Tutorial 05 – 30/01/2025 @ 8:00 AM

Problem – 1 Show that following series

$$f(x) = \frac{1}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty} \cos(nx_0)\cos(nx) + \sum_{n=1}^{\infty} \sin(nx_0)\sin(nx)$$

where $x_0 \in [-\pi, \pi]$ $x \in [-\pi, \pi]$.

Show the $f(x)$ represent Dirac δ function $\delta(x - x_0)$

To show $\delta(x-x_0)$ is a Dirac delta function, we have to show, for an arbitrary function $f(x)$

$$\int_{-\pi}^{\pi} f(x)\,\delta(x-x_0)\,dx = f(x_0)$$

If $f(x)$ is L^2 function, we can write

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n\cos nx + b_n \sin nx)$$

where

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$\int f(x)\,\delta(x-x_0)\,dx$$

$$= \int_{-\pi}^{\pi} f(x) \left[\frac{1}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty} \cos nx \cos nx_0 + \frac{1}{\pi}\sum_{n=1}^{\infty} \sin nx \sin nx_0 \right] dx$$

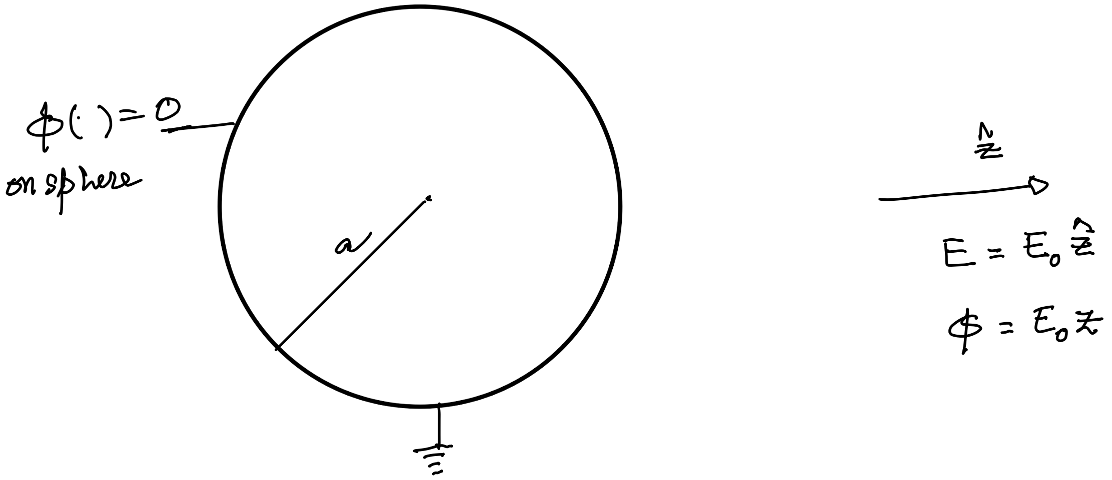
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx + \frac{1}{\pi}\sum_{n=1}^{\infty} \int_{-\pi}^{\pi} f(x) \cos nx \cos nx_0 \, dx$$

$$+ \frac{1}{\pi}\sum_{n=1}^{\infty} \int_{-\pi}^{\pi} f(x) \sin nx \sin nx_0 \, dx$$

$$= a_0 + \sum_{n=1}^{\infty} a_n \cos nx_0 + \sum_{n=1}^{\infty} b_n \sin nx_0$$

$$= f(x_0)$$

Problem - 2 > A conducting ground-sphere of radius 'a' is place uniform electric field E_0. Find the potential outside sphere.



The field at $r \to \infty$ is $E_0 \hat{z}$

The potential $\phi_\infty = -E_0 z$

We use the polar coordinate and Explor azimutal symmetry

$$\phi_\infty = -E_0 r \cos\theta$$

Since there are no charges outside azimutally symmetric sol^n to laplace eq^n is given by

$$\phi = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

generally we assume $A_l = 0$ outside

that ~~it~~ will not satisfy boundary condition as $r^{-(l+1)} \to 0$ as

$r \to \infty$ $\qquad P_1(\cos\theta) = \cos\theta$

$A_l \neq 0$ and

$$A_1 r\cos\theta = -E_0 r\cos\theta$$

$$-E_0 = A_1$$

$$\phi = -E_0 r\cos\theta + \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$$

$$\phi(r=a,\theta) = 0$$

$$0 = -E_0 a\cos\theta + \frac{B_1\cos\theta}{r^2} = 0$$

All B_l for $l \neq 1$ are zero

$$-E_0 a + \frac{B_l}{a^2} = 0$$

$$B_l = E_0 a^3$$

$$\phi = -E_0 r\cos\theta + \frac{E_0 a^3}{r^2}\cos\theta$$