Tutorial - 6. 18/09/2024 (1)

A Lagrangian is given by

$$\boxed{\mathcal{L} = \frac{1}{2} m R^2 \left(\dot{\theta}^2 + \sin^2\theta\, \omega^2\right) - mgR(1-\cos\theta)}$$

θ is generalized coordinate

ω, R, m and g are constant.

- find the Hamiltonian.

$\cancel{p = \frac{\partial \mathcal{L}}{\partial \theta} = mR^2\dot{\theta}}$

- what is the total energy, is it same as Hamiltonian.

$$H = p_\theta \dot{\theta} - \mathcal{L}$$

$$p_\theta = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mR^2\dot{\theta} \Rightarrow \dot{\theta} = \frac{p_\theta}{mR^2}$$

$$H = mR^2\dot{\theta}^2 - \frac{1}{2} mR^2\left(\dot{\theta}^2 + \sin^2\theta\, \omega^2\right) + mgR(1-\cos\theta)$$

we need to replace $\dot{\theta}$ by p in the

$$H = \frac{1}{2} m R^2 \dot{\theta}^2 - \frac{1}{2} m R^2 \sin^2\theta\, \omega^2 + mgR(1-\cos\theta)$$

$$H = \frac{1}{2} m R^2 \frac{p_\theta^2}{m^2 R^4} - \frac{1}{2} m R^2 \sin^2\theta\, \omega^2 + mgR(1-\cos\theta)$$

$$H = \frac{1}{2} \frac{p_\theta^2}{mR^2} - \frac{1}{2} m R^2 \sin^2\theta\, \omega^2 + mgR(1-\cos\theta)$$

The total energy is sum of K.E & potential energy

$$E = T + V$$

$$E = \frac{1}{2} m R^2 (\dot{\theta}^2 + \sin^2\theta\, \omega^2) + mgR(1-\cos\theta)$$

Let us replace $\dot{\theta}$ by p and compare with H

$$E = \frac{1}{2}\frac{p_\theta^2}{mR^2} + \frac{1}{2}mR^2\sin^2\theta\,\omega^2 + mgR(1-\cos\theta)$$

which is not same as H !.

(4)

Q2) Let x be the generalized coordinate

$$\mathcal{L}(x) = \frac{m}{2}\dot{x}^2 - \frac{k}{2}(x - v_0 t)^2$$

* find the second order Lagrange EOM.

* Find the Hamiltonian and Hamiltonian EOM

* ~~H~~ is the Hamiltonian is constant?

LEOM is

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x} \qquad \frac{\partial \mathcal{L}}{\partial x} = -k(x - v_0 t)$$

EOM is

$$m\ddot{x} + k(x - vt) = 0$$

$$H = p_x \dot{x} - \mathcal{L}$$

$$p_x = m\dot{x} \qquad \dot{x} = \frac{p_x}{m}$$

$$H = p_x\dot{x} - \frac{m}{2}\dot{x}^2 + \frac{k}{2}(x - v_0 t)^2$$

$$H = \frac{p_x^2}{2m} + \frac{k}{2}(x - v_0 t)^2$$

H is not constant as

$$\frac{\partial H}{\partial t} \neq 0.$$