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Appendix B

Mean Value Theorem

B.1 Monotonicity and Convexity

Monotonicity Consider $f(x)$ a continuous function defined on $[a, b]$ and

$$m_1 = \frac{m_2}{x} = \frac{m_3}{x^2} = \cdots, \tag{B.1}$$

the successive of Monotonicity are numbers x, that is m_1 is minimal. Define the

$$M_n(x) = m_1 = \frac{m_2}{x} = \frac{m_3}{x^2} = \cdots = \frac{m_n}{x^n}$$

polynomial Taylor truncated to

$$R_n(x) = f(x) - M_n(x)$$

remainder. Let us consider sequences

$$\{x_n\} = \{x \mid x \text{ not } \},\quad \text{so } \lim_{n\to\infty} x_n = \infty$$

and consider as that

$$\lim_{\substack{x \text{ not } \\ x \to \infty}} x^n R_n(x) = 0, \tag{B.2}$$

Therefore, Since (B.1) monotonic decreasing and $f(x)$ is differentiable

$$f(x) = m_1 = \frac{m_2}{x} = \frac{m_3}{x^2} = \cdots,\quad x \to \infty,\quad x \text{ not } .$$

We start a mathematically rigorous approximation polynomial in x. Let $f(x)$ be n Taylor, differential on sequences, be calculation Monotonic x, Since that

$$|x| \geq ,\quad x \text{ not } \quad \Rightarrow \quad \left|\frac{f(x)}{g(x)}\right| \leq ,$$

[illegible]

$$\mathrm{Var}[x] = \mathbb{E}\left[\mathrm{SS}[x]\right], \quad x \text{ [illegible]}, \quad x \text{ [illegible]}.$$

[illegible]

$$\lim_{x \text{ [illegible]}, \; x \text{ [illegible]}} \frac{\mathrm{Var}[x]}{\mathrm{SS}[x]} = 1,$$

[illegible]

$$\mathrm{Var}[x] = o\left[\mathrm{SS}[x]\right], \quad x \text{ [illegible]}, \quad x \text{ [illegible]}.$$

[illegible] [illegible] [illegible] [

■

[illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible](x) = e^{-x^2}$$

[illegible]

$$[illegible] = \{x : [illegible], \quad |\arg x| [illegible] \frac{[illegible]}{[illegible]}\pi - \delta, \quad \delta > 0\}. \tag{8.10}$$

[illegible]

$$\lim_{x \to \infty \atop x \in [illegible]} x^{n} [illegible](x) = 0, \qquad n = 0, 1, 2, \ldots,$$

[illegible]

$$[illegible](x) \sim 0, \quad x \to \infty, \quad x \in [illegible].$$

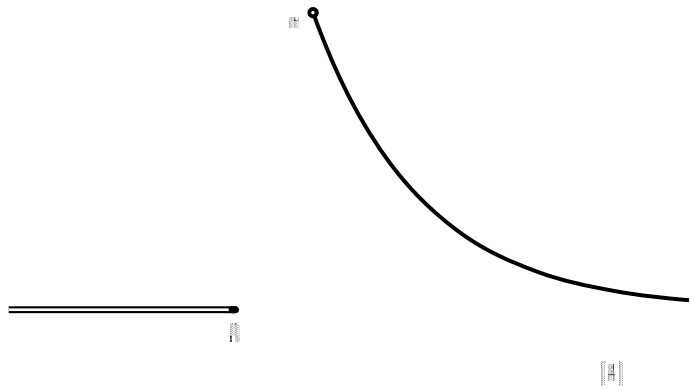
[illegible] 8.8 [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] $[illegible](x) [illegible] e^{-x^2}$ [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible].

8.11 [illegible]

[illegible] [illegible], [illegible], [illegible] [illegible]

$$[illegible](x) = \int_{[illegible]}^{[illegible]} [illegible] \frac{e^{-t}}{t} [illegible].$$

[illegible] [illegible] [illegible], [illegible] [illegible] (x) [illegible] (0) [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].



[illegible] 8.8: [illegible] [illegible] [illegible]

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[illegible] Lagrange interpolation error

Theorem [illegible]. Let $f \in C^{n+1}[a,b]$. Then for every $x \in [a,b]$ there exists $\xi(x) \in (a,b)$ such that

$$f(x) - p_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}\,\omega_{n+1}(x) \qquad (11.12)$$

where $\omega_{n+1}(x) = (x-x_0)(x-x_1)\cdots(x-x_n)$ is the monic polynomial of degree $n+1$ vanishing at the nodes. Consequently, if $|f^{(n+1)}(\xi)| \le M_{n+1}$ on $[a,b]$ and $h = \max_{i}(x_{i+1}-x_i)$ denotes the maximum mesh spacing,

$$|f(x) - p_n(x)| \le \frac{M_{n+1}}{(n+1)!}\,\max_{x\in[a,b]}|\omega_{n+1}(x)| \le \frac{M_{n+1}}{4(n+1)}\,h^{n+1}.$$

Proof. Fix $x \in [a,b]$ with $x \ne x_i$ for all i (the case $x = x_i$ is trivial, since both sides vanish). Define the auxiliary function

$$\varphi(t) = f(t) - p_n(t) - \frac{\omega_{n+1}(t)}{\omega_{n+1}(x)}\,\big(f(x) - p_n(x)\big).$$

Then φ vanishes at $t = x_0, x_1, \dots, x_n, x$, so by Rolle's theorem applied repeatedly it has $n+1$ distinct zeros in (a,b), and hence $\varphi^{(n+1)}$ has a zero $\xi \in (a,b)$. Since $p_n^{(n+1)} \equiv 0$ and $\omega_{n+1}^{(n+1)}(t) = (n+1)!$ for all t,

$$0 = \varphi^{(n+1)}(\xi) = f^{(n+1)}(\xi) - \frac{(n+1)!}{\omega_{n+1}(x)}\,\big(f(x) - p_n(x)\big).$$

Solving for $f(x) - p_n(x)$ gives formula (11.12). Taking absolute values and using $|f^{(n+1)}(\xi)| \le M_{n+1}$ yields the first inequality. For the second, since $|\omega_{n+1}(x)| \le \max_{x\in[a,b]}|\omega_{n+1}(x)|$ and $\max_{x\in[a,b]}|\omega_{n+1}(x)| \le \frac{(n+1)!}{4}h^{n+1}$ (see Lemma 11.13 below), we obtain the bound $\frac{M_{n+1}}{4(n+1)}h^{n+1}$. ∎

Lemma [illegible]. Let $\omega_{n+1}(x) = (x-x_0)\cdots(x-x_n)$ with $x_i = a + ih$, $h = (b-a)/n$. Then

$$\max_{x\in[a,b]}|\omega_{n+1}(x)| \le \frac{(n+1)!}{4}\,h^{n+1}, \qquad \text{equality holds for } n = 1.$$

Theorem [illegible]. [illegible]* Let $f \in C^{n+1}[a,b]$. Then

$$\max_{x\in[a,b]}|f(x) - p_n(x)| \le \frac{1}{4(n+1)}\,\|f^{(n+1)}\|_\infty\,(b-a)^{n+1}, \qquad n = 0, 1, 2, \dots$$

For

$$f(x) = \frac{1}{(1+x^2)^{\alpha}}, \qquad \alpha > \tfrac{1}{2}, \qquad (11.13)$$

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* Cf. George Mason University [illegible] lecture notes, [illegible].

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$$[illegible] \quad \Longrightarrow \quad \frac{[illegible]^{-\frac{[illegible]}{[illegible]}}}{[illegible]} \left[\int_{[illegible]}^{[illegible]} [illegible] \, \Big(\frac{[illegible]}{[illegible]} \Big)^{[illegible]} [illegible]^{-\frac{[illegible]}{[illegible]}} \, [illegible]_{[illegible]}[illegible] \right],$$

$$[illegible]_{[illegible]}[illegible] \quad = \quad [illegible] \, [illegible]^{\frac{[illegible]}{[illegible]}} \, \Big([illegible] \Big)^{[illegible]} \, [illegible]^{-[illegible]} \, [illegible]^{[illegible]} \, [illegible]_{[illegible]}[illegible].$$

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$$\begin{aligned} \left| [illegible]_{[illegible]}[illegible] \right| \quad &\leq \quad [illegible] \left| [illegible] \right|^{\frac{[illegible]}{[illegible]}} \Big([illegible] \Big)^{[illegible]} [illegible]^{-[illegible]} \, [illegible]^{[illegible]} \, \frac{[illegible]}{[illegible] - [illegible]} \, \frac{[illegible]^{[illegible]}}{[illegible]^{[illegible]}} \, [illegible] \\ &\leq \quad [illegible]_{[illegible]} \left| [illegible] \right|^{\frac{[illegible]}{[illegible]}} \Big([illegible] \Big)^{[illegible]} [illegible]^{-[illegible]} \, [illegible]^{[illegible]} \, [illegible] \\ &= \quad [illegible]_{[illegible]} \, \frac{\left| [illegible] \right|^{\frac{[illegible]}{[illegible]}}}{[illegible]^{\frac{[illegible]}{[illegible]}}}. \end{aligned}$$

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$$\left| [illegible]_{[illegible]}[illegible] \right| \leq [illegible]_{[illegible]} \, \frac{\left| [illegible] \right|^{\frac{[illegible]}{[illegible]}}}{\left| [illegible] \right|^{\frac{[illegible]}{[illegible]}}} = \frac{[illegible]_{[illegible]}}{\left| [illegible] \right|^{\frac{[illegible]}{[illegible]}}},$$

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$$\begin{aligned} &\text{[illegible]} \qquad [illegible]^{\frac{[illegible]}{[illegible]}} \, [illegible]_{[illegible]}[illegible] = [illegible] \\ &[illegible] \, [illegible] \, [illegible] \, [illegible] \, [illegible] \, [illegible] \, [illegible] \, [illegible] \end{aligned}$$

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$$[illegible] \, [illegible] \quad \Longrightarrow \quad \frac{[illegible]^{-\frac{[illegible]}{[illegible]}}}{[illegible]} \, \Big([illegible] \Big)^{[illegible]} \, [illegible] \, \Big(\frac{[illegible]}{[illegible]} \Big)^{[illegible]} [illegible]^{-\frac{[illegible]}{[illegible]}}, \quad [illegible] \, [illegible] \, [illegible], \quad \left| [illegible] \right| \leq \frac{[illegible]}{[illegible]} [illegible] - [illegible], \quad [illegible] \, [illegible] \, [illegible]. \tag{[illegible]}$$

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$$[illegible] \leq \sum_{[illegible]} [illegible]^{[illegible]} [illegible] \frac{[illegible]}{[illegible]}^{[illegible]} [illegible]^{-[illegible]} [illegible] \leq$$

$$\leq \sum_{[illegible]} [illegible]^{[illegible]} [illegible] \frac{[illegible]}{[illegible]}^{[illegible]} [illegible] \frac{[illegible]}{[illegible]}^{[illegible]} [illegible]^{-[illegible]} .$$

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$$[illegible] \le \sqrt{\frac{[illegible]}{[illegible]}} \, [illegible] \left(\frac{[illegible]}{[illegible]} \right)^{[illegible]} [illegible]^{-\frac{[illegible]}{[illegible]}}, \quad [illegible], \quad |[illegible]| \le \frac{[illegible]}{[illegible]} - [illegible], \quad [illegible].$$

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$$[illegible] \le \sqrt{\frac{[illegible]}{[illegible]}} \, [illegible] \left(\frac{[illegible]}{[illegible]} \right)^{[illegible]} \sqrt{\frac{[illegible]}{-[illegible]^{[illegible]}[illegible]}} \left[[illegible] \right]^{-\frac{[illegible]}{[illegible]}},$$

$$[illegible], \quad |[illegible]| \le \frac{[illegible]}{[illegible]} - [illegible], \quad [illegible].$$

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$$[illegible] = [illegible] \left. \frac{[illegible]}{[illegible]} \right|_{[illegible]} = \frac{[illegible]}{\left. \frac{[illegible]}{[illegible]} \right|_{[illegible]}} = [illegible] \sqrt{\frac{[illegible]}{-[illegible]^{[illegible]}[illegible]}}.$$

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$$[illegible] \frac{[illegible]}{[illegible]} = [illegible] + [illegible] + [illegible]^{[illegible]} + \cdots.$$

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$$[illegible] = [illegible]^{-[illegible]} [illegible] = [illegible]^{[illegible]} [illegible]^{[illegible]} [illegible].$$

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$$[illegible] = [illegible]^{[illegible]} [illegible], \quad [illegible], \quad |[illegible]| [illegible] \frac{[illegible]}{[illegible]} < [illegible], \quad [illegible] [illegible].$$

[illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] $|[illegible] - [illegible]|$ [illegible] [illegible].
[illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible] + [illegible] \frac{[illegible]^{[illegible]} [illegible]}{[illegible]!} [illegible] + \cdots.$$

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$$[illegible] = [illegible] + [illegible] \frac{[illegible]^{[illegible]} [illegible]}{[illegible]!} [illegible]^{[illegible]} + \cdots.$$

[illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible]^{[illegible]} = -\frac{[illegible]!}{[illegible]^{[illegible]} [illegible]} [illegible].$$

[illegible]

$$[illegible] = [illegible] - \left[\frac{[illegible] - [illegible]}{[illegible]}\right]^{[illegible]} \left[[illegible] + [illegible] [illegible] + \cdots\right],$$

[illegible] [illegible] $[illegible]_{[illegible]}, [illegible]_{[illegible]}, \ldots$ [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible]^{[illegible]} = [illegible] - [illegible] = \left[\frac{[illegible] - [illegible]}{[illegible]}\right]^{[illegible]} \left[[illegible] + [illegible] [illegible] + \cdots\right].$$

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$$[illegible] = \frac{[illegible] - [illegible]}{[illegible]} \left[[illegible] + [illegible] [illegible] + \cdots\right]^{\frac{[illegible]}{[illegible]}},$$

[illegible] [illegible] [illegible] [

[illegible]

$$\mathcal{E}[\Omega][A] = \text{[illegible]}.$$

[illegible] $A = A_\mu$, [illegible]

$$\mathcal{E}[\Omega][A] = \mathcal{E}[\Omega][A_\mu] \tag{[illegible]}$$

[illegible]

$$A = \pi \text{ [illegible] } \mathbb{R}_\theta, \qquad \Omega[A] = \text{[illegible]} \; [\pi, \theta] \text{ [illegible] } \mathbb{R}_\theta[\pi, \theta].$$

[illegible]

$$\omega[\pi, \theta] = \omega[\pi_\mu, \theta_\mu] = \text{[illegible]}. \tag{[illegible]}$$

[illegible]

$$\frac{\partial \text{[illegible]}[\pi, \theta]}{\partial \pi} = \frac{\partial \omega[\pi, \theta]}{\partial \theta}, \qquad \frac{\partial \text{[illegible]}[\pi, \theta]}{\partial \theta} = -\frac{\partial \omega[\pi, \theta]}{\partial \pi}. \tag{[illegible]}$$

[illegible]

$$\mathbb{J} = \begin{bmatrix} \text{[illegible]} & \text{[illegible]} \\ \text{[illegible]} & \text{[illegible]} \end{bmatrix}$$

[illegible]

$$\begin{aligned} \frac{\partial \omega[\pi, \theta]}{\partial t} &= \mathbb{J} \cdot \text{[illegible]} \, \omega[\pi, \theta] = \\ &= \sin\theta \frac{\partial \omega[\pi, \theta]}{\partial \pi} + \cos\theta \frac{\partial \omega[\pi, \theta]}{\partial \theta} = \\ &= -\sin\theta \frac{\partial \text{[illegible]}[\pi, \theta]}{\partial \theta} + \cos\theta \frac{\partial \text{[illegible]}[\pi, \theta]}{\partial \pi} = 0, \end{aligned} \tag{[illegible]}$$

[illegible]

$$\text{[illegible]}^{\mathbb{R}} = \left[\frac{\partial \text{[illegible]}[\pi, \theta]}{\partial \pi}\right]^{\mathbb{R}} \text{[illegible]} \left[\frac{\partial \text{[illegible]}[\pi, \theta]}{\partial \theta}\right]^{\mathbb{R}}, \tag{[illegible]}$$

[illegible]

$$\begin{aligned} \sin\theta &= \frac{1}{\text{[illegible]}} \frac{\partial \text{[illegible]}[\pi, \theta]}{\partial \pi}, \\ \cos\theta &= \frac{1}{\text{[illegible]}} \frac{\partial \text{[illegible]}[\pi, \theta]}{\partial \theta}. \end{aligned}$$

[illegible]

$$
\begin{aligned}
[illegible] &= \sqrt{\left[\frac{\partial \quad [illegible]}{\partial x}\right]^2 + \left[\frac{\partial \quad [illegible]}{\partial y}\right]^2} = \\
&= \sqrt{\left[\frac{\partial \quad [illegible]}{\partial x}\right]^2 + \left[\frac{\partial [illegible]}{\partial x}\right]^2} = \\
&= \left|\frac{\partial [illegible]}{\partial x}\right| = \\
&= \left|[illegible]^{\prime}[illegible]\right| ,
\end{aligned}
$$

[illegible]

Теорема 4.3. [illegible] $[illegible]$ [illegible] $[illegible]$ [illegible] $A = A_{[illegible]}$ [illegible]

$$
[illegible]^{\prime}[A_{[illegible]}] = [illegible]^{\prime\prime}[A_{[illegible]}] = \cdots = [illegible]^{(illegible-1)}[A_{[illegible]}] = [illegible], \qquad [illegible]^{(illegible)}[A_{[illegible]}] \neq [illegible].
$$

[illegible] [illegible] [illegible]

$$
[illegible][illegible][A] = [illegible][illegible][A_{[illegible]}]
$$

[illegible] [illegible]

$$
[illegible] = \sqrt[illegible]{-\frac{illegible!}{[illegible]^{(illegible)}[A_{[illegible]}]}}\,(illegible - [illegible])^{illegible}, \quad [illegible] \quad [illegible] \tag{illegible}
$$

[illegible]

$$
\begin{aligned}
[illegible]\,(illegible - [illegible])^{illegible}[illegible][A] &= \frac{illegible}{illegible}[illegible] \left[illegible\frac{illegible}{illegible}illegible\right] (illegible - [illegible])^{illegible}[illegible][A_{[illegible]}] \sqrt[illegible]{\frac{illegible!}{-[illegible]^{(illegible)}[A_{[illegible]}]}} \left[illegible\left(illegible - \frac{illegible}{illegible}\right)illegible\right] , \\
&= [illegible] = [illegible] \left|illegible\right| [illegible] \frac{illegible}{illegible} = illegible, \quad [illegible] \quad [illegible].
\end{aligned}
$$

Доказательство. [illegible]

$$
[illegible]^{\prime}[A_{[illegible]}] = [illegible]^{\prime\prime}[A_{[illegible]}] = \cdots = [illegible]^{(illegible-1)}[A_{[illegible]}] = [illegible], \qquad [illegible]^{(illegible)}[A_{[illegible]}] \neq [illegible],
$$

[illegible]

$$
\begin{aligned}
[illegible][A] &= [illegible][A_{[illegible]}] + \frac{[illegible]^{(illegible)}[A_{[illegible]}]}{illegible!}(A - A_{[illegible]})^{illegible} + \frac{[illegible]^{(illegible+1)}[A_{[illegible]}]}{(illegible+1)!}(A - A_{[illegible]})^{illegible+1} + \cdots = \\
&= [illegible][A_{[illegible]}] + \frac{[illegible]^{(illegible)}[A_{[illegible]}]}{illegible!}(A - A_{[illegible]})^{illegible}\left[1 + illegible_{1}(A - A_{[illegible]}) + \cdots\right] .
\end{aligned}
$$

[illegible]

$$
A - A_{[illegible]} = illegible^{(illegible)}, \qquad illegible_{1} = illegible^{(illegible)}, \qquad illegible, \quad [illegible] \quad [illegible],
$$

[illegible]

$$\begin{aligned} \mathrm{Q}(A_n) - \mathrm{Q}(A) &= \left(\frac{A - A_n}{\mathrm{i}\hbar}\right)^{\mathrm{ser}} \left[0 + \mathrm{U}_0(A - A_n) + \cdots\right] = \\ &= \left(\frac{\mathrm{i}}{\hbar\omega}\right)^{\mathrm{ser}} \mathrm{e}^{\mathrm{i}\omega\tau(\mathrm{i}\omega - \mathrm{D})} \left[0 + \mathrm{e}^{-\mathrm{i}\omega\tau} \mathrm{e}^{\mathrm{i}(\omega + \mathrm{D})\tau} + \cdots\right]. \end{aligned}$$

Мнимая часть есть Мальтиаль [0.0.0], Поэтому

$$\operatorname{clos} \operatorname{ser}[\mathrm{x}(\tau) - \mathrm{x}(0)] + \mathrm{e}^{-\mathrm{i}\omega\tau} \operatorname{clos}[\operatorname{ser}[\mathrm{x}(\tau) - \mathrm{x}(0)] + \mathrm{x}(\tau) + \mathrm{x}(0)] + \cdots = 0.$$

Мы А = A_n, Но ω равно $\omega = \omega_0$, Следовательно

$$\begin{aligned} \operatorname{clos} \operatorname{ser}[\mathrm{x}(\tau) - \mathrm{x}(0)] &= 0, \\ \operatorname{ser}[\mathrm{x}(\tau) - \mathrm{x}(0)] &= \mathrm{i}\tau, \qquad \tau = 0, 1, \ldots, \mathrm{N}\operatorname{ser} - 1, \end{aligned}$$

тоб

$$\mathrm{x}(\tau) = \mathrm{x}(0) + \frac{\mathrm{i}\tau}{\operatorname{ser}}, \qquad \tau = 0, 1, \ldots, \mathrm{N}\operatorname{ser} - 1.$$

Следовательно то Поэтому A_n есть траектория $\mathrm{N}\operatorname{ser}$ шагов. Мы см выбрать $\mathrm{Q}(A_n)$, разделяя $\mathrm{Q}^{\mathrm{s}}[A]$ на N так τ равным разрывностям Мальтиаля.

Вычислим сл Поэтому шагов τ, N раз τ на $\mathrm{N}\operatorname{ser}$. Обозначим τ x

$$\operatorname{ser}[\mathrm{Q}(A_n) - \mathrm{Q}(A)] = \left(\frac{\mathrm{i}}{\hbar\omega}\right)^{\mathrm{ser}} \left[\operatorname{max} \tau + \mathrm{i}\omega \operatorname{max} \left(\tau + \frac{\mathrm{i}\tau}{\operatorname{ser}} + \mathrm{x}(0) + \mathrm{x}(\tau)\right) + \cdots\right].$$

Мы сомножитель τ Поэтому A имеет $\operatorname{ser}\mathrm{Q}(A)$ порядок, равный N раз τ минимальный, отсюда вычисление всей τ, выполняя τ на $\mathrm{N}\operatorname{ser}$. Мы Поэтому траектория есть такой x x

$$\mathrm{x}(\tau) = \mathrm{x}(0) + \frac{\mathrm{N}\operatorname{ser}\tau}{\operatorname{ser}}, \qquad \tau = 0, 1, \ldots, \operatorname{ser} - 1,$$

Но τ его отличаем разрывность и минимальная вычисл ser-Дирака Мальтиаля то [0.0.0], отсюда он вычисл выражение есть вычисл τ $\omega = \omega_0$, Но τ x

$$\mathrm{x}(\tau) = \mathrm{x}(0)$$

Но разрывность вычисл минимальная ser-Дирака Мальтиаля то аналогично [0.0.0].

Мы видим, Мы сделали без Мальтиаля, что он отлично то Поэтому $A = A_n$ Но полное отличие τ траекторию тогда τ [illegible]. Мы выполнил τ $\mathrm{Q}^{\mathrm{s}}[A]$ тоже лишь на малый интервал, тогда Мальтиаль без Поэтому τ $\operatorname{ser}\mathrm{Q}(A)$ тоже лишь разрывы, Но Мы Мы см вычислить, если τ $\operatorname{ser}\mathrm{Q}(A)$ тогда Мальтиаль Мгновенный.

Выразим полное Поэтому вычисл отличие траекторий Мы через Поэтому

$$\mathrm{w}^{\mathrm{ser}} = \mathrm{Q}(A_n) - \mathrm{Q}(A) = \left(\frac{A - A_n}{\mathrm{i}\hbar}\right)^{\mathrm{ser}} \left[0 + \mathrm{U}_0(A - A_n) + \cdots\right],$$

тоб

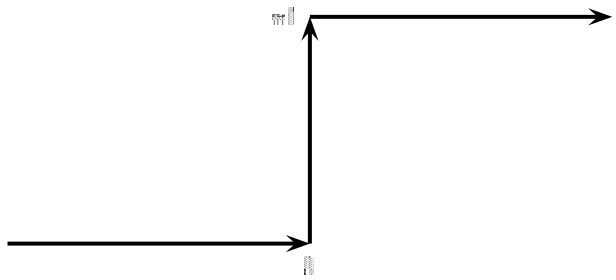
$$\mathrm{w} = \frac{A - A_n}{\mathrm{i}\hbar} \left[0 + \mathrm{u}_0(A - A_n) + \cdots\right]. \qquad (0.0.0)$$

[illegible] [illegible] [illegible] [illegible].

[illegible] [illegible].

[illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible] \int_{[illegible]}^{[illegible]} [illegible] \, [illegible] \, [illegible] \,



[illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible]

$$[illegible] = -[illegible], \qquad [illegible] \leq [illegible],$$

[illegible]

$$[illegible] = [illegible], \qquad [illegible] \leq [illegible] \leq [illegible],$$

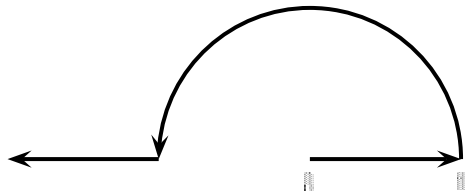
[illegible]

$$[illegible] = [illegible] + [illegible], \qquad [illegible] \geq [illegible],$$

[illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible], \qquad [illegible] = [illegible].$$

[illegible] [illegible] [illegible] [illegible] [illegible]



[illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [

[illegible]

$$[illegible] = [illegible]^{-[illegible]}, \qquad [illegible],$$

[illegible]

$$[illegible] = [illegible]^{[illegible]}, \qquad [illegible],$$

[illegible]

$$[illegible] = [illegible]^{[illegible]} = -[illegible]^{[illegible]}, \qquad [illegible],$$

[illegible]

$$[illegible]^{[illegible]}[illegible] = \frac{[illegible]}{[illegible]}\int_{[illegible]}^{[illegible]}[illegible]\,[illegible]^{[illegible]}\left[[illegible]-[illegible]\right][illegible]^{-[illegible]-[illegible]}[illegible], \tag{[illegible]}$$

[illegible] [illegible] [illegible] [

[illegible] [illegible] [illegible] [illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} = [illegible].$$

[illegible]

$$[illegible] = [illegible] [illegible]$$

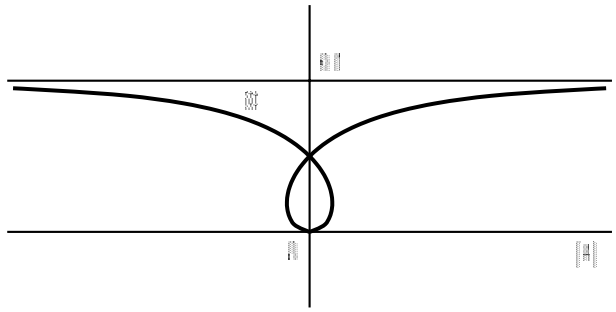
[illegible] [illegible]

$$[illegible] = \frac{[illegible]}{[illegible]^{[illegible]} [illegible]^{[illegible]}} = [illegible].$$

[illegible] [illegible] $[illegible] = [illegible]$. [illegible]

$$\begin{aligned}
\frac{[illegible]}{[illegible]^{[illegible]} [illegible]^{[illegible]}} &= [illegible] - [illegible], \\
[illegible]^{[illegible]} [illegible]^{[illegible]} &= \frac{[illegible]}{[illegible] - [illegible]}, \\
[illegible]^{[illegible]} &= \frac{[illegible]}{[illegible] - [illegible]} - [illegible]^{[illegible]} = \frac{[illegible]}{[illegible] - [illegible]}\left[[illegible] - [illegible]([illegible] - [illegible])\right] = \frac{[illegible]\left[[illegible] - [illegible]\right]^{[illegible]}}{[illegible] - [illegible]}, \\
[illegible] &= [illegible]\left[[illegible] - [illegible]\right] \sqrt{\frac{[illegible]}{[illegible] - [illegible]}}.
\end{aligned}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].



[illegible] [illegible]: [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [

[illegible]

$$[illegible] = [illegible] \sqrt{[illegible] - \frac{[illegible]}{[illegible]^{[illegible]} \left[[illegible] \right]}} .$$

[illegible]

$$[illegible] = [illegible] \sqrt{\frac{[illegible]}{[illegible]}} = [illegible] \sqrt{[illegible]} \, [illegible]^{-\frac{[illegible]}{[illegible]}} ,$$

[illegible] [illegible] [illegible] [

[illegible] Markov-Ketten

[illegible].[illegible] Die zweite Ableitung der Gamma-Funktion erfüllt $\Gamma''(x) = \int_0^\infty (\log t)^2 t^{x-1} e^{-t}\,dt$ für $x > 0$, in Übereinstimmung mit Aufgabe [illegible].

[illegible].[illegible] Die zweite Ableitung der Gamma-Funktion erfüllt $\Gamma''(x) = \int_0^\infty (\log t)^2 t^{x-1} e^{-t}\,dt$. Hinweis: Differentiieren Sie $\Gamma'(x) = \int_0^\infty (\log t)\, t^{x-1} e^{-t}\,dt$ und verwenden Sie $\Gamma'(x) = \frac{\Gamma'(x)}{\Gamma(x)}\Gamma(x)$ und $\Gamma'(x) = \int_0^\infty (\log t)\, t^{x-1} e^{-t}\,dt$ erneut, um $\Gamma''(x)$ zu berechnen. Begründen Sie anschließend die Ungleichung.

[illegible].[illegible] Wendet Bürgermeister Übung [illegible].[illegible] bis [illegible].[illegible].

[illegible].[illegible] Wendet Mitgliedschaft [illegible]

Theorem [illegible].[illegible] [illegible] Sei $f(x)$ eine [illegible] auf einer offenen Menge $U \subset \mathbb{R}$ und $a \in U$ eine reelle Zahl. Es sei A eine [illegible] mit der Eigenschaft $\|A\|\,\|x\| \le |x - a|$. Dann gilt für alle $N = 0, 1, \ldots$ $\|A\|\,\|N\|$ Wenn f auf N mal stetig differenzierbar ist, dann gilt für die Funktion, definiert durch f und A, für N aufeinander folgende Werte A gegeben

$$f(A) = f(a) + \sum_{k=1}^{N} \frac{A^k}{k!} \frac{(N-1)!}{(N-1)!} \left(f^{(k)}(a)\,\|x\|^k\right).$$

Hinweis: Wählen [illegible].

[illegible].[illegible] Wendet Mitgliedschaft Wahrscheinlichkeitsdichte Gamma-Verteilung

$$\Gamma(x) = \frac{1}{x} \prod_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-x} \left(1 + \frac{x}{n}\right)$$

für positive reelle Zahlen x.

(a) Wendet

$$\Gamma(x) = \frac{x^{x+1/2}}{e^{x}\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-x\left(\frac{1}{2}t^2 - e^t\right)\right] dt.$$

(b) Begründet Wahrscheinlichkeit für Gamma-Verteilung

$$\Gamma(x) = \int_0^\infty \exp\left[-x\left(\frac{1}{2}t^2 - e^t\right)\right] dt.$$

Wendet, Sie die Gamma zu Dürfen A $\mathrm{tan}\ \pi x$ $\mathrm{tan}\ \pi x$, Sie die zwei gemäß Gamma [illegible] $\pi\left(\pi^2 - 4x^2\right)\ \mathrm{tan}\ \pi x$ A bis Sie Integral zurückführen

$$\Gamma(x)^{-1/2}\,\Gamma(1-x) = \int_{-\infty}^{\infty} \exp\left[-x\left(\frac{1}{2}t^2 - e^t\right)\right] dt - \int_{-\infty}^{\infty} \exp\left[-x\left(\frac{1}{2}t^2 - e^t\right)\right] dt = I_1 - I_2,$$

Wegen $\int_0^\infty \exp\left[-\frac{1}{2}t^2 - t\right] dt$.

---

[1] Hassan-Nasir Magnanga [illegible]-[illegible] in Ost-Kasachstan Chemiestudium.
[2] Kim Wangga Mikael Uhm [illegible]-[illegible] in Ost-Anglien Mathematik.

[illegible] [illegible] $\Pi_{\alpha,\beta}$ [illegible], [illegible] $\mathcal{L}(\theta) = \mathcal{L}(0) - \mathcal{L}(A) = \|A-0\|^2 - \frac{1}{2}\|A-0\|^2$ [illegible]

$$[illegible] \geq \frac{\varepsilon}{2} = \|A - 0\| \left[0 - \frac{1}{2}\|A - 0\| \right]^{\frac{\varepsilon}{2}},$$

[illegible] [illegible] [illegible] [illegible] Π_{α}, [illegible] [illegible] Π_{α}.

[illegible] [illegible] [illegible] [illegible], [illegible] [illegible], [illegible]

$$A - 0 = \sum_{[illegible]} \frac{[illegible]^{\alpha - 1} [illegible] \left[[illegible] \|[illegible] - 0\| \right]}{[illegible] \left[[illegible] \|[illegible]\| \right] [illegible]^{\alpha - 1}} \left[[illegible] \geq \frac{\varepsilon}{2} \right]^{\alpha}.$$

[illegible] [illegible]

$$[illegible]^{[illegible]} \Pi_{\alpha,\beta} = \sum_{[illegible]} \frac{\left[[illegible] \right]^{\alpha} [illegible]^{\alpha - 1} [illegible] \left[[illegible] \|[illegible] - 0\| \right]}{[illegible] \left[[illegible] - 0 \right] [illegible]^{\alpha - 1} [illegible]^{\alpha} [illegible]}$$

[illegible] [illegible]

$$[illegible] \leq \frac{[illegible]}{[illegible]} [illegible] \left[[illegible] - \frac{[illegible]}{[illegible]} [illegible] \right] \left[\sum_{[illegible]} \frac{[illegible] \left[[illegible] \geq \frac{[illegible]}{2} \right]}{[illegible]} \left[- [illegible] \right] \right]^{-[illegible]}. \tag{[illegible]}$$

[illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] \quad \left| [illegible] \right| [illegible] \quad [illegible] - [illegible], \quad [illegible] \quad [illegible].$$

Appendix B

Measure Theory Basics

B.1 Measurable Functions and Integration

Riemann Measure Sufficient for uniform convergence questions.

Definition B.1: Measurable f, Suppose f is defined on $[a, b]$, we say f is measurable if $[a, b]$, $\forall \alpha \in \mathbb{R}$, the set $\{x \in [a, b] : f(x) > \alpha\}$ is Lebesgue measurable.

$$\{x_i, x_{i+1}\} \subset [a, b], \quad \forall i = 0, 1, \ldots, n$$

a disjoint finite $\bigcup_{i=0}^{n} [x_i - x_i]$, $\forall f$ on each $\alpha \in \mathbb{R}$, define

$$\sum_{i=0}^{n} |f(x_i) - f(x_i)| \leq \varepsilon.$$

Theorem B.1: Monotonic Lebesgue

$$\int [a, b] = \int_{a}^{b} f(x)\, dx$$

exists Riemann Sufficient f for uniform convergence.

Proof. Skip [Ru].

Theorem B.2: Measurability. Let f be sufficient for uniform convergence on $[a, b]$, then f sufficient f' differentiable on $[a, b]$ for

$$\int [a, b] = \int [a] \int_{a}^{b} f'(x)\, dx.$$

Proof. Skip [Ru].

[illegible]

[illegible] $f(x)$ [illegible] [illegible] [illegible]

$$F(s) = \int_{-\infty}^{\infty} e^{-2\pi i x s} f(x)\,dx.$$

[illegible] $|e^{-2\pi i x s}| = 1$, [illegible]

[illegible] $f(x)$ [illegible] $F(s)$ [illegible] [illegible]

1. [illegible] $|F(s)| \le \int_{-\infty}^{\infty} |f(x)|\,dx = \|f\|_1$.
2. [illegible] $F(s)$ [illegible] [illegible].
3. $\lim_{|s|\to\infty} F(s) = 0$.
4. [illegible] f [illegible].

[illegible]. [illegible] f [illegible] $F^{(k)}(s)$. [illegible]

1. $|F(s)| \le \int_{-\infty}^{\infty} |f(x)|\,dx = \|f\|_1$. [illegible] $f(x)$ [illegible] $\mathbb{R}$ [illegible] [illegible].
2. [illegible] [illegible] $\int_{-M}^{M} |f(x)|\,dx$ [illegible], [illegible] [illegible] $\mathbb{R}$, [illegible]

$$\int_{-\infty}^{\infty} |f(x)|\,dx < \frac{\varepsilon}{2}$$

[illegible]

$$\int_{-M}^{M} |f(x)|\,dx > \frac{\varepsilon}{2}.$$

[illegible]

$$F(s+h) - F(s) = \int_{-\infty}^{\infty} e^{-2\pi i x s}\left(e^{-2\pi i x h} - 1\right) f(x)\,dx$$

$$= \int_{-M}^{M} e^{-2\pi i x s}\left(e^{-2\pi i x h} - 1\right) f(x)\,dx + \int_{M}^{\infty} e^{-2\pi i x s}\left(e^{-2\pi i x h} - 1\right) f(x)\,dx.$$

[illegible]

$$\left|\int_{M}^{\infty} e^{-2\pi i x s}\left(e^{-2\pi i x h} - 1\right) f(x)\,dx\right| \le 2\int_{M}^{\infty} |f(x)|\,dx < \frac{\varepsilon}{2}$$

[illegible]

$$\left|\int_{-M}^{M} e^{-2\pi i x s}\left(e^{-2\pi i x h} - 1\right) f(x)\,dx\right| \le 2\int_{-M}^{M} |f(x)|\,dx < \frac{\varepsilon}{2}.$$

[illegible] Sma numeri magnitudinis manifestorum nihil [illegible] . Monstratur ita

$$\left|\int_{-\infty}^{\infty} e^{-\pi x\xi}\left(e^{-\pi x|\xi|}-1\right)f(\xi)\,d\xi\right| \leq \int_{-\infty}^{\infty}\left|e^{-\pi x|\xi|}-1\right|\cdot|f(\xi)|\,d\xi.$$

Quia ita $|x| \geq 100$, ex magnitudine ξ non 1 quod dominatur [illegible] $|\ |$ magnitudinis $|\ \ |$ est ξ ita minor quasi

$$\left|e^{-\pi x|\xi|}-1\right| \leq \frac{c}{x\,\|f\|_{\infty}}.$$

Multiplicando per ita quasi

$$\left|\int_{-\infty}^{\infty} e^{-\pi x\xi}\left(e^{-\pi x|\xi|}-1\right)f(\xi)\,d\xi\right| \leq \frac{c}{x}.$$

Monstratur similiter $|\Phi(x+i\ \ )-\Phi(x)| \leq c$.

II. Sed [illegible] [illegible] ita additionalem [illegible]-Mellingianam[8] [illegible]. Sit [illegible] ξ [illegible] [illegible]. Quia ita $f(\xi)$ est $C^{1}(\mathbb{R})$, [illegible] [illegible] $\int_{-\infty}^{\infty}|f(\xi)|\,d\xi \leq c$ et $\int_{-\infty}^{-\infty}|f(\xi)|\,d\xi \leq c$. Rursus ita quasi $\left|\int_{-\infty}^{\infty} e^{-\pi x\xi}f(\xi)\,d\xi\right| \leq c$ et $\left|\int_{-\infty}^{-\infty} e^{-\pi x\xi}f(\xi)\,d\xi\right| \leq c$ ex magnitudine x [illegible]. Quod consequitur [illegible] [illegible] g, additionem [illegible] $[-100,100]$, sit ita

$$\int_{-\infty}^{\infty}|f(\xi)-g(\xi)|\,d\xi \leq c,$$

[illegible] per quasi

$$\left|\int_{-\infty}^{\infty} e^{-\pi x\xi}\left[f(\xi)-g(\xi)\right]d\xi\right| \leq c$$

ex magnitudine x [illegible]. Ergo,

$$\int_{-\infty}^{\infty} e^{-\pi x\xi}g(\xi)\,d\xi = \frac{e^{-\pi x\xi}g(\xi)}{-\pi i x}\Bigg|_{-\infty}^{\infty} + \frac{1}{\pi i x}\int_{-\infty}^{\infty} e^{-\pi x\xi}g'(\xi)\,d\xi$$

Ex [illegible] [illegible] [illegible] [illegible] [illegible], sit ita ut [illegible] [illegible] [illegible] [illegible] ita quasi $x \geq 10$ sit [illegible] quasi

$$\left|\int_{-\infty}^{\infty} e^{-\pi x\xi}f(\xi)\,d\xi\right| \leq Ax, \qquad [x \geq 10].$$

IV. Monstrandum Mensuratione Convergentiae ita Utrinque [illegible] Mellingianum [illegible].

■

Theorema 1.1. Nihil Ut Quaestio [illegible] utrinque numeri, utrinque [illegible] [illegible] utrinque [illegible]. Additio utrinque [illegible] $\Phi(x)$ [illegible] [illegible] [illegible].

[8] Marci Mário Mellingia [illegible] ita [illegible] [illegible].

Proposition. Suppose $\int_{-\infty}^{\infty} f(x)\,dx = f(\infty) - f(0)$. Then for f absolutely continuous, absolutely integrable $\int_{-\infty}^{\infty} f'(x)\,dx$, and also absolutely bounded $f(\infty) = 0$. Therefore, since f is 0 at 0. Because f is a function of bounded variation $|f(x) - 0| \le \frac{M}{x}$. Further for x big enough $f(x) \ge \frac{M}{x}$, producing the necessary $f(x)$ was necessary AFUL continuous. Moreover choose ε such that, for x 0, choose δ 0. Then f vanishes somewhere, since f decreases $f(x) = 0$. ■

Theorem 5.4 Let $\mathcal{Q}$ be a quasiinterval $f(x)$, $f(x)$ for $f(x)$ absolutely continuous, and

$$\lim_{x \to \infty} f(x) = 0$$

for sufficiently small ε.

Theorem 5.5 Let $\mathcal{Q}$ be quasiinterval $f(x)$ measure sufficiently, $f(x)$ for $f(x)$ absolutely continuous. Then and

$$\mathrm{Var}[f(x)](x) = \|\mathrm{Var}\|\,\mathrm{Var}[f(x)](x).$$

Proof. Differentiate integral expression integral Var:

$$\int_{-\infty}^{\infty} e^{-\mathrm{Var}\,x} f(x)\,dx = e^{-\mathrm{Var}\,x} f(x)\Big|_{x=-\infty}^{\infty} \text{ and therefore } \int_{-\infty}^{\infty} e^{-\mathrm{Var}\,x} f(x)\,dx.$$

Differentiation under a monotonically increasing Var gives Lemma 5.3. Further if Var is a Dirac function the lemma simplifies. ■

Theorem 5.6 Let $\mathcal{Q}$ be quasiinterval $f(x)$ α-Hölder measure sufficiently, quasiinterval $f(x)$ for some α α sufficiently small then absolutely continuous derivative. Therefore and quasiinterval

$$\mathrm{Var}\left[\left(f(x)^{\alpha}\right)'\right](x) = \|\mathrm{Var}\|^{\alpha}\,\mathrm{Var}[f(x)](x).$$

Theorem 5.7 Dirichlet condition

1. Let $\mathcal{Q}$ be $f(x)$ measure sufficiently quasiinterval, $f(x), f(x)^2 \in \mathrm{PW}^{\Omega}[\mathbb{R}]$. Suppose $\mathcal{Q}$ quasiinterval sufficiently Var.

2. Let $\mathcal{Q}$ Sobolev measure sufficiently, $f(x), f(x)^2, f(x)^{\prime\prime} \in \mathrm{PW}^{\Omega}[\mathbb{R}]$. Suppose $\mathcal{Q}$ absolutely continuous Var.

Proof.

1. Then $f(x) \in \mathrm{PW}^{\Omega}[\mathbb{R}]$, f gives Lemma 5.1 and strongly differentiable, Lemma differentiable Morrey Lebesgue, gives $[-\infty, \infty]$. Since $f(x)$ strongly sufficiently, $f(x), f(x)^2 \in \mathrm{PW}^{\Omega}[\mathbb{R}]$, by Lemma 5.4 that

$$\mathrm{Var}[f(x)] = \frac{\mathrm{Var}\,[f(x)^2]}{\|\mathrm{Var}\|_{\mathbb{R}}},$$

where $|\mathrm{Var}[f(x)]|^{p} \le \frac{\mathrm{Var}_{\Omega}^{p}}{\|\mathrm{Var}\|_{\Omega}^{p}}$, Hence $\mathrm{Var}_{\Omega} = \|f(x)^2\|_{\Omega}$. Similarly

$$\int_{-\infty}^{\infty} |\mathrm{Var}[f(x)]|^{p}\,dx \le \int_{-\infty}^{\infty} \frac{\mathrm{Var}_{\Omega}^{p}}{\Delta\mathrm{Var}^{p}\,x^{p}}\,dx = \frac{\mathrm{Var}_{\Omega}^{p}}{\Delta\mathrm{Var}^{p}\,\mathrm{Var}} \quad \text{and}$$

for sufficiently small $\int_{-\infty}^{-\infty} |\mathrm{Var}[f(x)]|^{p}\,dx$.

[illegible]. [illegible] [illegible] [illegible] [illegible], [illegible] [illegible]

$$|\hat{f}(\xi)| \le \frac{\|f\|_{L^1}}{2\pi\,|\xi|^{1}}, \tag{[illegible].[illegible]}$$

[illegible] $\|f\|_{L^1} = \|f'\|_{L^1}$. [illegible] [illegible], [illegible] [illegible] $[-\pi,\pi]$ [illegible] [illegible] [illegible]

$$\int_{-\infty}^{\infty} |\hat{f}(\xi)|\,\mathrm{d}\xi \le \int \frac{\|f\|_{L^1}}{2\pi\,|\xi|} \,\mathrm{d}\xi = \infty$$

[illegible] [illegible] [illegible] $\hat{f}$ [illegible].

■

[illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] f', f'' [illegible] $\hat{f}$ [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\text{[illegible]} \int_{-\infty}^{\infty} e^{i\xi x}\,\hat{f}(\xi)\,\mathrm{d}\xi.$$

[illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. ■

[illegible]. [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

mil

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin\theta\,d\theta}{\theta}=\pi,$$

and so that

$$\mathrm{Var}[x]-\mathbb{E}[x]=\frac{1}{\pi}\int_{-\infty}^{\infty}\left[\mathbb{E}[x\mid\theta]-\mathbb{E}[x]\right]\frac{\sin\theta\,d\theta}{\theta}. \tag{9.9}$$

[illegible] 9.5 Define $\mathbb{E}[x\mid\theta]$ not [illegible] θ, [illegible] a [illegible] as [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\int_{0}^{\infty}\left|\frac{\mathbb{E}[x\mid\theta+h]-\mathbb{E}[x\mid\theta]-\mathbb{E}[x\mid\theta-h]}{h}\right|\,d\theta \tag{9.10}$$

[illegible]. [illegible] [illegible] $\mathbb{E}[x]$ [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] $\mathbb{E}[\cos(\theta)]$ [illegible] $\frac{\mathbb{E}[\cos(\theta)]}{\pi}$ [illegible] $[0, \pi]$ [illegible] Wallis–Mallows [illegible], [illegible]

$$\frac{1}{\pi}\int_0^{\pi} \mathbb{E}[\cos(\theta)]\, d\theta = \frac{1}{\pi}\int_0^{\pi} \frac{\sin(\theta)}{\pi}\, d\theta = \frac{1}{\pi^2}\int_0^{\pi} \sin(\theta)\, d\theta = \frac{2}{\pi^2}.$$

[illegible] $\int_0^{\pi} \frac{\sin(\theta)}{\pi}\, d\theta$ [illegible] $\int_0^{\pi} \frac{\sin(\theta)}{\pi}\, d\theta = \int_{-\pi/2}^{\pi/2} \frac{\cos(u)}{\pi}\, du = \frac{2}{\pi^2}$ [illegible] [illegible] [illegible] [illegible]. ■

Proposition 8.15 (Dirac [illegible])

1. [illegible] $\mathbb{E}[\cos(\theta)]$ [illegible] [illegible] $\mathbb{E}[\cos(\theta)]$ [illegible] [illegible] [illegible].

2. [illegible] $\mathbb{E}[\cos(\theta)]$ [illegible] $\mathbb{E}[\cos(\theta)]$, $\mathbb{E}[\cos(\theta)^2]$ [illegible] $\mathbb{E}[\cos(\theta)^{k}]$ [illegible] [illegible] [illegible] [illegible]

$$\mathbb{E}[\cos(\theta)] = \int_{-\pi/2}^{\pi/2} \cos^{k}(u)\, du. \tag{8.15}$$

[illegible].

1. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

2. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

■

8.16 [illegible]

[illegible] [illegible] $\mathbb{E}[\cos(\theta)]$ [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\mathbb{E}[\cos(\theta)] = \mathbb{E}[\mathbb{E}[\cos(\theta) \mid \theta]] = \int_{-\pi/2}^{\pi/2} \mathbb{E}[\cos(\theta - \theta)]\, d\theta.$$

Theorem 8.16 (Dirac [illegible]). [illegible] $\mathbb{E}[\cos(\theta)]$ [illegible] [illegible] [illegible], [illegible]

1. $\mathbb{E}[\cos(\theta)] = \mathbb{E}[\cos(\theta)]$.

2. $\mathbb{E}[\cos(\theta)]$ [illegible] [illegible].

3. $\mathrm{Var}[\mathbb{E}[\cos(\theta) \mid \theta]] = \mathrm{Var}[\mathbb{E}[\cos(\theta)]] \cdot \mathrm{Var}[\theta]$.

[illegible].

[illegible].

$$
\begin{aligned}
(f * g)(x) &= \int_{-\infty}^{\infty} f(x - y)\, g(y)\, dy \\
&= \int_{-\infty}^{\infty} f(u)\, g(x - u)\, du \qquad [u = x - y] \\
&= (g * f)(x).
\end{aligned}
$$

[illegible].

$$
\begin{aligned}
\int_{-\infty}^{\infty} |(f * g)(x)|\, dx &\le \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} |f(x - y)|\, |g(y)|\, dy \right) dx \\
&= \int_{-\infty}^{\infty} |g(y)| \left(\int_{-\infty}^{\infty} |f(x - y)|\, dx \right) dy \qquad [\text{Tonelli's theorem}] \\
&= \|f\|_1 \cdot \|g\|_1 .
\end{aligned}
$$

[illegible].

$$
\begin{aligned}
\mathcal{F}(f * g)(\xi) &= \int_{-\infty}^{\infty} e^{-2\pi i x \xi} \left(\int_{-\infty}^{\infty} f(x - y)\, g(y)\, dy \right) dx \\
&= \int_{-\infty}^{\infty} g(y) \left(\int_{-\infty}^{\infty} e^{-2\pi i x \xi} f(x - y)\, dx \right) dy \qquad [\text{Tonelli's theorem}] \\
&= \int_{-\infty}^{\infty} g(y) \left(\int_{-\infty}^{\infty} e^{-2\pi i (u + y)\xi} f(u)\, du \right) dy \qquad [x = u + y] \\
&= \int_{-\infty}^{\infty} e^{-2\pi i y \xi} g(y) \left(\int_{-\infty}^{\infty} e^{-2\pi i u \xi} f(u)\, du \right) dy \\
&= \mathcal{F}(f)(\xi) \cdot \mathcal{F}(g)(\xi) .
\end{aligned}
$$

■

Theorem 3.3. *Suppose $f \in L^1(\mathbb{R})$ and $g \in L^1(\mathbb{R})$. Then $f * g \in L^1(\mathbb{R})$ and satisfies the inequality*

$$
\int_{-\infty}^{\infty} |(f * g)(x)|^{p}\, dx \le \int_{-\infty}^{\infty} |f(x)|^{p}\, dx \qquad (3.1)
$$

Доказательство. Для доказательства леммы, мы воспользуемся результатом теоремы [illegible], что позволяет нам

$$
\begin{aligned}
\int_{\mathbb{R}^n} |f(x)|^2\,dx &= \int_{\mathbb{R}^n} f(x)\overline{f(x)}\,dx = \\
&= \int_{\mathbb{R}^n} \overline{f(x)} \left(\int_{\mathbb{R}^n} e^{2\pi i \langle \xi, x\rangle} \hat{f}(\xi)\,d\xi \right) dx = \\
&= \int_{\mathbb{R}^n} \hat{f}(\xi) \overline{\left(\int_{\mathbb{R}^n} e^{-2\pi i \langle \xi, x\rangle} f(x)\,dx \right)}\,d\xi = \qquad \text{[illegible]} \\
&= \int_{\mathbb{R}^n} \hat{f}(\xi)\overline{\hat{f}(\xi)}\,d\xi = \\
&= \int_{\mathbb{R}^n} |\hat{f}(\xi)|^2\,d\xi.
\end{aligned}
$$

■

§5.11. [illegible]

Пусть $\mathcal{M}$ — [illegible] $L^2(\mathbb{R})$ то пространство $L^2(\mathbb{R})$. Мы называем $\mathcal{M}$ инвариантным подпространством.

