# Probability and Statistics

Part-8

## MA2103 - 2023

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# Let's toss some coins



We know $P(H) = 1/2$ and$P(T) = 1/2$

If we toss two coins at same time, we following possibility, and

All events are equally-likely and we

We have$P(H, H) = P(H, T)$

$= P(T, H) = P(T, T)$

$= 1/4$

If we toss two coins, what is the prob. that at least one of the two coin is tails, given that the first flip is heads

$P(A = \text{at least one of the coins is tails}) = 3/4$

$P(B = \text{the first coins is heads}) = 2/4 = 1/2$

We have already peeped at the first coins there are two possibilities or the condition that first should be heads, reduces the domain to only two possibilities {HH} and {HT}. Out of which only possibility is second coins is head.

Answer to the question is$1/2$

# Conditional probability

Here we are looking at conditional probability, where first a event has already happened, and denoted by $P(A \mid B)$, probability of occurring event when the event B has already occurred

$P(A)$ prob. at least one of the coins is tails

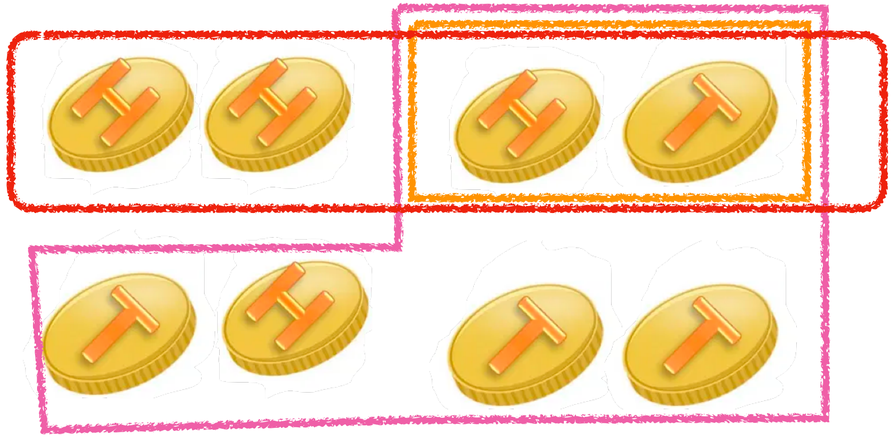
$P(B)$ prob. the first coins is heads

$P(A \cap B)$ prob. at least one coin is tail prob. of first coin is head

The conditional prob of A given event B is true is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(A \cap B) = 1/4}{P(B) = 1/2} = \frac{1}{2}$$



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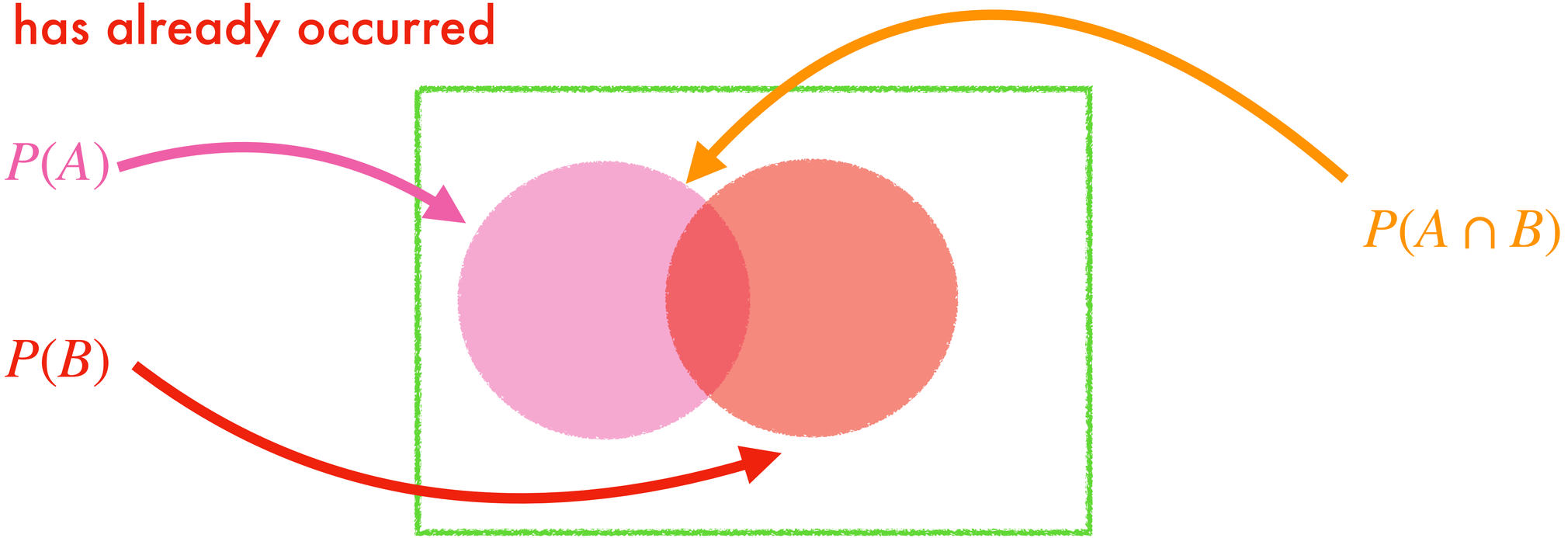


