Tutorial 03 - MA2103 - 2023

⊗ Solve the following ODE

$$\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + \omega^2 y = A\cos\Omega t$$

Make plot of solution!

The solution to inhomogeneous is given by

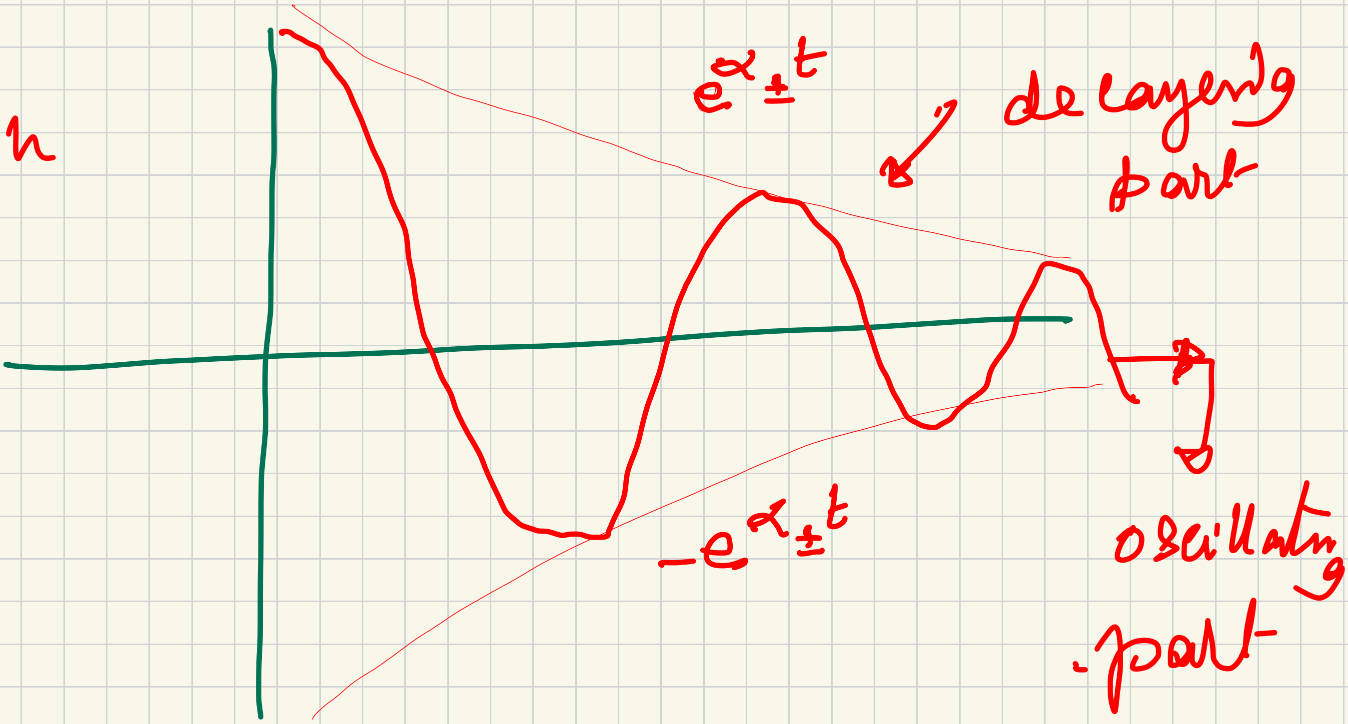
$$y = y_h + y_p$$

where y_h is general sol^n to homogenous part of ODE and y_p is particular sol^n to ODE

$$y_h = Ae^{\alpha_+ t} + Be^{\alpha_- t}$$

where $\alpha_\pm = -\frac{\gamma}{2} \pm \frac{1}{2}\sqrt{\gamma^2 - 4\omega^2}$

plot y_h



Particular Sol^n. y_p can be written as

$$y_p = F \cos(\Omega t + \phi_0)$$

We determine F and ϕ_0 by substituting in ODE.

$$y_p' = -F\Omega \sin(\Omega t + \phi_0)$$

$$y_p'' = -F\Omega^2 \cos(\Omega t + \phi_0)$$

Substituting, we get

$$-F\Omega^2\cos(\Omega t+\phi_0) - F\nu\Omega\sin(\Omega t+\phi_0)$$
$$+ F\omega^2\cos(\Omega t+\phi_0) = A\cos\Omega t$$

Simplifying :

$$F(\omega^2-\Omega^2)\cos(\Omega t+\phi_0)$$
$$- F\nu\Omega\sin(\Omega t+\phi_0)$$
$$= A\cos\Omega t$$

Let us write

$$e\cos\sigma = F(\omega^2-\Omega^2)$$

$$e\sin\sigma = F\nu\Omega$$

Square and add above eqn

$$e^2 = F^2\left[(\omega^2-\Omega^2)^2 + \nu^2\Omega^2\right]$$

$$\sigma = \tan^{-1}\left[\frac{\nu\Omega}{(\omega^2-\Omega^2)}\right]$$

with this we simplify

$$\ell \cos(\Omega t + \phi_0 - \sigma) = A\cos\Omega t$$

$$\ell = A$$

$$F = \frac{A}{\left[(\omega^2-\Omega^2)^2 + \nu^2\Omega^2\right]^{1/2}}$$

$$\phi_0 = \tan^{-1}\left[\frac{\nu\Omega}{(\omega^2-\Omega^2)}\right]$$

F for small Ω ; $\omega \gg \Omega$

we approximate F

$$F = \frac{A}{\omega^2\left[\left(1-\frac{\Omega^2}{\omega^2}\right)^2 + \frac{\nu^2}{\Omega^2}\right]^{1/2}}$$

to lowest order

$$F \propto \frac{1}{\omega^{-2}}$$

when $\Omega > \omega$

$$F = \frac{1}{\Omega^2\left[\left(1-\frac{\omega^2}{\Omega^2}\right)^2 + \left(\frac{\gamma}{\Omega}\right)^2\right]^{1/2}}$$

to lowest order

$$F \propto' \frac{1}{\Omega^2}$$

Very close to ω i.e. $\omega \simeq \Omega$

$$\omega^2 - \Omega^2 = 0$$

$$F = \frac{1}{\gamma\Omega} \simeq \frac{1}{\gamma\omega}$$

plot of y_p