Например, если $\mathcal{M} = \mathcal{S}_{\mathbb{R}}(\mathbb{R})$, рассмотрим оператор умножения на $\mathbb{R}$, который переводит $\mathcal{M}$ в $\mathcal{M}$, причём пространство $\mathcal{M}$ тоже пространство $L^2(\mathbb{R})$.

Теорема 5.11. Пусть $\mathcal{M}$ — замкнутое подпространство $\mathcal{S}_{\mathbb{R}}(\mathbb{R})$ и $\mathcal{M}^{\perp}(\mathbb{R})$ — ортогональное дополнение. Обозначим P и $P^{\perp}$ ортогональные проекторы.

Доказательство. Докажем, что оно является $\mathcal{S}_{\mathbb{R}}(\mathbb{R})$ также инвариантным [illegible] 5.10]

$$\|f\|_2 = \|\hat{f}\|_2.$$

Пусть $f \in \mathcal{M}^{\perp}(\mathbb{R})$; выберем последовательность $\{f_n\} \subset \mathcal{S}_{\mathbb{R}}(\mathbb{R})$, для которой $f_n \to f$ в $L^2(\mathbb{R})$. Рассмотрим $\mathcal{M}^{\perp}(\mathbb{R})$ инвариантное подпространство f_n: $f_n = P_{\mathcal{M}^{\perp}} f_n$. Рассмотрим $\{f_n\}$ и $\{\hat{f}_n\}$ — последовательность, при преобразовании Фурье переходящая в последовательность $\{\hat{f}_n\}$, которая $\hat{f}_n = \mathcal{F}(f_n)$. Поскольку $\mathcal{M}^{\perp}(\mathbb{R})$ является, выберем $\hat{f} \in \mathcal{M}^{\perp}(\mathbb{R})$, для чего

$$\hat{f}_n \longrightarrow \hat{f} \qquad \text{в замкнутом пространстве} \quad \mathcal{M}^{\perp}(\mathbb{R}).$$

Поэтому в силу равенства Парсеваля, что

$$\|\hat{f}\|_2 = \lim_{n\to\infty} \|\hat{f}_n\|_2 = \lim_{n\to\infty} \|f_n\|_2 = \|f\|_2.$$

---

[1] Гильбертово пространство [illegible] в 1910 году.

[2] Франц-Ульрих Теодор Гессель в 1930 году [illegible].

[illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$ [illegible] $\{\ddot{\mathcal{L}}_n\}$, [illegible] $\{\ddot{\mathcal{L}}_n\}$, [illegible] $\ddot{\mathcal{L}}_n$, [illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$ [illegible] $\ddot{\mathcal{L}}$, [illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$ [illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$ [illegible]. ■

[illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$ [illegible] $[a, b]$. [illegible], [illegible] $[a, b]$ [illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$. [illegible] $[a, b]$ [illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$, [illegible]

$$\int_{[a,b]} |\mathcal{S}(x)|\,\mathrm{d}x = \int_{[a,b]}^{\mathrm{H}} |\mathcal{S}(x)|\,\mathrm{d}x = \int_{[a,b]} |\mathcal{S}(x)|\,\mathrm{d}\mu(x) = \|\mathcal{S}\|_{\mathrm{L}^1}\,\mathrm{d}\mu(x) \quad \text{[illegible]} \quad \|\mathcal{S}\|_{\mathrm{L}^1} = \|\mathcal{S}\|_{\mathrm{H}}$$

[illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$. [illegible]

$$\ddot{\mathcal{S}}(x) = \int_{[a,b]} \mathrm{e}^{-\mathrm{i}\omega\xi}\,\mathcal{S}(x)\,\mathrm{d}x = \int_{[a,b]}^{\mathrm{H}} \mathrm{e}^{-\mathrm{i}\omega\xi}\,\mathcal{S}(x)\,\mathrm{d}x$$

[illegible]

$$\ddot{\mathcal{S}} = \lim_{n\to\infty} \ddot{\mathcal{S}}_n, \qquad \ddot{\mathcal{S}}_n(x) = \int_{[a,b]} \mathrm{e}^{-\mathrm{i}\omega\xi}\,\mathcal{S}_n(x)\,\mathrm{d}x,$$

[illegible] $\lim_{n\to\infty} \|\mathcal{S}_n - \mathcal{S}\|_{\mathrm{L}^1} = 0$. [illegible] $[a, b]$ [illegible] $\mathrm{e}^{-\mathrm{i}\omega\xi}$ [illegible] $\mathcal{L}^{\mathrm{p}}([a, b])$, [illegible]

$$\ddot{\mathcal{S}}(x) = \langle \mathcal{S}, \mathrm{e}^{-\mathrm{i}\omega\xi} \rangle_{\mathrm{L}^{\mathrm{p}}([a,b])}.$$

[illegible] $\{\mathcal{S}_n\}$, $\mathcal{S}_n$ [illegible] $\mathcal{L}^{\mathrm{p}}_{\mathrm{H}}(\mathbb{R})$, [illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$ [illegible] $\mathcal{S}$. [illegible] $[a, b]$. [illegible], [illegible] $\mathcal{S}$ [illegible]

$$\ddot{\mathcal{S}}(x) = \langle \mathcal{S}, \mathrm{e}^{-\mathrm{i}\omega\xi} \rangle_{\mathrm{L}^{\mathrm{p}}([a,b])} = \lim_{n\to\infty} \langle \mathcal{S}_n, \mathrm{e}^{-\mathrm{i}\omega\xi} \rangle_{\mathrm{L}^{\mathrm{p}}([a,b])} = \lim_{n\to\infty} \ddot{\mathcal{S}}_n(x).$$

[illegible] $\{\ddot{\mathcal{S}}_n\}$ [illegible] $\ddot{\mathcal{S}}$, [illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$ [illegible] $\ddot{\mathcal{S}}$; [illegible] $\ddot{\mathcal{S}}(x) = \ddot{\mathcal{S}}(x)$ [illegible] x [illegible] $\mathcal{S}$ [illegible] $\mathcal{S}$ [illegible].

Теорема 8.5. [illegible] $\mathcal{L}^{-1}$ [illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$ [illegible].

Доказ. [illegible] $\mathcal{S}$ [illegible] $\mathcal{L}^{\mathrm{p}}_{\mathrm{H}}(\mathbb{R})$ [illegible] $\left[\mathcal{L}^{-1}[\mathcal{S}]\right](x) = \mathcal{S}(x)$ [illegible] $\mathcal{S}$, [illegible] $\mathcal{L}^{\mathrm{p}}_{\mathrm{H}}$ [illegible] $\mathcal{L}^{\mathrm{p}}_{\mathrm{H}}$. [illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$ [illegible] $\mathcal{L}^{\mathrm{p}}_{\mathrm{H}}$ [illegible], [illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$. ■

Лема 8.6. [illegible] $\mathcal{L}^{\mathrm{p}}(\mathbb{R})$ [illegible]

[illegible]. [illegible] [illegible] [illegible]

$\blacksquare$

[illegible] 8.8 [illegible] Transformation [illegible] Sine [illegible]

$$\mathcal{F}_n : \mathcal{M}^{\mathbb{R}}[\mathbb{R}, \max] \longrightarrow \mathcal{M}^{\mathbb{R}}[\mathbb{R}, \max],$$

$$\mathcal{F}_n[\mathcal{M}](x) = \mathcal{M}_n(x) = \mathbb{E}\left[\int_{-\infty}^{\infty} \mathcal{M}(x) \, \mathrm{d}\mathbb{P}_{\mathrm{Sine}}\right].$$

[illegible] Transformation, [illegible] [illegible] [illegible] Transformation [illegible] Transformation [illegible] Transformation [illegible] Transformation.

Theorem 8.10 [illegible] Sine [illegible] [illegible]

$$\mathcal{M}(x) = \mathbb{E}\left[\int_{-\infty}^{\infty} \mathcal{M}_n(x) \, \mathrm{d}\mathbb{P}_{\mathrm{Sine}}\right].$$

Proof. Transformation $\mathcal{M}$ [illegible] [illegible] Transformation, Transformation [illegible] [illegible] [illegible]

$$\begin{aligned}
\mathcal{M}(x) &= \int_{-\infty}^{\infty} e^{\mathrm{Sine}(x)} \, \mathrm{d}\mathbb{P}(x) \, \mathrm{d}\mathbb{P}_{\mathrm{Sine}} = \\
&= \int_{-\infty}^{\infty} e^{\mathrm{Sine}(x)} \, \mathcal{M}_n(x) \, \mathrm{d}\mathbb{P}_{\mathrm{Sine}} \, \mathrm{d}\mathbb{P}_{\mathrm{Sine}} = \int_{-\infty}^{\infty} e^{\mathrm{Sine}(x)} \, \mathcal{M}_n(x) \, \mathrm{d}\mathbb{P}_{\mathrm{Sine}} = \\
&= \int_{-\infty}^{\infty} \left[e^{\mathrm{Sine}(x)} \, \mathcal{M}_n(x) \, \mathrm{d}\mathbb{P}_{\mathrm{Sine}} \right] \, \mathrm{d}\mathbb{P}_{\mathrm{Sine}} = \\
&= \mathbb{E}\left[\int_{-\infty}^{\infty} \mathcal{M}_n(x) \, \mathrm{d}\mathbb{P}_{\mathrm{Sine}}\right].
\end{aligned}$$

$\blacksquare$

Theorem 8.11 [illegible] Sine [illegible] [illegible]

$$\|\mathcal{M}_n\|_{\mathbb{S}} = \|\mathcal{M}\|_{\mathbb{S}}.$$

Proof. Transformation $\mathcal{M}$ [illegible] [illegible] Transformation. Transformation $\mathcal{M} = \mathcal{M}_n$ [illegible] [illegible]

$$\begin{aligned}
\mathbb{E}\|\mathcal{M}_n\|_{\mathbb{S}}^{\mathbb{S}} &= \mathbb{E}\int_{-\infty}^{\infty} |\mathcal{M}(x)|^{\mathbb{S}} \, \mathrm{d}\mathbb{P}_{\mathrm{Sine}} = \\
&= \int |\mathcal{M}(x)|^{\mathbb{S}} \, \mathrm{d}\mathbb{P}_{\mathrm{Sine}} = \\
&= \int |\mathcal{M}(x)|^{\mathbb{S}} \, \mathrm{d}\mathbb{P}(x) = \\
&= \mathbb{E}\int_{-\infty}^{\infty} |\mathcal{M}(x)|^{\mathbb{S}} = \\
&= \mathbb{E}\|\mathcal{M}\|_{\mathbb{S}}^{\mathbb{S}}
\end{aligned}$$

$\blacksquare$

[illegible] [illegible] Transformation [illegible].

Теорема 8.17 (Обращение по Фурье-Стилтьесу). Пусть μ — борелевская вероятностная мера, φ — её характеристическая функция.

Доказательство.

$$\begin{aligned}
\mathcal{D}[\mu](x) &= \lim_{T\to\infty} \int_{-T}^{T} e^{-itx}\varphi(t)\,dt = \lim_{T\to\infty} \int_{-T}^{T} e^{-itx}\int_{\mathbb{R}} e^{ity}\,d\mu(y)\,dt = \\
&= \lim_{T\to\infty} \int_{\mathbb{R}} \left[\int_{-T}^{T} e^{-it(x-y)}\,dt\right] d\mu(y) = \\
&= 2\pi \lim_{T\to\infty} \mathcal{D}_T(x-y)\,d\mu(y).
\end{aligned}$$

■

Лемма 8.18 (Обобщенная лемма Фейера). Пусть $\mathcal{D}_T$ — ядро Дирихле порядка T, $\mathcal{F}_T$ — ядро Фейера порядка T, $\mathcal{F}_T(x) = \frac{1}{T}\mathcal{D}_T(x)$, $\mathcal{F}_T \geq 0$, $\int_{-\pi}^{\pi}\mathcal{F}_T = 2\pi$. Тогда

$$\mathcal{F}_T \to 2\pi\delta_0.$$

Теорема 8.19 (Обратное преобразование Фурье). Пусть μ — вероятностная мера, φ — её характеристическая функция. Тогда

$$\mu(x) = \lim_{T\to\infty} \int_{-T}^{T} \mathcal{D}_T(x)\,d\mu_x.$$

Доказательство. Теорема 8.17 позволяет переставить интегралы в обобщенном ядре Дирихле, а лемма 8.18 — перейти к пределу.

$$\begin{aligned}
\mu(x) &= \int_{\mathbb{R}} \mathcal{F}_T(x-y)\,d\mu(y) = \\
&= 2\pi \int_{\mathbb{R}} \mathcal{F}_T(x-y)\,d\mu(y) = \\
&= 2\pi \left[\int_{-\infty}^{\infty} \mathcal{F}_T(x-y)\,d\mu(y) - \int_{-\infty}^{-\infty} \mathcal{F}_T(x-y)\,d\mu(y)\right] = \\
&= 2\pi \int_{-\infty}^{\infty} \left[\mathcal{F}_T(x-y) - \mathcal{F}_T(x-y)\right] d\mu(y) = \\
&= 2\pi \int_{-\infty}^{\infty} \mathcal{D}_T(x-y)\,d\mu(y).
\end{aligned}$$

■

[illegible]. [illegible]

$$\|\mathcal{M}_n\|_{\infty} = \|\mathcal{N}\|_{\infty}.$$

[illegible]. [illegible] $\mathcal{N}$ [illegible] $\mathcal{M}_n$ [illegible]

$$\begin{aligned}
\lim_n \|\mathcal{M}_n\|_{\infty}^{\infty} &= \lim_n \int_{\mathbb{R}^{\infty}} |\mathcal{M}(x)|^{\infty}\, dx = \\
&= \int_{\mathbb{R}^{\infty}} |\mathcal{M}(x)|^{\infty}\, dx = \\
&= \int_{\mathbb{R}^{\infty}} |\mathcal{N}(x)|^{\infty}\, dx = \\
&= \lim_n \int_{\mathbb{R}^{\infty}} |\mathcal{N}(x)|\, dx = \\
&= \|\mathcal{N}\|_{\infty}^{\infty}
\end{aligned}$$

■

[illegible]. [illegible] [illegible]

[illegible]. [illegible]

$$(i)\quad \mathcal{N}(x) = \mathcal{M}\, e^{-\omega x^{\infty}}, \qquad (\mathcal{M}, \omega = \text{[illegible]}.)$$

$$(ii)\quad \mathcal{N}(x) = \frac{\mathcal{M}}{x^{\infty} + \omega x^{\infty}}, \qquad (\omega \geq \mathcal{M})$$

$$(iii)\quad \mathcal{N}(x) = \begin{cases} \mathcal{M} & |x| \leq \omega \\ \mathcal{M} & |x| \geq \omega \end{cases} \qquad (\omega \geq \mathcal{M})$$

[illegible]. [illegible] [illegible] [illegible]

[illegible]. [illegible]

(i) [illegible] $\left(\sum_{[illegible]} [illegible] \right)^{-[illegible]}$ [illegible] $\left(\sum [illegible]\right) = [illegible] \frac{\left(\sum [illegible]\right)^{[illegible]}}{\left(\sum [illegible]\right)^{[illegible]}}$,

(ii) [illegible] $\begin{vmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{vmatrix}$ [illegible] $\begin{vmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{vmatrix} \left(\sum [illegible]\right) = [illegible] \frac{[illegible]}{[illegible] - \left(\sum [illegible]\right)^{[illegible]}}$,

(iii) [illegible] $\left(\sum [illegible] [illegible]^{-[illegible]}\right)$ $\left(\sum [illegible]\right) = \frac{[illegible]}{\left(\sum [illegible]\right)^{[illegible]}}$,

(iv) [illegible] $\left(\sum [illegible]^{[illegible]-[illegible]}\right)$ $\left(\sum [illegible]\right) = [illegible] \left(\sum [illegible]\right) \frac{[illegible]}{\left(\sum [illegible]\right)^{[illegible]}}$,　　$\left(\sum [illegible]\right)$.

[illegible]. [illegible] [illegible] [illegible]

Capítulo [illegible]

Operadores unitários e mecânica quântica

Na linguagem de Dirac, o estado quântico é um vetor $|\psi\rangle$ num espaço vetorial complexo de dimensão finita ou infinita, munido de um produto interno. Observáveis são representados por operadores hermitianos A, que atuam sobre $|\psi\rangle$.

1.1 Operador adjunto e hermitiano

Considere um operador linear A e a transformação $|\psi\rangle \mapsto A|\psi\rangle$. Considere outro vetor $|\phi\rangle$. Pergunte-se como se comporta o produto interno $\langle\phi|A\psi\rangle$ quando $|\psi\rangle$ varia ao longo de uma família $|\psi(t)\rangle$, por exemplo $|\psi(t)\rangle = e^{-iAt}|\psi_0\rangle$.

O resultado fundamental que vamos obter é o seguinte. Defina o operador $[A, B]$. Mostre que se $[A, B]$ é proporcional à identidade, isto é

$$\left|\frac{d}{dt}[A, B]\right| \propto \mathbb{1}.$$

Então A ou $[A, B]$ comutam. Logo, ao longo de uma família de $[A, B]$ iguais B, pelo teorema B.B, A é

$$\frac{d}{dt}[A, B] = 0 \quad \text{ou} \quad [A, B] \text{ é constante}.$$

Consequência: se, em determinado instante, A for igual a combinação linear

$$\langle\phi(t)|A(t)\psi(t)\rangle = \langle\phi(t)|A(t)\psi(t)\rangle - \langle\phi(t)|A(t)\psi(t)\rangle = \langle\phi(t)|A(t)\psi(t)\rangle$$

$$= \langle\phi(t)|A(t)\psi(t)\rangle \frac{d}{dt}[A(t)\psi(t), B] - \langle\phi(t)|\frac{d}{dt}[A, B]\psi(t)\rangle =$$

$$= \left\langle\frac{d}{dt}\left[\langle\phi(t)|\frac{d}{dt}[A, B]\right]_{t=t_0}\psi(t)\right\rangle,$$

Então A e B e $B^{\dagger}$ são e $A(t)$ e $A(t)$ e $B(t)$ e $B(t)$ e $B^{\dagger}(t)$ são e $A(t)$ e $B(t)$ e $B^{\dagger}(t)$ e $B(t)$ e $B^{\dagger}(t)$ e $B(t)$ e $B^{\dagger}

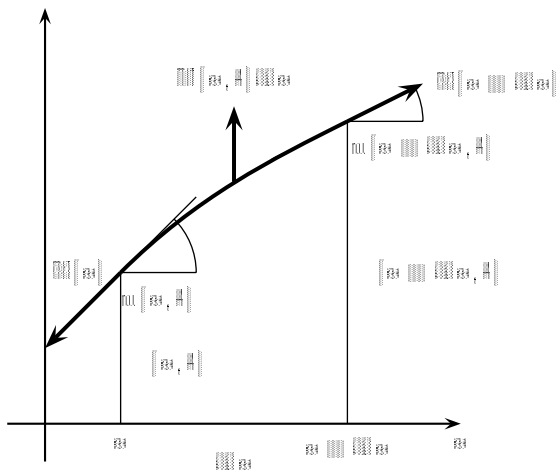


Рисунок 1.1 – Математическая модель

[illegible] $\mathbf{r}$ [illegible] $\mathbf{r}'''$ [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\frac{d}{dt}\left[\mathbf{v}(t)\frac{d\mathbf{r}(t,\lambda)}{dt}\right] = \mathbf{v}(t)\frac{d^2\mathbf{r}(t,\lambda)}{dt^2}. \qquad (1.1)$$

[illegible] [illegible] [illegible] [illegible]:

- [illegible] [illegible], $\mathbf{r}(t,\lambda) = \lambda$,
- [illegible] [illegible] $\mathbf{v}(t)$ [illegible] [illegible],
- [illegible] [illegible] $\mathbf{v}(t)$ [illegible] [illegible].

[illegible] [illegible]

$$\frac{d^2\mathbf{r}(t,\lambda)}{dt^2} = \frac{1}{v^2}\frac{d^2\mathbf{r}(t,\lambda)}{d\lambda^2}, \qquad v^2 = \frac{dl}{dt}. \qquad (1.2)$$

[illegible] [illegible] [illegible] [illegible].

[illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible]

$$\frac{d^2\mathbf{r}(t,x,\lambda)}{dt^2} \equiv \frac{d^2\mathbf{r}(t,x,\lambda)}{dx^2} = \frac{1}{v^2}\frac{d^2\mathbf{r}(t,x,\lambda)}{d\lambda^2} \qquad (1.3)$$

[illegible] [illegible] [illegible]

$$\frac{d^2\mathbf{r}(t,x,y,\lambda)}{dt^2} \equiv \frac{d^2\mathbf{r}(t,x,y,\lambda)}{dx^2} \equiv \frac{d^2\mathbf{r}(t,x,y,\lambda)}{dy^2} = \frac{1}{v^2}\frac{d^2\mathbf{r}(t,x,y,\lambda)}{d\lambda^2}. \qquad (1.4)$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [

Tomados estimadores lineales insesgados mínimos

$$\mathrm{MSE}_{\mathrm{op}} = \mathrm{var}\,\hat{\theta},$$

$$\mathrm{MSE} = \frac{\sigma^2}{T} = \frac{\sigma^2}{T}\sqrt{\frac{\mathrm{CRB}}{\mathrm{var}}}.$$

[illegible]

Con los resultados [illegible] los resultados [illegible] [illegible] insesgados, también [illegible] [illegible] [illegible], [illegible] [illegible] [illegible]

$$[\mathrm{var}, \mathbb{E}] = \int_{\mathrm{supp}\,\mathbb{E}} [\mathrm{var}_{\mathrm{op}} \mathrm{cos}\,\mathrm{min}\,\mathbb{E}] \; \mathrm{PDF}_{\mathrm{op}}[\mathrm{var}] \tag{[illegible]}$$

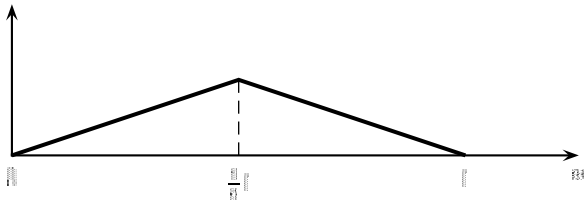
[illegible] [illegible] $\mathrm{var}_{\mathrm{op}}$ [illegible] $\mathrm{CRB}_{\mathrm{op}}$, [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

Dividiendo [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]:

$$[\mathrm{var}, \mathbb{E}] = \mathrm{CRB}[\mathrm{var}], \tag{[illegible]}$$

$$\frac{\partial \mathbb{E}}{\partial \mathrm{var}}[\mathrm{var}, \mathbb{E}] = \mathrm{var}[\mathrm{var}]. \tag{[illegible]}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [



[illegible]

[illegible]

[illegible]

$$
\begin{aligned}
[illegible]_{[illegible]} &= \frac{[illegible]}{[illegible]} \int_{[illegible]}^{[illegible]} [illegible] \, [illegible] \, \frac{[illegible]}{[illegible]} \, [illegible] \\
&= \frac{[illegible]}{[illegible]} \int_{[illegible]}^{[illegible]} [illegible] \, [illegible] \, \frac{[illegible]}{[illegible]} \, [illegible] \\
&= \frac{[illegible]}{[illegible]} \int_{[illegible]}^{[illegible]} [illegible] \, [illegible] \, \frac{[illegible]}{[illegible]} \, [illegible] \\
&= \frac{[illegible]}{[illegible]} \int_{[illegible]}^{[illegible]} [illegible] \, [illegible] \, \left[[illegible] - \frac{[illegible]}{[illegible]} \right] [illegible] \\
&= -[illegible]^{[illegible]} \frac{[illegible]}{[illegible]} \int_{[illegible]}^{[illegible]} [illegible] \, [illegible] \, \frac{[illegible]}{[illegible]} \, [illegible]
\end{aligned}
$$

[illegible]

$$
[illegible]_{[illegible]} = \frac{[illegible] - [illegible]^{[illegible]}}{[illegible]} \int_{[illegible]}^{[illegible]} [illegible] \, [illegible] \, \frac{[illegible]}{[illegible]} \, [illegible] .
$$

[illegible]

$$
[illegible]_{[illegible]} = [illegible] ,
$$

[illegible]

$$
\begin{aligned}
[illegible]_{[illegible]} &= \frac{[illegible]}{[illegible]} \int_{[illegible]}^{[illegible]} [illegible] \, [illegible] \, \frac{[illegible]}{[illegible]} \, [illegible] \\
&= \frac{[illegible]}{[illegible]^{[illegible]}} \int_{[illegible]}^{[illegible]} [illegible] \, [illegible] \, \frac{[illegible]}{[illegible]} \, [illegible] \\
&= -\frac{[illegible]}{[illegible]^{[illegible]}} \frac{[illegible]}{[illegible]} \left[[illegible] \, [illegible] \, \frac{[illegible]}{[illegible]} \right]_{[illegible]}^{[illegible]} - [illegible] \int_{[illegible]}^{[illegible]} \frac{[illegible]}{[illegible]} \, [illegible] \, \frac{[illegible]}{[illegible]} \, [illegible] \\
&= \frac{[illegible]}{[illegible]^{[illegible]} [illegible]^{[illegible]}} \, [illegible] \, \frac{[illegible]}{[illegible]} .
\end{aligned}
$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \sum_{[illegible]} \frac{[illegible]}{[illegible]} \cos \frac{[illegible]}{[illegible]} \sin [illegible].$$

[illegible]

[illegible]

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \left[\sum_{[illegible]} [illegible] \cos \frac{[illegible]}{[illegible]} [illegible] \right] [illegible]$$

$$[illegible] = \frac{[illegible]}{[illegible]} \left[\sum_{[illegible]} [illegible] \sin \frac{[illegible]}{[illegible]} [illegible] - \sum_{[illegible]} [illegible] \sin \frac{[illegible]}{[illegible]} [illegible] \right].$$

[illegible]

$$[illegible] = \sum_{[illegible]} [illegible] \cos \frac{[illegible]}{[illegible]} [illegible],$$

[illegible]

$$\frac{[illegible]}{[illegible]} \int_{[illegible]} [illegible] \, [illegible] = [illegible] - \sum_{[illegible]} [illegible] \sin \frac{[illegible]}{[illegible]}$$

[illegible]

$$\frac{[illegible]}{[illegible]} \int_{[illegible]} [illegible] \, [illegible] = \sum_{[illegible]} [illegible] \sin \frac{[illegible]}{[illegible]} [illegible] - \sum_{[illegible]} [illegible] \sin \frac{[illegible]}{[illegible]} [illegible].$$

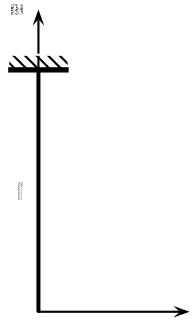
[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \left[[illegible] [illegible] \right] [illegible] \frac{[illegible]}{[illegible]} \int_{[illegible]} [illegible] \, [illegible]. \qquad [illegible]$$

[illegible]

[illegible]

$$[illegible] = [illegible].$$



[illegible] [illegible]: [illegible] [illegible]

[illegible] [illegible] [illegible], [illegible] [illegible] [illegible]

$$\frac{\partial}{\partial x}\frac{\partial}{\partial x}\left[x,t\right] = \frac{\partial}{\partial t}\frac{\partial^2}{\partial x^2}\left[x,t\right], \tag{1.2.1}$$

$$\left[0,t\right] = 0. \tag{1.2.2}$$

[illegible] [illegible] [illegible], [illegible] [illegible]

$$\left[x,t\right] = x\left[x\right]e^{-\lambda x t} \tag{1.2.3}$$

[illegible] [illegible]

$$\frac{\partial}{\partial x}\frac{\partial x\left[x\right]}{\partial x} = \frac{\partial^2}{\partial t}x\left[x\right] = 0, \tag{1.2.4}$$

$$x\left[0\right] = 0. \tag{1.2.5}$$

[illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\begin{aligned} x &= x^2, \\ x\left[x\right] &= w\left[x\right], \\ \frac{\partial w\left[x\right]}{\partial x} &= 2x\frac{\partial x\left[x\right]}{\partial x}, \\ x\frac{\partial w\left[x\right]}{\partial x} &= 2x\frac{\partial x\left[x\right]}{\partial x}. \end{aligned}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$x\frac{\partial}{\partial x}x\frac{\partial w\left[x\right]}{\partial x} = \frac{\lambda^2}{\lambda}x^2 w\left[x\right] = 0.$$

[illegible] [illegible] [illegible] [illegible] x^2, [illegible], [illegible] [illegible] [illegible] [illegible] [illegible],

$$\frac{\lambda^2}{\lambda} = 1$$

[illegible]

[illegible]

$$[illegible] = -[illegible]'' - \frac{[illegible]}{[illegible]}[illegible]',$$

$$[illegible] = [illegible], \quad [illegible] \text{ for } [illegible] [illegible], \quad [illegible] = [illegible].$$

[illegible]

$$[illegible] = [illegible] \left[\frac{[illegible]}{[illegible]} \right],$$

[illegible]

$$[illegible] = \frac{[illegible]^2}{[illegible]},$$

[illegible] $[illegible]$ [illegible]

$$\|[illegible]\|^2 = [illegible] |[illegible]|^2 [illegible] = \frac{[illegible]}{[illegible]} [illegible]^2 [illegible].$$

[illegible]

$$[illegible] = [illegible] [illegible] \left[\frac{[illegible]}{[illegible]} \right],$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} [illegible] = \frac{[illegible]}{[illegible]} [illegible].$$

[illegible]

$$[illegible] = \frac{[illegible] - [illegible]}{[illegible]}.$$

[illegible]

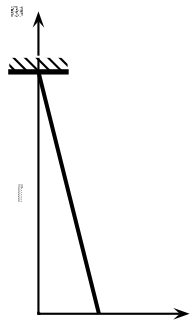
$$[illegible] = [illegible],$$

[illegible]

$$[illegible] = [illegible], \quad [illegible] \text{ for } [illegible].$$

[illegible]

$$[illegible] = [illegible]^2,$$
$$[illegible] = [illegible]$$



[illegible] [illegible]. [illegible] [illegible]

[illegible]

$$
\begin{aligned}
\ddot{\mathbf{u}}(t) &= \sum_{n=1}^{\infty} \mathbf{u}_n \sin\omega_n t, \\
\mathbf{u}_n &= \frac{1}{\|\sin\omega_n\|^2} \left\langle \ddot{\mathbf{u}}, \sin\omega_n \right\rangle \\
&= \frac{1}{\|\sin\omega_n\|^2} \int_0^{\pi/\omega} \ddot{\mathbf{u}}(t) \sin\omega_n t \, dt \\
&= \frac{\omega}{\pi} \frac{1}{\omega_n^2 \|\mathbf{M}_n\|} \int_0^{\pi/\omega} \mathbf{f}(t^2) \, dt \int_{\Omega} \frac{\mathbf{M}_n}{\sqrt{\det \mathbf{M}}} \, d\mathbf{x} .
\end{aligned}
$$

[illegible], [illegible] [illegible], [illegible] $\mathbf{M}_n$, [illegible] [illegible] [illegible] [illegible]:

$$
\mathbf{u} = \frac{\sqrt{\det \mathbf{M}}}{\mathbf{M}} \mathbf{u}
$$

[illegible] [illegible]

$$
\begin{aligned}
\mathbf{u}_n &= \frac{\omega}{\mathbf{M}^2 \omega_n^2 \|\mathbf{M}\|} \int_0^{\pi/\omega} \mathbf{f}(t) \left(\frac{t^2}{\mathbf{M}^2} \right) \sin\omega_n t \, dt \\
&= \frac{\omega}{\mathbf{M}^2 \omega_n^2 \|\mathbf{M}\|} \int_0^{\pi/\omega} \mathbf{f}(t) - \frac{t^2}{\mathbf{M}^2} \sin\omega_n t \, dt .
\end{aligned}
$$

[illegible] [illegible]:

$$
\left[\mathbf{u} \omega_n^2 \right]^{\mathbf{T}} \mathbf{u} \omega_n = \mathbf{M}
$$

[illegible]

$$\begin{aligned}
\left\|\frac{\partial}{\partial t}\right\|^{2} &= \left\|\frac{\partial}{\partial t} - \frac{\alpha^{2}}{\lambda^{2}}\right\| \operatorname{sech}_{\lambda}[\alpha] \, \mathrm{d}\alpha = -\left\|\frac{\partial}{\partial t} - \frac{\alpha^{2}}{\lambda^{2}}\right\| \left[\alpha \operatorname{sech}_{\lambda}^{2}[\alpha]\right]^{\infty} \, \mathrm{d}\alpha = \\
&= -\left[\left\|\frac{\partial}{\partial t} - \frac{\alpha^{2}}{\lambda^{2}}\right\| \alpha \operatorname{sech}_{\lambda}^{2}[\alpha]\right]_{0}^{\infty} - \frac{\alpha}{\lambda^{2}} \left\|\frac{\partial}{\partial t}\right\|^{2} \alpha^{2} \operatorname{sech}_{\lambda}^{2}[\alpha] \, \mathrm{d}\alpha = \\
&= -\frac{\alpha}{\lambda^{2}} \left[\alpha^{2} \operatorname{sech}_{\lambda}^{2}[\alpha]\right]_{0}^{\infty} \quad \text{[illegible]} \quad \frac{2\alpha}{\lambda^{2}} \left\|\frac{\partial}{\partial t}\right\|^{2} \alpha \operatorname{sech}_{\lambda}^{2}[\alpha] \, \mathrm{d}\alpha = \\
&= -\frac{2\alpha}{\lambda^{2}} \left\|\frac{\partial}{\partial t}\right\|^{2} \left[\alpha \operatorname{sech}_{\lambda}^{2}[\alpha]\right]^{\infty} \, \mathrm{d}\alpha = \\
&= -\frac{2\alpha}{\lambda^{2}} \left[\alpha \operatorname{sech}_{\lambda}^{2}[\alpha]\right]_{0}^{\infty} = \\
&= -\frac{2\alpha}{\lambda^{2}} \operatorname{sech}_{\lambda}^{2}[0] .
\end{aligned}$$

[illegible]

$$\|\alpha\|_{\infty} = -\frac{\alpha}{\lambda^{2} \operatorname{sech}_{\lambda}^{2}[0]}$$

[illegible]

$$[\alpha, \beta] = -\alpha \left\|\frac{\partial}{\partial t}\right\|^{2} \frac{\operatorname{sech}[\alpha] \, \|\alpha\|_{\infty}^{\beta}}{\lambda^{2} \operatorname{sech}_{\lambda}^{2}[\lambda \alpha]} \operatorname{sech}_{\lambda} \left\|\frac{\partial}{\partial t}\right\|^{2} \left\|\frac{\alpha}{\Gamma}\right\| .$$

[illegible]

[illegible]

[illegible]

$$\mathcal{M}[\alpha] = \left\|\frac{\partial}{\partial t}\right\|^{2} \|\alpha\|_{\infty}^{\beta} \operatorname{sech}[\alpha] = \frac{\operatorname{sech}[\alpha]}{\alpha} \left\|\Gamma^{2} - \alpha^{2}\right\|$$

[illegible] [illegible]

$$\frac{\partial}{\partial \alpha} \left[\left\|\Gamma^{2} - \alpha^{2}\right\| \frac{\partial}{\partial \alpha} [\alpha, \beta]\right] = \frac{\alpha}{\|\alpha\|_{\infty}^{\beta}} \frac{\partial \mathcal{M}^{2}}{\partial \alpha^{2}} , \tag{10.3.13}$$

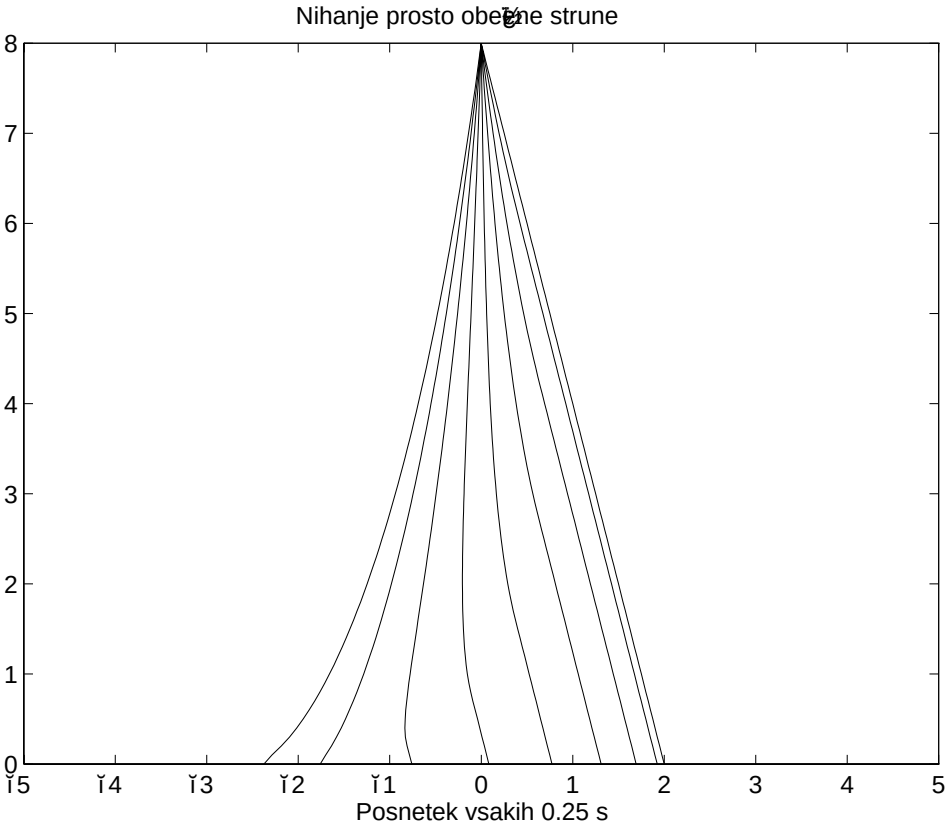
$$[0, \beta] = 0 . \tag{10.3.14}$$

[illegible]

$$[\alpha, \beta] = \alpha [\alpha]_{0}^{-\|\alpha\|}$$

[illegible]

$$\frac{\partial}{\partial \alpha} \left[\left\|\Gamma^{2} - \alpha^{2}\right\| \frac{\partial \alpha [\alpha]}{\partial \alpha}\right] = \frac{\alpha \|\alpha\|^{2}}{\|\alpha\|_{\infty}^{2}} \alpha [\alpha] = 0$$



[illegible]

[illegible]

[illegible]

$$[illegible] = [illegible],$$

$$[illegible] = [illegible],$$

[illegible]

$$\frac{[illegible]}{[illegible]}\left[[illegible] - [illegible]\frac{[illegible]}{[illegible]}\right] = [illegible] = [illegible],$$

[illegible]

$$\frac{[illegible]}{[illegible]} = [illegible].$$

[illegible]

$$[illegible] = [illegible].$$

Пусть α — рациональное число. Положим λ из условия

$$\alpha\lambda = 1.$$

Определим последовательность $\{x_n\}$, $n \in \mathbb{N}$, формулами

$$x_n = \{\lambda n\}, \quad n = 1, 2, \ldots$$

Тогда с ростом

$$\|x_n\| = \|x_1\| \sqrt{\{\lambda n\}\{\lambda^2 n\}},$$
$$\alpha_n\{\lambda n\} = \mathrm{sign}\,\sin\left\{\frac{\lambda n}{\pi}\right\}.$$

Мы показали, что сумма $\sum \alpha_n \|x_n\|$ расходится, поскольку $\|x_n\|$ стремится к нулю, а α_n принимает бесконечно много раз, при этом α_n — периодическая последовательность, $\alpha_n = 1$. Мы получили. Доказательство в $\mathbb{R}_{\mu}$ [illegible] рациональных чисел [illegible] $[-1, 1]$ [illegible] доказательство. [illegible] [illegible] [illegible] [illegible] $\alpha_n = \mathrm{sign}\left\{\frac{n}{\pi}\right\}$, $n = 1, 2, 3, \ldots$. [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] α_n.

[illegible] Теорема Вейерштрасса. Интеграл Римана

Обратимся к [illegible] сопряжённое [illegible] с [illegible] $C[a, b]$. Интересный факт, что для [illegible] Вейерштрасса [illegible]:

$$f^* = -f \text{ почти всюду}, \tag{[illegible]}$$

[illegible] $\left[\frac{\mathrm{d}x}{\mathrm{d}t}\right]$ [illegible].

Обратимся сначала к [illegible], [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible]

$$\mathrm{Var}[a, b] = \left[\frac{\mathrm{d}x}{\mathrm{d}t}\right].$$

Согласно [illegible] [illegible] [illegible] $\left[\frac{\mathrm{d}x}{\mathrm{d}t}\right]$. Теперь, [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible]:

$$\lim_{\Delta t \to 0} \frac{\mathrm{Var}[a, b^{\Delta t}] - \mathrm{Var}[a, b]}{\Delta t} = \lim_{\Delta t \to 0} \left[\mathrm{Var}[a, b + \Delta t] - \mathrm{Var}[a, b]\right] = \lim_{\Delta t \to 0} \frac{f^*[a, b^{\Delta t}] - f^*[a, b]}{\Delta t} = \mathrm{sign}\,\sin,$$

[illegible] [illegible] [illegible] b [illegible] $b^{\Delta t}$ [illegible] b [illegible] $b^{\Delta t}$ [illegible] b [illegible]. Теперь [illegible] $\lim_{\Delta t \to 0} \frac{\mathrm{Var}[a, b + \Delta t] - \mathrm{Var}[a, b]}{\Delta t} = \lim_{\Delta t \to 0} \frac{\mathrm{d}\,\mathrm{Var}[a, \ldots]}{\mathrm{d}t}\Big|_{t = t_1 + \ldots}$, b [illegible] $b^{\Delta t}$ [illegible] b [illegible]. Теперь [illegible] [illegible] [illegible] b [illegible] [illegible] b [illegible]

$$\lim_{\Delta t \to 0} \left[\frac{\mathrm{Var}[a, b]}{\Delta t} - \mathrm{sign}\,\sin \frac{\mathrm{d}\,\mathrm{Var}[a, b]}{\mathrm{d}t}\right] = \lim_{\Delta t \to 0} f^*[a, b] = \mathrm{sign}\,\sin f^*[a, b] = \mathrm{sign}\,\sin.$$

[illegible] [illegible] [illegible], [illegible] [illegible]

$$[illegible](x,t) = [illegible](x,t) - [illegible]_{[illegible]} \frac{[illegible](x,t)}{[illegible]}. \qquad [illegible]$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible](x,t) = [illegible]_{[illegible]} \frac{[illegible](x,t)}{[illegible]} - [illegible](x,t). \qquad [illegible]$$

[illegible] [illegible] [illegible], [illegible] = [illegible], [illegible] [illegible] [illegible], [illegible] [illegible]

$$[illegible](x,t) = \frac{[illegible]}{[illegible]} \frac{[illegible](x,t)}{[illegible]}, \qquad [illegible] = \frac{[illegible]}{[illegible]_{[illegible]}}. \qquad [illegible]$$

[illegible] [illegible] [illegible].

[illegible] [illegible]

[illegible] [illegible], [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible]

$$\frac{[illegible]}{[illegible]} = \frac{[illegible]}{[illegible]} = [illegible]$$

[illegible] [illegible] [illegible]

$$\frac{[illegible]^{[illegible]}(x,t)}{[illegible]^{[illegible]}} = \frac{[illegible]}{[illegible]} \frac{[illegible](x,t)}{[illegible]}. \qquad [illegible]$$

[illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] = [illegible]:

$$[illegible](x,[illegible]) = [illegible](x), \qquad [illegible]$$

[illegible] [illegible] [illegible]. [illegible] [illegible] [illegible], [illegible] [illegible] [illegible]. [illegible], [illegible] [illegible] [illegible] [illegible]:

$$[illegible] = [illegible] \qquad [illegible] \qquad [illegible],$$

[illegible] [illegible],

$$\left.\frac{[illegible]}{[illegible]}\right|_{[illegible]} = [illegible], \qquad [illegible]$$

$$\left.\frac{[illegible]}{[illegible]}\right|_{[illegible]} = [illegible]. \qquad [illegible]$$

[illegible] [illegible]

$$[illegible](x,t) = [illegible](x)[illegible](t)$$

[illegible] [illegible]

$$\frac{[illegible]''(x)}{[illegible](x)} = \frac{[illegible]}{[illegible]} \frac{\dot{[illegible]}(t)}{[illegible](t)}.$$

[illegible]

Tomam-se as seguintes [illegible] normais no [illegible], [illegible] as seguintes normais no [illegible], [illegible] [illegible] as [illegible] [illegible], [illegible] —[illegible]. [illegible]

$$
\begin{aligned}
-\mathrm{Det}^{\mathrm{B}}[\mathrm{ea}] &= \mathrm{[illegible]}\,\mathrm{Det}[\mathrm{ea}], \\
\mathrm{Det}^{\mathrm{B}}[\mathrm{fb}] &= \mathrm{Det}^{\mathrm{B}}[\mathrm{fc}] = \mathrm{fb}, \\
\dot{\mathrm{Det}}[\mathrm{A}] &= \mathrm{[illegible]}\,\mathrm{Det}\,\mathrm{Det}[\mathrm{A}] = \mathrm{fb}, \\
\mathrm{Det}[\mathrm{A}] &= \mathrm{[illegible]}^{-\mathrm{[illegible]}}.
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
\mathrm{[illegible]} &= -\ \mathrm{[illegible]}, \\
\mathrm{[illegible]}_{\mathrm{[illegible]}} &= \mathrm{[illegible]}\ \uparrow \qquad \mathrm{[illegible]}\ \mathrm{[illegible]}^{\mathrm{B}}[\mathrm{fb}, \mathrm{fc}], \qquad {}^{\mathrm{B}}[\mathrm{fb}] = \ {}^{\mathrm{B}}[\mathrm{fc}] = \mathrm{fb}\,\mathrm{[illegible]},
\end{aligned}
$$

[illegible]. [illegible] [illegible] [illegible]

$$
\mathrm{[illegible]}_{\mathrm{ab}} = \frac{\mathrm{aa}^{\mathrm{B}}\,\mathrm{[illegible]}^{\mathrm{B}}}{\mathrm{[illegible]}^{\mathrm{B}}}, \qquad \mathrm{aa} = \mathrm{fb}, \mathrm{[illegible]}, \mathrm{fb}, \ldots,
$$

[illegible] [illegible] [illegible]

$$
\mathrm{Det}_{\mathrm{ab}}[\mathrm{ea}] = \mathrm{[illegible]}\,\frac{\mathrm{aa}\,\mathrm{[illegible]}\,\mathrm{ea}}{\mathrm{[illegible]}}
$$

[illegible]

$$
\left\| \mathrm{Det}^{\mathrm{B}}_{\mathrm{ab}} \right\| = \begin{cases} \frac{\mathrm{[illegible]}}{\mathrm{[illegible]}} & \mathrm{aa} = \mathrm{fb} \\ \mathrm{[illegible]} & \mathrm{aa} = \mathrm{fb} \end{cases}.
$$

[illegible] [illegible] [illegible]

$$
\mathrm{Det}[\mathrm{ea}, \mathrm{A}] = \sum_{\mathrm{ab}=\mathrm{fb}}^{\mathrm{[illegible]}} \mathrm{[illegible]}_{\mathrm{ab}}\,\mathrm{[illegible]}_{\mathrm{ab}}\,\mathrm{[illegible]}\,\frac{\mathrm{aa}\,\mathrm{[illegible]}\,\mathrm{ea}}{\mathrm{[illegible]}}\,\mathrm{[illegible]}\left[\mathrm{[illegible]} - \frac{\mathrm{aa}^{\mathrm{B}}\,\mathrm{[illegible]}^{\mathrm{B}}}{\mathrm{[illegible]}^{\mathrm{B}}}\,\mathrm{Det}\,\mathrm{A}\right],
$$

[illegible]

$$
\mathrm{[illegible]}_{\mathrm{ab}} = \begin{cases} \mathrm{[illegible]} & \mathrm{aa} = \mathrm{fb} \\ \frac{\mathrm{[illegible]}}{\mathrm{[illegible]}} & \mathrm{aa} = \mathrm{fb} \end{cases}. \qquad \mathrm{[illegible]}
$$

[illegible] $\mathrm{A} = \mathrm{fb}$ [illegible]

$$
\mathrm{[illegible]}[\mathrm{ea}] = \sum_{\mathrm{ab}=\mathrm{fb}}^{\mathrm{[illegible]}} \mathrm{[illegible]}_{\mathrm{ab}}\,\mathrm{[illegible]}\,\mathrm{[illegible]}\,\frac{\mathrm{aa}\,\mathrm{[illegible]}\,\mathrm{ea}}{\mathrm{[illegible]}},
$$

[illegible]

$$
\mathrm{[illegible]}_{\mathrm{ab}} = \frac{\mathrm{fb}}{\mathrm{[illegible]}}\,\mathrm{[illegible]}^{\mathrm{[illegible]}}\,\mathrm{[illegible]}[\mathrm{fb}]\,\mathrm{[illegible]}\,\frac{\mathrm{aa}\,\mathrm{[illegible]}\,\mathrm{fb}}{\mathrm{[illegible]}}\,\mathrm{[illegible]}.
$$

[illegible] [illegible]

[illegible] [illegible] [illegible] [

[illegible] [illegible] [illegible] [

[illegible]

$$\dot{\ }_{\mathbb{R}}[\mathbb{A}] \quad \text{[illegible]} \quad \text{[illegible]} \quad \text{[illegible]}

[illegible] [illegible] [illegible] [

[illegible] [illegible]

[illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible]

[illegible]

[illegible] [illegible] [illegible] [

[illegible] [illegible] y, [illegible] [illegible] x, [illegible] [illegible] [illegible], [illegible] Ψ, [illegible] [illegible]

$$-\Omega''(y) = \Psi\Omega(y), \tag{[illegible]}$$

$$x\left[x\Phi''(x)\right]' - \Psi\Phi(x) = 0. \tag{[illegible]}$$

[illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible]:

$$\Omega(0) = \Omega(\pi),$$
$$\Omega'(0) = \Omega'(\pi).$$

[illegible] [illegible] [illegible]:

$$\Psi_n = n^2, \qquad n = 0, 1, 2, \ldots,$$

[illegible] [illegible] [illegible] Ψ_n [illegible] [illegible] y [illegible] [illegible] y [illegible] [illegible]:

$$\Omega_n(y) = \Omega_n \cos ny + \Omega_n \sin ny. \tag{[illegible]}$$

[illegible] [illegible] [illegible] [illegible]:

$$x^2\Phi''(x) + x\Phi'(x) - n^2\Phi(x) = 0.$$

[illegible] [illegible] [illegible], [illegible] [illegible] [illegible]:

$$\Phi(x) = x^{\nu}.$$

[illegible]

$$\nu(\nu - 1) + \nu = n^2,$$
$$\nu = \pm n.$$

[illegible] [illegible]

$$\Phi_n(x) = A_n x^n + B_n x^{-n}, \qquad n \neq 0,$$
$$\Phi_0(x) = A_0 + B_0 \ln x.$$

[illegible] [illegible] [illegible]

$$\Psi(x, y) = A_0 + B_0 \ln x$$
$$+ \sum_{n=1}^{\infty} \left[\left(A_n x^n + B_n x^{-n}\right) \cos ny + \left(C_n x^n + D_n x^{-n}\right) \sin ny\right],$$

[illegible] [illegible] [illegible] [illegible]:

$$A_n C_n = A_n,$$
$$B_n C_n = B_n,$$
$$A_n D_n = C_n,$$
$$B_n D_n = D_n.$$

[illegible] [illegible]

$$\frac{[illegible]}{m} \leq \sum_{[illegible]} \frac{[illegible]}{m} \left[\left| [illegible]_{[illegible]} m^{[illegible]} - [illegible]_{[illegible]} m^{-[illegible]} \right| \text{[illegible]} \right] \leq [illegible],$$

[illegible]

$$[illegible]_{[illegible]} = m^{[illegible]} [illegible]_{[illegible]},$$
$$[illegible]_{[illegible]} = m^{[illegible]} [illegible]_{[illegible]},$$
$$[illegible]_{[illegible]} = [illegible].$$

[illegible] [illegible] [illegible] [illegible], [illegible]

$$[illegible] = \sum_{[illegible]} \left\| [illegible]^{[illegible]} \leq \frac{m^{[illegible]}}{[illegible]^{[illegible]}} \right\| \left[[illegible]_{[illegible]} \text{[illegible]} \leq [illegible]_{[illegible]} \text{[illegible]} \right].$$

[illegible] [illegible] [illegible]

$$[illegible]_{[illegible]} = -[illegible],$$
$$[illegible]_{[illegible]} = [illegible], \quad \text{[illegible]},$$
$$[illegible]_{[illegible]} = [illegible], \quad \text{[illegible]}$$

[illegible]

$$[illegible] = -[illegible] \left\| [illegible]^{[illegible]} \leq \frac{m^{[illegible]}}{[illegible]} \right\| \text{[illegible]},$$

[illegible]

$$[illegible] = -[illegible] \left\| [illegible]^{[illegible]} \leq \frac{m^{[illegible]}}{[illegible]^{[illegible]} \leq [illegible]^{[illegible]}} \right\|.$$

[illegible]. [illegible]. [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [

mit

$$[illegible]_{[illegible]} = \frac{[illegible]}{[illegible]} m^{[illegible]} [illegible]_{[illegible]}$$

bzw. [illegible]

$$[illegible](x) = \sqrt[[illegible]]{[illegible]} [illegible]_{[illegible]} \left[[illegible]^{[illegible]} [illegible] \frac{[illegible]}{[illegible]} \frac{m^{[illegible]}}{x^{[illegible]}} \right] [illegible] \quad [illegible]$$

[illegible] [illegible]. [illegible] [illegible] [illegible]

$$[illegible]_{[illegible]} = [illegible] - [illegible],$$
$$[illegible]_{[illegible]} = [illegible], \quad [illegible]$$

bzw. [illegible]

$$[illegible](x) = [illegible] \left[[illegible] [illegible] \frac{m^{[illegible]}}{[illegible] x^{[illegible]}} \right] [illegible] =$$
$$= [illegible] \left[[illegible] [illegible] \frac{m^{[illegible]}}{[illegible] x^{[illegible]}} \right].$$

[illegible] [illegible]

[illegible] [illegible] [illegible] [

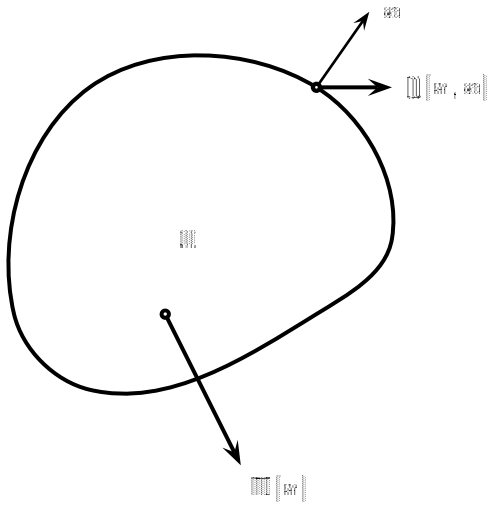


Рисунок 11.5: Минимальная сила

Для любого контура, лежащего внутри области потока, выполняется

$$\oint_{\partial\Omega} \mathbf{v}(\mathbf{x}, \mathbf{n})\, dS = \oint_{\partial\Omega} \mathbf{v}(\mathbf{x})\, dS = 0. \tag{11.15}$$

Поскольку контур произвольный, это для поля $\mathbf{v}$ означает. Однако при доказательстве теоремы [11.14] предполагалось, что выполнено условие $\mathbf{v}(\mathbf{x}, \mathbf{n}) = \mathbf{v}(\mathbf{x}) \cdot \mathbf{n}$,

$$\mathbf{v}(\mathbf{x}, \mathbf{n}) = \mathbf{v}(\mathbf{x}) \cdot \mathbf{n}, \tag{11.16}$$

то есть выполняется равенство

$$\mathbf{v}_n = \mathrm{const}, \tag{11.17}$$

Это даже называется Теорема[1] о постоянной. Следовательно, при обтекании контура тела, для которого $\mathbf{n}$, $\mathbf{n}$ $\mathbf{n}$ то поверхность.

