Lecture Five

Anti-bunching and Entanglement

Contents

Part one: Anti-Bunching

Part two: Entanglement

Part One

Anti-Bunching

5-1 Classical Correlation functions (Recap)

$$I = \left\langle \left|E_1(t)\right|^2 \right\rangle + \left\langle \left|E_2(t+\tau)\right|^2 \right\rangle + 2\,\mathrm{Re}\left\langle E_1(t)E_2(t+\tau)\right\rangle \ldots.(5.1)$$

$$G^{(1)}(\tau) = \left\langle E(t)E^*(t+\tau)\right\rangle \ldots.(5.2)$$

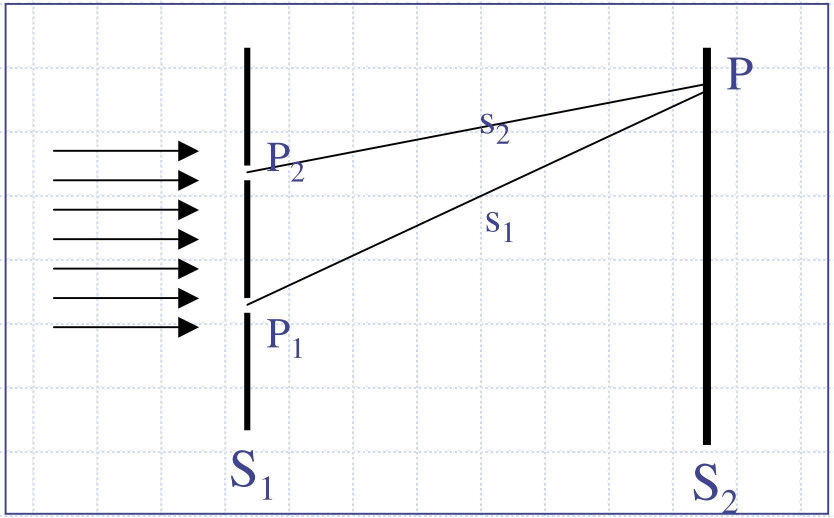
$$g^{(1)}(\tau) = \frac{\left\langle E\ (\tau)E^*(t+\tau)\right\rangle}{\left\langle \left|E(t)\right|^2\right\rangle} \ldots\ldots.(5.3)$$

$$I\ = I_1 + I_2 \pm 2\sqrt{I_1 I_2}\,\mathrm{Re}\,g^1(\tau) \ldots..(5.4)$$

$$V = \frac{\langle I(r)\rangle_{\max} - \langle I(r)\rangle_{\min}}{\langle I(r)\rangle_{\max} + \langle I(r)\rangle_{\min}}$$

$$= \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

$$= \frac{2I}{2I} g^1(\tau) = g^1(\tau) \ldots\ldots.(5.5)$$



Laser light

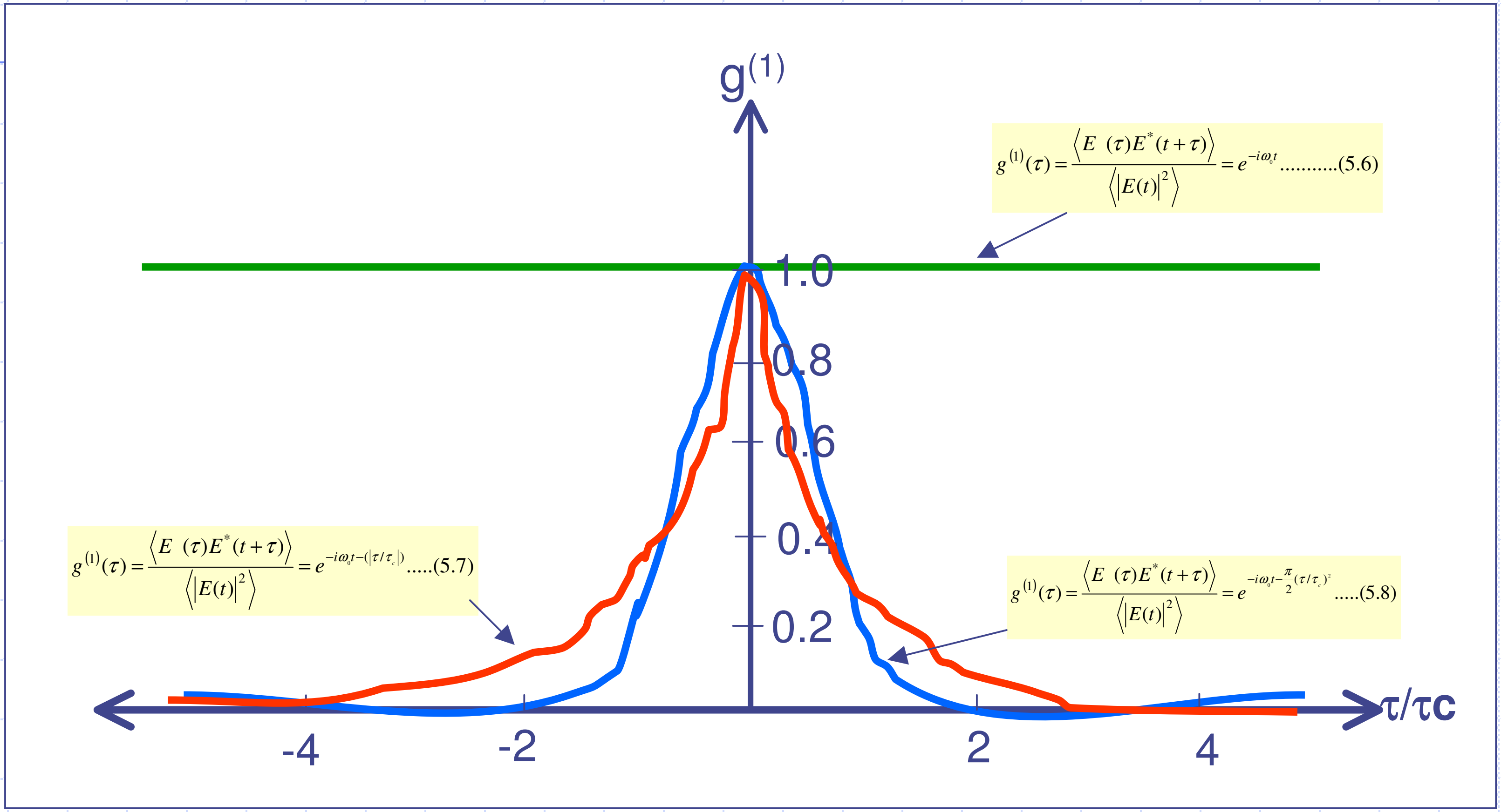
$$g^{(1)}(\tau) = \frac{\langle E\ (\tau)E^{*}(t+\tau)\rangle}{\langle |E(t)|^{2}\rangle} = e^{-i\omega_0 t} \text{..........}(5.6)$$

Collision broadened chaotic light (Lorentzian line shape)

$$g^{(1)}(\tau) = \frac{\langle E\ (\tau)E^{*}(t+\tau)\rangle}{\langle |E(t)|^{2}\rangle} = e^{-i\omega_0 t-(|\tau/\tau_c|)} \text{.....}(5.7)$$

Thermal broadened chaotic light (Doppler spectral line)

$$g^{(1)}(\tau) = \frac{\langle E\ (\tau)E^{*}(t+\tau)\rangle}{\langle |E(t)|^{2}\rangle} = e^{-i\omega_0 t-\frac{\pi}{2}(\tau/\tau_c)^2} \text{.....}(5.8)$$

