Mach–Zehnder interferometer

The Mach–Zehnder interferometer is a particularly simple device for demonstrating interference by division of amplitude. A light beam is first split into two parts by a beamsplitter and then recombined by a second beamsplitter. Depending on the relative phase acquired by the beam along the two paths the second beamsplitter will reflect the beam with efficiency between 0 and 100%. The operation of a Mach–Zehnder interferometer is often used as an example in quantum mechanics because it shows a clear path-choice problem. However, it is not at all obvious at first glance that it works as claimed, until reflection phase shifts are considered in detail.

The question of how a Mach–Zehnder interferometer works arose out of reading through A-level Research and Analysis essays. Because our students only study division-of-wavefront interferometers and not division of amplitude they had simply accepted the standard explanation given on websites such as www.qubit.org. On that site, the basic interferometer is shown as in figure 1 and it is explained that interference between the two paths ensures that the photon always strikes detector A. If one of the paths is lengthened then the interference can be altered to ensure that all photons strike detector B.

Whilst this seems eminently plausible (indeed, so plausible that we all accepted it without too much worry) it is grossly misleading. First of all, consider the phase of the photon on following each of the two paths, the lower and upper. Initially we shall assume that there is no phase shift on reflection or transmission.

The phase on reaching the second beamsplitter is simply the path length divided by the wavelength, multiplied by 2π. On recombination at the beamsplitter, if the two paths are of equal length, then the phases are equal. So which path shows constructive interference, the path towards A or B? The answer is unresolved. In fact, the entire situation is symmetrical with respect to the two detectors and should one path allow constructive interference, so will the other. Similarly if one path suffers destructive interference, so does the other. This violates conservation of energy.

Phase shifts on reflection

Clearly there is a false assumption and the obvious place to look is the phase shift on reflection. A standard piece of physics lore is that on transmission a wave picks up no phase shift, but on reflection it picks up a phase shift of π. So now let's investigate the problem with that in mind. We shall break the problem into two parts: first the path from the source to the second beamsplitter, and then the final stretch from the second beamsplitter to the detectors A and B.

On the lower path, the beam undergoes one transmission and one reflection before the second beamsplitter—a total phase shift of π. On the upper path there are two reflections—a total phase shift of 2π. Now if we continue on to detector A, the lower path makes one more reflection and the upper path one transmission. So now each path has a phase shift of 2π and they will interfere constructively. All well and good? Until we look at the path to detector B. Now the lower path makes one more transmission, picking up a total phase shift of π. The upper path makes a further reflection, so its total phase shift is 3π. The

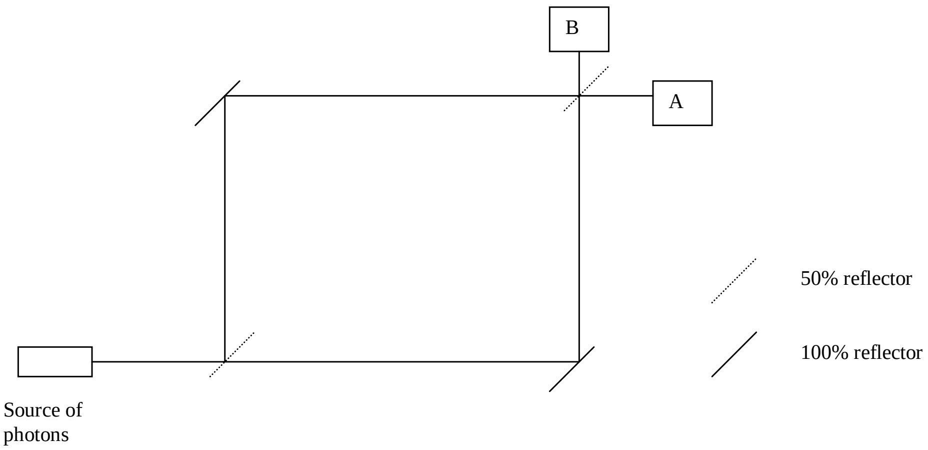

Figure 1. Diagram of a simple Mach–Zehnder interferometer, ignoring the thickness of the beamsplitters.