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# Another Example

A random card is drawn from a deck of 52 - cards. What is the probability the cards is a King, given that the card is red

$P(A)$ Prob. that randomly drawn card is King $= 4/52$

$P(B)$ Prob. that a given a card is red $= 26/52$

We need $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$P(A \cap B)$ prob that card read and king $= 2/52$

$$P(A \mid B) = \frac{2/52}{26/52} = \frac{2}{26} = \frac{1}{13}$$

# Independent events

Two events are independent if $P(A \mid B) = P(A)$

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# Let's play around with the Conditional probability

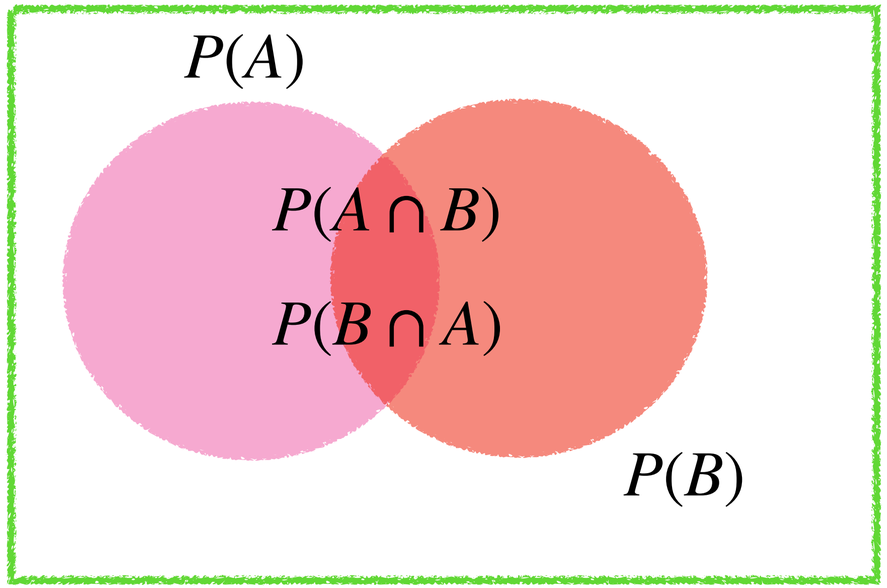
We have two events $A$ and $B$, and know $P(A)$, $P(B)$, and $P(A \ B)$, can we figure out $P(B \ A)$

