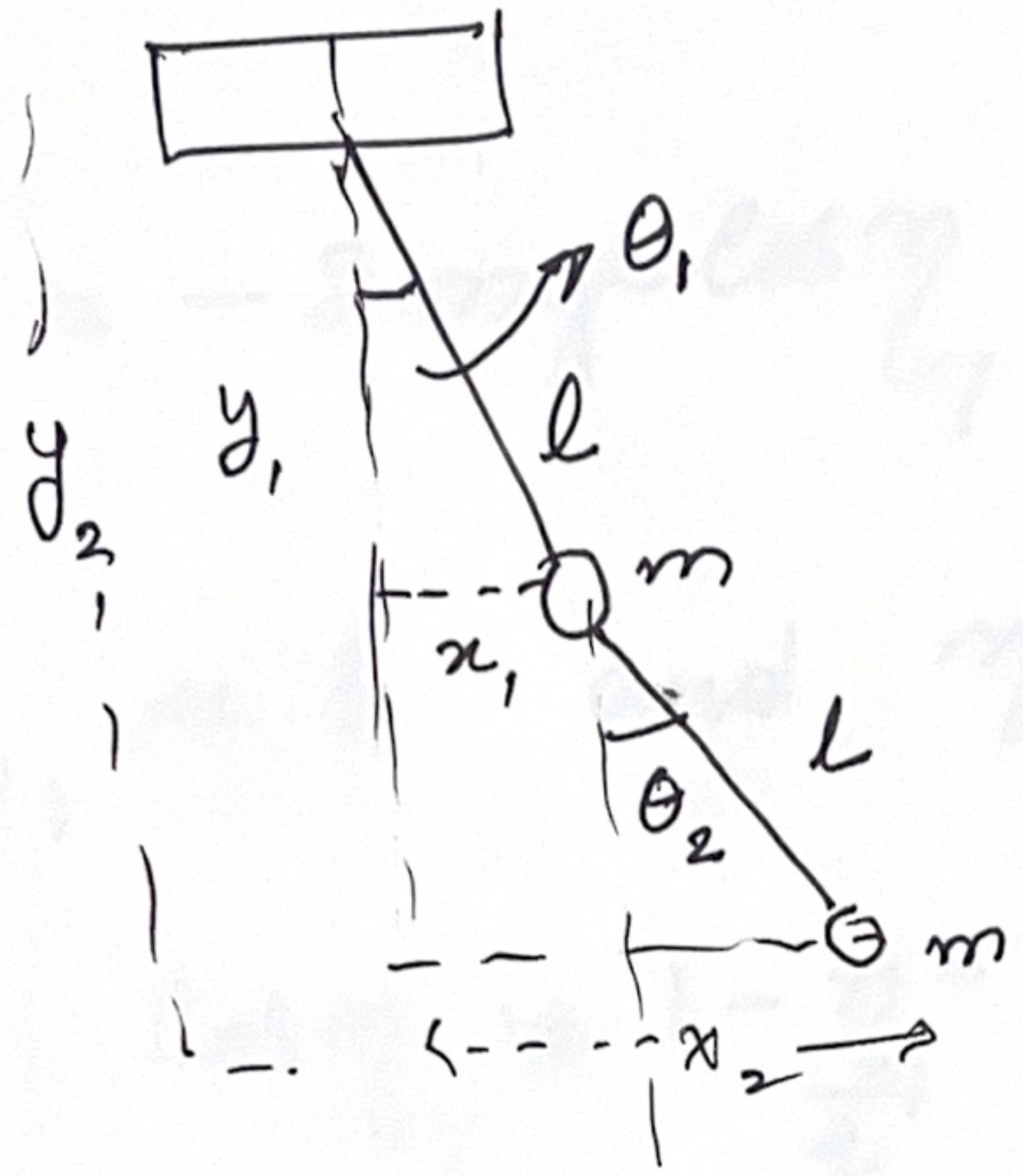
Problem Set 8 :