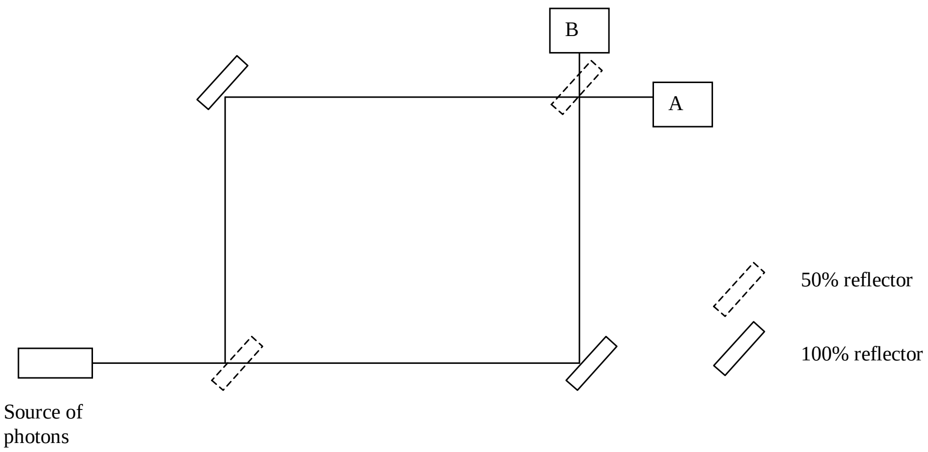

Figure 2. More sophisticated version of the Mach–Zehnder interferometer diagram. Here the thicknesses of the beamsplitters and the reflecting surfaces are indicated.

difference is 2π and again we expect constructive interference.

So that is the problem—the Mach–Zehnder interferometer as presented does not work. The devil in this case is in the detail of the phase shifts on reflection—the story is not as simple as we commonly make out.

Resolution of the problem

How is a beamsplitter actually made? Usually it is a piece of glass with a dielectric or metal coating on the front surface. Light striking it from the front has a 50% (or any other value, depending on the coating) chance of being reflected, and a 50% chance of being transmitted. In the case of a dielectric (non-conducting and non-magnetic) a reflection does indeed induce a phase shift of π, whereas a transmitted photon picks up no phase shift. However, the key to the problem lies in what happens to a photon approaching the beamsplitter from behind. There it first enters the glass (ignoring the small chance of reflection off the air–glass interface) and has a 50% chance of reflecting off the dielectric coating whilst within the glass. Here is the crux of the matter—*that* reflection does not induce a phase change. Given that, let us once again examine the phase shifts on the two paths.

Refer to figure 2 and notice that the second beamsplitter is arranged so that the dielectric is on the right-hand surface. This is purely to simplify the overall analysis—it makes no difference to the underlying physics.

We shall use the following definitions: l_1 and l_2 are the total path lengths for the light travelling from the source to the detector for the upper and lower paths respectively. When the light passes through the glass of the beamsplitters it picks up an extra phase shift which we shall call $2\pi t/\lambda$. This simplifies the maths a little, but note that t is not the thickness of the beamsplitters by this definition. In fact t is the *optical path length* through the beamsplitter, which takes account of the actual distance travelled (the beam passes through at an angle) and the refractive indices of the glass and the coatings.

The upper path picks up the following phase shifts on the way to detector A: π at the first reflection, π at the second (100%) reflection, nothing at the transmission, $2\pi l_1/\lambda$ for the distance travelled, and $2\pi t/\lambda$ for the extra phase picked up in traversing the glass substrates where the wavelength is reduced. This gives a total of

$$2\pi + 2\pi\left(\frac{l_1 + t}{\lambda}\right).$$

The lower path, also on its way to A, picks up a phase shift of π off the 100% reflector, π at the second beamsplitter, a phase shift of $2\pi l_2/\lambda$ for the distance travelled, and an extra phase shift of $2\pi t/\lambda$ from passing through the glass substrate at the first beamsplitter. The phase difference between the two paths is

$$\begin{aligned} &2\pi + 2\pi\left(\frac{l_1 + t}{\lambda}\right) - 2\pi - 2\pi\left(\frac{l_2 + t}{\lambda}\right) \\ &\quad = 2\pi\left(\frac{l_1 - l_2}{\lambda}\right) = \delta \end{aligned}$$

where δ is the phase shift due to the difference in the path lengths.

Similarly, we can calculate the phase difference between the two paths on their way to detector B. We obtain

$$\begin{aligned} &2\pi + 2\pi\left(\frac{l_1 + 2t}{\lambda}\right) - \pi - 2\pi\left(\frac{l_2 + 2t}{\lambda}\right) \\ &\quad = \pi + 2\pi\left(\frac{l_1 - l_2}{\lambda}\right) = \pi + \delta. \end{aligned}$$

Now it is clear that when $\delta = 0$ there is constructive interference on the path to A and destructive on the path to B. By varying δ, this condition can be changed so as to vary the probability of arrival at either detector from 0 to 1.

