PH3201 Class Test 2. **9** April 2025 10:10-10:50

1. A 3d gas of non-interacting Bosons with spin $S^{(z)} \equiv S = \{+1,0,-1\}$ has the Hamiltonian

$H = \sum_i \frac{p_i^2}{2m} - \mu_0 B S_i$ where B is a constant magnetic field in the z-direction.

(a) Calculate the number of Bosons N_+, N_0, N_- in three different spin states for some temperature>>0.

(b) Find the average magnetisation. (Write the answer in terms of functions given below) [7 **Marks**]

2. Given a Hamiltonian $H(\{q_i, p_i\})$ show that $\left\langle q_i \frac{\partial H}{\partial q_i} \right\rangle = \frac{1}{\beta}$ [4 **Marks**]

3. Let a classical system given by the Hamiltonian H_0 has Helmholtz Free energy $F_0 = F(H_0)$. Now consider a Hamiltonian $H = H_0 + \lambda H_1$. Obtain a series expansion of the free energy $F(H)$ written in terms of the cumulants of H_1 and up to the quadratic power of λ. [4 **Marks**]

$$f_{5/2}(z) = \frac{4}{\sqrt{\pi}} \int_0^\infty dx\ x^2\ ln[1 + ze^{-x^2}]$$

$$f_{3/2} = z \frac{\partial}{\partial z} f_{5/2}(z)$$

$$g_{5/2}(z) = -\frac{4}{\sqrt{\pi}} \int_0^\infty dx\ x^2\ ln[1 - ze^{-x^2}]$$

$$g_{3/2} = z \frac{\partial}{\partial z} g_{5/2}(z)$$

$$\lambda = \sqrt{\frac{2\ \pi\ \hbar^2}{m\ k_B\ T}}$$

$$\sigma = \frac{\pi^2\ k_B^4}{60\ \hbar^3\ c^2}$$

$$\int_0^\infty \frac{x^3\ dx}{e^x - 1} = \frac{\pi^4}{15}$$