Root Finding

Part-12

MA2103 - 2023

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Root's are very interesting

we have to find x such that $f(x) = 0$.

Which is not possible analytically except for a very few special case. Here numerical methods are important

Let's Look at Another problem

Equation of motion for a simple particle, in $1-D$, with potential $V(x)$

We use energy conservation $E = K.E + P.E$

$$E = \frac{m}{2}v^2 + V(x)$$

$$\frac{2}{m}\left[E - V(x)\right] = v^2$$

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Find the position x_0 at which velocity $v = 0$

In the problem $V(x) = x - 12x^2 + x^4$

There are two way to solve the problem:

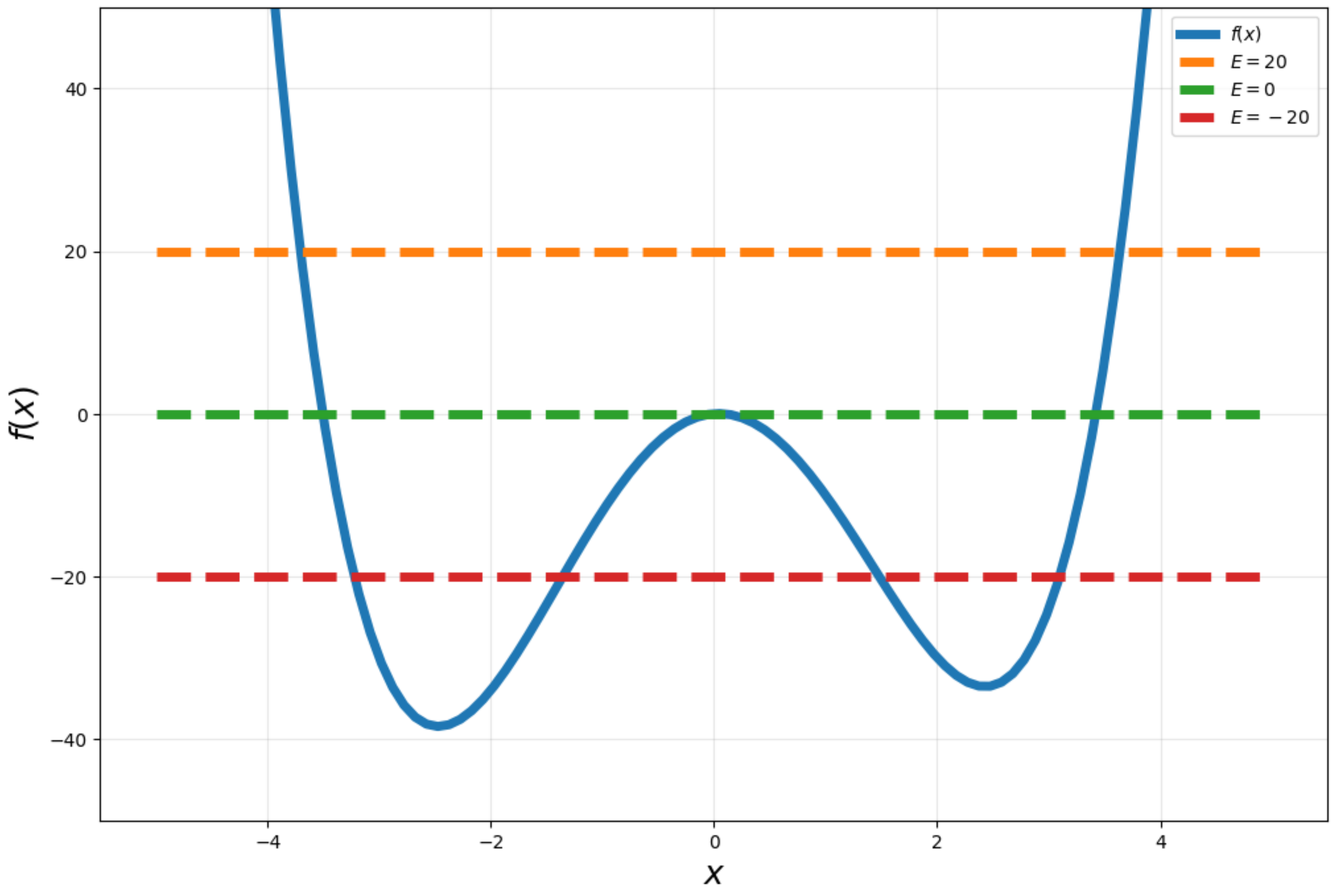
- ✦given E and find the x_0 $\Longrightarrow$ solve x_0 such that

  $$E - x_0 + 12{x_0}^2 - {x_0}^4 = 0$$

- ✦given $\{x_1, v_1\}$find x_0 $\Longrightarrow$ Find $E_1 = \frac{m}{2}{v_1}^2 + x_1 - 12{x_1}^2 + {x_1}^4$

