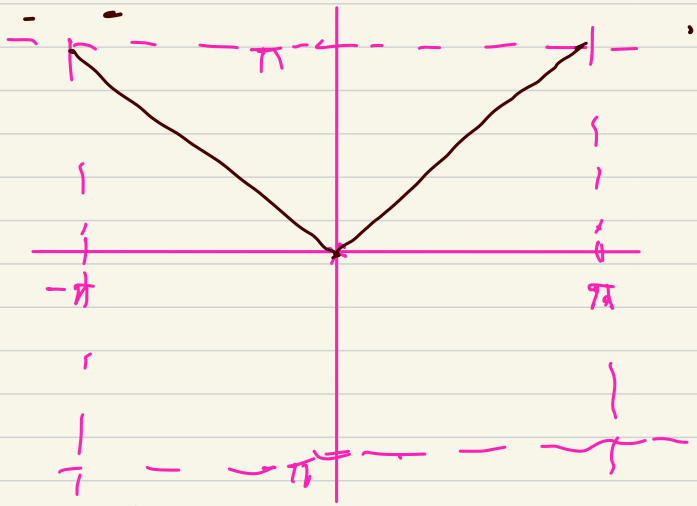
MA2103 - 2023 Tutorial 2

(1) Express $f(x) = \begin{cases} x & 0 < x < \pi \\ -x & -\pi < x < 0 \end{cases}$

in term of Fourier Series and plot first two terms



$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{0} -x\,dx + \frac{1}{2\pi}\int_{0}^{\pi} x\,dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi} x\,dx = \frac{1}{2\pi}x^2\Big|_0^{\pi}$$

$$a_0 = \frac{\pi}{2}$$

function is and hence b_n are zero, only Cos series survive

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} -x\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\cos nx\,dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x\cos nx\,dx$$

$$\frac{2}{\pi}\left[\frac{-1+\cos n\pi}{n^2}\right]_0^{\pi}$$

$$\frac{2}{\pi}\left[\frac{-1+\cos n\pi}{n^2}\right] + 0$$

$$= -\frac{4}{\pi n^2} \quad \text{for odd}$$

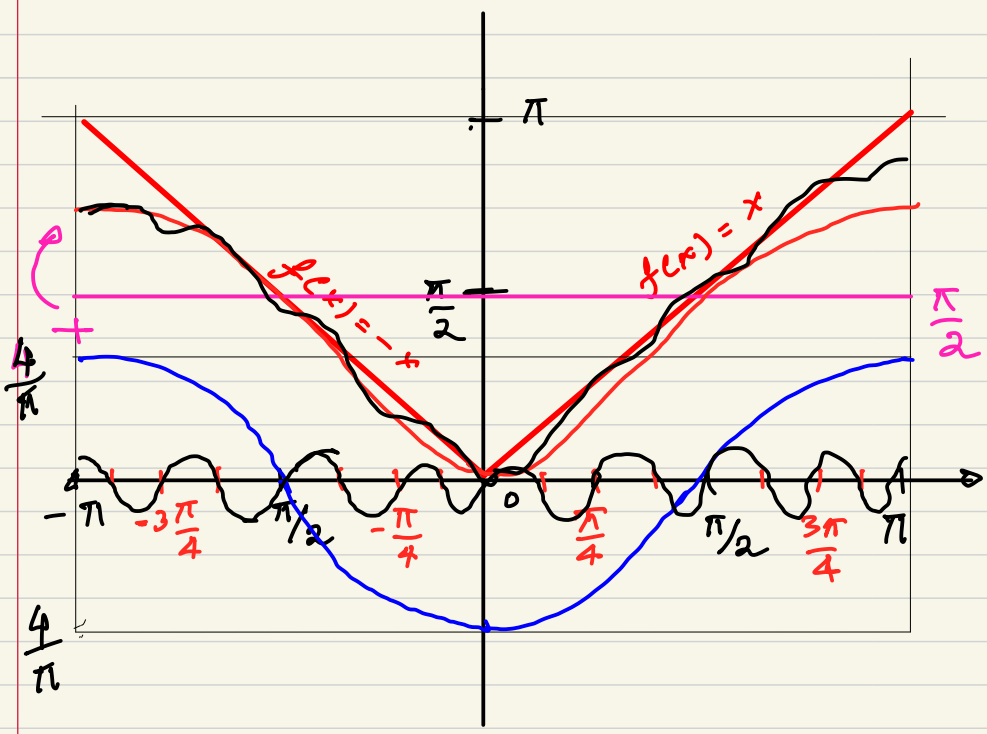
$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=\text{odd}} \frac{1}{n^2}\cos nx$$

or

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=0}^{\infty}{}' \frac{\cos(2n+1)x}{(2n+1)^2}$$

0^1: $f(x) = \frac{\pi}{2} - \frac{4}{\pi}\cos x$

0^2: $f(x) = \frac{\pi}{2} - \frac{4}{\pi}\cos x - \frac{4}{\pi 9}\cos 3x$


π
π/2
f(x) = −x
f(x) = x
π/2
4/π
−π
−3π/4
π/2
−π/4
0
π/4
π/2
3π/4
π
4/π