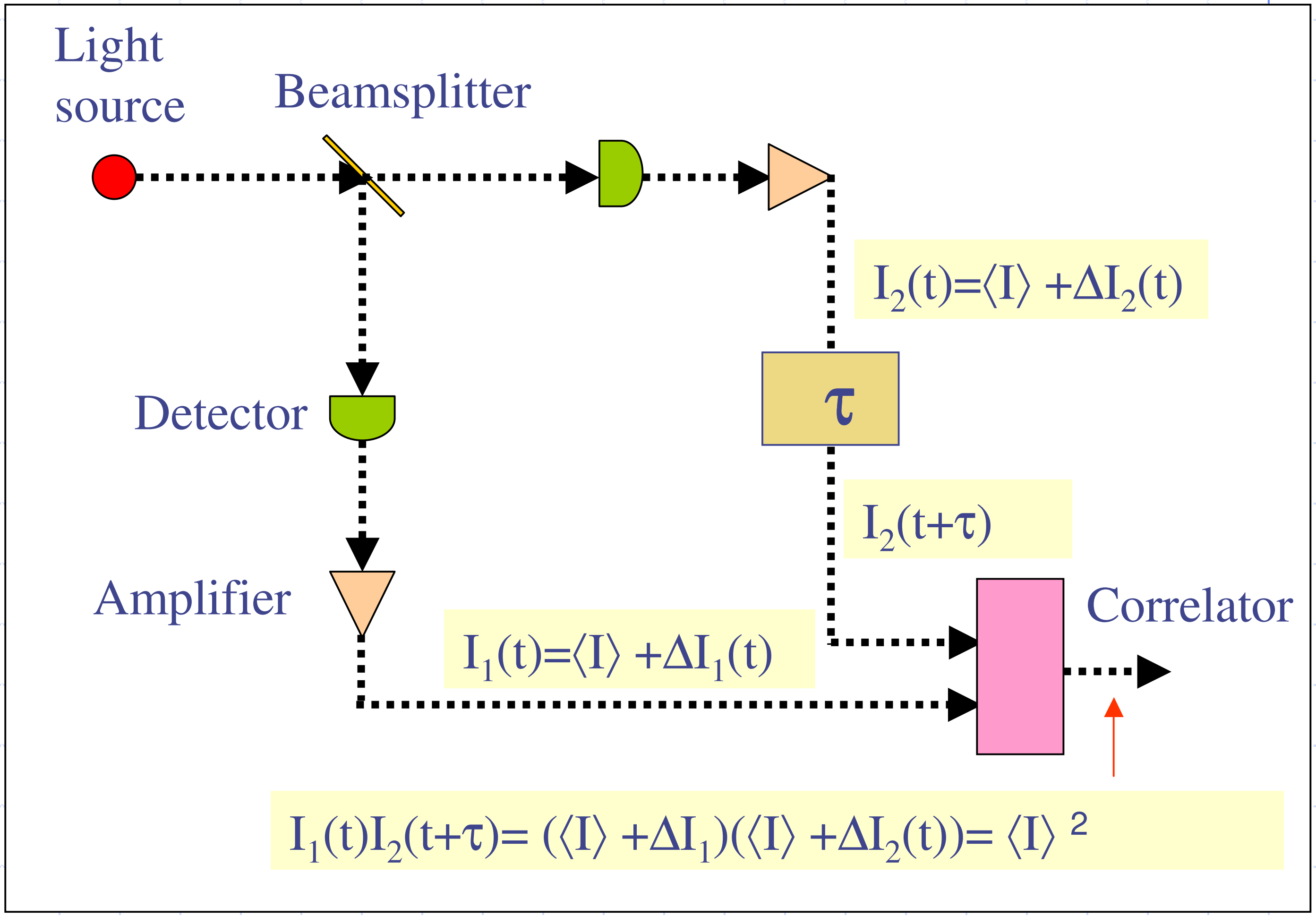


$g^{(1)}$
$g^{(1)}(\tau)=\frac{\langle E(\tau)E^{*}(t+\tau)\rangle}{\langle |E(t)|^{2}\rangle}=e^{-i\omega_0 t}$...........(5.6)
1.0
0.8
0.6
0.4
0.2
$g^{(1)}(\tau)=\frac{\langle E(\tau)E^{*}(t+\tau)\rangle}{\langle |E(t)|^{2}\rangle}=e^{-i\omega_0 t-(|\tau/\tau_c|)}$.....(5.7)
$g^{(1)}(\tau)=\frac{\langle E(\tau)E^{*}(t+\tau)\rangle}{\langle |E(t)|^{2}\rangle}=e^{-i\omega_0 t-\frac{\pi}{2}(\tau/\tau_c)^2}$.....(5.8)
-4
-2
2
4
τ/τc

$$G^{(2)}(r_1,t_1;r_2,t_2) = \left\langle E(r_1,t_1)E(r_2,t_2)E^*(r_1,t_1)E^*(r_2,t_2)\right\rangle ....(5.9)$$

$$G^{(2)}(\tau) = \left\langle I_1(t) I_2(t+\tau)\right\rangle ....(5.10)$$

$$g^{(2)}(\tau) = \frac{\left\langle I(t) I(t+\tau)\right\rangle}{\left\langle I(t)\right\rangle^2} ....(5.11) / (4.6)$$



In general for

*Monochromatic light (delta function line shape)

$$g^{(2)}(\tau)=1.........(5.12)\ /\ (4.9)$$

**Doppler spectral distribution light (Gaussian line shape)

$$g^{(2)}(\tau)=1+\exp\left[-\pi(\tau/\tau_c)^2\right]...........(5.13)/(4.10)$$

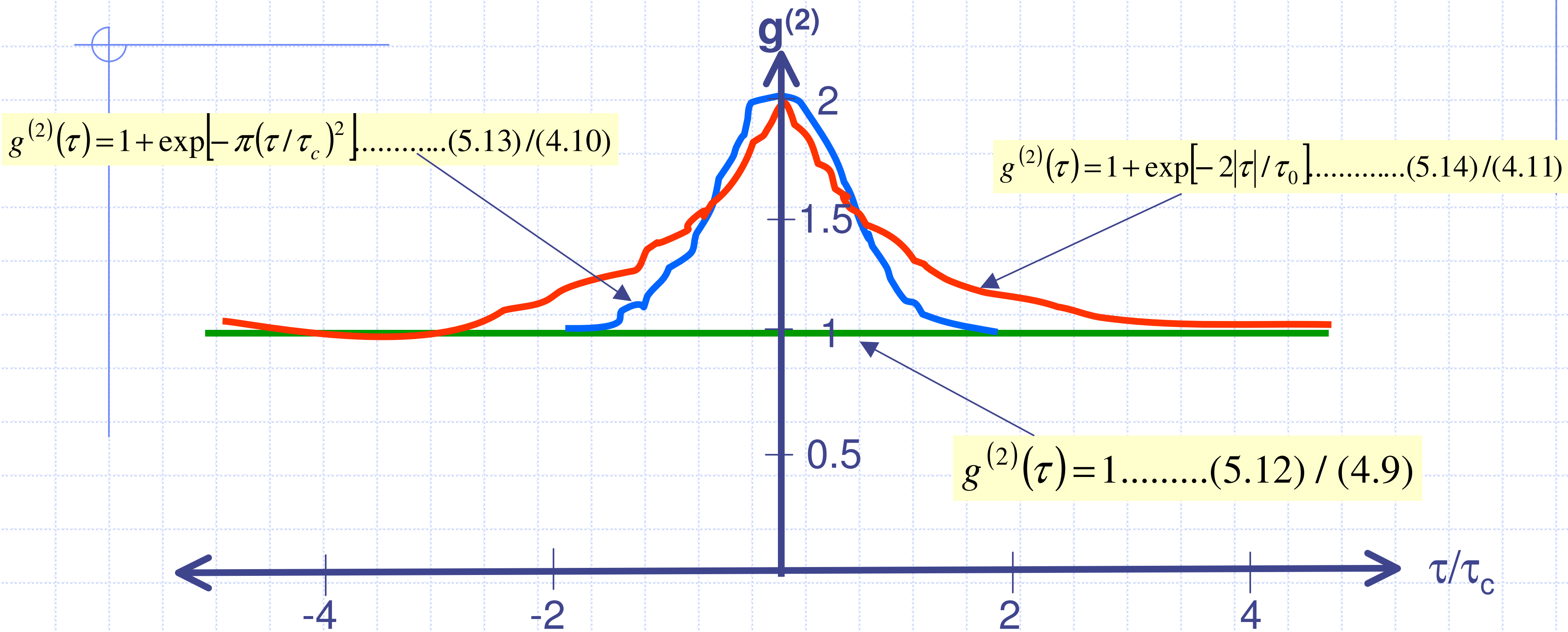
(Discharge lamp)

***Collision spectral distribution light(Lorentzian line shape)

$$g^{(2)}(\tau)=1+\exp\left[-2|\tau|/\tau_0\right]............(5.14)/(4.11)$$

In general for all chaotic light one may approximate this as

$$g^{(2)}(\tau)=1+\left|g^{1}(\tau)\right|^{2}........(5.15)$$


$g^{(2)}$
2
1.5
1
0.5
-4
-2
2
4
τ/τ_c
$g^{(2)}(\tau)=1+\exp\left[-\pi(\tau/\tau_c)^2\right]$............(5.13)/(4.10)
$g^{(2)}(\tau)=1+\exp\left[-2|\tau|/\tau_0\right]$............(5.14)/(4.11)
$g^{(2)}(\tau)=1$.........(5.12) / (4.9)

5-2-Quantum coherence functions

A light mode in a cavity (Spatial angular frequency , temporal angular frequency , polarization)= Quantized as a Harmonic oscillator + creation and annihilation operator to add or subtract photons

Quantum mechanically the varying electric field vector at point **r** now are becoming an operator. A linearly polarized field operator **E**(**r**,t) can be represented in terms of the positive and negative frequency parts as,

$$\hat{E}(\boldsymbol{r},t) = \hat{E}^{(+)}(\boldsymbol{r},t) + \hat{E}^{(-)}(\boldsymbol{r},t) \qquad (5.16)/(4.26)$$

where,

$$\hat{E}^{(-)}(\boldsymbol{r},t) = \sum E_0 \; u_k(r)\hat{a}^+ e^{i\omega_k t}$$

$$\hat{E}^{(+)}(\boldsymbol{r},t) = i\sum_{k,\vec{\varepsilon}} \frac{\hbar\omega_k}{2\varepsilon_0 L^3}\vec{\varepsilon}\hat{a}_k e^{i(k.r-\omega_k t)} = \sum_k E_{ok} u_{0k}(r)\hat{a}e^{-i\omega t}$$

$$\hat{E}^{-}(r,t) = \hat{E}^{+}(r,t)^*$$

== a^+ creation operator

== a annihilation operator

The qunatum mechanical first-– order degree of coherence at the position ***r*** is then ,

$$g^{(1)}(\ ,\tau)=\frac{\left\langle E^{(-)}(\ ,t)E^{(+)}(\ ,t+\tau)\right\rangle}{\sqrt{\left\langle E^{(-)}(\ ,t)E^{(+)}(\ ,t)\right\rangle\left\langle E^{(-)}(\ ,t+\tau)E^{(+)}(\ ,t+\tau)\right\rangle}}\text{...........}(5.17)/(4.39)$$

The qunatum mechanical second order degree of coherence at the position ***r*** is then ,

$$g^{(2)}(\ ,\tau)=\frac{\left\langle E^{(-)}(\ ,t)E^{(-)}(\ ,t+\tau)E^{(+)}(\ ,t+\tau)E^{(+)}(\ ,t)\right\rangle}{\left\langle E^{(-)}(\ ,t)E^{(+)}(\ ,t)E^{(-)}(\ ,t+\tau)E^{(+)}(\ ,t+\tau)\right\rangle}\text{............}(5.18)/(4.40)$$

Statistically stationary field ,

Note,: the ordering in the denominator
is not necessarily normal

For radiation field consisting of only a single mode.

Expanding E(+) and E(-) , with cancellations we get

$$g^{(1)}(\tau) = \frac{\langle a^{+} a(t+\tau) \rangle}{\langle a^{+} a \rangle} \text{...............}(5.19)/(4.41)$$

$$g^{(2)}(\tau) = \frac{\langle a^{+}(t) a^{+}(t+\tau) a(t+\tau) a(t) \rangle}{\langle a^{+} a \rangle^{2}} \text{.................}(5.20)/(4.42)$$

Any classical stationary random function of time must obey Schwartz inequality

$$\left|\left\langle I(\ ,t)I(\ ,t+\tau)\right\rangle\right|^2 \leq \left\langle I^2(\ ,t)\right\rangle\left\langle I^2(\ ,t+\tau)\right\rangle \text{..........}(5.21)$$

In quantum coherence theory , the corresponding inequality is obtained by replacing the product of intensities within the angle brackets by the corresponding normally ordered operators, i.e.,

$$\left|\left\langle :I(\ ,t)I(\ ,t+\tau):\right\rangle\right|^2 \leq \left\langle :I^2(\ ,t):\right\rangle\left\langle :I^2(\ ,t+\tau):\right\rangle \text{.......}(5.22)$$

where , : : represents normal ordering , i.e. the creation operators to the left and the annihilation operators to the right.

