Ordinary Differential Equations

Runge-Kutta Methods

Part-16

MA2103 - 2023

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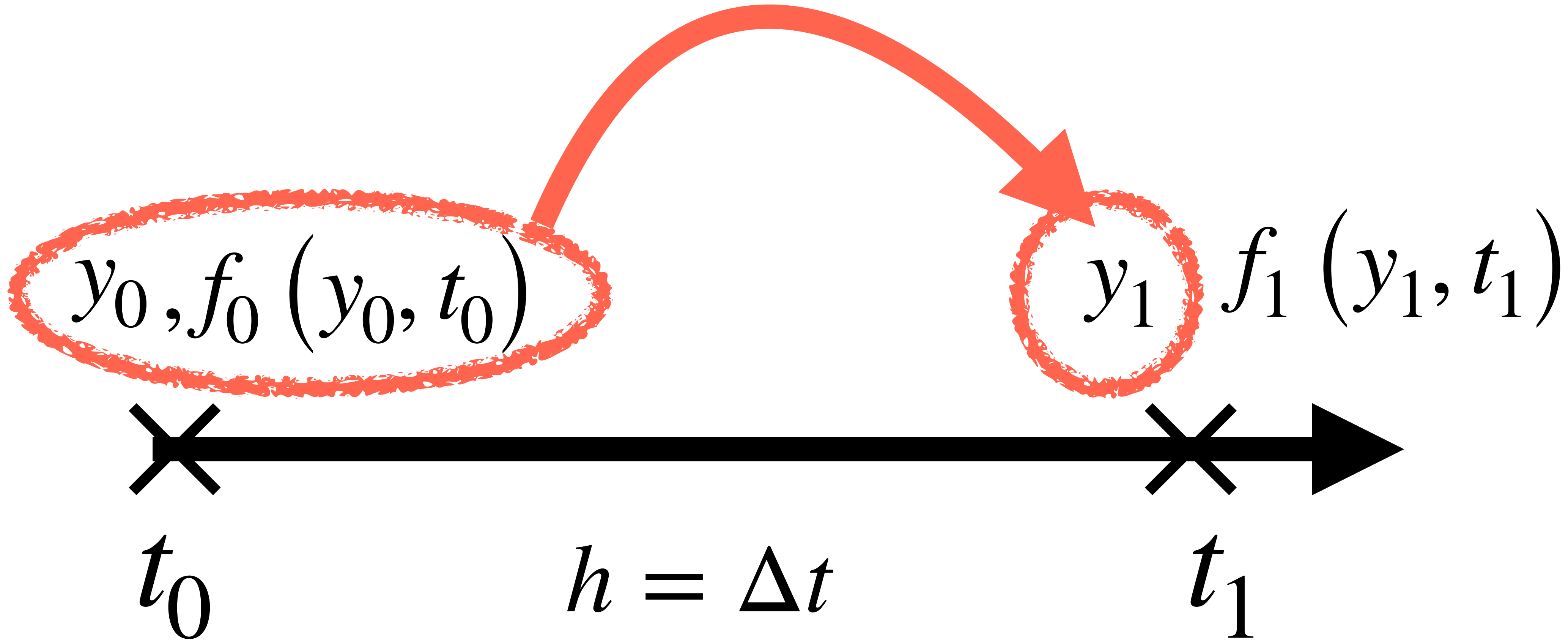
Numerical solution to ODE

