PH4209 Assigment — 01
22MS037

ALGORITHM

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Divide population of size N equally between two types of individuals A and B represented by the
numbers 0 & 1
Define mutation rates u1 and u2
Start loop over generations (total =T)
  Start loop over entire population (size=N)\
  Check if the individual chosen is of type 0 or type 1
    If type 0, mutate individual from 0 to 1 with probability u1
      Pick a random number r between 0 and 1
      If r < u1, mutate individual from 0 to 1, else leave unchanged
    If type 1, mutate individual from 1 to 0 with probability u2
      Pick a random number r between 1 and 0
      If r < u2, mutate individual from 1 to 0, else leave unchanged
  Close loop over population
  Calculate frequency of type 0 and type 1 in the population
  Record generation versus frequency data
Close loop over generations
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Problem 1 — Run simulations using $u_1 = 0.003$ and $u_2 = 0.001$ for 3 different population sizes $N = 50, 1000, 10000$ for $T = 2000$ generations.

We consider a population of size N with two types of individuals: type A and type B.

$$A \underset{u_1}{\overset{u_2}{\leftrightharpoons}} B \tag{1}$$

where u_1 is the mutation rate from type A to type B, and u_2 is the mutation rate from type B to type A. The master equation is ,

$$\begin{aligned} \frac{\mathrm{d}f_A}{\mathrm{d}t} &= u_2 f_B - u_1 f_A = -f_A(u_1 + u_2) + u_2 \\ \frac{\mathrm{d}f_B}{\mathrm{d}t} &= u_1 f_A - u_2 f_B = -f_B(u_1 + u_2) + u_1 \end{aligned} \tag{2}$$

The steady state solution is given by,

$$f_A^{\mathrm{ss}} = \frac{u_2}{u_1 + u_2} \quad ; \quad f_B^{\mathrm{ss}} = \frac{u_1}{u_1 + u_2} \tag{3}$$

For $u_1 = 0.003$ and $u_2 = 0.001$, we have,

$$f_A^{\mathrm{ss}} = 0.25 \quad ; \quad f_B^{\mathrm{ss}} = 0.75 \tag{4}$$

We run simulations for $N = 50, 1000, 10000$ for $T = 2000$ generations. The results are shown below:

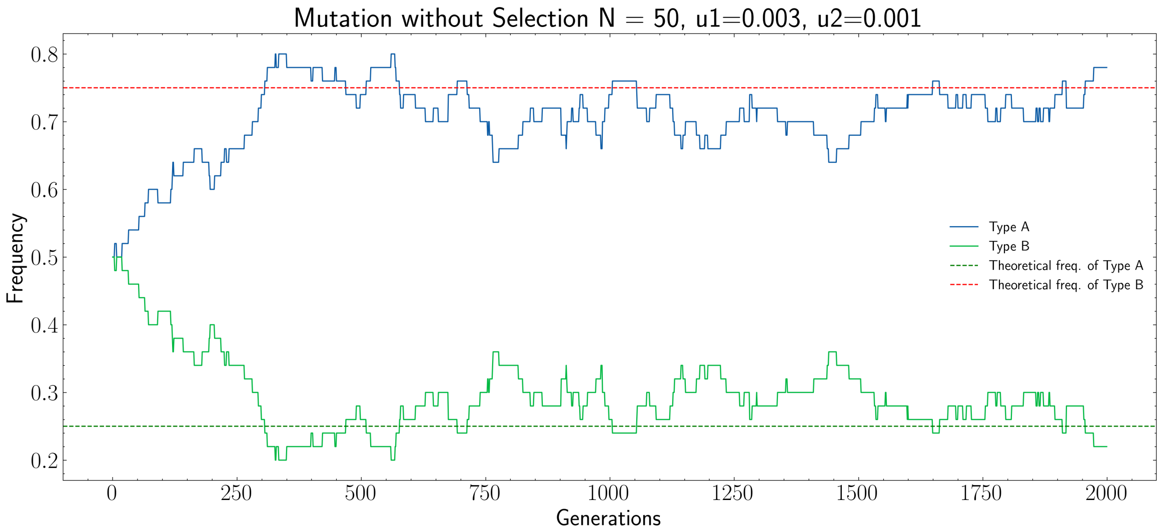

Figure 1: Simulation results for $N = 50$, with rates $u_1 = 0.003$ and $u_2 = 0.001$. The solid lines represent the steady state fractions $f_A^{ss} = 0.25$ and $f_B^{ss} = 0.75$ for 2000 generations

