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0.1 (a) $f(n): n-100 \quad g(n): n-200$

$$\lim_{x \to \infty} \frac{n-100}{n-200} = 1$$

$$\lim_{x \to \infty} \frac{n-200}{n-100} = 1$$

$\boxed{f = \Theta(g)}$

(b) $f(n) = n^{1/2} \quad g(n): n^{2/3}$

$$\lim_{x \to \infty} \frac{n^{1/2}}{n^{2/3}} = \frac{\frac{1}{2}n^{-1/2}}{\frac{2}{3}n^{-1/3}} = n^{-1/2}n^{1/3} = n^{-1/6} = 0$$

$$\lim_{x \to \infty} \frac{n^{2/3}}{n^{1/2}} = \frac{\frac{2}{3}n^{-1/3}}{\frac{1}{2}n^{-1/2}} = n^{-1/3}n^{+1/2} = n^{1/6} = \infty$$

$\boxed{f = O(g)}$

(c) $f(n) = 100n + \log n \quad g(n): n + (\log n)^2$

$$\lim_{x \to \infty} \frac{100n + \log n}{n + (\log n)^2} = 100$$

$$\lim_{x \to \infty} \frac{n + (\log n)^2}{100n + \log n} = 1/100$$

$\boxed{f = \Theta(g)}$

(d) $f(n): n\log n \quad g(n): 10n\log 10n$

$$\lim_{x\to\infty} \frac{n\log n}{10n\log 10n} = \frac{\cancel{n}\frac{1}{\cancel{n}} + \log n}{\cancel{10n}\frac{1}{\cancel{n}} + 10\log 10n}$$

$\frac{1}{10x} \cdot 10$

$$\lim_{x\to\infty} = \frac{\log n}{10 + 10\log 10n} = \frac{\frac{1}{n}}{\frac{10}{n}} = \frac{1}{10}$$

$$\lim_{x\to\infty} \frac{10n\log 10n}{n\log n} = \frac{\frac{10n}{n} + 10\log 10n}{\cancel{n}\frac{1}{\cancel{n}} + \log n}$$

$$\lim_{x\to\infty} = \frac{10 + 10\log 10n}{\log n} = \frac{\frac{n}{10}}{\frac{1}{n}} = 10$$

$\boxed{f = \Theta(g)}$

(e) $f(n) = \log 2n \quad g(n) = \log 3n$

$$\lim_{x\to\infty} = \frac{\log 2n}{\log 3n} = \frac{\frac{1}{n}}{\frac{1}{n}} = 1$$

$$\lim_{x\to\infty} \frac{\log 3n}{\log 2n} = \frac{\frac{1}{n}}{\frac{1}{n}} = 1$$

$\boxed{f = \Theta(g)}$

(f) $f(n) = 10\log n \quad g(n) = \log(n^2)$

$$\lim_{x\to\infty} \frac{10\log n}{\log(n^2)} = \frac{\frac{10}{n}}{\frac{2}{n}} = 5$$

$$\lim_{x\to\infty} \frac{\log(n^2)}{10\log n} = \frac{\frac{2}{n}}{\frac{10}{n}} = 1/5$$

$$f = \Theta(g)$$

(g) $f(n) = n^{1.01} \quad g(n) = n\log^2 n$

$$\lim_{x\to\infty} \frac{n^{1.01}}{n\log^2 n} = \frac{n^{.01}}{\log^2 n} = \frac{.01n^{-.99}}{2\log n \cdot \frac{1}{n}} = \frac{.01n^{.01}}{2\log n}$$

$$= \frac{.001n^{-.99}}{2\frac{1}{n}} = \frac{.001n^{.01}}{2} = \infty$$

$$\lim_{x\to\infty} \frac{n\log^2 n}{n^{1.01}} = \frac{\log^2 n}{n^{.01}} = \frac{2\log n \cdot \frac{1}{n}}{.01n^{-.99}}$$

$$= \frac{2\log n}{.01n^{.01}} = \frac{\frac{2}{n}}{.001n^{-.99}} = \frac{2}{.001n^{.01}} = 0$$