Here we solve of set of ordinary differential equations of the for

$$\frac{dy_i}{dt} = f(y_i, t) \qquad \text{for} \qquad i = 1, 2, \cdots N$$

$$\frac{dy}{dt} \approx \frac{y(t+h) - y(t)}{h}$$

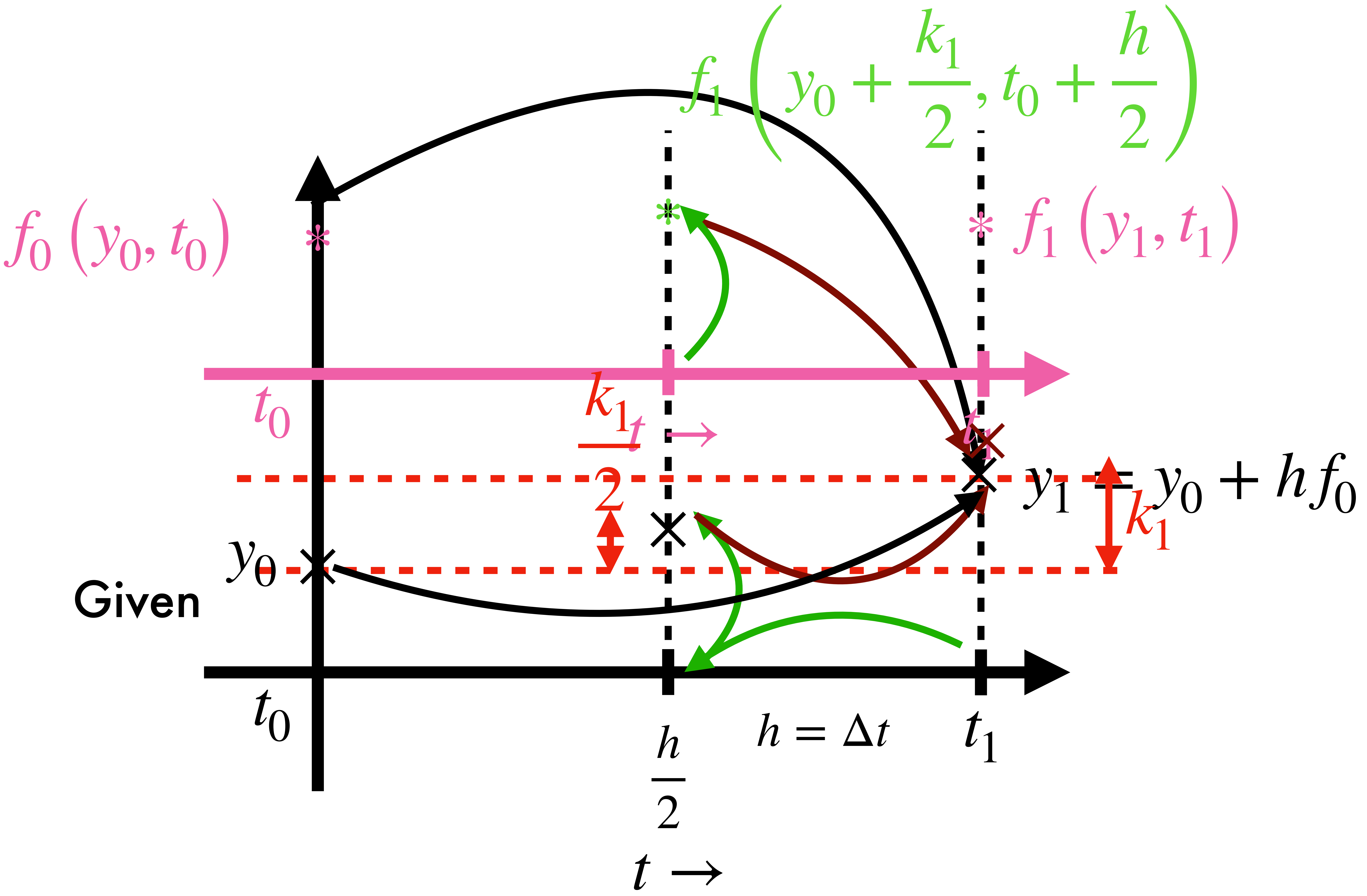


Unfortunately, this is not accurate enough!, we need higher order!

We could higher order or higher point formula to reduce error further

But one need to provide more points at the initial point $\{y_0, y_1, y_2\}$ at $\{t_0, t_1, t_2\}$