It follows from the definition of $g^{(2)}(\tau)$,

$$g^{(2)}(\tau) = \frac{\left\langle a^+(t)a^+(t+\tau)a(t+\tau)a(t)\right\rangle}{\left\langle a^+a\right\rangle^2} \text{.................}(5.23)$$

that for statistically stationary fields , this inequality can be recast in the following simple form,

$$g^{(2)}(\tau) \leq g^{(2)}(0)\ldots\ldots\ldots\ldots(5.24)$$

1. Obeyed by by thermal and coherent light
2. From the definition of $g^{(2)}(\tau)$, that it is a measure of the photon correlations between some time t and a latter time $t+\tau$
3. Satisfying the inequality
4. $g^{(2)}(\tau)< g^{(2)}(0)$ for $\tau< \tau_c$,
5. Photons exhibit excess correlations for times less than the correlation time τ_c

4. This leads to **photon bunching**

5. Meaning the photons tend to distribute themselves lly in bunches rather that at random ,

6. When such a light falls on a photodetector more photon pairs are detected close together than further apart

7. Thermal fields are examples of photon bunching.

In certain quantum optical systems , the inequality (eq.(5.24))may be violated with the result

$$g^{(2)}(0) = 1 - \frac{1}{n} \longrightarrow n \geq 2$$

$$g^{2}(0) = 0 \longrightarrow n < 2$$

$$g^{2} < 1$$

$$g^{(2)}(\tau) > g^{(2)}(0)......... \quad ...(5.25)$$

$$g^{(2)}(0) = \frac{\langle a^{+}aaa^{+}\rangle - \langle a^{+}a\rangle}{\langle a^{+}a\rangle^{2}}$$

$$-\mathrm{var}\,iance = \langle \hat{n}^{2}\rangle - \langle \hat{n}\rangle^{2}$$

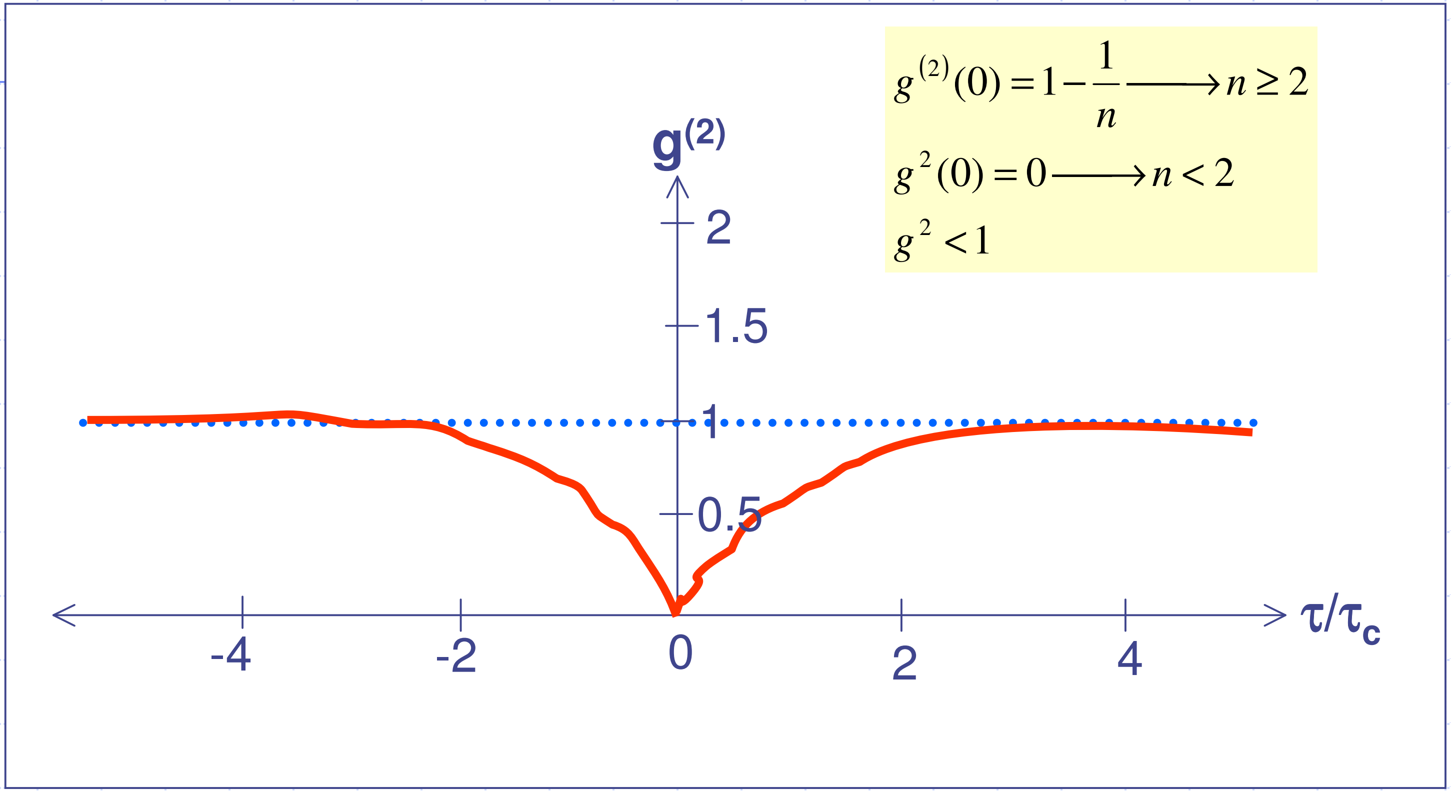
$$g^{2}(0) = 1 + \frac{\sigma^{2} - \langle \hat{n}\rangle}{\langle \hat{n}\rangle^{2}}$$

This will correspond to the phenomenon of photon anti-bunching

For coherence state the variance equalls the mean

For number state the varianvc is zero

This is the opposite effect , in which fewer photons pairs are detected close together than further apart



$g^{(2)}(0) = 1 - \frac{1}{n} \longrightarrow n \geq 2$
$g^{2}(0) = 0 \longrightarrow n < 2$
$g^{2} < 1$
$g^{(2)}$
2
1.5
1
0.5
-4
-2
0
2
4
τ/τ_c

5-3 -Photon bunching and anti – bunching

According to the second order correlation function, distribution , ,and fluctuation light is classified into three categories,

1. Bunched light
2. Random light
3. Anti – bunched light

Light Type	statistics	Behavior	Fluctuation	$g^{(2)}(0)$
Incoherent + coherent	**Super - Poissonian**	**bunched**	$> (\tilde{n})^{(1/2)}$	> 1
coherent	**Poissonian**	**random**	$(\tilde{n})^{(1/2)}$	1
Pure quantum state -	**Sub - Poissonian**	**anti - bunched**	$< (\tilde{n})^{(1/2)}$	< 1

5-3-1. Random light (Poissonian light)

In Poissonian light , the timing between photons is random because of the random nature of the Poissonian statistics

$g^{(2)}(\tau)$ must be the same for all values of τ

$g^{(2)}(\tau) = 1$

coherent state light is a manifestation of the randomness of the Poissonian photon statistics: Super positions of many number states with different n .

A Stabilized laser output consists of pulses of extremely regular amplitude.! Not absolutely true.

Coherent states contain a very large number of photons that their number fluctuations are negligible on a macroscopic scale

$$|\alpha\rangle = e^{\alpha a^+}|0\rangle e^{\frac{-|\alpha^2|}{2}} ....(2.30)$$

The mean number of photons

$$\bar{n} = \langle\alpha|\hat{n}|\alpha\rangle = \langle\alpha|\hat{a}^+\hat{a}|\alpha\rangle = |\alpha|^2 ....(2.33)$$

$$P(n) = |\langle n|\alpha\rangle|^2 = \langle n|\alpha\rangle\langle\alpha|n\rangle = \frac{|\alpha|^{2n}}{n!}e^{-|\alpha|^2}$$

$$= \frac{\langle n\rangle^n e^{-\langle n\rangle}}{n!} ....(2.34)$$

$$(\Delta n)^2 = \langle\hat{n}^2\rangle - \langle\hat{n}\rangle^2$$

$$\hat{n}^2 = \hat{a}^+\hat{a}\hat{a}^+\hat{a}$$

$$\hat{a}^+\left(\hat{a}^+\hat{a}+1\right)\hat{a} = \hat{a}^+\hat{a}^+\hat{a}\hat{a} + \hat{a}^+\hat{a}$$

$$\langle\alpha|\hat{n}^2|\alpha\rangle = |\alpha|^4 + |\alpha|^2$$

$$(\Delta n)^2 = |\alpha|^2 = \bar{n} ....(2.35)$$

5-3-2 Bunched light (super-Poissonian)

,

1. The photons come out in bunches rather than with random spacing between them
2. If a photon is detected at t=0 , there is a higher probability of detecting another photon at short times than at long times
3. $g^{(2)}(\tau)$is expected to be larger for small values of τ than for long values , so that $g^{(0)}(\tau) > g^{(\infty)}(\tau)$

5-3-3 Anti – bunched light

1. The photons come out with regular gaps between them rather than with a random spacing.

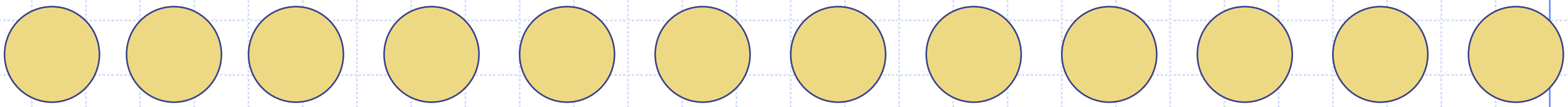
2. The photocount rate will therefore be more regular than for light source with Poissonian statistics

3. Smaller fluctuations in the count rate

4. The statistics are sub – Poissonian

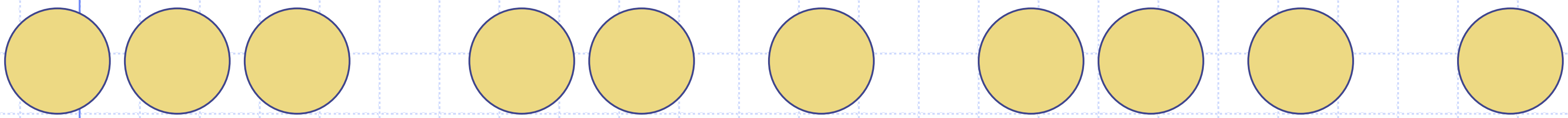
Number state |n>: A quantum state in which the number of phtons (n) is an integer.

Except for the vacuum number state |0> they r hard to get many techniques r suggested and some r implemented , next section .



