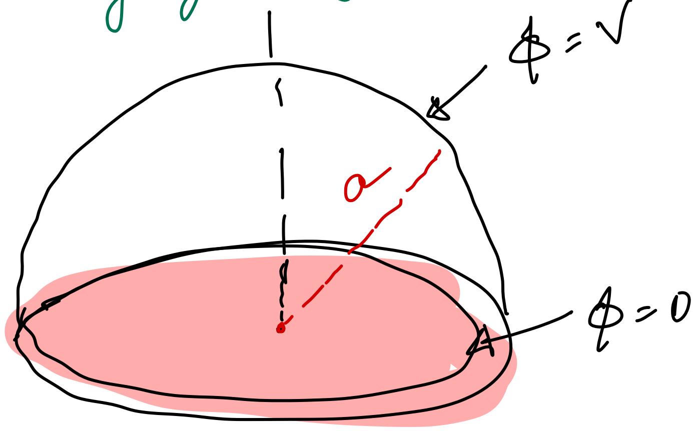
Problem-1) find the electrostatic potential with following geometry.




A hemispherical surface maintained at potential $\Phi = V$, with $\theta = \pi/2$ plane maintained at $\phi = 0$ find the potential inside the hemispherical region [we can assume azimuthal symmetry)

The Sol$^n$ is given by

$$\Phi(r,\theta) = \sum_{l=0}^{\infty} \left[c_l r^l + b_l r^{-(l+1)}\right] P_l(\cos\theta)$$

when $\theta = \frac{\pi}{2} \Rightarrow \cos\theta = 0$

$P_l(0) = 0$ for odd $l$'s

The Sol$^n$ becomes

$$\sum_{l=\text{odd}}^{\infty} \left[c_l r^l + d_l r^{-(l+1)}\right] P_l(\cos\theta)$$

we are looking for interior Sol$^n$ $\Rightarrow d_l = 0$

$$\Phi(r,\theta) = \sum_{l = odd}{}' A_l V \left(\frac{r}{a}\right)^l P_l(\cos\theta)$$

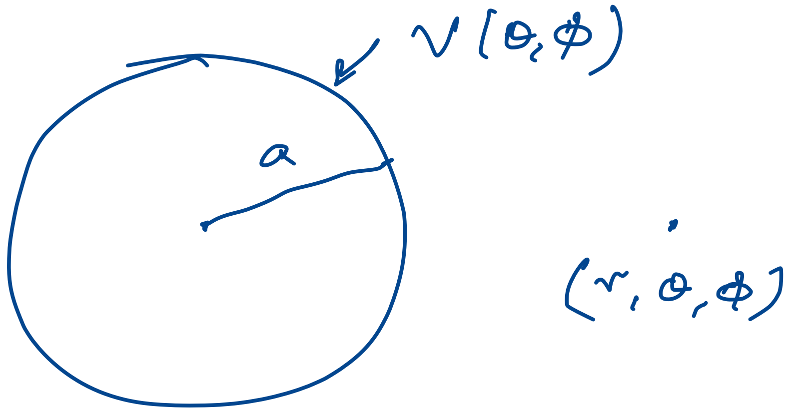
when $r = a$

$$V = \sum_{odd}{}' A_l V \; P_l(\cos\theta) \Rightarrow$$

$$1 = \sum_{odd}{}' A_l P_l(\cos\theta)$$

$$A_l = \frac{2l+1}{2} \int_0^1 P_l(x)\, dx$$

Problem 2 : A sphere of radius 'a' is maintained at potential $V(\theta,\phi)$. find the potential outside the sphere. There no charge outside



$$\nabla^2 \phi = 0 \quad \text{outside}$$

$$\phi(r=a,\theta,\phi) = V(\theta,\phi)$$

$$\phi = \sum \left(\frac{r}{a}\right)^{-(l+1)} A_{lm} Y_{lm}(\theta,\phi)$$

$r=a$

$$V(\theta,\phi) = \sum A_{lm} Y_{lm}(\theta,\phi)$$

Any function on sphere $(\theta,\phi)$ can be expanded in terms of $Y_{lm}$

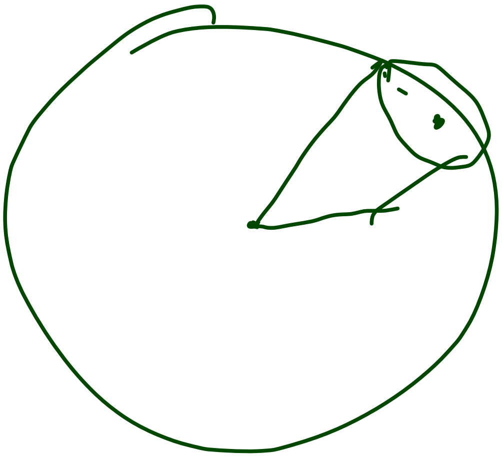
Let's say $g(\theta,\phi)$

$$g(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{lm} Y_{lm}(\theta,\phi)$$

$$A_{\ell m} = \int d\Omega \, Y^*_{\ell m}(\theta,\phi)\, g(\theta,\phi)$$

$\downarrow$

$\sin\theta \, d\theta \, d\phi$





$\sin\theta \, d\theta \, d\phi$