

### Classtest 1 of 3

Instructor: Dr. Rajesh Kumble Nayak

6:00 PM, 1 September 2023.

Duration: 50 Minutes.

- Answer all the question.
- All question carry equal marks.
- No calculators are allowed!

Q - 1: Plot the function

$$f(x) = \frac{1}{1-x} + \frac{1}{1+x},$$

In an appropriate domain.

$$f(x) = \frac{1}{1-x} + \frac{1}{1+x} = \frac{1+x+1-x}{1-x^2} = \frac{2}{1-x^2}$$

$$f(x) = \frac{2}{1-x^2} ; = \frac{2}{(1-x)(1+x)}$$

domain  $(-\infty, \infty]$

Singular point  $x = \pm 1$

at  $\pm \infty \rightarrow 0$  ; at 0  $\rightarrow 2$

at  $\pm 1$  ; let us first look

at  $x_0 = -1 - \epsilon$

$$f(x) = \frac{2}{(1-x)(1+x)} \quad (1-x) = 1 - (-1-\epsilon) = 2+\epsilon$$

$$(1+x) = 1 - 1 - \epsilon = -\epsilon$$

$$f(x) \longrightarrow -\infty$$

$$\text{as } \epsilon \longrightarrow 0 \text{ at } -1$$

side to at -1

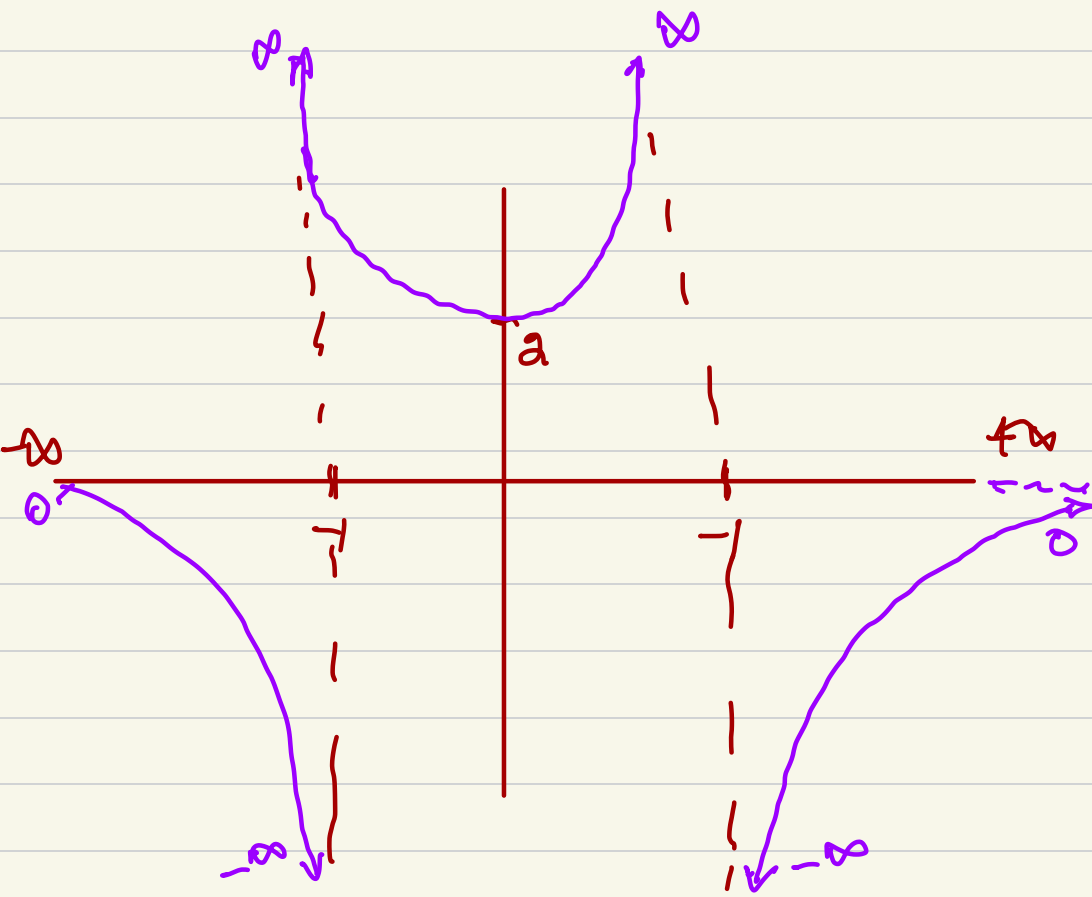
let us write  $x = -1 + \epsilon$

