HW 7

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5.4

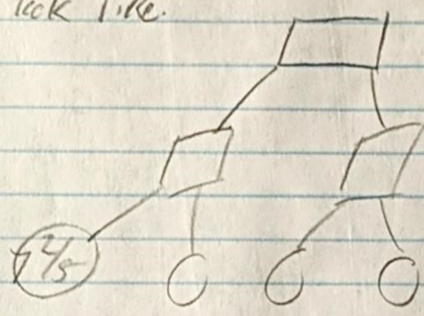
1.) Start with n connected components without any edges

2.) Pick an edge and merge two connected components. Reducing the number of connected components by 1. No other edges exist between the two components.

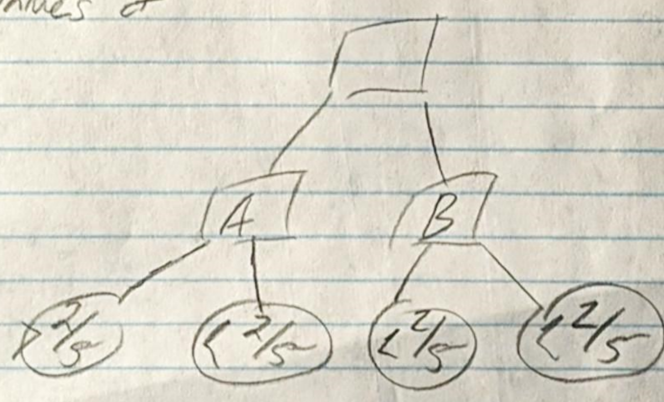
3.) does this exactly n - k times and you are left with a graph with n-k edges, n vertices, and k connected components

5.10 Suppose not that $T \cap H$ is not contained in some MST of H. This means that there is an cross edge in H MST that is lighter in T. But, if there is an edge lighter in $T \cap H$ and H is a subgraph of G, that means that edge should be in G. If that edge is in G, then T should have that lighter cross edge as it would drop other crossing edge & thus creating a contridiction that edge would not be in T as it will appear in the set $T \cap H$.

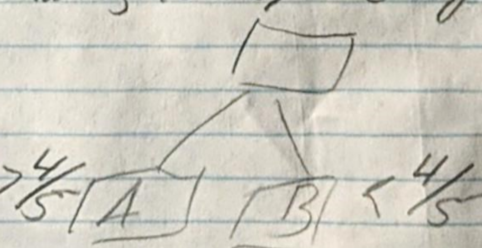
5.16(a) Suppose not, that a frequency more than 2/5 cannot have a codeword of the length that 1. That means that the character cannot be a child of the root node. The graph of the encoding would look like:



In order to have >2/5 selected where that is, that means it must the two lowest frequency during the Huffman encoding. We can select values of

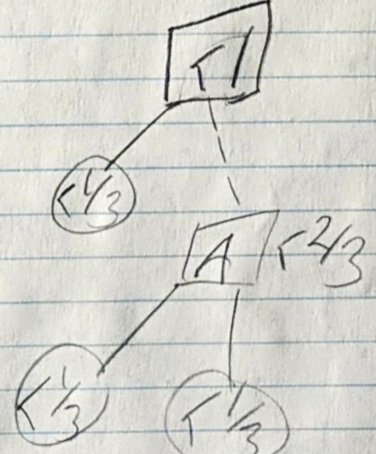


When we have then add two the two root children, A and B, we get



In order to have $>2/5$ in where it it at in the tree, the value of B must be greater than $2/5$ for it to not get paired with our frequency. Then we have other character frequencies that are lower than $2/5$ from them to be paired with each other and our highest frequency greater that $2/5$. Then we add them together according to Huffman encoding and get A and B values, $> 4/5$ and $< 4/5$. Then when adding A and B together, we get a number that is greater than 1, creating a contradiction that $> 2/5$ cannot have a codeword of the length of 1 as it is not possible. Have a sum greater than 1 breaks Huffman encoding as it does not change the frequency. Thus proving that having a frequency greater than $2/5$, there is a codeword that will have the length of 1.

(b)



Suppose not that all characters occur with frequency less than $1/3$, there is a codeword of 1. The tree would be set up like above. This takes the element less than a $1/3$ and adds with sum of A with weights of other nodes with the frequency less than $1/3$. adding them together creates the sum of ≤ 1. This is a contradiction since the sum of all weights should be 1.