Let's look at double pendulum



$$T = \frac{1}{2}(2m)\, l^2 \dot{\theta}_1^2 + \frac{1}{2} m l^2 \dot{\theta}_2^2 + m l^2 \cos(\theta_1 - \theta_2)\dot{\theta}_1 \dot{\theta}_2$$

$$V = -2mgl\cos\theta_1 - mgl\cos\theta_2$$

it is st. forward to show that

~~Eq~~ stable equilibrium point is

$$\theta_{10}^* = 0 \quad \text{and} \quad \theta_{20} = 0$$

we switch to η_1 and η_2

$$\theta_1 = \eta_1 \qquad \theta_2 = \eta_1$$

$$T = ml^2\dot{\eta}_1^2 + \frac{1}{2}ml^2\dot{\eta}_2^2 + ml^2\cos(\eta_1-\eta_2)\dot{\eta}_1\dot{\eta}_2$$

$$V = -2mgl\cos\eta_1 - mgl\cos\eta_2$$

$\eta_1 \ll 1$ and $\eta_2 \ll 1$

$$\cos\eta_1 \simeq 1-\frac{\eta_1^2}{2}$$

$$\cos\eta_2 \simeq 1-\frac{\eta_2^2}{2}$$

$$\cos(\eta_1-\eta_2) \simeq 0$$

with this we get

$$T \simeq ml^2\dot{\eta}_1^2 + \frac{1}{2}ml^2\dot{\eta}_2^2 + ml^2\dot{\eta}_1\dot{\eta}_2$$

$$V \simeq \cancel{-3mgl}^{\,0} + mgl\eta_1^2 + \frac{1}{2}mgl\eta_2^2$$

$$T = \begin{pmatrix} 2ml^2 & ml^2 \\ ml^2 & ml^2 \end{pmatrix}$$

$$V = \begin{pmatrix} 2mgl & 0 \\ 0 & mgl \end{pmatrix}$$

The EOM is

$$\det(V - \omega^2 T) = 0$$

$$\begin{vmatrix} 2ml(g - l\omega^2) & -ml^2\omega^2 \\ -ml^2\omega^2 & ml(g - l\omega^2) \end{vmatrix} = 0$$

$$\Rightarrow 2(g - l\omega^2)^2 - l^2\omega^4 = 0$$

$$\Rightarrow \omega^4 l^2 - 4\omega^2 gl + 2g^2 = 0$$

$$\omega_1^2 = \frac{4gl + \sqrt{16g^2l^2 - 8g^2l^2}}{2l^2}$$

$$\omega_2^2 = \frac{4gl - \sqrt{16g^2l^2 - 8g^2l^2}}{2l^2}$$

$$\omega_1^2 = \frac{g}{l}\left(2 + \sqrt{2}\right) \quad \omega_2^2 = \frac{g}{l}\left(2 - \sqrt{2}\right)$$

$\rightarrow$ one can need to get the Eigen vectors to get the amplitude of these ~~Ea~~ ~~So~~ Oscillations.

Problem 2:

Find oscillation frequency and Amplitude ~~of~~ for small oscillation

with

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$$V = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_2 - x_1)^2 + \frac{k_3}{2} x_2^2$$

here $m_1 = 2m$, $m_2 = m$

equilibrium point at $x_1 = 0$ $x_2 = 0$

$$T = \frac{1}{2} m_1 \dot{\eta}_1^2 + \frac{1}{2} m_2 \dot{\eta}_2^2$$

$$V = \frac{1}{2} k_1 \eta_1^2 + \frac{1}{2} k_2 (\eta_2 - \eta_1)^2 + \frac{k_3}{2} \eta_2^2$$

$$T = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

$$V = \begin{pmatrix} 5k & -k \\ -k & 3k \end{pmatrix}$$

Eqn of motion is

$$\det \left| V - \omega^2 T \right| = 0$$

But $m_1 = 2m$ & $m_2 = m$

$$\begin{vmatrix} 5k - 2m\omega^2 & -k \\ -k & 3k - m\omega^2 \end{vmatrix} = 0$$

$$(5k - 2m\omega^2)(3k - m\omega^2) - k^2$$

$$2m^2\omega^4 - 11km\omega^2 + 14k^2 = 0$$

we get $\quad \omega_1^2 = \dfrac{2k}{m}$

$$\omega_2^2 = \frac{7k}{2m}$$

To find the Eigen vector

$$\omega^2 T \cancel{A} = V \cancel{A}$$

$$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\frac{7k}{2m} \begin{pmatrix} 2m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 5k & -k \\ -k & 3k \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\frac{7k}{\cancel{2m}} \cancel{2m}\, a_1 = 5k a_1 - k a_2$$

$$7a_1 = 5a_1 - a_2$$

$$\frac{7k}{2\cancel{m}} \cdot \cancel{m}\, a_2 = -k a_1 + 3k a_2$$

$$7a_2 = -2a_1 + 6a_2$$

$$2a_1 + a_2 = 0 \qquad a_2 = -2a_1$$

$$\vec{a}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \qquad \vec{a}_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$