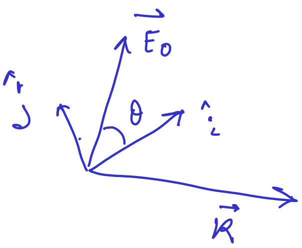
Polarization

Consider the plane formed by the displacement vector and the wavevector $\vec{k}$. If the displacement vector remains in the same direction as one moves along $\vec{k}$, we have a linearly polarized light.

$\vec{E} = \vec{E}_0 \cos(kz - \omega t)$ $\qquad$ $\vec{E}_0$ is not a function of z or t.



We choose the axes as shown in the figure.

$$\Rightarrow \vec{E}_0 = E_0 \cos\theta \, \hat{i} + E_0 \sin\theta \, \hat{j}$$

$$\text{and} \quad \vec{E} = \underbrace{\left(E_0 \cos\theta \, \hat{i} + E_0 \sin\theta \, \hat{j}\right)}_{\text{amplitude.}} \cos(kz - \omega t)$$

Where we choose x axis such that xz is the plane of incidence.

$$= \left(E_0^p \, \hat{i} + E_0^s \, \hat{j}\right) \cos(kz - \omega t)$$

amplitude of

- E_0^p: p-type, p-polarized, p-component
- E_0^s: s-type, s-polarized, s-component

Continued in the next page.

Now, Fresnel's equations

$$\frac{E_R^p}{E_I^p} = \frac{\alpha - \beta}{\alpha + \beta} \qquad \frac{E_T^p}{E_I^p} = \frac{2}{\alpha + \beta}$$

$$\boxed{\frac{E_R^s}{E_I^s} = \frac{1 - \alpha\beta}{1 + \alpha\beta} \qquad \frac{E_T^s}{E_I^s} = \frac{2}{\alpha + \beta}}$$

← not done in the class but is quite straight forward.

Fresnel's equations along with the decomposition to "s" and "p" types provide complete description.

* <u>Case</u> $\alpha = \beta \Rightarrow E_R^p = 0$ and $E_R^s \neq 0$ → <u>Brewster's angle</u>

$\Rightarrow$ Reflected light is linearly polarized.

How polarizers work:

<u>Explanation</u> based on free electrons in "wire".

Now, consider

$$\vec{E} = E_0 \hat{i} \cos(kz - \omega t) = \frac{E_0}{2}\left\{\hat{i}\cos(kz-\omega t) + \hat{j}\sin(kz-\omega t)\right\}$$
$$+ \frac{E_0}{2}\left\{\hat{i}\cos(kz-\omega t) - \hat{j}\sin(kz-\omega t)\right\}$$

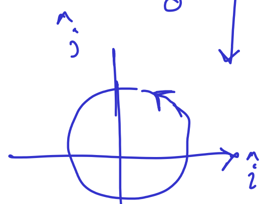
$$= \frac{E_0}{2}\left\{\hat{i}\cos(kz-\omega t) + \hat{j}\cos(kz-\omega t - \pi/2)\right\}$$
$$+ \frac{E_0}{2}\left\{\hat{i}\cos(kz-\omega t) + \hat{j}\cos(kz-\omega t + \pi/2)\right\}$$

using Complex notation

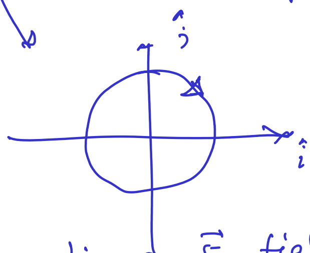
$$= \frac{E_0}{2}\left\{\hat{i}\, e^{i(kz-\omega t)} + \hat{j}\, e^{-i\pi/2} e^{i(kz-\omega t)}\right\}$$
$$+ \frac{E_0}{2}\left\{\hat{i}\, e^{i(kz-\omega t)} + \hat{j}\, e^{+i\pi/2} e^{i(kz-\omega t)}\right\}$$

$$= \frac{E_0}{2}\left\{(\hat{i} + i\hat{j})\, e^{i(kz-\omega t)}\right\}$$
$$+ \frac{E_0}{2}\left\{(\hat{i} - i\hat{j})\, e^{i(kz-\omega t)}\right\}$$

