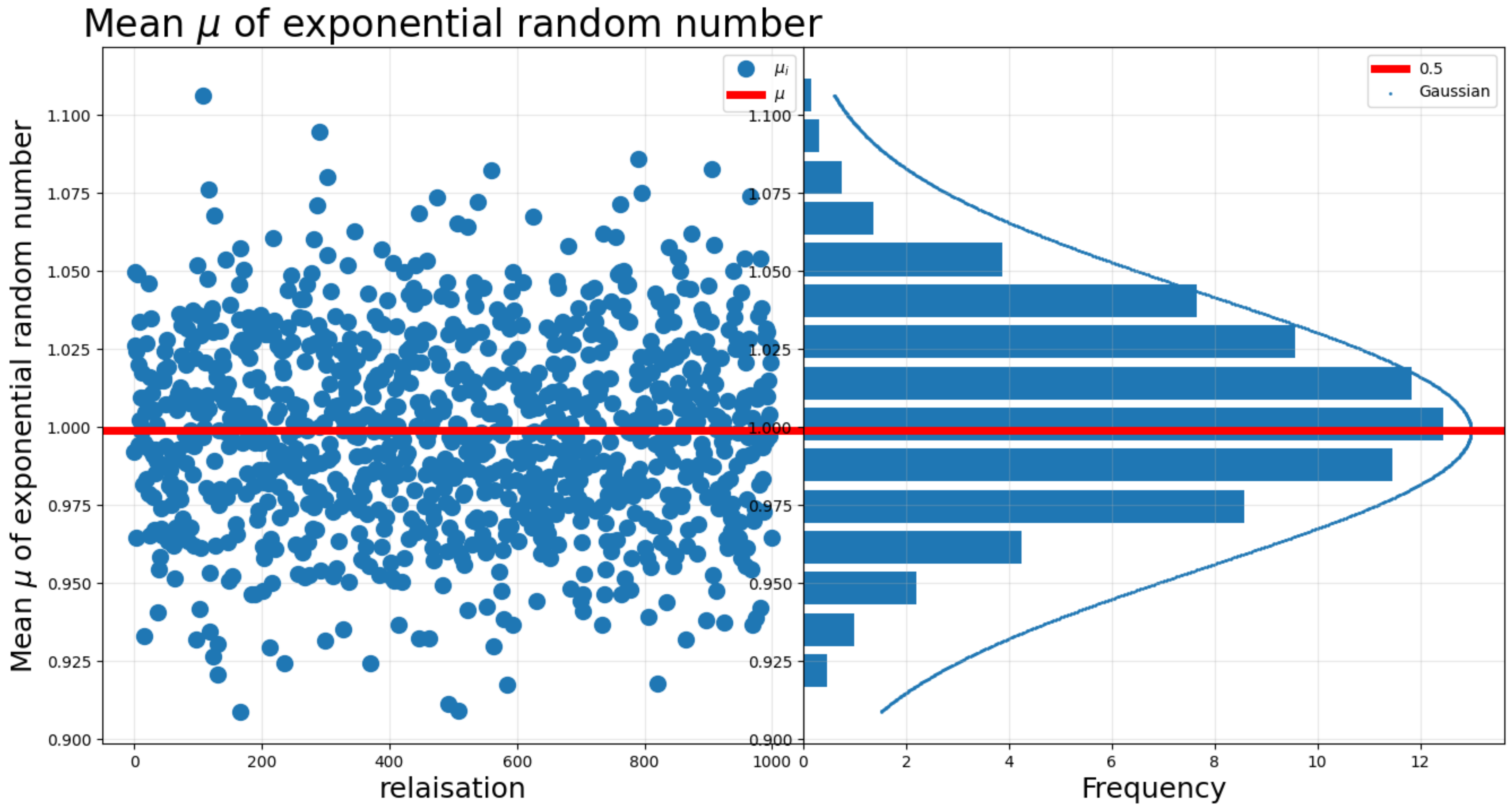
$$\mu = 0.4999356183743621$$

$$\sigma^2 = 8.650722858995328 \times 10^{-05}$$

$$\sigma^2 = \frac{1}{12N} = 8.6484110868161 2 \times 10^{-05}$$

Means of Exponential random number

Means of different realisation, μ_i of a random distribution is a random number!