2^{nd} Order Runge - Kutta Method



2^{nd} Order Runge - Kutta Method

1. We start with given t_n and y_n we can compute $f_n = f\left(y_n, t_n\right)$

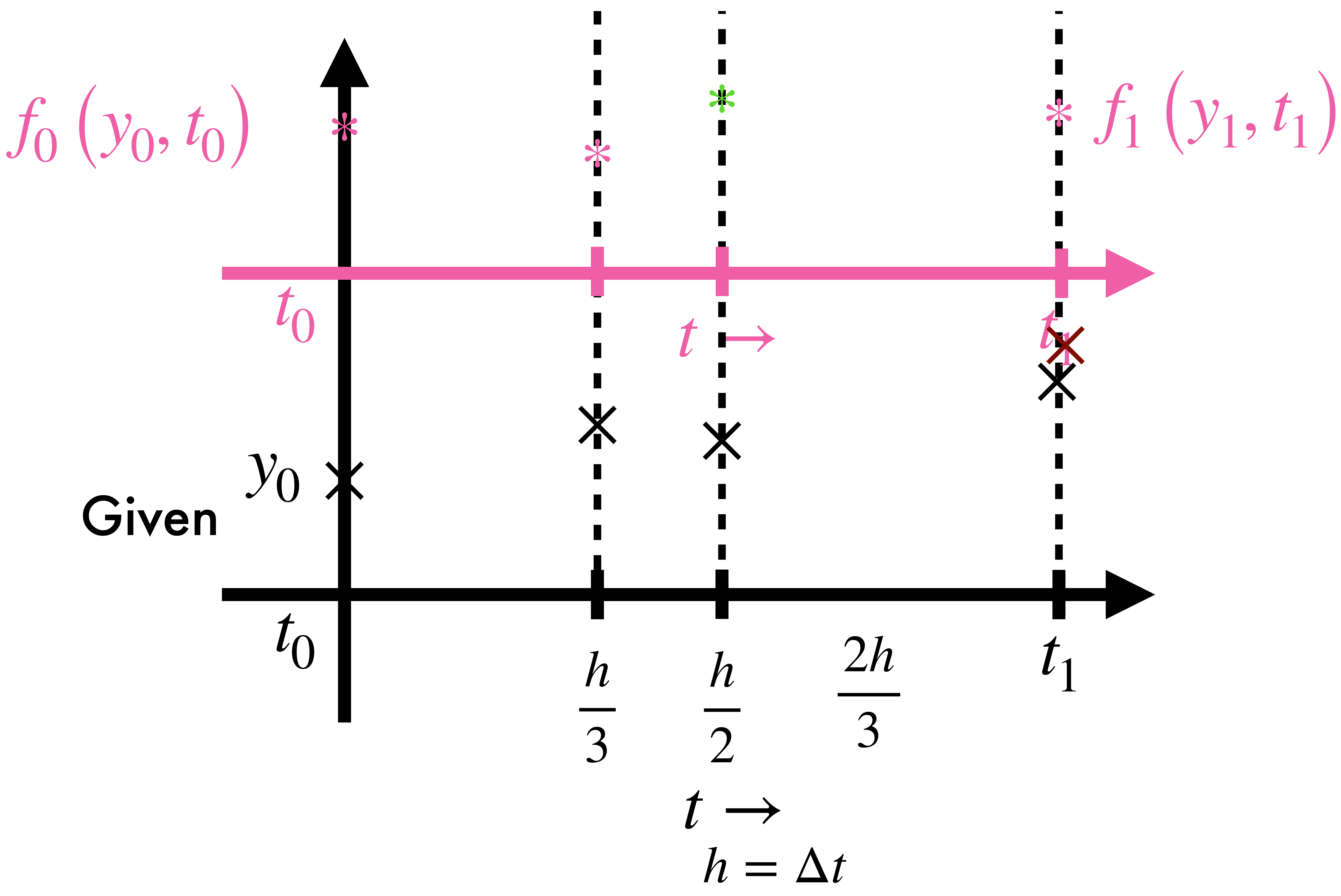
   The first order computation gives $y_n + hf\left(y_n, t_n\right)$, we are not interested in it

2. Compute $k_1 = hf\left(y_n, t_n\right)$
3. compute $k_2 = hf\left(y_n + \dfrac{k_1}{2},\ t_n + \dfrac{h}{2}\right)$
4. finally $y_{n+1} = y_n + k_2 + \mathcal{O}\left(h^3\right)$
5. Repeat step 2 to 4, N times

This is result is accurate up to $\mathcal{O}\left(h^3\right)$

Higher order Runge - Kutta Method

Runge-Kutta can be extended to higher orders.



4^{th} Order Runge-Kutta Method

1. Starting with given t_n and y_n we can compute $f_n = f\left(y_n, t_n\right)$
2. Compute $k_1 = hf\left(y_n, t_n\right)$
3. compute $k_2 = hf\left(y_n + \dfrac{k_1}{2},\ t_n + \dfrac{h}{2}\right)$
4. compute $k_3 = hf\left(y_n + \dfrac{k_2}{2},\ t_n + \dfrac{h}{2}\right)$
5. compute $k_4 = hf\left(y_n + k_3,\ t_n + h\right)$
6. finally $y_{n+1} = y_n + \dfrac{k_1}{6} + \dfrac{k_2}{3} + \dfrac{k_3}{3} + \dfrac{k_4}{6} + \mathcal{O}\left(h^5\right)$

Repeat till steps 1 to 6 for N iteration