Rumblings

OQS Notes

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Contents

1. Quantum Probability

These are just some important stuff and loose collection of notes of the Chapter on Quantum Probability from The Theory of Open Quantum Systems by H.-P. Breuer and F. Petruccione.

1.1. Spectral Theorem

Consider a self-adjoint operator A acting on a separable Hilbert space $\mathcal{H}$. Then, there exists a unique one parameter mutually commuting spectral family $\{E_\lambda\}_{\lambda\in\mathbb{R}}$ of projection operators acting on $\mathcal{H}$ such that

$$A = \int_{\mathbb{R}} \lambda dE_\lambda \tag{1.1}$$

where E_λ satisfy,

1. $\|E_{r'}\psi\| \leq \|E_r\psi\|$ for $r' \leq r$ and for all $\psi \in \mathcal{H}$. This can equivalently be written as
$$E_{r'} \leq E_r \text{ for } r' \leq r \tag{1.2}$$
.
2. The spectral family is right-continuous, i.e.,
$$\lim_{\varepsilon\longrightarrow 0+} E_{r+\varepsilon} = E_{r_0} \text{ for all } r_0 \in \mathbb{R} \tag{1.3}$$
.
3. $$\lim_{r\longrightarrow -\infty} E_r = 0 \tag{1.4}$$
and
$$\lim_{r\longrightarrow \infty} E_r = I \tag{1.5}$$
, where I is the identity operator on $\mathcal{H}$.

A much better and concise explanation can be found for normal operators in [1].

1.2. Basic Postulates of Quantum Mechanics

Consider a statistical ensemble ε of identically prepared quantum systems, $S^1, S^2, ..., S^N$. The postulates are,

1. Under certain conditions, a complete characterization of this ensemble can be given by a normalized state vector $|\psi\rangle \in \mathcal{H}$, which is a Hilbert space associated with the quantum system S.
2. Measureable quantities are represented by linear, self-adjoint operators A acting on $\mathcal{H}$. The outcome of a measurement of the observable A performed on the ensemble ε is a random variable R with a cumulative distribution function $F_{R(x)}$ defined by
$$F_{A(r)} = \langle E_r|\psi|E_r\rangle \tag{1.6}$$

where E_r is the spectral family of the operator A.One can easily verify from the Spectral theorem above that this indeed is a valid cumulative distribution function.

References

[1] D. Halmrast, "Spectral Decomposition of Quantum-Mechanical Operators," 2017. [Online]. Available: https://web.math.ucsb.edu/~dhalmrast/resources/halmrast-math-thesis.pdf