Electromagnetic waves in matter

For linear and homogeneous medium (no free charge or current) the Maxwell's relations are

$$\vec{\nabla}\cdot\vec{E} = 0 \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{B} = \mu\epsilon\frac{\partial\vec{E}}{\partial t}$$

$\mu \rightarrow$ permeability

$\epsilon \rightarrow$ permittivity

In vacuum

$\epsilon \rightarrow \epsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

$\mu \rightarrow \mu_0 = 4\pi \times 10^{-7}\ C^{-2}\ N\ T^2$

$\epsilon = \epsilon_r \epsilon_0$, where $\epsilon_r \rightarrow$ dielectric constant

$\mu \sim \mu_0$, for most linear, homogeneous media

Linear: $\vec{P}$ = Polarization (induced) $= \epsilon_0 \chi_e \vec{E}$ $\leftarrow$ linear on $\vec{E}$

$\vec{M}$ = Magnetization (induced) $= \mu_0 \chi_m \vec{H}$ $\leftarrow$ linear on $\vec{H}$

Homogeneous: ϵ and μ do not depend on $\vec{r}$.

$\mu_0(\vec{H}+\vec{M}) = \vec{B}$

$\Rightarrow \mu_0(1+\chi_m)\vec{H} = \vec{B}$

$\Rightarrow \vec{H} = \frac{1}{\mu}\vec{B}$

with $\mu = \mu_0(1+\chi_m)$

To construct the wave equation we use,

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu\epsilon\frac{\partial\vec{E}}{\partial t}\right)$$

As, $\vec{\nabla}\cdot\vec{E} = 0$, we have,

$$\boxed{\nabla^2\vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}} \Rightarrow \boxed{c^2 = \frac{1}{\mu\epsilon}}$$

Also,

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\times\left(\mu\epsilon\frac{\partial\vec{E}}{\partial t}\right) = \mu\epsilon\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = \mu\epsilon\frac{\partial}{\partial t}\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$\Rightarrow \boxed{\nabla^2\vec{B} = \mu\epsilon\frac{\partial^2\vec{B}}{\partial t^2}} \Rightarrow \boxed{c^2 = \frac{1}{\mu\epsilon}}$$

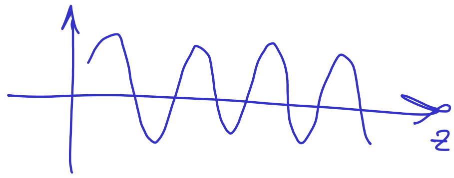
$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = ?$$

$$\vec{\nabla} \times \vec{A} \equiv \epsilon_{ijk}\, \partial_j A_k \quad \text{using Levi-Civita symbols}$$

$$\Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \epsilon_{ijk}\, \partial_j\, \epsilon_{klm}\, \partial_l A_m$$
$$= \epsilon_{ijk}\, \epsilon_{klm}\, \partial_j \partial_l A_m$$
$$= \epsilon_{kij}\, \epsilon_{klm}\, \partial_j \partial_l A_m$$
$$= (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\, \partial_j \partial_l A_m$$
$$= \partial_j \partial_i A_j - \partial_j^2 A_i$$
$$= \partial_i\, \partial_j A_j - \partial_j^2 A_i$$
$$\equiv \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$$

Consider monochromatic plane wave solutions.

$$\vec{E} = \vec{E}_0\, e^{i(kz - \omega t)} = \vec{E}_0\, f(z,t)$$



and

$$\vec{B} = \vec{B}_0\, e^{i(kz - \omega t)} = \vec{B}_0\, f(z,t)$$

[Note: no phase lag between $\vec{E}$ and $\vec{B}$, as dictated by Faraday's law]

Now, $\vec{\nabla} \cdot \vec{E} = 0 \Rightarrow E_{0z} = 0$
and $\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow B_{0z} = 0$ } The waves are transverse!

