Probability and Statistics

Part-9

MA2103 - 2023

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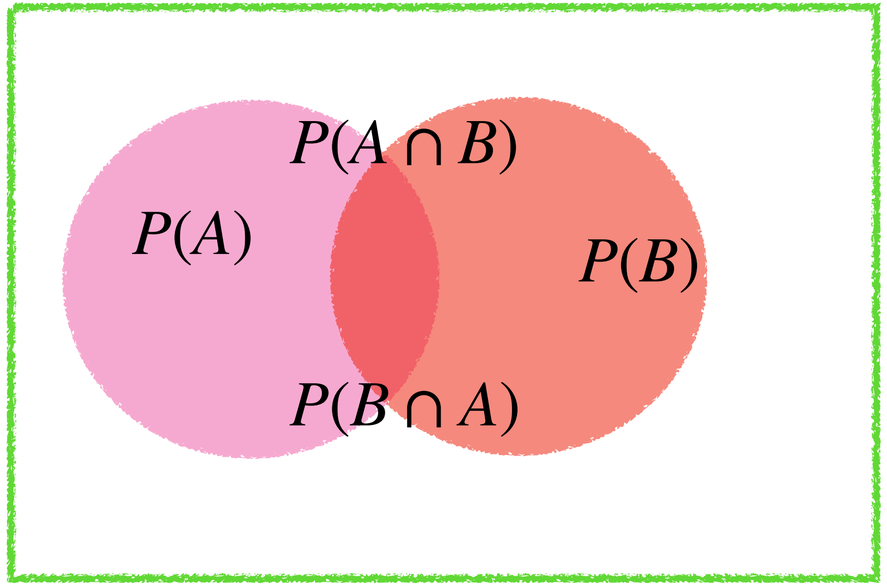
Bayes Theorem

We have two events A and B, and know $P(A)$, $P(B)$, and $P(A \mid B)$, can we figure out $P(B \mid A)$

From this we get $P(A \mid B) = \dfrac{P(B \mid A)P(A)}{P(B)}$

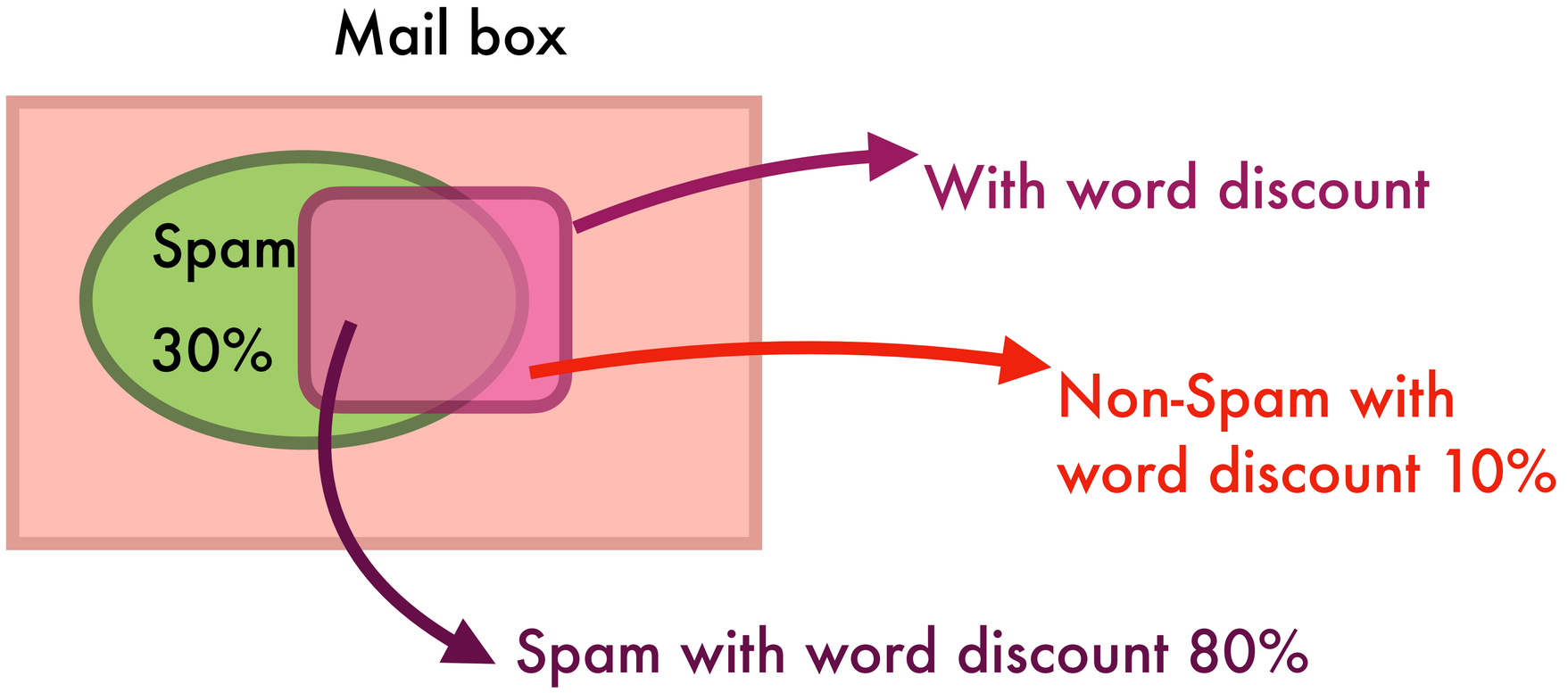


Example

30% of mails I get are spam mails. 80% of spam mails contain word “discount”.