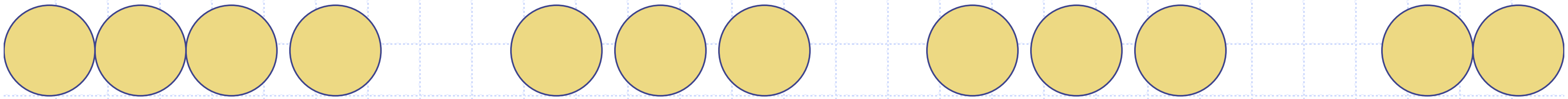
Number state

A state with definite number of photons , quiet light



Coherent state

Superposition of many number states with different numbers of photons for each number state



Thermal state

Incoherent mixture of many number states with different numbers of photons for each number state, Noisy light

First experiments for observation of anti-bunching:

1. **The first experiment was demonstrated in 1977 by Kimble et al by**
2. **Source:attenuated sodium atomic beam**
3. Flow : one atom presence in the excitation focus at any time

Excitation :(cw)tunable dye laser stabilized few percent in intensity and 1 MHz in frequency

Transition: ($3^2S_{1/2}$,F=2) to ($3^2S_{1/2}$,F=2)only allowed transition

Preparation :optical prepumping with circularly polarized light in a weak magnetic field

Sodium are prepared in the state ($3^2S_{1/2}$,F=2, m_F=2)

Allowed transition : only ($3^2P_{3/2}$,F=3, m_F=3)

The resonant optical field power density in which the atom experiences : 70 mW/cm^2 – corresponding to a Rabi

frequency, Ω = 2-2.5*A (A is the Einstein coefficient)

Light path : split in two , to two pm tubes

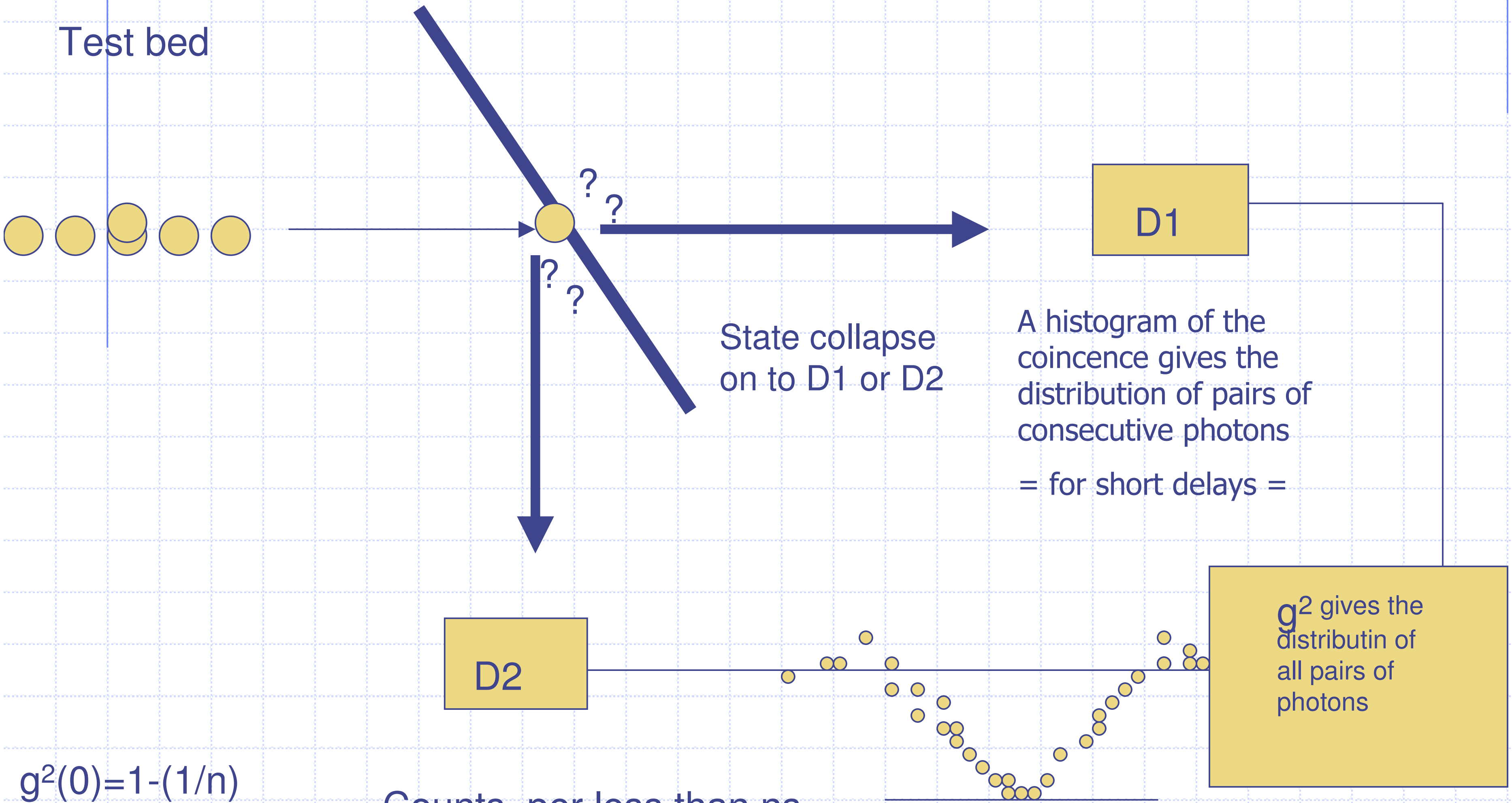
Time bin : 0.5 ns +Correlator.

5-4 Single photon sources(photon guns)

A source which delivers photons one by one ,i.e. one at a time.

Not possiple with one detector, limitations ,? Resolution ?better than flourescence lifetime = does not exist

Test bed



State collapse on to D1 or D2

A histogram of the coincence gives the distribution of pairs of consecutive photons

= for short delays =

g^2 gives the distributin of all pairs of photons

$g^2(0)=1-(1/n)$

Counts per less than ns time bin

0 anti-bunched

Single photon generation mechanisms

- Faint light pulses (next slide)
- Entangled photons (part 2 of this lecture)
- Single emitting species
- Quantum dot
- Colour centre (Nitrogen vacant)

macroscopic

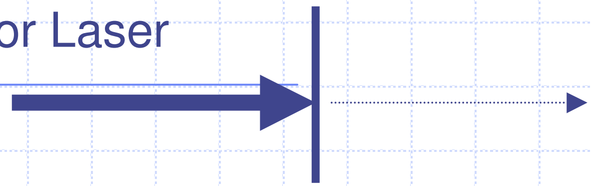
microscopic sources

In gas phase +

Molecules and quantun dots in condensed matter :
second photons via the first excited state , back relxation to this intermediate state is very fast = no emission = results behavoiour = two level system

Highly attenuated light(faint light pulses)

LED or Laser light



$$P(n) = \frac{\bar{n}^{n}}{n!} e^{-\bar{n}} \qquad n = 0,1,2,\ldots\ldots. \qquad \ldots\ldots(5.26)/(4.20)$$

Laser light = coherent state light –governed by Poisson statistics

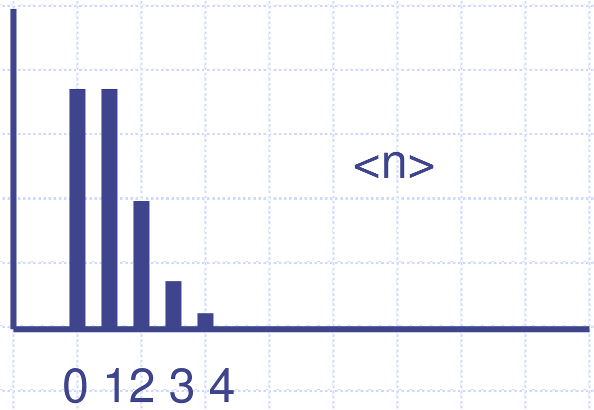
It is not a real single PHOTON source

Attenuating the laser pulse just lower the probability of having two or more photons at a time

Lower mean number of photons

Optimum at <n>=0.1 for qc

Most pulses r empty



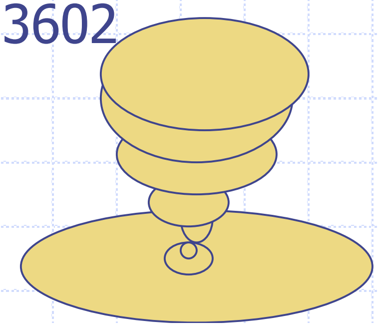
Microcavities

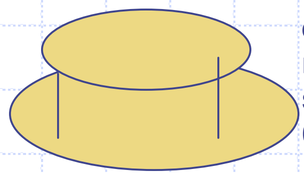
Vacuum decay rate

$$d_k^{ij} = -\frac{m_{ij}.\hat{E}_{0k}}{\hbar} ....(5.27)/(3.11)$$

$$\gamma_0 = \frac{1}{emissionlifetime} = \omega^3 m_{eg} / 3\pi\varepsilon_0 \hbar c^3$$

- Remember lecture three , spontaneous transition is a result of interaction of the emitting species and the em mode environment = both = system
- Playing with environment sp emission can be enhanced or suppressed
- Cavity Quantum electrodynamics (CQED)
- Couple the emitting species to high Q cavity
- Lecture 3 JC model – coupling parameter d=.. $qualityfactor = Q = \omega / \gamma_{cav}$
- Depending on $d/\gamma_{0,}\gamma_{cav.}$ = strong coupling and week coupling
- Recent results , Electrical pumping
- P.Michler ,et al Science 290, 22282 (2000)
- M.Pelton et al ., PRL 89,233602
- Microcavity in between two brag reflectors d 0.6 um h4.2 um





InAs in InGaAs quantum dots microdisc cavities structure 5um d-0.5 h

Single Molecule

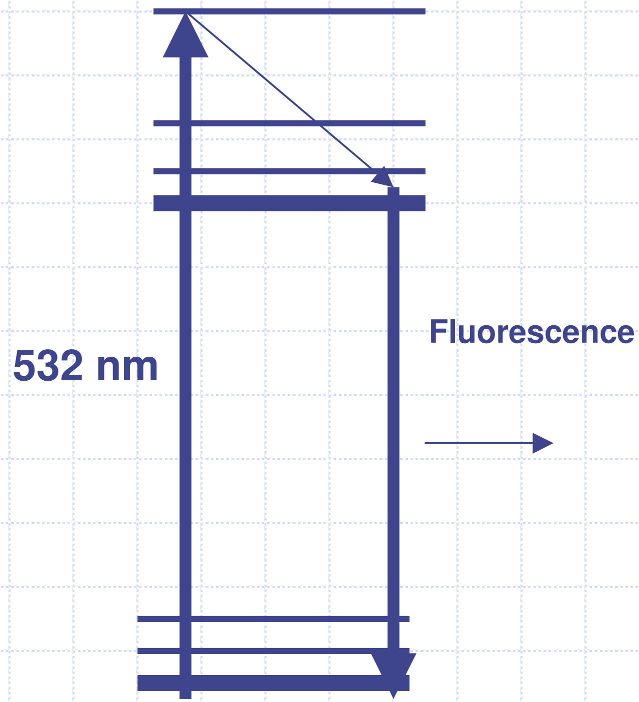
Emitting species = Molecules

Crystal flakes few microns thickness

10-11 of terrylene per mole of p–terphenyle

terrylene in the crystal 579nm with unity qy

Excitation of the single molecules -532 nm with 1.5 uW



Lonis, Moermer Nature , 407,491 (2000)