Also, $\vec{\nabla} \times \vec{E}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ E_{0x} f(z,t) & E_{0y} f(z,t) & 0 \end{vmatrix}$$

$$= \hat{i}\left(-ik\, E_{0y}\, f(z,t)\right) + \hat{j}\left(+ik\, E_{0x}\, f(z,t)\right) = -\frac{\partial \vec{B}}{\partial t}$$

$$= \hat{i}\left(+i\omega\, B_{0x}\, f(z,t)\right) + \hat{j}\left(+i\omega\, B_{0y}\, f(z,t)\right)$$

$$\Rightarrow \left.\begin{aligned} -k E_{0y} &= \omega B_{0x} \\ \text{and} \quad k E_{0x} &= \omega B_{0y} \end{aligned}\right\} \Rightarrow \vec{B} = \frac{k}{\omega}(\hat{k} \times \vec{E}_0)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ E_{0x} & E_{0y} & 0 \end{vmatrix} = -E_{0y}\,\hat{i} + E_{0x}\,\hat{j}$$

So, we have,

$$\vec{B}_0 = \frac{1}{c}\,\hat{k} \times \vec{E}_0 \Rightarrow B_0 = \frac{k}{\omega} E_0 = \frac{1}{c} E_0$$

$$\Rightarrow \vec{B} = \frac{1}{c}(\hat{k} \times \vec{E})$$

$\hat{k}$ ↑ direction of propagation

← We write $\vec{B}$ in terms of $\vec{E}$ and help reduce the complexity of the situation.

Energy density and flux (intensity)

$$u = \frac{1}{2}\left(\epsilon E^2 + \frac{1}{\mu} B^2\right)$$

Use,

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{C}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{C}\times\vec{A})$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C} = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

$$\begin{aligned}
B^2 &= \frac{1}{c^2}(\hat{k}\times\vec{E})\cdot(\hat{k}\times\vec{E}) \\
&= \frac{1}{c^2}\vec{E}\cdot\left[(\hat{k}\times\vec{E})\times\hat{k}\right] \\
&= \frac{1}{c^2}\vec{E}\cdot\left[\hat{k}\times(\vec{E}\times\hat{k})\right] \\
&= \frac{1}{c^2}\vec{E}\cdot\left\{(\hat{k}\cdot\hat{k})\vec{E} - \cancel{(\hat{k}\cdot\vec{E})}^{=0}\,\hat{k}\right\} \\
&= \frac{1}{c^2}E^2
\end{aligned}$$

$$\Rightarrow u = \frac{1}{2}\left(\epsilon E^2 + \frac{1}{\mu}\frac{E^2}{c^2}\right) = \frac{1}{2}\left(\epsilon E^2 + \frac{1}{\mu}\cdot\epsilon\mu E^2\right) = \epsilon E^2$$

E executes sinusoidal oscillation in time.

$$\Rightarrow \langle u\rangle = \langle \epsilon E^2\rangle = \frac{1}{2}\epsilon E_0^2$$

Now, Poynting vector is

$$\begin{aligned}
\vec{S} &= \frac{1}{\mu}\vec{E}\times\vec{B} = \frac{1}{\mu}\vec{E}\times\frac{1}{v}(\hat{k}\times\vec{E}) \\
&= \frac{1}{\mu v}\left\{\hat{k}\,\vec{E}\cdot\vec{E} - \vec{E}(\vec{E}\cdot\hat{k})\right\} \\
&= \epsilon v E^2\,\hat{k}
\end{aligned}$$

$$\mu = \frac{1}{\epsilon v^2}$$

Energy flux is

$$\langle\vec{S}\rangle = v\epsilon\langle E^2\rangle\,\hat{k} = \frac{1}{2}\epsilon v E_0^2\,\hat{k} = I\,\hat{k}$$

I ↑ <u>intensity</u>

Boundary conditions

We begin with the Maxwell's equations in the integral form.