We know $P(A \ B) = \dfrac{P(A \cap B)}{P(B)}$

We know $P(B \ A) = \dfrac{P(B \cap A)}{P(A)}$

$$P(A \cap B) = P(B \cap A)$$

We can reverse this and write




From this we get $P(B \ A) = \dfrac{P(A \ B)P(B)}{P(A)}$

This is called Bayes' theorem

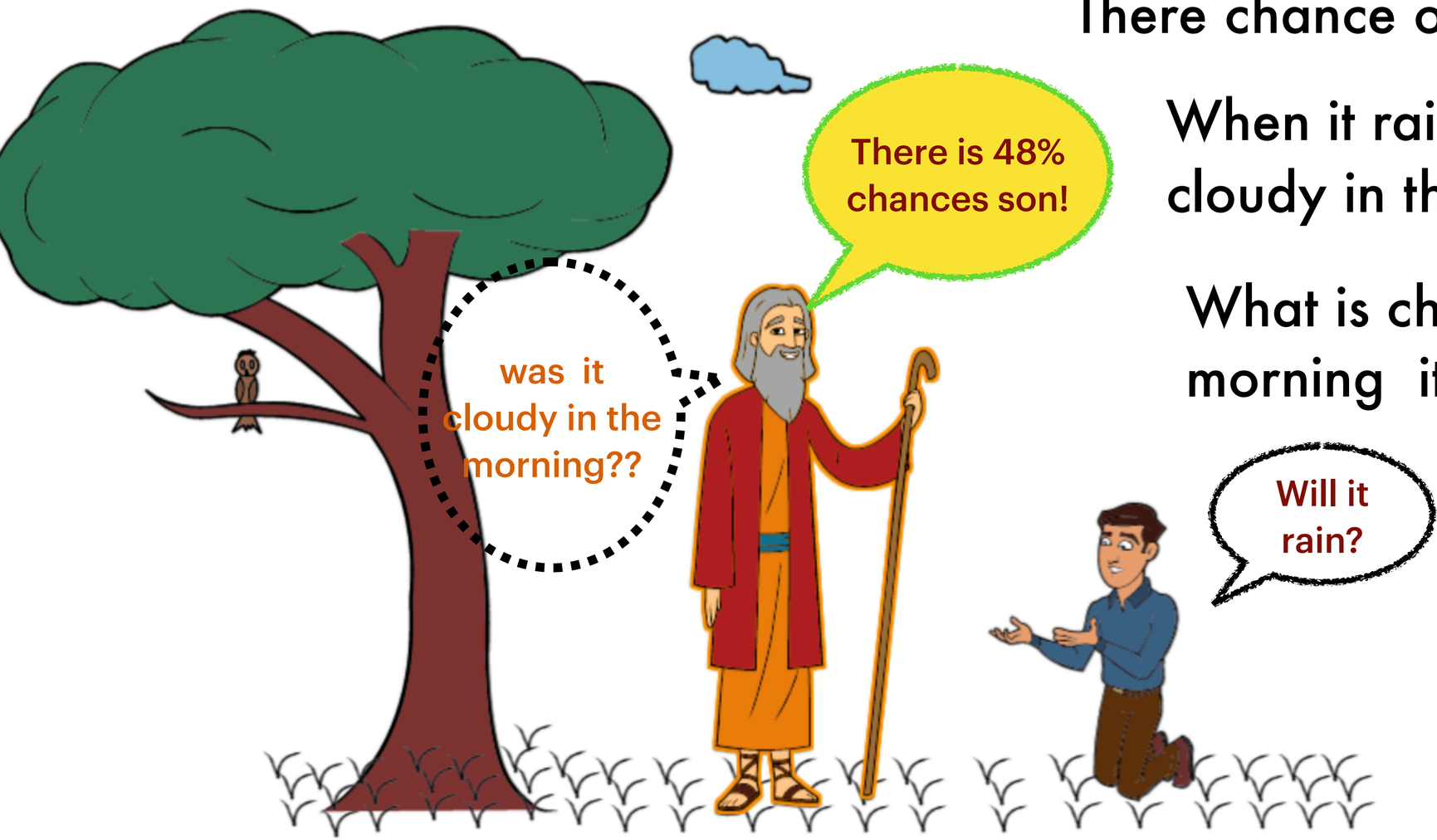
# Will it rain today ?

There is chance rain on any day is $15\,\%$

There chance of cloudy day $20\,\%$

When it rains $80\,\%$ of days where cloudy in the morning

What is chance if it is cloudy in the morning it would rain ?



# Will it rain today ?

There is chance rain on any day is 15 %  $P(rainy) = .15$

There chance of cloudy day 20 %  $P(cloudy) = .2$

When it rains 80 % of days where cloudy in the morning  $P(cloudy \mid rainy) = .8$

What is chance if it is cloudy in the morning it would rain ?

Let's use Bayes' theorem

$$P(rain \mid cloudy) = \frac{P(cloudy \mid rainy)P(rainy)}{P(cloudy)} = \frac{0.8 \times 0.15}{0.2} = 0.48$$