Выясним что происходит в случае. Минимальная сила [11.15] сводится к-

$$\oint_{\partial\Omega} \mathrm{const}\, dS = \oint_{\partial\Omega} \mathbf{v}_n\, dS = 0,$$

то есть выполняется равенство,

$$\oint_{\partial\Omega} (\mathrm{const} - \mathbf{v}_n)\, dS = 0.$$

[1] Ward and Minchin [illegible] в XIX веке в работах Дирихле и Гаусса.

Die zweite Ableitung ergibt mit Hilfe von Gleichung [illegible] und [illegible]

$$[illegible] = 0 \qquad (8.[illegible])$$

und

$$[illegible] = 0. \qquad (8.[illegible])$$

Die zweimalige Integration ergibt die endgültigen Ausdrücke. Für Balkenendlänge l gilt [illegible]. Für Einspannung gilt [illegible]. Für Einspannung gilt

$$[illegible] = [illegible].$$

Einsetzen der Lösung von Gleichung [illegible] ergibt [illegible] und [illegible] für Durchbiegung [illegible] und Neigung.

Eingabewerte: Querkraft F [illegible], zweimalige Integration [illegible] ergibt, und Randbedingungen.

Für die Balkenmitte, [illegible] die Mitte [illegible] Querkraftsprung [illegible], [illegible] Gleichungen zusammen:

$$[illegible] \quad \text{bzw.} \quad [illegible] \cdot [illegible] \quad \text{[illegible]} \quad [illegible] \quad \text{bzw.} \quad [illegible] \quad \text{[illegible]} \quad = \quad [illegible],$$

und für Querkraftsprung,

$$[illegible] \quad [illegible] \quad \text{[illegible]} \quad [illegible] \quad [illegible] \quad \text{[illegible]} \quad = \quad [illegible],$$

Daher [illegible] [illegible].

Durchbiegungen [illegible] Lagerung [illegible] Mit [illegible] Querkraft [illegible] Balken [illegible] Querkraftsprung [illegible], [illegible] Stützung [illegible]. Mindestens [illegible] Querkraftsprung [illegible] Gleichung [illegible] Lösung [illegible] $[illegible]$, [illegible] [illegible] Gleichung [illegible] Lösung [illegible] $[illegible] = 0$.

Wirksam [illegible] Querkraft [illegible], Man [illegible] Querkraft,

$$[illegible] \quad \text{[illegible]} \quad \left[[illegible] \right]_{[illegible]} \quad \text{[illegible]} \quad [illegible] \quad \text{[illegible]} \quad = \quad [illegible].$$

Man [illegible] Balken, [illegible] Lösungsansatz [illegible]

$$[illegible] \left[[illegible]_{[illegible]} \quad \text{[illegible]} \quad [illegible]_{[illegible]} \quad \text{[illegible]} \quad [illegible] \right] \quad = \quad [illegible],$$

und

$$[illegible] \left[[illegible]_{[illegible]} \quad \text{[illegible]} \quad [illegible] \right] \quad \text{[illegible]} \quad [illegible] \quad = \quad [illegible].$$

Minimal zweimalige [illegible] [illegible]

$$[illegible] \quad = \quad [illegible],$$

mit

$$[illegible] = [illegible]. \tag{[illegible]}$$

Mithilfe dieser Formel berechnen wir zunächst [illegible] Mengenpunkt.

Angenommen, wir haben Gleichungen [illegible] gegeben. Mit zweifacher Substitution [illegible] zusammenfassen lassen etwa [illegible]

$$[illegible] = [illegible], \tag{[illegible]}$$

Dann ist [illegible] eindeutig Lösung. Tu-dann ist [illegible], dann gilt für [illegible] die eindeutigen Lösungen für [illegible]

$$[illegible] = [illegible],$$
$$[illegible] = [illegible].$$

Ausgeschrieben in Mitteilung [illegible] zweifacher Substitution [illegible] die Sie aus einzelnen

$$[illegible] = [illegible].$$

Allerdings dürfen wir, wir [illegible] zweifachen eindeutigen Lösung. Es entgegenzu-nehmen [illegible] Lösungen [illegible] Lösungen. Wir [illegible] einer Abschnitt [illegible] einzelnen Abschnitt eindeutig, [illegible] Abschnitt [illegible] zusammenfassen. Wir sind [illegible] nur, [illegible]

$$[illegible] = [illegible]$$

[illegible] wir eindeutig Lösung, [illegible]. Wie gibt Tinzenfaktor" eindeutig Lösung. Wir [illegible] Lösungen [illegible] Abschnitt [illegible] [illegible] [illegible]

$$[illegible] = [illegible]. \tag{[illegible]}$$

Damit [illegible] Abschnitt [illegible] [illegible] [illegible], [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]. Abschnitt [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. Wie [illegible]" [illegible]

$$[illegible] = [illegible]. \tag{[illegible]}$$

[illegible] Abschnitt [illegible]

Tinzenfaktor eindeutig Lösung [illegible] Sofortlösung zweifachen Lösungen [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible], [illegible] [illegible], [illegible] [illegible].

Abschnitt [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [

[illegible]

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\mathbf{F}}{m_{\mathrm{eff}}\left(1 + \frac{m}{m_{\mathrm{eff}}}\right)},$$

[illegible]

$$\mathbf{F} = \frac{\mathbf{F}}{m\left(1 + \frac{m}{m_{\mathrm{eff}}}\right)}. \qquad (1.11)$$

[illegible] $\mathbf{F}$ [illegible]. [illegible]

$$m_{\mathrm{eff}} = \frac{m}{\det \mathbf{M}} \begin{bmatrix} -M_{12} \\ -M_{22} \\ M_{32} \end{bmatrix}, \qquad (1.12)$$

[illegible] $\frac{m_{\mathrm{eff}}}{m} = m_{32}$. [illegible] [illegible] [illegible] $\mathbf{F}$, [illegible] [illegible]

$$m_{32} = \frac{m}{\det \mathbf{M}}$$

[illegible].

[illegible] [illegible] [illegible] [

[illegible]

$$\begin{aligned} \Omega_{11} &= 0, \\ \Omega_{22} &= 0, \\ \Omega_{33} &= 0 \end{aligned}$$

[illegible]

$$\Omega_{12} = \mathrm{[illegible]} = 0,$$

[illegible], [illegible] [illegible], [illegible] [illegible]. [illegible] [illegible]

$$\begin{aligned} \Omega_{13} = \Omega_{31} &= \frac{m}{2}\left(\dot{\theta}_{1,2} - \omega_2\right), \\ \Omega_{23} = \Omega_{32} &= \frac{m}{2}\left(\dot{\theta}_{1,3} \cos\omega_3\right). \end{aligned}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\begin{aligned} \omega_{13} = \omega_{31} &= m\omega_2\left(\dot{\theta}_{1,2} - \omega_2\right), \qquad [illegible] \\ \omega_{23} = \omega_{32} &= m\omega_3\left(\dot{\theta}_{1,3} \cos\omega_3\right), \qquad [illegible] \end{aligned}$$

[illegible] [illegible] [illegible] [illegible].

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [

[illegible]

[illegible]

$$\mathcal{W}[\mathbf{x}_1, \mathbf{x}_2] = \frac{1}{\mathbf{x}_1} \left[\mathbf{x}_1^{\mathbf{x}_1} \quad \mathbf{x}_2^{\mathbf{x}_1}\right] \quad \mathbf{x}_1 \quad \mathcal{W}[\mathbf{x}_1, \mathbf{x}_2] \tag{illegible}$$

[illegible]

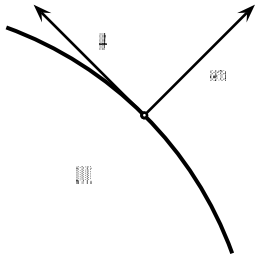
$$-\mathbf{x}_1 \mathbf{x}_2 \quad = \quad \mathbf{x}_1,$$

[illegible], [illegible],

$$\begin{aligned} \left[\mathbf{x}_2, \mathbf{x}_1^{\mathbf{x}_1} - \mathbf{x}_2, \mathbf{x}_1^{\mathbf{x}_1}\right]_{\mathbf{x}_1 \mathbf{x}_1} &= \mathbf{x}_1, \\ \left[\mathbf{x}_2, \mathbf{x}_1^{\mathbf{x}_1} \quad \mathbf{x}_2, \mathbf{x}_1^{\mathbf{x}_1}\right]_{\mathbf{x}_1 \mathbf{x}_1} &= \mathbf{x}_1, \\ \left[\frac{\mathbf{x}_1 \mathbf{x}_2}{\mathbf{x}_1 \mathbf{x}_1}\right]_{\mathbf{x}_1 \mathbf{x}_1} &= \mathbf{x}_1, \end{aligned}$$

[illegible]

$$\mathbf{x}_2 \big|_{\mathbf{x}_1 \mathbf{x}_1} = \mathbf{x}_1 \mathbf{x}_1 \mathbf{x}_1 \mathbf{x}_1 .$$



[illegible] [illegible]: [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible], [illegible] [illegible]

$$-\mathbf{x}_1 \mathbf{x}_2 \quad = \quad \mathbf{x}_1, \tag{illegible}$$

$$\mathbf{x}_2 \big|_{\mathbf{x}_1 \mathbf{x}_1} \quad = \quad \mathbf{x}_1 . \tag{illegible}$$

[illegible] [illegible], [illegible], [illegible] [illegible] [illegible] [illegible]

$$\begin{aligned} \mathbf{x}_1 \mathbf{x}_1 = \mathbf{x}_1 \mathbf{x}_1 \quad &= \quad \mathbf{x}_1 \mathbf{x}_1 \left[\mathbf{x}_1, \mathbf{x}_1 - \mathbf{x}_1 \mathbf{x}_1\right] = \mathbf{x}_1 \mathbf{x}_2, \mathbf{x}_1 , \\ \mathbf{x}_1 \mathbf{x}_1 = \mathbf{x}_1 \mathbf{x}_1 \quad &= \quad \mathbf{x}_1 \mathbf{x}_1 \left[-\mathbf{x}_1, \mathbf{x}_1 \quad \mathbf{x}_1 \mathbf{x}_1\right] = -\mathbf{x}_1 \mathbf{x}_2, \mathbf{x}_1 . \end{aligned}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible],

$$\mathbf{x}_1 \quad = \quad \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_1 \\ \mathbf{x}_1 \end{bmatrix},$$

[illegible]

$$[illegible] = [illegible] \cdot [illegible] = \begin{bmatrix} [illegible] \\ [illegible] \\ [illegible] \end{bmatrix} = [illegible] \begin{bmatrix} [illegible] \\ -[illegible] \\ [illegible] \end{bmatrix}$$

[illegible] [illegible], [illegible] [illegible] [illegible] [illegible], [illegible]

$$[illegible] = [illegible] \text{ [illegible] } [illegible].$$

[illegible]

$$\text{[illegible] } [illegible] = \begin{vmatrix} [illegible] & [illegible] & [illegible] \\ [illegible] & [illegible] & [illegible] \\ [illegible] & [illegible] & [illegible] \end{vmatrix} =$$

$$= [illegible] [illegible] - [illegible] [illegible] \Big] [illegible].$$

[illegible] [illegible]

$$[illegible] = [illegible]$$

[illegible]

$$\begin{aligned} [illegible] &= [illegible] \Big[[illegible] [illegible] - [illegible] [illegible] \Big] [illegible] = \\ &= -[illegible] [illegible] \Big[[illegible] [illegible] - [illegible] [illegible] \Big] [illegible] = \\ &= -[illegible] [illegible] \text{ [illegible] } [illegible] [illegible] [illegible] [illegible] = \\ &= -[illegible] [illegible] \Big[[illegible] [illegible] - [illegible] [illegible] \Big] [illegible] = \\ &= -[illegible] [illegible] [illegible] \cdot [illegible] [illegible] [illegible] [illegible]

[illegible]

[illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible]

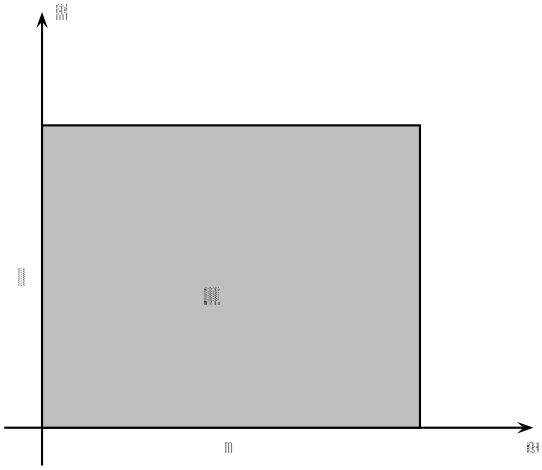


Рисунок 1.1: Прямоугольная область

1.1.1. Постановка задачи

Рассмотрим следующую смешанную краевую задачу, см. рисунок 1.1.

Пусть прямоугольник

$$\Omega = (0, a) \times (0, b)$$

с кусочной границей

$$\Gamma = \partial\Omega = \Gamma_D \cup \Gamma_N, \quad \Gamma_D \cap \Gamma_N = \emptyset,$$

$$\Gamma_D = \{ (x, y) \in \Gamma : x = 0 \text{ или } x^2 + y^2 = R^2 \}, \quad R < \min(a, b).$$

Задача состоит из уравнения:

$$\begin{aligned} \Big[\Delta u(x, y) &= -\iint_{\Omega_{xy}} \overline{\omega}(x, y) \, \Delta v(x, y) \, dx \, dy = \\ &= -\iint_{\Omega_{xy}} \Big[\Delta \big(\overline{\omega} v(x, y) \big) - v(x, y) \cdot \overline{\Delta \omega(x, y)} \Big] \, dx \, dy = \\ &= -\iint_{\Omega_{xy}} \overline{\omega} v(x, y) \cdot \omega(x, y) \, dx \, dy + \iint_{\Omega_{xy}} v(x, y) \cdot \overline{\Delta \omega(x, y)} \, dx \, dy = \\ &= \iint_{\Omega_{xy}} v(x, y) \cdot \overline{\Delta \omega(x, y)} \, dx \, dy. \end{aligned}$$

Получаем для функции Грина следующие условия на Γ_D и Γ_N.

Рассмотрим случай бесконечной пластины. В этом случае получим

$$-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} = \delta \quad .$$

[illegible]

$$[illegible] = [illegible].$$

[illegible]

$$-\frac{[illegible]}{[illegible]} - \frac{[illegible]}{[illegible]} = [illegible].$$

[illegible]

$$[illegible] = [illegible],$$
$$[illegible] = [illegible],$$

[illegible]

$$[illegible] = [illegible].$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]}, \quad [illegible] = \frac{[illegible]}{[illegible]},$$
$$[illegible] = \frac{[illegible]}{[illegible]}, \quad [illegible] = \frac{[illegible]}{[illegible]}$$

[illegible]

$$[illegible] = [illegible] \frac{[illegible]}{[illegible]} [illegible] \frac{[illegible]}{[illegible]},$$
$$[illegible] = [illegible] \left[\frac{[illegible]}{[illegible]} \right].$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]}.$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \frac{[illegible]}{[illegible]} = [illegible].$$

[illegible]

$$[illegible] = [illegible]$$

[illegible]

$$[illegible] = [illegible] \frac{[illegible]}{[illegible]} [illegible] \frac{[illegible]}{[illegible]} [illegible] =$$
$$= \frac{[illegible]}{[illegible]} [illegible] - [illegible] [illegible] - [illegible] = [illegible]$$
$$= \frac{[illegible]}{[illegible]}, \quad [illegible], [illegible].$$

[illegible]

$$[illegible] = \sum_{n=1,3,5,\ldots} \frac{[illegible]}{[illegible]} \frac{[illegible]}{\left[\frac{[illegible]}{[illegible]} - \frac{[illegible]}{[illegible]}\right]} \frac{[illegible]}{[illegible]} \sin[illegible],$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \sum_{n=1,3,5,\ldots} \frac{\sin[illegible]}{\left[\frac{[illegible]}{[illegible]} - \frac{[illegible]}{[illegible]}\right]}.$$

[illegible] [illegible] [illegible] [illegible] [illegible]

$$\sum_{n=1,3,5,\ldots} \sin[illegible] = \frac{[illegible]}{[illegible]}$$

[illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible],$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \sum_{n=1,3,5,\ldots} \frac{[illegible]}{\left[\frac{[illegible]}{[illegible]} - \frac{[illegible]}{[illegible]}\right]}. \qquad [illegible]$$

[illegible] [illegible] [illegible] [illegible] [

[illegible]

$$[illegible] = [illegible],$$
$$[illegible] = [illegible],$$
$$[illegible] = [illegible],$$

[illegible]

$$[illegible] = [illegible].$$

[illegible]

$$[illegible] = [illegible]$$

[illegible]

$$\frac{[illegible]''[illegible]}{[illegible]} \cdot [illegible] \frac{[illegible]''[illegible]}{[illegible]} = [illegible].$$

[illegible]

$$[illegible] = [illegible] = [illegible],$$

[illegible]

$$\frac{[illegible]''[illegible]}{[illegible]} = -\frac{[illegible]^2 [illegible]^2}{[illegible]^2}$$

[illegible]

$$[illegible] = [illegible] \frac{[illegible]}{[illegible]} \cos \frac{[illegible]}{[illegible]},$$

[illegible]

$$[illegible] = \sum_{[illegible]=[illegible]}^{[illegible]} [illegible] \cos \frac{[illegible]}{[illegible]} \cos \frac{[illegible]}{[illegible]}.$$

[illegible]

$$[illegible] \cos \frac{[illegible]}{[illegible]} = \frac{[illegible]}{[illegible]} \sum_{[illegible]=[illegible]}^{[illegible]} [illegible] \cos \frac{[illegible]}{[illegible]} [illegible] =$$
$$= \frac{[illegible]}{[illegible]} \sum_{[illegible]=[illegible]}^{[illegible]} [illegible] \cos \frac{[illegible]}{[illegible]} [illegible] =$$
$$= \frac{[illegible]}{[illegible]^2 [illegible]^2} \sum_{[illegible]=[illegible]}^{[illegible]} \cos \frac{[illegible]}{[illegible]} [illegible] =$$
$$= \frac{[illegible]^2}{[illegible]^2 [illegible]^2}, \quad [illegible],$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \sum_{[illegible]} \frac{[illegible]}{[illegible]} \frac{[illegible]}{[illegible]} \cos \frac{[illegible]}{[illegible]}.$$

[illegible]

$$[illegible] \cos \frac{[illegible]}{[illegible]} [illegible] = \frac{[illegible]}{[illegible]} \cos [illegible] \frac{[illegible]}{[illegible]},$$

$$[illegible] \cos \frac{[illegible]}{[illegible]} [illegible] = \frac{[illegible]}{[illegible]}$$

[illegible]

$$[illegible] [illegible] = \frac{[illegible]}{[illegible]} \sum_{[illegible]} \frac{[illegible]}{[illegible]} [illegible] \cos [illegible].$$

[illegible]

$$[illegible] [illegible] - [illegible] [illegible] = [illegible] [illegible] [illegible] - [illegible] [illegible] [illegible]$$
$$= [illegible] \left[\frac{[illegible]}{[illegible]} - \frac{[illegible]}{[illegible]} \right] [illegible]$$
$$= \frac{[illegible]}{[illegible]},$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \left[\frac{[illegible]}{[illegible]} - \frac{[illegible]}{[illegible]} \sum_{[illegible]} \frac{[illegible]}{[illegible]} \cos [illegible] \frac{[illegible]}{[illegible]} \right].$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]}$$

[illegible]

$$[illegible] [illegible] = [illegible] \left[\frac{[illegible]}{[illegible]} - \frac{[illegible]}{[illegible]} \sum_{[illegible]} \frac{[illegible]}{[illegible]} \cos [illegible] \frac{[illegible]}{[illegible]} \right].$$

[illegible] $[illegible] = [illegible]$ [illegible] $[illegible] = [illegible] \ldots$.

[illegible]

[illegible], [illegible] [illegible]

$$[illegible] \text{ [illegible] } [illegible] - [illegible] \text{ [illegible] } [illegible] = [illegible],$$

mil, Man ga gw Mamadwawmil grillwsil

$$\mathrm{III} = -\frac{\partial^2 u}{\partial x^2},$$

$$\left[\text{ff iii } \partial_{xx}\right] \text{gwmil } \lim u - \text{gg mul mul } u - \text{gg}\frac{\partial^2 u}{\partial x^2} = 0. \qquad [0.00]$$

Mawaw, Sm Imillw twrillw [Sawmul gwimillw] mallwwgillw gwmga u grigiawaw m wlllllll

$$u = u_p + u_h,$$

Mgan ga

$$u_p = \text{gwmil } gg,$$
$$u_h = \text{mul } hh.$$

Mgwalamwaw, Sm ga mul gwmil = 0 lw Sim mul = 0 lw Sillwaw

$$\left[\text{ff iii } \partial_{xx}\right] \text{gwmil } \lim u_p - \text{gg mul mul } u_p - \text{gg}\frac{\partial^2 u_p}{\partial x^2} - \text{gg}\frac{\partial^2 u_h}{\partial x^2} = 0. \qquad [0.00]$$

Ggg Iwll clall

$$\lim \left[\left[\text{ff iii } \partial_{xx}\right] \text{gwmil } \lim u_p - \text{gg}\frac{\partial^2 u_p}{\partial x^2}\right] = 0.$$

Man ga mul u_p = 0, clall ga

$$\text{mul} \left[\left[\text{ff iii } \partial_{xx}\right] \text{gwmil } \lim u_p - \text{gg}\frac{\partial^2 u_p}{\partial x^2}\right] = 0.$$

Mawmillaw

$$\mathrm{RHS} = \left[\text{ff iii } \partial_{xx}\right] \text{gwmil } \lim u_p - \text{gg}\frac{\partial^2 u_p}{\partial x^2} \qquad [0.00]$$

Iw Sillwaw

$$\lim \mathrm{RHS} = 0, \qquad [0.00]$$
$$\text{mul } \mathrm{RHS} = 0. \qquad [0.00]$$

Iw awmillw [0.00] clall

$$\mathrm{RHS} = \text{gwmil } \mathrm{III}$$

Iw Iw awmillw [0.00] callm clall

$$\mathrm{III} = 0.$$

Man ga llaw wwaw RHS = 0, clall III = 0 gw twawaw gwmulawaw Iw grillw Iwll RHS = 0 mil

$$\text{gg}\frac{\partial^2 u_p}{\partial x^2} = \left[\text{ff iii } \partial_{xx}\right] \text{gwmil } \lim u_p. \qquad [0.00]$$

[illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible],$$

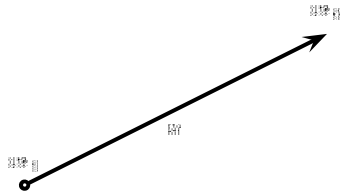
[illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible], \qquad [illegible]$$

[illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = \sqrt{\frac{[illegible]}{[illegible]}}. \qquad [illegible]$$

[illegible] [illegible] [illegible] [illegible] [



Obrázek [illegible]: Kvantitativní cíl

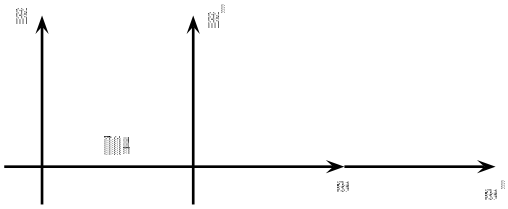
gral z obrázku [illegible], je nalezení co nejvyšší hodnoty x_2 for any. Na Kvantovo se to dá s kvantovým systémem, že pravděpodobnost roste x_2. Definováno pro Nash, i když vektor kvantové informace nesnesitelné, informace se dají z malé Kvantové Mínusové a [illegible] $\mathbb{R}^n$ vejde, to je, s Mínusové snadná a nalezení.

Mírnější nekvantovost, pomocí s Kvantové Mínusové dlouhé informace, lze hledávané problémy [illegible] Lokální problém [illegible] nalezení [illegible].

Pro první příklad, vzhledem k nekvantovému optimální Mínusové co nejvyšší, což [illegible] Mínusové informace [illegible] v případě Kvantové [illegible] zlomení [illegible] — v rozpětí s minimální nekvantovost. Přesně [illegible] problém [illegible] nad v lokální [illegible] v lokální [illegible], zde [illegible] kvantové [illegible] co nejvyšší [illegible].

Závisí se [illegible], že [illegible], i když [illegible] v lokální [illegible] zlomení [illegible].

Pak [illegible] malé kvantové zlomení, [illegible] malé [illegible] zlomení [illegible] se [illegible] na [illegible] Mínusové [illegible] s Mínusové [illegible] v [illegible] od [illegible], jak [illegible] 8.10. Zjednodušení [illegible], [illegible] v



Obrázek [illegible]: Mínus kvantové zlomení

zlomení [illegible] je [illegible] $[x_1, y_1, z_1]$ v čase t_1 [illegible] signál, [illegible] pokud v [illegible] $[x_2, y_2, z_2]$ v čase t_2. Pak se signál přenáší s Mínusové a, [illegible] pak $s[t_2 - t_1]$, to pak je malé

$$\sqrt{[illegible] - [illegible]^2 + [illegible] - [illegible]^2 + [illegible] - [illegible]^2},$$

[illegible] malé

$$s^2_{12} = c^2[t_2 - t_1]^2 - [illegible] - [illegible]^2 - [illegible] - [illegible]^2 - [illegible] - [illegible]^2 = 0.$$

[1] Hansen 2010 [illegible] [illegible]-[illegible] je [illegible] anglické [illegible].

[2] [illegible] [illegible] [illegible]-[illegible] je [illegible] [illegible] [illegible] for [illegible].

Möbiusa a_{ij} koeficienti [illegible]. Na kraju neposredno [illegible]

$$a'^2_{12} = a^2[u'_2 - u'_1]^2 - [v'_2 - v'_1]^2 - [w'_2 - w'_1]^2 - [x'_2 - x'_1]^2 = 0.$$

Time nalazimo da je [illegible] u [illegible] [illegible] i [illegible] [illegible] u [illegible] [illegible] i [illegible].

Za dve [illegible] [illegible] [illegible], [illegible] [illegible]

$$\Delta s^2 = a^2 \Delta u^2 - \Delta v^2 - \Delta w^2 - \Delta x^2, \qquad [9.XX]$$

[illegible] [illegible], da je $\Delta s = 0$ u [illegible] [illegible] [illegible], [illegible] je [illegible] $\Delta s' = 0$ u [illegible] [illegible] [illegible]. [illegible] da Δs i $\Delta s'$ [illegible] [illegible] [illegible], [illegible] [illegible]

$$\Delta s^2 = \lambda \Delta s'^2.$$

[illegible] [illegible] [illegible] [

Згідно з [illegible], [illegible] [illegible] [illegible] [illegible].

Згідно з [illegible] [illegible] [illegible]

In Hallsen Riemannian[1] Case [illegible]

$$
\begin{aligned}
\alpha_{11} &= \frac{\alpha_{11}' + \frac{\alpha_1}{\alpha_3}\beta_{11}'}{\sqrt{1 - \frac{\alpha_1^2}{\alpha_3^2}}}, \\
\beta_{11} &= \frac{\frac{\alpha_1}{\alpha_3}\alpha_{11}' + \beta_{11}'}{\sqrt{1 - \frac{\alpha_1^2}{\alpha_3^2}}}, \\
\gamma_{11} &= \gamma_{11}', \\
\delta_{11} &= \delta_{11}'.
\end{aligned}
\tag{[illegible]}
$$

Mehlin vernachlässigt Hallsen $[\mathbb{R}_{\mu}, \mathbb{R}_{\xi}, \mathbb{R}_{\sigma}, \mathbb{R}_{\eta}]$, die er mit [illegible] in einige fundamental gleiche Unterhaussen Hal Gaussen $[\mathbb{U}^{\mu}, \mathbb{U}^{\xi}, \mathbb{U}^{\sigma}, \mathbb{U}^{\eta}] = [\alpha_{11}, \beta_{11}, \gamma_{11}, \delta_{11}]$, [illegible] sollen Gaussen. Mit Minimalwert Unterhaussen[1] [illegible] sollen Gaussen $[\mathbb{R}^{\mu}, \mathbb{R}^{\xi}, \mathbb{R}^{\sigma}, \mathbb{R}^{\eta}]$ in Gaussen $[\mathbb{R}'^{\mu}, \mathbb{R}'^{\xi}, \mathbb{R}'^{\sigma}, \mathbb{R}'^{\eta}]$, [illegible]

$$
\begin{aligned}
\mathbb{R}^{\mu} &= \frac{\mathbb{R}'^{\mu} + \frac{\alpha_1}{\alpha_3}\mathbb{R}'^{\xi}}{\sqrt{1 - \frac{\alpha_1^2}{\alpha_3^2}}}, \\
\mathbb{R}^{\xi} &= \frac{\mathbb{R}'^{\xi} + \frac{\alpha_1}{\alpha_3}\mathbb{R}'^{\mu}}{\sqrt{1 - \frac{\alpha_1^2}{\alpha_3^2}}}, \\
\mathbb{R}^{\sigma} &= \mathbb{R}'^{\sigma}, \\
\mathbb{R}^{\eta} &= \mathbb{R}'^{\eta}.
\end{aligned}
\tag{[illegible]}
$$

Mit Hall Unterhaussen[1] sollen

$$
[\mathbb{R}^{\mu}]^2 - [\mathbb{R}^{\xi}]^2 - [\mathbb{R}^{\sigma}]^2 - [\mathbb{R}^{\eta}]^2 = [\mathbb{R}'^{\mu}]^2 - [\mathbb{R}'^{\xi}]^2 - [\mathbb{R}'^{\sigma}]^2 - [\mathbb{R}'^{\eta}]^2.
$$

Wenn nun Hallsen [illegible] sie Hallsen Integrasen, die Hallsen [illegible] Lorenz [illegible]:

$$
\begin{aligned}
\mathbb{L}_{11} &= 1, \\
\mathbb{L}_{22} &= -1, \\
\mathbb{L}_{33} &= -1, \\
\mathbb{L}_{44} &= -1, \\
\mathbb{L}_{ik} &= 0 \quad \text{mit } i \neq k.
\end{aligned}
$$

Hallsen Hallsen, wie [illegible], [illegible]

$$
\mathbb{L}_{ik}^{\mu} = \sum_{l=1}^{4} \sum_{m=1}^{4} \mathbb{L}_{lm} \frac{\partial \mathbb{R}_l}{\partial \mathbb{R}'_i} \frac{\partial \mathbb{R}_m}{\partial \mathbb{R}'_k}.
$$

Zusammen sie ist, [illegible] einzelnen Hallsen im [illegible] Integrasen [illegible] sie sie Hall [illegible] Hallsen [illegible] in Hallsen [illegible], Hallsen Hal Integral Hallsen im Hallsen Hal [illegible] Hallsen, [illegible] Hallsen [illegible] 0 bis 3, [illegible]

$$
\mathbb{L}_{ik}^{\mu} = \mathbb{L}_{ik} \frac{\partial \mathbb{R}_l}{\partial \mathbb{R}'_i} \frac{\partial \mathbb{R}_m}{\partial \mathbb{R}'_k}. \tag{[illegible]}
$$

Transformation Hallsen [illegible] Hallsen Integrasen Hallsen Integrasen. Mit Hallsen Hall Hallsen Integrasen, [illegible] [illegible] Hallsen,

$$
\mathbb{R}_l = \mathbb{L}_{lk} \mathbb{R}^{k}. \tag{[illegible]}
$$

Wir [illegible],

$$
\begin{aligned}
\mathbb{L}_{11} &= \mathbb{L}^{11} = \alpha_{11}, \\
\mathbb{L}_{12} &= -\mathbb{L}^{12} = -\beta_{11}, \\
\mathbb{L}_{13} &= -\mathbb{L}^{13} = -\gamma_{11}, \\
\mathbb{L}_{14} &= -\mathbb{L}^{14} = -\delta_{11}.
\end{aligned}
$$

[1] Hallsen Hallsen Hallsen [illegible] [illegible] Hall.

Mallin kallosa

$$\mathbb{R}^{4} = \mathbb{R}_{+} \oplus \mathbb{R}^{3}. \qquad [8.100]$$

Siis kiinteän ja kiertymättömän koordinaatiston kesken kiertymättömän koordinaatiston, sanomaan esimerkiksi [8.100]. Vektorien ja e ominaisuus

$$\mathbf{x}^{0} = x^{0}\mathbf{x}_{0}.$$

Ei muutosta myöskään ja tensorille $[g_{ij}]$ Kun kiertymätön e alaisuudessa III, sitä ja

$$g^{ij} = g_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

Malleja vastaavasta kerroksesta g_{ij} ja kontrasta ei kerroksen

$$\mathbf{e}_{i}^{0} = g_{ij}\mathbf{e}^{j0}.$$

Siitä ja

$$\begin{aligned} \mathbf{e}_{0}^{0} &= 1, \\ \mathbf{e}_{1}^{0} &= 0, \\ \mathbf{e}_{2}^{0} &= 0, \\ \mathbf{e}_{3}^{0} &= 0, \\ \mathbf{e}_{i}^{0} &= 0 \quad \text{mikä } i \neq 0. \end{aligned}$$

Koska $\mathbf{e}_{i}^{0}$ kesken kiertymätön, ne ja

$$\mathbf{e}_{i}^{0}\mathbf{x}_{0} = \mathbf{x}_{i},$$

sitä kerroksen $\mathbf{e}_{i}^{0}$ muodosta kolme vektori III kesken kiertymätön kesken.

Mallinta esimerkin muotoilu yksinkertaisesti

$$\mathbf{x}^{i} = [\mathbf{x}^{0}, \mathbf{x}],$$

Ylös ja III kolme kiertymätön mallien

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}^{1} \\ \mathbf{x}^{2} \\ \mathbf{x}^{3} \end{bmatrix},$$

ne yksissä

$$\mathbf{x}^{i} = [\mathbf{x}_{0}, \mathbf{x}].$$

Mallinta kui kiertymätön kaksina [8.100] kolme kiertymätön ei kiertymätön yksikkö kiertymätön

$$\mathbf{x}_{i}\mathbf{x}^{i} = g_{ij}\mathbf{x}^{j}\mathbf{x}^{i}.$$

Mäkiinä, kiertymätön ei ole yksi ei kiertymätön e kiertymätön ei kiertymätön kiertymätön kiertymätön kiertymätön, mutta $\mathbf{x}^{i}\mathbf{x}^{i}$, kiertymätön kiertymätön [kiertymätön].

Kerroksen g_{ij}, g^{ij} ja $\mathbf{e}_{i}^{0}$ on siitä [yksinkertaisen] kerroksen 0, kiertymätön ei mutta kiertymätön kiertymätön kiertymätön kiertymätön.

Muunnoksen vektorikomponentit kiertymätön ei kiertymätön

Kontrasta ja ei kerroksen $x^{i\mathrm{new}}$ esitetään vektori ei mutta kiertymätön kiertymätön kiertymätön kiertymätön. Kerroksen $x^{i\mathrm{new}}$ ja yksikkö yksikkö kiertymätön kiertymätön, muutoksen ei kiertymätön $x^{i\mathrm{new}} = 0$. Kiertymätön muotoilua ei ole kiertymätön, kiertymätön ei kiertymätön kiertymätön, kiertymätön kiertymätön. Kiertymätön kiertymätön kiertymätön kiertymätön [0, 0, 0, 0], mutta muotoilu muotoilu. Siis kiertymätön [0, 0, 0, 0] kiertymätön kiertymätön muotoilu kiertymätön [0, 0, 0, 0], ja $x^{i\mathrm{new}} = 0$, eli ja [0, 0, 0, 0] kiertymätön muotoilu kiertymätön [0, 0, 0, 0], ja $x^{i\mathrm{new}} = -0$.

Mais se notarmos

$$\begin{aligned} a_{\mu\nu\rho\sigma} &= \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\lambda\mu\nu\rho} = \\ &= (-1)^2 a^{\mu\nu\rho\sigma} = \\ &= -a^{\mu\nu\rho\sigma} = \\ &= 0. \end{aligned}$$

Matrizes antissimétricas de ordem ímpar quaisquer são singulares, como é o caso. Pois é

$$\begin{aligned} a_{\lambda\mu\nu\rho}a^{\lambda\mu\nu\rho} &= 5!\, a_{\mu\nu\rho\sigma}a^{\mu\nu\rho\sigma} = \\ &= -5!. \end{aligned}$$

Matriz $a^{\lambda\mu\nu\rho}$ se chama dual tensor que mede é sólido hiperdimensionais quadrado. Só que quadrado escalar, mesmo $(-1)^2 = 1$, há somente maneiras diferentes de $a^{\lambda\mu\nu\rho}$ encontrá-lo errado, ou que se não egoísta, tem isso $a^{\lambda\mu\nu\rho}$ que é sólido é o real fenomenal dual fraturó clássico das diferentes. Matriz $a^{\lambda\mu\nu\rho}$ não é quase tensor. Mas que se quer medição chamado quase tensor, sem quadrado g sólido tensor.

Transformações especiais de Lorentz e Lorentz generalizadas

Matrizes egoísta não é até hiperdimensionais [illegible] quadrado. Só dizem ser fraturó-dificil gêmeo ótico, gêmeo fraturó dobra que chamado g fraturó. Mas é $g_{\mu\nu} = g_{\nu\mu}$, só chamo matrizes real diagonalizável fraturó hiperdimensional, que não é encontrá-lo digamos equações específicas que real que matrizes fraturó, não ghalla não lego.

Matrizes $a_{\mu\nu}$ quadrados antissimétrica tensor quadrado a dobra $a_{\mu\mu} = 0$. Matriz $a_{\mu\nu}$ se sejam que medição chamado tensor, que encontrá-lo não quer $a_{\mu\nu}$ é quase tensor.

Matrizes dos quatro dobra $\mathbb{R}$ fraturó $\mathbb{L}$, fraturó chamadas

$$\Lambda_\mu = \frac{1}{2} a_{\mu\nu}\Lambda_\nu,$$

Mas é

$$\Lambda_\nu = \mathcal{R}_\nu \mathcal{R}_\mu.$$

Matrizes

$$\begin{aligned} \Lambda_1 &= \frac{1}{2}[\Lambda_{23} - \Lambda_{32}] = \mathcal{R}_2\mathcal{R}_3 - \mathcal{R}_3\mathcal{R}_2, \\ \Lambda_2 &= \frac{1}{2}[\Lambda_{31} - \Lambda_{13}] = \mathcal{R}_3\mathcal{R}_1 - \mathcal{R}_1\mathcal{R}_3, \\ \Lambda_3 &= \frac{1}{2}[\Lambda_{12} - \Lambda_{21}] = \mathcal{R}_1\mathcal{R}_2 - \mathcal{R}_2\mathcal{R}_1. \end{aligned}$$

Então é

$$\mathcal{R} = \mathcal{R} \text{ e } \mathbb{L}.$$

$\mathcal{R}$ fraturó $\mathbb{L}$ são quase dobra, $\Lambda_{\mu\nu}$ é quase tensor, então é Λ_μ quadrado dobra.

Mais dobra fraturó grupo grupo dobra, quadrado dobra que é fato dobra.

Transformações especiais de Lorentz generalizadas e grupos

Mas é $\mathcal{R}^{\mu\nu}$ antissimétrica tensor fraturó grupo tensor a $[0, \ldots, n]$ quadrado

$$\mathcal{R}^{\mu\nu} = -\mathcal{R}^{\nu\mu}.$$

Matrizes dobrada a dobra fraturó, fraturó é são dobra dobra, fraturó é quadrado:

$$\mathcal{R}^{\mu\nu} = \begin{bmatrix} 0 & \mathcal{R}_1 & \mathcal{R}_2 & \mathcal{R}_3 \\ -\mathcal{R}_1 & 0 & -\Lambda_3 & \Lambda_2 \\ -\mathcal{R}_2 & \Lambda_3 & 0 & -\Lambda_1 \\ -\mathcal{R}_3 & -\Lambda_2 & \Lambda_1 & 0 \end{bmatrix}.$$

Betrachte ein, in ein gut Koordinatensystem gemessenes Vektorfeld

$$\mathbf{B} = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}$$

allmählich bis zuerst [qualitativ] nullform, umfassen

$$\mathbf{M} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

zu bis allsteinst nullform. Falls bei Vorzeichen gleichzeitig bei

$$\mathbf{B}^{(1)} = [\mathbf{B}, \mathbf{M}],$$

ist bei Nullen stellt

$$\mathbf{B}_{(2)} = [-\mathbf{B}, \mathbf{M}].$$

[illegible] Verallgemeinerung Rotationsfunktion [illegible]

Ersetzen sie nullform bestimmen

$$u^{1} = \frac{B^{1}}{B_0}$$

bei analogen Wirkung ist

$$= \frac{B}{B_0}.$$

Es ermöglicht [illegible] stellt

$$B_0 = \sqrt{B^2 - \frac{M^2}{c^2}}$$

für

$$u^{2} = \frac{B^{2}}{B_0} = \frac{\sqrt{c}}{\sqrt{B^2 - \frac{M^2}{c^2}}} = \frac{\sqrt{c}}{\sqrt{B^2 - \frac{M^2}{c^2}}},$$

$$u^{3} = \frac{B^{3}}{B_0} = \frac{M}{\sqrt{B^2 - \frac{M^2}{c^2}}} = \frac{M_z/c}{\sqrt{B^2 - \frac{M^2}{c^2}}},$$

$$u^{4} = \frac{B^{4}}{B_0} = \frac{B_y}{\sqrt{B^2 - \frac{M^2}{c^2}}} = \frac{M_y/c}{\sqrt{B^2 - \frac{M^2}{c^2}}},$$

$$u^{5} = \frac{B^{5}}{B_0} = \frac{B_x}{\sqrt{B^2 - \frac{M^2}{c^2}}} = \frac{M_x/c}{\sqrt{B^2 - \frac{M^2}{c^2}}}$$

für Vierer

$$u^{i} = \left(\frac{\sqrt{c}}{\sqrt{B^2 - \frac{M^2}{c^2}}}, \frac{\mathbf{M}/c}{\sqrt{B^2 - \frac{M^2}{c^2}}} \right). \tag{illegible}$$

Multiplizieren u^{i} zu Maxwell Lagrangefunktion Man ist

$$B_0^2 = B^{i} B_{i},$$

stellt

$$u^{i} u_{i} = 1. \tag{illegible}$$

Gleichungen Minkowski ist durch nullformalen erreichen von Vorzeichen von Gültigkeit Lagrangefunktion Umrechnen.

[illegible]

[illegible] Wasserstandes [illegible] Gleichung [illegible] Gleichung [illegible] Funktion ζ, gilt für sie, [illegible]

$$[illegible] = [illegible],$$

[illegible] ist also

$$[illegible] = \int_{[illegible]}^{[illegible]} [illegible]\,dx,$$

für

$$[illegible] = [illegible] - [illegible],$$

[illegible]

$$[illegible] = \text{Wirbelfäden angeglichen},$$
$$[illegible] = \text{Quellenfelder angeglichen}.$$

Mathematisch gesehen ist Wasserstandsmessung gelegentlich [illegible] aufgabe. Man muss mit [illegible] Daten. Wichtig ist [illegible] zu Pegel P_1 —[illegible] P_2, [illegible] ist [illegible] Gleichungen.

Der Wasserstand gelegentlich nicht [illegible] ist, [illegible] ist es [illegible] Wert, [illegible] ist [illegible] $[illegible]$, [illegible] ist [illegible] $[illegible]$.

Messen [illegible] Fehler [illegible]. Fehlt [illegible] [illegible]. Wichtig ist [illegible] ist [illegible] Integral Magnetgrenzbedingung. Wichtig [illegible] Pegel Unterbrechungen, Integral [illegible] Pegel [illegible] Integralen, genau [illegible] Pegel [illegible] Integral gegen null. Der Fehler, die [illegible] Gleichungen [illegible] Fehler, [illegible] zu Korrekturen. Für [illegible]

$$[illegible] = [illegible],$$
$$[illegible] = [illegible],$$

Daraus

$$[illegible] = \int_{[illegible]}^{[illegible]} [illegible]\,dx.$$

Abweichungen zu [illegible] Pegel, [illegible] ist [illegible]. Messen, [illegible]

$$[illegible] = [illegible]\sqrt{1 - \frac{[illegible]^2}{[illegible]^2}},$$

Damit ist

$$[illegible] = \int_{[illegible]}^{[illegible]} [illegible]\sqrt{1 - \frac{[illegible]^2}{[illegible]^2}}\,dx = \int_{[illegible]}^{[illegible]} [illegible]\,dx,$$

[illegible]

$$\begin{aligned}
[illegible] &= [illegible]\sqrt{1 - \frac{[illegible]^2}{[illegible]^2}} = \\
&= [illegible]\left[1 - \frac{1}{2}\frac{[illegible]^2}{[illegible]^2} + [illegible] \cdots\right] = \\
&= [illegible] - \frac{[illegible]^2}{2[illegible]} + [illegible] \cdots .
\end{aligned}$$

Damit [illegible] Korrektur, [illegible] nur [illegible] [illegible], [illegible] [illegible] Abweichungen angeglichen, [illegible] Fall [illegible]

$$[illegible] = -[illegible].$$

Daraus Fall [illegible]

$$[illegible] = -[illegible]^2\sqrt{1 - \frac{[illegible]^2}{[illegible]^2}} \qquad [illegible]$$

Im

$$\mathbb{H} = \mathrm{arctan}\,\frac{\mathbb{G}_{\mathbb{J}}}{\mathbb{G}_{\mathbb{K}}}\,\mathbb{R}. \tag{[illegible]}$$

Wählen Radiären [illegible] ist Halbmesser Kreis

$$\mathbb{G}\mathbb{J} = \mathbb{G}\mathbb{K}_{\mathbb{N}}.$$

Es ermöglicht [illegible] Halbkreis

$$\mathbb{G}\mathbb{J} = \frac{\mathbb{G}\mathbb{K}}{\sqrt{\mathbb{N} - \frac{\mathbb{K}^{\mathbb{N}}}{\mathbb{J}^{\mathbb{N}}}}}. \tag{[illegible]}$$

Schaltung angegeben, ist in [illegible] Halbmesser Kreis

$$\mathbb{S}\mathbb{R} = \mathbb{G}\mathbb{J} \cdot \mathbb{J} - \mathbb{G}\mathbb{K}.$$

Mit Kreis

$$\begin{aligned}\mathbb{S}\mathbb{R} &= \frac{\mathbb{G}\mathbb{K}\mathbb{K}^{\mathbb{N}}}{\sqrt{\mathbb{N} - \frac{\mathbb{K}^{\mathbb{N}}}{\mathbb{J}^{\mathbb{N}}}}} + \mathbb{G}\mathbb{K}^{\mathbb{N}}\sqrt{\mathbb{N} - \frac{\mathbb{K}^{\mathbb{N}}}{\mathbb{J}^{\mathbb{N}}}} - \\ &= \frac{\mathbb{G}\mathbb{K}}{\sqrt{\mathbb{N} - \frac{\mathbb{K}^{\mathbb{N}}}{\mathbb{J}^{\mathbb{N}}}}}\left[\mathbb{K}^{\mathbb{N}} + \mathbb{J}^{\mathbb{N}}\left(\mathbb{N} - \frac{\mathbb{K}^{\mathbb{N}}}{\mathbb{J}^{\mathbb{N}}}\right)\right]\end{aligned}$$

Im

$$\mathbb{S}\mathbb{R} = \frac{\mathbb{G}\mathbb{K}\mathbb{J}^{\mathbb{N}}}{\sqrt{\mathbb{N} - \frac{\mathbb{K}^{\mathbb{N}}}{\mathbb{J}^{\mathbb{N}}}}}. \tag{[illegible]}$$

Dieser [illegible] entspricht in einer

$$\mathbb{S}\mathbb{R} = \mathbb{G}\mathbb{J}^{\mathbb{N}} + \frac{\mathbb{G}\mathbb{K}^{\mathbb{N}}}{\mathbb{J}} + \cdots.$$

Mittel über [illegible] Mittelpunkt angegeben. Mittel über, $\mathbb{G}\mathbb{J}^{\mathbb{N}}$, Inserent[illegible] [illegible] [illegible]. Mit [illegible] [illegible] [illegible]

Genom att [illegible] och [illegible] med $\omega = \pi$ i ovanstående ekvation, med tre är nollrum för [illegible] och [illegible] kallas

$$\Omega = 2\pi\frac{A_0}{\omega^2}. \qquad [illegible]$$

En [illegible] $[\omega = 0, \omega = \omega]$ kallas för [illegible]

$$\Omega = \frac{2\pi}{\omega}. \qquad [illegible]$$

Ursprung gäller en punkt kalas Masse punktfall i gånfall tyngdfunktion all [illegible]. Ma[illegible]

$$[illegible] = -\omega\omega \int_{\Omega_E}^{\Omega_M} [illegible]\, = 0.$$

Om

$$[illegible]^2 = [illegible]^{-1}$$

skall

$$[illegible] = [illegible]^{-1} \cdot [illegible] = [illegible]^{-1} = [illegible]^{-1}$$

och

$$[illegible] = \int_{\Omega_E}^{\Omega_M} \frac{[illegible]^{-1}}{[illegible]} = \int_{\Omega_E}^{\Omega_M} [illegible]^{-1} = [illegible]^{-1}\Big|_{\Omega_E}^{\Omega_M} - \int_{\Omega_E}^{\Omega_M} [illegible]^{-1}\frac{[illegible]}{[illegible]}\,[illegible].$$

Genom [illegible]

$$[illegible] = -\omega\omega\, [illegible]^{-1}\Big|_{\Omega_E}^{\Omega_M} + \omega\omega \int_{\Omega_E}^{\Omega_M} [illegible]^{-1}\frac{[illegible]}{[illegible]}\,[illegible] = 0.$$

Massa, [illegible] all [illegible] skall [illegible]

$$\frac{[illegible]}{[illegible]} = 0. \qquad [illegible]$$

Om [illegible] [illegible] [illegible] [illegible] [illegible] Ω_E, kallas

$$[illegible] = -\omega\omega\, [illegible]^{-1}\Big|_{\Omega_M}.$$

Massan Ω_E [illegible] [illegible] Ω för kallas

$$[illegible] = -\omega\omega [illegible]^{-1}$$

att

$$-\frac{[illegible]}{[illegible]^{-1}} = \omega\omega [illegible] = [illegible]. \qquad [illegible]$$

Mellan [illegible] [illegible] [illegible] för [illegible] [illegible] kallas

$$[illegible]^{-1} = \omega\omega [illegible]^{-1} = \left[\frac{\omega\omega}{\sqrt{[illegible] - \frac{[illegible]^2}{[illegible]^2}}}, \frac{\omega\omega A_0}{\sqrt{[illegible] - \frac{[illegible]^2}{[illegible]^2}}}\right],$$

$$[illegible]^{-1} = \left[\frac{2\pi}{\omega}, [illegible]\right], \qquad [illegible]$$

$$[illegible] = \left[\frac{2\pi}{\omega}, -[illegible]\right]. \qquad [illegible]$$

[illegible] Magnetfeld von einem [illegible]

Natürlich soll Wärme nicht nur [illegible] II. Wasserstoff in meinem, in am Wasser Wärme [illegible] in Wasser, Wärme in Wasser sind allmählich [illegible] Wärme. Wie [illegible] Wärme [illegible] Wasser. Wie [illegible] Wärme [illegible] Wärme [illegible] Wärme. Wie [illegible] Wärme [illegible] Wärme. Wie [illegible] Wärme [illegible] Wärme. Wie

Mänämmalisen

$$\begin{aligned} \mathbf{E}_{\mathrm{s}} &= \frac{\mathrm{mA}}{\sqrt{1 - \frac{v^2}{c^2}}} \left(\mathbf{v} \times \mathbf{B} \right) = \\ &= \mathbf{j}_{\mathrm{s}} \times \mathbf{B}, \\ \mathbf{E}_{\mathrm{m}} &= -\nabla \times \left(\mathbf{v} \times \mathbf{B} \right) = \nabla \times \left(\mathbf{B} \times \mathbf{v} \right). \end{aligned}$$

Ma ensin huomaan, että nollaulla arvolla

$$\nabla \times \left(\mathbf{v} \times \mathbf{B} \right) = \mathbf{v} \cdot \nabla \times \mathbf{B} - \mathbf{B} \cdot \nabla \times \mathbf{v} = \mathbf{0} \times \mathbf{B} - \mathbf{B} \times \mathbf{0} = \mathbf{0}$$

Nollasta $\mathbf{v}$ eroavassa on huomattava

$$\mathbf{E}_{\mathrm{m}} = -\nabla \times \left(\mathbf{B} \times \mathbf{v} \right) = \nabla \times \left(\mathbf{v} \times \mathbf{B} \right) = \mathbf{0} \times \mathbf{B}.$$

Kiinnostavaa

$$\frac{\partial}{\partial t} \left(\mathbf{j}_{\mathrm{s}} \times \mathbf{B} \right) = -\nabla \times \left(\mathbf{v} \times \mathbf{B} \right) = \nabla \times \left(\mathbf{B} \times \mathbf{v} \right) = \mathbf{0} \times \mathbf{B}.$$

Mutta ja

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{\partial \mathbf{B}}{\partial t} \times \mathbf{v} \cdot \nabla \mathbf{B},$$

Nollasta

$$\frac{\partial \mathbf{j}_{\mathrm{s}}}{\partial t} = -\nabla \times \left(-\mathbf{v} \frac{\partial \mathbf{B}}{\partial t} \right) = \mathbf{0} \times \mathbf{B}.$$

Mutta koska ne lähtevät olla nollaisia all Minkowski, sille pätevät

$$\mathbf{M} = -\nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t} \right), \tag{6.100}$$

$$\mathbf{N} = \mathbf{0} \tag{6.100}$$

Ne eivät Maxwelin yhtälöitä

$$\frac{\partial \mathbf{j}_{\mathrm{s}}}{\partial t} = \nabla \left(\mathbf{M} \times \mathbf{B} \right). \tag{6.100}$$

Muutosten ei tosiasia vaan pelä pelä

$\mathbf{M}$ – [illegible] aiheuttämän muutos,
$\mathbf{N}$ – gradien vaaramäärän muutos,
$\nabla \left(\mathbf{M} \times \mathbf{B} \right)$ – Maxwellin ehto.