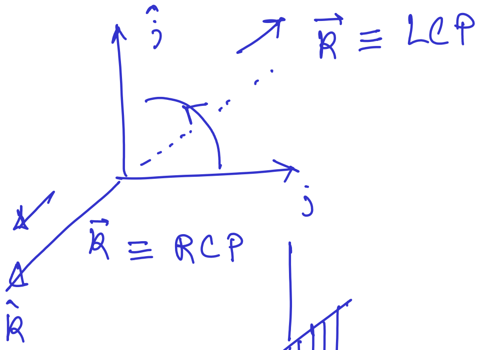
$$= \vec{E}_+ + \vec{E}_-$$

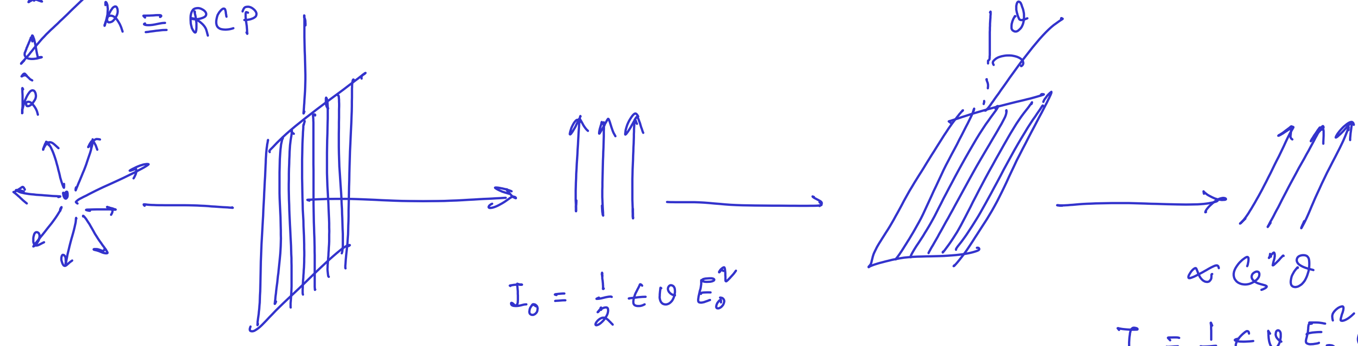


tip of $\vec{E}$ field at a given t as one moves along z → RCP



tip of $\vec{E}$ field at a given t as one moves along z → LCP



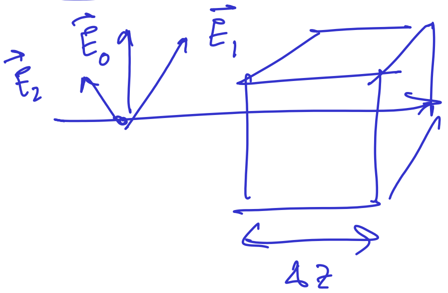
⊛ 

$I = \frac{1}{2} \epsilon v E_0^2 \cos^2\theta$

Malus's law.

⊛ Reflection of RCP gives LCP

⊛ Retardation plates (has two optical axes with diff. n values).



$$\frac{\omega}{k} = v = \frac{c}{n} \Rightarrow k = n\frac{\omega}{c}$$

$$\Rightarrow \text{Phase lag} = k\Delta z = n\frac{\omega}{c}\Delta z$$

Lag between two axes

$$= n_1 \frac{\omega}{c}\Delta z - n_2 \frac{\omega}{c}\Delta z$$

$$= \Delta n \frac{\omega}{c} \cdot \Delta z = \Delta n \frac{2\pi}{\lambda_0} \cdot \Delta z$$