I get 10% of non-spam mail also contain word “discount”.

Now I have mail in box which contain word “discount”. What is the probability it is spam!



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$P(A)$ the probability mail is spam $P(A) = 0.3$ and Non-spam 70%

$P(B)$ is the probability that mail has word “discount”

ie. 80% of 30% and 10% of 70% = $0.3 \times 0.8 + 0.1 \times 0.7 \ = 0.31$

$P(B \ A)$ Prob of spam mail that contain word “discount” $P(A \ B) = 0.8$

$P(A \ B)$ Prob of mail containing word “discount” is spam $P(B \ A) = ?$

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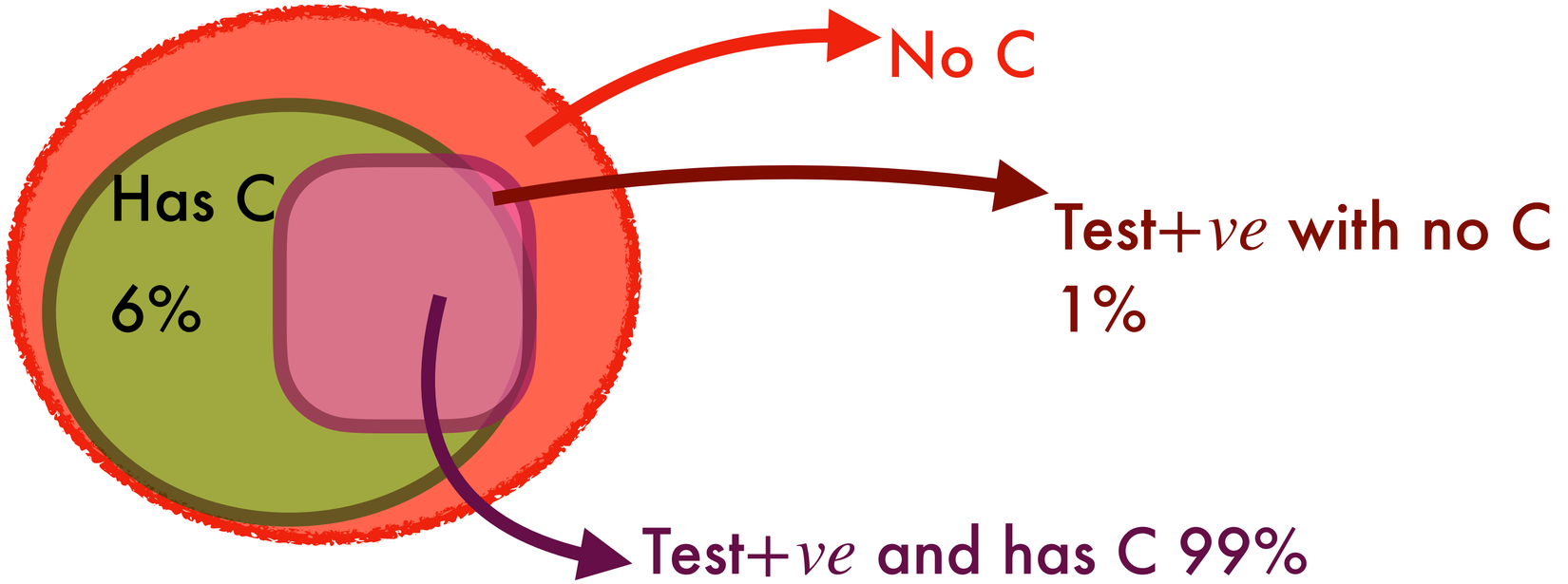
Let's use Bayes' theorem $P(A \ B) = \frac{P(B \ A)P(A)}{P(B)} = \frac{0.8 \times 0.3}{0.31} \ = 0.7742$

Example

This is a more practical example. Let us look at the cancer test. Test for a disease never fool proof. Let's say a cancer diagnostic test has 99% accuracy.

From general statistic, we know there is a 6% chance of any person getting caner!

Now question is, if a cancer diagnostic test comes with +ve result, what is probability that the person really has Cancer.



Test fails 1% of times

that means some of the people who has no cancer shown to have cancer by the test

$P(A)$ the probability a person has 'C' $P(A) = 0.06$ and Non-C 94%

$P(B \mid A)$ Prob of of test turning +ve is $P(A \mid B) = 0.99$

prob test giving +ve result for person not having 'C' is 1% of 94%

$P(B)$ prob of test giving +ve result is

Test resulting in +ve result is

06% of 99% and 94% of 1% = $0.06 \times 0.99 + 0.94 \times 0.01 = 0.0688$

$P(A \mid B)$ Prob of +ve test person has C $P(B \mid A) = ?$

Let's use Bayes' theorem $P(A \mid B) = \dfrac{P(B \mid A)P(A)}{P(B)} = \dfrac{0.99 \times 0.06}{0.0688} = 0.86337$