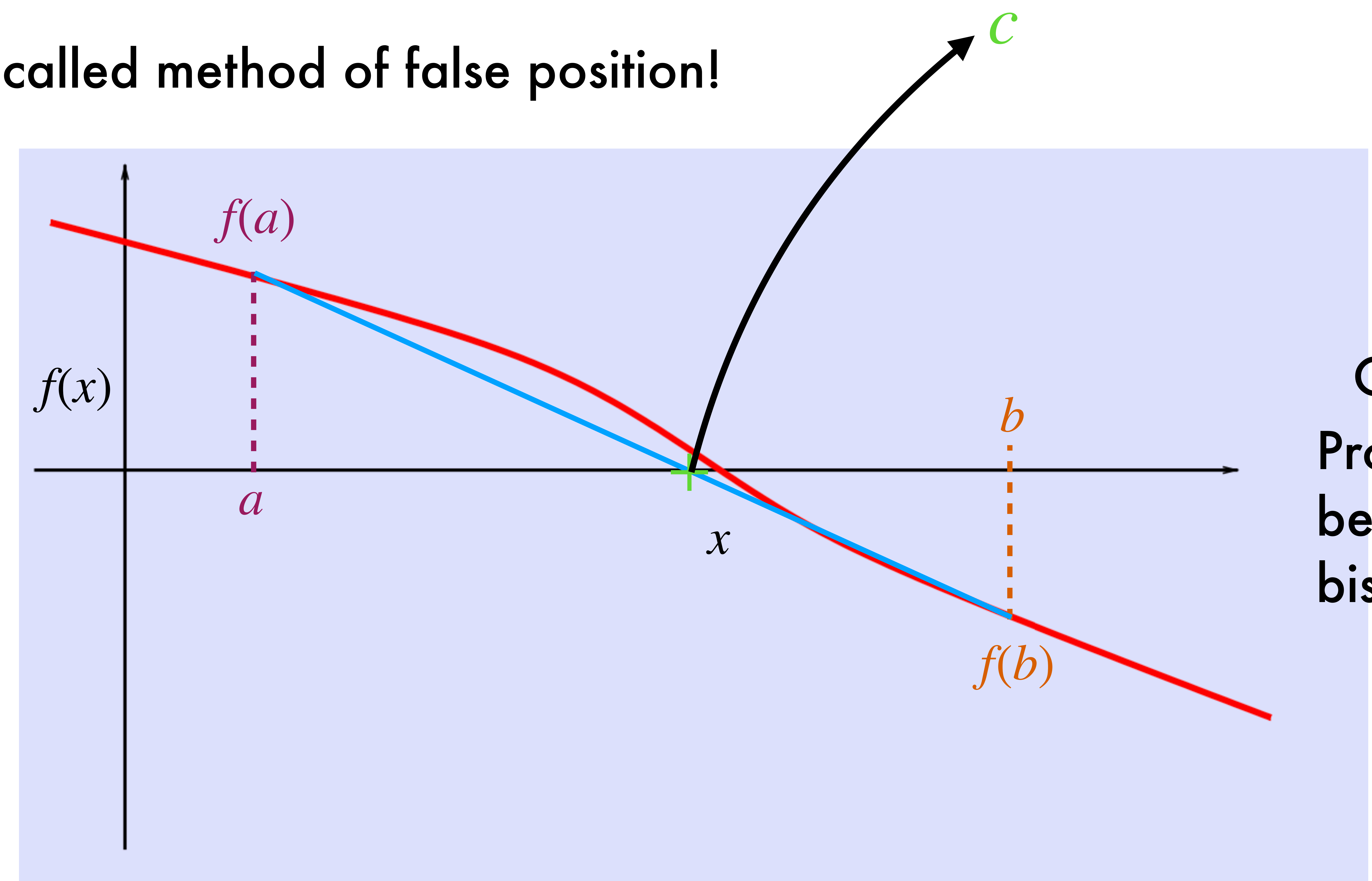
  solve x_0 such that $E_1 - x_0 + 12{x_0}^2 - {x_0}^4 = 0$

Graphical Method



The Regular Falsi Method

This is called method of false position!