$$\boxed{\phi = 2\pi \cdot \Delta n \cdot \frac{\Delta z}{\lambda_0}}$$

$\phi \to \pi/2 \to$ quarterwave plate $\to$ linear to circular

$\phi \to \pi \to$ half-wave plate $\to$ RCP $\rightleftarrows$ LCP

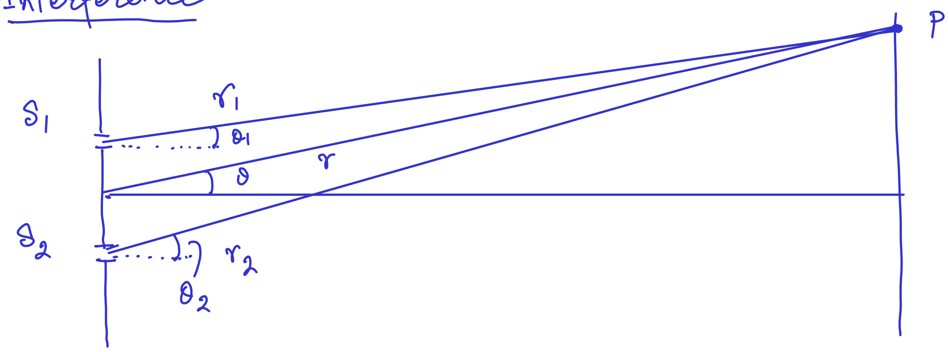
Interference and diffractions

Interference → Multiple discrete sources

diffractions → Continuous source

Both rely on the superposition principle.

Interference



S_1, S_2 are sources. Both emit EM waves with the same amplitude A.

EM waves from S_1 and S_2 arrive at P.

E field at P is given by,

$$E = A e^{i(kr_1 - \omega t + \phi_1)} + A e^{i(kr_2 - \omega t + \phi_2)}$$

$$= A e^{-i\omega t}\left[e^{i(kr_1 + \phi_1)} + e^{i(kr_2 + \phi_2)} \right]$$

$$= A e^{-i\omega t} e^{i\left(k\frac{r_1+r_2}{2} + \frac{\phi_1+\phi_2}{2}\right)} \left[e^{i\left(k\cdot\frac{r_1-r_2}{2} + \frac{\phi_1-\phi_2}{2}\right)} + e^{-i\left(k\frac{r_1-r_2}{2} + \frac{\phi_1-\phi_2}{2}\right)} \right]$$

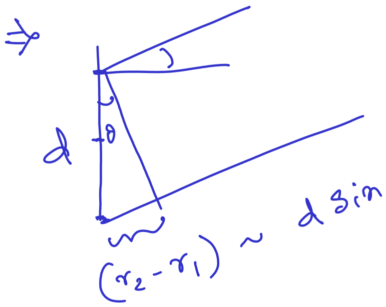
$$= A e^{-i\omega t} e^{i(kr + \phi_{av})}\, 2\cos\left(k\frac{\Delta r}{2} + \frac{1}{2}\Delta\phi\right)$$

$\Delta\phi = \phi_1 - \phi_2$

$\Delta r = r_1 - r_2$

Let, $\theta_1 \sim \theta_2 \sim \theta \implies$ Far-field approx

$\Rightarrow$



$$\text{Intensity} \propto |E|^2$$

$$\propto 4A^2\cos^2\left(\tfrac{1}{2}kd\sin\theta + \tfrac{1}{2}\Delta\phi\right)$$

For coherent sources, $\Delta\phi = 0$

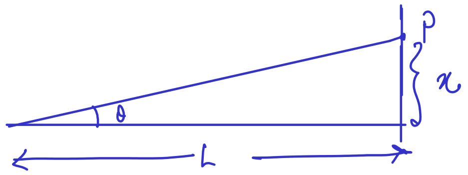
$$\Rightarrow \text{Intensity} \propto 4A^2\cos^2\left(\pi\frac{d}{\lambda}\sin\theta\right)$$

In general, minima occur when,

$$\tfrac{1}{2}kd\sin\theta = \boxed{\pi\frac{d}{\lambda}\sin\theta = (2n+1)\frac{\pi}{2}} \qquad n = 0, 1, 2, \dots$$

maxima occur when,

$$\tfrac{1}{2}kd\sin\theta = \boxed{\pi\frac{d}{\lambda}\sin\theta = n\pi} \qquad n = 0, 1, 2, \dots$$



For small θ,

$$\sin\theta \sim \tan\theta \sim \theta \sim \frac{x}{L}$$

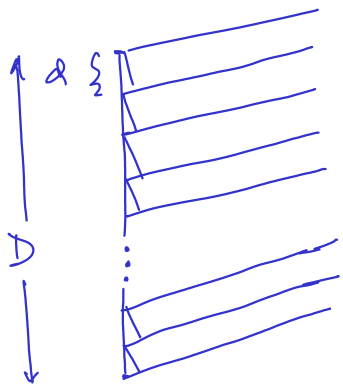
$$\Rightarrow \text{Intensity} \propto 4A^2 \cos^2\left(\frac{\pi d}{\lambda}\cdot\frac{x}{L}\right)$$

At $x=0$, central maxima.

