22. August 2023

Linear Algebra I (MA2102)

Assignment 2

Instructor: Soumya Bhattacharya

Teaching assistants: Md. Nurul Molla, Rajiv Mishra, Abisek Dewan

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Exercise 1. (10 points)
Find the dimension of $\operatorname{Span}_{\mathbb{R}} S$, where

$$S := \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} \right\}.$$

Exercise 2. (5+5 points)
(i) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ x & y & z \end{bmatrix}$ and let $V = \{(x, y, z) \in \mathbb{R}^3 : \det(A) = 0\}$. Show that V is a vector space and find the dimension of V.

(ii) Let $V = \{(x_1 - x_2 + x_3, \, x_1 + x_2 - x_3) : (x_1, x_2, x_3) \in \mathbb{R}^3\}$. Show that V is a vector space and find the dimension of V.

Exercise 3. (10 points)
Let A be an $n \times n$ real matrix such that the sum of the entries of each column of A is λ. Show that λ is an eigenvalue of A.

Exercise 4. (12 points)
For a prime p, find the cardinality of $\mathrm{GL_n}(\mathbb{F}_p)$.

Exercise 5. (12 points)
Let V be a finite dimensional real vector space and let $W_1, W_2, \ldots, W_n$ be proper subspaces of V. Show that

$$V \neq \bigcup_{i=1}^{n} W_i.$$

Maximum score: 50 points. You may answer as many questions as you wish. If the sum of your total score exceeds 49, you shall get the maximum score. Please mention your name, roll no. and **group** in your answersheet. Please submit your answersheet by 11:59 p.m. on **31.08.2023** in the DMS mailbox for MA2102, which is designated with your group name.