PH3203 Term Paper

Squeezed States of Light

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Hello this is an intro, fuck you ohs shit thus shit is slow as fuck

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Hello this is a literature review

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We have already seen that we quantize our electric field as,

$$\boldsymbol{E}(r,t) = \boldsymbol{E}^{+}(r,t) + \boldsymbol{E}^{-}(r,t) \tag{1}$$

where $\boldsymbol{E}^{+}(r,t)$ is the positive frequency part and $\boldsymbol{E}^{-}(r,t)$ is the negative frequency part. We will define some semblance of what coherence is in Quantum Optics. To do this we define the first order correlation function as

$$G^{(1)}(r_1, t_1; r_2, t_2) = \langle \boldsymbol{E}^{-}(r_1, t_1)\boldsymbol{E}^{+}(r_2, t_2)\rangle = \mathrm{Tr}[\rho \boldsymbol{E}^{-}(r_1, t_1)\boldsymbol{E}^{+}(r_2, t_2)] \tag{2}$$

In general we can define the nth order correlation functions. In order to write notation compactly, let us write $\left(\boldsymbol{r_j}, t_j\right) = x_j$. So our nth order correlation function is given to be

$$G^{(n)}(x_1, x_2, ..., x_n, x_{n+1}, ..., x_{2n}) = \langle \boldsymbol{E}^{-}(x_1)...\boldsymbol{E}^{-}(x_n)\boldsymbol{E}^{+}(x_{n+1})...\boldsymbol{E}^{+}(x_{2n})\rangle \tag{3}$$

We use a certain normalization convention for the 1st order correlation function. We define the normalized correlation function as

$$g^{(1)}(x_1, x_2) = \frac{G^{(1)}(x_1, x_2)}{\sqrt{G^{(1)}(x_1, x_1)G^{(1)}(x_2, x_2)}} \tag{4}$$

Generalizing this, we can define the normalized nth order normalized correlation function as

$$g^{(n)}(x_1, x_2, ..., x_n, x_{n+1}, ..., x_{2n}) = \frac{G^{(n)}(x_1, x_2, ..., x_n, x_{n+1}, ..., x_{2n})}{\prod_{i=1}^{2n} G^{(1)}(x_i, x_i)} \tag{5}$$

Using some properties of the trace, we can show that, specifically that $\mathrm{tr}\left[\rho A^{\dagger} A\right]$ is positive definite, $|g^{1}(x_1, x_2)| \leq 1$. Note that this constraint is not for $n \geq 1$, but since we are generalizing from the first order correlation, we still call it a normalized correlation function.

The main point of this article to talk about squeezed states of light. To talk about these states, we need to talk about the second order normalized correlation function. The second order normalized correlation function is defined as

$$g^{(2)}(x_1, x_2, x_3, x_4) = \frac{G^{(2)}(x_1, x_2, x_3, x_4)}{\sqrt{G^{(1)}(x_1, x_1)G^{(1)}(x_2, x_2)G^{(1)}(x_3, x_3)G^{(1)}(x_4, x_4)}} \tag{6}$$

We need to use some properties of the correlation functions.

1. Permutation of the first half $(x_1, ..., x_n)$ and the second half $(x_{n+1}, ..., x_{2n})$ individually, of the correlation function does not change the value of the correlation function. This is because when we quantize the electric field, we end up with a bunch of decoupled harmonic oscillators, so for any two oscillators the commutation relation is $\left[a_i, a_j^\dagger\right] = \delta_{ij}$, so the correlation function is invariant under permutation of the first half and the second half.
2. **pls elaborate on this pt** If the field is nth order coherent it must satisfy the following condition $g^{(j)}\left(x_1, x_2, ..., x_j, x_j, ..., x_1\right) = 1 \quad \forall j \leq n$. Classically we only use first order coherence to mean coherence. If the field is nth order coherent, then we get

$$G^{(j)}\left(x_1, x_2, ..., x_j, x_j, ..., x_1\right) = \prod_{i=1}^{j} G^{(1)}(x_i, x_i) \quad \forall j \leq n \tag{7}$$

Physically this means

5 Squeezed states

To check for squeezed states we are interested in the second order correlation function. The second order correlation function, with parameters x_1, x_2 is given by

$$\begin{aligned} g^2(x_1, x_2) &= \frac{G^{(2)}(x_1, x_2, x_1, x_2)}{G^{(1)}(x_1, x_1)G^{(1)}(x_2, x_2)} \\ &= \frac{\langle \boldsymbol{E}^-(x_1)\boldsymbol{E}^-(x_2)\boldsymbol{E}^+(x_1)\boldsymbol{E}^+(x_2)\rangle}{G^{(1)}(x_1, x_1)G^{(1)}(x_2, x_2)} \end{aligned} \tag{8}$$

We note that $\boldsymbol{E}^+$ is an anhilation operator which reduces photon number and $\boldsymbol{E}^-$ is a creation operator which increases photon number. So we can write $N_E = \boldsymbol{E}^-\boldsymbol{E}^+$ is a number operator which counts the number of photons. If the electric fields are classical , the number N_E is a representation of the intensity of the light. So we can write the second order correlation function as

$$g^2(x_1, x_2) = \frac{\left\langle : N_{E(x_1)} N_{E(x_2)} : \right\rangle}{\left\langle N_{E(x_1)} \right\rangle \left\langle N_{E(x_2)} \right\rangle} \tag{9}$$

where, $: X :$ represents the normal ordering of the operator X. The normal ordering of an operator is defined as the ordering of the operators such that all the creation operators are to the left of the annihilation operators.

For example, for an operator $M = a^\dagger a b^\dagger b$, the normally ordered operator is

$$: M := a^\dagger b^\dagger a b \tag{10}$$

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Here, we consider time $t_1 = t$ and $t_2 = t + \tau$, and consider that we have stationary fields. So we can write the second order correlation function for $t_1 = 0, t_2 = \tau$ as

$$g^2(\tau) = g^2(0, \tau) = \frac{\left\langle : N_{E(0)} N_{E(\tau)} : \right\rangle}{\left\langle N_{E(0)} \right\rangle \left\langle N_{E(\tau)} \right\rangle} \tag{11}$$

Claim: For coherent states of light,

$$g^2(0) = 1 \tag{12}$$

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Proof: We see that the coherent states are defined as the eigenstates of the annihilation operator $a = E^+$, where fix the electric field in some polarization direction. So we can write

$$a|\alpha\rangle = \alpha|\alpha\rangle \tag{13}$$

where α is a complex number. We can now calculate

$$\begin{aligned} g^2(0) &= \frac{\left\langle : N_{E(x,0)} : \right\rangle}{\left\langle N_{E(x,0)} \right\rangle^2} \quad = \frac{\left\langle : a^\dagger a a^\dagger a : \right\rangle}{\left\langle a^\dagger a \right\rangle^2} \\ &= \frac{\left\langle a^\dagger a^\dagger a a \right\rangle}{\left\langle a^\dagger a \right\rangle^2} \quad = \frac{\langle \alpha | a^\dagger a^\dagger a a | \alpha \rangle}{|\alpha|^2} \\ &= \frac{|\alpha|^2}{|\alpha|^2} = 1 \end{aligned} \tag{14}$$

6 References