$$\boxed{f = \Omega(g)}$$

(h) $f(n): n^2/\log n \quad g(n): n\log^2 n$

$$\lim_{x\to\infty} \frac{n^2/\log n}{n\log^2 n} = \frac{\frac{n(2\log(n)-1)}{\log^2(n)}}{\log(n)(\log(n)+2)}$$

$$\lim_{x\to\infty} \frac{n(2\log(n)-1)}{\log^3(n)(\log(n)+2)} = \infty$$

$$\lim_{x\to\infty} \frac{n\log^2 n}{n^2/\log n} = \frac{\log(n)(\log(n)+2)}{\frac{n(2\log(n)-1)}{\log^2(n)}}$$

$$= \frac{\log^3(n)(\log(n)+2)}{n(2\log(n)-1)} = 0$$

$$\boxed{f = \Omega(g)}$$

(i) $f(n) = n^{.01}$ $g(n) = (\log n)^{10}$

$$\lim_{x \to \infty} \frac{n^{.01}}{(\log n)^{10}} = \frac{.01n^{-.99}}{10(\log n)^{9} \cdot \frac{1}{n}} = \frac{.01n^{.01}}{10(\log n)^{9}}$$

$$= \frac{.001n^{-.99}}{90(\log n)^{8} \frac{1}{n}} = \frac{.001n^{.01}}{90(\log n)^{8}} = \dots \frac{n^{.01}}{(\log n)^{2}}$$

$$\frac{.01n^{-.99}}{2\log n \, \frac{1}{n}} = \frac{.01n^{.01}}{2\log n} = \frac{.01n^{-.99}}{\frac{1}{n}} = .01n^{.01} = \infty$$

$$\lim_{x \to \infty} \frac{(\log n)^{10}}{n^{.01}} = \dots \frac{1/n}{.01n^{-.99}} = \frac{1}{.01n^{.01}} = 0$$

$$\boxed{f = \Omega(g)}$$

(j) $f(n) = (\log n)^{\log n}$ $g(n) = n/\log n$

With the Common Sense Rule, an exponent will grow faster than a polynomial. Therefore $f = \Omega(g)$ because g will be the lower bound of f.

(k) $f(n) = \sqrt{n}$ $g(n) = (\log n)^3$

$$\lim_{x \to \infty} = \frac{n^{1/2}}{(\log n)^3} = \frac{\frac{1}{2}n^{-1/2}}{3(\log n)^2 \frac{1}{n}} = \frac{\frac{1}{2}n^{1/2}}{3(\log n)^2} =$$

$$\frac{\frac{1}{4}n^{-1/2}}{6(\log n)\frac{1}{n}} = \frac{\frac{1}{4}n^{1/2}}{6(\log n)} = \frac{\frac{1}{8}n^{-1/2}}{6/n} =$$

$$\frac{\frac{1}{8}n^{1/2}}{6} = \infty$$

$$\lim_{x \to \infty} = \frac{n^{1/2}}{(\log n)^3} = \dots \frac{6}{\frac{1}{8}n^{1/2}} = 0$$

$f = \Omega(g)$

(l) $f(n) = n^{1/2}$ $g(n) = 5^{\log_2 n}$

$= n^{\log_2 5}$

$= n^{2.32}$

Since $1/2 < 2.32$, $n^{2.32}$ dominates $n^{1/2}$ therefore by the Common Sense Rules, $f = O(g)$ and g is the upper bound of f

(m) $f(n) = n2^n$ $g(n) = 3^n$

By the Common Sense Rule, $2^n < 3^n$, so 3^n dominates, becoming the upper bound of f $\boxed{f = O(g)}$

(n) $f(n) = 2^n$ $g(n) = 2^{n+1}$

$= 2^n \cdot 2^1$

Both function have the exponent, so no function dominates making both the limits 2 and 1/2, therefore $\boxed{f = \Theta(g)}$

(o) $f(n) = n!$ $g(n) = 2^n$

Factorial grows faster than exponential, therefore $n!$ dominates 2^n, so g will be the lower bound of f

$\boxed{f = \Omega(g)}$

(p) $f(n) = (\log n)^{\log n}$ $g(n) = 2^{(\log_2 n)^2}$

$= n^{\log(\log n)}$ $= 2^{\log^2 n / \log^2 2}$

Exponent with grow faster than ~~a~~ polynomial, so $\boxed{f = O(g)}$ from the Common sense Rules.

