Problems on Open Set and Closed set

1. Let E be a non-empty bounded above set. Show that $\sup(E)$ is a limit point of E.

2. We denote E' as the set of all the limit points of E. We define the closure of E is the set $\overline{E} = E \cup E'$. Show that

i. Show that $\overline{E}$ is closed.

ii. Show that E is closed iff $E = \overline{E}$.

iii. Show that $\overline{E} \subset F$ for every closed set F, such that $E \subset F$. Thus $\overline{E}$ is the smallest closed set containing E.

3. Let E^o denote the set of all interior points of E.

i. Show that E^o is open.

ii. Show that E is open iff $E = E^o$.

iii. Show that $G \subset E^o$ for every open set G and $G \subset E$. Thus E^o is the largest open set contained in E.

4. Find the E' of the following sets

i. $\{\frac{1}{n} + \frac{1}{m} : n, m \in \mathbb{N}\}$

ii. $\{x \in \mathbb{Q} : x^2 + 5x - 6 < 0\}$

5. A point $x \in \mathbb{R}$ is said to be a boundary point of $A \subset \mathbb{R}$ if every neighbourhood of x contains points in A and points in A^c. Let ∂A denote the collection of all boundary points of A. Show that

i. $\partial A = \partial A^c$,

ii. A is open iff $\partial A = \Phi$,

iii. A is closed iff $\partial A \subset A$.