# Tutorial - 4

① Solve the problem for Lagrangian

$$\mathcal{L} = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$$

where $V(r) = \frac{1}{2}kr^2$

$$\mathcal{L} = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{k}{2}r^2$$

Lagrangian is independent of $\theta$ and $t$ and we have two conserved quantities.

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = p_\theta = mr^2\dot{\theta}^2 = L_z \quad \text{—— ①}$$

$$E = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{k}{2}r^2$$

From ① we get $\dot{\theta} = \frac{L_z}{mr^2}$

$$E = \frac{m}{2}\left(\dot{r}^2 + \frac{L_z^2}{m^2r^2}\right) + \frac{kr^2}{2}$$

$$\frac{2}{m}\left(E - \frac{kr^2}{2}\right) = \dot{r}^2 + \frac{L_z^2}{m^2r^2}$$

$$\dot{r}^2 = \frac{2}{m}\left(E - \frac{kr^2}{2}\right) - \frac{L_z^2}{m^2 r^2}$$

$$\dot{r}^2 = \frac{2}{m}\left[E - \frac{kr^2}{2} - \frac{L_z^2}{2mr^2}\right]$$

$$\dot{\theta} = \frac{L_z}{mr^2}$$

are soln to the EOM

However, we will not have are r and θ as function of ~~the~~ time

2 ~~The Lagrangian for/A~~ charge particle with charge 'q' and mass 'm' is subjectid a electric magnetic field E and B

E is given by $E = -\nabla\phi - \frac{\partial A}{\partial t}$

B is given by $B = -\nabla \times A$

The Lagrangian for such a

system can be given by

$$L = \frac{1}{2} m[\dot{x}^2 + \dot{y}^2 + \dot{z}^2] - q\phi + q A \cdot V$$

Show that the Lagrange eqn of motion reproduces the Lorentz force equation ~~F = ma = q[E ×~~

$$F = ma = q[E + V \times B]$$

Since coordinate system is cartesian we can ~~do~~ find EOM for $x$ other's are similar

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial \dot{x}} = \frac{\partial}{\partial \dot{x}}\left[\frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - q\phi + q(A_x \dot{x} + A_y \dot{y} + A_z \dot{z})\right]$$

$$\Rightarrow \frac{\partial L_e}{\partial \dot{x}} = (m\dot{x} + qA_x)$$

$$\frac{d}{dt}\left(\frac{\partial L_e}{\partial \dot{x}}\right) = m\ddot{x} + q\frac{dA_x}{dt}$$

$$\frac{\partial L_e}{\partial x} = -q\frac{\partial \phi}{\partial x} + q\frac{\partial A_x}{\partial x}\dot{x} + q\frac{\partial A_y}{\partial x}\dot{y} + q\frac{\partial A_z}{\partial z}\dot{z}$$

EOM is

$$m\ddot{x} + q\left[\frac{\partial A_x}{\partial t} + \cancel{\frac{\partial A_x}{\partial x}\dot{x}} + \frac{\partial A_x}{\partial y}\dot{y} + \frac{\partial A_x}{\partial z}\dot{z}\right]$$

$$+ q\frac{\partial \phi}{\partial x} - \cancel{q\frac{\partial A_x}{\partial x}\dot{x}} - q\frac{\partial A_y}{\partial x}\dot{y} - q\frac{\partial A_z}{\partial z}\dot{z}$$

$$m\ddot{x} + q\frac{\partial \phi}{\partial x} + q\frac{\partial A_x}{\partial t} + \left(\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x}\right)q\dot{y} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial z}\right)q\dot{z}$$

it can be shown to be $0 = m\ddot{x} + E_x q + (v \times B)_x q$