IISERK

Probability 1

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1 Introduction

This is a first course in Probability covering the following topics.

- Basic Axioms of Probability
- Random Variables
- Inequalities

The reference we will be using is *A First Course in Probability* by Sheldon Ross.

2 The Birthday problem

Problem (The Birthday Problem)**.** Given n randomly chosen people in a room, what is the probability that two people share a birthday? A similar restatement can be given that the probability that people share a birthday is above 50%, what will be the value of n (i.e. no of people in the room)?

A somewhat counterintuitive answer for the restatement is that we only need 23 randomly chosen people in a room to get the probability of a birthday match to be above 50%. We will actually plot a graph and show the probability for n people.

Proof. The proof of the answer is quite simple. Let E be the event that two people in the n people share a birthday. Then we can easily calculate E^c.

$$\begin{aligned} P(E^c) &= \frac{365}{365} \cdot \frac{364}{365} \cdots \frac{365-n}{365} \\ &= \frac{{}^{365}P_n}{365^{23}} \\ P(E) &= 1 - P(E^c) \\ &= 1 - \frac{{}^{365}P_n}{365^{23}} \end{aligned}$$

If we put n = 23 we get P(E^c) $\approx$ 0.507 which proves our result.

A graph $P(E^c)$ vs n is shown below. We can extend to something called the Probabilistic Pigeonhole Principle.

Theorem .1 (The Probabilistic Pigeonhole Principle)**.** Given n balls and m bins, and the probability that atleast two balls are in the same bin is given by the inequality:

$$n > \frac{1}{2} + \sqrt{2m \log\left(\frac{1}{1-p}\right) + \frac{1}{4}} \tag{1}$$

Proof. We use a fairly random inequality $e^{-x} \geq 1 - x \quad \forall x \in \mathbb{R}$.

3 Axioms of Probability

3.1 Definitions

- *Random Experiment:* A random experiment is an experiment where a certain outcome has no effect on other outcomes when repeated multiple times.
- *Mutually Exclusive:*
- *Exhaustive:*
- *Sample Space* Ω*:* A sample space is the set of all outcomes of a Random Experiment.
- *Event Space* ε*:*

3.2 Axioms

- If $A \in \varepsilon$,then $A^c \in \varepsilon$.
- ε is closed under countable union.
- Let $P : \varepsilon \to [0, 1]$ be a function st if A and B are events s.t. $A, B \in \varepsilon$ where $A \cap B = \phi$ then $P(A \cup B) = P(A) + P(B)$.
  In general, if $A_1, \ldots, A_n$ are mutually exclusive, then

$$P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$$

4 Run of heads

5 Boole's Inequality

6 Bon-Ferroni's Inequality

7 Conditional Probability

8 Monty Hall