

January 5, 2024

PROBABILITY I

MA2202

ASSIGNMENT 1

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Exercise 1. (10 points)

Show that if you keep on throwing a fair die, the probability of eventual occurrence of the outcome six is 1.

Exercise 2. (10 points)

Suppose (Ω, \mathcal{E}, P) be a given probability space and $A_1, A_2, \dots, A_n \in \mathcal{E}$. Then show that

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = \sum_{i=1}^n P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i \cap A_j) + \dots + (-1)^{n-1} P(A_1 \cap \dots \cap A_n)$$

Exercise 3. (10 points)

To the choice of each $n \in \mathbb{N}$, could you assign a probability $P(n) > 0$ such that the following conditions hold?

- (a) $P(m) \neq P(n)$ for all $m, n \in \mathbb{N}$.
- (b) The probability of choosing an odd positive integer is the same as the probability of choosing an even positive integer.

Justify your answer!

Exercise 4. (10 points)

Seven students of IISER Kolkata went to participate in an event at IISER Mohali. They booked AC 3-tier tickets from Howrah to Chandigarh in Netaji Express, which has three AC 3-tier coaches. Every such coach has eight coupes (i.e. compartments), each coupe containing eight berths. If the berths were allocated randomly, find the probability of at least two among the seven students being allocated berths in the same coupe.

Exercise 5. (10 points)

Carrom is played with a red, nine black and nine white coins (and a striker) on a square board with a pocket in each corner. If all these coins are scattered randomly (none of them being in any pocket) on a 29 inch \times 29 inch carrom board, show that the probability of at least two of the coins being less than three inches apart is greater than $1/2$.

Maximum score: 50 points. Please mention your name, roll no. and **group** in your answersheet. Please submit your answersheet by 11:59 p.m. on **January 18, 2022** in the DMS mailbox for MA2102, which is designated with your group name. Submit only your final answersheet! **Multiple answersheets or corrections to the original answersheet will not be accepted, irrespective of the time of their submission.**