Root Finding

Part-12

MA2103 - 2023

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Root's are very interesting

we have to find x such that $f(x) = 0$.

Which is not possible analytically except for a very few special case. Here numerical methods are important

Tip of Iceberg vs ship

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Which is not possible analytically except for a very few special case. Here numerical methods are important

Floating Sphere

Archimedes's principle: Buoyancy force is equal to the weight of the replaced liquid.

Floating Iceberg

What portion

First we assum

Floating Iceberg

What portion iceberg is above water ?

Archimedes's principle: Buoyancy force is equal to the weight of the replaced liquid.

Volume of the sphere be $V_{\bigcirc} = \frac{4}{3}\pi r^3$

Let the Volume of water displaced be $V_{\sim}$

Case 1

$V_{\bigcirc} \gg V_{\sim}$

Case 2

$V_{\bigcirc} \approx V_{\sim}$

Case 3

$V_{\bigcirc} \ll V_{\sim}$

Case 1

$V_{\bigcirc} \gg V_{\sim}$

Case 2

$V_{\bigcirc} \approx V_{\sim}$

Case 3

$V_{\bigcirc} \ll V_{\sim}$

The force acting is gravity and buoyancy of water

At static equilibrium $\rho_s g V_{\bigcirc} = \rho_{\sim} g V_{\sim}$

ρ_s density of sphere $\rho_{\sim}$ density of water

The volume V_h of water displaced when a sphere is submerged to a depth h is

$$V_h = \frac{\pi}{3} h^2 \left(3\, r - h\right)$$

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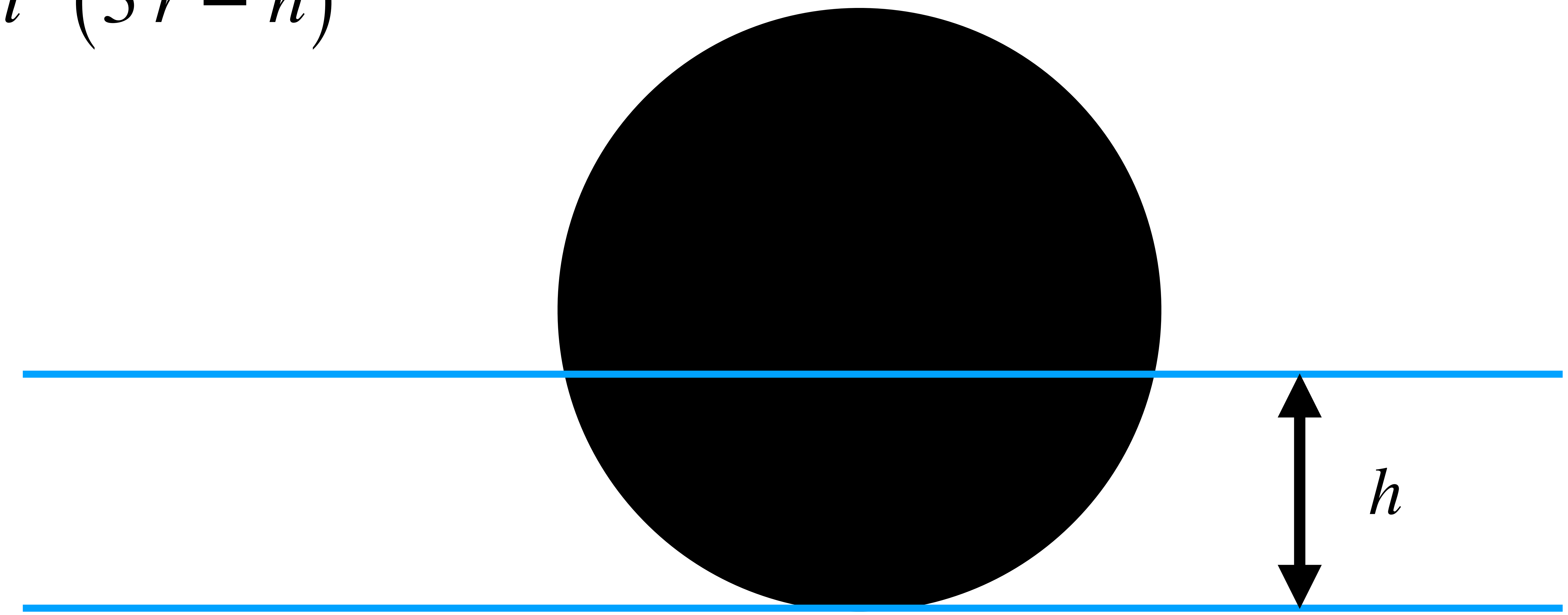


Applying Archimedes's principle, we get condition for h as:

$$h^3 - 3rh^2 + 4\frac{\rho_s}{\rho_\sim}r^3 = 0$$

Density of water 1 in what ever unit.