(i) $\oint_S \vec{D} \cdot d\vec{a} = Q_{\text{free, enclosed}}$

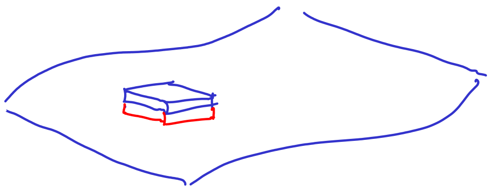
(ii) $\oint_S \vec{B} \cdot d\vec{a} = 0$

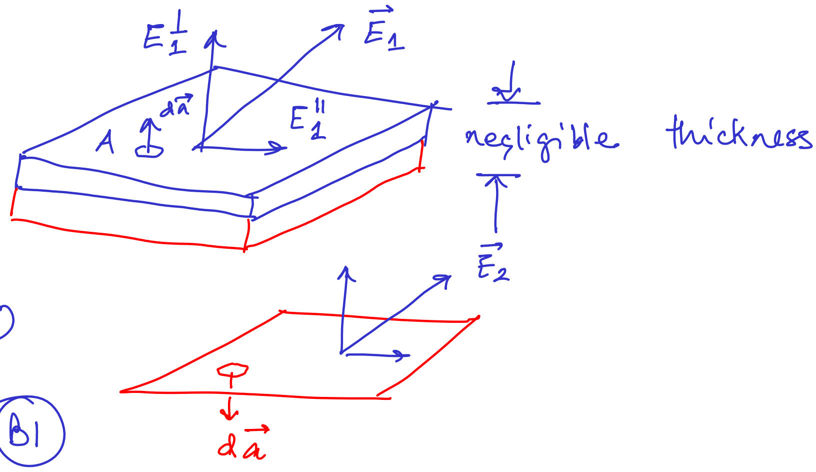
(iii) $\oint_P \vec{E} \cdot d\vec{l} = -\dfrac{d}{dt}\int_S \vec{B} \cdot d\vec{a}$

(iv) $\oint_P \vec{H} \cdot d\vec{l} = I_{\text{free, enclosed}} + \dfrac{d}{dt}\int_S \vec{D} \cdot d\vec{a}$

From, (i)

We use a thin Gaussian box around the boundary.





$$\epsilon_1 E_1^{\perp} A - \epsilon_2 E_2^{\perp} A = 0$$

$$\Rightarrow \boxed{\epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}} \quad \text{---} \quad (B1)$$

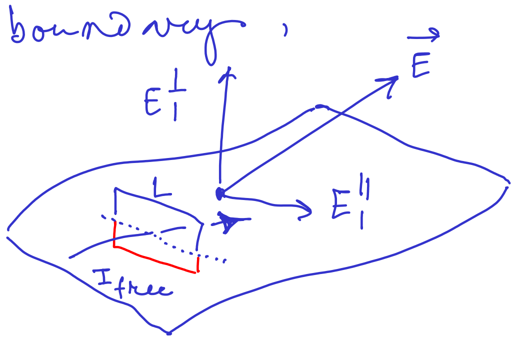
From (ii),

Exactly the same way, from (i) above we get,

$$\boxed{B_1^{\perp} = B_2^{\perp}} \quad \text{-----} \quad (B2)$$

From (iii)

We use a narrow rectangular loop around the boundary,



$$E_1^{\parallel} L - E_2^{\parallel} L = 0$$ ← For narrow loop the flux vanishes.

$$\Rightarrow \boxed{E_1^{\parallel} = E_2^{\parallel}} \quad \text{---} \quad (B3)$$