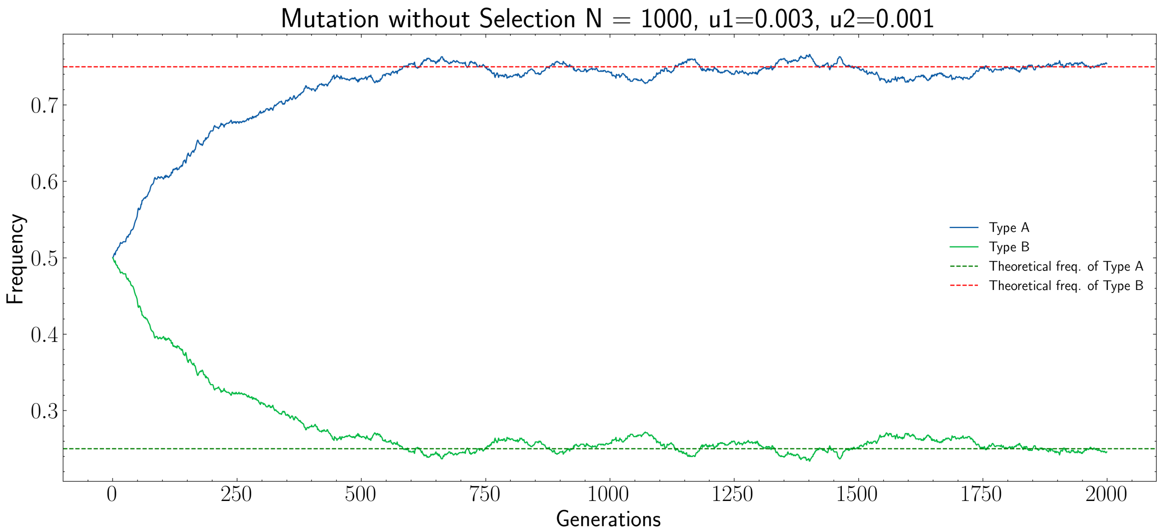

Figure 2: Simulation results for $N = 1000$, with rates $u_1 = 0.003$ and $u_2 = 0.001$. The solid lines represent the steady state fractions $f_A^{ss} = 0.25$ and $f_B^{ss} = 0.75$ for 2000 generations

