HOW do we get D= KbT/….

$$\Delta x = V_{0,x}\Delta t + \frac{f}{2m}(\Delta t)^2 \quad \text{— (1)}$$

$$\langle \Delta x\rangle = \cancel{\nearrow}\,0 + \left\langle \frac{f}{2m}(\Delta t)^2\right\rangle$$

$$= \frac{f}{2m}(\Delta t)^2 \quad \text{— (2)}$$

$$V_{drift} = \frac{f}{2m/\Delta t} = f/\zeta$$

$$= \frac{f}{K_BT/D} \qquad \therefore \zeta D = K_BT$$

Einstein's relation

$$\langle (\Delta x - \langle \Delta x\rangle)^2\rangle = 2D\Delta t$$

Even for unbiased random walk

from (1): $= \left\langle \left(V_{0x}\Delta t + \frac{f}{2m}\Delta t^2 - \frac{f}{2m}\Delta t^2\right)^2\right\rangle$

$$= \langle V_{0x}^2\rangle \Delta t^2$$

$$\therefore \langle V_{0,x}^2\rangle = \frac{2D}{\Delta t} \quad \text{— (3)}$$

But $\frac{1}{2}m\langle V_{0,x}^2\rangle = \frac{1}{2}K_BT$

$$\therefore \langle V_{0,x}^2\rangle = \frac{K_BT}{m} \quad \text{— (4)}$$

3, 4 : $\boxed{\frac{2m}{\Delta t} = \frac{K_BT}{D}}$

$$\boxed{f = 6\pi\eta R v}$$

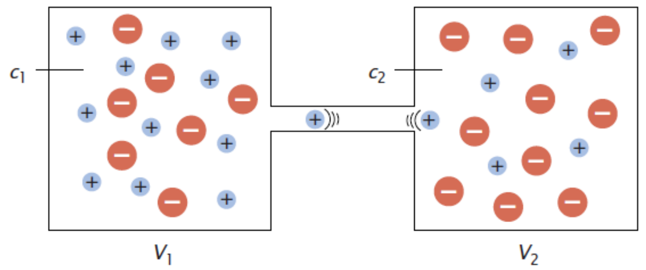
From Stoke's Law

$$f/v = 6\pi\eta R = \frac{K_BT}{D}$$

$$\therefore \boxed{D = \frac{K_BT}{6\pi\eta R}}$$

Nelson 4.1.4

How Thermal energy **governs accessibility of microstates of different energy at equilibrium:** Boltzmann Distribution



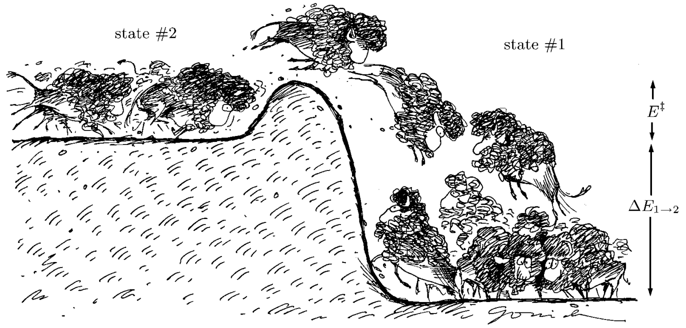
deterministic

$$p(E_i) = \frac{1}{Z} e^{-E_i/k_B T}$$

thermal

$$Z = \sum_{i=1}^{N} e^{-E_i/k_B T}$$

$$\frac{c_1}{c_2} = \frac{p_1}{p_2} = \frac{e^{-zeV_1/k_B T}}{e^{-zeV_2/k_B T}}.$$

