# Homework 3 Solutions

October 6, 2019

## 1

Show $\log(n!) \geq cn\log n$ for some $c > 0$ when $n$ is an odd natural number greater than some $n_0 > 0$.

### Solution

$$\begin{aligned}
n! &= n(n-1)\cdots[\frac{n+1}{2}+1]\cdot\frac{n+1}{2}\cdot[\frac{n+1}{2}-1]\cdots 5\cdot 4\cdot 3\cdot 2\cdot 1 && (1)\\
&= \left\{n(n-1)\cdots[\frac{n+1}{2}+1]\cdot\frac{n+1}{2}\cdot\right\}\left\{[\frac{n+1}{2}-1]\cdots 2.5\cdot 2\cdot 2\cdot 3\cdot 2\cdot 2\right\} \quad (n \geq 10) && (2)\\
&> \left[\underbrace{\frac{n+1}{2}\cdot\frac{n+1}{2}\cdots\frac{n+1}{2}}_{(n+1)/2 \text{ terms}}\right]\left[\underbrace{2\cdots 2\cdot 2\cdot 2\cdot 2\cdot 2}_{(n+1)/2 \text{ terms}}\right] && (3)\\
&= (n+1)^{\frac{n+1}{2}} && (4)
\end{aligned}$$

Thus, we have when $n$ is odd and $n > n_0 = 10$

$$\log n! > \frac{1}{2}(n+1)\log(n+1) > \frac{1}{2}n\log n$$

Therefore, we have

$$\log n! \geq cn\log n$$

at $n > n_0 = 10$ and $c = \frac{1}{2}$.

# Exercise 2.15

In our median-finding algorithm (Section 2.4), a basic primitive is the split operation, which takes as input an array S and a value v and then divides S into three sets: the elements less than v, the elements equal to v, and the elements greater than v. Show how to implement this split operation in place, that is, without allocating new memory.

## Solution

This problem can be solved by using swap rather than allocating new memory. Do a linear search of an input array S, once found elements lower than v, swap current element with element at suitable position of S. Also, swap elements if found element larger than v.

Pseudocode:

Split-In-Place$(S, n, v)$

If $S.length \leq 1$, return $S$.
Let $c = 1$, $d = S.length$.
**for** $i = 1$ **to** $S.length$:
    If $S[i] < v$: swap $S[i]$ and $S[c]$, $c = c + 1$.
    If $S[i] > v$: swap $S[i]$ and $S[d]$, $d = d - 1$.

$S[1 \ldots c-1]$ are elements $< v$, $S[d+1 \ldots n]$ are elements $> v$. Other elements are equal to $v$.

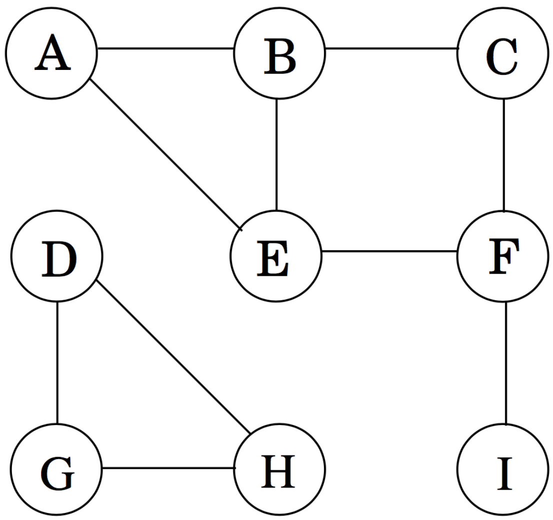
This algorithm search all elements of $S$ and save all elements $< v$ to $S[1 \ldots c-1]$, save all elements $> v$ to $S[d+1 \ldots n]$. The left elements can only be equal to $v$.

# Exercise 3.1

Perform a depth-first search on the following graph; whenever there's a choice of vertices, pick the one that is alphabetically first. Classify each edge as a tree edge or back edge, and give the pre and post number of each vertex.

## Solution

Back edges=$\{AE, BE, DH\}$
The rest of the edges are tree edges.




| Node | Pre | Post |
|---|---|---|
| A | 1 | 12 |
| B | 2 | 11 |
| C | 3 | 10 |
| D | 13 | 18 |
| E | 5 | 6 |
| F | 4 | 9 |
| G | 14 | 17 |
| H | 15 | 16 |
| I | 7 | 8 |