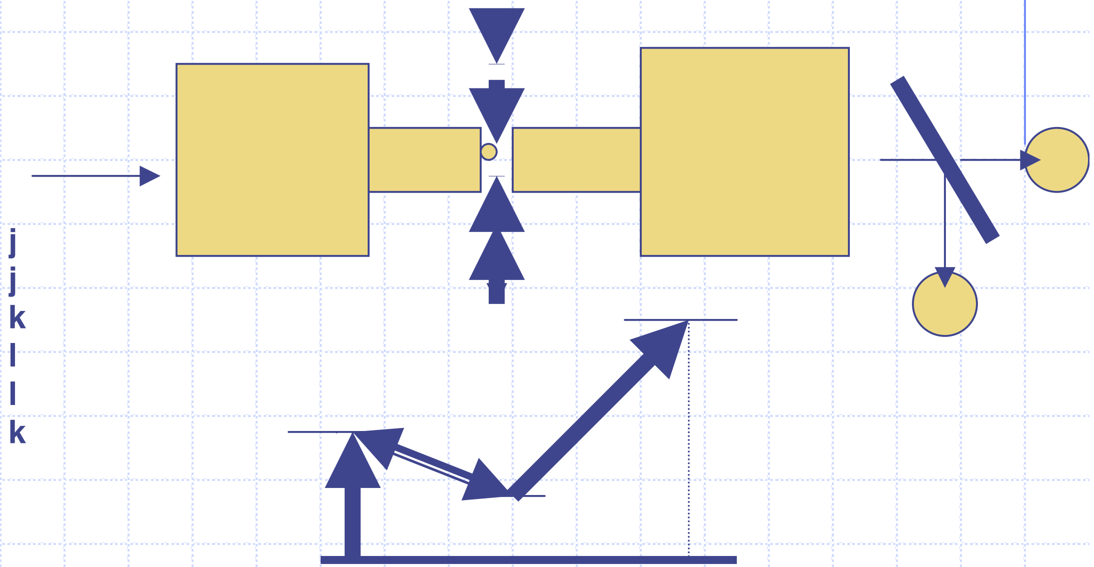
Single atom or ions

Single Cesium atom trapped within the mode of an optical cavity Quantum yield =1

Trapping and cooling are must to avoid Doppler and collision effects

1.4 x10 per each trapped atom

D2 line at 852.4 nm



Deterministic Generation of Single Photons from One Atom Trapped in a Cavity

J. McKeever, A. Boca, A. D. Boozer, R. Miller, J. R. Buck, A. Kuzmich, H. J. Kimble*

Science 303, 1992 (2004)

Deterministic Generation of Single Photons from One Atom Trapped in a Cavity

J. McKeever, A. Boca, A. D. Boozer, R. Miller, J. R. Buck, A. Kuzmich, H. J. Kimble*

Nitrogen vacant

Colour centre

room and low temperature operation

Host crystal =diamond

Easy to produce

C. Kurtstiefer et al .

PRL 85 290 (2000)

The ground and the optically excited states r triplet

Broad band

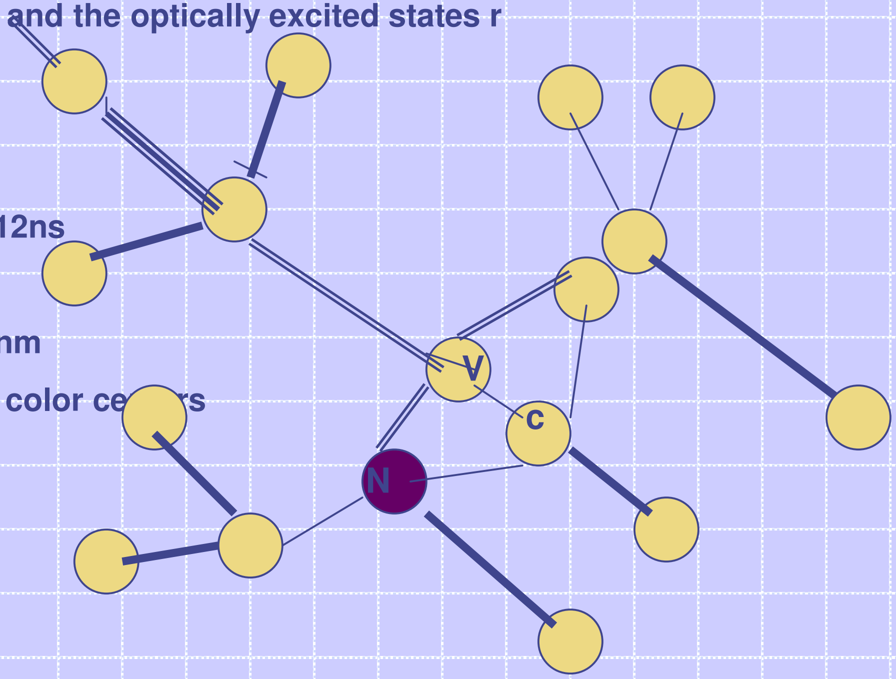
640-720 nm

Sp lifetime 12ns

Qy =1

Size 50-100nm

One or few color ce rs



Possible Applications

- Measurement of weak absorption
- Quantum optics predictions testing (locality and reality issues)
- Random number generation
- Quantum information :

  quantum cryptography, quantum computers and teleportation

Part Two

Entanglement

5-6 Definition of entangled states

Two systems

Two Hilbert spaces

$$|a_0 b_1\rangle = |a_0\rangle \otimes |b_1\rangle \longrightarrow H_a \otimes H_b = H_c$$

$$= |1\rangle \otimes |0\rangle = |10\rangle$$

Generalization for two orthonormal bases

$$\sum_i c_i |\phi_i\rangle \otimes \sum_j k_j |\vartheta_i\rangle$$ Separable states

For certain cases the complex coefficients may have the form as

$$\sum_{i,j} c_{ij} |\phi_i\rangle \otimes |\vartheta_j\rangle$$

Not-Separable states

Entangled states

Example

Two electrons (electron 1 and electron 2) composite system each has its own two dimension Hilbert space regarding its spin states basis vectors +1/2 and –1/2 , where the state vector of the entire system may be written as

$$|\psi\rangle_{1,2} \neq |\psi\rangle_1 \otimes |\psi\rangle_2$$

$$|\Psi_{1,2}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\rangle \otimes |\downarrow_2\rangle - |\downarrow_1,\rangle \otimes |\uparrow_2\rangle\right) \qquad (5.28)$$

It is not possible to assign any spin value to either of the two electrons y

5-6-1 Entangled photons generation

Polarization entangled photon pair – Two directions

SPDC = spontaneous parametric down conversion

Bell states

$$|\psi\pm\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|V\rangle_2 \pm |V\rangle_1|H\rangle_2\right) = \frac{1}{\sqrt{2}}\left(|HV\rangle \pm |VH\rangle\right)$$

$$|\phi\pm\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|H\rangle_2 \pm |V\rangle_1|V\rangle_2\right) = \frac{1}{\sqrt{2}}\left(|HH\rangle \pm |VV\rangle\right) \quad \text{........(5.29)}$$