Density of Ice 0.92 in the same unit.

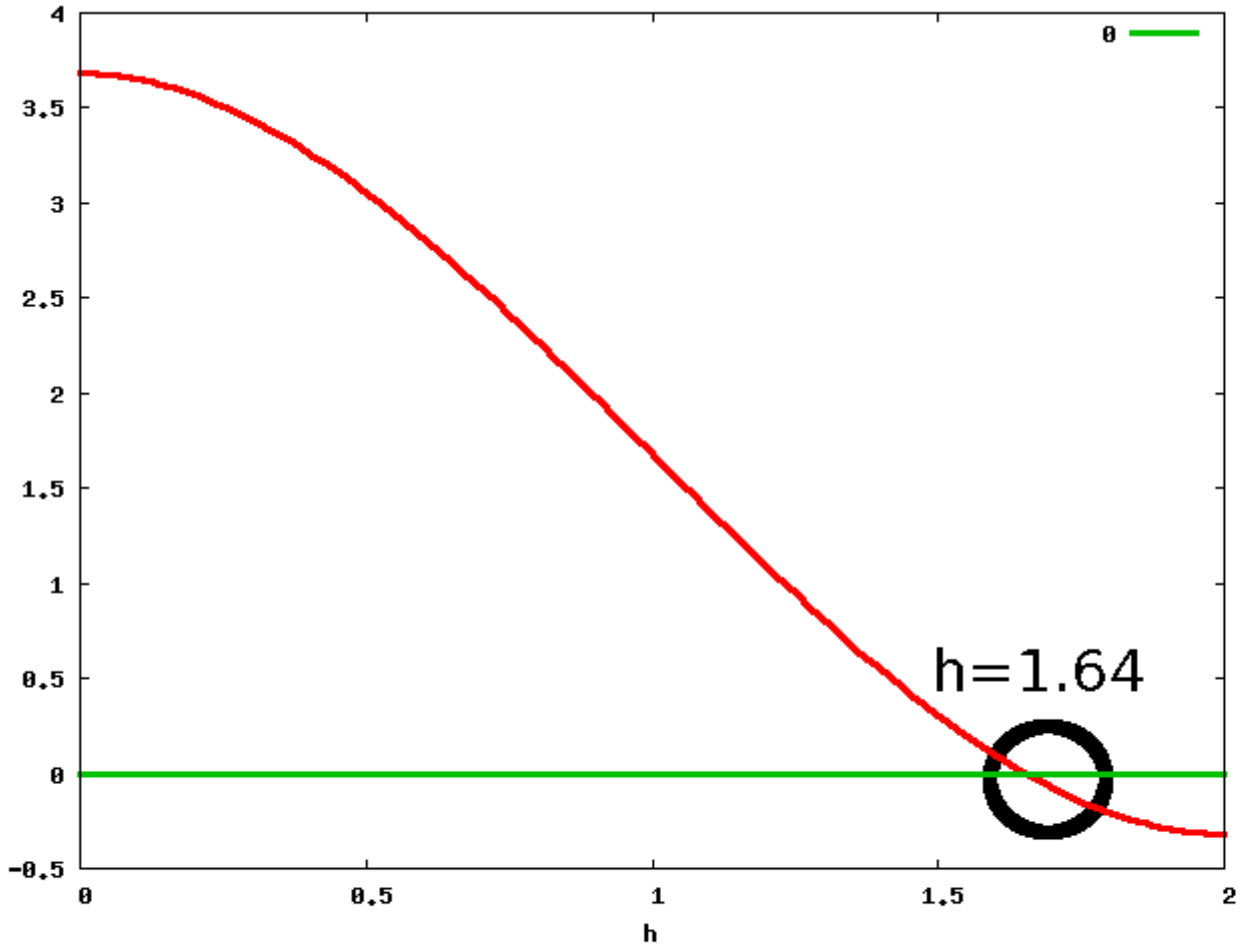
let r=1 in the same what ever unit.

what is the height of h in the same unit ?

we need solution for h with $h^3 - 3h^2 + 4 \times 0.95 = 0$ $\qquad h \in [0, 2]$

- We need a root finding method.

