PH3102 Quantum Mechanics   Assignment 3

Instructor: Dr. Siddhartha Lal   Autumn Semester, 2024

Start Date: August 20, 2024   Submission Deadline: August 27, 2024 .
Submit your answers to the Tutor at the start of the tutorial.

Q1. Heisenberg and Schrödinger pictures. [10 marks]

Let $|0\rangle$ and $|1\rangle$ be the ground and first excited states of a one-dimensional simple harmonic oscillator respectively.

(a) Construct a linear combination of $|0\rangle$ and $|1\rangle$ such that $\langle x \rangle$ is as large as possible.

(b) Suppose the oscillator is in the state constructed above at $t = 0$. What is the state vector for $t > 0$ in the Schrödinger picture?

(c) Evaluate the expectation value $\langle x \rangle$ as a function of time $t > 0$ using:

    (i) the Schrödinger picture, and

    (ii) the Heisenberg picture.

(d) Evaluate $\langle (\Delta x)^2 \rangle$ as a function of time using either picture.

Q2. HUP for simple harmonic oscillator [10 marks]

Consider a simple harmonic oscillator (SHO) described by the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2 \ .$$

  i. Calculate the product $\Delta x \Delta p$ of the uncertainties in position (x) and momentum (p) in the ground state of the SHO, where the uncertainty in an operator is defined as $(\Delta O)^2 = \langle O^2 \rangle - \langle O \rangle^2$, $\langle \cdot \rangle$ being the expectation value. Comment on whether this is consistent with the Heisenberg uncertainty principle (HUP).
    Hint: use the raising (a_+) and lowering (a_-) operators.

  ii. Argue that the average energy of the system in a given state can be written as

$$E = \frac{1}{2m}\langle p^2 \rangle + \frac{1}{2}m\omega^2 \langle x^2 \rangle \ .$$

    Now, by using the HUP between x and p you obtained above, computed a lower bound for the energy E. Then use a well-known property of the minimum of a function to obtain the minimum possible value of the energy.