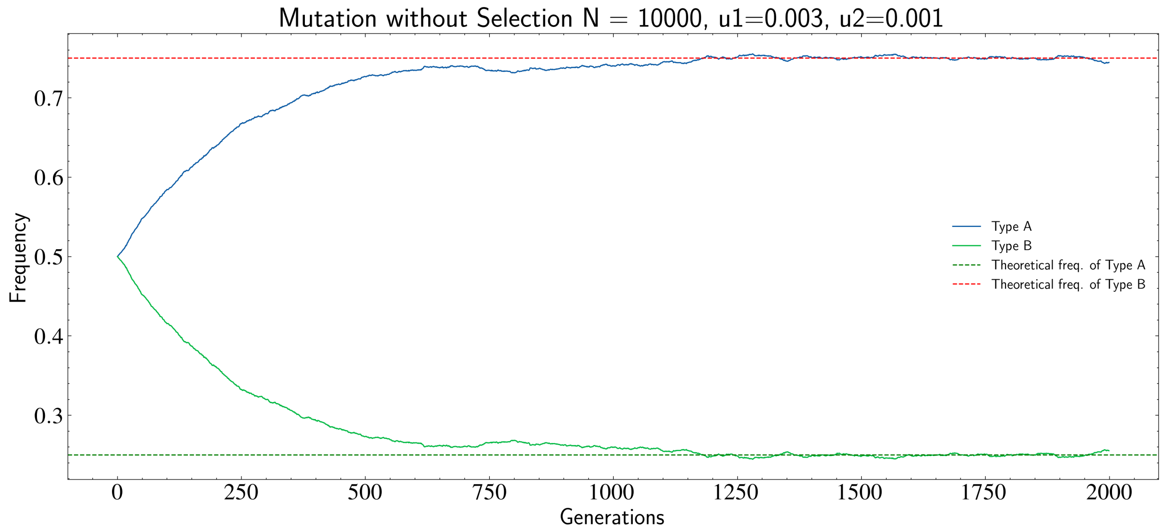

Figure 3: Simulation results for $N = 10000$, with rates $u_1 = 0.003$ and $u_2 = 0.001$. The solid lines represent the steady state fractions $f_A^{ss} = 0.25$ and $f_B^{ss} = 0.75$ for 2000 generations

We see that the fluctuations around the steady state fractions decrease with increasing population size N. One expects complete deterministic behavior in the limit $N \longrightarrow \infty$.

Problem 2 — Repeat simulation for $N = 1000$ using $u_1 = 0.07$ and $u_2 = 0.001$; Use $T = 2000$ in both cases.

The steady state fractions for $u_1 = 0.07$ and $u_2 = 0.001$ are given by,

$$f_A^{ss} = \frac{u_2}{u_1 + u_2} = 0.0141 \quad ; \quad f_B^{ss} = \frac{u_1}{u_1 + u_2} = 0.9859 \tag{5}$$

We run simulations for $N = 1000$ for $T = 2000$ generations. The results are shown below:

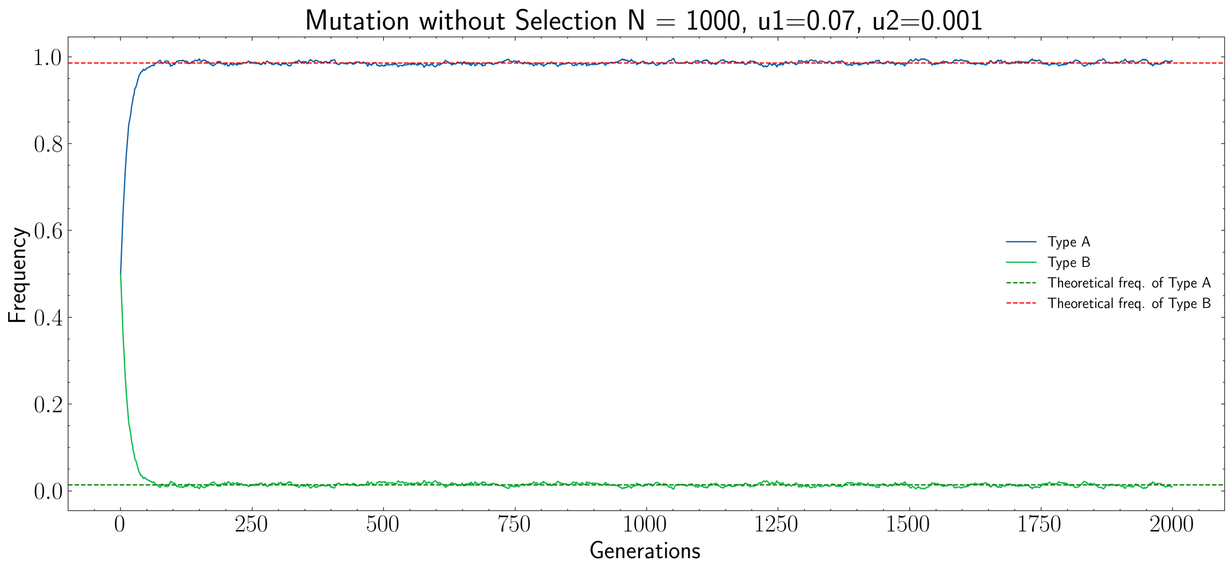

Figure 4: Simulation results for $N = 1000$, with rates $u_1 = 0.07$ and $u_2 = 0.001$. The solid lines represent the steady state fractions $f_A^{ss} = 0.0141$ and $f_B^{ss} = 0.9859$ for 2000 generations