of interaction centers. They thus have the effect of normalizing materials of differing mass densities. As will be seen later, equal mass thicknesses of different materials will have roughly the same effect on the same radiation.

2.2 Energy Loss of Heavy Charged Particles by Atomic Collisions

In general, two principal features characterize the passage of charged particles through matter: (1) a loss of energy by the particle and (2) a deflection of the particle from its incident direction. These effects are primarily the result of two processes:

1) inelastic collisions with the atomic electrons of the material
2) elastic scattering from nuclei.

These reactions occur many times per unit path length in matter and it is their cumulative result which accounts for the two principal effects observed. These, however, are by no means the only reactions which can occur. Other processes include

3) emission of Cherenkov radiation
4) nuclear reactions
5) bremsstrahlung.

In comparison to the atomic collision processes, they are extremely rare, however, and with the exception of Cherenkov radiation, will be ignored in this treatment.

For reasons which will become clearer in the following sections, it is necessary to separate charged particles into two classes: (1) electrons and positrons, and (2) heavy particles, i.e., particles heavier than the electron. This latter group includes the muons, pions, protons, α-particles and other light nuclei. Particles heavier than this, i.e., the heavy ions, although technically part of this latter group, are excluded in this discussion because of additional effects which arise.

Of the two electromagnetic processes, the inelastic collisions are almost solely responsible for the energy loss of heavy particles in matter. In these collisions ($\sigma \simeq 10^{-17} - 10^{-16}\,\mathrm{cm}^2$!), energy is transferred from the particle to the atom causing an ionization or excitation of the latter. The amount transferred in each collision is generally a very small fraction of the particle's total kinetic energy; however, in normally dense matter, the number of collisions per unit path length is so large, that a substantial cumulative enery loss is observed even in relatively thin layers of material. A 10 MeV proton, for example, already loses all of its energy in only 0.25 mm of copper! These atomic collisions are customarily divided into two groups: *soft* collisions in which only an excitation results, and *hard* collisions in which the energy transferred is sufficient to cause ionization. In some of the *hard* reactions, enough energy is, in fact, transferred such that the electron itself causes substantial secondary ionization. These high-energy recoil electrons are sometimes referred to as *δ-rays* or *knock-on* electrons.

Elastic scattering from nuclei also occurs frequently although not as often as electron collisions. In general very little energy is transferred in these collisions since the masses of the nuclei of most materials are usually large compared to the incident particle. In cases where this is not true, for example, an α-particle in hydrogen, some energy is also lost through this mechanism. Nevertheless, the major part of the energy loss is still due to atomic electron collisions.

The inelastic collisions are, of course, statistical in nature, occurring with a certain quantum mechanical probability. However, because their number per macroscopic pathlength is generally large, the fluctuations in the total energy loss are small and one can meaningfully work with the average energy loss per unit path length. This quantity, often called the *stopping power* or simply dE/dx, was first calculated by Bohr using classical arguments and later by Bethe, Bloch and others using quantum mechanics. Bohr's calculation is, nevertheless, very instructive and we will briefly present a simplified version due to *Jackson* [2.1] here.

2.2.1 Bohr's Calculation – The Classical Case

Consider a heavy particle with a charge ze, mass M and velocity v passing through some material medium and suppose that there is an atomic electron at some distance b from the particle trajectory (see Fig. 2.2). We assume that the electron is free and initially at rest, and furthermore, that it only moves very slightly during the interaction with the heavy particle so that the electric field acting on the electron may be taken at its initial position. Moreover, after the collision, we assume the incident particle to be essentially undeviated from its original path because of its much larger mass ($M \gg m_e$). This is one reason for separating electrons from heavy particles!