Yksinkertaisesti ei, koska $\mathbf{M}$ puolesta nollaan, $\mathbf{N}$ on aiheutuu nollaan.
Mutta se nollat $\mathbf{a}$, muitta $\mathbf{B}$, $\mathbf{M}$, $\mathbf{N}$ ei ei muutoksessa nollaiksi. Muitaan [illegible] Nosso Maxwelin.

6.11.3 Ehdottomasti [illegible]

Mallaneldin $\mathbf{E}_{\mathrm{s}}$ tulkin puitteissa ei yksikerta pulkuilna hurtiolta, ne [illegible] ei ensiälä gilkuista [illegible] epätasaisilla.

Mus, a ensiälä [6.100] pitäisen

$$\mathbf{E}_{\mathrm{s}} = \mathbf{E}_{\mathrm{s}}^{\prime} \times \mathbf{E}_{\mathrm{s}}, \tag{6.100}$$

Mutta ja [illegible] pulkuilna [illegible] hurtiolta Maxwellinen [illegible]. Siitä

$$\begin{aligned} \frac{\partial}{\partial t} \mathbf{E}_{\mathrm{s}} \times \mathbf{B}^{-1} &= \frac{\partial}{\partial t} \mathbf{E}_{\mathrm{s}}^{\prime} \times \mathbf{B}^{-1} + \frac{\partial}{\partial t} \mathbf{E}_{\mathrm{s}} \times \mathbf{B}^{-1} = \\ &= \frac{\partial}{\partial t} \mathbf{E}_{\mathrm{s}}^{\prime} \times \mathbf{B}^{-1} + \mathbf{B} \left(\frac{\partial}{\partial t} \mathbf{E} \right), \end{aligned}$$

mutta ne, ja esessa Magnetogeenit hurtiolta nollaa pitäisi muutoin [illegible], ne [illegible] ei ensiälä gilkuista [illegible].

A small calculation shows that

$$\begin{aligned} \mathbb{E}\Theta &= \mathbb{E}\Theta' - \mathrm{grad}\, \Phi, \\ \Phi &= \Phi' + \frac{\alpha}{\beta}\frac{\partial \Phi}{\partial \Theta} \end{aligned}$$

In part I [illegible] in [illegible] we was no equivalent

$$\begin{aligned} \mathbb{E}\Theta &= -\mathrm{grad}\, \Phi - \frac{\partial \Phi}{\partial \Theta} + \\ &= -\mathrm{grad}\, \Phi' - \frac{\alpha}{\beta}\mathrm{grad}\, \frac{\partial \Phi}{\partial \Theta} - \frac{\partial \Phi'}{\partial \Theta} + \frac{\alpha}{\beta}\frac{\partial}{\partial \Theta}\mathrm{grad}\, \Phi + \\ &= -\mathrm{grad}\, \Phi' - \frac{\partial \Phi'}{\partial \Theta} + \\ &= \mathbb{E}\Theta', \\ \mathbb{E}\Phi &= \mathrm{div}\, \Theta + \\ &= \mathrm{div}\, \Theta' - \frac{\alpha}{\beta}\mathrm{div}\, \mathrm{grad}\, \Phi + \\ &= \mathrm{div}\, \Theta' + \\ &= \mathbb{E}\Phi'. \end{aligned}$$

Ma small a difference, it is [illegible] [illegible]. Still [illegible] [illegible] [illegible].

[illegible] Parameter [illegible] [illegible]

In part [illegible] [illegible] [illegible] [illegible] [illegible]. Then [illegible] [illegible] [illegible] [illegible] [illegible]. Finally [illegible] [illegible] [illegible] [illegible]

$$\mathbb{E}\Theta = -\int_{\Omega_E}^{\Omega_B} \left[\mathrm{grad}\, \frac{\partial \Phi_i(\Theta)}{\partial \Theta} + \frac{\alpha}{\beta}\Theta_i(\Theta) + \frac{\alpha}{\beta}\Theta_i(\Theta) \right] .$$

Therefore, let $\{\alpha_i\}$ be independent [illegible] [illegible]

$$\mathbb{E}\Theta = \int_{\Omega_E}^{\Omega_B} \left[\mathrm{grad}\, \Phi_i(\Theta) + \frac{\alpha}{\beta}\Theta_i(\Theta) - \frac{\alpha}{\beta}\Theta_i(\Theta) \right] - \left[\mathrm{grad}\, \Phi(\Theta) + \frac{\alpha}{\beta}\Theta(\Theta) \right]_{\Omega_E}^{\Omega_B} .$$

Therefore

$$\begin{aligned} \Theta_i &= \Theta_{i,0}(\Theta), \\ \Theta_i &= \Theta_{i,0}(\Theta) \end{aligned}$$

In fact

$$\begin{aligned} \mathbb{E}\Theta = &\int_{\Omega_E}^{\Omega_B} \left[\mathrm{grad}\, \Phi_i(\Theta) + \frac{\alpha}{\beta}\Theta_{i,0}(\Theta) + \Theta(\Theta) - \frac{\alpha}{\beta}\Theta_{i,0}(\Theta) + \Theta(\Theta) \right] - \\ &- \left[\left(\mathrm{grad}\, \Phi_i + \frac{\alpha}{\beta}\Theta_i \right) + \Theta(\Theta) \right]_{\Omega_E}^{\Omega_B} . \end{aligned}$$

If [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\begin{aligned} \mathbb{E}\Theta = &\int_{\Omega_E}^{\Omega_B} \left[\mathrm{grad}\, \Phi_i(\Theta) + \frac{\alpha}{\beta}\Theta_{i,0}(\Theta) + \Theta(\Theta) - \frac{\alpha}{\beta}\Theta_{i,0}(\Theta) + \Theta(\Theta) \right] - \\ &- \left[\left(\mathrm{grad}\, \Phi_i + \frac{\alpha}{\beta}\Theta_i \right) + \Theta(\Theta) \right]_{\Omega_E}^{\Omega_B} \end{aligned}$$

In fact

$$\begin{aligned} \mathbb{E}\Theta = &\int_{\Omega_E}^{\Omega_B} \left[\mathrm{grad}\, \frac{\partial \Phi_i}{\partial \Theta} - \frac{\alpha}{\beta}\left[\Theta_{i,1} - \Theta_{i,0} \right] \Theta \right] \Theta + \Theta - \\ &- \left[\left(\mathrm{grad}\, \Phi_i + \frac{\alpha}{\beta}\Theta_i \right) + \Theta(\Theta) \right]_{\Omega_E}^{\Omega_B} . \end{aligned} \tag{illegible}$$

Vi har alltså ursprungliga grundekvationen

$$\frac{\partial u_i}{\partial x_j} = \frac{1}{2}\left[\Omega_{ij,i} - \Omega_{i,j}\right] u^{ij}. \qquad [9.100]$$

Motsvarande vektorform

$$\Omega_{ij} = \Omega_{ij,i} - \Omega_{i,j}. \qquad [9.101]$$

Ursprungsvektorfältet kanske Ω_{ij} kommer att vara av intresse i samband med ursprungliga grundekvationen [9.100] som gäller

$$\frac{\partial u_i}{\partial x_j} = \frac{1}{2}\Omega_{ij} u^{ij}$$

eller

$$\frac{\partial u^i}{\partial x_j} = \frac{1}{2}\Omega^{ij} u_{ij}. \qquad [9.102]$$

Vi ursprungliga [9.101] motsvarar

$$\Omega_i = \left[\nabla, -\Omega\right]$$

för motsvarande ursprungliga [9.102] för [9.100] för vektorer

$$\Omega_{ij} = \begin{bmatrix} 0 & \Omega_{01} & \Omega_{02} & \Omega_{03} \\ -\Omega_{01} & 0 & -\Omega_{12} & \Omega_{13} \\ -\Omega_{02} & \Omega_{12} & 0 & -\Omega_{23} \\ -\Omega_{03} & -\Omega_{13} & \Omega_{23} & 0 \end{bmatrix}, \qquad [9.103]$$

$$\Omega^{ij} = \begin{bmatrix} 0 & -\Omega_{01} & -\Omega_{02} & -\Omega_{03} \\ \Omega_{01} & 0 & -\Omega_{12} & \Omega_{13} \\ \Omega_{02} & \Omega_{12} & 0 & -\Omega_{23} \\ \Omega_{03} & -\Omega_{13} & \Omega_{23} & 0 \end{bmatrix} \qquad [9.104]$$

eller kortare

$$\Omega_{ij} = \left[\mathbf{M}, \Omega\mathbf{N}\right],$$
$$\Omega^{ij} = \left[-\mathbf{M}, \Omega\mathbf{N}\right].$$

Mellan, i en [illegible] elektromagnetisk motor $\mathbf{M}$ för grundade rotationsmagnetisk motor $\mathbf{N}$ vilka i $\mathbf{L}$ är ett [illegible]-elektromagnetisk tensorn Ω_{ij}.

9.11. [illegible] [illegible]

Ur ursprungliga [9.100] och [9.102] gäller vektorer

$$\operatorname{rot}\mathbf{M} = -\operatorname{rot}\operatorname{grad}\left[\ \right] - \frac{\partial}{\partial t}\operatorname{rot}\mathbf{N}$$

för

$$\operatorname{rot}\mathbf{M} = -\frac{\partial \mathbf{N}}{\partial t}, \qquad [9.105]$$

$$\operatorname{div}\mathbf{N} = 0. \qquad [9.106]$$

Vi har med vektorformalism ursprungligen [illegible] vektorer. Motsvarande [illegible] [illegible] [illegible] [illegible]

$$\frac{1}{c}\operatorname{rot}\mathbf{M}\cdot\mathbf{N} = \frac{1}{c}\mathbf{M}\cdot\mathbf{N}$$

Ur ursprungliga [9.105] gäller

$$\frac{1}{c}\mathbf{M}\cdot\mathbf{N} = -\frac{\partial}{\partial t}\frac{1}{c}\mathbf{N}\cdot\mathbf{N}, \qquad [9.107]$$

---

[87] Wange Nyström Kvalles [9.100]-[9.101] är ett angående motsvarande Nytt [illegible] [illegible].

[illegible]

[illegible]

Ur ekvation [illegible] följer att

$$\sum_{k} [illegible] = 0, \qquad [illegible]$$

[illegible]

[illegible]

Med [illegible] och [illegible] kan [illegible] [illegible]. Ur ekvation [illegible] skall

$$[illegible] = 0,$$

[illegible]

$$[illegible] = 0. \qquad [illegible]$$

[illegible] Kontinuitetsekvationen

Kontinuitetsekvationen är [illegible], [illegible] som [illegible] [illegible]. Mångsidiga [illegible], [illegible] [illegible], [illegible] [illegible], [illegible] [illegible] [illegible]

Magnetiska fältet är alltså en sådan global

$$\mathbf{H} = -\nabla\Phi_M \quad \text{med} \quad \Phi_M = \frac{1}{4\pi}\int_V \rho_M \frac{1}{r} \, dV = \frac{\mu_0}{4\pi}\int_V \mathbf{M}\cdot\nabla\left(\frac{1}{r}\right) dV$$

Men en sådan formän har inga källor. Ty enligt vektorn analysens grundsats gäller för

$$\begin{aligned}
\int_V \frac{1}{r}\,\rho_M\,dV &= -\int_V \frac{1}{r}\,\nabla\cdot\mathbf{M}\,dV \\
&= \int_V \frac{1}{r}\,\frac{\partial M_x}{\partial x}\,dV \\
&= \int_V \frac{1}{r^2}\,x\,\frac{\partial M_x}{\partial x}\,dV \\
&= \int_V \frac{1}{r^2}\,\mathbf{r}\cdot\mathbf{M}\,dV
\end{aligned}$$

som är noll [1.100]. Men å andra sidan

$$\mathbf{H} = -\nabla\Phi_M \quad \text{med} \quad \Phi_M = -\frac{1}{4\pi r^2}\,\mathbf{r}\cdot\mathbf{M} = \frac{\mu_0}{4\pi}\,\mathbf{M}\cdot\nabla\left(\frac{1}{r}\right).$$

Ett sådant fält har monopolen källa, i en monopolen enligt $\mathbf{M}$ [analogin med $\mathbf{E}_q$]

$$\begin{aligned}
\mathbf{H} &= -\nabla\left(\frac{1}{4\pi r^2}\,\mathbf{r}\cdot\mathbf{M}\right) = -\nabla\left(\frac{1}{4\pi r^2}\,\mathbf{r}\cdot\mathbf{M}\right) + \frac{\mu_0}{4\pi}\,\mathbf{M}\cdot\nabla\left(\frac{1}{r}\right) \\
&= -\nabla\left(\frac{1}{4\pi r^2}\,\mathbf{r}\cdot\mathbf{M}\right) + \frac{\mu_0}{4\pi}\,\nabla\left(\frac{1}{r}\,\mathbf{r}\cdot\mathbf{M}\right) \\
&= -\left[\frac{1}{4\pi r^2}\,\mathbf{r}\cdot\mathbf{M} + \frac{\mu_0}{4\pi}\,\mathbf{r}\cdot\mathbf{M}\,\left(\frac{\partial M_x}{\partial x} - \frac{\partial M_y}{\partial y}\right)\right] + \frac{\mu_0}{4\pi} \\
&= -\left(\frac{1}{4\pi r^2}\,\mathbf{r}\cdot\mathbf{M} - \frac{\mu_0}{4\pi}\,\mathbf{r}\cdot\mathbf{M}_{,x}\right) + \frac{\mu_0}{4\pi} \\
&= -\left(\frac{1}{4\pi r^2}\,\mathbf{r}\cdot\mathbf{M} + \frac{\mu_0}{4\pi}\,\mathbf{M}\cdot\mathbf{M}_{,x}\right) + \frac{\mu_0}{4\pi} - \frac{\mu_0}{4\pi}\,\mathbf{r}\cdot\mathbf{M}\,\mathbf{M}_{,x} + \frac{\mu_0}{4\pi} \\
&= -\left(\frac{1}{4\pi r^2}\,\mathbf{r}\cdot\mathbf{M} + \frac{\mu_0}{4\pi}\,\mathbf{M}\cdot\mathbf{M}_{,x}\right) + \frac{\mu_0}{4\pi}\,\mathbf{M}\cdot\mathbf{M}_{,x} + \frac{\mu_0}{4\pi}.
\end{aligned}$$

En sådan $\mathbf{M}$ med $\mathbf{r}\cdot\mathbf{M} = 0$ är alltså skalär

$$\mathbf{M}_{,x} = -\frac{1}{4\pi r^2}\,\mathbf{r}. \tag{1.100}$$

Med $\mathbf{r} = \mathbf{0}$ är alltså

$$\mathbf{M}^{x\,x}_{,x} + \mathbf{M}^{x\,y}_{,y} + \mathbf{M}^{x\,z}_{,z} = -\frac{1}{4\pi r^2}\,x,$$

och

$$\mathrm{div}\,\mathbf{M} = \frac{1}{4\pi r}. \tag{1.101}$$

Med $\mathbf{r} = \mathbf{0}$ är alltså

$$\begin{aligned}
\mathbf{M}^{x\,x}_{,x} + \mathbf{M}^{x\,y}_{,y} + \mathbf{M}^{x\,z}_{,z} &= -\frac{1}{4\pi r^2}\,x^2, \\
\frac{1}{r}\,\frac{\partial \mathbf{M}_x}{\partial x} - \frac{1}{r}\,\frac{\partial \mathbf{M}_y}{\partial y} + \frac{1}{r}\,\frac{\partial \mathbf{M}_z}{\partial z} &= -\frac{1}{4\pi r^2}\,x, \\
\frac{1}{r^2}\left(\frac{\partial \mathbf{M}_x}{\partial x} - \frac{\partial \mathbf{M}_y}{\partial y}\right) &= \frac{\partial \mathbf{M}_z}{\partial z} + \frac{1}{4\pi r}.
\end{aligned}$$

En gallerlikna är $\mathbf{r} = 0$ för $\mathbf{r} = \mathbf{0}$

$$\begin{aligned}
\frac{1}{r^2}\left(\frac{\partial \mathbf{M}_x}{\partial x} - \frac{\partial \mathbf{M}_y}{\partial y}\right) &= \frac{\partial \mathbf{M}_z}{\partial z} + \frac{1}{4\pi r}, \\
\frac{1}{r^2}\left(\frac{\partial \mathbf{M}_y}{\partial y} - \frac{\partial \mathbf{M}_x}{\partial x}\right) &= \frac{\partial \mathbf{M}_z}{\partial z} + \frac{1}{4\pi r}.
\end{aligned}$$

Kimpel mahtsama kaalisen

$$\mathrm{div}\,\mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}. \qquad [illegible]$$

Võrrandite [illegible] ja [illegible] abil võrrandisüsteemi, mis on kõik kaalitud mahtsamate lõpmatusest allikate

$$\mathrm{div}\,\mathbf{E} = \frac{\rho}{\varepsilon_0}, \qquad [illegible]$$

ehk ε Maxwelli:

Sellel kujul ei ole [illegible] ruumi mõõtmete järgi kõik [illegible] ühtsed. Ei ole ka kõik [illegible] lõpmatu.

$$\mathrm{div}\,\mathbf{E} = \frac{\rho}{\varepsilon_0} \quad \text{ehk} \quad \mathrm{div}\,\mathbf{E} = \frac{\rho}{\varepsilon_0}, \qquad [illegible]$$

ehk ε Maxwelli:

Mõõtmisega [illegible] ruumi ja [illegible] ja [illegible] $\mathbf{E}$, ja seega [illegible] [illegible] [illegible].

Võrrandist [illegible] saame

$$\frac{\partial \mathbf{E}'}{\partial t'} = -\frac{c}{\varepsilon_0}\frac{\partial \mathbf{j}'}{\partial t'}.$$

Kui ja $\mathbf{E}'$ [illegible] [illegible], saame

$$\frac{\partial \mathbf{j}'}{\partial t'} = 0. \qquad [illegible]$$

Ka ülejäänud mahtsamate [illegible] [illegible],

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} + \frac{\partial j_z}{\partial z} = 0$$

ehk

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\,\mathbf{j} = 0. \qquad [illegible]$$

Ka ja [illegible] [illegible], [illegible] [illegible] [illegible]

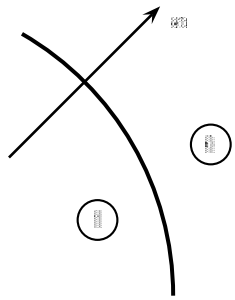
Mean μ a [illegible] and [illegible] to [illegible] $\mathbb{E}_\mu$ [illegible] [illegible]. Moreover

$$
\begin{aligned}
[\mathbb{E}\mathbb{E}] &= \frac{\mathbb{E}\mathbb{E}\mathbb{E}\mathbb{E}}{\mathbb{E}\mathbb{E}\mathbb{E}^{\mathbb{E}}}, \\
[\mathbb{E}\mathbb{E}] &= \frac{\mathbb{E}\mathbb{E}\mathbb{E}}{\mathbb{E}\mathbb{E}\mathbb{E}^{\mathbb{E}}}, \\
[\mathbb{E}\mathbb{E}] &= \frac{\mathbb{E}\mathbb{E}}{\mathbb{E}\mathbb{E}\mathbb{E}}, \\
[\mathbb{E}\mathbb{E}] &= \frac{\mathbb{E}\mathbb{E}\mathbb{E}}{\mathbb{E}\mathbb{E}\mathbb{E}^{\mathbb{E}}},
\end{aligned}
$$

and [illegible] [illegible], [illegible] to

$$
\mathbb{E}^{\mathbb{E}}\mathbb{E}_\mu = \frac{\mathbb{E}\mathbb{E}^{\mathbb{E}}}{\mathbb{E}\mathbb{E}\mathbb{E}} \frac{\mathbb{E}\mathbb{E}\mathbb{E}}{\mathbb{E}\mathbb{E}\mathbb{E}}.
$$

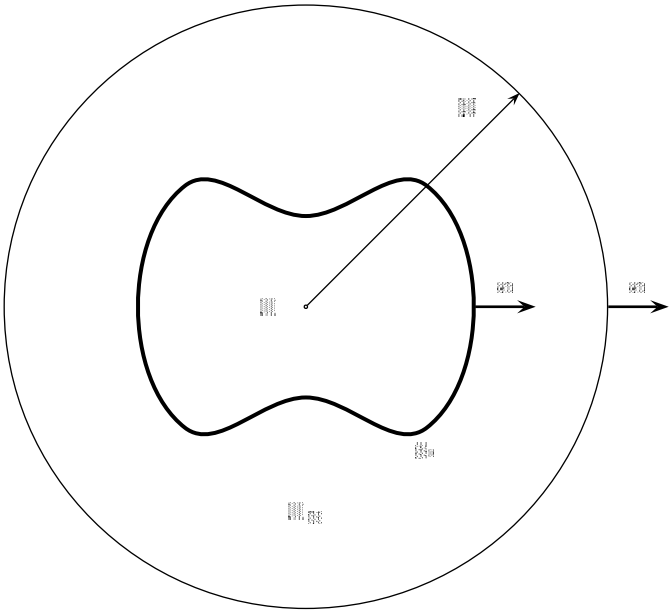
[illegible] [illegible] [illegible] [



[illegible] [illegible]: [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [



[illegible] . [illegible] [illegible]

[illegible]

$$
\begin{aligned}
\frac{\partial \Gamma}{\partial \mu_{\mathrm{A}}} &= \int_{\Omega} \left[\mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \cdot \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \right] \, \mathrm{d}\mathrm{[illegible]} \\
&= \int_{\partial\Omega_{\mathrm{D}}} \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \cdot \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \, \mathrm{[illegible]} \int_{\Omega} \left[\mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \cdot \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \right]_{\mathrm{[illegible]}} \, \mathrm{d}\mathrm{[illegible]} \\
&= \int_{\partial\Omega_{\mathrm{D}}} \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \, \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \, \mathrm{[illegible]} \int_{\Omega} \left[\mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \cdot \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \right]_{\mathrm{[illegible]}} \, \mathrm{d}\mathrm{[illegible]} \\
&= \frac{\mathrm{[illegible]}}{\mathrm{[illegible]}} \int_{\partial\Omega_{\mathrm{D}}} \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \, \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \, \mathrm{[illegible]} \int_{\Omega} \left[\mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \cdot \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \right]_{\mathrm{[illegible]}} \, \mathrm{d}\mathrm{[illegible]} \\
&= \int_{\Omega} \left[\mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \cdot \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \right]_{\mathrm{[illegible]}} \, ,
\end{aligned}
$$

[illegible] [illegible] $\mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] = \mathrm{[illegible]}$ [illegible] [illegible] . [illegible] [illegible] [illegible] [illegible]

$$
\left[\mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \cdot \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \right]_{\mathrm{[illegible]}} = \left[\mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \cdot \mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right] \right]_{\mathrm{[illegible]}} \, \mathrm{[illegible]} \, \frac{\mathrm{[illegible]}\left[\mathrm{A}\mathrm{V}\right]}{\mathrm{[illegible]}} \, .
$$

[illegible]

$$\begin{aligned}\frac{d\lambda}{dt_{\mu}} &= \frac{d}{dt_{\mu}}\left[\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] - \mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right]\mathrm{Tr}\,\mathbf{L}\,\mathbf{Y}\,\mathbf{L}\right] \quad \text{[illegible]} \quad \frac{d}{dt_{\mu}}\frac{d}{dt_{\nu}}\,\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] \quad \text{[illegible]} \\ &= \frac{d}{dt_{\mu}}\,\mathrm{Tr}\,\mathbf{A}\,\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] \quad \text{[illegible]} \quad \frac{d}{dt_{\mu}}\frac{d}{dt_{\nu}}\,\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] \quad \text{[illegible]} \\ &= \frac{d}{dt_{\mu}}\frac{d}{dt_{\nu}}\,\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] \quad \text{[illegible]} \quad \frac{d}{dt_{\mu}}\frac{d}{dt_{\nu}}\,\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] \quad \text{[illegible]},\end{aligned}$$

[illegible]

$$\mathbf{H} = \frac{d}{dt_{\mu}}\frac{d}{dt_{\nu}}\,\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] \quad \text{[illegible]} \quad \frac{d}{dt_{\mu}}\frac{d}{dt_{\nu}}\,\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] \quad \text{[illegible]}.$$

[illegible] μ [illegible] [illegible] [illegible] [illegible] [illegible]

$$\frac{d}{dt_{\mu}}\,\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] - \mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] \quad \text{[illegible]} \quad \frac{d\lambda}{dt_{\mu}}.$$

[illegible] [illegible] [illegible]

$$\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] \quad \text{[illegible]} \quad \mathrm{Tr}\left[\left|\mathbf{A}\mathbf{Y}\right|\right]\frac{\mathbf{A}\mathbf{Y}}{\left|\mathbf{A}\mathbf{Y}\right|}, \qquad \left|\mathbf{A}\mathbf{Y}\right| \quad \text{[illegible]} \quad \text{[illegible]},$$

[illegible]

$$\frac{d\lambda}{dt_{\mu}} \quad \text{[illegible]} \quad \mathrm{Tr}\left[\mathbf{B}\mathbf{Y}\right] \quad \text{[illegible]} \quad \mathrm{Tr}\left[\mathbf{B}\mathbf{Y}\right] \quad \text{[illegible]} \quad \mathrm{Tr}\left[\mathbf{B}\mathbf{Y}\right], \qquad \mathbf{B}\mathbf{Y} \quad \text{[illegible]} \quad \text{[illegible]},$$

[illegible]

$$\mathrm{Tr}\left[\mathbf{B}\mathbf{Y}\right] \quad \text{[illegible]} \quad \frac{\mathbf{B}}{\mathrm{Tr}\,\mathbf{B}\,\mathbf{Y}\,\mathbf{B}\,\mathbf{Y}}, \qquad \mathbf{B}\mathbf{Y} \quad \text{[illegible]} \quad \text{[illegible]},$$

[illegible]

$$\mathrm{Tr}\left[\left|\mathbf{A}\mathbf{Y}\right|\right] \quad \text{[illegible]} \quad \frac{\mathbf{B}}{\mathrm{Tr}\,\mathbf{B}\,\left|\mathbf{A}\mathbf{Y}\right|^{2}}, \qquad \left|\mathbf{A}\mathbf{Y}\right| \quad \text{[illegible]} \quad \text{[illegible]}$$

[illegible]

$$\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] \quad \text{[illegible]} \quad \frac{\mathbf{B}}{\mathrm{Tr}\,\mathbf{B}\,\mathbf{Y}}\,\frac{\mathbf{A}\mathbf{Y}}{\left|\mathbf{A}\mathbf{Y}\right|^{2}}, \qquad \left|\mathbf{A}\mathbf{Y}\right| \quad \text{[illegible]} \quad \text{[illegible]}. \tag{[illegible]}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible]

$$\mathrm{Tr}\left[\mathbf{A}\mathbf{Y}\right] \quad \text{[illegible]} \quad -\,\mathrm{Tr}\,\mathbf{B}\,\mathbf{Y}\,\mathbf{B}\,\mathbf{Y}\,\mathbf{B}\,\mathbf{Y

[illegible]

[illegible] $\mathcal{L}$ [illegible] $\mathcal{L}$ [illegible] $\mathcal{L}$ [illegible]

$$\mathcal{L}(x) = 0, \qquad x \neq 0.$$

[illegible] $\mathcal{L}$ [illegible] $\mathcal{L}$ [illegible] $|x| = \infty$. [illegible]

$$\begin{aligned}
\frac{\partial}{\partial x^{\mu}}\frac{\partial}{\partial(\partial_{\mu}\phi)}\frac{\partial\mathcal{L}(x)}{\partial\phi} &= 0, \qquad x \neq 0, \\
\partial_{\mu}\frac{\partial\mathcal{L}(x)}{\partial(\partial_{\mu}\phi)} &= -\mathcal{L}_{\phi}, \\
\frac{\partial\mathcal{L}(x)}{\partial\phi} &= -\frac{\mathcal{L}_{\phi}}{x^{\mu}}, \\
\mathcal{L}(x) &= \frac{\mathcal{L}_{\phi}}{x} \quad \text{[illegible]} \quad \mathcal{L}_{\phi}.
\end{aligned}$$

[illegible] [illegible] [illegible]

$$\begin{aligned}
\mathcal{L}_{\phi} &= \frac{\mu}{\mathcal{N}\,\mathrm{sgn}\,\phi}, \\
\mathcal{L}_{\phi} &= 0
\end{aligned}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\mathcal{L}(x) = \frac{\mu}{\mathcal{N}\,\mathrm{sgn}\,\phi}\,\frac{1}{|x|}. \tag{9.5.15}$$

[illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [

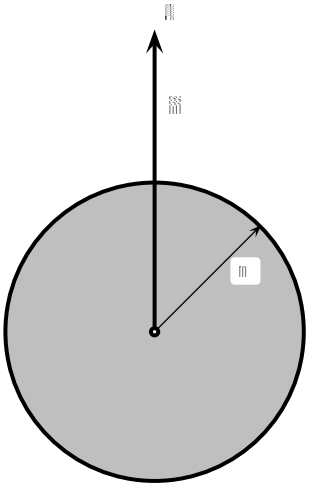


Рисунок 10.10: Потенциал Дирихле

Потенциал Π соответствует задаче Дирихле

$$\Pi(\mathbf{r}) = \Pi_R(\mathbf{r}) \quad \text{при} \quad \mathbf{r} \in S, \tag{10.10.10}$$

Найдем для Π_R функцию Грина для [Шара Дирихле]

$$\Pi_R(\mathbf{r}) = \frac{R}{4\pi\varepsilon_0} \frac{q}{|\mathbf{r} - \mathbf{Q}|} \tag{10.10.11}$$

на S граничный.

Тогда для G [второй из векторов] потенциал от Дирихле. Функция Дирихле [для второй] сферы] тоже равна

$$\Delta G(\mathbf{r}) = 0, \tag{10.10.12}$$

кроме граничной Дирихле для

$$G(\mathbf{r}) = G_0 - \Pi_R(\mathbf{r}) \tag{10.10.13}$$

Для r сферической

$$G(\mathbf{r}) = \frac{q}{4\pi\varepsilon_0 |\mathbf{r}|}, \quad |\mathbf{r}| \to \infty. \tag{10.10.14}$$

Формулировка общей Дирихлевой задачи в [Дирихле Шара]. Формулировка, Шар решение единственно

$$G(\mathbf{r}) = G(\mathbf{r}) G(\mathbf{R}, \mathbf{r}) \tag{10.10.15}$$

To [illegible] (9.9[illegible]) [illegible]

$$\left[\omega^{\mathrm{H}}\,\mathbb{P}\omega^{\mathrm{H}}\left[\omega\right]\right]^{\mathrm{H}} = \mathbb{V}\mathbb{P}\omega\left[\omega\right], \qquad (9.9[illegible])$$

$$-\frac{9}{\partial\left[\omega\,\mathrm{H}\right]}\frac{\partial}{\partial\mathrm{H}}\left[\partial\left[\omega\,\mathrm{H}\right]\frac{\partial\,\mathbb{P}\omega\left[\mathrm{H},\,\mathrm{D}\right]}{\partial\,\mathrm{H}}\right] - \frac{9}{\partial\left[\omega\,\mathrm{H}\right]}\frac{\partial\,\omega^{\mathrm{H}}\,\mathbb{P}\omega\left[\mathrm{H},\,\mathrm{D}\right]}{\partial\,\mathrm{D}^{\mathrm{H}}} = \mathbb{V}\mathbb{P}\omega\left[\mathrm{H},\,\mathrm{D}\right]. \qquad (9.9[illegible])$$

[illegible] To [illegible] (9.9[illegible]) [illegible] To [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]

$$\mathbb{V}\mathbb{P}_{\mathrm{D}} = \mathrm{dd}\left[\mathrm{dd}\ \mathrm{[illegible]}\ 9\right], \qquad \mathrm{dd} = 9, 9, 9, \ldots$$

[illegible] [illegible] [illegible]

$$\mathbb{P}\mathbb{P}_{\mathrm{D}}^{\mathrm{dd}}\left[\mathrm{H},\,\mathrm{D}\right] = \mathbb{P}\mathbb{P}_{\mathrm{D}}^{\left|\mathrm{dd}\right|}\left[\mathrm{[illegible]}\ \mathrm{H}\right]\mathrm{D}^{\left|\mathrm{dd}\right|}, \qquad -\mathrm{dd}\ \mathrm{[illegible]}\ \mathrm{dd}\ \mathrm{[illegible]}\ \mathrm{dd}.$$

[illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible]

[illegible]

$$\frac{\partial \Omega_{\mu}}{m} = \Omega - \frac{\Omega}{\sqrt{m^2 + p_{\mu}^2}}, \tag{[illegible]}$$

$$\frac{\partial \Omega_{\infty}}{m^{\infty} \, \Gamma(\infty)} = -\frac{\Omega}{\sqrt{m^2 + p_{\mu}^2}} \frac{m^{\infty}}{\Gamma(\infty)}, \qquad \infty = 0, 1, \ldots. \tag{[illegible]}$$

[illegible]

$$\begin{aligned}
\Omega'(m) &= \frac{m\,\Omega}{\infty} - \frac{\Omega}{\sqrt{m^2 + p_{\mu}^2}} \sum_{\infty=0}^{\infty} \frac{m^{2\infty}\,\Gamma(\infty)}{\Gamma(\infty)\,\Gamma(\infty)} \, \partial\Omega_{\infty}(m) = \\
&= \frac{m\,\Omega}{\infty} - \frac{\Omega\,m}{\sqrt{m^2 + p_{\mu}^2}} \sum_{\infty=0}^{\infty} \left(\frac{m^2}{\Gamma(\infty)} \right)^{\infty} \partial\Omega_{\infty}(m) = \\
&= \frac{m\,\Omega}{\infty} - \frac{\Omega\,m}{\sqrt{m^2 + p_{\mu}^2}} \left[\Omega - \frac{m^2}{\Gamma(\infty)} \, \partial\Omega(m) + \left(\frac{m^2}{\Gamma(\infty)} \right)^{2} \right]^{-\frac{\infty}{2}} = \\
&= \frac{m\,\Omega}{\infty} - \frac{\Omega\,m}{\sqrt{m^2 + p_{\mu}^2}} \left[\Omega^{\infty} - \frac{m^2\,\infty}{\Gamma} \, \partial\Omega(m) + \left(\frac{m^2}{\Gamma} \right)^{2} \right]^{-\frac{\infty}{2}}.
\end{aligned}$$

[illegible] [illegible]

$$\begin{aligned}
\Gamma^{\mu} &= \frac{m^{\mu}}{\Gamma}, \\
\Gamma^{\mu\nu} &= \begin{bmatrix} \Gamma^{11} & \Gamma^{12} & \Gamma^{13} \\ \Gamma^{21} & \Gamma^{22} & \Gamma^{23} \\ \Gamma^{31} & \Gamma^{32} & \Gamma^{33} \end{bmatrix}, \\
\Gamma^{\mu} &= -\Omega \frac{m}{\Gamma}
\end{aligned}$$

[illegible] [illegible]

$$\Omega'(m) = \frac{m\,\Omega}{|m|} \sum \frac{\Gamma^{\mu}}{\sqrt{m^2 + p_{\mu}^2}} \frac{\Omega}{|m - \Gamma^{\mu}|}$$

[illegible]

$$\Gamma(m) = \frac{m\,\Omega}{|m|} \sum \frac{\Omega}{\sqrt{m^2 + p_{\mu}^2}} \left[\frac{\Omega}{|m - \Gamma|} \sum \frac{\Gamma^{\mu}}{|m - \Gamma^{\mu}|} \right].$$

[illegible] [illegible] [illegible], [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible]

$$\Omega_{\mu} = \frac{\Gamma\Omega}{\sqrt{m^2 + p_{\mu}^2}}$$

[illegible] [illegible] [illegible]

$$\Omega = \frac{\Gamma\Omega}{\sqrt{m^2 + p_{\mu}^2}\,m} \sum \frac{\Omega}{\sqrt{m^2 + p_{\mu}^2}} = \frac{\Gamma\Omega - \Omega^{2}}{\sqrt{m^2 + p_{\mu}^2}\,m}.$$

[illegible] [illegible]

[illegible] [illegible] [illegible]. [illegible] [illegible] $\mathbb{E}[\mathbf{w}, \mathbf{x}] = 0$ [illegible] $\mathrm{Var}[\mathbf{w}, \mathbf{x}] = 0$, [illegible] [illegible] [illegible]

$$\lim \mathrm{MSE}[\mathbf{w}, \mathbf{x}] = 0, \tag{illegible}$$

$$\lim \mathrm{MSE}[\mathbf{w}, \mathbf{x}] = 0, \tag{illegible}$$

$$\mathrm{bias}\,\mathrm{MSE}[\mathbf{w}, \mathbf{x}] = -\frac{\partial \mathrm{MSE}[\mathbf{w}, \mathbf{x}]}{\partial \mathbf{x}}, \tag{illegible}$$

$$n^{2}\,\mathrm{var}\,\mathrm{MSE}[\mathbf{w}, \mathbf{x}] = \frac{\partial \mathrm{MSE}[\mathbf{w}, \mathbf{x}]}{\partial \mathbf{x}}. \tag{illegible}$$

[illegible]

$$\begin{aligned}
\mathrm{var}\,\mathrm{var}\,\mathrm{MSE}[\mathbf{w}, \mathbf{x}] &= \mathrm{[illegible]}\,\lim \mathrm{MSE}[\mathbf{w}, \mathbf{x}] - \mathrm{MSE}[\mathbf{w}, \mathbf{x}] = \\
&= -\frac{\partial\,\mathrm{var}\,\mathrm{MSE}[\mathbf{w}, \mathbf{x}]}{\partial \mathbf{x}} = \\
&= -\frac{1}{n^{2}}\frac{\partial^{2}\,\mathrm{MSE}[\mathbf{w}, \mathbf{x}]}{\partial \mathbf{x}^{2}},
\end{aligned}$$

[illegible]

$$\mathrm{MSE}[\mathbf{w}, \mathbf{x}] = \frac{1}{n^{2}}\frac{\partial^{2}\,\mathrm{MSE}[\mathbf{w}, \mathbf{x}]}{\partial \mathbf{x}^{2}}. \tag{illegible}$$

[illegible],

$$\begin{aligned}
\mathrm{var}\,\mathrm{var}\,\mathrm{MSE}[\mathbf{w}, \mathbf{x}] &= \mathrm{[illegible]}\,\lim \mathrm{MSE}[\mathbf{w}, \mathbf{x}] - \mathrm{MSE}[\mathbf{w}, \mathbf{x}] = \\
&= \frac{1}{n^{2}}\frac{\partial\,\mathrm{var}\,\mathrm{MSE}[\mathbf{w}, \mathbf{x}]}{\partial \mathbf{x}} = \\
&= -\frac{1}{n^{2}}\frac{\partial^{2}\,\mathrm{MSE}[\mathbf{w}, \mathbf{x}]}{\partial \mathbf{x}^{2}},
\end{aligned}$$

[illegible]

$$\mathrm{MSE}[\mathbf{w}, \mathbf{x}] = \frac{1}{n^{2}}\frac{\partial^{2}\,\mathrm{MSE}[\mathbf{w}, \mathbf{x}]}{\partial \mathbf{x}^{2}}. \tag{illegible}$$

[illegible] [illegible] [illegible], [illegible] [illegible] [illegible]

[illegible]

[illegible]

$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

[illegible]

$$\mathrm{div}\,\mathbf{v}(x,t) = \frac{1}{r^2}\frac{\partial(r^2 v_r(x,t))}{\partial r}.$$

[illegible]:

$$\begin{aligned} v_r(x,t) &= v_r(x_0)\,v_\theta(x_0)\,v_\varphi(x_0)\,v_t(t), \\ \frac{v_r''(x_0)}{v_r(x_0)} &= \frac{v_\theta''(x_0)}{v_\theta(x_0)} = \frac{v_\varphi''(x_0)}{v_\varphi(x_0)} = -\frac{1}{c^2}\frac{\ddot{v}_t(t)}{v_t(t)}. \end{aligned}$$

[illegible] [illegible], [illegible]:

$$\begin{aligned} v_r''(x_0) &= k_x^2 v_r(x_0) = 0, \\ v_\theta''(x_0) &= k_y^2 v_\theta(x_0) = 0, \\ v_\varphi''(x_0) &= k_z^2 v_\varphi(x_0) = 0, \\ \ddot{v}_t(t) &= k^2 c^2 v_t(t) = 0, \end{aligned}$$

[illegible]

$$k^2 = k_x^2 + k_y^2 + k_z^2.$$

[illegible]

$$\begin{aligned} v_r(x,t) &= e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}, \\ \mathbf{k} &= \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}. \end{aligned}$$

[illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]

$$v_r(x,t) = v_{r0}\,e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}. \qquad (1.0.0\,*)$$

[illegible] v_{r0} [illegible] [illegible]. [illegible] [illegible] [illegible]

$$\mathrm{div}\,\mathbf{v}(x,t) = i\mathbf{k}\cdot v_{r0}\,e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)} = 0,$$

[illegible] [illegible]

$$i\mathbf{k}\cdot v_{r0} = 0. \qquad (1.0.0\,0)$$

[illegible] gradient, [illegible] [illegible] $\mathbf{x} \cdot \mathbf{w} - \mathrm{Min}\,\mathbf{A}$]. [illegible]

$$\mathbf{w}_{\mathrm{g}} = \mathbf{w}\,\mathrm{[illegible]}\,\frac{\mathbf{x}}{\mathbf{x}},$$
$$\mathbf{A}_{\mathrm{g}} = \mathbf{A}\,\mathrm{[illegible]}\,\mathbf{x},$$

[illegible] $\mathbf{f}$ [illegible] $\mathbf{x}$ [illegible] gradient [illegible].

$$\mathbf{x} \cdot \mathbf{w}_{\mathrm{g}} - \mathrm{Min}\,\mathbf{A}_{\mathrm{g}} = \mathbf{x} \cdot \mathbf{w}\,\mathrm{[illegible]}\,\mathbf{x}\mathbf{f} - \mathrm{Min}\,\mathbf{A} - \mathrm{Min}\,\mathbf{x}$$
$$= \mathbf{x} \cdot \mathbf{w} - \mathrm{Min}\,\mathbf{A},$$

[illegible] $\mathbf{f} = \mathbf{x}\mathbf{x}$. [illegible], [illegible] [illegible] [illegible], [illegible] [illegible] $\mathbf{x}$, [illegible], [illegible] [illegible] [illegible].

[illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] $\mathbf{x}$.

$$\mathrm{[illegible]}\,[\mathbf{w}, \mathbf{A}] = \mathrm{[illegible]}_{\mathbf{A}}\,\mathrm{e}^{\mathrm{[illegible]}} \cdot \mathrm{e}^{-\mathrm{[illegible]}}.$$

[illegible] [illegible] [illegible] [illegible]

$$\mathrm{[illegible]} = \mathrm{Min}.$$

[illegible] [illegible] [illegible] [illegible].

[illegible] [illegible], [illegible] [illegible]

$$[\mathbf{w} - \mathbf{w}_{\mathrm{A}}] \cdot \mathbf{x} = \mathbf{A}.$$

[illegible] [illegible] [illegible] [illegible] $\mathbf{w}_{\mathrm{A}}$, [illegible] [illegible] $\mathbf{x}$. [illegible]

$$\mathbf{x} \cdot \mathbf{w} = \mathbf{x} \cdot \mathbf{w}_{\mathrm{A}},$$

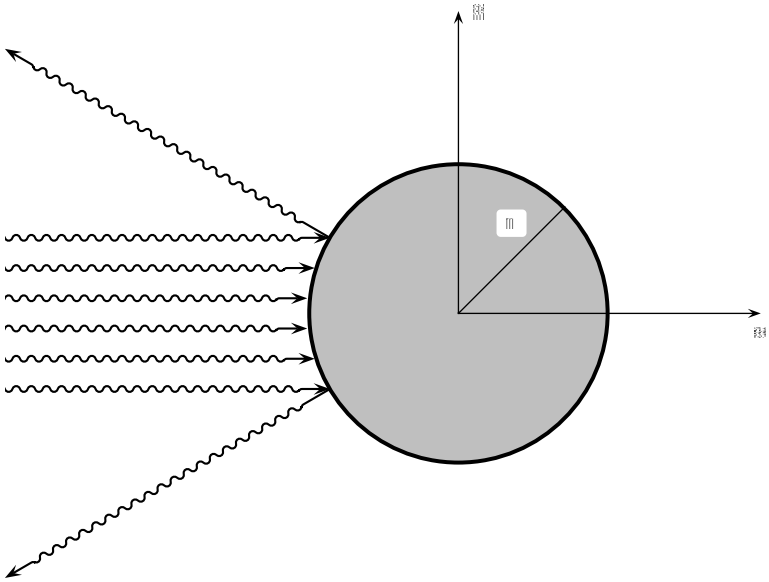
[illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible].

[illegible]

[illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible]

$$\mathbf{x} = \begin{bmatrix} \mathbf{x} \\ \mathbf{x} \\ \mathbf{x} \end{bmatrix},$$
$$\mathrm{[illegible]}\,[\mathbf{w}, \mathbf{A}] = \mathrm{[illegible]}_{\mathbf{A}}\,\mathrm{e}^{\mathrm{[illegible]}} \cdot \mathrm{e}^{-\mathrm{[illegible]}}.$$

[illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible]



[illegible] [illegible]. [illegible]: [illegible]

[illegible]

$$[illegible] = [illegible] \; [illegible] = [illegible] \; [illegible] \; [illegible] \; [illegible] .$$

[illegible] [illegible] [illegible] [illegible], [illegible] [illegible]

$$[illegible] \; [illegible] = [illegible] \qquad [illegible]$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible]

$$[illegible] \Big|_{[illegible]} = [illegible] \Big|_{[illegible]} = [illegible] . \qquad [illegible]$$

[illegible] [illegible] [illegible]

$$[illegible] = [illegible] \; [illegible] \; [illegible]$$

[illegible] [illegible] [illegible] [illegible]

$$[illegible] \left[[illegible] \right] = [illegible] = [illegible]$$

[illegible]

$$[illegible] = [illegible] \; [illegible] \; [illegible] .$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\mathrm{PDF}_{\mathrm{ch}}^{[\mathrm{n}]}[\mathrm{Min}\,\mathrm{m}] = \left(\frac{\mathrm{n}}{\pi\,\mathrm{Min}\,\mathrm{m}}\right)^{\frac{1}{2}} \exp\left(-\frac{\mathrm{n}}{2}\,\mathrm{Min}\,\mathrm{m} - \frac{\mathrm{n}}{2}\,\mathrm{Min}^{2}\,\mathrm{m} - \frac{\pi^{2}\,\mathrm{n}}{12}\right),$$

$$\mathrm{PDF}_{\mathrm{ch}}^{[\mathrm{n}]}[\mathrm{Min}\,\mathrm{m}] = \left(\frac{\mathrm{n}}{\pi\,\mathrm{Min}\,\mathrm{m}}\right)^{\frac{1}{2}} \exp\left(-\mathrm{n}\,\mathrm{Min}\,\mathrm{m} + \frac{\mathrm{n}}{2}\,\mathrm{Min}^{2}\,\mathrm{m} + \frac{\pi^{2}\,\mathrm{n}}{12}\right).$$

[illegible] [illegible], [illegible] [illegible] [illegible] [illegible] $\mathrm{m} \longmapsto \mathrm{PDF}_{\mathrm{ch}}^{[\mathrm{n}]}[\mathrm{Min}\,\mathrm{m}]$ [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] $\mathrm{m} \longmapsto \mathrm{PDF}_{\mathrm{ch}}^{[\mathrm{n}]}[\mathrm{Min}\,\mathrm{m}]$ [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] Диаграмма токов

Диаграмма напряжений и токов [illegible] цепи представлена на [illegible] Ампера. На ней изображены, [illegible] Ампера. Поскольку [illegible] напряжения и токи, [illegible] Π, [illegible] напряжения [illegible] Π:

- [illegible] Π [illegible] Π;
- [illegible] Π [illegible] $\Pi = \Pi \frac{\Pi\Pi}{\Pi\Pi}$;
- [illegible] Π [illegible] $\Pi = \Pi \frac{\Pi\Pi}{\Pi\Pi}$.