(q) $f(n) = \sum_{i=1}^{n} i^k = \frac{n^{k+1}-1}{n-1}$ $g(n) = n^{k+1} = n^k \cdot n$

Since both are polynomials to the k there will both be $f = O(g)$ and $f = \Omega(g)$, therefore $\boxed{f = \Theta(g)}$

0.2 $g(n) = 1 + c + c^2 + \dots c^n = \frac{c^{n+1}-1}{c-1}$

$c < 1$

(a) $\lim_{x \to \infty} \frac{c^{n+1}-1}{c-1} = \frac{0-1}{c-1} = \frac{1}{1-c} = g(n)$

$\lim_{x \to \infty} c^{n+1} = 0$

$g = \Theta(1) = \lim_{n \to \infty} g(n) / 1 = \frac{1}{1-c}$

$\lim_{n \to \infty} 1 / g(n) = 1-c$

(b) $\Theta(n)$ $c=1$ $g(n) = n$

$\lim_{x \to \infty} \frac{g(n)}{n} = \frac{n}{n} = 1$ $g = \Theta(n)$ if $c=1$

$\lim_{x \to \infty} \frac{n}{g(n)} = \frac{n}{n} = 1$

c) $\Theta(c^n)$ if $c>1$ $\quad \frac{c^{n+1}-1}{c-1} = g(n)$

$$\lim_{x\to\infty} \frac{g(n)}{c^n} = \frac{\frac{c^{n+1}-1}{c-1}}{c^n} = \frac{c^n}{c^n} = 1$$

$$\lim_{x\to\infty} \frac{c^n}{g(n)} = \frac{c^n}{\frac{c^{n+1}-1}{c-1}} = \frac{c^n}{c^n} = 1$$

$g = \Theta(c^n)$

0.3

(a) $F_n \geq 2^{0.5n}$ $\quad n \geq 6$

$F_6 = F_5 + F_4$ $\quad F_n = F_{n-1} + F_{n-2}$, $F_0 = 0$, $F_1 = 1$

$F_0 = 0$ $\quad F_2 = 0+1 = 1$ $\quad F_4 = 1+2 = 3$

$F_1 = 1$ $\quad F_3 = 1+1 = 2$ $\quad F_5 = 3+2 = 5$

$F_6 = 5+3 = 8$

$2^{.5(6)} = 2^3 = 8$

$8 = 8, n = 6$

$n = 7$

$F_7 = 8+5 = 13$

$2^{.5(7)} = 2^{3.5} \approx 11.3$

$13 > 11.3, n = 7$

$cn-2c$

(b) $F_n \leq 2^{cn}$

$F_{n-1} + F_{n-2} \leq 2^{c(n-1)} + 2^{c(n-2)}$

$2^{cn} = 2^{cn}\left(2^{-1c} + 2^{-2c}\right)$

$1 = 2^{-c} + 2^{-2c} \quad x = 2^{-c}$

$1 = x + x^2$

$0 = x^2 + x - 1$

$\frac{-1 \pm \sqrt{1-4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$

$2^{-c} = \frac{-1 \pm \sqrt{5}}{2}$ let $n = 6$

$F_6 = 8$

$-c = \log_2\left(\frac{-1 \pm \sqrt{5}}{2}\right)$

$8 \leq 2^{.7(6)}\left(2^{-.7} + 2^{-.7 \cdot 2}\right)$

$c = -\log_2\left(\frac{-1 \pm \sqrt{5}}{2}\right)$

$\leq 18.4(.62 + .39)$

$c \approx .6942 \approx .7$

$8 \overset{?}{\leq} 18.4$, for $n = 6$

(c) So we found that $2^{.4n} \geq F_n$, then that means that $2^{.4n}$ grows faster than F_n and it dominates it for all n

Then we when take the limit of F_n and $2^{.4n}$, we get:

$$\lim_{k \to \infty} \frac{F_n}{2^{.4n}} = 0$$

$$\lim_{k \to \infty} \frac{2^{.4n}}{F_n} = \infty$$

Therefor $F_n = \Omega(2^{.4n})$.