PH3102 Quantum Mechanics Assignment 6

Instructor: Dr. Siddhartha Lal Autumn Semester, 2024

Start Date: September 11, 2024 Submission Deadline: September 17, 2024 .

Submit your answers to the Tutor at the start of the tutorial.

Q1. Spherical Harmonics [5 marks]

The wave function of a particle subjected to a spherically symmetrical potential $V(r)$ is given by

$$\psi(x, y, z) = (x + y + 3z)f(r)$$

a) Is ψ an eigenfunction of L^2 ? If so, what is the l-value (i.e., the good quantum number associated with L^2)? If not, what are the possible values of l we may obtain when L^2 is measured?
b) What are the probabilities for the particle to be found in various m_l states?
c) Suppose it is known somehow that $\psi(x, y, z)$ is an energy eigenfunction with eigenvalue E. Indicate how we may find the spherically symmetrical potential $V(r)$.

Q2. Spin Measurement [5 marks]

Find the eigenvalues and eigenstates of the spin operator $\vec{S}$ of an electron in a direction given by a unit vector $\vec{n}$ that lies in the XZ plane. Also find out the probability of measuring $\langle S_z \rangle = \frac{\hbar}{2}$.

Q3. Adding two angular momentum $j_1 = 3/2$ and $j_2 = 1/2$. [10 marks]

(a) Compute all eigenvectors of the total angular momentum $\vec{J} = \vec{J_1} + \vec{J_2}$ where $j_1 = \frac{3}{2}$ and $j_2 = \frac{1}{2}$.
(b) Compute $\vec{J_1} \cdot \vec{J_2}$ in the various possible states $|j, j_z\rangle$?