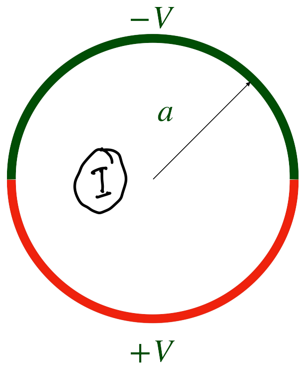
Tutorial - 03 23/01/2025 8:00 AM

Problem 1 : find electric potential inside (region (I)) for the boundary condition given in the figure



$\nabla^2 \psi = 0$ with boundary condition

$$\psi(\rho = a, \phi) = \begin{cases} -V & \text{if } 0 \phi \leq \pi \\ +V & \text{if } \pi < \phi \leq 2\pi \end{cases}$$

Sol^n: We need to solve $\nabla^2 \phi = 0$

plane polar coordinate is best suited for this problem

The Laplacian in is given by

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \psi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \psi}{\partial \phi^2} = 0 \quad \text{——— (1)}$$

We make an ansatz that $\psi = R(\rho)\, P(\phi)$

From all the stories from class, we know

$$R(\rho) \Rightarrow \rho^n \text{ or } \rho^{-n} \quad \text{and} \quad P(\phi) \propto \sin n\phi \text{ or } \cos n\phi$$

we have $\rho \to 0$ in the domain and no $\rho \to \infty$ hence

$$R(\rho) \propto \left(\frac{\rho}{a}\right)^n$$ we can scale ρ readjust coefficients

There no suitable condition choose sin or cos hence

we write Sol^n as

$$\psi(\rho, \phi) = \sum_{n=0}^{\infty} b_n \left(\frac{\rho}{a}\right)^n \sin(n\phi + \phi_0)$$

Now for the boundary condition $\rho \to a$ $\psi \begin{cases} -V & \phi < \pi \\ +V & \phi < 2\pi \end{cases}$

$$\begin{matrix} -V \\ +V \end{matrix} = \psi(\rho = a, \phi) = \sum_{n=1}^{\infty} b_n \sin(n\phi + \phi_0)$$

This is a Fourier Series $\phi_0 = 0$ only can be expressed in terms of $\sin n\phi$

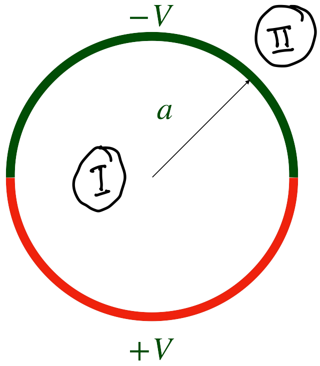
$$\int_0^{2\pi} \psi(\rho = a, \phi) \sin m\phi \, d\phi = \sum_{n=1}^{\infty} b_n \int_0^{2\pi} \sin(n\phi) \sin m\phi \, d\phi$$

from this one can find the b_n's using orthogonality of $\sin m\phi$!

Problem 2: find electric potential outside (region II) for the boundary conditions given in the figure

$\nabla^2 \psi = 0$ with boundary condition

$$\psi(\rho = a, \phi) = \begin{cases} -V & \text{if } 0 \, \phi \leq \pi \\ +V & \text{if } \pi < \phi \leq 2\pi \end{cases}$$



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sol^n Same as before but only r^{-n} contribute!