April 9, 2024

Probability I

MA2202

Assignment 6

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Exercise 1. (5 points)
Let $X \sim \mathrm{N}(0,1)$. Show that the MGF of X is given by $M_X(t) = e^{t^2/2}$.

Exercise 2. (10 points)
(i) Let $X_1, X_2, \ldots, X_n$ be iid discrete random variables with common PMF f. Let $m \leq n$, be a positive integer and let $t_1, t_2, \ldots, t_m$ be m distinct points in the range of X_1. Determine the probability

$$P(\{X_1, X_2, \ldots, X_n\} \subseteq \{t_1, t_2, \ldots, t_m\}).$$

Exercise 3. (5 points)
Show that for $x \in (0,1)$ and $j \in \{1, 2, \ldots, n\}$, we have

$$\sum_{r=j}^{n} \binom{n}{r} x^r (1-x)^{n-r} = n \binom{n-1}{j-1} \int_0^x t^{j-1} (1-t)^{n-j} dt.$$

Exercise 4. (10 points)
Let there be a coin such that the probability of obtaining a head when the coin is tossed is p. Let H_n and T_n denote the number of heads and tails in n tosses of the coin. Given $\epsilon > 0$, show that

$$\mathbb{P}\Big(2p - 1 - \epsilon \leq \frac{1}{n}(H_n - T_n) \leq 2p - 1 + \epsilon\Big) \to 1 \quad \text{as} \quad n \to \infty.$$

Exercise 5. (20 points)
Show that if two states of a Markov chain are in the same communication class, either both of them are recurrent, or both of them are transient.

Full marks: 50. Please mention your name, roll no. and **group** in your answersheet. Please submit your answersheet by 11:59 p.m. on **April 23, 2024** in the DMS mailbox for MA2202, which is designated with your group name. Submit only your final answersheet! **Multiple answersheets or corrections to the original answersheet will not be accepted, irrespective of the time of their submission.**