Graphical method



Bisection method

For a function $f(x)$ continuous in the interval $[a, b]$, there exists at least one root in the interval (a, b) **if $f(a)f(b) < 0$**

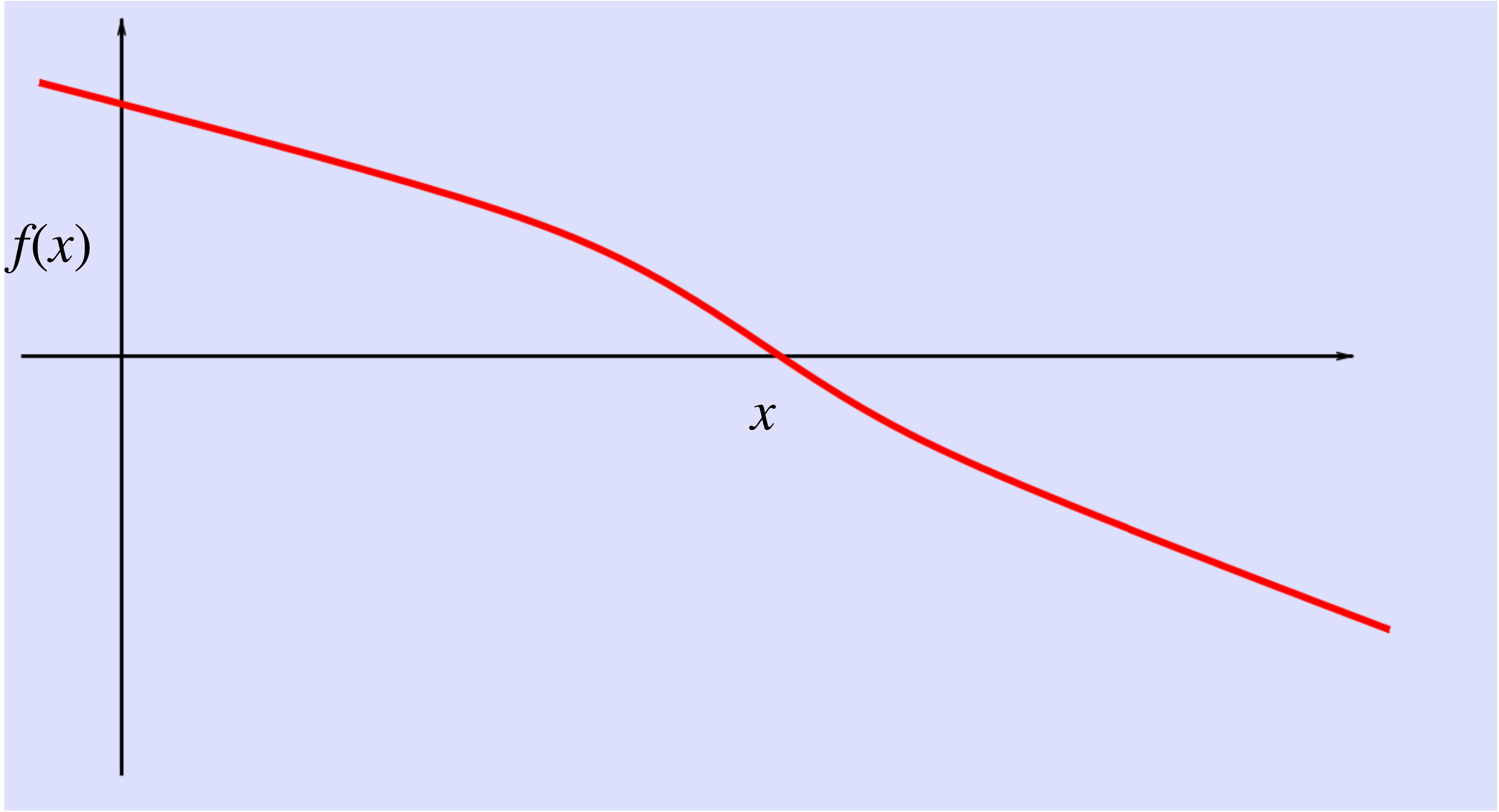
a and b such that $f(a)f(b) < 0$

$$c = \frac{1}{2}(a + b)$$