From (iv),

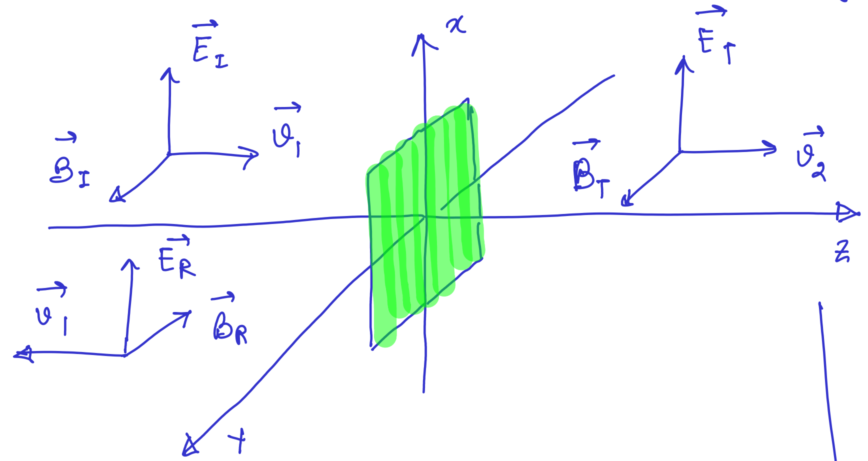
we use the narrow loop above to get,

$$\frac{1}{\mu_1} B_1^{\parallel} L - \frac{1}{\mu_2} B_2^{\parallel} L = 0$$ ← no free current and using (i) (no free charge).

$$\Rightarrow \boxed{\frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_1^{\parallel}} \quad \text{---- (B4)}$$

We shall use (B1) to (B4) to get the laws of geometric optics.

Consider the following boundary



$$\vec{E}_I = E_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{x}$$

$$\vec{B}_I = B_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{y}$$

$$= \frac{1}{v_1} E_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{y}$$

$$\vec{E}_R = E_{0R}\, e^{i(k_1 z - \omega t)}\, \hat{x}$$

$$\vec{B}_R = -\frac{1}{v_1} E_{0I}\, e^{i(-k_1 z - \omega t)}\, \hat{y}$$

and similarly $\vec{E}_T$ & $\vec{B}_T$

(B1) and (B2) is trivilly zero.

From (B3) we get,

$$E_{0I} + E_{0R} = E_{0T}$$

and from (B4) we get,

$$\frac{1}{\mu_1}\left(\frac{1}{v_1} E_{0I} - \frac{1}{v_1} E_{0R}\right) = \frac{1}{\mu_2}\frac{1}{v_2} E_{0T} = \frac{1}{\mu_2 v_2}\left(E_{0I} + E_{0R}\right)$$

$$\Rightarrow \left(\frac{1}{\mu_1 v_1} + \frac{1}{\mu_2 v_2}\right) E_{0R} = \left(\frac{1}{\mu_1 v_1} - \frac{1}{\mu_2 v_2}\right) E_{0I}$$

$$\Rightarrow E_{0R} = \frac{1 - \frac{\mu_1 v_1}{\mu_2 v_2}}{1 + \frac{\mu_1 v_1}{\mu_2 v_2}} \cdot E_{0I} = \frac{1-\beta}{1+\beta} E_{0I}$$

where, $\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$ | using, $\frac{n_1}{n_2} = \frac{v_2}{v_1}$

$$\Rightarrow \quad E_{0T} = (1+R)E_{0I} = \left(1 + \frac{1-\beta}{1+\beta}\right)E_{0I} = \left(\frac{2}{1+\beta}\right)E_{0I}$$

Intensity of a light beam

$$= \boxed{\frac{1}{2}\epsilon v E_0^2}$$

$$\Rightarrow \quad R = \frac{I_R}{I_I} = \frac{E_{0R}^2}{E_{0I}^2} = \left(\frac{1-\beta}{1+\beta}\right)^2 \sim \left(\frac{1-n_2/n_1}{1+n_2/n_1}\right)^2$$

as $\mu \approx \mu_0$ for most materials

$$= \left(\frac{n_1-n_2}{n_1+n_2}\right)^2$$

$$T = \frac{I_T}{I_I} = \frac{\epsilon_2 v_2 E_{0T}^2}{\epsilon_1 v_1 E_{0I}^2} = \frac{n_2^2\, n_1}{n_1^2\, n_2}\,\frac{(2n_1)^2}{(n_1+n_2)^2}$$

$$= \frac{4n_1 n_2}{(n_1+n_2)^2}$$

$$\frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$\frac{\epsilon_1}{\epsilon_2} = \frac{\epsilon_1\mu}{\epsilon_2\mu} = \frac{v_2^2}{v_1^2} = \frac{n_1^2}{n_2^2}$$

So, $R + T = 1$.