$$1-x = 1 - (-1 + \epsilon) = 2 - \epsilon$$

$$1+x = 1 - 1 + \epsilon = \epsilon$$

$$f(x) = \frac{1}{(2-\epsilon)(\epsilon)} \longrightarrow +\infty$$

as  $x \longrightarrow -1$  from right



Q - 2: Find the Fourier series expansion of the function

$$f(x) = \begin{cases} \sin x & x > 0 \\ -\sin x & x < 0 \end{cases},$$

Make plot of function  $f(x)$ , first two individual terms and the partial sum of two terms.

the function is even function around zero

so  $b_n = 0$  for all  $n$

Let us find  $a_0$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$= \frac{1}{2\pi} \int_{-\pi}^0 -\sin x dx + \int_0^{\pi} \sin x dx$$

$$= \frac{2}{2\pi} \int_0^{\pi} \sin x dx = \frac{4}{2\pi} = \frac{2}{\pi}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^0 \sin x \cos nx dx + \frac{1}{\pi} \int_0^{\pi} \sin x \cos nx dx$$

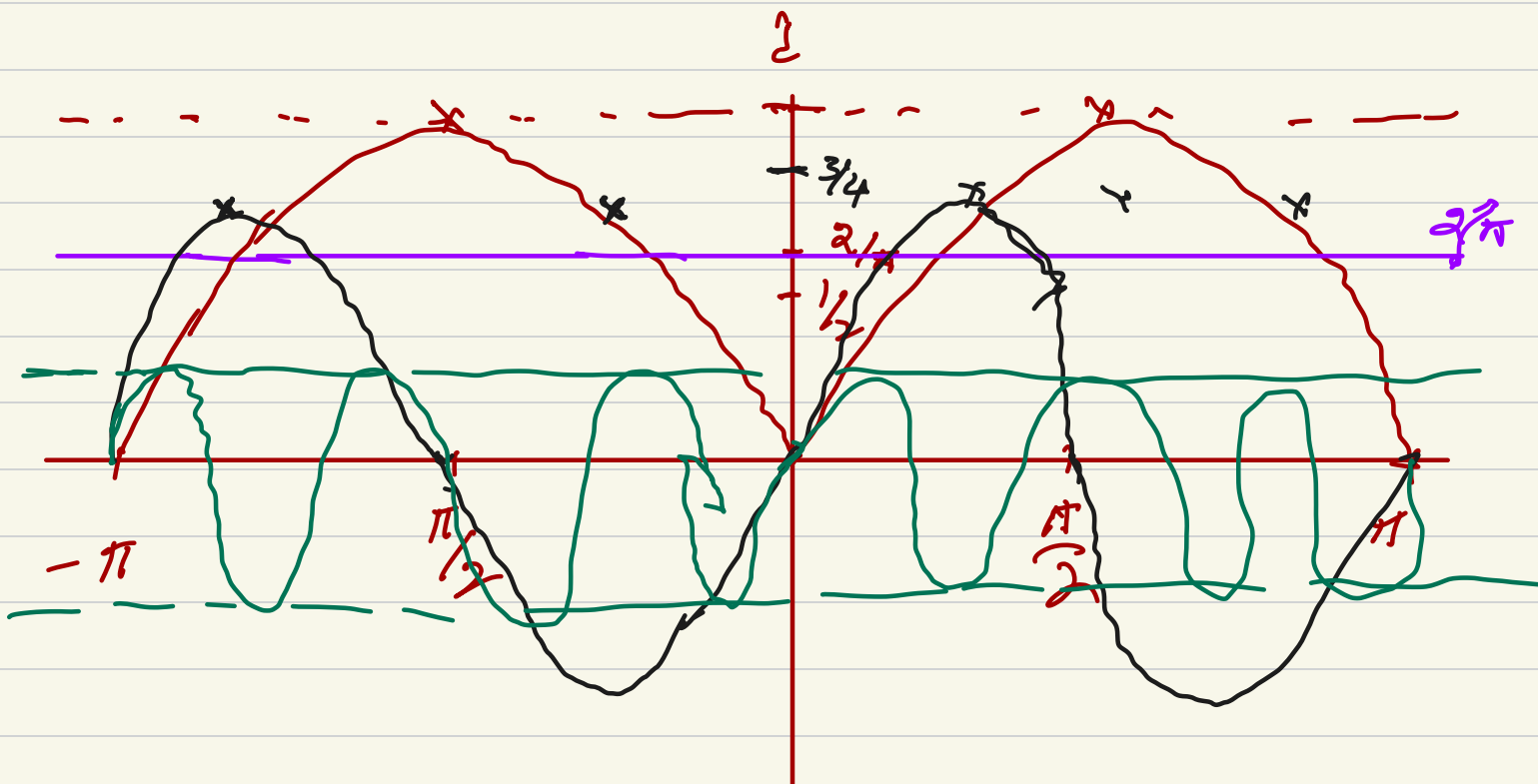
$$a_n = \frac{1 + \cos n\pi}{1 - n^2} + \frac{1 + \cos n\pi}{1 - n^2}$$