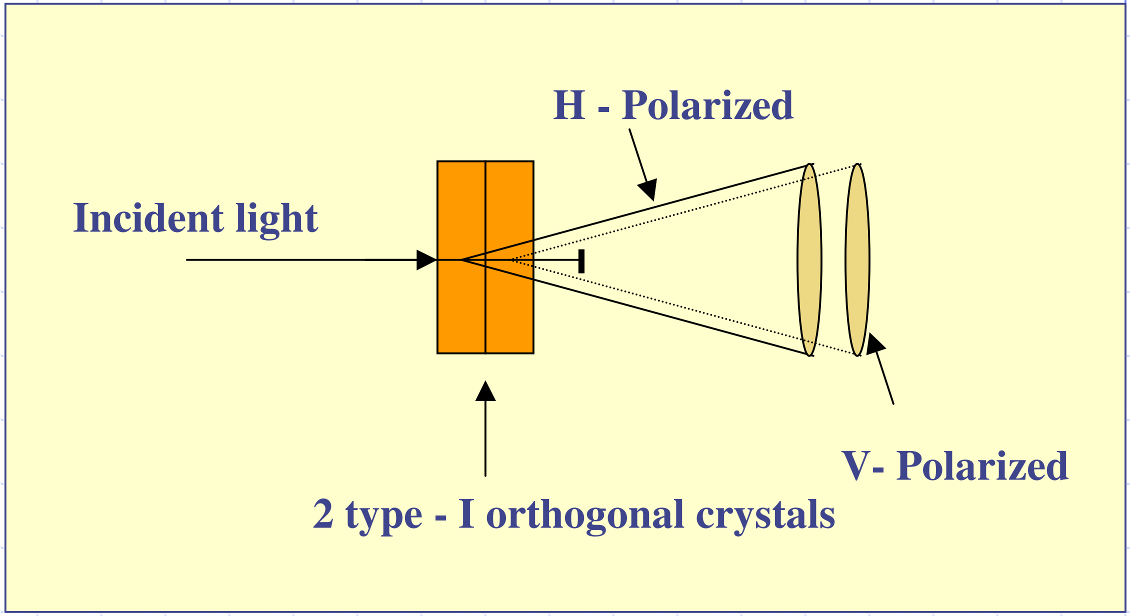
$$\omega = \omega_1 + \omega_2$$

$$k = k_1 + k_2$$

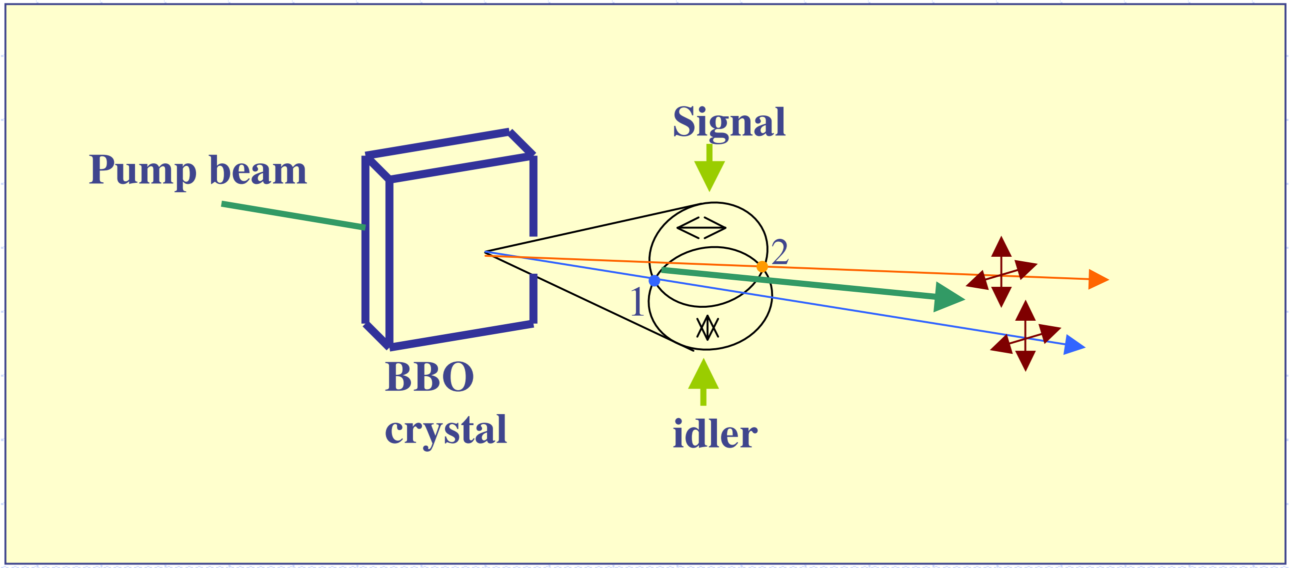
Non linear crystal:

KDP,BBO,LBO,$LiNbO_3$

Type –I Parametric down conversion



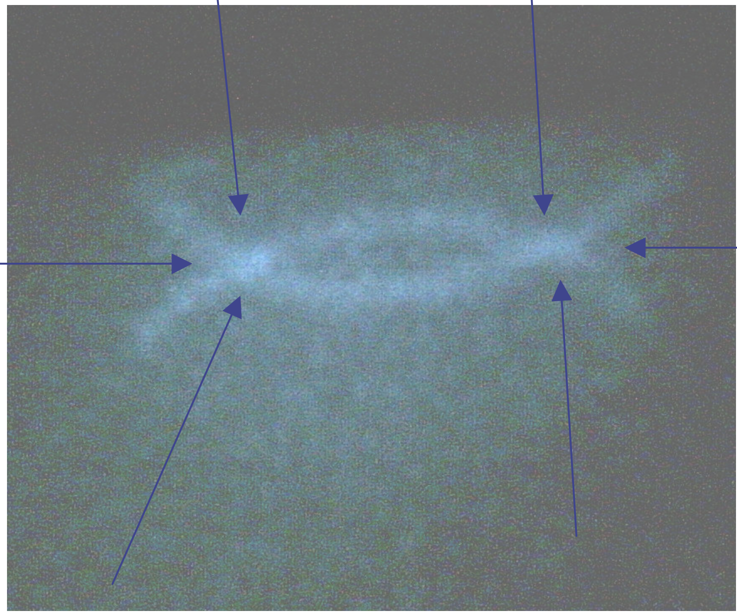
Type –II Parametric down conversion



$\beta - BariumBorate(BBo)$

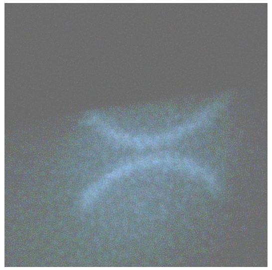
Spontaneous parametric down conversion

1. BBO crystal : Double – band anti - reflection coating at 810-nm and 405 nm for minimizing the external reflection of the pump beam at the air – crystal surface + increasing no. of photons down converted
2. BBO crystal size = 4 mm (cube) , compensation crystals size = 2 mm
3. Pumping source : CW violett laser diode with λ=405.2 nm, output power = 18 Mw
4. The pump beam is focused to a waist W_0=40µm
5. The opening angle of the intersection points of the signal and idler cones = 6^0
6. the corresponding angles of the crystal optical axes are , $\theta=42.6^0$ and $\phi=30^0$



Type II

Type I



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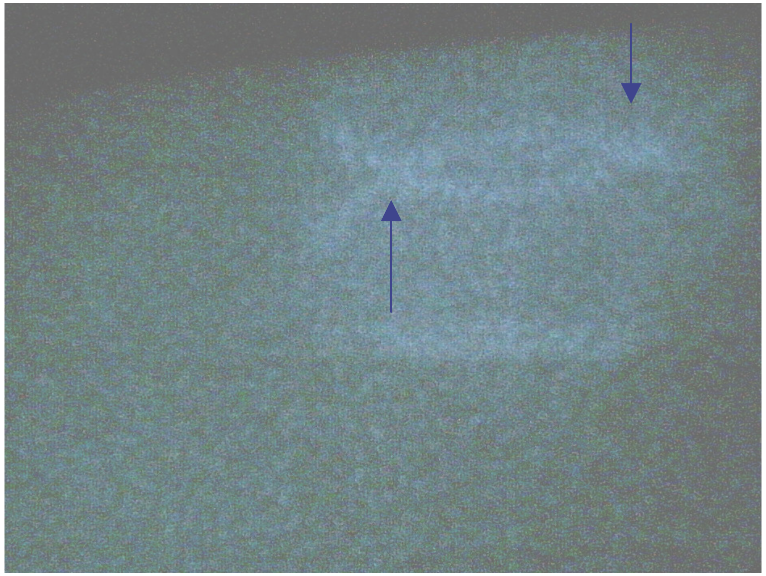

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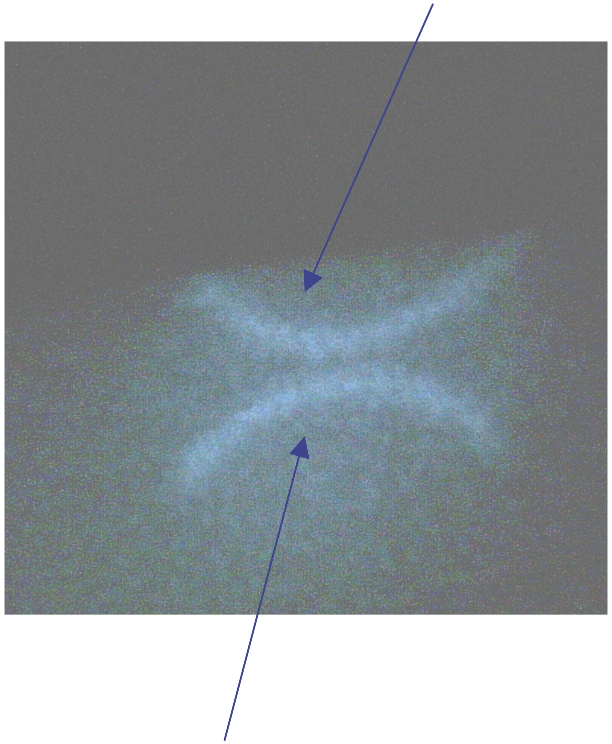


Type II

Type I

5-7. **A problem in hand :**

Say the possible outcomes of measurements |i>

$$|\psi\rangle = \sum_i c_i |i\rangle.......(5.30)$$

and

$$c_i = \langle i | \psi \rangle.......... ..(5.31)$$

$$P_i = |c_i|^2.......... .....(5.32)$$

Example a photon with two possible spin values –h and +h(vertical and horizontal polarization); an electron which may have two spin states spin up or spin down , atomic system with two or more levels ,

A measurement will force the system to collapse into one of the states .

The problem: what is the mechanism that cause the choice of that particular state from all possible states

Solvay 1927

Does the measurement process pushes the particle to collapse in one of the possible states (properties) of the particle or the state (property)of the particle is already prescribed before the measurement as that goes with our notion of reality and locality.

Einstein-Podolsky –Rosen - EPR Paradox:1935

Based on the concept of entanglement

Quantum mechanics is an incomplete theory because it violates the notion of reality and locality.

How?

The thought experiment they suggested predicts unacceptable results if quantum mechanics rules r followed

Almost 15 Years later , David Bohm suggested a thought experiment which could be realized 1951

Experiment:

Source: Two spin ½ particles (Diatomic molecule, Hg_2 molecule) with 0 net angular momentum

Photo-dissociation : Taking the z axis as reference we have two Hg atoms with spin, ½ $\uparrow$ and $-1/2 \downarrow$

State vector :

$$|\Psi_{1,2}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1, \downarrow_2\rangle - |\downarrow_1, \uparrow_2\rangle\right) \text{..........} (5.33)$$

Measurement :pass atom 1 through SGA oriented in the z direction

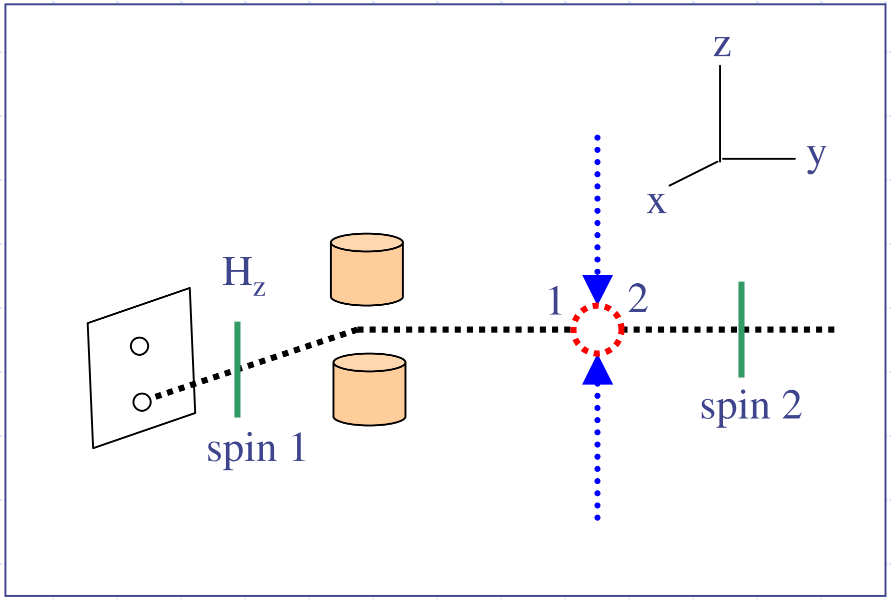
Results: deflection to either z or $-z$, let we have a deflection to +z i.e.