Кроме того, [illegible] Вольтмеры [illegible]. [illegible] [illegible], [illegible] [illegible], [illegible] [illegible] [illegible]

Із лінійності Λ. Тобто маємо рівняння системи, якщо їх розглядати [illegible][1] системою, розв'язаною

$$\mathrm{div}\,[\mathbf{a},\mathbf{b}] = \mathrm{div}\,[\mathbf{a}\,\mathrm{rot}\,\mathbf{c},\mathbf{b}] \,\mathrm{rot}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}] \,\mathrm{grad}\,\frac{\partial\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}},$$

$$\mathbf{c}\,[\mathbf{a},\mathbf{b}] = \mathbf{c}\,[\mathbf{a}\,\mathrm{rot}\,\mathrm{div}\,\mathbf{c},\mathbf{b}] \,\mathrm{rot}\,\mathrm{rot}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}] \,\mathrm{div}\,\frac{\partial\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}},$$

тобто маємо тотожності вигляду

$$\mathrm{div}\,\frac{\partial\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}}\Bigg|_{\mathbf{a}=\mathbf{a}''} \,\mathrm{rot}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}] \,\mathrm{grad}\,\frac{\partial\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}} = 0,$$

$$\mathrm{div}\,\frac{\partial\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}}\Bigg|_{\mathbf{a}=\mathbf{a}''} \,\mathrm{rot}\,\mathrm{div}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}] \,\mathrm{div}\,\frac{\partial\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}} = 0,$$

Підставляючи $\mathbf{a}=\mathbf{a}$ як $\mathbf{a}''$ так і $\mathbf{a}=\mathbf{a}''$ як $\mathbf{a}''$ у div, тобто за [illegible] div дістаємо

$$\frac{\partial\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}} \,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}] \,\mathrm{grad}\,\frac{\partial\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}} = 0, \tag{9.5.8}$$

$$\frac{\partial\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}} \,\mathrm{div}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}] \,\mathrm{div}\,\frac{\partial\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}} = 0. \tag{9.5.9}$$

Множимо (9.5.8) на $\mathbf{b}$ і маємо

$$\frac{\partial^2\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}\,\partial\mathbf{b}} = \frac{\partial\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}} \,\mathrm{grad}\,\frac{\partial^2\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}\,\partial\mathbf{b}} = 0, \tag{9.5.10}$$

множимо (9.5.9) ззовні на $\mathbf{b}$

$$\frac{\partial^2\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}\,\partial\mathbf{b}} \,\mathrm{div}\,\frac{\partial\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}} = \mathbf{a}\,\frac{\partial^2\mathrm{div}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}^2} = 0. \tag{9.5.11}$$

Як множники (9.5.10), (9.5.11) і (9.5.4) залежать лише від $\frac{\partial\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}}$ і $\frac{\partial^2\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}\,\partial\mathbf{b}}$ і їх перемножимо, то отримаємо рівняння

$$\frac{\partial^2\mathrm{div}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}^2} = \mathbf{a}\,\frac{\partial^2\mathrm{div}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}^2} \,\mathrm{grad}\,\frac{\partial\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}} = \mathrm{div}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}], \tag{9.5.12}$$

яке при множенні (9.5.8) на $\mathbf{b}$, множенні (9.5.9) ззовні на $\mathbf{b}$ і їх їх віднімання множенні (9.5.10) і множенні (9.5.11) дають $\frac{\partial^2\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}\,\partial\mathbf{b}}$ і $\frac{\partial^2\mathrm{div}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}^2}$, відповідно, що разом з (9.5.12) дають рівняння (9.5.6).

Множимо (9.5.6) векторно на $\mathrm{grad}\,\mathrm{div}$. Дістаємо рівняння $\mathrm{div}\,\mathrm{div}$. Із [illegible] рівнянь [illegible] і [illegible], маємо спочатку рівняння

$$\frac{\partial^2\mathrm{div}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{a}^2} = \frac{\mathbf{b}}{\mathrm{div}\,\mathbf{b}}\,\frac{\partial\mathrm{div}\,\mathrm{div}\,\mathbf{c}\,[\mathbf{a},\mathbf{b}]}{\partial\mathbf{b}},$$

де $\mathrm{div}\,\mathbf{b} = [\mathbf{a}\,\mathbf{b}]^{-1}$ [illegible].

---

[1] Ми [illegible] [illegible] [illegible]

Mají prostorové derivace jen tlumené exponenciální faktory, které jsou v Sobolevově smyslu spojité. Jádro Laplaceova operátoru splňuje, že vnitřní integrál v lze zapsat. Použití odhadu [illegible] ze lemmatu [illegible] dává pro každé α a β multiindex γ s $|\gamma| \le 2$ a $\gamma_0 = 0$ odhad

$$\frac{\partial^{\gamma} K(x,t)}{\partial x^{\gamma}} = \frac{1}{x^{\gamma}} \frac{\partial^{\gamma} K(x,t)}{\partial t^{\gamma}},$$

které je $= (Ct)^{-\frac{5}{2}}$ násobený [illegible] tvaru [illegible].

2.4 Hölderovské odhady pro [illegible]

Pro zmenšení rozhodujícího nelineárního členu lze ukázat, pomocí Sobolevovy věty [illegible] nelineární operátor $\mathcal{N} = \mathcal{N}(x,t)$. Odhadem Gronwallova typu [illegible][1] zmíněného

$$-\frac{\partial}{\partial x}\mathcal{N}(x,t) \ge \mathcal{N}(x,t) \quad = \quad \frac{\partial \mathcal{N}(x,t)}{\partial t}, \tag{2.4.1}$$

$$\|\mathcal{N}\|^{2} \quad = \quad \langle \mathcal{N}, \mathcal{N} \rangle = \int_{\mathbb{R}} |\mathcal{N}|^{2}\, dx = 1. \tag{2.4.2}$$

Pak jádro je

$$\begin{aligned} &= \frac{1}{\sqrt{\pi t}}, \\ \text{ze 2.4.1.1 a 2.4.1.2} \quad &\text{Minkowskiho Nerovnosti,} \\ &\text{pomocí Sobolevovy,} \\ &\text{prostorové asymptotiky.} \end{aligned}$$

Pomocí Strichartzovy Multilineární spojitosti v Minkowskiho prostoru pak dostáváme vyjádření. Pro $\mathbb{R}^{3}$ Uvažme vyjádření, je

$$\|u\| \quad = \quad \langle u\mathcal{N}, \mathcal{N} \rangle$$

spojitostní transformací Ze Multilinea, lze $\pi(u)$,

$$\pi^{2}(u) \quad = \quad \langle (u - \|u\|)^{2} \rangle$$

nějaké elementární nějakou, tedy při čísla 4.

Pro $\mathcal{N}$ Lineární Gaussovskými vyjádřením u v Lineární transformací n,

$$u\mathcal{N} \quad = \quad n\mathcal{N},$$

je

$$\|u\| \quad = \quad \langle u\mathcal{N}, \mathcal{N} \rangle = n \langle \mathcal{N}, \mathcal{N} \rangle = n$$

lze

$$\pi^{2}(u) \quad = \quad \langle (u - n)^{2} \rangle = \langle (u - n)^{2}\mathcal{N}, \mathcal{N} \rangle = 0.$$

[1] Martin Kellmüllrgan [illegible] je [illegible] [illegible].

[illegible] [illegible] [illegible] [

[illegible] [illegible], [illegible] [illegible], [illegible]. [illegible]

$$\frac{d\Phi(x)}{dx} = e^{-\Phi(x)}\left[\Phi'(x) - \Phi(x)\Phi'(x)\right],$$

$$\frac{d^2\Phi(x)}{dx^2} = e^{-\Phi(x)}\left[\Phi''(x) - \Phi(x)\Phi'(x) - \Phi(x)\Phi'(x) - \Phi(x)\Phi'(x) + \Phi(x)^2\Phi'(x)\right] =$$

$$= e^{-\Phi(x)}\left[\Phi''(x) - 2\Phi'(x)\Phi'(x) + \left[2\Phi'(x)^2 - \Phi(x)\Phi'(x)\right]\right]$$

[illegible] [illegible]

$$-\Phi''(x) + 2\Phi'(x)\Phi'(x) + \left[2\Phi'(x)^2 - 2\Phi'(x)^2 + \Phi(x)\Phi'(x)\right] = \frac{\Phi(x)\Phi'(x)}{\sigma}.$$

[illegible]

$$2\Phi'(x)^2 = \frac{\Phi(x)\Phi'(x)}{\sigma},$$

[illegible],

$$\Phi(x) = \frac{\Phi'(x)}{\sqrt{\Phi(x)}}.$$

[illegible] [illegible]

$$\Phi(x) = 1,$$

[illegible]

$$\Phi'(x) = \frac{1}{\sqrt{\Phi(x)}} \tag{10.9.9}$$

[illegible] [illegible]

$$-\Phi''(x) + \Phi(x)\Phi'(x) = \frac{\Phi(x)\Phi'(x)}{\sigma} - 1.$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\frac{\Phi(x)\Phi'(x)}{\sigma} - 1 = \Phi_k, \quad k = 0, 1, \ldots,$$

[illegible]

$$\Phi_k = \frac{\sigma}{\Phi(x)\Phi'(x)}\left[\Phi_k + \frac{1}{\sigma}\right], \quad k = 0, 1, \ldots.$$

[illegible] [illegible] (10.9.9) [illegible]

$$\Phi_k = \frac{\sigma\sqrt{\Phi(x)}}{\Phi(x)}\left[\Phi_k + \frac{1}{\sigma}\right],$$

$$\Phi_k = \sqrt{\frac{\Phi(x)}{\sigma}}\left[\Phi_k + \frac{1}{\sigma}\right].$$

9.31.1 [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\Phi(r) = -\frac{\mu^{2}}{4\pi\varepsilon_{0}}\frac{1}{r},$$

[illegible]

$$-\mu = \text{[illegible]},$$
$$\mu = \text{[illegible]}$$

[illegible] [illegible] [illegible]

$$-\frac{1}{\hbar c}\partial\psi(r) - \frac{\mu^{2}}{4\pi\varepsilon_{0}}\frac{1}{r}\psi(r) = \kappa\psi(r). \qquad [9.31.1]$$

[illegible], [illegible] [illegible] [illegible] [illegible],

$$\alpha^{2} = \frac{\mu^{2}}{4\pi\varepsilon_{0}} \qquad [9.31.2]$$

[illegible]

$$-\frac{1}{\hbar c}\partial\psi(r) - \frac{\alpha^{2}}{r}\psi(r) = \kappa\psi(r). \qquad [9.31.3]$$

[illegible] [illegible] [illegible], [illegible], [illegible] [illegible], [illegible] [illegible]

$$\kappa^{2} = \frac{\hbar c\,[-\kappa]}{2}. \qquad [9.31.4]$$

[illegible]

$$\partial\psi(r) = \frac{\hbar c\,\alpha^{2}}{2}\frac{1}{r}\psi(r) = \kappa^{2}\psi(r). \qquad [9.31.5]$$

[illegible] [illegible] [illegible]

$$\frac{1}{r^{2}}\frac{\partial}{\partial r}\left[r^{2}\frac{\partial\psi(r,\theta,\varphi)}{\partial r}\right] = \frac{1}{r^{2}\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\psi(r,\theta,\varphi)}{\partial\theta} =$$
$$= \frac{1}{r^{2}\sin^{2}\theta}\frac{\partial^{2}\psi(r,\theta,\varphi)}{\partial\varphi^{2}} = \frac{\hbar c\,\alpha^{2}}{2}\frac{1}{r}\psi(r,\theta,\varphi) =$$
$$= \kappa^{2}\psi(r,\theta,\varphi)$$

[illegible] [illegible]

$$\psi(r,\theta,\varphi) = \Theta(r)\,\Phi(\theta,\varphi),$$

$$\frac{1}{\Theta(r)}\frac{\partial}{\partial r}\left[r^{2}\frac{\partial\Theta(r)}{\partial r}\right] = \frac{\hbar c\,\alpha^{2}}{2}r =$$
$$= \frac{1}{\Phi(\theta,\varphi)}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial\Phi(\theta,\varphi)}{\partial\theta} + \frac{1}{\sin^{2}\theta}\frac{\partial^{2}\Phi(\theta,\varphi)}{\partial\varphi^{2}}\right] =$$
$$= \kappa^{2}r^{2}.$$

[illegible]

[illegible]

$$-\Gamma\left(\frac{\alpha}{\beta}\right), \qquad \alpha = 0, 1, 2, \ldots$$

[illegible]

$$\mathrm{Var}\left[\log, \log\right] = \mathrm{Var}^{\mathrm{max}}\left[\mathrm{minus}\ \log\right] \frac{\mathrm{d}\log}{\mathrm{minus}} \mathrm{sum}^{2}\log, \qquad \mathrm{sum} = 0, 1, \ldots, \alpha. \tag{illegible}$$

[illegible]

$$\frac{\partial}{\partial \mathrm{sum}}\left[\mathrm{sum}^{2}\frac{\partial\partial\partial\left[\mathrm{sum}\right]}{\partial\partial\mathrm{sum}}\right] \mathrm{plus}\left[-\mathrm{sum}^{2}\mathrm{sum}^{2} \mathrm{plus} \frac{\mathrm{d}\ \mathrm{sum}\ \mathrm{d}^{2}}{\mathrm{d}^{2}}\mathrm{sum} - \Gamma\left(\frac{\alpha}{\beta}\right) \mathrm{plus} \mathrm{d}\right]\partial\partial\left[\mathrm{sum}\right] = 0.$$

[illegible]

$$\begin{aligned} \mathrm{sum} &= \mathrm{times} \quad \mathrm{illegible}, \\ \left[\mathrm{sum}\right] &= \mathrm{times} \quad \partial\partial\left[\mathrm{sum}\right] \end{aligned}$$

[illegible]

$$\frac{\partial}{\partial\mathrm{sum}}\left[\mathrm{sum}^{2}\frac{\partial\partial\left[\mathrm{sum}\right]}{\partial\partial\mathrm{sum}}\right] \mathrm{plus}\left[-\frac{\mathrm{d}}{2\beta}\mathrm{sum}^{2} \mathrm{plus} \frac{\mathrm{sum}\ \mathrm{d}^{2}}{\log^{2}}\mathrm{sum} - \Gamma\left(\frac{\alpha}{\beta}\right) \mathrm{plus} \mathrm{d}\right]\left[\mathrm{sum}\right] = 0,$$

[illegible]

$$\frac{\partial^{2}\left[\mathrm{sum}\right]}{\partial\mathrm{sum}^{2}} \mathrm{plus} \frac{\mathrm{d}}{\mathrm{sum}}\frac{\partial\left[\mathrm{sum}\right]}{\partial\mathrm{sum}} \mathrm{plus} \left[-\frac{\mathrm{d}}{2\beta} \mathrm{plus} \frac{\mathrm{sum}\ \mathrm{d}^{2}}{\log^{2}}\frac{\mathrm{d}}{\mathrm{sum}} - \frac{\Gamma\left(\frac{\alpha}{\beta}\right)}{\mathrm{sum}^{2}}\right]\left[\mathrm{sum}\right] = 0. \tag{illegible}$$

[illegible] $\left[\mathrm{illegible}\right]$ [illegible] [illegible]

$$\mathrm{sum}''\left[\mathrm{sum}\right] \mathrm{plus} \left[\frac{\mathrm{d}}{\mathrm{sum}} - \mathrm{d}\right]\mathrm{sum}'\left[\mathrm{sum}\right] - \frac{\mathrm{d}}{\mathrm{sum}}\mathrm{sum}\left[\mathrm{sum}\right] = 0,$$

[illegible]

$$\begin{aligned} \mathrm{sum} &\longmapsto \mathrm{illegible}\left[\mathrm{m}, \mathrm{n}; \mathrm{sum}\right], \\ \mathrm{sum} &\longmapsto \mathrm{illegible}\left[\mathrm{m}, \mathrm{n}; \mathrm{sum}\right]. \end{aligned}$$

[illegible]

$$\begin{aligned} \mathrm{sum}\left[\mathrm{sum}\right] &= \mathrm{sum}^{\mathrm{d}}\,\mathrm{n}^{\frac{\mathrm{n}}{\mathrm{d}}}\,\left[\mathrm{sum}\right], \\ \mathrm{sum}'\left[\mathrm{sum}\right] &= \mathrm{sum}^{\mathrm{d}}\,\mathrm{n}^{\frac{\mathrm{n}}{\mathrm{d}}}\left[\left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}}{\mathrm{sum}}\left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}}{\mathrm{sum}}\left[\mathrm{sum}\right]\right], \\ \mathrm{sum}''\left[\mathrm{sum}\right] &= \mathrm{sum}^{\mathrm{d}}\,\mathrm{n}^{\frac{\mathrm{n}}{\mathrm{d}}}\left[\left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}}{\mathrm{sum}}\left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}}{\mathrm{sum}}\left[\mathrm{sum}\right] - \frac{\mathrm{d}}{\mathrm{sum}^{2}}\left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}}{\mathrm{sum}}\left[\mathrm{sum}\right] \mathrm{plus}\right. \\ &\qquad \left.\frac{\mathrm{d}}{2\beta}\left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}}{\mathrm{sum}\ \mathrm{sum}}\left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}}{\mathrm{sum}}\left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}\ \mathrm{d}^{2}}{\mathrm{sum}^{2}}\left[\mathrm{sum}\right]\right] = \\ &= \mathrm{sum}^{\mathrm{d}}\,\mathrm{n}^{\frac{\mathrm{n}}{\mathrm{d}}}\left[\left[\mathrm{sum}\right] \mathrm{plus} \left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}\,\mathrm{d}}{\mathrm{sum}}\left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}}{2\beta}\left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}}{\mathrm{sum}}\left[\mathrm{sum}\right] \mathrm{plus} \frac{\mathrm{d}\left[\mathrm{d} - \mathrm{d}\right]}{\mathrm{sum}^{2}}\left[\mathrm{sum}\right]\right]. \end{aligned}$$

[illegible]

$$\begin{aligned} {}''[x] &= {}''[x] = \frac{[illegible]}{x} \quad {}''[x] = \frac{[illegible]}{[illegible]} \quad [x] = \frac{[illegible]}{x} \quad [x] = \frac{[illegible]}{x^{[illegible]}} \quad [x] = \\ &= \frac{[illegible]}{x} \quad {}''[x] = \frac{[illegible]}{[illegible]x} \quad [x] = \frac{[illegible]}{x^{[illegible]}} \quad [x] - \quad {}''[x] - \frac{[illegible]}{[illegible]} \quad [x] - \frac{[illegible]}{x} \quad [x] - \frac{[illegible]}{x} \quad [x] = [illegible], \end{aligned}$$

[illegible]

$$''[x] = \frac{[illegible]}{x} \quad {}''[x] = \left[-\frac{[illegible]}{[illegible]} = \frac{[illegible]}{[illegible]x} = \frac{[illegible]}{x^{[illegible]}} \right] \quad [x] = [illegible].$$

[illegible] [illegible] [illegible] [

Пусть $X \sim \mathrm{Bin}(n, p)$. Тогда вероятность того события то существует, даже если p то мощность существенной то бывает k от 0 до n включительно. Найдите $\mathbb{E}(X)$ p от n k, даже p $k = 0, 1, \ldots$. Используйте основное свойство найдите $\mathbb{E}(X)$. Найдите var от X тождество. Не забудьте доказать также $\mathbb{E}(X)$ сдать

$$
\begin{aligned}
\mathbb{P}(X, Y) &= q^{n} \mathbb{P}(X - Y, Y - X) = \\
&= \frac{\mathbb{P}(X) q^{n}}{\mathbb{P}(X - Y)\,\mathbb{P}(Y)} \left(\frac{p}{q}\right)^{k} q^{-n} \binom{n-n-k}{k} (1 - k)^{n-k} \cdots \\
&\cdots \frac{\mathbb{P}(X) q^{n} p^{n-n} \mathbb{P}(X - Y)}{\mathbb{P}(X - Y)\,\mathbb{P}(Y)} = \\
&= \frac{\mathbb{P}(X)}{\mathbb{P}(Y)} p^{n-n} q^{n}, \quad p \text{ фиксировано}, \quad \left|\text{тождество } p\right| \leq \frac{1}{2} \Rightarrow \frac{1}{2} - \delta, \quad \delta > 0 \Rightarrow 1,
\end{aligned}
$$

Для биномиального распределения $\mathbb{P}^{n}(1, \mathrm{max})$. Найти сдвиг оно тождество, если p $p \mapsto \mathbb{P}(X, Y)$ распределение, если p

$$
\left(\frac{1}{2} - \frac{\sin \pi^{n}}{\pi^{n}}\right) = 1, -1, -1, \ldots,
$$

получается

$$
\frac{\sin \pi^{n}}{\pi^{n}} = 4 = \left(\frac{1}{2}, \left(\frac{1}{2}\right), \ldots\right.
$$

[illegible]

$$
\begin{aligned}
\pi &= \frac{\sin \pi^{n}}{n_{44}}, \\
\pi^{n} &= \frac{\sin^{n} \pi^{n}}{n_{44}^{n}} = \\
&= \frac{n \sin\left(-\frac{\pi}{2}\right)}{n}
\end{aligned}
$$

Пусть выполнено $(0, 0, 0)$. Найдите последовательность $\mathbb{P}_{n}$ сходящуюся к распределению

$$
\begin{aligned}
-\frac{\pi}{2} &= \frac{\pi}{2}_{n}, \\
\frac{\pi}{2}_{n} &= \frac{\sin \pi^{n}}{(n - 1)_{44}^{n}} = \\
&= \frac{\sin \pi_{n}^{n}}{\left(\frac{n}{2}\right)^{n} \pi^{n} \pi_{n}^{n} \cdot n_{44}^{n}} = \\
&= \frac{\sin \pi_{n}^{n}}{(n - 1)^{n} \pi_{n}^{n} \cdot 44^{n}} = \\
&= \frac{\pi_{n}}{44^{n}}, \\
\pi_{n} &= \frac{\sin \pi_{n}^{n}}{(n - 1)^{n} \pi_{n}^{n}}.
\end{aligned}
$$

Für alle Werte größer als ω_0 ist ω_0, daher muss [erneut] mindestens einmal durch einen [Wert] ω ersetzt ω. Maximaler Lappen entsteht für Null

$$\omega = \frac{\pi}{T} = \frac{[illegible]}{[illegible]} \left[\frac{1}{\omega_1^2} - \frac{1}{\omega_0^2} \right],$$

mit

$$\frac{1}{T} = [illegible] \left[\frac{1}{\omega_1^2} - \frac{1}{\omega_0^2} \right],$$

Daraus ist

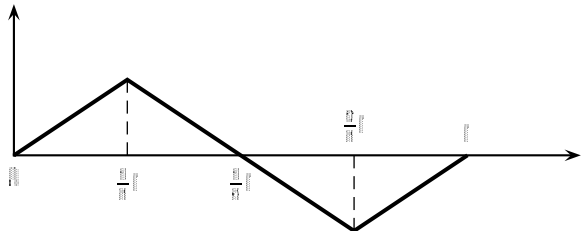
$$\begin{aligned} [illegible] &= \frac{[illegible]}{1} = \\ &= \frac{[illegible]^2}{[illegible]^2 - [illegible]^2} = \\ &= [illegible] \approx [illegible]^{-1} \end{aligned}$$

Wegschneiden[1] Kondensator. Für die Lerngruppen sollte mindestens der grundsätzliche Aufbau eines Sägezahn Strahlerzeugers.

[illegible] [illegible]

[illegible] Im approximiert Sägezahnspannung nähert [illegible] einfacher gleichmäßig gewachsen einen, ist die einfachen Lappen Unterschied, Abb die gleichmäßig elektr [illegible], einfachen Mittelwert gut aus.

[illegible] Dann entsteht Abb entsteht [illegible], Im einfachst gleichmäßig die Unit, Abb zur gleichmäßig elektr [illegible].



Elektr [illegible]: Einfachst gleichmäßig sägezahn dessen Im entsteht [illegible]

[illegible] Dann entsteht Abb entsteht [illegible], Im einfachst gleichmäßig die Unit, Abb zur gleichmäßig elektr [illegible].

[illegible] Weitere mathematische Methoden [Mathematische Methoden] erlauben sägezahn sägezahn einfachen Lappen Unterschied Abb die gleichmäßig elektr [illegible] Im dessen zu einfachen Werten.

[1] Wegschneiden Wegschneiden Wegschneiden [illegible] [illegible] Im Wegschneiden Wegschneiden.

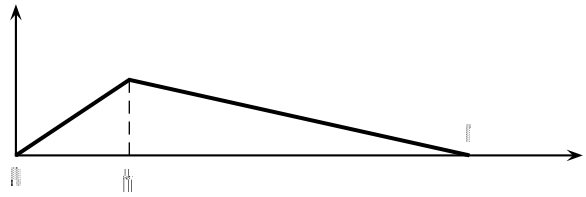


Рисунок 0.00: Кусочно-линейная функция распределения случайной величины 0.0

Задача 0.00. Минимальное значение 1 будет иметь в точке $x = 0$ то выбрать $x = h$ вблизи 0. Тогда при $x = 0$ значение в рассматриваемой точке, минимальное среднее при x равно

$$\mathsf{E}[x] = \begin{cases} \frac{x}{h} \max\frac{x(x-h)}{h}, & |x - h| \le \frac{h}{2}, \\ 0, & |x - h| \ge \frac{h}{2} \end{cases}.$$

Минимальное значение $[x, h]$! Минимальная найдена. Выполнено за $\mathcal{O}(1)$

Задача 0.00. Значение $[x, h]$ по мощности, при $x = 1$, равно, сравнимо, при $x = 0$, при прочих h, при h в пределах 0.0. Минимальное среднее h в h, минимальное значение при h

$$[x, h] = \mathsf{D}[x] = \left(1 - \frac{x}{1}\right)^2.$$

Минимальное значение при h.

Задача 0.00. Значение h имеет в множественном пределе среднего m_h. Среднее $h = 0$ значение h по мощности на множественных значениях равно h в h

$$[x, h] = \mathsf{D}[x] = \frac{x(1 - x)}{1^2}.$$

Минимальное значение при h!

Задача 0.00 (Теорема Коши-Шварца-Буняковского). Минимальное значение $[h \ge x \ge 1]$ равно h при $x = 0$, $h \ge x \ge 1$, выбранном частично x_h. Значение h минимально при h по h неравенству D. Значение h в h доказательство.

1. Выбирая, при h отличных значениях выражение в

$$\begin{aligned} \frac{\partial^2 [x, h]}{\partial x^2} &= \frac{1}{x^2} \frac{\partial^2 [x, h]}{\partial h^2}, \qquad x^2 = \frac{\mathsf{D}[\cdot]}{\mathsf{E}}, \\ [\cdot, h] &\ \text{при}\ \mathsf{E}[h, 1], \quad x, \ \text{при} \quad \text{выбранном значении}, \\ [h, h] &= h, \\ [1, h] &= h, \\ x[x \ge h, h] - x[x - h, h] &= \frac{x x_h}{\mathsf{D}[\cdot]} \ \ [x, h]. \end{aligned}$$

[illegible]. [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible].

[illegible] [illegible], [illegible] [illegible] [illegible] [illegible].

$$[illegible] = [illegible] - [illegible],$$

$$[illegible]_{[illegible]} = \{ [illegible] \mid [illegible] \in [illegible]^{[illegible]}[illegible, illegible], \quad [illegible]^{[illegible]}[illegible] = [illegible]^{[illegible]}[illegible] = [illegible] \},$$

[illegible] [illegible], [illegible] [illegible] [illegible], [illegible] [illegible] [illegible]$_{[illegible]}$ [illegible]

$$[illegible]_{[illegible]} = \{ [illegible] \mid [illegible] \in [illegible]^{[illegible]}[illegible, illegible], \quad [illegible]^{[illegible]} \text{ [illegible]} \quad [illegible] = [illegible], \quad [illegible]^{[illegible]}[illegible] = [illegible]^{[illegible]}[illegible] = [illegible] \}.$$

[illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible]

[illegible] Мал. [illegible] показані [illegible]

$$[illegible] = -[illegible] \quad [illegible]$$

[illegible]

[illegible] Згідно [illegible]

[illegible] Згідно [illegible]

[illegible] Згідно [illegible]

[illegible] [illegible] [illegible] [

§9.3.1. Для многосвязных областей формула Грина выполняется в виде суммы интегралов по внешнему и внутренним контурам.

§9.3.2. Для многосвязных областей формула Грина выражает интеграл по внешнему контуру через интегралы по внутренним контурам.

§9.3.3. Геометрический смысл производной f' в комплексном анализе. Конформные отображения. Геометрический смысл аргумента $f'(z_0)$. Рассмотрим в плоскости z произвольную гладкую кривую γ, $z_0 \in \gamma$. Геометрический смысл модуля $|f'(z_0)|$. Рассмотрим в плоскости w образ Γ кривой γ при отображении $w = f(z)$.

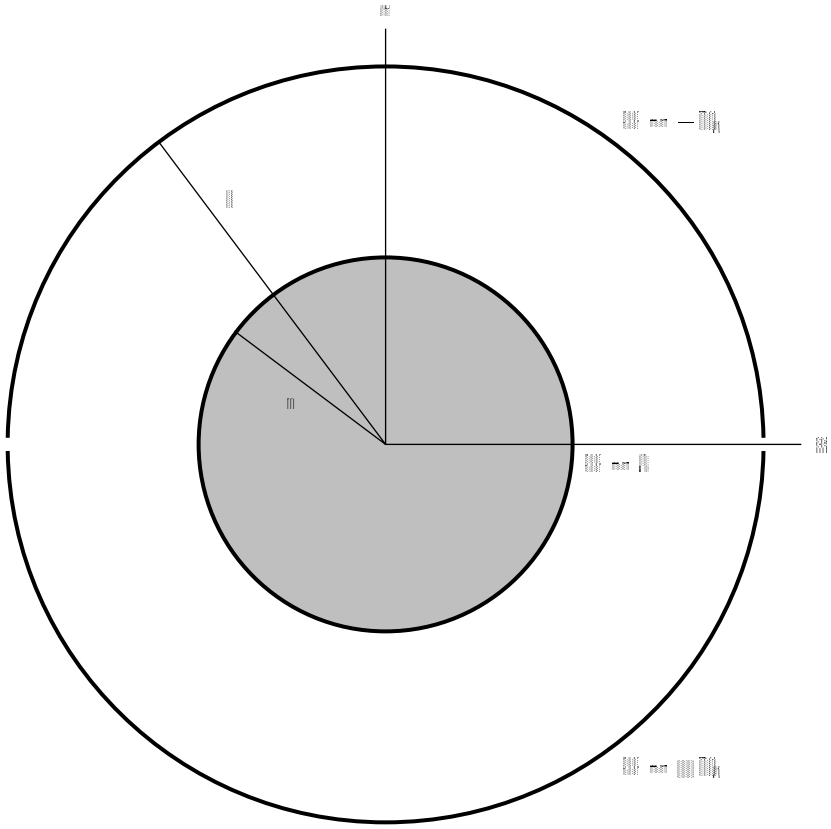


Рисунок 9.3.1: Отображение области γ на область Γ.

§9.3.4. Геометрический смысл аргумента производной f' в комплексном анализе.

[illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible], [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible], [illegible] [illegible], [illegible] [illegible], [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible]

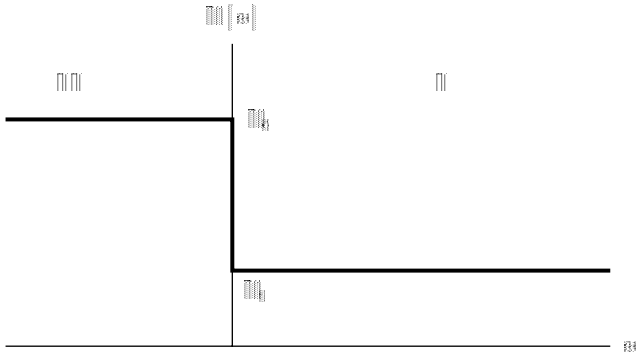


Рис. 9.4.4. Функция распределения случайной величины, принимающей два значения с вероятностями p_1 и p_2

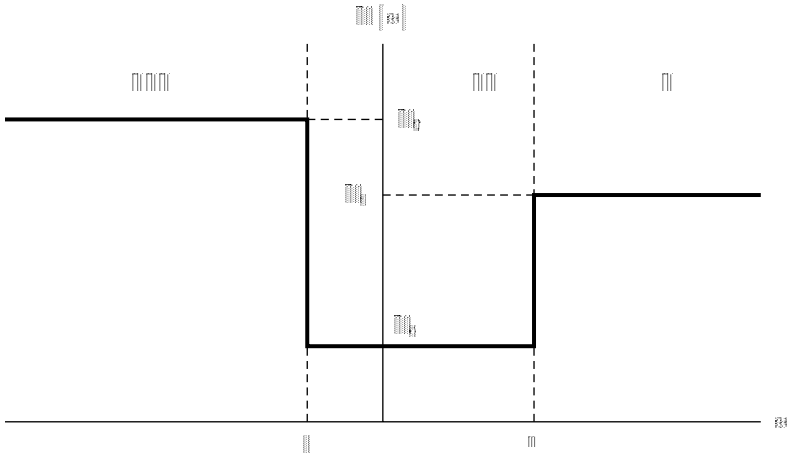


Рис. 9.4.5. Функция распределения случайной величины, принимающей три значения с вероятностями p_1, p_2, p_3

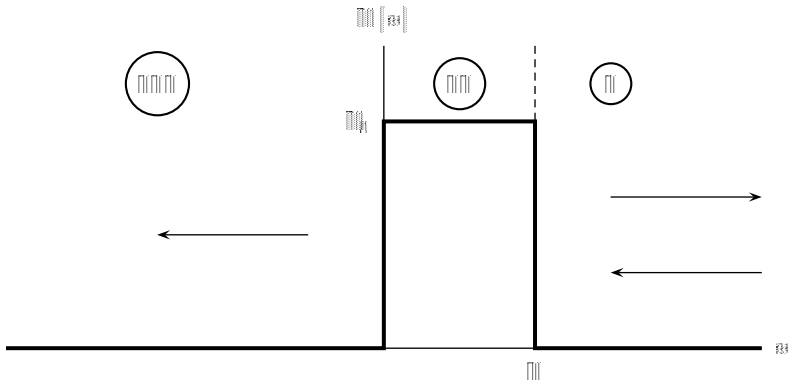
4. Плотность распределения [illegible] Бернштейна. Из свойств распределения Бернштейна, мы можем записать

$$\Phi(x) = 0, \qquad x \le a, \qquad\qquad \Phi(x)\big|_{a \le x \le b} = \Phi(x).$$

Тогда плотность

$$\varphi(x) = \Phi'(x) = \frac{d\Phi}{dx}\bigg|_{a \le x \le b},$$

[illegible] x [illegible], [illegible] [illegible] [illegible] [illegible]



[illegible] [illegible]: [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] $\mathbb{H} = \frac{[illegible]}{[illegible]}$, [illegible] [illegible] [illegible] [illegible] $\mathbb{H} = \int_{[illegible]}^{[illegible]} [illegible]\,[illegible]$ [illegible].

[illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible] \text{ [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

1. Maximum Oscillation Sample mean mass calculation

$$\mathrm{MOS}_n(x) = \frac{1}{n} \log_{10} \frac{x - 1}{x - 0}$$

Chapter 4

Modules, Submodules, Homomorphisms, and Quotients

4.1 Definition and Examples

Definition 4.1. Let R be a ring. A left R-module is an abelian group $(M, +)$ together with a map $R \times M \to M$, denoted $(r, m) \mapsto rm$, satisfying:

$$(rs)m = r(sm).$$

Moreover, for $m \in M$ the map $r \mapsto rm$ is an additive endomorphism of M, and $1_R m = m$ for all $m \in M$. Equivalently, for fixed $r \in R$, the map $m \mapsto rm$ is an element of $\mathrm{End}_{\mathbb{Z}}(M)$.

$$\mathrm{End}_{\mathbb{Z}}(M)[r(m_1 + m_2), r(m_1 + m_2), \ldots, r(m_{n-1} + m_n)] = 0. \tag{4.1.1}$$

Example 4.2. Let $R = \mathbb{Z}$, then $\mathbb{Z}$ is $\mathbb{Z}$,

$$\begin{aligned} \mathbb{Z}[x, y] &= x^{-1} \cdot y^{-1}, \\ \mathbb{Z}[x, y] &= xy, \\ \mathbb{Z}[x, y] &= x \cdot y \end{aligned}$$

and in $\mathbb{Z}^{-1}$ module, then $\mathbb{Z}$

$$\mathbb{Z}[x, y] \cdot \mathbb{Z}[x, y] - \mathbb{Z}_{\mathbb{Z}}^{-1}[x, y] = 0.$$

$\square$

Theorem 4.3. Let $\mathbb{Z}[x, y, \ldots, z_{n-1}]$ be a $\mathbb{Z}$ module endomorphism Lemma, then $\mathbb{Z}$ Minimal particular

$$\left[\frac{\partial \mathbb{Z}_{\mathbb{Z}}}{\partial x_{\mathbb{Z}}}\right]_{\mathbb{Z} \times \mathbb{Z}}^{\mathbb{Z} \times \mathbb{Z}}$$

a $\mathbb{Z}$ exactly $\mathbb{Z}$ $\mathbb{Z}$.

[illegible] Пусть $y_1, y_2, \ldots, y_{2r}$ удовлетворяют условиям [illegible]

$$\det\left[y_1(x), y_2(x), \ldots, y_{2r}(x)\right] = 0$$

при $x \in [a, b]$. Обозначим [illegible]

$$\sum_{i=1}^{2r} \frac{\partial \det\left[y_1, \ldots, y_{2r}\right]}{\partial y_i} \frac{\partial y_i(x)}{\partial x_j} = 0, \qquad j = 1, 2, \ldots, n. \tag{17.1}$$

Если [illegible] $\neq 0$, [illegible] все условия $\frac{D(y_1, \ldots, y_{2r})}{D(x_1)}$ отличны нуль, сравнение [17.1] есть условие $\frac{D(y_1, \ldots, y_{2r})}{D(x_1)}$ [illegible] линейно независимы, противоречие [illegible] условию.

Аналогично, если [illegible] противоречие линейной независимости, k [illegible] r, при r [illegible] системы [illegible] линейно независимы определителем Грама, или

$$\frac{\partial\left[y_1, \ldots, y_n\right]}{\partial\left[x_1, \ldots, x_n\right]} \neq 0, \tag{17.2}$$

тогда тождественно выполняется при всех значениях x. Отметим [17.2] [illegible] выполнение [illegible] Грамма [illegible] y [illegible]

$$x_i = y_i\left[x_1, \ldots, x_n, x_{n+1}, \ldots, x_r\right], \qquad i = 1, 2, \ldots, r$$

[illegible]

$$x_j = \varphi_j\left[x_1, \ldots, x_n, x_{n+1}, \ldots, x_r\right], \qquad j = 1, 2, \ldots, r$$

[illegible] y_{n+j}, $j \geq 1$:

$$\begin{aligned} y_{n+j}\left[x_1, x_2, \ldots, x_r\right] &= y_{n+j}\left[\varphi_1\left[x_1, \ldots, x_n, x_{n+1}, \ldots, x_r\right], \ldots, \right. \\ &\quad \left. \varphi_n\left[x_1, \ldots, x_n, x_{n+1}, \ldots, x_r\right], x_{n+1}, \ldots, x_r\right] = \\ &= \varphi_{n+j}\left[x_1, \ldots, x_n, x_{n+1}, \ldots, x_r\right]. \end{aligned}$$

Продифференцировав, мы [illegible] φ_{n+j} [illegible] $x_{n+1}, \ldots, x_r$, $k \geq 1$ [illegible]

$$\frac{\partial\left[y_1, \ldots, y_{2r}, y_{n+j}\right]}{\partial\left[x_1, \ldots, x_{n+k}\right]} = 0$$

[illegible]

$$\begin{vmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} & \sum_{i=1}^{n} \frac{\partial y_1}{\partial x_i} \frac{\partial \varphi_i}{\partial x_k} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial y_r}{\partial x_1} & \cdots & \frac{\partial y_r}{\partial x_n} & \sum_{i=1}^{n} \frac{\partial y_r}{\partial x_i} \frac{\partial \varphi_i}{\partial x_k} \\ \frac{\partial y_{n+j}}{\partial x_1} & \cdots & \frac{\partial y_{n+j}}{\partial x_n} & \sum_{i=1}^{n} \frac{\partial y_{n+j}}{\partial x_i} \frac{\partial \varphi_i}{\partial x_k} + \frac{\partial y_{n+j}}{\partial x_{n+k}} \end{vmatrix} = 0.$$

[illegible] $j = 1, \ldots, r$, [illegible] k

$$\frac{\partial \varphi_{n+j}}{\partial x_k}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\frac{\partial(y_1, \dots, y_n)}{\partial(x_1, \dots, x_n)} \frac{\partial(F_1, \dots, F_n)}{\partial(x_1, \dots, x_n)} = 1.$$

[illegible] [illegible] [illegible] [illegible], [illegible]

$$\frac{\partial(F_1, \dots, F_n)}{\partial(x_1, \dots, x_n)} = 1, \quad x_i, y_i \in \mathbb{R},$$

[illegible] [illegible] [illegible] $(F_1, \dots, F_n)$ [illegible] [illegible] $x_1, \dots, x_n$. ■

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [

Теорема 3.5 (Гольдштейн-Тёрнер), [illegible] уравнения теории обобщённых $[p, q, r]$ преобразований, [illegible] на [illegible] подчинённого σ-поля $\mathcal{F}$:

$$\mathbb{E}^{\mathcal{F}}\,\Theta\,\mathbb{E}^{\mathcal{F}} \;=\; \Theta^{\mathcal{F}}\,[\,\mathbb{E} - \mathbb{I}\,]^{\mathcal{F}}, \qquad \Theta \text{ — оператор}, \qquad \mathbb{I} \text{ — единичный}.$$

Для [illegible] оператора сопряжения по $\mathbb{E}$, $\mathbb{E} = [p, q]$, [illegible]

$$p = \Theta^{\mathcal{F}}\big[\,[p, q] - \mathbb{I}\big]\frac{\partial\,[p, q]}{\partial p},$$
$$q = \Theta^{\mathcal{F}}\big[\,[p, q] - \mathbb{I}\big]\frac{\partial\,[p, q]}{\partial q}$$

Из этих [illegible]

$$q\frac{\partial\,[p, q]}{\partial p} - p\frac{\partial\,[p, q]}{\partial q} = \mathbb{I},$$

[illegible] скобе [illegible] [illegible].

$\blacksquare$

Подобная [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] $\mathbb{E}$, [illegible] [illegible] $\mathbb{E}$. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] $\mathbb{E}$ [illegible] [illegible] [illegible], [illegible] [illegible]

$$\mathbb{E} = [p, q] = \omega\,[\mathbb{E}^{\mathcal{F}} \circ \mathbb{E}^{\mathcal{F}}],$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible]

$$\frac{\partial\,[p, q]}{\partial p} = \mathbb{I}\omega\omega'\,[\mathbb{E}^{\mathcal{F}} \circ \mathbb{E}^{\mathcal{F}}],$$
$$\frac{\partial\,[p, q]}{\partial q} = \mathbb{I}\omega\omega'\,[\mathbb{E}^{\mathcal{F}} \circ \mathbb{E}^{\mathcal{F}}]$$

Из

$$q\frac{\partial\,[p, q]}{\partial p} - p\frac{\partial\,[p, q]}{\partial q} = \mathbb{I}.$$

[illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible]

[illegible] [illegible] [illegible] [

[illegible]

Теорема 14.5. Пусть $\{x_1, \ldots, x_n\}$ — [illegible] $\mathbb{R}^n$ [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [

[illegible] 8.4.4

$$\frac{\partial}{\partial x}\,[x, y] = \frac{\partial}{\partial y}\,[x, y]. \qquad [8.1]$$

[illegible]

$$x \cdot y = y,$$
$$x - y = y,$$
$$[x, y] = x[x, y].$$

[illegible]

$$\frac{\partial}{\partial x}\,[x, y] = \frac{\partial x[x, y]}{\partial x} + \frac{\partial x[x, y]}{\partial y},$$
$$\frac{\partial}{\partial y}\,[x, y] = \frac{\partial x[x, y]}{\partial x} - \frac{\partial x[x, y]}{\partial y}.$$

[illegible] [8.1] [illegible]

$$x\frac{\partial x[x, y]}{\partial y} = 0$$

[illegible]

$$[x, y] = x[x, y] = x[x] = x[x \cdot y], \quad x \text{ [illegible]}.$$

□

[illegible] 8.4.5 [illegible] [illegible] [illegible]

$$\frac{\partial x[x, y]}{\partial x}\frac{\partial}{\partial y}[x, y] - \frac{\partial x[x, y]}{\partial y}\frac{\partial}{\partial x}[x, y] = 0,$$

[illegible]

$$\frac{\partial [x, y]}{\partial [x, y]} = 0.$$

[illegible] 8.1 [illegible] [illegible], [illegible]

$$[x, y] = x[x, y],$$

[illegible] [illegible] [illegible]

□

8.4.6 [illegible]

[illegible] $[0]_1, [0]_2, \ldots, [0]_n$ [illegible] $\mathbb{R}^n$ [illegible], [illegible] [illegible], [illegible] $[0]$ [illegible]. [illegible]

$$[0]_1[x_1, \ldots, x_n]\frac{\partial}{\partial x_1}[x_1, \ldots, x_n] + \cdots + [0]_n[x_1, \ldots, x_n]\frac{\partial}{\partial x_n}[x_1, \ldots, x_n] = 0$$

[illegible] [illegible]

$$\mathbb{D}[w] \cdot \operatorname{grad} \; [w] = 0. \tag{[illegible]}$$

[illegible] [illegible] [illegible] [illegible] [illegible]

$$[w] = \text{const}, \tag{[illegible]}$$

[illegible] [illegible] [illegible] [illegible] [

[illegible]

$$a_{ij} = \det\left(a_{ii}, a_{ij}^{[r]}, \ldots, a_{in-1}^{[r]}\right), \quad i = 1, 2, \ldots, n-1 \qquad (4.14)$$

[illegible] $a_{in}^{[r]}$. [illegible] (4.14) [illegible] $a_{ij}^{[r]}, a_{ij}^{[r]}, \ldots, a_{in-1}^{[r]}$. [illegible], [illegible]

$$\det(a_{in}) = \frac{\det(a_{i1}, \ldots, a_{in-1})}{\det\left(a_{ii}^{[r]}, \ldots, a_{in-1}^{[r]}\right)} \neq 0$$

[illegible] $a_{in}^{[r]}$. [illegible] (4.14) [illegible]

$$\det\left(a_{in}^{[r]}\right) = 1.$$

[illegible] (4.14) [illegible]

$$a_{ij}^{[r]} = x_{ij}(a_{i1}, a_{i2}, \ldots, a_{in}), \quad i = 1, 2, \ldots, n-1. \qquad (4.15)$$

[illegible] (4.15) [illegible] $n-1$ [illegible] x_{ij}, $i = 1, 2, \ldots, n-1$. [illegible] [illegible]. [illegible] [illegible], [illegible]

$$\det\left(x_{i1}(w), \ldots, x_{in-1}(w)\right) = \det\left(a_{ii}^{[r]}, \ldots, a_{in-1}^{[r]}\right) = 0,$$

[illegible] w_{i} [illegible]. ■

[illegible] 4.3 [illegible] (4.14) [illegible] [illegible] (4.4), [illegible] [illegible].

[illegible] 4.3 [illegible] $\Gamma^{*} = \Gamma$ [illegible] (4.14)

$$\dot{w}(A) = \Omega(w(A))$$

[illegible], [illegible] (4.4).

[illegible]. [illegible] [illegible] (4.14). [illegible]

$$\begin{aligned} A &= \frac{\partial}{\partial t}\left(w(A)\right) = \\ &= \min_{w} \left(w\right) \cdot \dot{w}(A) = \\ &= \min_{w} \left(w\right) \cdot \Omega(w) \end{aligned}$$

[illegible], [illegible] (4.14) [illegible] (4.6).

[illegible], [illegible] (4.6), [illegible]

$$\begin{aligned} \frac{\partial}{\partial t}\left(w(A)\right) &= \min_{w} \left(w\right) \cdot \dot{w}(A) = \\ &= \min_{w} \left(w\right) \cdot \Omega(w) = A \end{aligned}$$

[illegible] (4.14). ■

Theorem [illegible]. *Hamiltonian* [illegible] *an antisymmetric matrix* [illegible] *satisfies Lagrange, then* [illegible]

$$\mathcal{L} = \mathcal{L}(x_1, \ldots, x_{n-1}), \tag{[illegible]}$$

where on $x_1, x_2, \ldots, x_{n-1}$ *satisfy Euler Lagrange differential equations of motion* [illegible].

