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Probability I

MA2202

Assignment 3

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Exercise 1. (2+2+6 Points)
(a) Let $(\Omega, \mathcal{E}, P)$ be a probability space and let $A \subset \Omega$. Define $X : \Omega \to \mathbb{R}$ by

$$X(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{otherwise.} \end{cases}$$

In which of the following cases is X a random variable? Justify your answer!

$$(i)\ A \in \mathcal{E} \qquad (ii)\ A \notin \mathcal{E}.$$

(b) Let $(\Omega, \mathcal{E}, P)$ be a probability space with $\Omega = \{1, 2, 3, 4, 5\}$ and $\mathcal{E} = \{\emptyset, \Omega, \{1\}, \{2, 3, 4, 5\}\}$. Define $X \colon \Omega \to \mathbb{R}$ by

$$X(\omega) = \omega + 1$$

for all $\omega \in \Omega$. Is X a random variable? Justify your answer!

Exercise 2. (10 Points)
Let X be a random variable defined on $\{1, 2, \ldots, 10\}$ with PMF $f(x) = ax+b$ and expectation 7. Find a and b.

Exercise 3. (10 Points)
For what value of the constant c, the real valued function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \frac{c}{1 + (x - \theta)^2},$$

where θ is a real parameter, is a PDF of random variable X?

Exercise 4. (4+4 Points)
Let $p \in [0, 1]$, $a, b \in \mathbb{R}$ with $a > b$ and let X be a random variable such that

$$P(X = a) = p \ \text{ and } \ P(X = b) = 1 - p.$$

Find the expectation and variance of $\frac{X-b}{b-a}$.

Exercise 5. (4+4+4 Points)
A bag contains five coins, two of which are made of gold and the rest are made of silver. Consider the random experiment in which the coins are drawn out of the bag randomly, one after another, without replacement. Let X denote the number of draws until the last gold coin is drawn. Find the PMF, the CDF and the expectation of the random variable X.

Maximum score: 50 points. Please mention your name, roll no. and **group** in your answersheet. Please submit your answersheet by 11:59 p.m. on **Frbruary 8, 2024** in the DMS mailbox for MA2202, which is designated with your group name. Submit only your final answersheet! **Multiple answersheets or corrections to the original answersheet will not be accepted, irrespective of the time of their submission.**