The histogram fits well with Gaussian distribution!

$$p(x) = \frac{1}{\sigma\sqrt{2pi}} e^{-\left(\frac{x-\mu}{2\sigma}\right)^2}$$

We generated uniform random number with $\mu = 1$ and $\sigma^2 = 1$

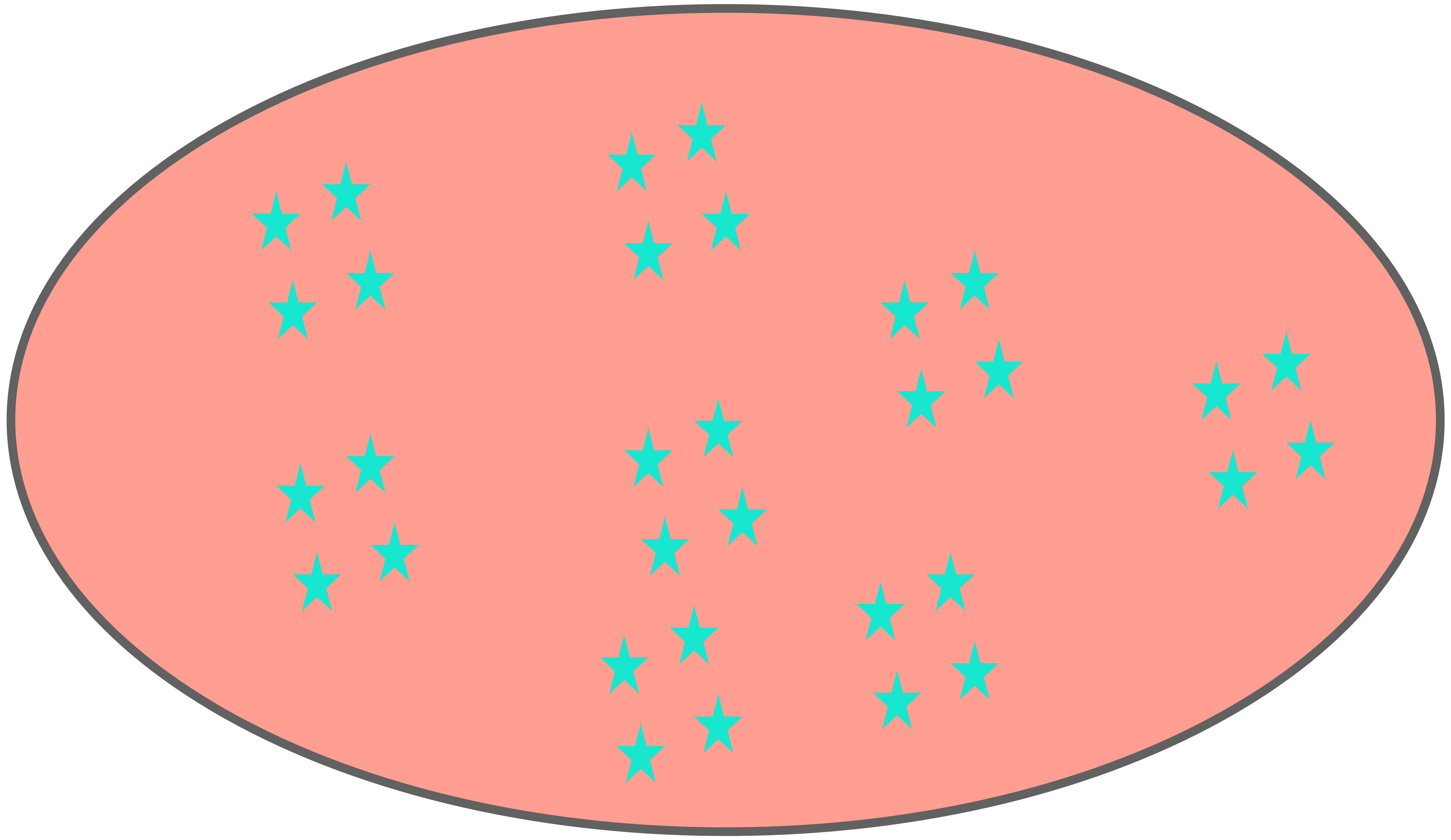
$$\mu = 0.998880541413627$$

$$\sigma^2 = 0.0008989152534278044$$

$$\sigma^2 = 1/N = 0.001$$

Application

Total population



Take sample with any distribution! find the mean μ and σ^2

repeat the sampling with same distribution several(large number of) times

We should able to estimate final mean and variance with central limit theorem