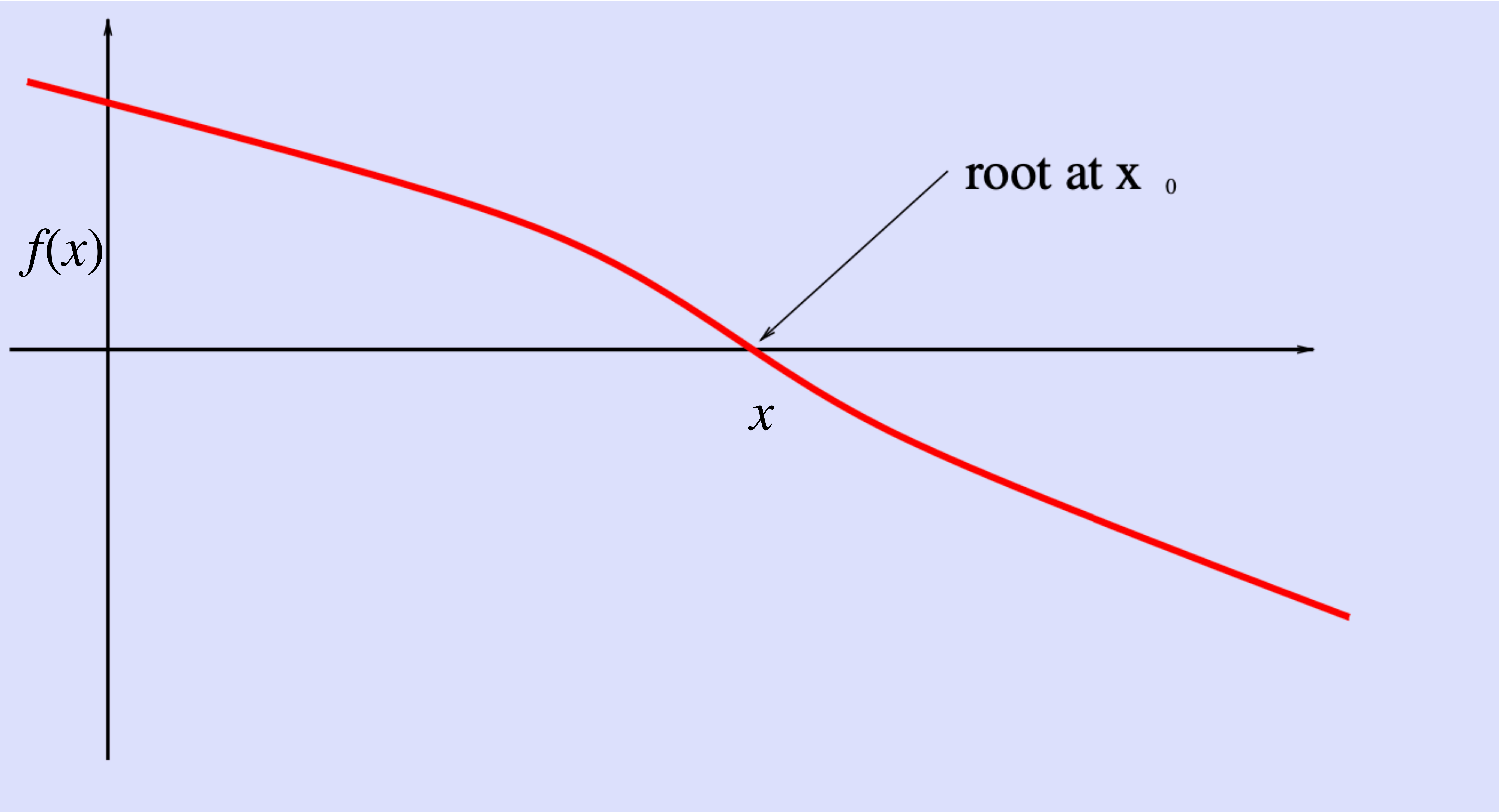
If $f(a)f(c) < 0$ then replace b by c

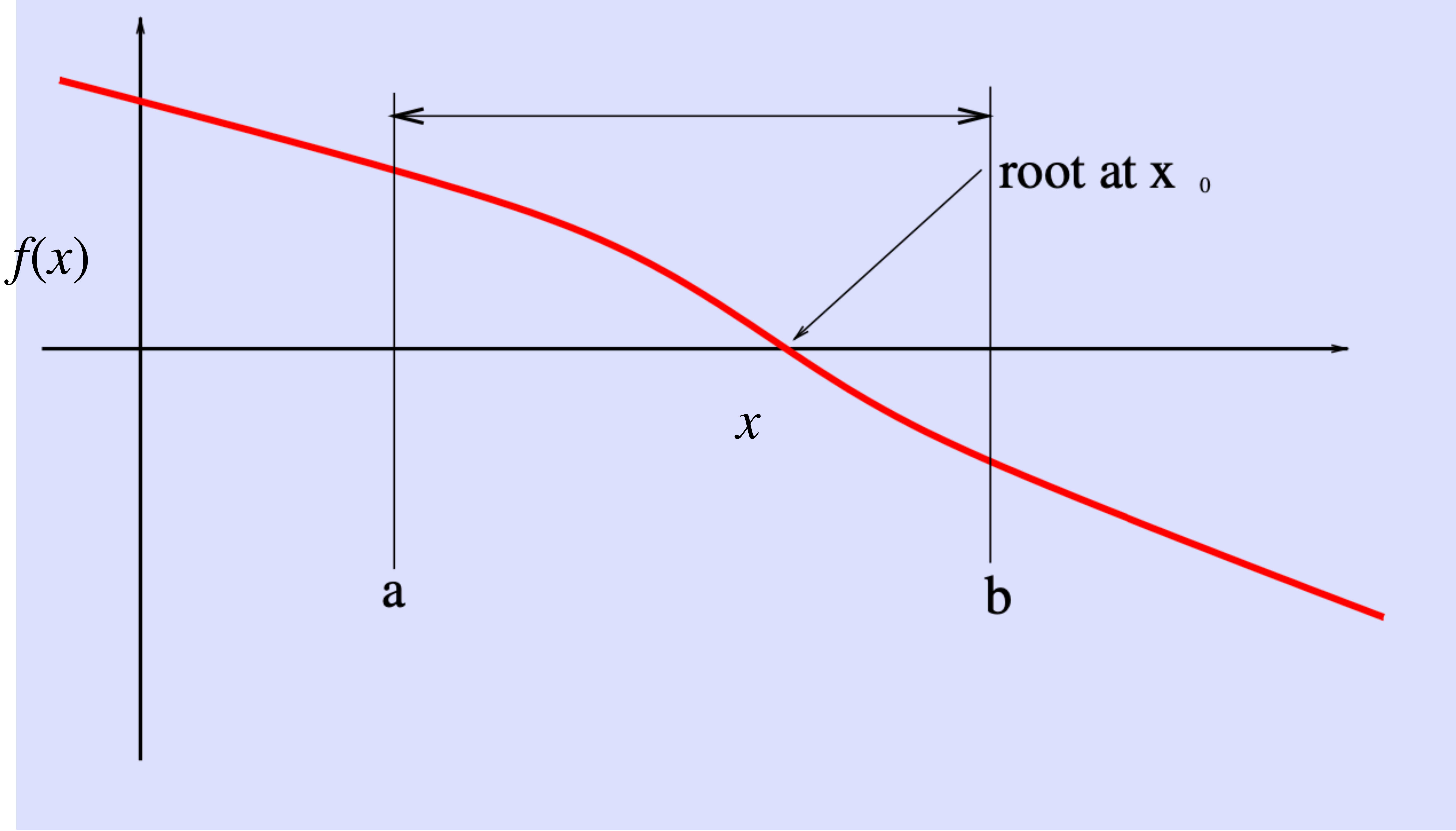
If $f(c)f(b) < 0$ then replace a by c