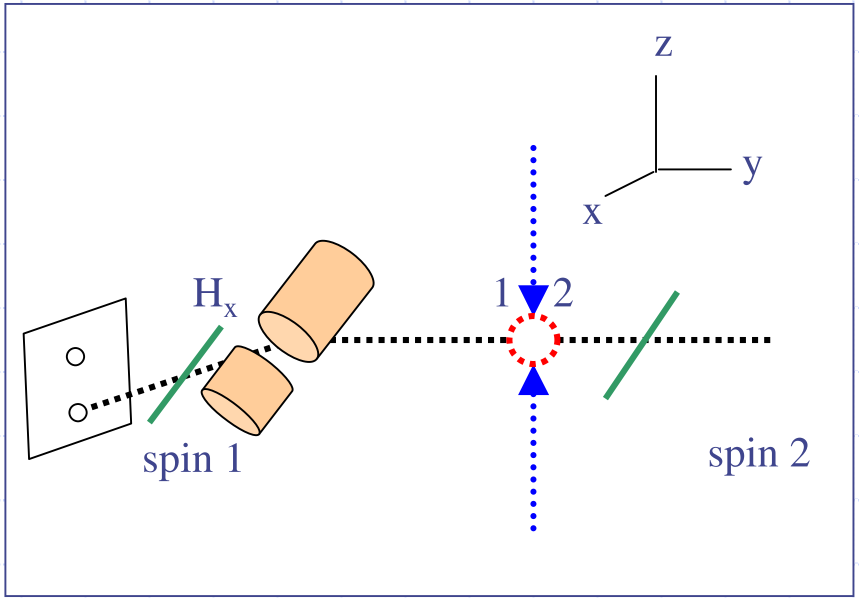
spin $\downarrow$

Second atom: sure will have spin $\uparrow$

Second atom again: pass it through SGA oriented in the x direction it will have a definite spin, either x or $-x$,

Gedanken experiment





The problem ?: we know for sure the values of z and x components of the spin of particle 2 (the second atom) = violation =A quantum state can not have definite values for two variable (incompatible variable – Heisenberg uncertainty principle, Non –commuting quantum observables))

There is an incompatibility between local realistic theories with Quantum Mechanics

How?

Let at time before the measurement process the state vector be

$$\left|\psi_{1,2}^{<}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow_1, \downarrow_2\right\rangle - \left|\downarrow_1, \uparrow_2\right\rangle\right)\text{..........}(5.34)$$

Now let atom 1, goes through SGA setup Z

Possible deflection : say $\downarrow$

Atom 2 state for sure will collapse into $\left|\psi_{2}^{>}\right\rangle = \left|\uparrow_2\right\rangle\text{..........}(5.35)$

According to EBR; since no interaction , and no effect in measuring atom 1 on atom 2 then the following must obeyed

<before measurement

>after measurement

$$\left|\psi_{2}^{<}\right\rangle = \left|\psi_{2}^{>}\right\rangle = \left|\uparrow_2\right\rangle\text{..........}(5.36)$$

Using SGA setup X

$$\left|\pm x\right\rangle \equiv \left|\pm\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle \pm \left|\downarrow\right\rangle\right)...............(5.37)$$

Total state vector

$$\left|\phi_{1,2}^{<}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|+_1 -_2\right\rangle - \left|-_1 +_2\right\rangle\right)..........(5.38)$$

Repeat same sequence

Atom one spin measurement :collapse into state

Second atom spin state : collapse into

$$\left|\phi_2^{>}\right\rangle = \left|+_2\right\rangle...........(5.39)$$

But according to the same argument led to eq.5.36 we must have

$$\left|\phi_2^{<}\right\rangle = \left|+_2\right\rangle............(5.40)$$

$$\left|\psi_2^<\right\rangle = \left|\psi_2^>\right\rangle = \left|\uparrow_2\right\rangle \text{..........(5.36)}$$

$$\left|\phi_2^>\right\rangle = \left|+_2\right\rangle \text{...........(5.39)}$$

$$\left|\phi_2^<\right\rangle = \left|+_2\right\rangle \text{............(5.40)}$$

Meaning

Possible to assign two different state vectors ↑ and ↓

to the same reality

unrealism and indeterminism

EPR reality criterion
If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity.

The core of their objection was quantum entanglement .

Their argument centered around the fact that quantum mechanics violated either the principle of **non – locality** or the principle of **reality.**

They argued that , as a result , quantum mechanics must be incomplete , and that quantum entanglement could be explained by missing hidden variables

5-8 Bell inequality

15 years later, J.S. Bell used Bohm thought experiment to set up and present an inequality which must be true for the acceptable in the normal sense of reality and locality .

He showed that no physical theory which is realistic and also local in a specified sense can agree with all of the statistical implications of Quantum Mechanics

Bell inequalities indicates that a physical criterion is possible found to distinguish the two theories .

Bell developed inequalities which , if violated , would clearly prove that quantum mechanics is correct and the hidden variable are not.

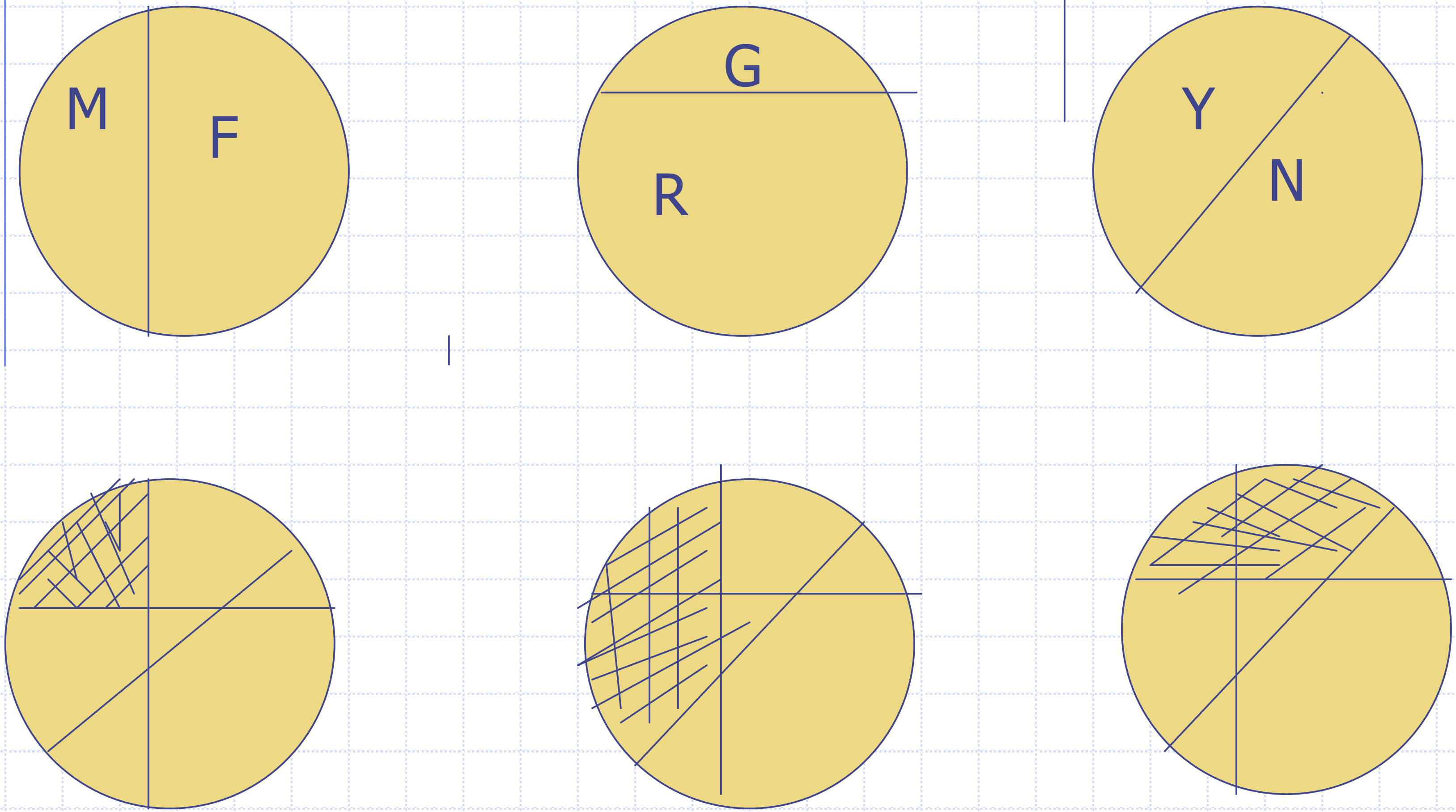
Simple version

Consider students in a classroom

M males and F females

G with green shirts and R with red shirts

Y of them have bicycle and N do not have i



$$N(M,G) \leq N(M,Y) + N(G,Y)$$

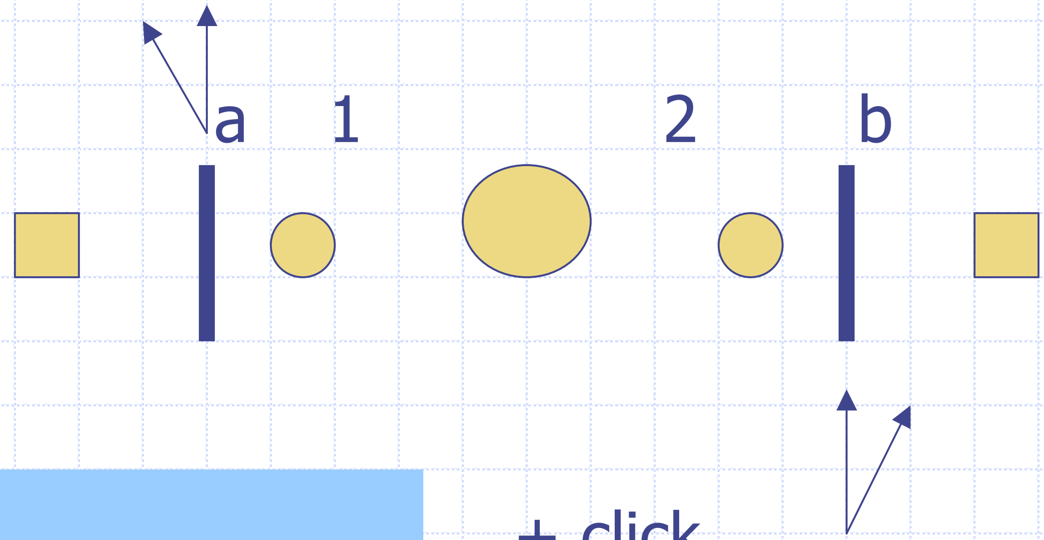
Less simple version

Similar as presented in Scully and Zubairy text book ,Quantum opticsonsidered

Say we have two particles , they are a product of a zero spin system splitting , they have anti-correlated values of spin projection along any given axis.

