PH4103 Term Paper

Measurement of sub-milliohm resistance of a copper wire using an SR830 Lock-in Amplifier

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Abstract

In this experiment, we make use of a lock-in amplifier to measure sub-milliohm resistance of a copper wire. To do this, we set up a two-probe measurement configuration where we pass a small AC current through the copper wire and measure the resulting voltage drop across it using the lock-in amplifier. We also characterize flicker noise (1/f noise) observed in resistors and compare it with theoretical estimates. We also detail the basic working principle of a lock-in amplifier in this report.

Contents

1. The Lock-In Amplifier

1.1. Introduction

A lock-in amplifier is a type of amplifier that can extract a signal from an extremely noisy input. Generally, the signal we are trying to extract has a known carrier signal, i.e. a known frequency. The signal to noise ratio upto which we can reliably detect the target signal depends on the dynamic reserve of the instrument. For our experiments, we used Stanford Research Systems' Model SR830 DSP Lock-In Amplifier. As a DSP (Digital Signal Processor) lock-in amplifier, the SR830 performs most of it's core funcitons digitally, leading to a better performance than it's analog competitors. In the following section, we describe the workings of this amplifier in greater detail.

1.2. Working Principle

The core function that the lock-in amplifier performs, is *Phase Sensitive Detection*. Let's say the input signal we provide is $V_i(t) = V_0 \sin(\omega_i t + \varphi_i)$. The lock-in multiplies this signal with another reference signal, often provided by an internal oscillator. Let's say the reference sinusoidal signal is given by $V_r(t) = V_1 \sin(\omega_r t + \varphi_r)$. Note that both ω_r and φ_r of this internal oscillator are tunable parameters. After multiplying these to signals, we get

$$
\begin{aligned}
V_{\text{psd}}(t) &= V_i(t)V_r(t) \\
&= V_0 V_1 \sin(\omega_i t + \varphi_i) \sin(\omega_r t + \varphi_r) \\
&= \frac{1}{2}V_0 V_1 [\cos[(\omega_i - \omega_r)t + (\varphi_i - \varphi_r)] - \cos[(\omega_i + \omega_r) + (\varphi_i + \varphi_r)]]
\end{aligned}
\tag{1.1}
$$

As we can see, we end up with two different sinusoidal components. If $\omega_i = \omega_r$, then we end up with an oscillating AC signal at frequency $2\omega_r$ and a DC offset signal. If we now send this signal through a low pass filter (or equivalently take a time average of the signal), we will end up with just the DC signal, which from our calculations turns out to be,

$$
V_{\text{lpf}} = \frac{1}{2}V_0 V_1 \cos(\Delta\varphi)
\tag{1.2}
$$

Where $\Delta\varphi = \varphi_i - \varphi_r$, can be adjusted to 0 by setting $\varphi_r = \varphi_i$. The output of the low pass filter can then be scaled down by $\frac{1}{2}V_r$, and we finally end up with the output voltage of $V_{\text{out}} = V_0$. Note that it's not necessary that the input signal is a pure sinusoid. Infact, most of the times it will not be one.
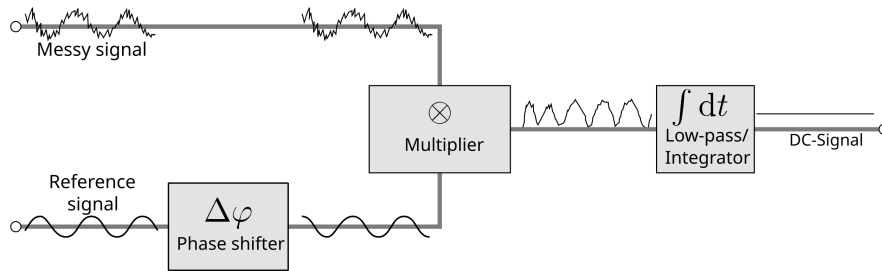