Repeat the process till you are happy

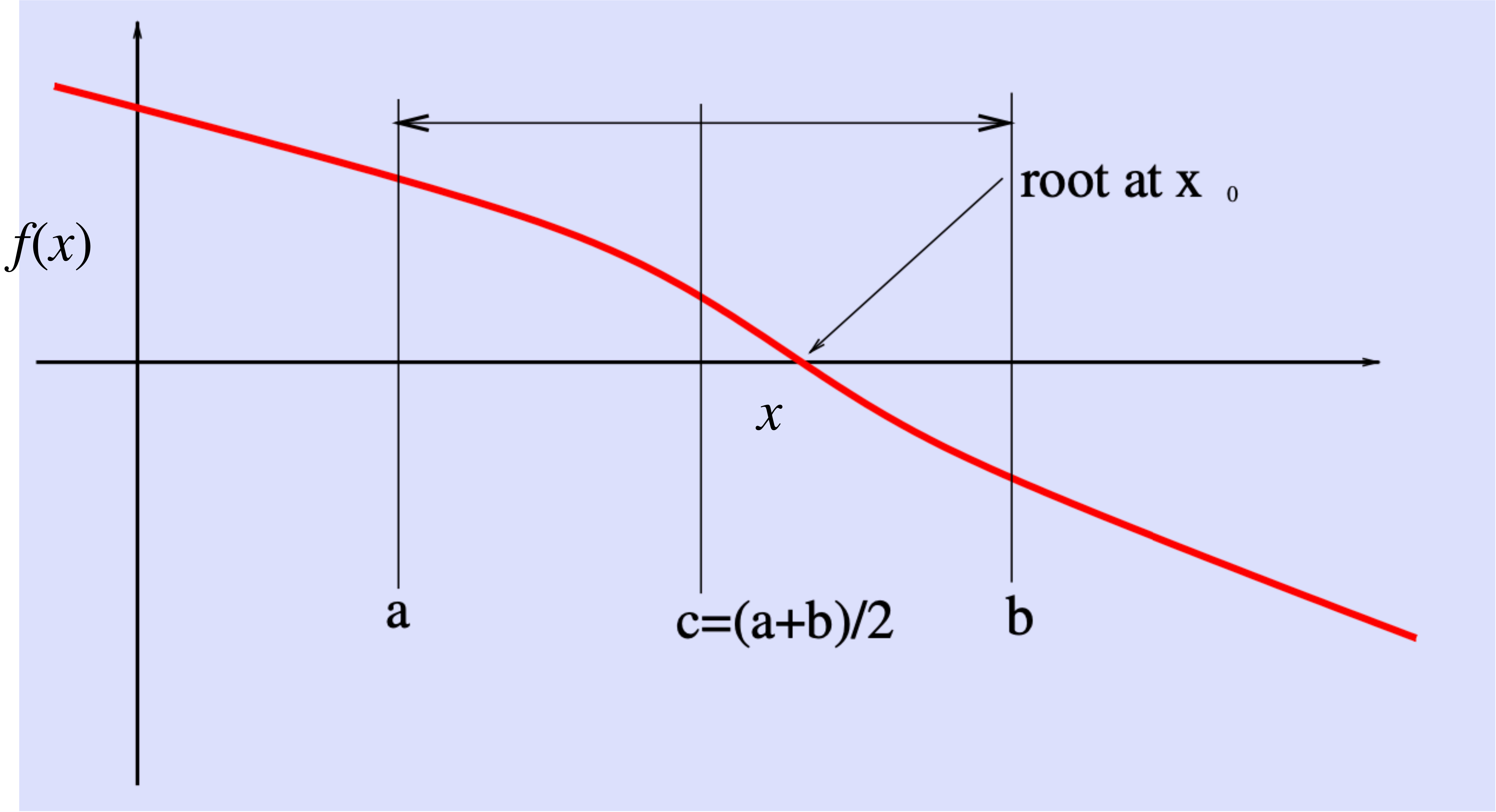


$f(x)$
x

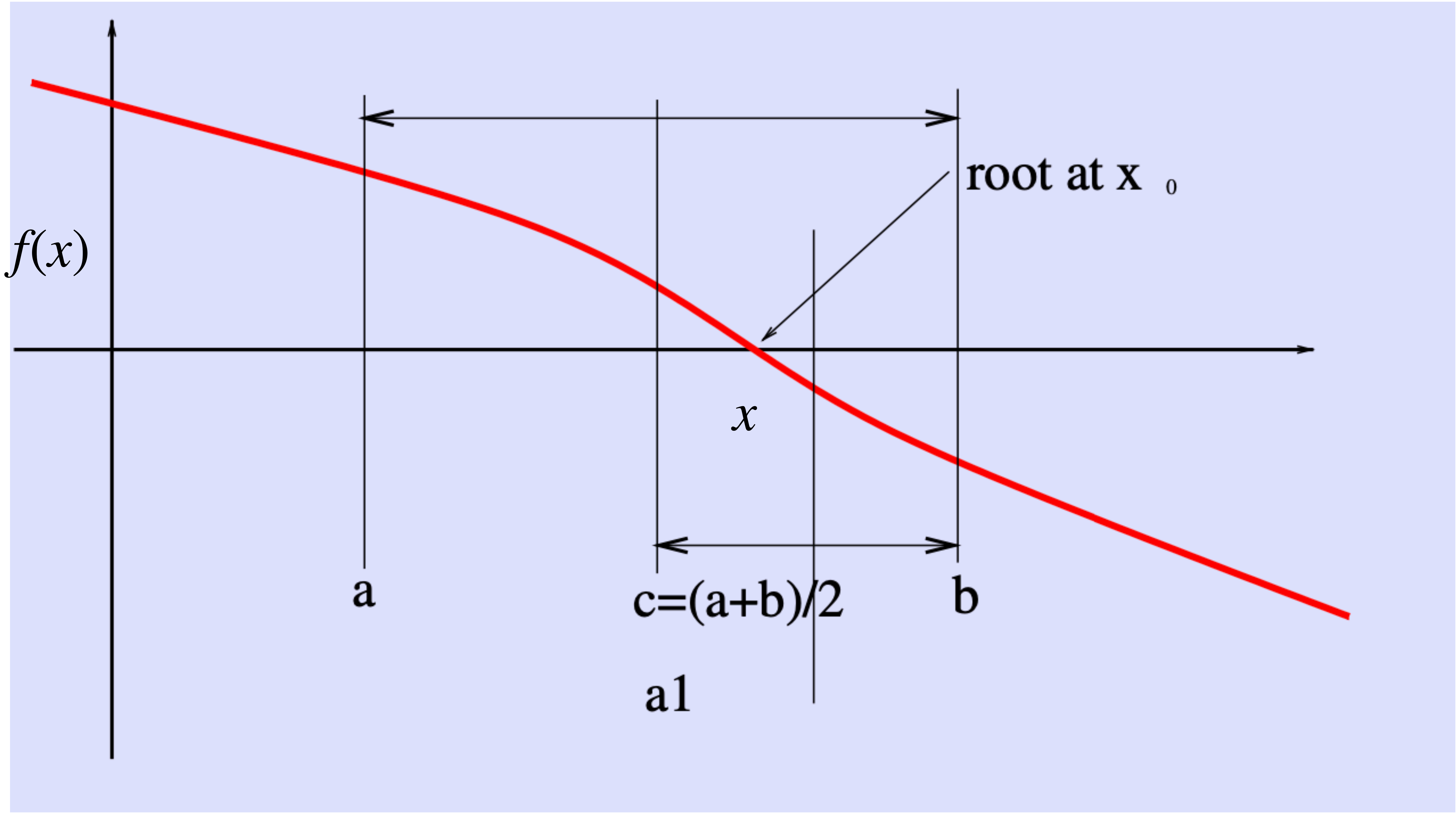

root at x 0
f(x)
x



a and b such that $f(a)f(b) < 0$



a and b such that $f(a)f(b) < 0$ $\quad\quad$ $c = \frac{1}{2}\left(a + b\right)$