Consider the setup shown

Stern-Gerlach System(SGA)

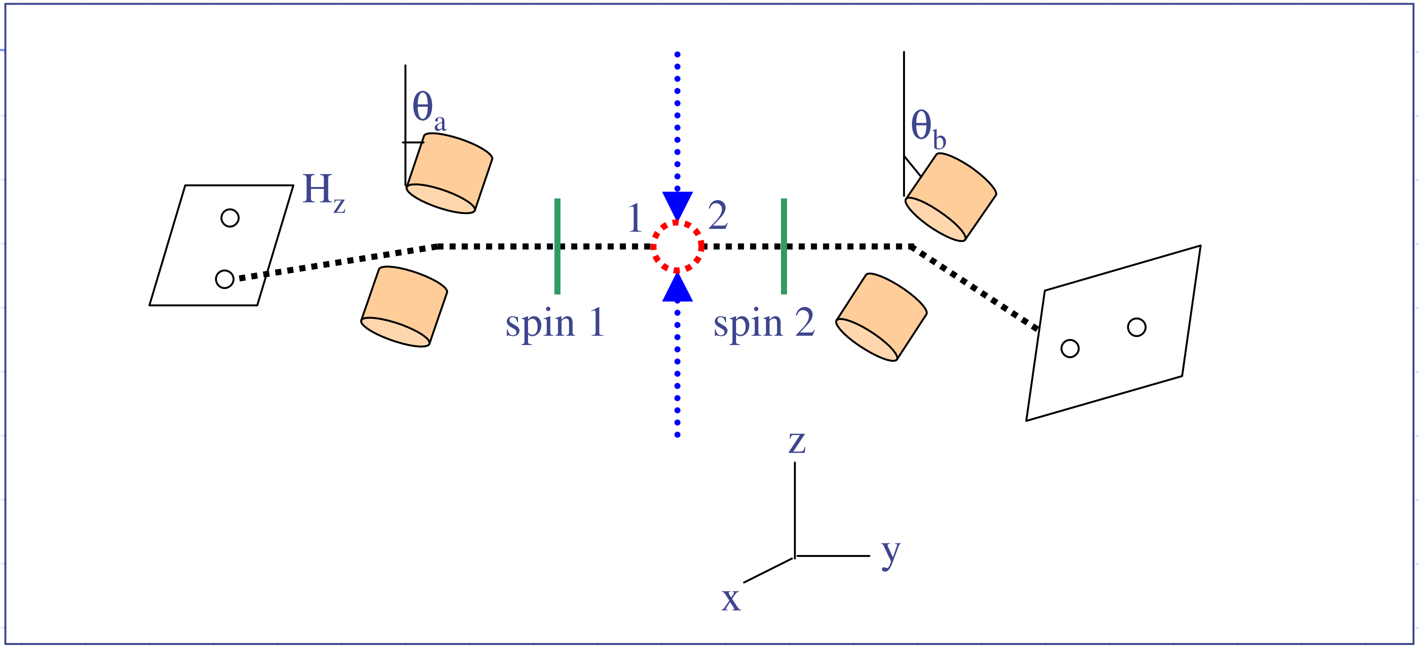


$$\begin{array}{c} \\ P_{ab} = P \end{array}\begin{array}{c} particle\ 1 \quad particle\ 2 \\ \left(+ - O \;\middle|\; - + O \right) \\ a\ b\ c \quad\quad a\ b\ c \end{array} \dots\dots\dots\dots (5.41)$$

+ click

- no click

-O



θ_a
θ_b
H_z
1
2
spin 1
spin 2
z
y
x

$$P_{bc} = P\left(\mathrm{O} + - \middle| \mathrm{O} - +\right) \ldots\ldots\ldots\ldots (5.42)$$

$$P_{ac} = P\left(+ \mathrm{O} - \middle| - \mathrm{O} +\right) \ldots\ldots\ldots\ldots (5.43)$$

$$
\begin{aligned}
P_{ab} &= P(+ - \mathrm{O} \mid - + \mathrm{O}) \\
&= P(+ - + \mid - + -) + P(+ - - \mid - + +) \text{..........} (5.44)
\end{aligned}
$$

$$
\begin{aligned}
P_{bc} &= P(\mathrm{O} + - \mid \mathrm{O} - +) \\
&= P(+ + - \mid - - +) + P(- + - \mid + - +) \text{..........} (5.45)
\end{aligned}
$$

$$
\begin{aligned}
P_{ac} &= P(+ \mathrm{O} - \mid - \mathrm{O} +) \\
&= P(+ + - \mid - - +) + P(+ - - \mid - + +) \text{..........} (5.46)
\end{aligned}
$$

$$P_{ab} + P_{bc} = P(+-+|-+-) + P(+--|-++) + P(++-|--+) + P(-+-|+-+) \ldots\ldots\ldots\ldots (5.47)$$

$$P_{ab} + P_{bc} = P_{ac} + P(+-+|-+-) + P(-+-|+-+) \ldots\ldots\ldots\ldots\ldots (5.48)$$

$$P_{ab} + P_{bc} \geq P_{ac} \ldots\ldots\ldots\ldots (5.49)$$

5-9 Quantum Bell inequality

The coefficients of the basis vector can be calculated state vector along that particular basis vector , inner product obey projection the basis vector

$$prpability\ amplitud = \langle \theta | \Psi \rangle$$

The probability that the state is into the angle which allows the electron to pass SGA oriented with θ

$$P_{\Psi}(\theta) = \left| \langle \theta | \Psi \rangle \right|^2 \text{..........}(5.50)$$

This state is obtained by rotating the spin state $|\uparrow\rangle$ by an angle θ

$$|\theta\rangle = e^{-i\theta\sigma_y/2} |\uparrow\rangle \text{............}(5.51)$$

Where Pauli matrices are given as

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{............}(5.52)$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{............}(5.53)$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{..........}(5.54)$$

The bra and ket vectors may be represented in matrix form as

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{.........}(5.56)$$

$$|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{.........}(5.57)$$

Using the projection operator

$$P_{\Psi}(\theta) = \langle \Psi | \theta \rangle \langle \theta | \Psi \rangle \text{.........}(5.58)$$

$$\rho_{\theta} = |\theta\rangle\langle\theta| \text{.........}(5.59)$$

$$\rho_{\theta} = e^{-i\theta\sigma_{y}/2} |\uparrow\rangle\langle\uparrow| e^{i\theta\sigma_{y}/2} \text{..........}(5.60)$$

Using the relation

$$e^{-i\theta\sigma_y/2}\left|\uparrow\right\rangle = \cos\frac{\theta}{2}\left|\uparrow\right\rangle + \sin\frac{\theta}{2}\left|\downarrow\right\rangle\text{........(5.61)}$$

We get

$$\rho_\theta = \frac{1}{2}\left(1+\sigma_z\cos\theta+\sigma_x\sin\theta\right)\text{...........(5.62)}$$

Thus for ab

$$P_{ab} = \left\langle\Psi_{1,2}\left|\rho_{\theta_a}^{(1)}\rho_{\theta_b}^{(2)}\right|\Psi_{1,2}\right\rangle\text{..........(5.63)}$$

$$P_{ab} = \frac{1}{4}\left[1-\cos\left(\theta_a-\theta_b\right)\right] = \frac{1}{2}\sin^2\left(\frac{\theta_a-\theta_b}{2}\right)\text{.........(5.64)}$$

Similarly we can derive for bc and ac combinations similar expressions

$$P_{ab} = \frac{1}{2}\sin^2\left(\frac{\theta_a-\theta_b}{2}\right)\text{...........(5.65)}$$

Checking whether bell inequality is satisfied

We write down

$$\frac{1}{2}\sin^2\left(\frac{\theta_a-\theta_b}{2}\right)+\frac{1}{2}\sin^2\left(\frac{\theta_b-\theta_c}{2}\right)\geq\frac{1}{2}\sin^2\left(\frac{\theta_a-\theta_c}{2}\right)\text{..........}(5.66)$$

consider the angles $\theta_a = 0, \theta_b = \pi/4$ and $\theta_c = \pi/2$

$$2\sin^2\frac{\pi}{8}\geq\sin^2\frac{\pi}{4}\text{............}(5.67)$$

$$or \quad 0.15\geq 0.25\text{.........}(5.68)$$

Quantum mechanics violates bell inequality based on reality and locality

5-10. Violation of Bell"s inequality

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- A. Aspect et al., *Experimental Tests of Realistic Local Theories via Bell's Theorem*, Phys. Rev. Lett. **47**, 460 (1981)
- **Tittel, 1997**: W. Tittel et al., *Experimental demonstration of quantum-correlation over more than 10 kilometers*, Phys. Rev. A, **57**, 3229 (1997)
- **Weihs, 1998**: G. Weihs, et al., *Violation of Bell's inequality under strict Einstein locality conditions*, Phys. Rev. Lett. **81**, 5039 (1998)
- **Kwiat, 1999**: P.G. Kwiat, et al., *Ultrabright source of polarization-entangled photons*, Physical Review A **60** (2), R773-R776 (1999)
- **Rowe, 2001**: M. Rowe et al., *Experimental violation of a Bell's inequality with efficient detection*, Nature **409**, 791 (2001)
- **García-Patrón, 2004**: R. García-Patrón, J. Fiurácek, N. J. Cerf, J. Wenger, R. Tualle Brouri, and Ph. Grangier, *Proposal for a Loophole-Free Bell Test Using Homodyne Detection*, Phys. Rev. Lett. **93**, 130409 (2004)

First experiment: about 15 years after Bell’’s paper on his inequality o

Aspect et al 1982:

Calcium atom beam

Cascade transition

4p2   1S0- 4s4p 1P1- 4s2 1S0

Two polarization correlated photons

551nm and 423nm

Excitation through two photons from 4s2 1S0 to 4p2 !S0

Rate :5*10 to power 7