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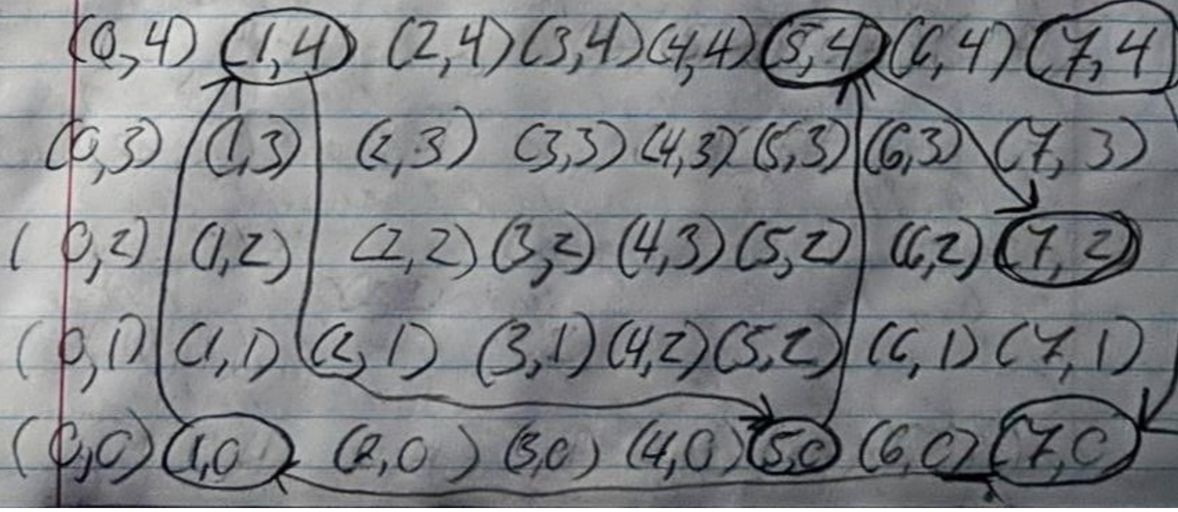
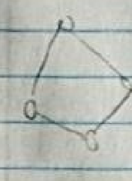
3.8 $(7/7, 4/4, 0/10) \to (7/7, 0/4, 4/10) \to$
$(1/7, 0/4, 10/10) \to (1/7, 4/4, 6/10) \to$
$(5/7, 0/4, 6/10) \to (5/7, 4/4, 2/10) \to$
$\to (7/7, 2/4, 2/10)$

lets state each a (a, b) where b is the amount of water that the 4-pint has and a the amount of water the 7-pint and an directed edge be the pouring the containers. As a restriction, each is one possible pour, therefore (1, 3) has no edge with (8, 1).



Constructing all the possible trees, we then can use DFS to find any that have a equal to 2 or b equal to 2.



3.26 a) (only if) We have an Eulerian tour exist on our graph. Now consider a vertex that is visited k times from different edges through the Eulerian tour. Since it is an Eulerian tour, there must be an exist edge paired with every arrival, thus the degree of the vertex is even.

(if) We can use induction on number of vertices of the graph

Base Case

if $|V| = 2$, and both vertices has an even degree, then it has an Eulerian Tour.

Let the statement be true for $|V| = n$. Then we can consider graph G with $n+1$ vertices such that all vertices have an even degree. Let u be a vertex in G with neighbor $i_1, i_2, \ldots i_{2k}$. Now consider G_1 where we remove u vertex and edges $(i_1, i_2) \ldots (i_{2k-1}, i_{2k})$ to G. Now G_1 has n vertices and even degree for each vertices since it is the same as G, has a Eulerian tour. Thus replace each extra edge of the form (i_{t-1}, i_t) with (i_{t-1}, u) followed by (u, i_t) thus making G having an Eulerian Tour.

b) To have an Eulerian path if and only if there are exactly two vertices that have an odd degree and the rest must have an even degree.