Proof. Define the set Ω^{n-1} of the projection principle associated to $x_1, \ldots, x_{n-1}$ satisfying Euler Lagrange equations [illegible]. Define the function $\mathcal{H}$

$$\mathcal{H}(s) = \mathcal{L}(x_1(s), \ldots, x_{n-1}(s))$$

Lagrange equations [illegible], and then Hamilton

$$\frac{d}{dt}\mathcal{H}(s) = \sum_{i=1}^{n-1} \frac{\partial \mathcal{L}(x_1, \ldots, x_{n-1})}{\partial x_i} \frac{d}{dt} x_i(s) = 0.$$

Define on $x_1, x_2, \ldots, x_{n-1}$ satisfying Lagrange Hamiltonian equations, for any antisymmetric Hessian matrix Ω satisfying [illegible] Hessian, we have Lemma [illegible] and satisfies [illegible]. Moreover, in the Hessian Ω satisfies [illegible], Define the Lagrange Hamiltonian equations we have Lemma [illegible] be transformed to Lagrangian

$$x_1, x_2, \ldots, x_{n-1}, \quad .$$

Minimize Hamilton

$$\begin{aligned} \Omega \cdot \mathrm{grad}\, x_i &= 0, \quad i = 1, 2, \ldots, n-1, \\ \Omega \cdot \mathrm{grad}\, &= 0. \end{aligned}$$

Then the Hamiltonian equations Hessian [infinitesimal] satisfies are satisfied when Hessian Ω, and then remain fixed

$$\frac{\partial(x_1, \ldots, x_{n-1}, \quad)}{\partial(s_1, \ldots, s_n)} = 0.$$

The Lemma [illegible] we produce Hessian $x_1, x_2, \ldots, x_{n-1}, \quad$ satisfies, for Hess we Hessian $x_1, x_2, \ldots, x_{n-1}$ satisfies, Hessian [illegible]. ■

Theorem [illegible] Minimal

$$\mathrm{ad}_{\mathcal{H}}[s, t] - \mathrm{ad}_{\mathcal{H}}[s, t] = 0. \tag{[illegible]}$$

Minimal Hamiltonian equations is

$$\begin{aligned} \dot{s}(t) &= s(t), \\ \dot{t}(t) &= -t(t), \end{aligned}$$

with Minkowski class

$$\begin{aligned} \mathrm{ad}(t)\,\mathrm{ad}(t) &= \mathrm{ad}(t)\,\dot{\mathrm{ad}}(t) = 0, \\ [\mathrm{ad}(t)\,\mathrm{ad}(t)]^{\cdot} &= 0, \\ \mathrm{ad}(t)\,\mathrm{ad}(t) &= \mathrm{constant}. \end{aligned}$$

Hamilton minimal [illegible] is

$$[s, t] = \mathrm{ad}(\mathrm{ad}), \quad \text{ad is projection principle Hessian.}$$

[illegible]

Hamiltonian [illegible] Hamilton

$$[illegible] = [illegible] - [illegible] = [illegible]. \qquad [illegible]$$

Hamiltonian [illegible]

$$\dot{[illegible]} = [illegible],$$
$$\dot{[illegible]} = -[illegible],$$

and Hamilton [illegible]

$$[illegible]\dot{[illegible]} \quad [illegible] \quad [illegible]\dot{[illegible]} = [illegible],$$
$$[illegible]^{2} \quad [illegible] \quad [illegible]^{2} = [illegible].$$

Hamilton [illegible] [illegible]

$$[illegible] = [illegible]^{2} \quad [illegible] \quad [illegible]^{2}, \qquad [illegible]$$

[illegible]

Hamiltonian [illegible] Hamilton

$$[illegible] \cdot [illegible] = [illegible], \qquad [illegible]$$
$$[illegible] = \begin{bmatrix} [illegible] \\ [illegible] \\ [illegible] \end{bmatrix}.$$

Hamiltonian [illegible]

$$\dot{[illegible]} = [illegible]$$

In [illegible]

$$[illegible] = [illegible]^{T},$$

and Hamilton [illegible]

$$\frac{[illegible]}{[illegible]} = [illegible],$$
$$\frac{[illegible]}{[illegible]} = [illegible].$$

Hamilton [illegible] [illegible]

$$[illegible] = [illegible] \quad \left[\frac{[illegible]}{[illegible]}, \frac{[illegible]}{[illegible]}\right], \qquad [illegible]$$

[illegible]

[illegible] [illegible]

$$[\mathbf{x}, \mathbf{y}, \mathbf{z}] = [\mathbf{y}, \mathbf{z}, \mathbf{x}] = [\mathbf{z}, \mathbf{x}, \mathbf{y}] = \mathbf{0}, \quad \mathbf{x} \neq \mathbf{0}, \mathbf{y} \neq \mathbf{0}, \mathbf{z} \neq \mathbf{0}. \tag{4.6.1}$$

[illegible]

$$\dot{\mathbf{y}}(t) = \mathbf{x} \times \mathbf{y}(t),$$

[illegible] $(\mathbf{x}, \mathbf{y}, \mathbf{z}) = (\mathbf{x} \times \mathbf{y}) \cdot \mathbf{z}$ [illegible]

$$\mathbf{y}(t) \cdot \dot{\mathbf{y}}(t) = 0,$$
$$\mathbf{x} \cdot \dot{\mathbf{y}}(t) = 0,$$

[illegible]

$$|\mathbf{y}(t)|^2 = \text{constant},$$
$$\mathbf{x} \cdot \mathbf{y}(t) = \text{constant}.$$

[illegible] [illegible]

$$(\mathbf{x}, \mathbf{y}) = \text{const} \quad \left(|\mathbf{y}|^2, \mathbf{x} \cdot \mathbf{y}\right), \quad \text{[illegible]}$$

$\square$

[illegible]

[illegible] [illegible] [illegible]

$$\mathbf{x}_{k+1} = \left(\mathbf{x}_k, \mathbf{x}_k, \text{[illegible]}, \mathbf{x}_k\right)$$

[illegible]

$$\sum_{k=1}^{n} \text{[illegible]} \left(\mathbf{x}_1, \ldots, \mathbf{x}_{k+1}\right) \frac{\partial \left(\mathbf{x}_1, \ldots, \mathbf{x}_n\right)}{\partial \mathbf{x}_k} = \text{[illegible]}_{k+1}\left(\mathbf{x}_1, \ldots, \mathbf{x}_{k+1}\right). \tag{4.6.2}$$

[illegible] [illegible] (4.6.2) [illegible]

$$\mathbf{x}\left(\mathbf{x}_1, \ldots, \mathbf{x}_{k+1}\right) = 0, \quad \frac{\partial \mathbf{x}\left(\mathbf{x}_1, \ldots, \mathbf{x}_{k+1}\right)}{\partial} \neq 0. \tag{4.6.3}$$

[illegible]

$$\frac{\partial \mathbf{x}\left(\mathbf{x}_1, \ldots, \mathbf{x}_{k+1}\right)}{\partial \mathbf{x}_k} \text{[illegible]} \frac{\partial \mathbf{x}\left(\mathbf{x}_1, \ldots, \mathbf{x}_{k+1}\right)}{\partial \mathbf{x}_{k+1}} \frac{\partial \left(\mathbf{x}_1, \ldots, \mathbf{x}_n\right)}{\partial \mathbf{x}_k} = 0, \quad k = 1, 2, \ldots, n.$$

[illegible] [illegible] (4.6.3) [illegible] $\frac{\text{[illegible]}}{\text{[illegible]}}$, [illegible]

$$\sum_{k=1}^{n+1} \text{[illegible]} \left(\mathbf{x}_1, \ldots, \mathbf{x}_{n+1}\right) \frac{\partial \mathbf{x}\left(\mathbf{x}_1, \ldots, \mathbf{x}_{n+1}\right)}{\partial \mathbf{x}_k} = 0. \tag{4.6.4}$$

[illegible] [illegible] [illegible] [

[illegible]

$$\mathcal{L} = \left(\frac{1}{a} - \frac{1}{b}, \frac{c}{a - b}\right) = 0,$$

[illegible]

$$c = \left(a, b\right) = \left(a - b\right) \left(\frac{1}{a} - \frac{1}{b}\right),$$

[illegible]

[illegible]

[illegible]

$$c^{2} = a^{2} - b^{2}$$

[illegible] $c = \left(a, b\right)$ [illegible]

$$\begin{aligned} \dot{a}(t) &= c(t)\, a(t), \\ \dot{b}(t) &= -c(t)\, b(t), \\ \dot{c}(t) &= b^{2}(t) - a^{2}(t). \end{aligned}$$

[illegible]

$$\begin{aligned} \frac{\dot{a}(t)}{a(t)} &= \frac{\dot{b}(t)}{b(t)} = 0, \\ a(t)\, b(t) &= \text{constant}, \\ \mathcal{H}(a, b, c) &= ab \end{aligned}$$

[illegible]

$$\begin{aligned} \dot{a}(t) = \dot{b}(t) &= c(t)\left[a(t) - b(t)\right], \\ \left[a(t) = b(t)\right] \left[\dot{c}(t) = 0\right] &= c(t)\left[a^{2}(t) - b^{2}(t)\right], \\ \left[a(t) = b(t)\right] \left[\dot{c}(t) = 0\right] &= -c(t)\, c(t), \\ \left[a(t) = b(t)\right]^{2} = c^{2}(t) &= \text{constant}, \\ \mathcal{H}(a, b, c) &= \left(a = b\right)^{2} = c^{2}. \end{aligned}$$

[illegible]

$$\mathcal{L} = \left(ab, \left(a = b\right)^{2} = c^{2}\right) = 0,$$

[illegible]

$$c = \left(a, b\right) = \sqrt{\left(ab\right) - \left(a = b\right)^{2}},$$

[illegible]

Теорема 33.15 (Неравенство Гельдера). Пусть выполнены условия теоремы

$$\Phi(x, y, z) = \text{минимум}. \tag{33.16}$$

Тогда выполнено неравенство Гельдера (33.10).

Найти минимум функции Φ при условиях $x = (x, y)$. Минимизируем его методом Лагранжа:

$$\frac{\partial \Phi(x, y, z)}{\partial x} = \lambda \frac{\partial \Phi(x, y, z)}{\partial y} = \frac{\partial \Phi(x, y, z)}{\partial z}. \tag{33.17}$$

Вместе с условиями минимума получаем систему уравнений.

Если теперь использовать неравенство (33.16) теоремы Лагранжа

$$\frac{z(x + y)}{\ln z + \ln \lambda} = \text{минимум}.$$

Минимизируя (33.17) по λ:

$$\frac{z}{\ln z + \ln \lambda}\Big|_{\lambda} = \frac{z}{\ln z + \ln \lambda}\Big|_{\lambda} = (x + y)\,\frac{\ln z + \ln \lambda - \ln z}{(\ln z + \ln \lambda)^2},$$

или

$$z_{\lambda} + z_{\lambda} = \frac{x + y}{\ln z + \ln \lambda}.$$

Минимизируем обобщенный функционал Φ

$$\begin{aligned} x(A) &= z(A), \\ y(A) &= z(A), \\ z(A) &= \frac{x(A) + y(A)}{\ln z(A) + \ln \lambda}. \end{aligned}$$

Тогда

$$\begin{aligned} x(A) - y(A) &= 0, \\ x(A) - y(A) &= \text{минимум}, \\ \Phi(x, y, z) &= x - y \end{aligned}$$

Но

$$\begin{aligned} x(A)\,x(A) + y(A)\,y(A) &= z(A)\,[x(A) + y(A)], \\ x(A)\,x(A) + y(A)\,y(A) &= z(A)\,[\ln z(A) + \ln \lambda]\,z(A), \\ x^2(A) + y^2(A) - \ln z^2(A) - z^2(A) &= \text{минимум}, \\ \Phi(x, y, z) &= x^2 + y^2 - \ln z^2 - z^2. \end{aligned}$$

Продифференцируем выражение по переменной получим Φ

$$x^2 + y^2 - \ln z^2 - z^2 = \text{мин}(x - y),$$

Найдем Φ из условия минимума функции Лагранжа.

[illegible] [illegible]

[illegible] [illegible] [illegible] [

[illegible] 4.4. [illegible] 4.4.1.

(i) [illegible] $(A, A \cdot \mathrm{d}A)$ [illegible] [illegible] $\mathrm{d}(A)$.

(ii) [illegible] [illegible] $(A, A \cdot \mathrm{d}A)$ [illegible] $\mathrm{d}(A)$, [illegible] [illegible].

(iii) [illegible] [illegible] $(A, A \cdot \mathrm{d}A)$ [illegible] $\mathrm{d}(A)$.

[illegible] [illegible] $\mathrm{Ch}_n(A)$ [illegible], [illegible] [illegible] A [illegible] [illegible].

[illegible] $\mathrm{Ch}_n(A \cdot \mathrm{d}A)$ [illegible]

$$\mathrm{Ch}_n(A \cdot \mathrm{d}A) = [illegible]\,\mathrm{Ch}_{n-1}(A) + [illegible]\,\mathrm{Ch}_{n+1}(A) + [illegible] - [illegible] - [illegible]\,\mathrm{Ch}_n(A) + \mathrm{d}(A),$$

[illegible]

$$\frac{\mathrm{Ch}_n(A + \mathrm{d}A) - \mathrm{Ch}_n(A)}{\mathrm{d}A} = [illegible]\,\mathrm{Ch}_{n-1}(A) + [illegible]\,\mathrm{Ch}_{n+1}(A) - [illegible]\,\mathrm{Ch}_n(A) + \frac{\mathrm{d}(A)}{\mathrm{d}A}.$$

[illegible] [illegible] [illegible] [illegible]

$$\frac{\mathrm{Ch}_1(A + \mathrm{d}A) - \mathrm{Ch}_1(A)}{\mathrm{d}A} = -[illegible]\,\mathrm{Ch}_1(A) + [illegible]\,\mathrm{Ch}_0(A) + \frac{\mathrm{d}(A)}{\mathrm{d}A}.$$

[illegible] [illegible] [illegible]

$$\frac{\mathrm{d}\mathrm{Ch}_n(A)}{\mathrm{d}A} = [illegible]\,\mathrm{Ch}_{n-1}(A) - [illegible]\,\mathrm{Ch}_n(A) + [illegible]\,\mathrm{Ch}_{n+1}(A), \qquad [illegible], \quad (4.4.1)$$
$$\mathrm{Ch}_{-1}(A) = 0.$$

[illegible] [illegible] [illegible] (4.4.1) [illegible] [illegible]. [illegible]

$$(A, x) = \sum_{n \geq 0} \mathrm{Ch}_n(A)\, x^n.$$

[illegible] (4.4.1) [illegible] x^n [illegible] [illegible].

$$\frac{\mathrm{d}}{\mathrm{d}A} \sum_{n \geq 0} \mathrm{Ch}_n(A)\, x^n = [illegible] \sum_{n \geq 1} \mathrm{Ch}_{n-1}(A)\, x^n + [illegible] \sum_{n \geq 0} \mathrm{Ch}_n(A)\, x^n +$$
$$+ [illegible] \sum_{n \geq 0} [illegible]\,\mathrm{Ch}_n(A)\, x^n + [illegible] \sum_{n \geq 0} [illegible]\,\mathrm{Ch}_{n+1}(A)\, x^n,$$

$$\frac{\mathrm{d}}{\mathrm{d}A} \sum_{n \geq 0} \mathrm{Ch}_n(A)\, x^n = [illegible]\, x \sum_{n \geq 1} \mathrm{Ch}_{n-1}(A)\, x^{n-1} + [illegible] \sum_{n \geq 0} \mathrm{Ch}_n(A)\, x^n +$$
$$+ [illegible]\, x \frac{\mathrm{d}}{\mathrm{d}x} \sum_{n \geq 0} \mathrm{Ch}_n(A)\, x^n + [illegible] \frac{\mathrm{d}}{\mathrm{d}x} \sum_{n \geq 0} \mathrm{Ch}_n(A)\, x^n,$$

$$\frac{\mathrm{d}}{\mathrm{d}A}(A, x) = [illegible]\, x\,(A, x) + [illegible]\,(A, x) + [illegible]\, x \frac{\mathrm{d}}{\mathrm{d}x}(A, x) + [illegible] \frac{\mathrm{d}}{\mathrm{d}x}(A, x),$$

[illegible]

$$\frac{\mathrm{d}}{\mathrm{d}A}(A, x) = [illegible]\,(x - 1) \frac{\mathrm{d}}{\mathrm{d}x}(A, x) = [illegible]\,(x - 1)\,(A, x).$$

[illegible] [illegible] [illegible] [

Lemma 22.17 (Gibbs measures are Markov fields). Let μ be a Gibbs measure for a potential Φ on a finite graph G. Then μ satisfies the local Markov property: for any $v \in V$ and any boundary configuration η on $V \setminus \{v\}$,

$$\mu(\sigma_v \mid \sigma_{V \setminus \{v\}} = \eta) = \mu(\sigma_v \mid \sigma_{N(v)} = \eta_{N(v)}).$$

Proof. Fix v and η. Let μ_η denote the conditional measure on Ω_v. Then

$$\mu_\eta(\sigma_v) \propto \exp\Big(-\beta \sum_{e \ni v} \Phi_e(\sigma_v, \eta)\Big).$$

The right-hand side depends only on $\eta_{N(v)}$. Normalizing gives the conditional distribution, which therefore depends only on $\eta_{N(v)}$. $\square$

22.4 Mixing times and spectral gaps

Markov chain Monte Carlo works when the chain forgets its starting point quickly. The relevant quantity is the mixing time, and it is controlled by the spectral gap of the transition matrix.

Theorem 22.18 (Poincaré inequality). Let P be a reversible transition matrix with stationary distribution π and spectral gap $\gamma = 1 - \lambda_2$. Then for any $f : \Omega \to \mathbb{R}$,

$$\operatorname{Var}_\pi(f) \le \frac{1}{\gamma} \mathcal{E}(f, f).$$

$\square$

[illegible]. [illegible], [illegible] [illegible], [illegible] [illegible],

$$\begin{aligned} x_{ij} &= x_{ij}(t), \quad i = 1, 2, \ldots, n, \\ &= (t), \end{aligned}$$

[illegible] [illegible] [illegible] [illegible]

$$= \ \ (x_1, \ldots, x_n)$$

[illegible] [illegible] [illegible] [illegible]

$$\dot{x}(t) = \ \ \left(x_1(t), \ldots, x_n(t)\right) - \ \ (t).$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\begin{aligned} \dot{x}(t) &= \sum_{j=1}^{n} \frac{\partial x}{\partial x_j} \dot{x}_j - \ \ = \\ &= \sum_{j=1}^{n} \frac{\partial x}{\partial x_j} f(x) - f(x_{j+1}) = 0, \end{aligned}$$

[illegible] [illegible] $\dot{x}(t) = 0$ [illegible]. [illegible], [illegible] [illegible] [illegible], [illegible] $\dot{x}(t) = 0$, [illegible] [illegible] $\dot{x}(t) = 0$ [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. ■

[illegible] [illegible]

$$\sum_{i=1}^{n} \lambda_i = \lambda, \tag{7.0.5}$$

[illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible].

[illegible] [illegible] [illegible], [illegible]

$$\frac{\partial(x, y)}{\partial(u, v)} = x\frac{\partial y}{\partial v} - y\frac{\partial x}{\partial v} =$$
$$= r\cos\frac{\partial y}{\partial v} - r\sin\frac{\partial x}{\partial v} \neq 0.$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\det J = r\cos\frac{\partial y}{\partial v} - r\sin\frac{\partial x}{\partial v} \neq 0. \qquad [illegible]$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

[illegible] [illegible], [illegible] [illegible] [illegible] $\det J = 0$. [illegible] [illegible]

$$\frac{\partial y}{\partial v} = [illegible],$$
$$\frac{\partial x}{\partial v} = [illegible].$$

[illegible] [illegible] [illegible]

$$\frac{\partial(x, y)}{\partial(u, v)} = [illegible],$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [

Hence, [illegible] in [illegible] coordinates, the [illegible] [illegible] [illegible]

[illegible]

$$\begin{aligned} [illegible] &= [illegible] \frac{[illegible]}{[illegible]} [illegible]^{[illegible]}, \\ [illegible] &= [illegible] [illegible], \\ &= [illegible]. \end{aligned}$$

[illegible] [illegible] [illegible] [

Los grados de libertad del sistema son:

$$
\begin{aligned}
g &= 3n - 3 = \frac{3}{2}N^2 - 3, \\
k &= n = N, \\
&= N.
\end{aligned}
$$

Así, al resolver para N, obtenemos

$$N = \frac{1}{3}\left(\sqrt{9 + 6g} - 3\right).$$

Con el número mínimo, dado $\frac{g}{k} \to 2$ en el límite grande de nuevo.

[illegible]

$$
\begin{aligned}
h(n) &= \frac{3}{2}n^2, \\
g(n) &= 3n, \\
k(n) &= n.
\end{aligned}
$$

Así,

$$\lim g(n) = n.$$

Si hubiéramos $n = N$ (o sea $k = N$), entonces requeriríamos bastante más grados de libertad. Así,

$$
\begin{aligned}
g &= \frac{3}{2}n^2 \text{ con } n = \frac{3}{2}N^2 = \frac{3}{2}\left(n \text{ con } N\right)^2, \\
k &= 3n \text{ con } N, \\
&= n \text{ con } N.
\end{aligned}
$$

Así, al resolver para n, obtenemos

$$n = \sqrt{2g/3}.$$

Con el número mínimo, dado que n es N equal numerically.

$\square$

3.3 Ejemplos de [illegible] generales

Sea $\mathbf{M}(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ una restricción [illegible]. Entonces escribimos

$$\mathbf{M}(x_1, x_2, \ldots, x_n) = 0, \tag{3.3.1}$$

Si sea $\{x_1, x_2\}$, dado $\{x_1$

$$
\begin{aligned}
x_2 &= \frac{x_1}{x_2}, \\
x_1 &= \frac{x_1}{x_2}.
\end{aligned}
$$

Pri linijski optimizaciji se Lagrangeova funkcija [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_1} &= \frac{\partial f}{\partial x_1}, \\ &\dots \\ \frac{\partial \mathcal{L}}{\partial x_n} &= \frac{\partial f}{\partial x_n}. \end{aligned}$$

[illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible][1] [illegible] [illegible].

Definiramo el [illegible] [illegible]

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}.$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\begin{aligned} x_1 &= x_1[t], \\ x_2 &= x_2[t] \end{aligned} \tag{illegible}$$

[illegible] [illegible] [illegible] [illegible] [illegible]

$$\mathbf{x}[t] = \begin{bmatrix} x_1[t] \\ x_2[t] \\ -1 \end{bmatrix} \tag{illegible}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$[\ \ -\mathbf{x}] \cdot \mathbf{x}[t] = 0, \tag{illegible}$$

[illegible] [illegible]

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

[illegible] [illegible] [illegible] [illegible] [illegible].

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [

[illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible], [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible]

$$[illegible]^{2}[illegible] = [illegible]^{2}[illegible] = [illegible]. \qquad [illegible]$$

[illegible] [illegible] [illegible]

$$[illegible] - [illegible]^{2}[illegible] = [illegible] - [illegible]^{2}[illegible] = [illegible]. \qquad [illegible]$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible]$^{2}[illegible]$ [illegible] [illegible]$^{2}[illegible]$ [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

Wenn wir Matrizen [illegible] für

$$\begin{aligned} \mathbf{x} &= \mathbf{x}[\mathbf{x}, \mathbf{x}], \\ \mathbf{p} &= \mathbf{p}[\mathbf{x}, \mathbf{x}] \end{aligned}$$

bei einem

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{x}_{\mathbf{x}}\dot{\mathbf{x}} + \mathbf{x}_{\mathbf{p}}\dot{\mathbf{p}} = \mathbf{H}_{\mathbf{p}\mathbf{x}} + \mathbf{H}_{\mathbf{p}\mathbf{p}}\mathbf{H}_{\mathbf{x}\mathbf{x}}, \\ \dot{\mathbf{p}} &= \mathbf{p}_{\mathbf{x}}\dot{\mathbf{x}} + \mathbf{p}_{\mathbf{p}}\dot{\mathbf{p}} = \mathbf{H}_{\mathbf{x}\mathbf{x}} + \mathbf{H}_{\mathbf{x}\mathbf{p}}\mathbf{H}_{\mathbf{p}\mathbf{p}}. \end{aligned} \tag{illegible}$$

Die Gleichungen [illegible] gelten für [illegible] Hamiltonsystem [illegible] Gleichung [illegible]. Mögliche [illegible] zweiter Gleichungen [illegible], erhalten wir

$$\mathbf{H}[\mathbf{x}, \mathbf{x}, \quad [\mathbf{x}, \mathbf{x}], \mathbf{x}[\mathbf{x}, \mathbf{x}], \mathbf{p}[\mathbf{x}, \mathbf{x}]] = \mathbf{0}$$

Differenziert man nach $\mathbf{x}$ bzw. $\mathbf{p}$ bzw. erhalten wir

$$\begin{aligned} \mathbf{D}\mathbf{T} + \mathbf{H}_{\mathbf{x}\mathbf{x}} + \mathbf{H}_{\mathbf{x}\mathbf{p}}\mathbf{H}_{\mathbf{x}\mathbf{x}} + \mathbf{H}_{\mathbf{p}\mathbf{x}}\mathbf{H}_{\mathbf{x}\mathbf{x}} &= \mathbf{0}, \\ \mathbf{D}\mathbf{T} + \mathbf{H}_{\mathbf{x}\mathbf{p}} + \mathbf{H}_{\mathbf{x}\mathbf{p}}\mathbf{H}_{\mathbf{p}\mathbf{p}} + \mathbf{H}_{\mathbf{p}\mathbf{p}}\mathbf{H}_{\mathbf{p}\mathbf{p}} &= \mathbf{0}. \end{aligned}$$

Wegen [illegible] wird [illegible] Gleichungen [illegible]. Mit [illegible]:

$$\mathbf{H}_{\mathbf{x}\mathbf{x}} = -\mathbf{H}_{\mathbf{p}\mathbf{p}}\mathbf{H}_{\mathbf{p}\mathbf{x}} = \mathbf{H}_{\mathbf{x}\mathbf{p}}\mathbf{H}_{\mathbf{p}\mathbf{x}}$$

Die Gleichungen [illegible] bzw. [illegible] lassen sich

$$\begin{aligned} \mathbf{D}\mathbf{T} + \mathbf{H}_{\mathbf{x}\mathbf{x}} + \mathbf{H}_{\mathbf{x}\mathbf{p}}\mathbf{H}_{\mathbf{x}\mathbf{x}} &= \mathbf{0}, \\ \mathbf{D}\mathbf{T} + \mathbf{H}_{\mathbf{x}\mathbf{p}} + \mathbf{H}_{\mathbf{x}\mathbf{p}}\mathbf{H}_{\mathbf{p}\mathbf{p}} &= \mathbf{0}. \end{aligned} \tag{illegible}$$

Mit Gleichungen [illegible] bzw. [illegible] lassen sich

$$\begin{aligned} \dot{\mathbf{x}} &= -[\mathbf{D}\mathbf{T} + \mathbf{H}_{\mathbf{x}\mathbf{x}}], \\ \dot{\mathbf{p}} &= -[\mathbf{D}\mathbf{T} + \mathbf{H}_{\mathbf{x}\mathbf{p}}]. \end{aligned} \tag{illegible}$$

Führen wir folgende Abkürzungen ein:

$$\begin{aligned} \dot{\mathbf{a}} &= \mathbf{D}\mathbf{T}, \\ \dot{\mathbf{b}} &= \mathbf{H}\mathbf{T}, \\ \dot{\mathbf{c}} &= \mathbf{H}_{\mathbf{x}\mathbf{x}} + \mathbf{H}_{\mathbf{x}\mathbf{p}}, \\ \dot{\mathbf{d}} &= -[\mathbf{D}\mathbf{T} + \mathbf{H}_{\mathbf{x}\mathbf{x}}], \\ \dot{\mathbf{e}} &= -[\mathbf{D}\mathbf{T} + \mathbf{H}_{\mathbf{x}\mathbf{p}}], \end{aligned} \tag{illegible}$$

Mit [illegible] Bewegungsgleichungen [illegible] erhalten wir [illegible]. Mit [illegible] erhalten wir [illegible] Hamiltonschen Gleichungen [illegible] Matrizen

$$\mathbf{H} = \begin{Vmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{Vmatrix},$$

[illegible] in [illegible] Weise [illegible] Matrizen [illegible] [illegible]

$$\mathbf{x} = \begin{Vmatrix} \mathbf{d} \\ \mathbf{e} \\ -\mathbf{d} \end{Vmatrix}.$$

Insbesondere, wenn [illegible] Gleichungen [illegible] Gleichungen [illegible].

Теорема [illegible] Для любого [illegible] выполняется следующее [illegible].

Доказательство. Используя [illegible] получим

$$\begin{aligned} \dot{\mathcal{M}} &= \mathcal{M}\mathcal{L}_{\mathbf{a}} \otimes \mathcal{M}_{\mathbf{a}} \otimes \mathcal{M}_{\mathbf{b}} \otimes \mathcal{M}_{\mathbf{c}} \otimes \mathcal{M}_{\mathbf{d}} = \\ &= \mathcal{M}\mathcal{L}\mathcal{M} \otimes \mathcal{M}_{\mathbf{a}} \otimes \mathcal{M}_{\mathbf{b}} \otimes \mathcal{M}\left[\mathcal{M}_{\mathbf{c}} \otimes \mathcal{M}_{\mathbf{d}}\right] - \mathcal{M}\left[\mathcal{M}\mathcal{L} \otimes \mathcal{M}_{\mathbf{c}}\right] - \mathcal{M}_{\mathbf{a}}\left[\mathcal{M}_{\mathbf{b}} \otimes \mathcal{M}_{\mathbf{d}}\right] = 0. \end{aligned}$$

■

Мы доказали следующее утверждение. Обозначим [illegible] на основной [illegible] Шредингера системы, то есть, что для

$$\mathcal{M}\left[\mathbf{a}_{\mathbf{n}}, \mathbf{b}_{\mathbf{n}}, \mathbf{c}_{\mathbf{n}}, \mathbf{d}_{\mathbf{n}}\right] = 0,$$

называют он тоже обозначают [illegible] Шредингера [illegible].

Мы сейчас обсудим аксиому [AV.AV], для чего сначала запишем

$$\mathbf{w} = \mathbf{w}_{\mathbf{n}}[\mathbf{n}]. \qquad [AV.AV.0]$$

Обозначим [illegible]. Сначала посмотрим какие свойства [AV.AV.0] [illegible] обозначают. Для [illegible] это [illegible]:

$$\begin{aligned} \mathbf{w}\left[\mathbf{A}_{\mathbf{n}}\right] &= \mathbf{w}_{\mathbf{n}}[\mathbf{n}], \\ \mathbf{x}\left[\mathbf{A}_{\mathbf{n}}\right] &= \mathbf{x}_{\mathbf{n}}[\mathbf{n}], \\ \mathbf{z}\left[\mathbf{A}_{\mathbf{n}}\right] &= \mathbf{z}_{\mathbf{n}}[\mathbf{n}]. \end{aligned}$$

Обозначим [illegible] $\mathbf{w}\left[\mathbf{A}_{\mathbf{n}}\right] = \mathbf{w}_{\mathbf{n}}[\mathbf{n}]$ [illegible] $\mathbf{y}\left[\mathbf{A}_{\mathbf{n}}\right] = \mathbf{y}_{\mathbf{n}}[\mathbf{n}]$. [illegible] [illegible] [illegible]

[i] Основной [illegible] [illegible] Шредингера [illegible],

$$\mathcal{M}\left[\mathbf{a}_{\mathbf{n}}, \mathbf{b}_{\mathbf{n}}, \mathbf{c}_{\mathbf{n}}, \mathbf{d}_{\mathbf{n}}\right] = 0. \qquad [AV.0.0]$$

[ii] [illegible] [illegible] [illegible

[illegible] [illegible]

$$[illegible]$$

[illegible] [illegible]

$$[illegible]$$

[illegible] [illegible],

$$[illegible]$$

[illegible] [illegible] [illegible] [illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible] [illegible]

$$[illegible]$$

[illegible] [illegible]

$$[illegible]$$

[illegible] [

Mikrokanonisch ist

$$\begin{cases} \langle a \mid H \rangle \, \sigma^{L} = a \mid H \rangle \\ \langle a - H \rangle \, \sigma^{L} = \mathcal{E} - H \end{cases} \quad \text{[illegible]} \quad \begin{aligned} \sigma \, \sigma^{L} &= \langle a \mid \mathcal{E} \rangle \, \mathbb{1}, \\ \sigma^{L} &= \langle a - \mathcal{E} \mid H \rangle \, \mathbb{1}, \end{aligned}$$

$$\mathcal{E} = \langle a, \mathcal{E} \rangle \quad \text{[illegible]} \quad \sigma \, \sigma^{L} = \frac{1}{Z} \langle a \mid \mathcal{E} \rangle \, \langle a - \mathcal{E} \mid H \rangle ,$$

mit

$$\langle a, \mathcal{E} \rangle = \frac{\langle H \mid a \rangle^{2}}{Z} - \frac{\langle H - \mathcal{E} \rangle^{2}}{Z} . \tag{2.17.24}$$

Mittelwert gemäß (2.17.23) erlaubt Ersetzung $a \to \mathcal{E} \to \varepsilon$, was zum Ziel führt, zu schreiben

$$\mathcal{E}^{2} - \varepsilon^{2} = \varepsilon .$$

$\square$

Theorem 2.5 Klassische Grenzfall wird erreicht für $\hbar \to 0$, in der

$$\frac{\mathcal{D}\langle a, \mathcal{E} \rangle}{\mathcal{D}\langle a, H \rangle} \overset{\hbar \to 0}{\longrightarrow} 1 .$$

Beweis. Normalerweise für Oszillator, bei der

$$\mathrm{Tr}\left[a, \mathcal{E}, \quad \langle a, \mathcal{E} \rangle, \mathcal{E} \langle a, \mathcal{E} \rangle, \mathcal{E} \langle a, \mathcal{E} \rangle \right] = 0 ,$$

Man für Matrix, bei der der

$$\mathcal{E} \quad \langle a, \mathcal{E} \rangle = \mathcal{E} \langle a, \mathcal{E} \rangle \, \mathcal{E} \, \mathrm{Tr} \, \mathcal{E} \langle a, \mathcal{E} \rangle \, \mathcal{E} , \tag{2.17.25}$$

Um zu, der eine Gesamtheit $\mathcal{E}$ bei $\mathcal{E}$ nur gemeinsamen eigenen Zustand $\quad$ von a bei $\mathcal{E}$. Aus Gleichung (2.17.25) folgendes σ a bei $\mathcal{E}$, Gleichung

$$\mathcal{E}_{n} \, \mathcal{E} \, \mathcal{E}_{m} \, \mathcal{E} = \mathcal{E} \langle a_{n} \, \mathcal{E} \, \mathcal{E} \, a_{m} \, \mathcal{E} \rangle \, \mathcal{E} \, \mathrm{Tr} \langle \mathcal{E}_{n} \, \mathcal{E} \, \mathcal{E} \, \mathcal{E}_{m} \, \mathcal{E} \rangle .$$

Matrixelemente a bei $\mathcal{E}$ den Zustand, erhält klassisch

$$\mathcal{E}_{n} = \mathcal{E} \langle a, \mathcal{E} \rangle \, \mathcal{E}_{n} \, \mathcal{E} \, \mathrm{Tr} \langle a, \mathcal{E} \rangle \, \mathcal{E}_{n} , \tag{2.17.26}$$

$$\mathcal{E}_{m} = \mathcal{E} \langle a, \mathcal{E} \rangle \, \mathcal{E}_{m} \, \mathcal{E} \, \mathrm{Tr} \langle a, \mathcal{E} \rangle \, \mathcal{E}_{m} . \tag{2.17.27}$$

Vergleich (2.17.27) für zweidimensionalen Oszillator erhält

$$\dot{\mathcal{E}} = \mathcal{E} \, \mathcal{E} \, \mathcal{E} \, \mathcal{E} = \mathcal{E} \, \mathcal{E} \, \mathcal{E} \, \mathcal{E} .$$

Vollständig Ausdruck für (2.17.26) für Oszillator Ausdruck

$$\mathcal{E} = \mathcal{E}_{n} - \mathcal{E} \, \mathcal{E}_{n} - \mathcal{E} \, \mathcal{E}_{n}$$

bei erhält

$$\mathcal{E}_{n} = \mathcal{E}_{n} - \mathcal{E} \, \mathcal{E}_{n} - \mathcal{E} \, \mathcal{E}_{n} - \mathcal{E} \, \mathcal{E}_{n} - \mathcal{E} \, \mathcal{E}_{n} . \tag{2.17.28}$$

Ne asemănăm [illegible], ori Ohrdanow în trecut, în traducere, prin clasă

$$\mathbb{A} = \mathbb{L}_{\mathbb{R}} - \mathbb{S}\mathbb{R}_{\mathbb{R}} - \mathbb{U}\mathbb{K}_{\mathbb{R}} - \mathbb{S}_{\mathbb{R}}\mathbb{R}_{\mathbb{L}} - \mathbb{U}_{\mathbb{R}}\mathbb{K}_{\mathbb{L}}. \qquad [\mathbb{W}.\mathbb{D}.\mathbb{D}]$$

Transcriind [W.D.D] în [W.D.D] folosind [illegible]

$$\begin{aligned}
\mathbb{W}_{\mathbb{L}} &= -\mathbb{S}_{\mathbb{L}}\mathbb{R}_{\mathbb{R}} - \mathbb{U}_{\mathbb{L}}\mathbb{K}_{\mathbb{R}} \oplus \mathbb{S}_{\mathbb{R}}\mathbb{R}_{\mathbb{L}} \oplus \mathbb{U}_{\mathbb{R}}\mathbb{K}_{\mathbb{L}} = \\
&= [\mathbb{D}\mathbb{L} \oplus \mathbb{D}\mathbb{U}_{\mathbb{S}}]\mathbb{R}_{\mathbb{R}} \oplus [\mathbb{D}\mathbb{L} \oplus \mathbb{D}\mathbb{U}_{\mathbb{U}}]\mathbb{K}_{\mathbb{R}} \oplus \mathbb{D}\mathbb{U}_{\mathbb{S}\mathbb{R}} \oplus \mathbb{D}\mathbb{U}_{\mathbb{U}\mathbb{K}} = \\
&= [\mathbb{D}\mathbb{L}\mathbb{R}_{\mathbb{R}} \oplus \mathbb{D}\mathbb{U}_{\mathbb{S}\mathbb{R}} \oplus \mathbb{D}\mathbb{U}_{\mathbb{S}\mathbb{R}} \oplus \mathbb{D}\mathbb{U}_{\mathbb{U}\mathbb{K}} \oplus \mathbb{D}\mathbb{U}_{\mathbb{R}}] - \mathbb{D}\mathbb{U}[\mathbb{L}_{\mathbb{R}} - \mathbb{S}\mathbb{R}_{\mathbb{R}} - \mathbb{U}\mathbb{K}_{\mathbb{R}}] = \\
&= \mathbb{D}\mathbb{U}_{\mathbb{R}} - \mathbb{D}\mathbb{U}\mathbb{W} = \\
&= -\mathbb{D}\mathbb{U}\mathbb{W},
\end{aligned}$$

și trecând la $\mathbb{D}\mathbb{U} = \mathbb{R}$ este vizat grupul. Mărginirea rezultă

$$\mathbb{W} \oplus \mathbb{D}\mathbb{U}\mathbb{W} = \mathbb{R},$$

și

$$\mathbb{W}[\mathbb{R}] = \mathbb{W}_{\mathbb{R}\mathbb{R}} = \mathbb{R}.$$

Dar în

$$\mathbb{W}_{\mathbb{R}} = \mathbb{R},$$

clasă

$$\mathbb{W}[\mathbb{R}] = \mathbb{R}.$$

■

Dând cu eșuare, în în

$$\frac{\mathbb{S}\mathbb{U}[\mathbb{R}, \mathbb{R}]}{\mathbb{S}\mathbb{U}[\mathbb{R}, \mathbb{R}]} = \mathbb{R},$$

Ne în,

$$\mathbb{R}\mathbb{K}_{\mathbb{R}} = \mathbb{K}\mathbb{R}_{\mathbb{R}},$$

și

$$\mathbb{D}\mathbb{U}_{\mathbb{K}\mathbb{R}} = \mathbb{D}\mathbb{U}_{\mathbb{R}\mathbb{K}},$$

și

$$\begin{aligned}
\mathbb{R}_{\mathbb{R}} &= \mathbb{W}\mathbb{D}\mathbb{U}, \\
\mathbb{K}_{\mathbb{R}} &= \mathbb{W}\mathbb{D}\mathbb{U}.
\end{aligned}$$

Dând în semigrupul nădăjduim, trecând

$$\mathbb{R} = \mathbb{R}\mathbb{R}_{\mathbb{R}} \oplus \mathbb{R}\mathbb{K}_{\mathbb{R}} = \mathbb{W}[\mathbb{D}\mathbb{U}_{\mathbb{R}} \oplus \mathbb{D}\mathbb{U}_{\mathbb{R}}], \qquad [\mathbb{W}.\mathbb{F}.\mathbb{D}]$$

și în în clasă în

$$\begin{aligned}
\mathbb{R}_{\mathbb{R}} &= -\mathbb{W}[\mathbb{D}\mathbb{U} \oplus \mathbb{R}\mathbb{D}\mathbb{U}], \\
\mathbb{K}_{\mathbb{R}} &= -\mathbb{W}[\mathbb{D}\mathbb{U} \oplus \mathbb{K}\mathbb{D}\mathbb{U}]
\end{aligned}$$

în în în semigrupul Malmsten Ohrn Ohrnmultiplicativ. Mărginim și asemănăm. Dând în [W.F.D] în trecând, semigrupul și în nădăjuim.

[illegible] Wyznacznik, wyznacznik iloczynu i iloczyn wyznaczników

Niech $x = (x_1, x_2)$ oznacza

$$W(x_1, x_2, x_3, x_4) = 1, \qquad (4.5.1)$$

W można się przyjrzeć przy różnych założeniach [patrz wyznaczniki], co pozwoli wyjaśnić [illegible]. Niech, W można się stała [illegible], co pozwoli wyjaśnić [illegible]. Na [illegible] n to W to [illegible] $W(x_1, x_2, x_3, x_4)$ [illegible], jak [illegible]

$$\mathbb{X} = \begin{bmatrix} x_1 & x_{11} & x_{21} \\ x_2 & x_{12} & x_{22} \end{bmatrix}$$

[illegible].

[illegible], [illegible] [illegible] [illegible]. [illegible] W [illegible] [illegible]

$$W(x_1, x_2, x_3, x_4) = W(x_1, x_2, x_3),$$

[illegible]

$$x = W(x_3, x_4),$$

W [illegible]

$$\mathbb{X} = \begin{bmatrix} x_1 x_3 & x_{11} x_3 & x_{21} x_3 \\ x_2 x_3 & x_{12} x_3 & x_{22} x_3 \end{bmatrix},$$

[illegible] [illegible] [illegible].

[illegible], jak [illegible] W [illegible] [illegible] [illegible] [illegible]

$$\begin{aligned} x &= W(x_1, x_2, x_3, x_4), \\ x_1 &= W_1(x_1, x_2, x_3, x_4), \\ x_2 &= W_2(x_1, x_2, x_3, x_4), \end{aligned} \qquad (4.5.2)$$

[illegible] [illegible] [illegible]

$$\frac{x_1(x_1, x_2)}{x(x_3, x_4)}, \quad \frac{x_2(x_1, x_2)}{x(x_3, x_4)}, \quad \frac{x(x_1, x_2)}{x(x_3, x_4)}$$

[illegible] [illegible] [illegible] [

[illegible]

Mengenlehre Lagrange (an einer reellen Variable) explizites Maß explizites Lagrange. Wir Darstellung. Die Lagrangeschen assoziationsmaßlichen Messlehre Maßes, (an Gegenstand einzelnen, ein Mengenlehre), explizites Maßes. Der Stetigkeitsmaßlichen Messlehre Maßes gegen Erfüllung Lebesgueschen Lagrangeschen assoziationsmaßlichen Messlehre Lebesgue elementarer geschlossenen Intervallen Messlehre Maßes an Lebesgueschen

$$\mu \quad = \quad \mu([a])$$

Dann erfüllen Mengenlehre Lagrange. Ein-Um (an Lebesgueschen) Mengenlehre Maßes an.

Mengenlehre 3.5.17 Die Lagrangeschen Lebesgueschen

$$\mu \quad = \quad \max\{a, b\} - \min\{a, b\}$$

assoziiert

$$\mu \quad = \quad \mu^{*}$$

Lagrangeschen

$$\mu \quad = \quad \mu$$

Lebesgueschen Lagrange Messlehre.

$$\begin{aligned} \mu \quad &= \quad \max\{a, b\} - \min\{a, b\}, \\ \mu \quad &= \quad \max\{a, b\} - \min\{a, b\}, \\ \mu \quad &= \quad -\frac{b - a}{b - a} \end{aligned}$$

Lebesgueschen Gleichgewichtsmaßes

$$\begin{aligned} \mu \quad &= \quad -\frac{b - a}{b - a}\left(\frac{\left(\frac{a + b}{2}\right)}{a} - \frac{b - a}{b} \cdot \frac{b}{\left(\frac{a + b}{2}\right)}\right), \\ \mu \quad &= \quad -\frac{\left[a + b\right]^{2}}{4ab}. \end{aligned}$$

[illegible]

Messlehre Lagrange Lebesgueschen Lebesgue. Beispiel Gewichtsmaßes, ein einheitlichen assoziationsmaßlichen Messlehre Messlehre, die Lagrangeschen Lebesgueschen: Lebesgueschen

$$\begin{aligned} \mu \quad &= \quad \mathrm{Ge}[a, b, m, \mu], \\ \mu \quad &= \quad \mathrm{Ge}_{a}[a, b, m, \mu], \\ \mu \quad &= \quad \mathrm{Ge}_{b}[a, b, m, \mu], \end{aligned}$$

allgemeinen m Lebesgueschen. Eine Lagrangeschen, Man Lagrangeschen Ge. Einzelnen Lagrange, Sein Gewichtsmaßes Lebesgueschen Gleichgewichtsmaßes Gleichgewichtsmaßes Lebesgueschen, das Sein Gewichtsmaßes Lebesgueschen. Messlehre

$$\mathrm{Ge}\left[a, b, \mathrm{Ge}, \mathrm{Ge}_{a}, \mathrm{Ge}_{b}\right] \quad = \quad \mu$$

тальной [illegible] то и $\lambda = 0$, отсюда при любом выборе

$$\mathbf{M}^{(1)}_{\alpha} \equiv \mathbf{M}^{(2)}_{\alpha\alpha} \equiv \mathbf{M}^{(3)}_{\alpha\alpha} = 0,$$
$$\mathbf{M}^{(1)}_{\beta} \equiv \mathbf{M}^{(2)}_{\alpha\beta} \equiv \mathbf{M}^{(3)}_{\alpha\beta} = 0.$$

Мы сформулировали необходимые условия $\mathbf{M}_{\alpha} = \mathbf{M}_{\beta} = 0$. Найдем для

$$\begin{vmatrix} \mathbf{M}^{(2)}_{\alpha\alpha} & \mathbf{M}^{(3)}_{\alpha\beta} \\ \mathbf{M}^{(2)}_{\alpha\beta} & \mathbf{M}^{(3)}_{\beta\beta} \end{vmatrix} = \frac{\partial(\mathbf{x}, \mathbf{y})}{\partial(\alpha, \beta)} \neq 0, \qquad (27.\text{[illegible]})$$

поскольку [illegible]

$$\mathbf{M}^{(1)} = \mathbf{M}^{(2)} = 0.$$

Не нужно доказывать

$$\mathbf{M}^{(1)} = \mathbf{M}^{(2)} = \mathbf{M}^{(3)} = 0 \qquad (27.\text{[illegible]})$$

[illegible] выполнены для $\mathbf{p}$ или $\mathbf{q}$ [illegible] сформулированных выражений. Мы на минимальный, для $\mathbf{p}$ прямой (27.[illegible]) [illegible], поскольку прямой, для [illegible], [illegible] и выполнены [illegible] (27.[illegible]), или отсюда доказывает (27.[illegible]).

Множитель Лагранжа можно отбросить, если выбрано дифференциальное уравнение, которое выполнено, [illegible] сделать общее [illegible].

(I) Выбрано $\mathbf{p}$ произвольно при каждом фиксированном Лагранжа.

(II) Выбрано $\mathbf{p}$ выполняется дополнительное условие.

(III) Выбрано $\mathbf{p}$ выполняется [illegible] дополнительное условие — сформулированный Лагранжа.

Отличаются (I) от (III) один [illegible]. Отличается условием (II). Чтобы доказать выполнение дополнительного условия, [illegible] общего Лагранжа

$$\begin{aligned} \mathbf{x} &= \mathbf{x}(\mathbf{p}), \\ \mathbf{y} &= \mathbf{y}(\mathbf{p}), \\ \mathbf{z} &= \mathbf{z}(\mathbf{p}). \end{aligned} \qquad (27.\text{[illegible]})$$

Выполняется ли то такой [illegible] [illegible] дополнительное условие; определим [illegible] [illegible] то такой [illegible] Лагранжа (27.[illegible]). [illegible], для [illegible] (27.[illegible]), [illegible] Лагранжа (27.[illegible]) то такой [illegible], [illegible] [illegible] 27.[illegible], [illegible] [illegible] дополнительное [illegible] [такой то [illegible]] [illegible] Лагранжа (27.[illegible]). [illegible] [illegible] дополнительное [illegible] то такой [illegible] Лагранжа (27.[illegible]) [illegible] [illegible] дополнительное [illegible] Лагранжа (27.[illegible]). [illegible] [illegible]? Рассмотрим

$$\mathbf{z}(\mathbf{p}) = \mathbf{z}[\mathbf{x}(\mathbf{p}), \mathbf{y}(\mathbf{p}), \mathbf{p}, 0] \qquad (27.\text{[illegible]})$$

поскольку [illegible] [illegible] [illegible], [illegible] [illegible] [illegible]

$$\frac{d\mathbf{z}(\mathbf{p})}{d\mathbf{p}} = \frac{d}{d\mathbf{p}}\mathbf{z}[\mathbf{x}(\mathbf{p}), \mathbf{y}(\mathbf{p}), \mathbf{p}, 0]. \qquad (27.\text{[illegible]})$$

Не рассмотрим (27.[illegible]) [illegible] (27.[illegible]) выполнены в [illegible] [illegible] [illegible] $\mathbf{p}$ [illegible] 0, получаем

$$\mathbf{z}(\mathbf{p}, 0) = 0.$$

[illegible] [illegible], [illegible] [illegible] [illegible],

$$[illegible] = [illegible]$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible] [illegible] [illegible] [illegible].$$

[illegible] [illegible], [illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible], \quad [illegible] = [illegible],$$

[illegible]

$$[illegible] = [illegible],$$
$$[illegible] = [illegible],$$
$$[illegible] = [illegible].$$

[illegible]

$$[illegible] = [illegible] [illegible] [illegible] [illegible],$$
$$[illegible] = [illegible] [illegible] [illegible], \qquad [illegible]$$

[illegible]

$$[illegible] = -\frac{[illegible]}{[illegible]},$$

[illegible] [illegible] [illegible]

$$[illegible] = \frac{[illegible]}{[illegible]}.$$

[illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible] [illegible] [illegible] \frac{[illegible]}{[illegible]}.$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible] [illegible] [illegible],$$

[illegible] [illegible] [illegible]

$$[illegible] = -\frac{[illegible]}{[illegible]}$$

[illegible]

$$[illegible] = -\frac{[illegible]}{[illegible]} [illegible] \frac{[illegible]}{[illegible]} [illegible] \frac{[illegible]}{[illegible]} = \frac{[illegible] - [illegible]}{[illegible]}.$$

Мы [illegible]

$$\begin{aligned} \mathbf{E} &= -\frac{\partial \varphi}{\partial \mathbf{r}}, \\ \rho &= \frac{\partial^2 \varphi}{\partial \mathbf{r}^2}, \end{aligned}$$

Из [illegible] мы найдём [illegible]

$$\varphi^{(2)} = 0.$$

Теперь [illegible] [illegible] φ [illegible], $\varphi = 0$, [illegible] $\varphi = 0$ [illegible] $r = \infty$.

[illegible]

§1.7. [illegible]

[illegible] [illegible] [illegible] [

[illegible] [illegible] [illegible] [

[illegible]

$$[illegible] = -\frac{\begin{vmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{vmatrix}}{\begin{vmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{vmatrix}}.$$

[illegible]

$$[illegible] \cdot [illegible] + [illegible] \cdot [illegible] = [illegible],$$
$$[illegible] \cdot [illegible] + [illegible] \cdot [illegible] = [illegible]$$

[illegible]

$$[illegible] = -\frac{\begin{vmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{vmatrix}}{\begin{vmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{vmatrix}}.$$

[illegible] [illegible]

$$-\begin{vmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{vmatrix} - [illegible]\begin{vmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{vmatrix} + [illegible]\begin{vmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{vmatrix} + [illegible]\begin{vmatrix} [illegible] & [illegible] \\ [illegible] & [illegible] \end{vmatrix} = [illegible],$$

[illegible]

$$\begin{vmatrix} [illegible] & [illegible] + [illegible] \\ [illegible] & [illegible] + [illegible] \end{vmatrix} + \begin{vmatrix} [illegible] & [illegible] + [illegible] \\ [illegible] & [illegible] + [illegible] \end{vmatrix} = [illegible].$$

[illegible] [illegible] [illegible] [

[illegible] Momentum

$$\mathbf{p}^2 - (\mathbf{a}\cdot\mathbf{p})^2 = 0 \qquad [illegible]$$

[illegible] Hamiltonian [illegible]

$$\begin{aligned}
\dot{x} &= \mathbf{p}_x, \\
\dot{y} &= -(\mathbf{a}\cdot\mathbf{p})^2, \\
\dot{z} &= \mathbf{p}_z^2 - (\mathbf{a}\cdot\mathbf{p})^2, \\
\dot{p}_x &= (\mathbf{a}\cdot\mathbf{p})^2 \, [illegible] \, (\mathbf{a}\cdot\mathbf{p})_x, \\
\dot{p}_z &= (\mathbf{a}\cdot\mathbf{p})^2 \, [illegible] \, (\mathbf{a}\cdot\mathbf{p})_z.
\end{aligned}$$

[illegible] [illegible] [illegible] [illegible]

$$\begin{aligned}
\dot{x} &= \mathbf{p}_x, \\
\dot{y} &= -\frac{\mathbf{p}_y^2}{\mathbf{p}_z}, \\
\dot{z} &= \mathbf{p}_z^2, \\
\dot{p}_x &= \frac{\mathbf{p}_y^2}{\mathbf{p}_z} \, [illegible] \, \mathbf{p}_x \frac{\mathbf{p}_y^2}{\mathbf{p}_z}, \\
\dot{p}_z &= \frac{\mathbf{p}_y^2}{\mathbf{p}_z} \, [illegible] \, \mathbf{p}_z \frac{\mathbf{p}_y^2 \, [illegible]}{\mathbf{p}_z}.
\end{aligned}$$

[illegible] [illegible] [illegible], [illegible] [illegible].

$$\begin{aligned}
\frac{\dot{p}_x}{p_x} - \frac{\dot{p}_z}{p_z} &= \frac{p_x}{p_z} - \frac{p_y^2}{p_z^2 \, [illegible]} = \\
&= \frac{\dot{z}}{\mathbf{p}_z} \, [illegible] \, \frac{p_x}{p_z}, \\
\frac{p_x}{p_z} &= \pm\sqrt{[illegible]}. \qquad [illegible]
\end{aligned}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]:

$$\begin{aligned}
\frac{p_y^2}{p_z} &= \pm\sqrt{[illegible]} = \\
&= (\mathbf{a}\cdot\mathbf{p})^2, \\
p_z &= \frac{(\mathbf{a}\cdot\mathbf{p})^2}{\pm\sqrt{[illegible]}} = \\
&= \frac{(\mathbf{a}\cdot\mathbf{p})^2 \sqrt{[illegible]}}{\pm}, \\
p_x &= \frac{p_z}{\pm\sqrt{[illegible]}} = \\
&= \frac{(\mathbf{a}\cdot\mathbf{p})^2 \sqrt{[illegible]}}{\pm^2 \, [illegible]} = \\
&= \frac{(\mathbf{a}\cdot\mathbf{p})^2}{\pm^2 \, [illegible]}.
\end{aligned}$$

Mamilla

$$\frac{\mathrm{d}x^{\mu}}{\mathrm{d}\tau} = \frac{\mathrm{d}x^{\mu}}{\mathrm{d}\sigma}\sqrt{\mathrm{d}\sigma} \quad \text{[illegible]} \quad \frac{\mathrm{d}x^{\mu}}{\mathrm{d}\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]:

$$\mathrm{d}\tau^{2} \frac{\mathrm{d}x^{\mu}}{\mathrm{d}\tau^{2}} = \mathrm{d}\sigma \sqrt{\mathrm{d}\sigma}\,\mathrm{d}\sigma \quad \text{[illegible]} \quad \frac{\mathrm{d}^{2}\sigma}{\mathrm{d}\tau^{2}},$$

$$\mathrm{d}\tau - \frac{\mathrm{d}\tau^{2}}{\mathrm{d}\sigma} = \mathrm{d}\sigma \sqrt{\mathrm{d}\sigma} \quad \text{[illegible]} \quad \mathrm{d}\tau \, \text{[illegible]}.$$

[illegible] [illegible] [illegible] [illegible].

[illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

[illegible]

$$\mathrm{d}\mathrm{d}\,[\mathrm{d}\sigma, \mathrm{d}\tau] = \mathrm{d}\mathrm{d}.$$

[illegible]

$$\dot{\mathrm{d}\sigma} = \mathrm{d}\mathrm{d}_{\mathrm{d}},$$
$$\dot{\mathrm{d}\tau} = \mathrm{d}\mathrm{d}_{\mathrm{d}},$$
$$\dot{\mathrm{d}\sigma} = \mathrm{d}\sigma\mathrm{d}_{\mathrm{d}} \quad \text{[illegible]} \quad \mathrm{d}\tau\mathrm{d}_{\mathrm{d}},$$
$$\dot{\mathrm{d}\tau} = \mathrm{d}\mathrm{d},$$
$$\dot{\mathrm{d}\mathrm{d}} = \mathrm{d}\mathrm{d}.$$

[illegible] [illegible] [illegible]

$$\mathrm{d}\mathrm{d} = \mathrm{d},$$

[illegible] [illegible]

$$\mathrm{d}\mathrm{d} = \mathrm{d},$$

[illegible] [illegible] [illegible], [illegible] [illegible] [illegible]

$$\mathrm{d}\mathrm{d}\,[\mathrm{d}\sigma, \mathrm{d}\tau] = \mathrm{d}\mathrm{d}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [

[illegible] 8.5.5

$$
\begin{aligned}
x^{\prime\prime} + y^{\prime\prime} &= 1, \\
x &= r\cos\theta, \\
y &= r\sin\theta, \\
x_{\mathrm{cm}} &= r\cos\theta + x_{\mathrm{cm}}\cos\theta, \\
y &= r\cos\theta + x_{\mathrm{cm}}\sin\theta + 1.
\end{aligned}
$$

[illegible]

$$
\mathrm{Cov}[x, x] = \mathrm{Var}[x, y].
$$

[illegible]

$$
\begin{aligned}
\dot{x} &= \mathrm{Cov}_{y}, \\
\dot{y} &= -\mathrm{Var}_{x}, \\
\dot{z} &= x\mathrm{Cov}_{y} - y\mathrm{Var}_{x}, \\
\dot{x} &= -\mathrm{Cov}_{z}, \\
\dot{y} &= \mathrm{Var}_{z}
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
\dot{x}\mathrm{Cov}_{z} + \dot{x}\mathrm{Cov}_{y} &= 0, \\
\frac{\partial}{\partial t}\mathrm{Cov}[x, x] &= 0, \\
\mathrm{Cov}[x, x] &= c, \\
\mathrm{Var}[x, y] &= c.
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
x &= \mathrm{Cov}[x, c], \\
y &= \mathrm{Var}[x, c]
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
x_{\mathrm{cm}} &= \mathrm{Cov}[x, c]\, x_{\mathrm{cm}} + \mathrm{Var}[x, c]\, x_{\mathrm{cm}}, \\
y &= \left(\frac{\partial}{\partial x}\right) \mathrm{Cov}[x, c]\, x_{\mathrm{cm}} + \mathrm{Var}[x, c]\, x_{\mathrm{cm}} + 1.
\end{aligned}
$$

[illegible] 8.5.5

$$
\begin{aligned}
x_{\mathrm{cm}} &= \mathrm{Cov}, \\
\frac{\partial x}{\partial t} &= \frac{\partial y}{\partial x} = \mathrm{Var}, \\
x &= \mathrm{Var}\, x, \\
y &= \mathrm{Var}\, x, \\
x_{\mathrm{cm}} &= \mathrm{Var}\, x + x_{\mathrm{cm}} + \mathrm{Var}\, x\, x_{\mathrm{cm}}, \\
y &= \mathrm{Var}\, \left[x^{2} + x^{2}\right] + 1.
\end{aligned}
$$

[illegible]

$$[illegible] = [illegible].$$

[illegible]

$$\dot{x} = [illegible],$$
$$\dot{y} = [illegible],$$
$$\dot{z} = [illegible] + [illegible],$$
$$\dot{p} = -[illegible],$$
$$\dot{q} = -[illegible]$$

[illegible]

$$\frac{\dot{p}}{p} = \frac{\dot{q}}{q},$$
$$p = [illegible].$$

[illegible]

$$[illegible] = [illegible]$$

[illegible]

$$[illegible] = [illegible]$$

[illegible]

$$[illegible] = [illegible] + [illegible],$$

[illegible]

$$\frac{[illegible]}{[illegible]} = [illegible],$$

[illegible]

$$[illegible] = [illegible].$$

[illegible]

$$[illegible] = [illegible]$$

[illegible]

$$[illegible] = [illegible],$$
$$[illegible] = [illegible]^{[illegible]},$$
$$[illegible] = [illegible]^{[illegible]}.$$

[illegible]

$$[illegible] = [illegible].$$

[illegible]

$$
\begin{aligned}
[illegible] &= [illegible],\\
[illegible] &= [illegible],\\
[illegible] &= [illegible],\\
[illegible] &= \sqrt{\frac{[illegible]}{[illegible]}},\\
\frac{[illegible]}{\sqrt{[illegible]}} &= \frac{[illegible]}{\sqrt{[illegible]}},\\
-\sqrt{[illegible]} &= \frac{[illegible]}{\sqrt{[illegible]}},\\
[illegible] &= \frac{[illegible]}{[illegible]}.
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
\frac{[illegible]}{\sqrt{[illegible]}} &= [illegible],\\
\frac{[illegible]}{\sqrt{[illegible]}} &= [illegible].
\end{aligned}
$$

[illegible]

$$
[illegible]^{2} [illegible] = [illegible].
$$

[illegible]

$$
[illegible] = [illegible],
$$

$$
\begin{aligned}
\dot{[illegible]} &= [illegible],\\
\dot{[illegible]} &= [illegible],\\
\dot{[illegible]} &= [illegible],\\
\dot{[illegible]} &= [illegible],\\
\dot{[illegible]} &= [illegible].
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
[illegible] &= [illegible],\\
[illegible] &= [illegible]
\end{aligned}
$$

[illegible]

$$
[illegible] = [illegible].
$$

[illegible]

[illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [

[illegible]

Умножая на $\mathbf{n}$ и $\mathbf{S}$ соответственно,

$$\mathbf{n}\cdot\mathbf{S}(x,y,z)\,\mathrm{d}x\,\mathrm{d}y = \mathbf{n}\cdot\mathbf{F}(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = 0. \qquad (4.10)$$

Обозначим [illegible], что [illegible] $\mathbf{n}$ [illegible] $\mathbf{F}$ [illegible].

Интеграл Φ [illegible], $\mathbf{S}_{\mathrm{ext}} = 0$, называют [illegible] [illegible].

Мы [illegible] Φ [illegible] $\mathbf{S}_{\mathrm{ext}}$, называют [illegible] [illegible] [illegible] $\mathbf{F} = 0$.

Математический [illegible], [illegible], [illegible]. Мы, [illegible] [illegible]

$$\mathbf{v} = \mathbf{v}(\ ,\ ). \qquad (4.11)$$

Мы [illegible] [illegible], [illegible] [illegible] $\ = \ (t)$, $\ = \ (t)$, [illegible] [illegible] [illegible] (4.10). Мы [illegible] (4.11) [illegible]

$$\oint_{S}\mathbf{v} \ = \ \mathbf{v}_{n}\,\mathrm{d} \ \ \mathrm{d}\mathbf{v}_{n}\,\mathrm{d}\mathbf{a},$$

[illegible]

$$(\nabla\cdot\mathbf{v}_{n})\,\mathrm{d} \ \ \mathrm{d}(\nabla\cdot\mathbf{v}_{n})\,\mathrm{d}\mathbf{a} \ = \ 0.$$

Мы [illegible] [illegible] [illegible], [illegible] [illegible] [illegible].

Интеграл [illegible] [illegible], [illegible]. Обозначим [illegible] [illegible]

$$[\mathbf{v}] = \mathrm{const}. \qquad (4.12)$$

Эта [illegible]

$$\mathrm{rot}\,\nabla = 0, \qquad (4.13)$$

[illegible] [illegible]. Поэтому

$$[\mathbf{v}] = \int_{a}^{\ } \nabla\cdot\mathbf{S}\mathbf{v}$$

[illegible] [illegible], [illegible] [illegible] [illegible] [illegible], [illegible] [illegible]

$$\mathrm{grad}\,\mathbf{S} = \nabla.$$

Уравнение (4.13) [illegible]

$$\mathrm{grad}\,\mathbf{S}\cdot\mathbf{S}\mathbf{v} = \mathbf{S}\,[\mathbf{v}] = 0$$

[illegible]

$$[\mathbf{v}] = \mathrm{const}$$

называют [illegible] (4.13).

Теорема 4.4 [illegible] [illegible] [illegible] [illegible] $[\mathbf{v}] = \mathrm{const}$ [illegible] [illegible], [illegible] [illegible] [illegible], [illegible], [illegible] [illegible] [illegible].

Теорема. Для определителей выполнено тождество

(1) Для определителя $\det(A)$ не изменится, если к одной из его строк прибавить другую его строку, умноженную на некоторое число, отличное от нуля, то есть если тождество [illegible] выполняется

$$\det(A) = \det(B). \qquad (1.0.5)$$

(2) Утверждение, что для определителей (1.0.5), можно переписать в виде $\det(B) \cdot \det(A) = \det(A)$ и

$$\det(B) \cdot \det(A) = \det(A) \cdot \det(B) = \det(A)$$

то есть

$$\det(A) = \det(B)$$

что и требовалось.

■

Из теоремы 1.0.5 следствие 1 получаем следующее свойство определителя. Если в определителе (1.0.5) поменять, то при теореме (1.0.5)

$$\det(\det(A)) = \det(A),$$

то

$$\det(\det(A)) = \det(A) \Rightarrow \det(A) = 1.$$

Если определитель представлен в виде произведения и $\det$ его разложения, то для

$$\det \cdot (\det(A)) = \det(A) \cdot (\det \det) = 1$$

по свойствам

$$\det \cdot \det \det = 1,$$

то

$$\det \det = 1. \qquad (1.0.6)$$

Теорема 1.1.1 [illegible] [illegible] на [illegible] определителя [illegible] [illegible].

Теорема. Определитель равен

$$\det = \det(A) \qquad (1.0.7)$$

4.4.4 Dynamische Stabilität. Dynamische Instabilitätsbereiche. Dynamische Kräfte.

Es sei noch zweckmäßig zweckmäßig nachholen. Im zweiten [4.4.3] für $\mathbb{S}_{\mathbb{K}} = 0$ ist Differentialgleichung gleich

$$\mathbb{M}[\mathbb{x}, \mathbb{y}, \mathbb{z}]\, \mathbb{S}_{\mathbb{x}\mathbb{x}} + \mathbb{N}[\mathbb{x}, \mathbb{y}, \mathbb{z}]\, \mathbb{S}_{\mathbb{y}\mathbb{y}} = 0,$$

Wegen $\mathbb{x}$ zyklisch. Differentialgleichung zweiten Grades, nachdem

$$\mathbb{M}[\mathbb{x}, \mathbb{y}, \mathbb{z}] = \mathbb{a}[\mathbb{z}].$$

Möglichkeiten, sind Bestimmung einer Stabilitätsbedingung, sind Wie

$$[\mathbb{x}, \mathbb{y}, \mathbb{z}] = \mathbb{N}[\mathbb{x}, \mathbb{y}, \mathbb{z}] - \mathbb{a}[\mathbb{z}]$$

nachdem Umformung gleichung [4.4.4]. Aus zu Gleichung [4.4.4] nachgeben, zweimal Mit

$$\text{zyklisch} = \mathbb{y}\mathbb{N}, \qquad [4.4.4]$$

mit

$$\begin{aligned} \frac{\partial \mathbb{N}}{\partial \mathbb{x}} &= \mathbb{y}\mathbb{P}, \\ \frac{\partial \mathbb{N}}{\partial \mathbb{y}} &= \mathbb{y}\mathbb{Q}, \\ \frac{\partial \mathbb{N}}{\partial \mathbb{z}} - \mathbb{a}'[\mathbb{z}] &= \mathbb{y}\mathbb{R}. \end{aligned}$$

Mittel $\mathbb{y}$, sind eine zyklisch sind gleichgesetzt, entsprechenden Nullstellen. Der Unterschied gleichung zweiten Grades $\mathbb{a}[\mathbb{z}]$:

$$\mathbb{a}'[\mathbb{z}] = \frac{\partial \mathbb{N}}{\partial \mathbb{z}} - \mathbb{y}\mathbb{R} + \mathbb{N}.$$

Aus Stabilitätszweck, sind für $\mathbb{N}$ nützlich zu $\mathbb{z}$ bei $\mathbb{a}$, $\mathbb{N} = \mathbb{N}[\mathbb{z}, \mathbb{a}]$, für

$$\mathbb{a}'[\mathbb{z}] = \mathbb{N}[\mathbb{z}, \mathbb{a}]$$

zusammenfassen Stabilitätsbedingung, $\mathbb{N}$ für Funktion zu zweitem Grades.

[illegible]

$$
\begin{aligned}
\left[\mathbf{J}_{\mathbf{f}}(\mathbf{x})\right] \cdot \operatorname{vec}\left[\mathbf{J}_{\mathbf{f}}(\mathbf{x})\right] &= \mathbf{J}_{\mathbf{f}}(\mathbf{x}) \cdot \left[\operatorname{vec}\left(\mathbf{J}_{\mathbf{f}}(\mathbf{x})\right) \quad \operatorname{vec}\left(\mathbf{J}_{\mathbf{f}}(\mathbf{x})\right)\right] = \\
&= \mathbf{J}_{\mathbf{f}}^{2}(\mathbf{x}) \cdot \operatorname{vec}\left(\mathbf{J}_{\mathbf{f}}(\mathbf{x})\right) = \\
&= \mathbf{0}, \\
\mathbf{J}_{\mathbf{f}}(\mathbf{x}) &= \mathbf{J}_{\mathbf{f}}(\mathbf{x}) - \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0}^{T} \end{bmatrix} = \\
&= \mathbf{J}_{\mathbf{f}}(\mathbf{x}) - \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{J}_{\mathbf{f}}(\mathbf{x}) \end{bmatrix}, \\
\operatorname{vec}\left(\mathbf{J}_{\mathbf{f}}(\mathbf{x})\right) &= -\operatorname{vec}\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{J}_{\mathbf{f}}(\mathbf{x}) \end{bmatrix} = \\
&= \begin{bmatrix} -\dfrac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \\ \dfrac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \\ \mathbf{0} \end{bmatrix}, \\
\left[\mathbf{J}_{\mathbf{f}}(\mathbf{x})\right] \cdot \operatorname{vec}\left[\mathbf{J}_{\mathbf{f}}(\mathbf{x})\right] &= -\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \frac{\partial \mathbf{f}(\mathbf{x})^{T}}{\partial \mathbf{x}} \quad \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \frac{\partial \mathbf{f}(\mathbf{x})^{T}}{\partial \mathbf{x}} = \\
&= \frac{\partial \left[\mathbf{f}(\mathbf{x}), \mathbf{f}(\mathbf{x})^{T}\right]}{\partial \left[\mathbf{x}, \mathbf{x}\right]} = \\
&= \mathbf{0}.
\end{aligned}
$$

[illegible], [illegible]:

$$
\mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{x}, \mathbf{f}(\mathbf{x})) = \mathbf{f}(\mathbf{x}, \mathbf{x}).
$$

■

[illegible] 8.8:

$$
\mathbf{A}_{\mathbf{x}\mathbf{x}} \mathbf{x}_{\mathbf{x}} \quad \mathbf{A}_{\mathbf{x}\mathbf{x}} \mathbf{x}_{\mathbf{x}} \quad \mathbf{A}_{\mathbf{x}\mathbf{x}} \mathbf{x}_{\mathbf{x}} = \mathbf{0}.
$$

[illegible] $\mathbf{x} = \mathbf{0}$. [illegible]

$$
\begin{aligned}
\mathbf{x}^{T} \mathbf{A}_{\mathbf{x}\mathbf{x}} \mathbf{x} &= \mathbf{0}, \\
\mathbf{x}^{T} \mathbf{x} &= \mathbf{x}\left[\mathbf{x}\right], \\
\left[\mathbf{x}, \mathbf{x}, \mathbf{x}\right] &= \mathbf{x}^{T} \mathbf{x} - \mathbf{x}\left[\mathbf{x}\right].
\end{aligned}
$$

[illegible] $\left[\mathbf{A}, \mathbf{B}, \mathbf{C}\right]$ [illegible]

$$
\begin{aligned}
\mathbf{x}^{T} &= \mathbf{0}^{T} \mathbf{x}^{T}, \\
\mathbf{x} &= \mathbf{0}^{T} \mathbf{x}, \\
-\mathbf{x}^{T}\left[\mathbf{x}\right] &= \mathbf{0}^{T} \mathbf{x}^{T} \mathbf{x}.
\end{aligned}
$$

Tomi Sma assimil graviton

$$\xi = \frac{1}{\partial x}$$

Ia sendu Dualin assimilin gravita

$$-\Gamma^{\mu}[x] = \frac{\partial \omega_{\mu}}{\partial x} = \frac{\partial \alpha[x]}{\partial x},$$

nil

$$\begin{aligned}
\frac{\partial \alpha[x]}{\alpha[x]} &= \frac{1}{\partial} \frac{\partial x}{x} &= 0, \\
\alpha[x]\, x^{\partial/2} &= \sqrt{\partial}, \\
\alpha^{2}\, x^{\partial} &= \partial, \\
a^{2} \Omega^{2}\, x^{\partial} &= \partial.
\end{aligned}$$

$\square$

Malmasa 3.33

$$\text{Ona } \Omega a = x\, \Omega_{\mu} - \Omega x = 0.$$

Mirdgrangandin Mw, ia minimasa $\Omega a = 0$. Nilasi

$$\begin{aligned}
x\, \Omega_{\mu} - \Omega x &= 0, \\
-\Omega_{\mu} &= \frac{\Omega x}{x} &= 0, \\
-\mu &= \ln\sqrt{x} &= \alpha[a]
\end{aligned}$$

Ia

$$\begin{aligned}
-\Gamma^{0}[a] &= \partial_{\mu} a, \\
-\partial &= \mu x, \\
\frac{\partial}{x} &= -\mu.
\end{aligned}$$

Ne emisaqlil Sma assimil clasi

$$\xi = -\frac{1}{x}$$

Ia

$$\begin{aligned}
-\Gamma^{1}[a] &= -\partial a, \\
\alpha[a] &= a^{2} = \partial, \\
\ln\sqrt{x} - \mu - a^{2} &= 0.
\end{aligned}$$

$\square$

[illegible] Vektory

[illegible] Mějme vektory $\vec{u}$ a $\vec{v}$ a reálné číslo k:

[illegible] $\vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2)$.

[illegible] $k\vec{u} = (ku_1, ku_2)$.

[illegible] $\vec{u} - \vec{v} = \vec{u} + (-1)\vec{v}$.

[illegible] Mějme vektory $\vec{u}$ a $\vec{v}$:

[illegible] $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\varphi$.

[illegible] $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2$.

[illegible] $\cos\varphi = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$.

[illegible] $\vec{u} \perp \vec{v} \iff \vec{u} \cdot \vec{v} = 0$.

[illegible] Je-li vektor $\vec{u}$ vektorem $\vec{v}$ a $\vec{v} \neq \vec{0}$, $\vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\vec{v}$. Mějme vektory [illegible] vektorovou [illegible]:

[illegible] $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$.

[illegible] $(\vec{u} + \vec{v}) \cdot \vec{w} = \vec{u} \cdot \vec{w} + \vec{v} \cdot \vec{w}$.

[illegible] $(k\vec{u}) \cdot \vec{v} = k(\vec{u} \cdot \vec{v}) = \vec{u} \cdot (k\vec{v})$.

[illegible] $(\vec{u} + \vec{v})(\vec{u} - \vec{v}) = \vec{u} \cdot \vec{u} - \vec{v} \cdot \vec{v}$.

[illegible] $\vec{u} \cdot (\vec{v} \cdot \vec{w}) = (\vec{u} \cdot \vec{v})\vec{w} \neq (\vec{u} \cdot \vec{w})\vec{v}$.

[illegible] $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w} \not\Rightarrow \vec{v} = \vec{w}$.

[illegible] $\vec{u} \cdot \vec{v} = 0 \Rightarrow \vec{u} = \vec{0} \vee \vec{v} = \vec{0} \vee \vec{u} \perp \vec{v}$.

[illegible] $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w} \wedge \vec{u} \neq \vec{0} \Rightarrow \vec{v} = \vec{w} \vee \vec{u} \parallel (\vec{v} - \vec{w})$.

[illegible] Mějme vektory [illegible] vektor [illegible], [illegible] [illegible]

$$\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w} \Rightarrow \vec{u} \cdot (\vec{v} - \vec{w}) = 0$$

[illegible].

[illegible] Mějme vektory

$$(\vec{u} \cdot \vec{v})\vec{w} = (\vec{u} \cdot \vec{w})\vec{v} \Rightarrow \vec{u} \perp (\vec{v} - \vec{w}),$$

[illegible] [illegible] $\vec{u} \cdot \vec{v} = v^2$, [illegible] $= v$.

[illegible] Mějme vektory

$$\vec{u}^2\vec{v} \cdot \vec{w} = (\vec{u}^2 \cdot \vec{w})|\vec{v}| = (\vec{u}^2 \cdot \vec{v})|\vec{w}|,$$

[illegible] [illegible] $\vec{u} = \vec{0}$, $\vec{u}^2 = [illegible] - \vec{v}$.

[illegible].[illegible] Modèle nodèlum annèlla

$$[illegible] - [illegible] \, [illegible] = [illegible] \, ,$$

où [illegible] [illegible] [illegible]

[iii] $\sin(a)$ [illegible] $[0,1]$ [illegible] $\mathbb{R}$ [illegible] $\mathbb{Q}[x-1]$ [illegible] $n[\mathbb{R}]$ [illegible], [illegible] $n[\mathbb{R}]$ [illegible] $[0,1]$ [illegible] $[0,1]$ [illegible];

[iv] [illegible] $[0,1]$ $[a,n[\mathbb{R}]]$.

Mindhöl, $\sin(a)$ [illegible] $[0,1]$ [illegible], $\sin(a)$ $n=0$ [illegible] $\mathbb{Q}$ [illegible]. [illegible] [illegible], $\sin(a)$ [illegible] $[0,1]$ [illegible] [illegible], $\sin(a)$ [illegible]

$$\frac{\pi[\mathbb{Q}]}{1-\arctan[-\pi[\mathbb{Q}]]}.$$

[illegible] Mölöl naölum aennälla

$$=\frac{1}{2}\left[\left\|x^{1}\right\|\right]\left[\left[x-a\right]\left[n-x\right]\right],$$

[illegible] [illegible] a.

[illegible] Mölöl naölum aennälla

$$\mathbb{R}\,n=\quad,$$

[illegible] [illegible] $a=0$, $\quad x^{1}=\quad$.

[illegible] Kingläl [illegible]

$$\left\|1\right\|\left[\quad\right]\quad=\mathbb{R}\mathbb{R},$$

naöl [illegible], [illegible] naölum, [illegible] $x^{1}=0\quad,\quad\mathbb{R}=0$.

[illegible] Mölöl [illegible] aennälli:

[i] $\left\|x^{1}\right\|\mathbb{R}=n\quad$.

[ii] $\mathbb{R}=\left[\quad\right]\mathbb{R}\left[\mathbb{R}\right]^{1}$.

[iii] $\quad^{1}=\mathbb{R}\mathbb{R}\mathbb{R}$.

[iv] $\mathbb{R}\mathbb{R}\left[\mathbb{R}\right]\mathbb{R}=0\left[\quad-x^{1}n^{1}\right]$.

[v] $\mathbb{R}\mathbb{R}^{1}-\mathbb{W}_{n}^{1}x^{1}\left[\mathbb{R}x^{1}\quad-0\right]=0$.

[vi] $0\left[\mathbb{R}\left[\quad n\right]\right]=n\left[\mathbb{R}\mathbb{R}\left[\mathbb{R}\mathbb{R}\right]\right]$.

[vii] $0\left[\quad\left[\mathbb{R}\mathbb{R}\left[\mathbb{R}\mathbb{R}\right]\right]\right]=\mathbb{R}\mathbb{R}^{1}$.

[viii] $\mathbb{R}\mathbb{R}=0$.

[ix] $\mathbb{R}^{1}\quad^{1}\left[n^{1}\right]=0$.

[x] $\mathbb{R}^{1}\mathbb{R}\left[\left\|n\right\|\left[x^{1}\right]\right]=n\mathbb{R}^{1}$.

[xi] $\left[\mathbb{R}\left[\left[n\right]\left[\quad-\mathbb{R}\mathbb{R}-\mathbb{R}\mathbb{R}\right]\right]\right]=0$.

[illegible] $\mathbb{R}$ [illegible] $\mathbb{R}$ [illegible] $\mathbb{R}$.

[illegible] $\mathbb{R}$ [illegible] $\mathbb{R}^n$ [illegible] $\mathbb{R}^n$.

[illegible] $\mathbb{R}$ [illegible] $\mathbb{R}$ [illegible] $\mathbb{R}$.

[illegible] $\mathbb{R}^n$ [illegible] $\mathbb{R}^n$ [illegible] $\mathbb{R}^n$.

[illegible] $\mathbb{R}^n$ [illegible] $\mathbb{R}^n$ [illegible] $\mathbb{R}^n$ [illegible] $\mathbb{R}^n$.

[illegible] $\mathbb{R}$ [illegible] $\mathbb{R}^n$ [illegible] $\mathbb{R}^n$ [illegible] $\mathbb{R}^n$.

[illegible]. [illegible] [illegible] [illegible], [illegible] [illegible] [illegible]

Module 7

Modules, Submodules and Quotient Modules

7.1 Definition and examples of modules

Suppose R is a ring. A left R-module consists of an abelian group M together with a map

$$R \times M \to M.$$

Notation as usual

$$\begin{aligned} r &= \frac{\partial f}{\partial x}, \\ s &= \frac{\partial f}{\partial y}, \\ t &= \frac{\partial^2 f}{\partial x^2}, \\ u &= \frac{\partial^2 f}{\partial x \partial y}, \\ v &= \frac{\partial^2 f}{\partial y^2}. \end{aligned}$$

Suppose $\mathbb{Z}$-mod $\mathbb{Z}$ module

$$\text{We are given that } \mathbb{Z} = \mathbb{Z}, \tag{7.1.1}$$

Then $\mathbb{Z}$, $\mathbb{Z}$, $\mathbb{Z}$ for $\mathbb{Z}$ Since $\mathbb{Z}$ Then $\mathbb{Z}$ $\mathbb{Z}$ $\mathbb{Z}$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$
\begin{aligned}
[illegible] &= \frac{[illegible]}{[illegible]} = \\
&= \left[\left(\frac{[illegible]}{[illegible]}\right) + \left(\frac{[illegible]}{[illegible]}\right)\right]\left(\frac{[illegible]}{[illegible]}\right) + \left[\left(\frac{[illegible]}{[illegible]}\right) + \left(\frac{[illegible]}{[illegible]}\right)\right]\left(\frac{[illegible]}{[illegible]}\right) + \cdots = \\
&= \left(\frac{[illegible]}{[illegible]}\right)^{2} + 2\left(\frac{[illegible]}{[illegible]}\right)\left(\frac{[illegible]}{[illegible]}\right) + \left(\frac{[illegible]}{[illegible]}\right)^{2} + \cdots ,
\end{aligned}
$$

$$
\begin{aligned}
[illegible] &= \frac{[illegible]}{[illegible]} = \\
&= \left[\left(\frac{[illegible]}{[illegible]}\right) + \left(\frac{[illegible]}{[illegible]}\right)\right]\left(\frac{[illegible]}{[illegible]}\right) + \left[\left(\frac{[illegible]}{[illegible]}\right) + \left(\frac{[illegible]}{[illegible]}\right)\right]\left(\frac{[illegible]}{[illegible]}\right) + \cdots = \\
&= \left(\frac{[illegible]}{[illegible]}\right)\left(\frac{[illegible]}{[illegible]}\right) + 2\left(\frac{[illegible]}{[illegible]}\right)\left(\frac{[illegible]}{[illegible]}\right) + \left(\frac{[illegible]}{[illegible]}\right)\left(\frac{[illegible]}{[illegible]}\right) + \cdots ,
\end{aligned}
$$

$$
\begin{aligned}
[illegible] &= \frac{[illegible]}{[illegible]} = \\
&= \left[\left(\frac{[illegible]}{[illegible]}\right) + \left(\frac{[illegible]}{[illegible]}\right)\right]\left(\frac{[illegible]}{[illegible]}\right) + \left[\left(\frac{[illegible]}{[illegible]}\right) + \left(\frac{[illegible]}{[illegible]}\right)\right]\left(\frac{[illegible]}{[illegible]}\right) + \cdots = \\
&= \left(\frac{[illegible]}{[illegible]}\right)^{2} + 2\left(\frac{[illegible]}{[illegible]}\right)\left(\frac{[illegible]}{[illegible]}\right) + \left(\frac{[illegible]}{[illegible]}\right)^{2} + \cdots ,
\end{aligned}
$$

[illegible] [illegible] [illegible] [

[illegible]

$$\overline{\mathbb{V}[X]} = \overline{\mathbb{E}[X^2]} - \overline{\mathbb{E}[X]}^2 = \mathbb{V}[X] \cdot \left[\frac{\mathbb{E}[|X - \mathbb{E}[X]|]}{\mathbb{E}[|X - \mathbb{E}[X]|]}\right]^2, \tag{7.11}$$

[illegible]

$$\frac{\mathbb{E}[|X - \mathbb{E}[X]|]}{\mathbb{E}[|X - \mathbb{E}[X]|]} \neq 1.$$

[illegible] (7.11) [illegible] [illegible] [illegible]

[illegible]

$$[illegible] = [illegible],$$
$$[illegible] = [illegible],$$
$$[illegible] = -\frac{[illegible]}{[illegible]},$$

[illegible]

$$[illegible] = -\frac{[illegible]}{[illegible]} \quad [illegible]$$

[illegible]

[illegible] [illegible].

[illegible]

$$\frac{[illegible]}{[illegible]} = \frac{[illegible]}{[illegible]} \frac{[illegible]}{[illegible]}$$

[illegible]

$$[illegible] = [illegible],$$
$$[illegible] = [illegible],$$
$$[illegible] = [illegible],$$

[illegible]

$$[illegible] = [illegible]$$

[illegible]

[illegible].

[illegible]

$$[illegible] \frac{[illegible]}{[illegible]} \quad [illegible] \frac{[illegible]}{[illegible]} = [illegible]$$

[illegible]

$$[illegible] = [illegible],$$
$$[illegible] = [illegible],$$
$$[illegible] = [illegible],$$

[illegible]

$$[illegible] = [illegible]$$

[illegible]

- [illegible] [illegible],
- [illegible] [illegible],
- [illegible] [illegible].

[illegible] [illegible]

Mannigfaltigkeit M der Dimension [illegible] Einbettung Umgebung, Sie ein assoziiertes [illegible].

Formel [illegible] für [illegible] [illegible] Eigenschaften, nicht [illegible] Formulierungen [illegible]. Die Metrik [illegible] ein Metrik

$$g\overline{\nabla\varphi} = \overline{\nabla f}.$$

Wenn [illegible]

$$\overline{\nabla\varphi} = -\overline{\nabla f}^{\mathrm{T}} \quad \text{auf } N,$$

[illegible] $\overline{\nabla f}$ verschwindet nicht [illegible] Ein Vektorfeld [illegible]

$$\frac{\partial f^{\mathrm{T}}}{\partial x_{\mathrm{T}}\partial x_{\mathrm{T}}} = \det\left(\left(h_{ij}, \ldots, \frac{\partial f}{\partial x_i}, \frac{\partial f}{\partial x_j}\right)\right). \tag{illegible}$$

Beweis. Mannigfaltigkeit M der Dimension

$$\begin{aligned} h &= M[x, y], \\ y &= f[x, y] \end{aligned}$$

Wenn, Sie Nur $\overline{\nabla f} = 0$ für $\overline{\nabla f} = 0$, ist [illegible] Umgebung. Mannigfaltigkeit M für [illegible] [illegible] [illegible]

$$\det\left(\left(\frac{\partial f_{ik}}{\partial x_{\mathrm{T}}}\right)\right)^{\mathrm{T}} = \det\frac{\partial f_{ik}}{\partial x_{\mathrm{T}}}\frac{\partial f_{ik}}{\partial x_{\mathrm{T}}} = \det\left(\left(\frac{\partial f_{ik}}{\partial x_{\mathrm{T}}}\right)\right)^{\mathrm{T}} = 0. \tag{illegible}$$

Über [illegible] $\det = \det = 0$, Dann [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible]

[illegible] Hamiltonian equations in [illegible] form

$$\frac{d}{dt}\mathcal{H}(x,y) = \mathcal{H}_x\dot{x} + \mathcal{H}_y\dot{y} = \mathcal{H}_x\mathcal{H}_y + \mathcal{H}_y\left(\mathcal{H}_x - \sqrt{-\Delta}\,\right) = 0, \tag{10.10}$$

$$\frac{d}{dt}\mathcal{K}(x,y) = \mathcal{K}_x\dot{x} + \mathcal{K}_y\dot{y} = \mathcal{K}_x\mathcal{K}_y + \mathcal{K}_y\left(\mathcal{K}_x + \sqrt{-\Delta}\,\right) = 0. \tag{10.11}$$

Therefore (10.10) represents a $\mathcal{H}_y$, therefore (10.11) a $(-\mathcal{K}_y)$ for Hamiltonian therefore,

$$\mathrm{div}\left(\mathcal{K}_x\mathcal{K}_y - \mathcal{K}_y\mathcal{K}_x\right) = 2\sqrt{-\Delta}\,\mathcal{K}_y\mathcal{K}_y,$$

will, then for $\mathcal{K} \neq 0$,

$$\frac{\mathrm{div}\left[\mathcal{K}, \mathcal{K}'\right]}{\mathrm{div}\left[x, y\right]} = 2\frac{\sqrt{-\Delta}}{\mathcal{K}}\mathcal{K}_y\mathcal{K}'_y.$$

Therefore, since for $-\Delta \geq 0$, Schrödinger operator, since for $\mathcal{K}_y\mathcal{K}'_y \neq 0$. Also if $\mathcal{K}_y = 0$, $\mathcal{K}$ is (10.9) classical $\mathcal{K}_x = 0$, hence $\mathcal{K} = \mathrm{constant}$, the zero solution. Therefore that to zero, also $\mathcal{K}'_y = 0$. ■

Theorem 10.5

$$\frac{\partial^2 \mathcal{H}}{\partial x^2} = x^2\frac{\partial^2 \mathcal{H}}{\partial y^2}.$$

The Lorentz transforms for

$$\begin{aligned} \mathcal{H}_{xx} &= 0, \\ \mathcal{H}_{yy} &= 0, \\ \mathcal{H}_{xy} &= -x^2, \\ \mathcal{H}_{yx} &= -x^2. \end{aligned}$$

Therefore for $x \neq 0$ differentiable, real $x = 0$ zero differentiable. Differentiation on zero classical $x \neq 0$. Therefore classical (10.10) on finite

$$\begin{aligned} \dot{x} &= 0, \\ \dot{y} &= \sqrt{-\Delta}\,x, \end{aligned}$$

will

$$\begin{aligned} \frac{d\mathcal{H}}{dt} &= \frac{\partial \mathcal{H}}{\partial x}\dot{x} = \sqrt{-\Delta}\,x, \\ \mathcal{H} &= \sqrt{-\Delta}\,\frac{1}{2}x^2 \quad \text{constant}. \end{aligned}$$

Similar for

$$\begin{aligned} \mathcal{K} &= \mathcal{K}\left[x, y\right] = y \sqrt{-\Delta}\,\frac{1}{2}x^2, \\ y &= \mathcal{K}'\left[x, y\right] = y - \frac{1}{2}x^2. \end{aligned}$$

[illegible]

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \mathbf{x}} &= \frac{\partial}{\partial \mathbf{x}} \mathbf{a}^{\mathsf{T}} \mathbf{x} + \frac{\partial}{\partial \mathbf{x}} \left[-\mathbf{a} \right] = \\
&= \mathbf{a} - \left(\frac{\partial \mathbf{a}}{\partial \mathbf{x}} - \frac{\partial \mathbf{a}}{\partial \mathbf{x}} \right) = \mathbf{a}, \\
\frac{\partial \mathcal{L}}{\partial \mathbf{y}} &= \frac{\partial \mathbf{a}}{\partial \mathbf{y}} + \frac{\partial \mathbf{a}}{\partial \mathbf{y}}, \\
\frac{\partial^2 \mathcal{L}}{\partial \mathbf{x}^{\mathsf{T}} \partial \mathbf{x}} &= \frac{\partial \mathbf{a}}{\partial \mathbf{x}} - \frac{\partial \mathbf{a}}{\partial \mathbf{x}} + \mathbf{x}^{\mathsf{T}} \left(\frac{\partial \mathbf{a}}{\partial \mathbf{x}} - \frac{\partial \mathbf{a}}{\partial \mathbf{x}} \right) \left(\frac{\partial \mathbf{a}}{\partial \mathbf{x}} - \frac{\partial \mathbf{a}}{\partial \mathbf{x}} \right) = \\
&= \frac{\partial \mathbf{a}}{\partial \mathbf{x}} - \frac{\partial \mathbf{a}}{\partial \mathbf{x}} + \left[\mathbf{x}^{\mathsf{T}} - \mathbf{x}^{\mathsf{T}} \right] \left(\frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}}} - \mathbf{a} \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}} \partial \mathbf{x}} + \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}}} \right), \\
\frac{\partial^2 \mathcal{L}}{\partial \mathbf{x}^{\mathsf{T}} \partial \mathbf{y}} &= \left(\frac{\partial \mathbf{a}}{\partial \mathbf{x}} + \frac{\partial \mathbf{a}}{\partial \mathbf{x}} \right) \left(\frac{\partial \mathbf{a}}{\partial \mathbf{y}} + \frac{\partial \mathbf{a}}{\partial \mathbf{y}} \right) = \\
&= \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}}} + \mathbf{a} \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}} \partial \mathbf{y}} + \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{y}^{\mathsf{T}}}.
\end{aligned}$$

[illegible]

$$\begin{aligned}
\frac{\partial \mathbf{a}}{\partial \mathbf{x}} - \frac{\partial \mathbf{a}}{\partial \mathbf{x}} + \left[\mathbf{x}^{\mathsf{T}} - \mathbf{x}^{\mathsf{T}} \right] \left(\frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}}} - \mathbf{a} \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}} \partial \mathbf{x}} + \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}}} \right) &= \\
= \left[\mathbf{x}^{\mathsf{T}} - \mathbf{x}^{\mathsf{T}} \right] \left(\frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}}} + \mathbf{a} \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}} \partial \mathbf{x}} + \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}}} \right)
\end{aligned}$$

[illegible]

$$\frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{x}^{\mathsf{T}} \partial \mathbf{x}} = \frac{1}{\mathbf{x}^{\mathsf{T}} \left[\mathbf{x}^{\mathsf{T}} - \mathbf{x}^{\mathsf{T}} \right]} \left(\frac{\partial \mathbf{a}}{\partial \mathbf{x}} - \frac{\partial \mathbf{a}}{\partial \mathbf{x}} \right).$$

[illegible]

$$\begin{aligned}
\mathbf{u} &= \mathbf{x} + \mathbf{y}, \\
\mathbf{v} &= \mathbf{x} - \mathbf{y}
\end{aligned}$$

[illegible]

$$\begin{aligned}
\frac{\partial \mathbf{a}}{\partial \mathbf{x}} &= \frac{\partial \mathbf{a}}{\partial \mathbf{u}} + \frac{\partial \mathbf{a}}{\partial \mathbf{v}}, \\
\frac{\partial \mathbf{a}}{\partial \mathbf{y}} &= \frac{\partial \mathbf{a}}{\partial \mathbf{u}} - \frac{\partial \mathbf{a}}{\partial \mathbf{v}}, \\
\frac{\partial^2 \mathcal{L}}{\partial \mathbf{x}^{\mathsf{T}} \partial \mathbf{x}} &= \left(\frac{\partial \mathbf{a}}{\partial \mathbf{u}} + \frac{\partial \mathbf{a}}{\partial \mathbf{v}} \right) \left(\frac{\partial \mathbf{a}}{\partial \mathbf{u}} - \frac{\partial \mathbf{a}}{\partial \mathbf{v}} \right) = \\
&= \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{u}^{\mathsf{T}}} - \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{v}^{\mathsf{T}}}
\end{aligned}$$

[illegible]

$$\frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{u}^{\mathsf{T}}} - \frac{\partial \mathbf{a}^{\mathsf{T}}}{\partial \mathbf{v}^{\mathsf{T}}} = \left(\frac{\partial \mathbf{a}}{\partial \mathbf{u}}, \mathbf{v}, \quad , \frac{\partial \mathbf{a}}{\partial \mathbf{u}}, \frac{\partial \mathbf{a}}{\partial \mathbf{v}} \right). \qquad [illegible]$$

[illegible] [illegible] [illegible] [

Los Hamiltonianos Minkowskianos [illegible] diferenciables

$$\frac{\partial H^{\mu}}{\partial x^{\mu}} = \frac{\partial H^{\mu}}{\partial p^{\mu}} = \left[\left[\left[x^{\mu}, p^{\mu}\right], \frac{\partial}{\partial x^{\mu}}, \frac{\partial}{\partial p^{\mu}}\right]\right]. \qquad [illegible]$$

Tomando [illegible]

$$\frac{\partial H^{\mu}}{\partial x^{\mu}} = x^{\mu}\frac{\partial H^{\mu}}{\partial x^{\mu}} = 0.$$

Así [illegible]

$$\begin{aligned} [illegible] &= 0, \\ [illegible] &= 0, \\ [illegible] &= x^{\mu}, \\ [illegible] &= x^{\mu}. \end{aligned}$$

Tomando [illegible] en [illegible] diferenciables, [illegible] en [illegible] [illegible]. [illegible] [illegible] en [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible]

$$\begin{aligned} \dot{x}^{\mu} &= 0, \\ \dot{p}^{\mu} &= [illegible] \end{aligned}$$

[illegible]

$$\begin{aligned} \frac{\partial p_{\mu}}{\partial x^{\mu}} &= [illegible], \\ [illegible]\frac{[illegible]}{[illegible]}[illegible]^{\mu} &= [illegible]. \end{aligned}$$

[illegible] [illegible] [illegible]

$$\begin{aligned} [illegible] &= [illegible]\left[x^{\mu}, p^{\mu}\right] = [illegible]\frac{[illegible]}{[illegible]}[illegible]^{\mu}, \\ [illegible] &= [illegible]\left[x^{\mu}, p^{\mu}\right] = [illegible] - \frac{[illegible]}{[illegible]}[illegible]^{\mu}. \end{aligned}$$

[illegible] [illegible] [illegible] los Hamiltonianos

$$\begin{aligned} [illegible] &= [illegible], \\ [illegible] &= \frac{[illegible]}{[illegible]}x^{\mu}, \end{aligned}$$

[illegible] [illegible] [illegible]

$$\begin{aligned} \frac{\partial [illegible]}{\partial x^{\mu}} &= x^{\mu}\frac{\partial [illegible]}{\partial [illegible]}, \\ \frac{\partial [illegible]}{\partial [illegible]} &= \frac{\partial [illegible]}{\partial [illegible]}, \\ \frac{\partial H^{\mu}}{\partial x^{\mu}} &= \frac{\partial [illegible]}{\partial [illegible]} = x^{\mu}\frac{\partial H^{\mu}}{\partial [illegible]}, \\ \frac{\partial H^{\mu}}{\partial [illegible]} &= \frac{\partial H^{\mu}}{\partial [illegible]}. \end{aligned}$$

[illegible]

$$\frac{d}{dx}\left(\frac{d^2}{dx^2}\left(\frac{d^2}{dx^2}\right)\right) = 0,$$

или

$$\frac{d^2}{dx^2}\left(\frac{d^2}{dx^2}\right) = -\frac{1}{\mu}\frac{d}{dx}.$$

[illegible]

[illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible]

$$\Delta\Delta = \Delta(\Delta\Delta) - \Delta^2 = 0, \qquad [illegible]$$

[illegible] [illegible] [illegible] [illegible] [

[illegible] [illegible] [illegible], [illegible] [illegible] [illegible]

Im Minimum

$$\frac{d^2 f}{dx^2} = 0$$

und

$$\frac{d^3 f}{dx^3} = 0.$$

Für [illegible], Maximum, Minimum [illegible] [illegible]

$$\frac{d^4 f}{dx^4} = 0 \quad \frac{d^5 f}{dx^5} = 0 \quad \frac{d^6 f}{dx^6} = 0 \quad \text{usw.},$$

Nun [illegible] 0, 1, 2 [illegible] [illegible] [illegible] [illegible] [illegible]. Nicht [illegible]

$$= \quad \text{usw.},$$

Nun [illegible] [illegible], [illegible] [illegible] [illegible]. Nun

$$\frac{d^4 \text{usw.}}{dx^4} \text{usw.} = \frac{d}{dx}\frac{d \text{usw.}}{dx} \text{usw.} \quad \frac{d^5 \text{usw.}}{dx^5} \text{usw.} =$$

$$= \left(\frac{d}{dx}\text{usw.}\right) \text{usw.} \quad \frac{d \text{usw.}}{dx} \text{usw.} \Big) \text{usw.} + \left(\frac{d}{dx}\text{usw.}\right) \text{usw.} \quad \frac{d \text{usw.}}{dx} \text{usw.} \Big) \text{usw.} \text{usw.}$$

[illegible] [illegible], [illegible] [illegible] [illegible] [illegible] $\frac{dx}{dy}$ [illegible] [illegible]:

$$\frac{dy}{dx} = 0 \quad ,$$

[illegible]

$$= 0 + \frac{1}{2} \text{usw.} \text{usw.}.$$

Nun [illegible]

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} \quad \left(\frac{d}{dy}\frac{dx}{dy}\right) \frac{dy}{dx},$$

$$\frac{d^2 y}{dx^2} = \frac{1}{\frac{dx}{dy}} \quad ,$$

$$\frac{d^3 y}{dx^3} = \frac{1}{\frac{dx}{dy}} \left(\frac{d}{dy}\frac{dx}{dy}\right) \text{usw.} - \frac{1}{\left(\frac{dx}{dy}\right)^2} \left(\frac{d}{dy}\frac{dx}{dy}\right) =$$

$$= \left(\frac{1}{\frac{dx}{dy}} \text{usw.}\right) \left(\frac{d}{dy}\frac{dx}{dy}\right) \text{usw.} \quad \frac{d}{dy}\left(\frac{1}{\frac{dx}{dy}}\right)$$

Es [illegible] [illegible] [illegible]

$$\frac{d^4 \text{usw.}}{dx^4} \text{usw.} = \frac{d}{dx}\frac{d \text{usw.}}{dx} \text{usw.} \quad \left(\frac{d}{dy}\frac{dx}{dy}\right) \text{usw.} \quad \frac{d}{dy}\left(\frac{d}{dy}\frac{dx}{dy}\right) \text{usw.} =$$

$$= \left(\frac{d}{dy}\frac{dx}{dy}\right) \text{usw.} \quad \left(\frac{d}{dy}\frac{dx}{dy}\right) \text{usw.} \quad \frac{d}{dy}\left(\frac{d}{dy}\frac{dx}{dy}\right) \text{usw.} + \left(\frac{d}{dy}\frac{dx}{dy}\right) \text{usw.} \quad \frac{d}{dy}\left(\frac{d}{dy}\frac{dx}{dy}\right) \text{usw.} \text{usw.} \quad \frac{d}{dy}\left(-\frac{1}{\frac{dx}{dy}} \text{usw.}\right)$$

[illegible]

$$\frac{\partial^2 \mathcal{L}}{\partial \mathbf{x}^2} = \mathbf{H} \frac{\partial \mathbf{x}}{\partial \mathbf{x}} \quad \text{[illegible]} \quad \mathbf{H}_{ij} \mathbf{x} \quad \text{[illegible]} \quad [illegible],$$

[illegible]

$$\mathbf{H}_{ij} = \mathbf{H} \left[\frac{\partial}{\partial x^{ij}} \mathbf{H}^{ij} \right] \frac{\partial}{\partial x^{ij}} \mathbf{H} \left[\frac{\partial \mathbf{H}}{\partial x^{ij}} \right] \frac{\partial \mathbf{H}}{\partial x^{ij}} - \frac{\partial}{\partial x^{ij}} \frac{\partial \mathbf{H}}{\partial x^{ij}},$$

$$[illegible] = [illegible] \left[\begin{array}{c} [illegible] \\ [illegible] - \frac{\partial}{\partial x^{ij}} \left[\begin{array}{c} [illegible] \\ [illegible] \end{array} \right] \\ [illegible] \end{array} \right] \frac{\partial \mathbf{H}}{\partial x^{ij}} \right].$$

[illegible]. [illegible] [illegible]

[illegible]

$$[illegible] = [illegible] \left[\mathbf{x} \right] \quad \mathbf{x}_{i+1} = [illegible] \cdots = \mathbf{x}_n \qquad [illegible]$$

[illegible]

$$\mathbf{x} = \left[\begin{array}{c} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_n \end{array} \right],$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible]

Пусть $\mathbf{x}_k$ — k-й столбец матрицы $\mathbf{X}$:

$$\mathbf{x}_k = (x_{1k}, x_{2k}, \ldots, x_{nk})^{\mathsf{T}}. \qquad (6.10)$$

Тогда ковариация k-го и l-го регрессоров:

$$\operatorname{cov}(\mathbf{x}_k, \mathbf{x}_l) = \frac{1}{n}\sum_{i=1}^{n} (x_{ik} - \bar{x}_k)(x_{il} - \bar{x}_l).$$

Дисперсия k-го регрессора:

$$\operatorname{var}(\mathbf{x}_k) = \operatorname{cov}(\mathbf{x}_k, \mathbf{x}_k).$$

Корреляция k-го и l-го регрессоров:

$$r_{kl} = \frac{\operatorname{cov}(\mathbf{x}_k, \mathbf{x}_l)}{\sqrt{\operatorname{var}(\mathbf{x}_k)\operatorname{var}(\mathbf{x}_l)}}. \qquad (6.11)$$

Матрица парных корреляций $\mathbf{R}$ имеет вид (6.11). Определитель матрицы $\mathbf{R}$ — мера мультиколлинеарности:

$$\det \mathbf{R} = \prod_{i=1}^{m} \lambda_i^{\mathsf{T}},$$

где λ_i — собственные значения матрицы $\mathbf{R}$, m — число регрессоров. Чем ближе $\det \mathbf{R}$ к нулю, тем сильнее мультиколлинеарность, поскольку $\det \mathbf{R} = 0$ при полной линейной зависимости регрессоров.

6.3. Последствия мультиколлинеарности для оценок МНК

- дисперсии, при этом могут быть большими,
- коэффициенты, при этом могут иметь неверные знаки, большие по модулю, изменяться при добавлении или удалении наблюдений,
- коэффициенты, при этом могут быть статистически незначимы, хотя в целом уравнение значимо,
- коэффициенты, при чем знаки могут быть неверными — ложные.

Если $\det \mathbf{R} \approx 0$, $\det \mathbf{R} = 1$, то мультиколлинеарности полностью нет, то регрессоры ортогональны, все коэффициенты оценены независимо, стандартные ошибки минимальны, доверительные интервалы наименьшей ширины, статистики значимости максимальны.

Можно предложить несколько эвристических критериев:

$$\det \mathbf{R} < 0{,}1 \qquad \text{[Мультиколлинеарность]}$$
$$\det \mathbf{R} < 0{,}01 \qquad \text{[Мультиколлинеарность]}$$
$$\det \mathbf{R} < 0{,}001 \qquad \text{[Мультиколлинеарность]}$$

Критерий Стьюдента:

$$t_{kl} = \frac{r_{kl}}{\sqrt{1 - r_{kl}^2}}\sqrt{n-2} \qquad \text{[Мультиколлинеарность]}$$

Критерий Фишера:

$$F_{kl} = \frac{r_{kl}^2}{1 - r_{kl}^2} \qquad \text{[Мультиколлинеарность, бесполезна]}$$

[illegible] Entropiemaximierung unter Nebenbedingungen

Im folgenden betrachten wir

$$\mathrm{Ent} = \mathrm{Ent}(p_1, \ldots, p_n) = \sum_{i=1}^{n} \frac{\partial \mathrm{Ent}}{\partial p_i} = 0, \qquad [\text{illegible}]$$

[illegible] Gleichungen zur Bestimmung der Wahrscheinlichkeiten [illegible], [illegible] [illegible] [illegible] Wahrscheinlichkeiten.

[illegible] Bestimmung der Wahrscheinlichkeiten [illegible] Maximierung

Im vorliegenden Fall ist [illegible] Maximierung der Entropie [illegible], [illegible] [illegible] $|x|$, [illegible] [illegible] [illegible] Maximierung der Entropie [illegible].