All of the physics is contained in this analysis. In practice, the beamsplitters may be of different thicknesses but this will simply add a fixed phase difference, as will placing the second beamsplitter the other way around.

A brief word on phase shifts

The rule about phase shifts derives from the Fresnel equations for reflection and transmission of a wave at a dielectric. These equations show that there is a phase change for a reflection when a wave reflects off a change from low to high refractive index but not when it reflects off a change from high to low. Usually a beamsplitter has a dielectric coating which is intermediate in value between the glass and the air, which will fulfil the conditions assumed in the analysis. In fact, the analysis is considerably more complicated when different polarizations are taken into account, and if a metallic (conducting) reflector is considered. However, in the absence of absorption it has to be the case that there is a phase difference of π between the two possible paths, because no other value can satisfy conservation of energy. A beamsplitter which is not 50/50 will simply prevent total constructive or destructive interference occurring, and this is also a possibility—indeed this property is used to improve the performance of such an interferometer in making so-called weak measurements, where the photon does not interact with an object in one of the paths.

Refractive index of air – dependence on pressure:

It may be shown that from the Lorenz-Lorentz equation that the refractive index of a mixture of nonpolar gases is given by the relation

$$\frac{n^2 - 1}{n^2 + 2} = \sum_i R_i \rho_i, \qquad (1)$$

in which R_i is the specific refraction and ρ_i the partial density of the ith component of the mixture. The specific refraction, which is invariant under changes in density to a high degree of approximation for the components of air at atmospheric pressures and hence may be evaluated from absolute measurements of refractivity vs wavelength at one density, is given by

$$R_i = [(n_i^2 - 1)/(n_i^2 + 2)](1/\rho_i) \qquad (2)$$

$$= \frac{4}{3}\pi(N_A/M_i)\alpha_i, \qquad (3)$$

in which n_i, M_i, and α_i are the refractive index at density ρ_i, the molecular weight, and the polarizability, respectively, of the ith component, and N_A is Avogadro's number.

The use of Eq. (1) to represent the refractive index of a gas mixture, in which the contribution of each component is given by the product of the partial density and a quantity R_i which depends only on wavelength, assumes that the Lorentz local field is correct and that the polarizabilities of the components are unchanged in the mixture. For atmospheric air it is sufficient to write Eq. (1) as the sum of three terms,

$$(n^2 - 1)/(n^2 + 2) = R_1\rho_1 + R_2\rho_2 + R_3\rho_3, \qquad (4)$$

in which R_1, R_2, and R_3 are the specific refractions of dry, $C0_2$-free air, of water vapor, and of carbon dioxide, respectively, and ρ_1, ρ_2, and ρ_3 are the corresponding partial densities. The dispersion curves of N_2, 0_2, and Ar are sufficiently similar that all three may be represented by the first term.

Reference: J. C. Owens, Applied Optics **6**, 51 (1967).

Experiment (wavelength of the laser = 650 nm, length of pressure cell = 3 cm):

1. Set up the apparatus as per Figs. 1 and 2.
2. Make sure that the fringes are clear in both arms A and B.
3. Insert the pressure cell in one of the interferometer arms. Note carefully the readings of the vacuum gauge of the hand-pump.
4. Measure the fringe shifts as a function of pressure of the vacuum cell by marking a reference point on the screen.

5. Calculate the slope of n (refractive index) vs pressure from the formula (be careful about your units):

$$\frac{n_i - n_f}{P_i - P_f} = \frac{\Delta m\, \lambda_o / d}{P_i - P_f}$$

where $\mathbf{P_i}$ = the initial air pressure

$\mathbf{P_f}$ = the final air pressure

$\mathbf{n_i}$ = the index of refraction of air at P_i

$\mathbf{n_f}$ = the index of refraction of air at P_f

Δ **m** = the number of fringes that passed the reference point during evacuation

λ_o = the wavelength of the laser light in vacuum

d = the length of the vacuum chamber (3.0 cm)

6. From the slope, find out the value of n for air at 1 atmosphere (assume n at vacuum to be 1).
7. Insert a lens in front of the laser. Now prove that the fringes obtained at outputs A and B are out of phase by π.