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Linear Algebra I (MA2102)

Mock Test II

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Exercise 1. (8 points)
Let A be an $n \times n$ matrix, all of whose row sums equal 1. Prove that for any positive integer m, each of the row sums of the matrix A^m is also equal to 1.

Exercise 2. (8 points)
Let V be a finite dimensional vector space and let W_1 and W_2 be two subspaces of V. Show that

$$(W_1 + W_2)^0 = W_1^0 \cap W_2^0.$$

Exercise 3. (4+8 points)
i) Let $\mathrm{M}_2(\mathbb{R})$ be the vector space of 2×2 real matrices, let $A := \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ and let T be a linear operator on $\mathrm{M}_2(\mathbb{R})$, defined by $T(M) = MA + AM$ for all $M \in \mathrm{M}_2(\mathbb{R})$. Find the trace of T.

ii) Let $\mathrm{M}_n(\mathbb{R})$ be the vector space of $n \times n$ real matrices. Let A be an $n \times n$ diagonal matrix with its (j, j)-th entry being equal to j. Let $T : \mathrm{M}_n(\mathbb{R}) \to \mathrm{M}_n(\mathbb{R})$ be defined by $T(M) = MA + AM$ for all $M \in \mathrm{M}_n(\mathbb{R})$. Find the trace of T. Is T diagonalizable? Justify your answer!

Exercise 4. (6+4 points)

Let $V = \mathbb{R}^4$ and let $f \in V^*$ be defined by $f(v) = v_1 + 2v_2 + 3v_3 + 4v_4$ for $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} \in V$.

i) Find an orthonormalbasis of $\ker f$ with respect to the dot product.
ii) Extend the orthonormalbasis of $\ker f$ to an orthonormalbasis of V.

Exercise 5. (8+4 points)
i) Show that an $n \times n$ Hermitian matrix A is positive definite if and only if all the eigenvalues of A are positive real numbers.
ii) Let M be an $n \times n$ real matrix with positive real eigenvalues. Is M necessarily positive definite? Justify your answer!

Full marks: 50