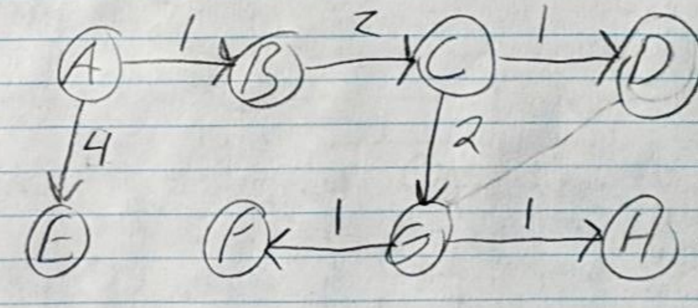
C) Every incoming edge must have an outgoing edge for each vertice.

4.1

a)

		A	B	C	D	E	F	G	H
0	(A)	0	∞	∞	∞	∞	∞	∞	∞
1	(B)	0	1	∞	∞	4	8	∞	∞
2	(C)	0	1	3	∞	4	7	7	∞
3	(d)	0	1	3	4	4	7	5	8
4	(e)	0	1	3	4	4	7	5	8
5	(G)	0	1	3	4	4	6	5	6
6	(F)	0	1	3	4	4	6	5	6
7	(h)	0	1	3	4	4	6	5	6
		0	1	3	4	4	6	5	6

(b)

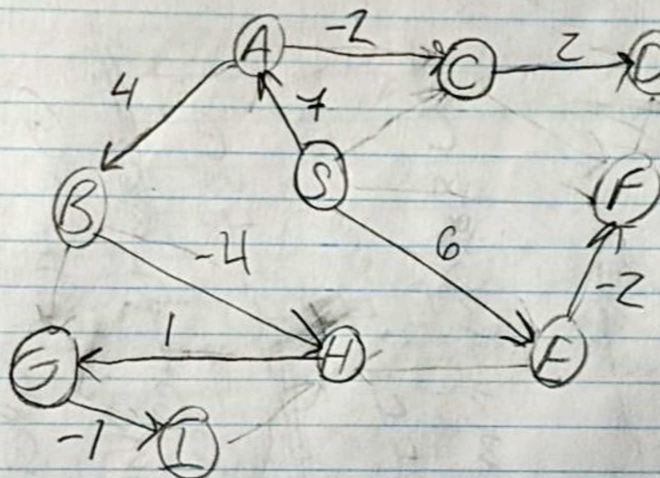


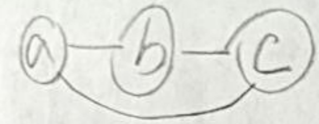
4.2

a)

	S	A	B	C	D	E	F	G	H	I
0	0	∞	∞	∞	∞	∞	∞	∞	∞	∞
1	0	7	∞	6	∞	6	5	∞	∞	∞
2	0	7	11	5	7	6	4	∞	9	∞
3	0	7	11	5	7	6	4	9	7	∞
4	0	7	11	5	7	6	4	8	7	7

b)





4.3 G has an adjacency matrix M. We can square M, M^2 and write D is the diagonal part of M^2 and P is the non-diagonal part of M^2, thus $M^2 = D + P$

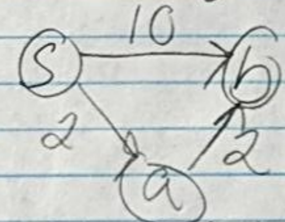
Then $Q = P^2$. Q_{aa} counts the number of ways to start with node a and end node a.

Also $R = MDM$, where R_{aa} counts the number of cycles of the abcba.

Then let $S = Q - R$ and S_{aa} is counts the number of simple 4-cycles that start and end with a. Thus the graph G has a simple 4-cycle if and only if the diagonal entries of S are nonzero

4.8 COUNTER Example.

No it will work



When we add the constants, the length from s-a-b becomes $4 + 2c$ and s-b becomes $10 + c$. When $c > 6$, we will return s-b as the shortest path.