$x = \frac{\lambda L}{2d}(2n+1)$ for $n = 0, 1, 2, \cdots$, minima

* Adding $\Delta\phi$ introduces shift of the pattern.

Diffraction.



For an extended source of width D, we divide it into N segments with $d = D/N$.
$\Rightarrow$ There are $(N+1)$ beams (see figure); we are counting them from the edges. N is large.

Amplitude at a far away point (small angle holds and all angles are approximated by the average angle) is given by,

$$E = \frac{A}{(N+1)} e^{i(kr-\omega t)}\left[1 + e^{ikd\sin\theta} + e^{ik2d\sin\theta} + \cdots + e^{ikNd\sin\theta}\right]$$

$$= \frac{A}{(N+1)} e^{i(kr-\omega t)} \left(\frac{e^{ikNd\sin\theta} - 1}{e^{ikd\sin\theta} - 1}\right)$$

$$= \frac{A}{(N+1)} e^{i(kr-\omega t)} \frac{e^{i\frac{kNd}{2}\sin\theta}}{e^{i\frac{kd}{2}\sin\theta}} \cdot \frac{e^{i\frac{kNd}{2}\sin\theta} - e^{-i\frac{kNd}{2}\sin\theta}}{e^{i\frac{kd}{2}\sin\theta} - e^{-i\frac{kd}{2}\sin\theta}}$$

$$= \frac{A}{(N+1)} \cdot e^{i(kr-\omega t)} e^{ik(N-1)\frac{d}{2}\sin\theta} \cdot \frac{\sin\frac{Nkd\sin\theta}{2}}{\sin\frac{kd\sin\theta}{2}}$$

$$\Rightarrow I \propto |E|^2 = \frac{A^2}{(N+1)^2} \cdot \frac{\sin^2\frac{Nkd\sin\theta}{2}}{\sin^2\frac{kd\sin\theta}{2}} = \frac{A^2}{(N+1)^2} \cdot \frac{\sin^2 N\phi}{\sin^2\phi}$$

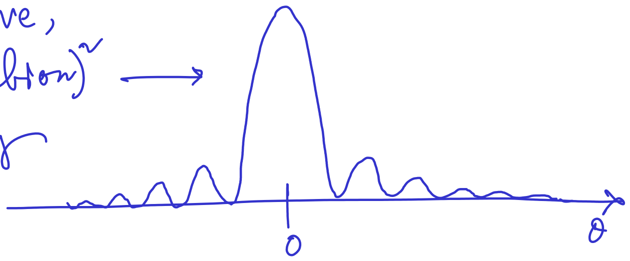
where, $\phi = \frac{kd\sin\theta}{2} = \pi\frac{d}{\lambda}\sin\theta$.

For very large N, we can have a small ϕ and $\sin\phi \sim \phi$ and $(N+1)^2 \sim N^2$

$$\Rightarrow \quad I \propto \frac{A^2}{N^2}\cdot\frac{\sin^2\left(\pi \frac{Nd}{2\lambda}\sin\theta\right)}{\left(\pi\frac{d}{2\lambda}\sin\theta\right)^2} = A^2\,\frac{\sin^2\left(\pi\frac{D}{2\lambda}\sin\theta\right)}{\left(\frac{\pi D}{2\lambda}\sin\theta\right)^2} \approx A^2\,\frac{\sin^2\left(\frac{\pi D}{2\lambda}\theta\right)}{\left(\frac{\pi D}{2\lambda}\theta\right)^2}$$

For small angle

So, we have, a $(\text{sinc function})^2$ like intensity distribution.



* Sinc function has a θ-dependent width
$\Rightarrow$ the beam "spreds" out as a result of the diffraction.