For, $n_1 = 1$, $n_2 = 1.5$, $R = 0.04$ and $T = 0.96$.

Wavevector

We have $\vec{E} = \vec{E}_0\, e^{i(kz - \omega t)}$

phase.

$kz \equiv k\hat{k} \cdot z\hat{k} \longrightarrow$ assumption the EM wave is propagating along z.

In general, we have

instead of $k\hat{k} \rightarrow k_x\hat{i} + k_y\hat{j} + k_z\hat{k} = \vec{k} = k\hat{k}$

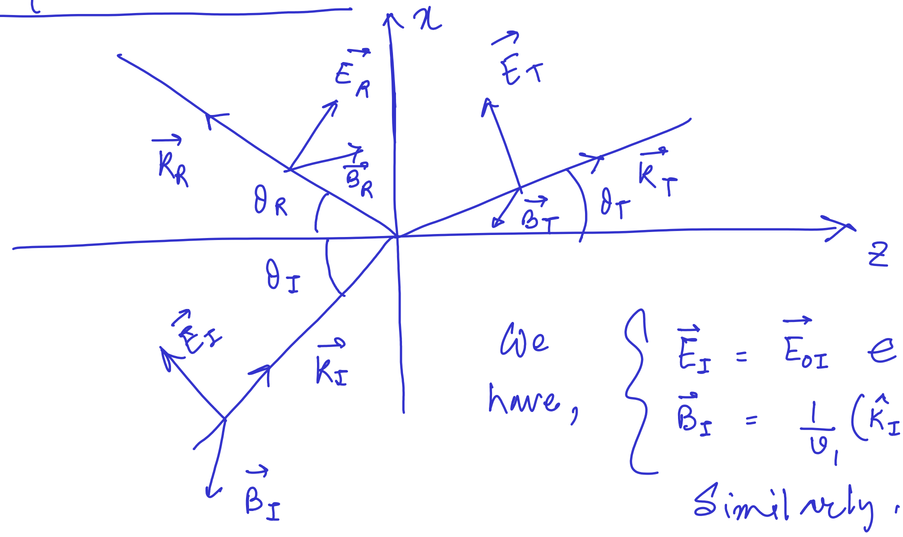
and $z\hat{k} \rightarrow x\hat{i} + y\hat{j} + z\hat{k} = \vec{r} = r\hat{r}$

$$\Rightarrow \quad \vec{E} = \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= \vec{E}_0\, e^{i(k\hat{k}\cdot\vec{r} - \omega t)}$$

and $\boxed{v = \frac{\omega}{k}}$

Oblique incidence



We have,
$$\begin{cases} \vec{E}_I = \vec{E}_{0I}\, e^{i(\vec{K}_I \cdot \vec{r} - \omega t)} \\ \vec{B}_I = \frac{1}{v_1}(\hat{K}_I \times \vec{E}_{0I})\, e^{i(\vec{K}_I \cdot \vec{r} - \omega t)} \end{cases}$$

Similarly, for R and T

We note,
$$k_I = \frac{\omega}{v_1} = k_R \quad \text{and} \quad K_T = \frac{\omega}{v_2} = \frac{v_1}{v_2} k_I = \frac{n_2}{n_1} k_I$$

Matching boundary conditions will lead to

$$(\ )_I\, e^{i(\vec{K}_I \cdot \vec{r} - \omega t)} + (\ )_R\, e^{i(\vec{K}_R \cdot \vec{r} - \omega t)} = (\ )_T\, e^{i(\vec{K}_T \cdot \vec{r} - \omega t)} \quad \text{at } z = 0$$