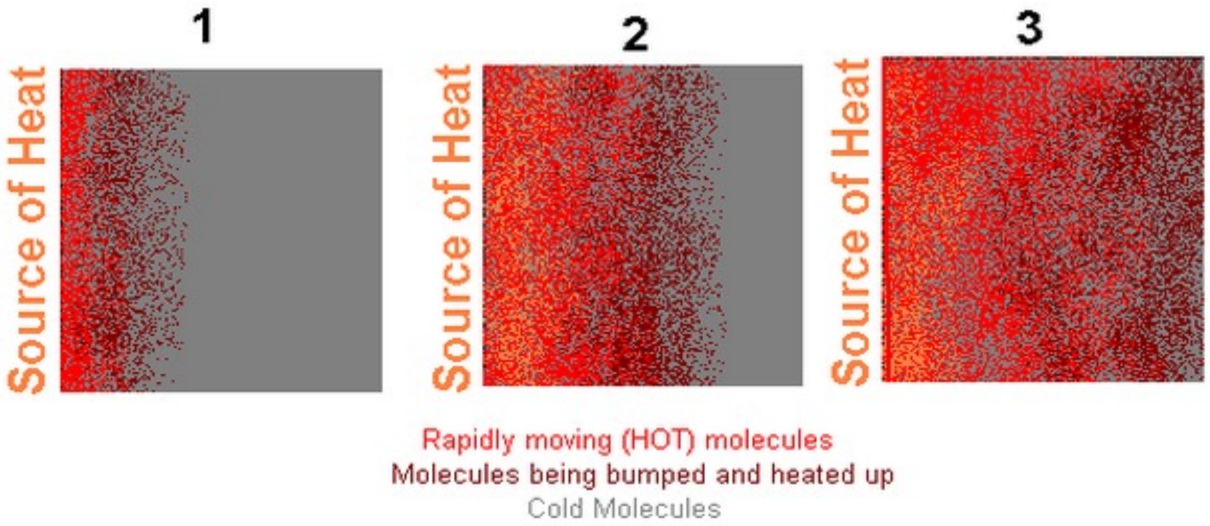


Nano world…incessant motion

- Thermal energy?
- $k_B T$
- Ideal gas law?
- How much is $k_B T$ at room temperature? $k_B = 1.38 \times 10^{-23}$ J/K.
  In Joules? In pN-nm?
- Equipartition theorem?

http://www.feynmanlectures.caltech.edu/I_39.html

HEAT


1
2
3
Source of Heat
Source of Heat
Source of Heat
Rapidly moving (HOT) molecules
Molecules being bumped and heated up
Cold Molecules

Heat as a form of energy

- Conservation of energy;
- Energy can be transformed
- Mechanical work on a system produces heat
- Once transformed to heat, transforming to other energies
- Heat = disorganized kinetic energy
- Free energy = Total energy – energy related to disorder/randomness in the system
- $F=E-TS$
- Equilibrium: F minimized
- Non-equilibrium: System evolves in the direction such that F is minimized