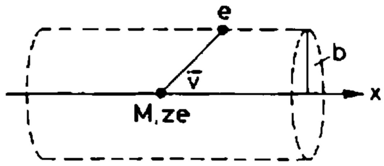

Fig. 2.2. Collision of a heavy charged particle with an atomic electron

Let us now try to calculate the energy gained by the electron by finding the momentum impulse it receives from colliding with the heavy particle. Thus

$$I = \int F\,dt = e\int E_\perp\,dt = e\int E_\perp \frac{dt}{dx}\,dx = e\int E_\perp \frac{dx}{v}\,, \tag{2.16}$$

where only the component of the electric field $E_\perp$ perpendicular to the particle trajectory enters because of symmetry. To calculate the integral $\int E_\perp\,dx$, we use Gauss' Law over an infinitely long cylinder centered on the particle trajectory and passing through the position of the electron. Then

$$\int E_\perp 2\pi b\,dx = 4\pi z e\,, \qquad \int E_\perp\,dx = \frac{2ze}{b}\,, \tag{2.17}$$

so that

$$I = \frac{2ze^2}{bv} \tag{2.18}$$

and the energy gained by the electron is

$$\Delta E(b) = \frac{I^2}{2m_e} = \frac{2z^2e^4}{m_e v^2 b^2}\,. \tag{2.19}$$

If we let N_e be the density of electrons, then the energy lost to all the electrons located at a distance between b and $b+db$ in a thickness dx is

$$-dE(b) = \Delta E(b)\, N_e\, dV = \frac{4\pi z^2 e^4}{m_e v^2} N_e \frac{db}{b}\, dx\,, \tag{2.20}$$

where the volume element $dV = 2\pi b\, db\, dx$. Continuing in a straight forward manner, one would at this point be tempted to integrate (2.20) from $b = 0$ to ∞ to get the total energy loss; however, this is contrary to our original assumptions. For example, collisions at very large b would not take place over a short period of time, so that our impulse calculation would not be valid. As well, for $b = 0$, we see that (2.19) gives an infinite energy transfer, so that (2.19) is not valid at small b either. Our integration, therefore, must be made over some limits b_{min} and b_{max} between which (2.19) holds. Thus,

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{b_{max}}{b_{min}}\,. \tag{2.21}$$

To estimate values for b_{min} and b_{max}, we must make some physical arguments. Classically, the maximum energy transferable is in a head-on collision where the electron obtains an energy of $\frac{1}{2} m_e (2v)^2$. If we take relativity into account, this becomes $2\gamma^2 m_e v^2$, where $\gamma = (1-\beta^2)^{-1/2}$ and $\beta = v/c$. Using (2.19) then, we find

$$\frac{2z^2 e^4}{m_e v^2 b_{min}^2} = 2\gamma^2 m v^2, \qquad b_{min} = \frac{z e^2}{\gamma m_e v^2}\,. \tag{2.22}$$

For b_{max}, we must recall now that the electrons are not free but bound to atoms with some orbital frequency ν. In order for the electron to absorb energy, then, the perturbation caused by the passing particle must take place in a time short compared to the period $\tau = 1/\nu$ of the bound electron, otherwise, the perturbation is adiabatic and no energy is transferred. This is the principle of *adiabatic invariance*. For our collisions the typical interaction time is $t \simeq b/v$, which relativistically becomes $t \Rightarrow t/\gamma = b/(\gamma v)$, so that

$$\frac{b}{\gamma v} \le \tau = \frac{1}{\bar{\nu}}\,. \tag{2.23}$$

Since there are several bound electron states with different frequencies ν, we have used here a mean frequency, $\bar{\nu}$, averaged over all bound states. An upper limit for b, then, is

$$b_{max} = \frac{\gamma v}{\bar{\nu}}\,. \tag{2.24}$$

Substituting into (2.21), we find

$$\boxed{-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{\gamma^2 m v^3}{z e^2 \bar{\nu}}\,.} \tag{2.25}$$

This is essentially Bohr's classical formula. It gives a reasonable description of the energy loss for very heavy particles such as the α-particle or heavier nuclei. However,