The oscillatory part must match on both sides.

$$\Rightarrow \quad \vec{K}_I \cdot \vec{r} = \vec{K}_R \cdot \vec{r} = \vec{k}_T \cdot \vec{r} \quad \text{when } z = 0$$

$$\Rightarrow \quad x K_{Ix} + y K_{Iy} = x K_{Rx} + y K_{Ry} = x K_{Tx} + y K_{Ty}$$

$\Rightarrow$ For $y = 0$, $\quad K_{Ix} = K_{Rx} = K_{Tx}$

and for $x = 0$, $\quad K_{Iy} = K_{Ry} = K_{Ty}$

Now, if we set $K_{Iy} = 0$ (choosing an axes for a given incidence),

we have $\quad K_{Ry} = K_{Ty} = 0$

$\Rightarrow$ $\vec{K}$ vectors are all on a single (xz in this case) plane $\longrightarrow$ <u>plane of incidence</u>.

<u>First law</u>.

Now, $\quad k_{Ix} = K_{Rx} = K_{Tx}$

$$\Rightarrow \quad k_I \sin\theta_I = k_R \sin\theta_R = k_T \sin\theta_T$$

$\Rightarrow \quad \theta_I = \theta_R$ as $k_I = k_R$ $\longrightarrow$ Second law / law of reflection

and $\quad \dfrac{\sin\theta_T}{\sin\theta_I} = \dfrac{K_I}{K_T} = \dfrac{n_1}{n_2}$ $\longrightarrow$ Third law / Snell's law.

Boundary conditions

(B1) $\epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp} \rightarrow$ for z

(B2) $B_1^{\perp} = B_2^{\perp} \rightarrow$ for z

(B3) $E_1^{\parallel} = E_2^{\parallel} \rightarrow$ for both x, y

(B4) $\frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_2^{\parallel} \rightarrow$ for both x, y

for the chosen boundary

(B1) $\Rightarrow \epsilon_1 \left(-E_{0I} \sin\theta_I + E_{0R} \sin\theta_R\right) = \epsilon_2 \left(-E_{0T} \sin\theta_T\right)$ .... (1)

(B2) $\Rightarrow 0 = 0$

(B3) $\Rightarrow \left(E_{0I} \cos\theta_I + E_{0R} \cos\theta_R\right) = E_{0T} \cos\theta_T$ .... (2)

(B4) $\Rightarrow \frac{1}{\mu_1}\frac{1}{v_1}\left(E_{0I} - E_{0R}\right) = \frac{1}{\mu_2}\frac{1}{v_2} E_{0T}$ .... (3)

(1) $\Rightarrow E_{0I} - E_{0R} = \frac{\epsilon_2}{\epsilon_1}\frac{\sin\theta_T}{\sin\theta_I} E_{0T} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1} E_{0T} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T}$ $\quad \epsilon_1 = \frac{1}{\mu_1 v_1^2}$

$= \beta E_{0T}$

(3) $\Rightarrow E_{0I} - E_{0R} = \frac{\mu_1 v_1}{\mu_2 v_2} E_{0T} = \beta E_{0T}$

(2) $\Rightarrow E_{0I} + E_{0R} = \frac{\cos\theta_T}{\cos\theta_I} E_{0T} = \alpha E_{0T}$

$$\boxed{E_{0R} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right) E_{0I}, \quad E_{0T} = \left(\frac{2}{\alpha + \beta}\right) E_{0I}}$$

Fresnel's equations

* Transmitted beam $\rightarrow$ always in-phase with the incident beam
* Reflected beam $\rightarrow$ either in-phase or out-of-phase with the incident beam

$\alpha = \beta \Rightarrow$ No reflection condition!

