Tutorial 1 Date 08/01/2025

P1) Suppose that, instead of the Coulomb force Law, one found experimentally that the force between any two charges q_1 and q_2 was

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{(1-\sqrt{\alpha r})}{r^2} \hat{e}_r \qquad \alpha = \text{constant}$$

1) find Electric field due to a charge at origin

2) Find $\oint \vec{E}\cdot d\vec{s}$ over a spherical Surface of radius r_1 with a point charge at center. Compare with the Coulomb result.

Sol^n 1) $\vec{E}$ is force per unit charge

$$\vec{E} = \frac{Q}{4\pi\epsilon_0} \cdot \frac{(1-\sqrt{\alpha r})}{r^2} \hat{e}_r$$

Sol^n 2)

$$E = \frac{Q}{4\pi\epsilon_0} \frac{(1-\sqrt{\alpha r})}{r^2} \hat{e}_r$$

for sphere with radius r_1

$$d\vec{s} = r^2 d\Omega\, e_r$$

$$\int \vec{E} \cdot d\vec{S} = \frac{Q}{4\pi\epsilon_0} \int \left(1 - \sqrt{\alpha r_1}\right) d\Omega$$

$$\boxed{\int \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0}\left(1 - \sqrt{\alpha r_1}\right)}$$

Problem 2:

Let $\vec{S}$ be a surface with surface charge density $\sigma(x)$. Let an electric field $\vec{E}_1$ is distributed on the top of surface and $\vec{E}_2$ be the electric field distributed at the bottom of the surface

(a) → show that the component of electric field in the direction of $\vec{S}$ discontinous across the surface

(b) Show that the component of electric field perpendicular to $\vec{S}$ is continous.

Continued →

$E^{\perp} \Rightarrow$ component of electric field along $\hat{n}$ $\quad E^{\perp} = E \cdot \hat{n}$

$\vec{E}^{\perp}$ on top $\quad E_1 \cdot \hat{n}$

$\vec{E}^{\perp}$ on bottom $-E_2 \cdot \hat{n}$

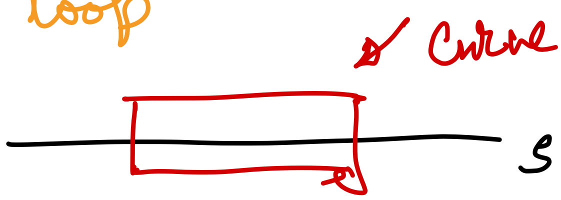
Using Gauss law $\int \vec{E} \cdot d\vec{s} = \frac{\Delta A \sigma}{\epsilon_0}$

$$\int (\vec{E}_1 \cdot \hat{n} - \vec{E}_2 \hat{n}) \Delta A = \frac{\Delta A \sigma}{\epsilon_0}$$

$$\boxed{\vec{E}_1 \cdot \hat{n} - \vec{E}_2 \hat{n} = \sigma / \epsilon_0}$$

use $\oint E \cdot dl = 0 \Rightarrow \nabla \times \vec{E} = 0$

take a close loop



from this we get $E^{\parallel}_{above} = E^{\parallel}_{below}$

Problem 3 : prove the following property of Dirac-δ function

$$\int f(x)\,\delta'(x-a)\,dx = -f'(a)$$

integration by parts

$$\int f(x)\,\delta(x-a)\,dx$$

$$\int \frac{d}{dx}\left[f(x)\,\delta(x-a)\right]dx = \int \underline{f'(x)\,\delta(x-a)}\,dx + \int f(x)\,\delta'(x-a)\,dx$$

$$0$$

$$f'(a) = \int f(x)\,\delta'(x-a)\,dx$$