Figure 1: Schematic diagram of a Phase Sensitive Detector (Credits: Zelbear on Wikipedia)

We can use the lock-in amplifier to extract all the fourier components of the target signal at the reference frequency. To do this, the lock-in performs the calculation we just described, twice in the two quadratures by using a 90° phase shifted copy of the reference signal. It's better explained by this schematic diagram shown below.
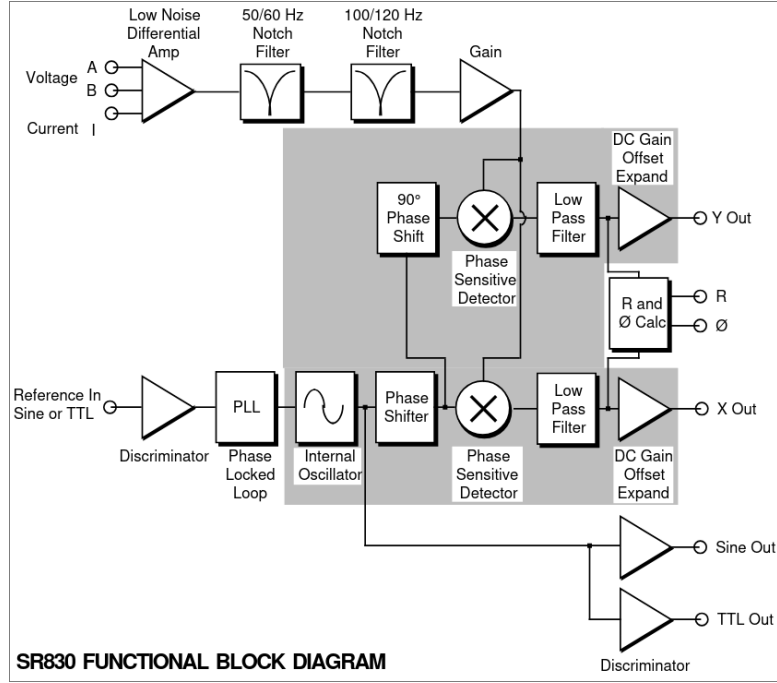

Figure 2: Schematic Diagram for the SR830 Lock-In Amplifier (Credits: Stanford Research Systems)

And the end of this process, we end up with two signals, X and Y, which correspond to the sin and cos quadratures of the signal respectively. The lock-in also calculates two other variables R and θ as

$$R = \sqrt{X^2 + Y^2} \tag{1.3}$$

$$\theta = \arctan\left(\frac{Y}{X}\right) \tag{1.4}$$

1.3. Core Blocks and Specifications of the SR830

In this section we describe the core blocks of the SR830 lock-in amplifier, along with the various specifications and available values for the adjustable parameters in those blocks. Below is an image of the front-panel of the SR830.
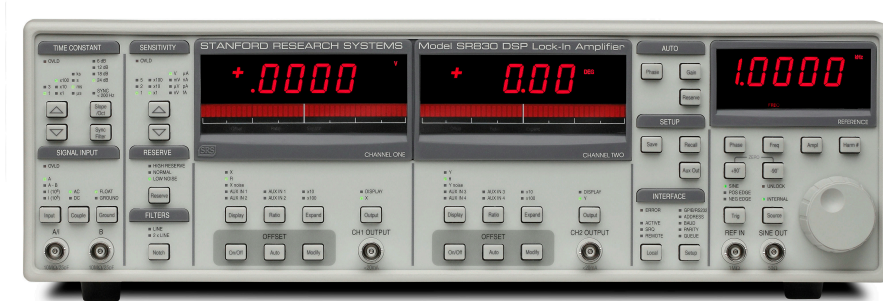

Figure 3: Front Panel of the SR830 (Credits: Stanford Research Systems)

1.3.1. Input

1.3.2. Reference

1.3.3. Filters

References