$$\boxed{\sin^2\theta_B = \frac{1 - \beta^2}{\left(\frac{n_1}{n_2}\right)^2 - \beta^2}}$$

$$\boxed{\alpha = \frac{\cos\theta_T}{\cos\theta_I} = \frac{\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_I}}{\cos\theta_I}}$$

$\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2}$

$n_1 \sin\theta_I = n_2 \sin\theta_T$

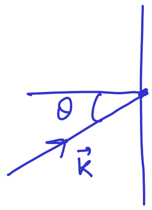
$\sin\theta_T = \frac{n_1}{n_2} \sin\theta_I$

$\Rightarrow \cos\theta_T = \sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_I\right)^2}$

Intensity

$$I \propto \underbrace{\tfrac{1}{2}\epsilon v E^2 \cos\theta}$$

Component of the beam perpendicular to the boundary



Intensity of the reflected wave,

$$\frac{I_R}{I_I} = \frac{\epsilon_1 v_1 E_{0R}^2}{\epsilon_1 v_1 E_{0I}^2} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2$$

Intensity of the transmitted wave,

$$\frac{I_T}{I_I} = \frac{\epsilon_1 v_1 E_{0T}^2 \cos\theta_T}{\epsilon_2 v_2 E_{0I}^2 \cos\theta_I} = \frac{\mu_1 v_1}{\mu_2 v_2}\cdot\frac{\cos\theta_T}{\cos\theta_I}\cdot\left(\frac{2}{1+\beta}\right)^2 = \beta\cdot\alpha\cdot\frac{4}{(1+\beta)^2}$$

* Brewster's angle

$\alpha = \beta \Rightarrow$ no reflection

$$\Rightarrow \frac{\cos\theta_T^0}{\cos\theta_I^0} = \frac{\mu_1 v_1}{\mu_2 v_2} \approx \frac{n_2}{n_1}$$

$$\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2} = \frac{1}{\beta}$$

$$\Rightarrow \frac{\sqrt{1-\sin^2\theta_T^0}}{\cos\theta_I^0} = \frac{\sqrt{1-\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_I^0}}{\cos\theta_I^0} = \frac{n_2}{n_1} = \beta$$

$$\Rightarrow \sin^2\theta_I^0 = \frac{\beta^2}{1+\beta^2} \Rightarrow \tan\theta_I^0 = \beta = \frac{n_2}{n_1} \Rightarrow \theta_I^0 = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

Total internal reflection

For $n_2 > n_1$, $\theta_T < \theta_I$

$$\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2}$$

For $n_2 < n_1$, $\theta_T > \theta_I$

For a particular θ_I^0 for $n_2 < n_1$,
if $\sin\theta_T^0 = 1$ i.e. $\theta_T^0 = \pi/2$, we have,

$$\sin\theta_I^0 = \frac{n_2}{n_1}\cdot\sin\theta_T^0 = \frac{n_2}{n_1} \Rightarrow \theta_I^0 = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

We get this again from

$$\frac{I_T}{I_I} = \frac{4\alpha\beta}{(\alpha+\beta)^2} \quad \text{with} \quad \alpha = \frac{\sqrt{1-\left(\frac{n_1}{n_2}\right)\sin^2\theta_I}}{\cos\theta_I} = 0$$

For α to be real,

$$\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_I \leq 1$$

or $\theta_I \leq \sin^{-1}\left(\frac{n_2}{n_1}\right)$

Equality $\longrightarrow$ threshold for total internal reflection

For all $\theta_I > \theta_I^0$, we have "exponential wave" instead of a regular transmission.

Also,

$$\frac{E_{0R}}{E_{0I}} = \frac{\alpha-\beta}{\alpha+\beta} = \frac{\frac{\cos\theta_T}{\cos\theta_I} - \frac{n_2}{n_1}}{\frac{\cos\theta_T}{\cos\theta_I} + \frac{n_2}{n_1}} = \left(\frac{n_1\cos\theta_T - n_2\cos\theta_I}{n_1\cos\theta_T + n_2\cos\theta_I}\right)$$

For "p-type" polarization