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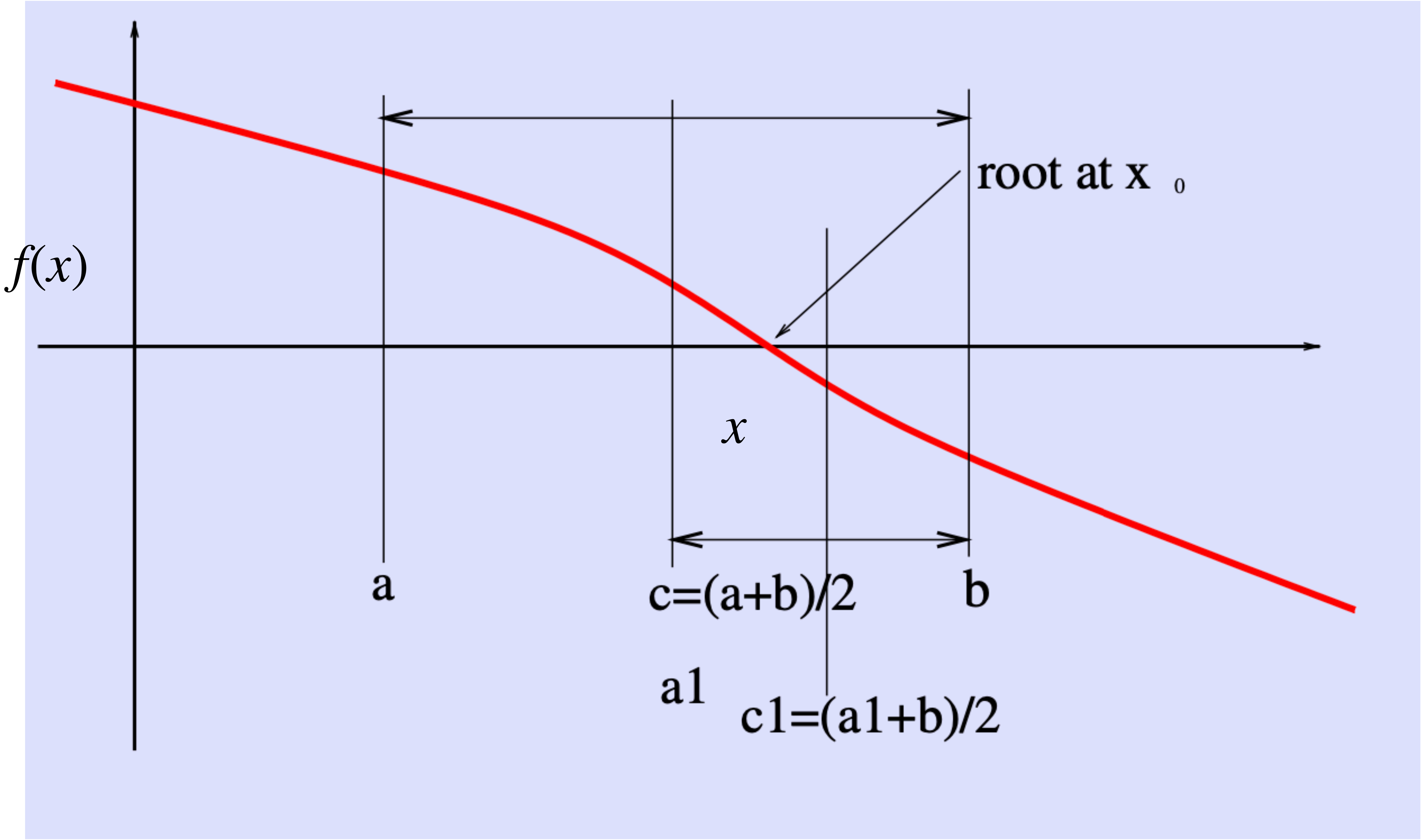

a and b such that $f(a)f(b) < 0$

Algorithm

1. start with interval $\left[a_0, b_0\right]$ such that $f(a_0) * f(b_0) < 0$
2. n th iteration is compute using:
3. mid point for each iteration is $c = \frac{1}{2}\left(a_n + b_n\right)$
4. if $f\left(a_n\right) f(c) < 0$ then $a_{n+1} = a_n$ and $b_{n+1} = c$ else $a_{n+1} = c$ and $b_{n+1} = b_n$
5. Repeat 3 to 4 till $\left|a_n - b_n\right| < \epsilon$ or n reached $ITMAX$
6. ϵ the error in the root.