Vector Space Axioms

Let F be a field, and let V be a set. Then $(V, +, \cdot)$ is a vector space over F if and only if the following axioms hold:

1. **Closure under addition:** For all $u, v \in V$, $u + v \in V$.
2. **Commutativity of addition:** For all $u, v \in V$, $u + v = v + u$.
3. **Associativity of addition:** For all $u, v, w \in V$, $(u + v) + w = u + (v + w)$.
4. **Existence of zero vector:** There exists an element $0 \in V$ such that for all $v \in V$, $v + 0 = v$.
5. **Existence of additive inverses:** For all $v \in V$, there exists an element $-v \in V$ such that $v + (-v) = 0$.
6. **Scalar multiplication by 1:** For all $v \in V$, $1 \cdot v = v$, where 1 is the multiplicative identity of F.
7. **Distributive property over addition:** For all a, $b \in F$ and $v \in V$, $a(v + w) = (a \cdot v) + (a \cdot w)$.
8. **Distributive property over scalar addition:** For all a, $b \in F$ and $v \in V$, $(a + b) \cdot v = (a \cdot v) + (b \cdot v)$.