$$= \frac{2(1 + \cos n\pi)}{1 - n^2}$$

$$= 0 \text{ for odd } n$$

$$= \frac{4}{1 - n^2} \text{ for even } n$$

$$f(x) = \frac{2}{\pi} - \frac{4}{3} \sin 2x - \frac{4}{15} \sin 4x$$



Q - 3: The Dirac delta function  $\delta(x)$  is an improper function with the property,

$$\int_a^b f(x)\delta(x-x_0)dx = \begin{cases} f(x_0) & \text{if } a < x_0 < b \\ 0 & \text{otherwise} \end{cases}$$

Assuming  $\delta(x-x_0)$  is a  $\mathbb{L}^2$  function provide a Fourier Series Expansion in the interval  $[-L, L]$  for some +ve  $L$ , and  $x_0 \in [-L, L]$ .

$$\text{Let } f(x) = \delta(x-x_0)$$

$$a_0 = \frac{1}{2\pi} \int_{-L}^L f(x) dx$$

$$= \frac{1}{2\pi} \int_{-L}^L \delta(x-x_0) dx = \frac{1}{2\pi}$$



$$a_0 = \frac{1}{2\pi}$$

$$a_n = \frac{1}{\pi} \int_{-L}^L f(x) \cos nx dx$$

$$= \frac{1}{\pi} \int_{-L}^L \delta(x-x_0) \cos nx dx$$

$$a_n = \frac{1}{\pi} \cos nx_0$$

$$\begin{aligned}
 b_n &= \frac{1}{\pi} \int_{-L}^L f(x) \sin nx \, dx \\
 &= \frac{1}{\pi} \int_{-L}^L \delta(x-x_0) \sin nx \, dx \\
 &= \frac{1}{\pi} \sin nx_0
 \end{aligned}$$


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$$a_0 = \frac{1}{2\pi}$$

$$a_n = \frac{1}{\pi} \cos nx_0$$

$$b_n = \frac{1}{\pi} \sin nx_0$$

$$\begin{aligned}
 \delta(x-x_0) &= \frac{1}{2\pi} + \frac{1}{\pi} \cos nx_0 \cos nx \\
 &\quad + \frac{1}{\pi} \sin nx_0 \sin nx
 \end{aligned}$$