Mechanical, Thermal Chemical Equilibrium

Nonequilibrium to equilibrium??? Minimization of energy

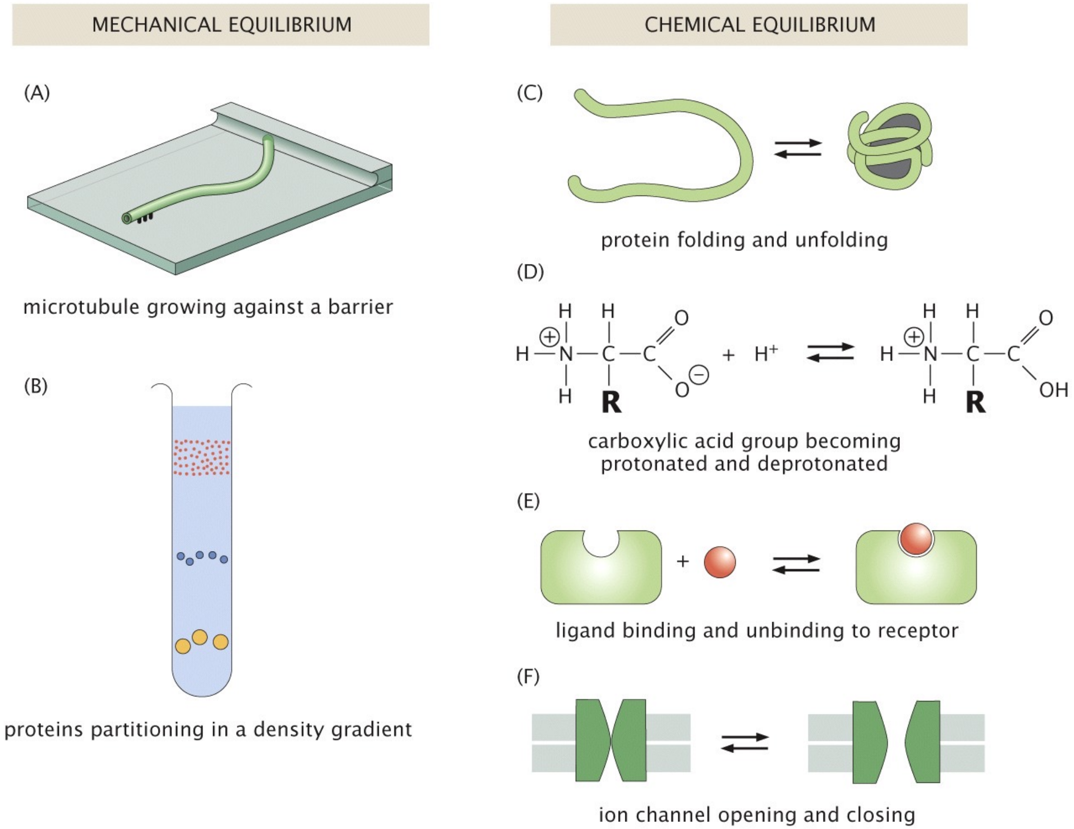


Figure 5.7 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

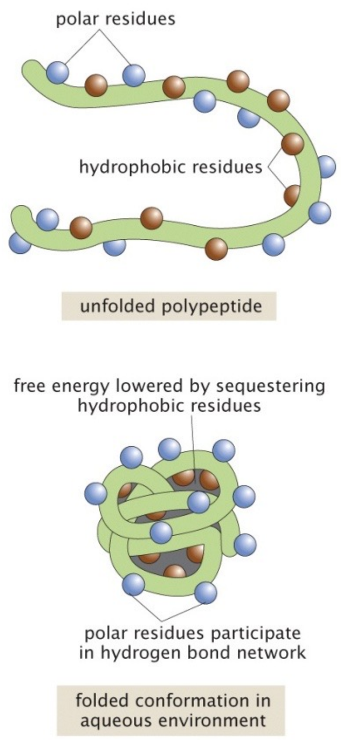


Figure 5.8 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

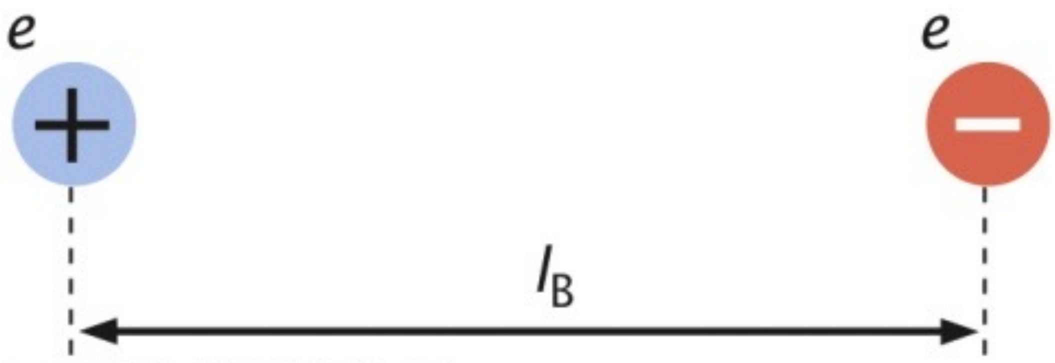


Figure 9.12 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

- Bjerrum Length
  - For two unit charges feeling coulomb's attraction/repulsion, what is the effect of k_BT? Allowing states otherwise improbable. At what lengthscales does thermal energy start overruling electrostatic energy for a pair of unit charges?
  - Monovalent ions in water at room temperature, l_B=________(find out)

$\frac{1}{4\pi\epsilon_0}$ = 9 x 10 ^{9}

Dielectric constant = 80

$$l_B = \frac{e^2}{4\pi\varepsilon k_B T}$$

e = 1.6 x 10 $^{-19}$

k_BT=4 x 10 $^{-21}$

Table 1: Length scales that emerge from the interplay of deterministic and thermal energies.

length scale name	energetic term	entropic term	equation	characteristic value	BNID
atmospheric concentration decay length	gravitational	occupation of spatial states	$mgh = k_BT \Rightarrow h_{th} = \frac{mg}{k_BT}$ m: mass h: height g: acceleration due to gravity	8 km	111406
persistence length	bending	number of states of polymer chain	$E \times I/\xi_p = k_BT \Rightarrow \xi_p = \frac{EI}{k_BT}$ E: Young's modulus I: moment of inertia	DNA: 50 nm actin: 17 µm microtubule: 1.4 mm	103112, 105505, 105534
Bjerrum length	electrostatic interaction	occupation of spatial states	$kq^2/l_B = k_BT \Rightarrow l_B = \frac{kq^2}{k_BT}$ q: charge k: Coulomb's constant	0.7 nm	106405
Debye length	electrostatic interaction	occupation of spatial states	$2c_\infty \lambda_D^2 q^2/\varepsilon_0 D = k_BT$ $\Rightarrow \lambda_D = \sqrt{\frac{\varepsilon_0 D k_B T}{2c_\infty q^2}}$ c_∞: salt conc. D: dielectric constant	1 nm (at 100mM monoionic conc.)	105902

http://book.bionumbers.org/what-is-the-thermal-energy-scale-and-how-is-it-relevant-to-biology/