

MA3105

Final Project

1. Explore the power method to find the dominant eigen value and eigen vector of a given matrix and describe the error estimate and convergence. Write a code to numerically compute the dominant eigen value of the given matrix A, plot the original eigen spectrum and comment on the drawbacks of the method. Compare eigenvalue computed with power method with theoretical eigen value calculated using eigen decomposition.

$$A = \begin{bmatrix} -7 & 13 & -16 \\ 13 & -10 & 13 \\ -16 & 13 & -7 \end{bmatrix}$$

2. Explore cubic spline interpolation technique and the associated error analysis. Write a code to interpolate the function $y=e^x$, $-1 \leq x \leq 1$, using spline interpolation and compare with linear and quadratic interpolation. Use evenly spaced nodes on $[-1, 1]$ to generate data points in sets of 10, 20 and 40 points and fit piecewise interpolation functions. Plot the results.
3. Explore least square approximation technique and discuss the drawbacks and possible improvements on the method. Write a code to approximate the function $y=e^{-x}$, $-1 \leq x \leq 1$, using the approximation method and compare with a suitable interpolation technique of your choice.
4. Describe the trapezium rule method for solving first order ordinary differential equations along with convergence and error analysis. Solve the following ODEs using trapezium method:
a) $dy/dx = y - x; y(0) = 2/3;$
b) $dy/dx = y - x^2; y(0) = 2/3;$
Change the initial condition of the differential equations and show the direction field graphically. Compare the error with that of Euler's method graphically.
5. Describe the Runge Kutta method (order 2 and order 4) for solving ordinary differential equations along with convergence and error analysis. Solve the following ODEs using RK2 and RK4:
 $dy/dx = y - x; y(0) = 2/3;$
 $dy/dx = y - x^2; y(0) = 2/3;$
Change the initial condition of the differential equations and show the direction field graphically.