$f(a)\ f(b) < 0$

Once we have c

Proceed as before in bisection method

The Regular Falsi method

For a function $f(x)$ continuous in the interval $[a, b]$, there exists at least one root in the interval (a, b) **if $f(a)f(b) < 0$ then we have root $\in [a, b]$**

1. Find c such that, $c = b - f(b)\dfrac{(b-a)}{f(b) - f(a)} = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$

If $f(a)f(c) < 0$ then replace b by c If $f(c)f(b) < 0$ then replace a by c

Another trick for stopping is put the condition $|f(c)| < \epsilon$

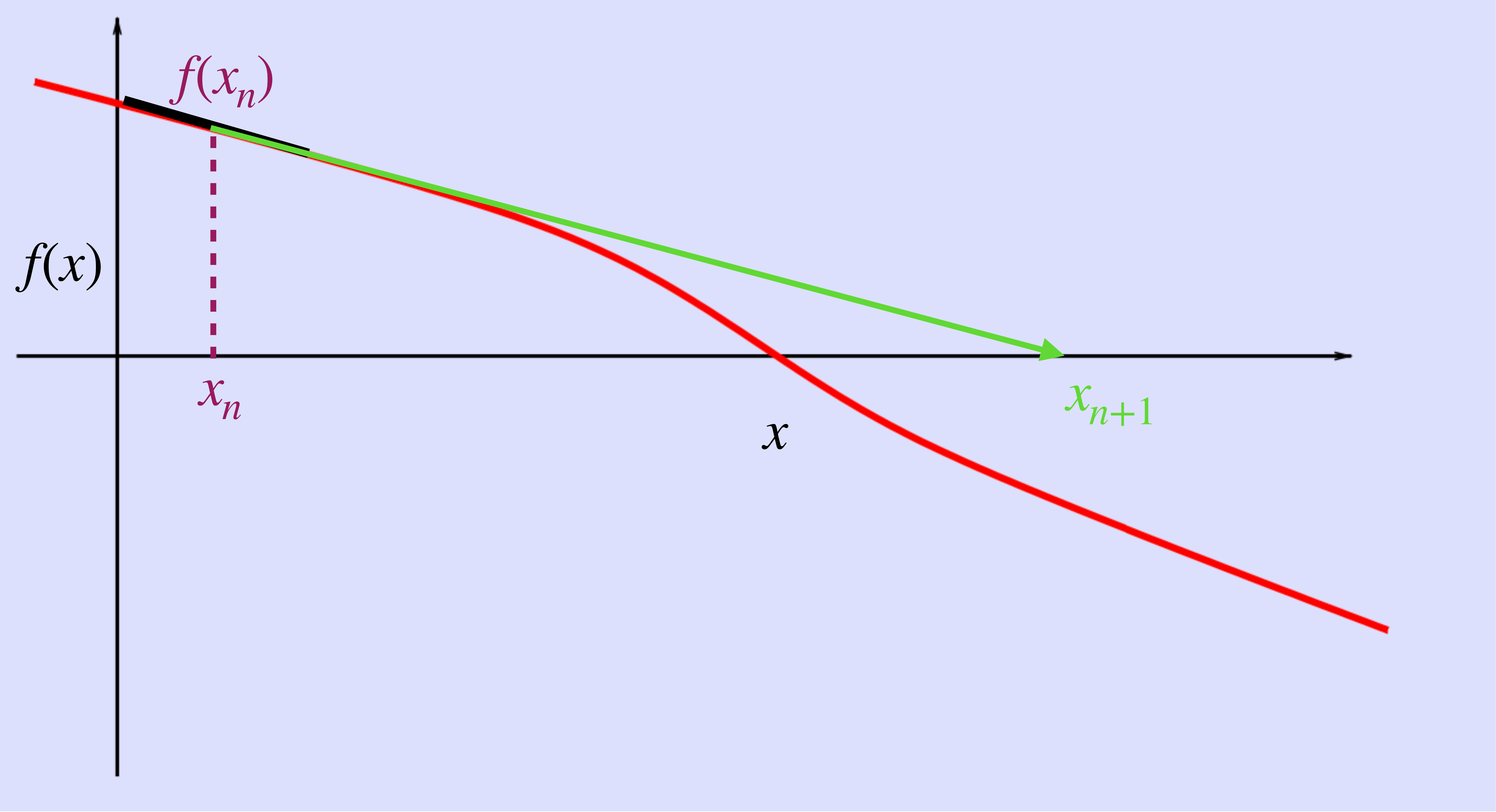
If $|f(c)| > \epsilon$ repeat the steps

Algorithm

1. start with interval $[a_0, b_0]$ such that $f(a_0) * f(b_0) < 0$
2. n th iteration is compute using:
3. next point for each iteration is $c = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$
4. if $f(a_n)f(c) < 0$ then $a_{n+1} = a_n$ and $b_{n+1} = c$ else $a_{n+1} = c$ and $b_{n+1} = b_n$
5. Repeat 3 to 4 till $|f(c)| < \epsilon$ or n reached $ITMAX$
6. ϵ the error in the root.

Newtons Method or Newton-Raphson Method

Approximate the function with st.line at $x = x_n$



Algorithm

1. start with point x_1 such that $f(x_1) \neq 0$

2. n th iteration is compute using:

3. next point for each iteration is $x_{n+1} = x_n - \dfrac{f\left(x_n\right)}{f'\left(x_n\right)}$

4.5. Repeat 3 to 4 till $\left|f(c)\right| < \epsilon$ or n reached $ITMAX$

6. ϵ the error in the root.