February 23, 2024

Probability I

MA2202

Assignment 4

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Exercise 1. (5+5 points)
Let X,Y and Z be iid positive random variables defined over the same sample space. Show that

$$\mathrm{E}\left(\frac{2X+Z}{X+Y+Z}\right)$$

exists and determine its value.

Exercise 2. (6+8 points)
Provide examples of a discrete and a continuous random variable such that their expectations do not exist.

Exercise 3. (10 points)
Show that if the n-th moment exists for some $n \in \mathbb{N}$, then so does the m-th moment for all $m \in \{1, 2, \ldots, n\}$.

Exercise 4. (6+6+4 Points)
Recall that for two random variables X and Y, we define

$$\mathrm{Cov}(X, Y) := E(XY) - E(X)E(Y).$$

(*i*) Show that $\mathrm{Cov}(X, Y) = E((X - E(X)(Y - E(Y))$.

(*ii*) For random variables $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$, show that

$$\mathrm{Cov}\left(\sum_{i=1}^{n} a_i X_i, \sum_{j=1}^{m} b_j Y_j\right) = \sum_{i=1}^{n}\sum_{j=1}^{m} a_i b_j \mathrm{Cov}(X_i, Y_j)$$

for all $a_i, b_j \in \mathbb{R}$.

(*iii*) Conclude from the above that for random variables $X_1, X_2, \ldots, X_n$, we have

$$\mathrm{Var}\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i^2 \mathrm{Var}(X_i) + 2 \sum_{1 \le i < j \le n} a_i a_j \mathrm{Cov}(X_i, X_j)$$

for all $a_i \in \mathbb{R}$.

Full marks: 50. Please mention your name, roll no. and **group** in your answersheet. Please submit your answersheet by 11:59 p.m. on **March 7, 2024** in the DMS mailbox for MA2202, which is designated with your group name. Submit only your final answersheet! **Multiple answersheets or corrections to the original answersheet will not be accepted, irrespective of the time of their submission.**