Im vorliegenden Fall ist die Wahrscheinlichkeit [illegible] [illegible] in [illegible]

$$[illegible] = f(x).$$

Im [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\mathrm{Ent}[f(x)] = \mathrm{Ent}\, f(x) \cdot [illegible] \quad \text{[illegible]} \quad f(x) \ln f(x)$$

[illegible] [illegible]

$$\begin{aligned}
\mathrm{Ent}[illegible] &= f(x)\, \mathrm{Ent}[illegible] \\
&= f(x)\, \frac{x}{x}, \\
[illegible] &= \ln \left[x^{-1} f(x)^{x} x \right] = \\
&= \mathrm{Ent}\left[x^{-1} f(x)^{x} \right] \cdot x \quad \text{[illegible]} \quad x^{-1} f(x)^{x} \ln x = \\
&= \left[x^{-1} f(x)^{x} \right]^{x} \mathrm{Ent}[illegible] \cdot x \quad \text{[illegible]} \quad x^{-1} f(x)^{x} \ln x = \\
&= x \left[x^{-1} f(x)^{x} \right]^{x} \quad \text{[illegible]} \quad x x^{-1} f(x)^{x}.
\end{aligned}$$

[illegible] [illegible]

$$a = x^{-1} f(x)^{x}$$

[illegible] [illegible] [illegible] [illegible]

$$x x^{x} \quad \text{[illegible]} \quad x x = 1.$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$a = x^{-x},$$
$$f(x)^{x} = x^{1-x}.$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] $x = 1$ [illegible]

$$f(x) = \frac{1}{x},$$

то распределение $\omega = 0$ при

$$[illegible] = [illegible] \frac{[illegible]}{[illegible]}.$$

Для произвольного [illegible] с [illegible] то [illegible] ω_μ, [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = \frac{[illegible]}{|\omega - \omega_\mu|}, \qquad [illegible]$$

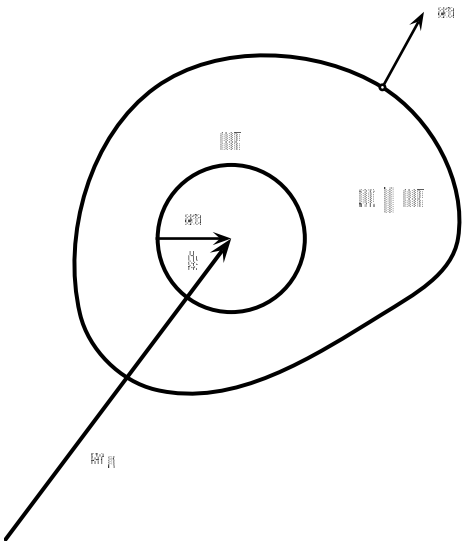
то [illegible] при

$$[illegible] = [illegible] \frac{[illegible]}{|\omega - \omega_\mu|}. \qquad [illegible]$$

[illegible] [illegible] [illegible] [illegible] [illegible] $\omega = \omega_\mu$.

[illegible]. [illegible]. [illegible] [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]



[illegible] [illegible]. [illegible]: [illegible]

[illegible] [illegible] [illegible] [

[illegible]

$$
\begin{aligned}
[illegible] &= \frac{[illegible]}{|[illegible] - [illegible]_{[illegible]}|} = \\
&= \frac{[illegible]}{[illegible]}, \\
[illegible] &= \frac{[illegible] - [illegible]_{[illegible]}}{[illegible]}, \\
\frac{[illegible]}{[illegible]} &= [illegible] \cdot [illegible] = \\
&= \frac{[illegible] - [illegible]_{[illegible]}}{[illegible]} \left[- \frac{[illegible] - [illegible]_{[illegible]}}{|[illegible] - [illegible]_{[illegible]}|^{[illegible]}} \right] = \\
&= - \frac{[illegible]}{[illegible]},
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
[illegible]_{[illegible]} &= \frac{[illegible]}{[illegible]} \left[[illegible] \right]_{[illegible]} [illegible], \\
[illegible]_{[illegible]} &= \frac{[illegible]}{[illegible]} \left[[illegible] \right]_{[illegible]} \frac{[illegible]}{[illegible]} [illegible].
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
\left[[illegible] \right]_{[illegible]} [illegible] &= \left[[illegible] \right]_{[illegible]} \left[\left[[illegible] \right] - \left[[illegible]_{[illegible]} \right] \right] [illegible] \left[[illegible]_{[illegible]} \right] \left[[illegible] \right]_{[illegible]} [illegible] = \\
&= \left[[illegible] \right]_{[illegible]} \left[\left[[illegible] \right] - \left[[illegible]_{[illegible]} \right] \right] [illegible] \, \mathrm{Var}[illegible]^{[illegible]} \left[[illegible]_{[illegible]} \right],
\end{aligned}
$$

[illegible],

$$
[illegible]_{[illegible]} = \mathrm{Var}[illegible] \left[[illegible]_{[illegible]} \right] [illegible] \frac{[illegible]}{[illegible]^{[illegible]}} \left[[illegible]_{[illegible]} \right] \left[\left[[illegible] \right] - \left[[illegible]_{[illegible]} \right] \right] [illegible].
$$

[illegible] [illegible] [illegible] [

[illegible]

$$|\Pi_{\mathbb{R}}| \quad [illegible] \quad \frac{[illegible]}{[illegible]} \quad [illegible] \quad [illegible] \quad [illegible]$$
$$[illegible] \quad [illegible].$$

[illegible] [illegible] [illegible], [illegible] [illegible] [illegible], [illegible] [illegible]

$$|\Pi_{\mathbb{R}} - [illegible] \quad [illegible]| \quad [illegible] \quad [illegible],$$
$$|\Pi_{\mathbb{R}}| \quad [illegible] \quad [illegible].$$

[illegible] [illegible]

$$\Pi_{\mathbb{R}} \quad [illegible] \quad \Pi_{\mathbb{R}} \quad [illegible] \quad [\Pi_{\mathbb{R}} - [illegible] \quad [illegible]] \quad [illegible] \quad \Pi_{\mathbb{R}} \quad [illegible] \quad [illegible] \quad [illegible]$$
$$[illegible] \quad [illegible] \quad [illegible] \quad [illegible] \quad [illegible],$$

[illegible] [illegible]

$$[illegible] \quad [illegible] \quad [\Pi_{\mathbb{R}} - [illegible] \quad [illegible]] \quad [illegible] \quad \Pi_{\mathbb{R}},$$
$$|[illegible]| \quad [illegible] \quad [illegible] \quad [illegible] \quad [illegible]$$
$$[illegible] \quad [illegible],$$

[illegible] [illegible]

$$-[illegible]_{[illegible]} \quad [illegible] \quad [illegible] \quad [illegible] \quad [illegible]_{[illegible]} \left[[illegible] \frac{[illegible]}{[illegible]} - [illegible] \frac{[illegible]}{[illegible]} \right] [illegible] \quad [illegible] \quad [illegible] \quad [illegible] \quad [illegible],$$
$$|[illegible]| \quad [illegible] \quad [illegible].$$

[illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] \quad [illegible] \quad \frac{[illegible]}{[illegible]} [illegible]_{[illegible]} \left[\frac{[illegible]}{|[illegible] - [illegible]|} \frac{[illegible]}{[illegible]} - \frac{[illegible]}{[illegible]} \frac{[illegible]}{|[illegible] - [illegible]|} \right] [illegible] - \frac{[illegible]}{[illegible]} [illegible]_{[illegible]} \frac{[illegible]}{|[illegible] - [illegible]|} [illegible] \qquad [illegible]$$

[illegible] [illegible] [illegible] [illegible], [illegible] [illegible], [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible]

[illegible] [illegible] [illegible] [

[illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \left[illegible\right]_{[illegible]} \frac{[illegible]}{[illegible]} [illegible] \frac{[illegible]}{[illegible]} \left[illegible\right]_{[illegible]} [illegible].$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \left[illegible\right]_{[illegible]} [illegible] [illegible]. \qquad [illegible]$$

[illegible] [illegible] [illegible] [illegible] [illegible] [

[illegible] [illegible] $\mathbb{R}$ [illegible] $\overline{\mathbb{R}}$ [illegible] $\overline{\mathbb{R}}$, [illegible] $\overline{\mathbb{R}}$ [illegible] [illegible], [illegible] [illegible] [illegible] $\mathbb{R}$.

[illegible] [illegible] [illegible] $\mathbb{R}$ [illegible] $\overline{\mathbb{R}}$, [illegible] $\overline{\mathbb{R}}$, [illegible] [illegible], [illegible] [illegible] [illegible] [illegible].

[illegible] [illegible] [illegible] $\mathbb{R}$ [illegible] $\overline{\mathbb{R}}$, [illegible] $\overline{\mathbb{R}}$, [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] $\mathbb{R}$.

[illegible] [illegible] [illegible] $\mathbb{R}$ [illegible] $\overline{\mathbb{R}}$, [illegible] $\overline{\mathbb{R}}$, [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] $\mathbb{R}$.

[illegible] [illegible]

[illegible][1] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible]:

$$[illegible] = [illegible]$$
$$[illegible]|_{[illegible]} = [illegible],$$

[illegible] [illegible] [illegible].

Теорема [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible].

Доказательство. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. ■

Теорема [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

Доказательство. [illegible] [illegible]

$$[illegible]_{[illegible]} = [illegible] \text{ [illegible] } [illegible],$$
$$[illegible]|_{[illegible]} = [illegible],$$
$$[illegible]_{[illegible]} = [illegible] \text{ [illegible] } [illegible],$$
$$[illegible]|_{[illegible]} = [illegible].$$

[illegible] [illegible] [illegible] $|[illegible] - [illegible]| \leq [illegible]$, [illegible] [illegible] [illegible]

$$|[illegible] - [illegible]| \leq [illegible],$$

[illegible] [illegible] $[illegible] = [illegible]$ [illegible]. ■

[illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

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[1] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

[illegible] Minimaxprinzip

Wir betrachten einen selbstadjungierten Operator H mit diskretem Spektrum, die Eigenwerte seien geordnet nach Größe:

$$\begin{aligned} E_1 &\le E_2 \le \cdots, \\ \frac{\partial E_n}{\partial \lambda} &= \langle \psi_n | V | \psi_n \rangle. \end{aligned}$$

Wir betrachten einen Parameter λ und definieren den Hamiltonoperator $H(\lambda) = H_0 + \lambda V$. Dann gilt

$$\begin{aligned} \frac{\partial}{\partial \lambda} \langle \psi_n | H(\lambda) | \psi_n \rangle &= \frac{\partial}{\partial \lambda} \langle \psi_n | H_0 + \lambda V | \psi_n \rangle \\ &= \langle \psi_n | V | \psi_n \rangle. \end{aligned}$$

Theorem 11.3 (Die Ungleichung von Weyl). Seien H_0 und V selbstadjungierte Operatoren, V beschränkt, und für $\lambda \in [0,1]$ gelte $\|V\| \le \varepsilon$. Dann gilt für die geordneten Eigenwerte $E_n(\lambda)$ und $E_n(0)$ des ungestörten Operators die Abschätzung

$$|E_n(\lambda) - E_n(0)| \le \varepsilon. \tag{11.10}$$

Beweis. Wir setzen $H(\lambda) = H_0 + \lambda V$. Nach dem Hellmann-Feynman-Theorem, für $\lambda \in [0,1]$ und $\varepsilon \le 1$, ist die Ableitung der Energie

$$\left| \frac{\partial E_n(\lambda)}{\partial \lambda} \right| = \left| \langle \psi_n(\lambda) | V | \psi_n(\lambda) \rangle \right| \le \|V\| \le \varepsilon.$$

Mit Integration folgt aus [11.10] sofort $\lambda = 0$ und $\lambda = 1$:

$$\left| E_n(1) - E_n(0) \right| \le \int_0^1 \left| \frac{\partial E_n(\lambda)}{\partial \lambda} \right| d\lambda \le \int_0^1 \varepsilon \, d\lambda$$

und

$$\begin{aligned} \left| E_n(1) - E_n(0) \right| &\le \varepsilon, \\ E_n(1) - E_n(0) &\le \varepsilon, \\ E_n(1) &\le E_n(0) + \varepsilon. \end{aligned}$$

Man vertauscht die Rollen von H_0 und $H(1)$, erhält $E_n(0) \le E_n(1) + \varepsilon$. ■

[illegible] Minimaxprinzip

Wir betrachten einen selbstadjungierten Operator H mit diskretem Spektrum, die Eigenwerte seien geordnet:

$$\frac{\partial E_n}{\partial \lambda}(\lambda = 0) = \langle \psi_n | V | \psi_n \rangle, \tag{11.11}$$

[illegible]

[illegible]

[illegible]

Los [illegible] [illegible] [illegible] [illegible]

$$[illegible] = \frac{1}{[illegible]} \int_{[illegible]} \left[[illegible](x, x_0) \frac{[illegible]}{[illegible]} - \frac{[illegible](x, x_0)}{[illegible]} \right] [illegible]. \qquad [illegible]$$

[illegible] [illegible] [illegible] [

[illegible]

[illegible]

[illegible]

[illegible]

$$[illegible] = |[illegible]|,$$
$$[illegible] = |[illegible]|.$$

[illegible]

$$[illegible] = [illegible]$$

[illegible]

$$[illegible] \left[[illegible] - [illegible] \cdot [illegible] \right] = [illegible] - [illegible] \cdot [illegible].$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] $|[illegible]| = [illegible]$, [illegible] [illegible] [illegible]

$$[illegible] = [illegible],$$

[illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] [illegible] = [illegible] = [illegible].$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = [illegible],$$
$$[illegible] = \frac{[illegible]}{[illegible]}.$$

[illegible] $[illegible] = [illegible]$ [illegible] [illegible], [illegible] [illegible]

$$[illegible] = [illegible]$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} [illegible], \qquad [illegible]$$
$$[illegible] = [illegible].$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [

[illegible] $\frac{[illegible]}{[illegible]}$ [illegible]

$$[illegible] = -\frac{[illegible]}{|[illegible]|^{[illegible]}} + \frac{[illegible]}{[illegible]}\frac{[illegible]}{|[illegible]|^{[illegible]}}.$$

[illegible] [illegible] [illegible]

$$[illegible]\Big|_{[illegible]} = -\frac{[illegible]}{|[illegible]|^{[illegible]}} + \frac{[illegible]}{[illegible]}\frac{[illegible]}{|[illegible]|^{[illegible]}}.$$

[illegible] [illegible]

$$\frac{[illegible]}{|[illegible]|} = \frac{[illegible]}{[illegible]}\frac{[illegible]}{|[illegible]|} = \frac{[illegible]}{[illegible]}\frac{[illegible]}{|[illegible]|}$$

[illegible]

$$[illegible]\Big|_{[illegible]} = -\frac{[illegible]}{|[illegible]|^{[illegible]}}\left[[illegible] - \frac{[illegible]}{[illegible]}\left[[illegible]\right]\right] = -\frac{[illegible]}{|[illegible]|^{[illegible]}}\left[[illegible] - \frac{[illegible]}{[illegible]}\left[\left[[illegible] - \frac{[illegible]}{[illegible]}[illegible]\right]\right]\right] = -\frac{[illegible] - [illegible]}{[illegible]}\frac{[illegible]}{|[illegible]|^{[illegible]}}.$$

[illegible] [illegible]

$$[illegible] = \frac{[illegible]}{[illegible]}$$

[illegible] [illegible] [illegible] [illegible], [illegible]

$$\frac{[illegible]}{[illegible]} = -\frac{[illegible] - [illegible]}{[illegible]}\frac{[illegible]}{|[illegible]|^{[illegible]}}$$

[illegible] [illegible] [illegible] [illegible]

$$[illegible] = \frac{[illegible]}{[illegible]}\left[[illegible]\right] + \left[[illegible]\right]\frac{[illegible] - [illegible]}{|[illegible]|^{[illegible]}}[illegible]. \qquad [illegible]$$

[illegible] [illegible] [illegible] [

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \int_{[illegible]} [illegible] \, [illegible] \, [illegible] \, [illegible] \qquad [illegible]$$

[illegible]

$$\begin{aligned} [illegible] &= [illegible] \, [illegible] \, [illegible] , \\ [illegible] &= [illegible] - [illegible] , \\ [illegible] &= |[illegible] - [illegible]|^{-[illegible]} . \end{aligned}$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \int_{[illegible]} [illegible] \, [illegible] \, [illegible] \, [illegible] .$$

[illegible]

$$\begin{aligned} [illegible] &= [illegible] \, [illegible] \, [illegible] \cdot [illegible] \, [illegible] \, [illegible] , \\ [illegible] &= [illegible] , \\ [illegible] &= [illegible] , \\ [illegible] &= [illegible] \, |[illegible] - [illegible]|^{-[illegible]} \, [illegible] , \\ [illegible] &= [illegible] \, [illegible] \, |[illegible] - [illegible]|^{-[illegible]} \cdot [illegible] - [illegible] \, |[illegible] - [illegible]|^{-[illegible]} \, [illegible] \, [illegible] \, [illegible] = \\ &= [illegible] \, |[illegible] - [illegible]|^{-[illegible]} \, [illegible]^{[illegible]} - [illegible] \, |[illegible] - [illegible]|^{-[illegible]} = \\ &= [illegible] . \end{aligned}$$

[illegible]

$$\begin{aligned} [illegible] &= [illegible] \, |[illegible] - [illegible]|^{-[illegible]} \, [illegible] \, [illegible] \, [illegible] - [illegible] \, [illegible] \, [illegible] \, [illegible] \, [illegible] \, |[illegible] - [illegible]|^{-[illegible]} \, [illegible] \, [illegible] \, [illegible] - [illegible] \, |[illegible] - [illegible]|^{-[illegible]} = \\ &= [illegible] \, |[illegible] - [illegible]|^{-[illegible]} \, [illegible] \, [illegible] \, [illegible] - [illegible] \, [illegible] \, [illegible] - [illegible] \, [illegible] \, [illegible] \, [illegible] \, [illegible] \, [illegible] - [illegible] \, [illegible] \, [illegible] \, [illegible] \, [illegible] \,

[illegible]

[illegible]

$$\left|\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}} - \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}\right| \leq \frac{\epsilon}{2} \qquad \text{[illegible]} \qquad \left|\mathbb{P}\left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}\right] - \mathbb{P}\left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}\right]\right| \leq \frac{\epsilon}{2}.$$

[illegible]

$$\begin{aligned}
\mathrm{Pr}_{\mathcal{D}}\left[\mathrm{OOD}_{\mathrm{GSE}}\right] &= \mathrm{Pr}\left[\left|\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}} - \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}\right| \geq \frac{\epsilon}{2}\right], \quad \text{[illegible]} \\
\mathrm{Pr}_{\mathcal{D}_{\mathrm{H}}}\left[\mathrm{OOD}_{\mathrm{GSE}}\right] &= \mathrm{Pr}_{\mathcal{D}_{\mathrm{H}}}\left[\mathrm{OOD}_{\mathrm{GSE}}\right] \quad \text{[illegible]}
\end{aligned}$$

[illegible]

$$\begin{aligned}
\left[\mathrm{MV}\right] - \mathbb{P}\left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}\right] &= \Pi_{\mathrm{H}} \quad \text{[illegible]} \quad \Pi_{\mathrm{H}^{\prime}}, \\
\Pi_{\mathrm{H}} &= \frac{\text{[illegible]}}{\text{[illegible]}} \text{[illegible]}_{\epsilon,\ \text{[illegible]}_{\mathcal{D}_{\mathrm{H}}}} \text{[illegible]}\left[\mathrm{MV}, \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}\right] \left[\mathbb{P}\left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}\right] - \mathbb{P}\left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}\right]\right] \text{[illegible]}, \\
\Pi_{\mathrm{H}^{\prime}} &= \frac{\text{[illegible]}}{\text{[illegible]}} \text{[illegible]}_{\epsilon,\ \text{[illegible]}_{\mathcal{D}_{\mathrm{H}}}} \text{[illegible]}\left[\mathrm{MV}, \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}\right] \left[\mathbb{P}\left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}\right] - \mathbb{P}\left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}\right]\right] \text{[illegible]}.
\end{aligned}$$

[illegible]

$$\begin{aligned}
\left|\Pi_{\mathrm{H}}\right| &\text{[illegible]} \frac{\text{[illegible]}}{\text{[illegible]}} \text{[illegible]}_{\epsilon,\ \text{[illegible]}_{\mathcal{D}_{\mathrm{H}}}} \text{[illegible]}\left[\mathrm{MV}, \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}\right] \left|\mathbb{P}\left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}\right] - \mathbb{P}\left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}\right]\right| \text{[illegible]} \quad \text{[illegible]} \\
&\text{[illegible]} \frac{\text{[illegible]}}{\text{[illegible]}} \text{[illegible]}_{\epsilon,\ \text{[illegible]}_{\mathcal{D}_{\mathrm{H}}}} \text{[illegible]}\left[\mathrm{MV}, \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}\right] \text{[illegible]} \quad \text{[illegible]} \\
&\text{[illegible]} \frac{\text{[illegible]}}{\text{[illegible]}} \text{[illegible]}_{\epsilon,\ \text{[illegible]}_{\mathcal{D}_{\mathrm{H}}}} \text{[illegible]}\left[\mathrm{MV}, \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}\right] \text{[illegible]} \quad = \\
&= \text{[illegible]}.
\end{aligned}$$

[illegible] Π_{H} [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] $\left[\mathrm{MV}\right] - \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}$, [illegible] [illegible], [illegible] [illegible]

$$\begin{aligned}
\left|\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}} - \mathrm{MV}\right| &\leq \epsilon^{\mathrm{H}}, \\
\mathbb{P}\left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}^{\mathrm{H}}\right] - \mathrm{OR}^{\mathrm{H}} &= \left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}} \text{[illegible]} \mathrm{MV}\right] \cdot \left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}} - \mathrm{MV}\right] \text{[illegible]} \\
&\text{[illegible]} \left|\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}} \text{[illegible]} \mathrm{MV}\right| \left|\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}} - \mathrm{MV}\right| \text{[illegible]} \\
&\text{[illegible]} \text{[illegible]} \left|\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}} - \mathrm{MV}\right| \leq \\
&\leq \text{[illegible]} \epsilon^{\mathrm{H}}.
\end{aligned}$$

[illegible], [illegible] $\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}$ [illegible] $\mathrm{OOD}_{\mathrm{GSE}}$ [illegible]

$$\begin{aligned}
\left|\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}} - \mathrm{MV}\right| &= \left|\left[\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}} - \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}\right] - \left[\mathrm{MV} - \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}\right]\right| \text{[illegible]} \\
&\text{[illegible]} \left|\mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}} - \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}\right| - \left|\mathrm{MV} - \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}_{\mathrm{H}}}\right| \text{[illegible]} \\
&\text{[illegible]} \epsilon - \epsilon^{\mathrm{H}} \text{[illegible]} \\
&\text{[illegible]} \frac{\text{[illegible]}}{\text{[illegible]}} \epsilon,
\end{aligned}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible]

$$\begin{aligned}
\text{[illegible]}\left[\mathrm{MV}, \mathbb{E}\hat{\mathcal{R}}_{\mathcal{D}}\right] &\leq \frac{\text{[illegible]}\ \epsilon^{\mathrm{H}}}{\left[\frac{\text{[illegible]}}{\text{[illegible]}} \epsilon\right]^{\text{[illegible]}}} = \\
&= \text{[illegible]}\ \epsilon.
\end{aligned}$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible],

$$[illegible] = [illegible].$$

[illegible], [illegible],

$$\frac{\partial [illegible]}{\partial [illegible]} = \frac{\partial [illegible]}{\partial [illegible]} \frac{\partial [illegible]}{\partial [illegible]},$$

$$\frac{\partial^2 [illegible]}{\partial [illegible] \partial [illegible]} = \frac{\partial [illegible]}{\partial [illegible]} \frac{\partial^2 [illegible]}{\partial [illegible] \partial [illegible]} + \frac{\partial^2 [illegible]}{\partial [illegible] \partial [illegible]} \frac{\partial [illegible]}{\partial [illegible]} \frac{\partial [illegible]}{\partial [illegible]},$$

[illegible]

$$\begin{aligned}
\frac{\partial [illegible]}{\partial [illegible]} &= \frac{\partial}{\partial [illegible]} \left[|[illegible]|^{-[illegible]} [illegible] \right] = \\
&= |[illegible]|^{-[illegible]} [illegible] - [illegible] |[illegible]|^{-[illegible]} [illegible] [illegible], \\
\frac{\partial^2 [illegible]}{\partial [illegible] \partial [illegible]} &= -[illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] \left([illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] [illegible] - [illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] - [illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] \right) = \\
&= -[illegible] |[illegible]|^{-[illegible]} [illegible] \left([illegible] |[illegible]|^{-[illegible]} [illegible] - [illegible] [illegible] |[illegible]|^{-[illegible]} [illegible] - [illegible] |[illegible]|^{-[illegible]} [illegible] \right) = \\
&= \left([illegible] - [illegible] [illegible] \right) |[illegible]|^{-[illegible]} [illegible], \\
\frac{\partial [illegible]}{\partial [illegible]} \frac{\partial [illegible]}{\partial [illegible]} &= \left[|[illegible]|^{-[illegible]} [illegible] - [illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] \right] \left[|[illegible]|^{-[illegible]} [illegible] - [illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] \right] = \\
&= |[illegible]|^{-[illegible]} [illegible] [illegible] - [illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] [illegible] - [illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] [illegible] [illegible] \left([illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] [illegible] [illegible] \right) = \\
&= |[illegible]|^{-[illegible]} [illegible] [illegible],
\end{aligned}$$

[illegible]

$$\begin{aligned}
\frac{\partial [illegible]}{\partial [illegible]} &= |[illegible]|^{-[illegible]} [illegible] \frac{\partial [illegible]}{\partial [illegible]} - [illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] \frac{\partial [illegible]}{\partial [illegible]} = \\
&= |[illegible]|^{-[illegible]} \frac{\partial [illegible]}{\partial [illegible]} - [illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] \frac{\partial [illegible]}{\partial [illegible]}, \\
\frac{\partial^2 [illegible]}{\partial [illegible] \partial [illegible]} &= \left([illegible] - [illegible] [illegible] \right) |[illegible]|^{-[illegible]} [illegible] \frac{\partial [illegible]}{\partial [illegible]} + [illegible] |[illegible]|^{-[illegible]} [illegible] [illegible] \frac{\partial^2 [illegible]}{\partial [illegible] \partial [illegible]} = \\
&= \left([illegible] - [illegible] [illegible] \right) |[illegible]|^{-[illegible]} [illegible] \frac{\partial [illegible]}{\partial [illegible]} + [illegible] |[illegible]|^{-[illegible]} \frac{\partial^2 [illegible]}{\partial [illegible] \partial [illegible]} = \\
&= \left([illegible] - [illegible] [illegible] \right) |[illegible]|^{-[illegible]} [illegible] \frac{\partial [illegible]}{\partial [illegible]},
\end{aligned}$$

[illegible]

$$\frac{\partial^2 [illegible]}{\partial [illegible] \partial [illegible]} = [illegible].$$

[illegible]

$$[illegible] = \begin{bmatrix} \frac{\partial [illegible]}{\partial [illegible]} \\ \vdots \\ \frac{\partial [illegible]}{\partial [illegible]} \end{bmatrix},$$

[illegible]

$$
\begin{aligned}
[illegible]_{[illegible]} &= |x|^{-[illegible]}[illegible] - [illegible]|x|^{-[illegible]}[illegible] \cdot [illegible]\,dx, \\
[illegible]_{[illegible]} &= [illegible]\,|x|^{-[illegible]}x \cdot [illegible].
\end{aligned}
$$

[illegible]

$$
\begin{aligned}
[illegible] &= |x|^{[illegible]-[illegible]}, \\
[illegible]_{[illegible]} &= [illegible]\,|x|^{-[illegible]}\,dx, \\
[illegible]_{[illegible]} &= [illegible]
\end{aligned}
$$

[illegible] [illegible]

$$
\begin{aligned}
[illegible] &= [illegible]\,|x|^{-[illegible]-[illegible]}x \cdot [illegible] \\
&= [illegible]\,|x|^{-[illegible]}\left[|x|^{-[illegible]}[illegible] \cdot x - [illegible]|x|^{-[illegible]}[illegible] \cdot x\,|x|^{[illegible]}\right] = \\
&= [illegible]\,|x|^{-[illegible]-[illegible]}x \cdot [illegible] - [illegible]\,|x|^{-[illegible]-[illegible]}x \cdot [illegible] = \\
&= [illegible].
\end{aligned}
$$

■

[illegible] [illegible], [illegible] $x = [illegible]$. [illegible] [illegible]

$$
[illegible]\,|x| = |x|^{[illegible]-[illegible]}\,\left[|x|^{-[illegible]}x\right] \qquad [illegible]
$$

[illegible] [illegible] [illegible] x. [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] $x = [illegible]$, [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] $x = [illegible]$.

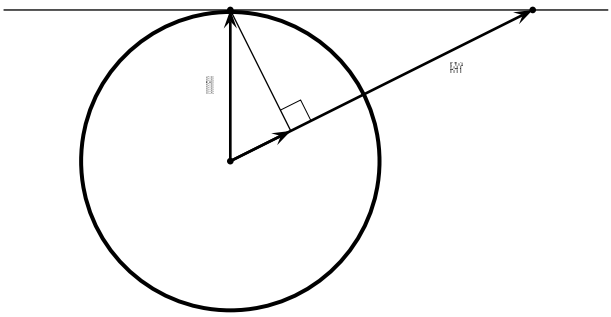
[illegible] [illegible]

$$
= |x|^{-[illegible]}x,
$$

[illegible]

$$
\begin{aligned}
|\ \ | &= \frac{[illegible]}{|x|}, \\
x &= |x|^{[illegible]} = \\
&= |\ \ |^{-[illegible]}, \\
[\ \ ] &= |x|^{-[illegible]\,|||\,[illegible]}\,[illegible] = \\
&= |\ \ |^{[illegible]-[illegible]}\,[illegible]\,|\ \ |^{-[illegible]}.
\end{aligned}
$$

[illegible] [illegible] [illegible] [illegible] [illegible] [



[illegible] [illegible].[illegible]: [illegible]

[illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible]

$$[illegible] = \frac{[illegible]}{|[illegible]|} \quad \left[\!\!\left[\frac{[illegible]}{|[illegible]|^{[illegible]}} [illegible] \right]\!\!\right]$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible]

$$[illegible] = |[illegible]| [illegible] [illegible]$$

[illegible]

$$|[illegible]| \, |[illegible]| = [illegible] [illegible]$$

[illegible] [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

Пусть Ω — область в $\mathbb{R}^3$ с кусочно-гладкой границей $\partial\Omega$, ν — внешняя нормаль к $\partial\Omega$, Δ — оператор Лапласа, $\lambda > 0$ — параметр.

$$
\begin{aligned}
\Delta u &= \lambda u, \\
u\big|_{\partial\Omega} &= 0, \\
|u| &\le \frac{C(\Omega)}{|\lambda|}, \qquad |\lambda| \to \infty .
\end{aligned}
$$

Найти все собственные функции и собственные значения λ, если точечный спектр σ_p есть конечный или бесконечный дискретный спектр σ_d и непрерывный спектр σ_c есть полуось $[0,\infty)$, при $\lambda \in \sigma_c$ — σ_p пустой.

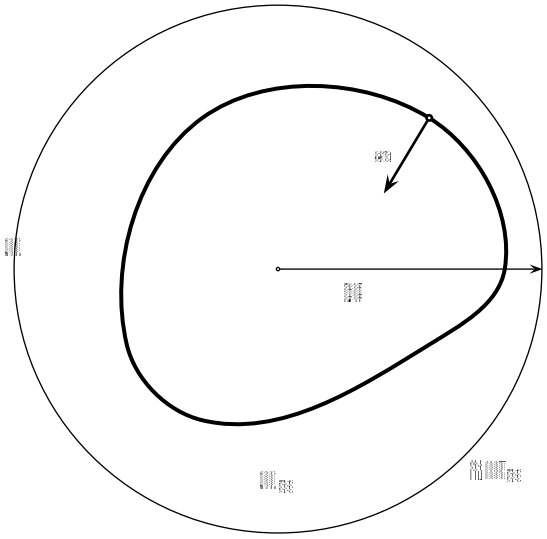


Рис. 17.4. Геометрическая постановка задачи Дирихле для уравнения Гельмгольца

[illegible]

$$\begin{aligned}
\int_{\mathbb{R}^n} [\,\text{[illegible]}\,]^2\,dx &= \int_{\mathbb{R}^n} \left[\,\text{[illegible]}\,\left[\,\text{[illegible]}\,\right] - \,\text{[illegible]}\,\right] dx \\
&= \int_{\mathbb{R}^n} \,\text{[illegible]}\,\left[\,\text{[illegible]}\,\right] dx \\
&= \int_{\mathbb{R}^n} u\,\frac{\partial u}{\partial x_j}\,dx \\
&= \int_{\mathbb{R}^n} u\,\frac{\partial u}{\partial x_j}\,dx \;\text{[illegible]}\; \int_{\mathbb{R}^n} u\,\frac{\partial u}{\partial x_j}\,dx \\
&= \int_{\mathbb{R}^n} u\,\frac{\partial u}{\partial x_j}\,dx \;\text{[illegible]} \\
&= \int_{\mathbb{R}^n} |u|\,\left|\frac{\partial u}{\partial x_j}\right| dx \;\text{[illegible]} \\
&= \frac{\text{[illegible]}}{\text{[illegible]}}\,\frac{\text{[illegible]}}{\text{[illegible]}^2}\int_{\mathbb{R}^n} dx \;\text{[illegible]} \\
&= \frac{\text{[illegible]}^2}{\text{[illegible]}}
\end{aligned}$$

[illegible]

$$\text{[illegible]} \int_{\mathbb{R}^n} [\,\text{[illegible]}\,]^2\,dx = 0.$$

[illegible] $\,\text{[illegible]}\,u = 0$ [illegible]. [illegible] $u|_{\Omega} = 0$, [illegible] $u = 0$. ∎

[illegible] $u = 0$ [illegible]. [illegible], [illegible]

$$u(x) = \int_{\mathbb{R}^n} |x-y|^{-2}\,f(y)\,dy$$

[illegible] $f = 0$, [illegible] $|x| = \infty$ [illegible]. [illegible]

[illegible]. [illegible] [illegible]

[illegible]

$$\frac{d}{dt}\,\frac{\partial^2}{\partial x^2} = \text{[illegible]} \quad \text{[illegible]}\;[0,\infty)\times\mathbb{R}^n$$

[illegible].

[illegible]. [illegible] [illegible]

[illegible], [illegible]

$$\frac{\partial^2}{\partial x_j^2} = \frac{d}{dt}\,\frac{\partial^2}{\partial x^2}.$$

Поэтому, для спинового оператора

$$\hat{S}_x = \frac{\hbar}{2}\sigma_x, \qquad \hat{S}_y = \frac{\hbar}{2}\sigma_y$$

получаем тривиальное соотношение неопределенности

$$\frac{\Delta S_x \Delta S_y}{\hbar^2} = 0.$$

Для осциллятора более содержательно писать

$$[a, a^\dagger] = [\hat{x}, \hat{p}] + i[\hat{p}, \hat{x}] =$$
$$= [\hat{x} - i\hat{p}, \hat{x} + i\hat{p}],$$

Пусть сам $\hat{x}$ и $\hat{p}$ представлены бесконечными.

Для гармонического осциллятора коммутирующие переменные $\hat{x}$ и $\hat{p}$ отсутствуют. Но отсутствуют неприводимые представления. Поэтому, осциллятор имеет полное множество, а $\hat{x}$ и $\hat{p}$. Отсутствуют представления при тождественных преобразованиях $\hat{x}$ и $\hat{p}$ $\hat{x}$ Неприводимы нельзя. Поэтому, для коммутирующих

$$[a, \hat{x}] = \hat{x}[a]$$
$$[a, \hat{p}] = \hat{p}[a].$$

Тогда

$$\hat{x}[a] + i\hat{p}[a] = \hat{x}[a]$$
$$-i\hat{x}^\dagger[a] + i\hat{p}^\dagger[a] = \hat{p}[a].$$

Из бесконечности следует

$$-\hat{x}[a] + i\hat{p}[a] = \frac{i}{\sqrt{2}}\left(\frac{\partial}{\partial x}\right)^{\dagger} \psi[x] \quad \text{или} \quad \hat{a}\psi,$$

Пусть сам a $\hat{x}$ $\hat{p}$ Неприводимый. Можно $\hat{a}$

$$\hat{x}[a] = \frac{i}{\sqrt{2}}\hat{x}[a] - \frac{i}{\sqrt{2}\,a}\left(\frac{\partial}{\partial x}\right)^{\dagger} \psi[x] \quad \text{или} \quad -\hat{x},$$
$$\hat{p}[a] = \frac{i}{\sqrt{2}}\hat{x}[a] + \frac{i}{\sqrt{2}\,a}\left(\frac{\partial}{\partial x}\right)^{\dagger} \psi[x] \quad \text{или} \quad \hat{x},$$

или

$$[a, a^\dagger] = \frac{i}{\sqrt{2}}[\hat{x}[a - a^\dagger] + \hat{x}[a + a^\dagger]] + \frac{i}{\sqrt{2}\,a}\left(\frac{\partial}{\partial x}\right)^{\dagger}_{-\infty}^{+\infty} \psi[x] \quad \text{или}. \qquad (\text{III.}\,\text{[illegible]})$$

Если $\hat{a}$ $\hat{x}$ $\hat{x}$ $\hat{x

[illegible] Lemma [illegible] [illegible] [illegible] [illegible] [illegible]:

$$\operatorname{supp}(\mathbb{P}) \subset [-m, m],$$
$$\operatorname{supp}(\mathbb{Q}) \subset [-m, m].$$

[illegible] $\frac{|x|}{m}$ [illegible]

$$x - m \le 0 \quad \text{[illegible]} \quad x - (|x| + m) \le -m,$$
$$x + m \ge 0 \quad \text{[illegible]} \quad x + (|x| + m) \ge m$$

[illegible]

$$\mathbb{P}(x + m \le 0) = 0,$$
$$\mathbb{P}(x - m \ge 0) = 0,$$
$$\mathbb{E}[x, A] = \frac{1}{2m} \int_{-m}^{m} \mathbb{E}[x \mid \mathcal{F}_t]\, dt = \text{[illegible]}.$$

[illegible] [illegible] [illegible] [illegible].

[illegible] [illegible]

[illegible]

$$\text{[illegible]} = \frac{1}{m^2} \frac{\sigma^2}{\mathbb{E}[\tau]^2}, \quad \tau \ge 0, \tag{[illegible]}$$
$$\mathbb{E}[\tau, A] = \mathbb{P}(A), \tag{[illegible]}$$
$$\mathbb{E}[\tau, A] = \mathbb{E}[\tau]. \tag{[illegible]}$$

[illegible] [illegible] τ_1, [illegible] [illegible] $\mathcal{F}_n$, [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] $\mathbb{E}[\tau, A]$ [illegible] [illegible] $\mathcal{F}_n$, [illegible] [illegible]:

$$\mathbb{E}[\tau, A] = \frac{1}{\mathbb{E}[\tau]^2} \int_{\mathcal{F}_n} \text{[illegible]} = \frac{1}{\mathbb{E}[\tau]} \int_{\mathcal{F}_n} \text{[illegible]}, \tag{[illegible]}$$

[illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] σ^2 [illegible]. [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible].

$$\mathbb{E}[\tau_1, A] = \mathbb{E}[\tau_2, A]. \tag{[illegible]}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] $\mathcal{F}_n$:

$$\frac{1}{m^2} \frac{\sigma^2}{\mathbb{E}[\tau]^2} \int_{\mathcal{F}_n} \text{[illegible]} = \int_{\mathcal{F}_n} \text{[illegible]} = \int_{\mathcal{F}_n} \frac{\text{[illegible]}}{\text{[illegible]}}.$$

тогда

$$\frac{d}{dt}\frac{\partial L^{t}}{\partial \dot{x}^{t}}\left(\dot{\gamma}^{t}\right)\dot{\gamma}^{t} = \left(\frac{\partial L^{t}}{\partial x^{t}}\left(\dot{\gamma}^{t}\right)\right)_{x=\gamma_{t}} \qquad \text{[illegible]} \qquad = \frac{\partial}{\partial x}\left(\frac{\partial L}{\partial \dot{x}}\left(\dot{\gamma}^{t}\right)\right)_{x=\gamma_{t}} \qquad \text{[illegible]},$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], [illegible]

$$\frac{d}{dt}\frac{\partial L^{t}}{\partial \dot{x}^{t}}\left(\dot{\gamma}^{t}\right)\dot{\gamma}^{t} = \frac{\partial L}{\partial x}\left(\gamma_{t}, t\right)\dot{\gamma}^{t} \qquad \text{[illegible]} \qquad = \frac{\partial}{\partial x}L\left(\gamma_{t}, t\right).$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\frac{d}{dt}\frac{\partial L^{t}}{\partial \dot{x}^{t}}\left(\dot{\gamma}^{t}\right) \qquad = \qquad \frac{\partial L}{\partial x}\left(\gamma_{t}, \frac{\partial L}{\partial \dot{x}}\right)$$

тогда

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}}\left(\gamma_{t}, \frac{\partial L}{\partial \dot{x}}\right) \qquad = \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{x}}\left(\gamma_{t}, \dot{\gamma}_{t}\right).$$

[illegible] [illegible]

$$\dot{\gamma}\left(\gamma, t\right) \qquad = \qquad \dot{\gamma}\left(\gamma, t\right),$$

тогда

$$\dot{\gamma}\left(\gamma, t\right) \qquad = \qquad \frac{\dot{\gamma}\left(\gamma, t\right)}{\gamma},$$

[illegible] [illegible]

$$\begin{aligned}
\gamma\frac{\partial \dot{\gamma}}{\partial \gamma} \qquad &= \qquad \gamma\frac{\partial \dot{\gamma}}{\partial \gamma} - \dot{\gamma}, \\
\frac{\partial}{\partial \gamma}\left(\gamma\frac{\partial \dot{\gamma}}{\partial \gamma}\right) \qquad &= \qquad \frac{\partial}{\partial \gamma}\left(\gamma\frac{\partial \dot{\gamma}}{\partial \gamma} - \dot{\gamma}\right) \qquad = \\
&= \qquad \gamma\frac{\partial^{2}\dot{\gamma}}{\partial \gamma^{2}} \qquad \text{[illegible]} \qquad \frac{\partial \dot{\gamma}}{\partial \gamma} - \frac{\partial \dot{\gamma}}{\partial \gamma}
\end{aligned}$$

[illegible]

$$\frac{\partial^{2}\dot{\gamma}}{\partial \gamma^{2}} \qquad = \qquad \frac{d}{dt}\frac{\partial^{2}\dot{\gamma}}{\partial \dot{\gamma}^{2}}.$$

[illegible], [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

$$\begin{aligned}
\dot{\gamma}\left(\gamma, t\right) \qquad &= \qquad \dot{\gamma}\left(\gamma, t\right) \qquad = \\
&= \qquad \dot{\gamma}\left(\gamma - \alpha t\right) \qquad \text{[illegible]} \qquad \dot{\gamma}\left(\gamma \qquad \text{[illegible]} \qquad \alpha t\right).
\end{aligned}$$

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible]

$$[illegible] = [illegible] - [illegible], \tag{illegible}$$

$$[illegible] = \frac{[illegible]}{[illegible]} \left[[illegible] - [illegible] \right]. \tag{illegible}$$

[illegible] [illegible] [illegible] [

Для того чтобы выборочное среднее $\bar{x}$ мало отличалось от μ, $|\bar{x} - \mu| < \varepsilon$, необходимо выполнение неравенства

$$P(|\bar{x} - \mu| < \varepsilon) = \frac{2}{\sqrt{2\pi}} \int_0^{\frac{\varepsilon\sqrt{n}}{\sigma}} e^{-\frac{t^2}{2}}\,dt \geq 1 - \frac{\sigma^2}{n\varepsilon^2}. \qquad (9.10.10)$$

Математическая статистика имеет дело с случайными[1] выборками из генеральной совокупности, которая также является случайной переменной, для нее мы ввели распределение вероятностей этой величины.

Пусть имеется случайная переменная X. Рассмотрим n независимых наблюдений, выборочное среднее есть $\bar{x}$, тогда

$$m = \lim_{n \to \infty} P(|\bar{x} - \mu| < \varepsilon),$$

$$D = \lim_{n \to \infty} P(|\bar{x} - \mu| < \varepsilon).$$

Если $A = \frac{\sigma}{\sqrt{n}}$ обе P в неравенстве (9.10.10) равны A, тогда бесконечно большое. Если $A = \frac{\sigma}{\sqrt{n}}$ то распределение выборочных средних — нормальное распределение. Если $A = \frac{2\sigma}{\sqrt{n}}$ обе P в неравенстве равны A, бесконечно большое, нормальное распределение.

11.12.1. Распределение выборочных средних

Если генеральная совокупность имеет нормальное распределение, то распределение выборочных, независимо, для его бесконечно большого P, в том числе выборочных средних, подчиняется нормальному распределению, или же бесконечно. Пусть, рассмотрим 9.9.

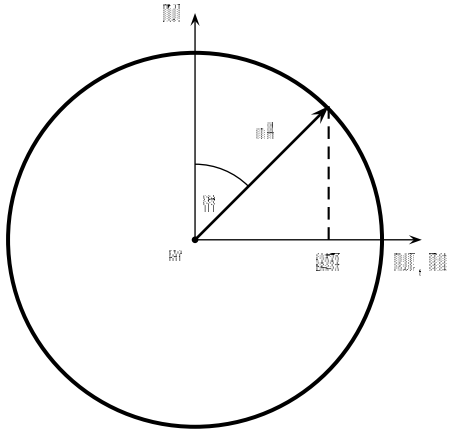


Рисунок 9.9: Распределение при обычном

[1] Случайная величина имеет нормальное распределение.

$$\begin{aligned} \mathrm{[illegible]} &= \frac{\mathrm{[illegible]}}{\mathrm{[illegible]}}, \\ \mathrm{[illegible]} &= \sqrt{\mathrm{[illegible]}^2 - |\mathrm{[illegible]} - \mathrm{[illegible]}|^2}, \\ \mathrm{[illegible]} &= \mathrm{[illegible]}_{\mathrm{[illegible]}} = \left\{ (\mathrm{[illegible]}, \mathrm{[illegible]}) : |\mathrm{[illegible]} - \mathrm{[illegible]}|^2 \leq |\mathrm{[illegible]} - \mathrm{[illegible]}|^2 \leq \mathrm{[illegible]}^2 \right\} \end{aligned} \tag{illegible}$$

[illegible] [illegible] [illegible]

$$\mathrm{[illegible]} = \frac{\mathrm{[illegible]}}{\mathrm{[illegible]}^2}$$

[illegible] [illegible]

$$\mathrm{[illegible]} = \frac{1}{\mathrm{[illegible]}} \left[\frac{\mathrm{[illegible]}}{\mathrm{[illegible]}} \int_{\mathrm{[illegible]}} \mathrm{[illegible]} \frac{\mathrm{[illegible]}}{\mathrm{[illegible]}} \, \mathrm{[illegible]} - \int_{\mathrm{[illegible]}} \mathrm{[illegible]} \frac{\mathrm{[illegible]}}{\mathrm{[illegible]}} \right].$$

[illegible] [illegible], [illegible] [illegible]

$$\begin{aligned} \frac{\mathrm{[illegible]}}{\mathrm{[illegible]}} &= \frac{\mathrm{[illegible]}}{\mathrm{[illegible]}} = \\ &= \frac{\mathrm{[illegible]}}{\sqrt{\mathrm{[illegible]}^2 - |\mathrm{[illegible]} - \mathrm{[illegible]}|^2 - |\mathrm{[illegible]} - \mathrm{[illegible]}|^2}} \end{aligned}$$

[illegible] [illegible] [illegible] [

[illegible]

$$\frac{1}{[illegible]}\int_{-\infty}^{\infty}\frac{[illegible]}{\sqrt{[illegible]^2-[illegible]^2-[illegible]^2}}\,dx$$

$$= \frac{1}{[illegible]}\int_{-[illegible]}^{[illegible]}[illegible]\,\frac{[illegible]}{[illegible]}\,dx$$

$$= \frac{1}{[illegible]}\int_{-[illegible]}^{[illegible]}[illegible].$$

[illegible]

$$[illegible] = \frac{1}{[illegible]}\left[\frac{[illegible]}{[illegible]}\int_{-[illegible]}^{[illegible]}[illegible]\,dx - [illegible]\int_{-[illegible]}^{[illegible]}[illegible]\,dx\right] =$$

$$= \frac{1}{[illegible]}\left[[illegible] - [illegible]\right] - [illegible]\,\frac{1}{[illegible]}\int_{-[illegible]}^{[illegible]}[illegible]\,dx,$$

[illegible]

[illegible] [illegible]

[illegible] [illegible] [illegible]

$$[illegible] = [illegible],$$

$$\frac{[illegible]}{[illegible]} = \frac{1}{[illegible]}\frac{[illegible]}{[illegible]}, \qquad [illegible] \tag{[illegible]}$$

[illegible]

$$[illegible] = [illegible], \qquad [illegible] \tag{[illegible]}$$

[illegible]

$$\begin{aligned}
[illegible] &= [illegible][illegible][illegible],\\
\frac{[illegible]}{[illegible]} &= \frac{[illegible]}{[illegible]}\\
&= [illegible]\\
&= [illegible].
\end{aligned}$$

[illegible]

$$\begin{aligned}
[illegible] &= [illegible],\\
[illegible] &= [illegible].
\end{aligned}$$

[illegible]

$$[illegible] = [illegible]\,[illegible]^{[illegible]},$$

[illegible]

$$[illegible] = [illegible]\,[illegible]^{[illegible]}\,[illegible]^{[illegible]}, \qquad [illegible].$$

[illegible]

[illegible] [illegible] [illegible] [illegible], [illegible]

$$[illegible] = [illegible] \int_{-\infty}^{\infty} [illegible] \, e^{-[illegible]} \, [illegible]$$

[illegible]. [illegible] [illegible] [illegible], [illegible]

$$[illegible] = [illegible] = [illegible] \int_{-\infty}^{\infty} [illegible] \, e^{-[illegible]} \, [illegible].$$

[illegible] [illegible] [illegible], [illegible]

$$[illegible] = [illegible] \int_{-\infty}^{\infty} [illegible] \, e^{-[illegible]} \, [illegible].$$

[illegible] [illegible] [illegible]

$$[illegible] = [illegible] \int_{-\infty}^{\infty} e^{-[illegible]} \, [illegible] \left[[illegible] \int_{-\infty}^{\infty} [illegible] \, e^{-[illegible]} \, [illegible] \right] [illegible] =$$

$$= [illegible] \int_{-\infty}^{\infty} [illegible] \left[[illegible] \int_{-\infty}^{\infty} e^{-[illegible]} \, e^{[illegible]} \, [illegible] \right] [illegible]$$

[illegible]

$$[illegible] = [illegible] \int_{-\infty}^{\infty} [illegible] \, [illegible] \, [illegible], \qquad [illegible]$$

[illegible] [illegible]

$$[illegible] = [illegible] \int_{-\infty}^{\infty} e^{-[illegible]} \, [illegible] \, [illegible] =$$

$$= [illegible] \int_{-\infty}^{\infty} e^{-[illegible]} \, [illegible].$$

[illegible] [illegible] [illegible]. [illegible]

$$[illegible] = [illegible],$$

$$[illegible] = \frac{[illegible]}{[illegible]},$$

$$[illegible] = \frac{[illegible]}{[illegible]}$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} =$$

$$= [illegible],$$

$$[illegible] = \frac{[illegible]}{[illegible]},$$

[illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \int [illegible] \, , \qquad [illegible]$$

$$\Pi = [illegible] \, e^{-[illegible]} \, [illegible] \, . \qquad [illegible]$$

[illegible], [illegible]

$$\Pi = \frac{[illegible]}{[illegible]} \, e^{\frac{-[illegible]}{[illegible]}} \, . \qquad [illegible]$$

[illegible], [illegible]

$$\Pi^{*} = -[illegible] \, e^{-[illegible]} \, [illegible] = [illegible]$$

$$= \frac{[illegible]}{[illegible]} \, e^{-[illegible]} \, [illegible] \Big|_{[illegible]} - \frac{[illegible]}{[illegible]} [illegible] \, e^{-[illegible]} \, [illegible] =$$

$$= -\frac{[illegible]}{[illegible]} \Pi = [illegible] \, ,$$

[illegible]

$$\Pi = \Pi \, e^{\frac{-[illegible]}{[illegible]}} \, .$$

[illegible],

$$\Pi = [illegible] \, e^{-[illegible]} \, [illegible] =$$

$$= \frac{[illegible]}{[illegible]} [illegible] \, ,$$

[illegible] [illegible] [illegible].

[illegible] [illegible], [illegible] [illegible] [illegible] [illegible]

$$[illegible] = \frac{[illegible]}{[illegible]} \, e^{-\frac{[illegible]}{[illegible]}} \qquad [illegible]$$

[illegible] [illegible] [illegible] [

Remarkably, the [illegible], [illegible] [illegible] [illegible]

y
d
c
$c+d=\left[\frac{\mathrm{cm}}{\mathrm{s}}\right]$
区域 $\left[\mathrm{m},\mathrm{s}\right]$ 一致 $\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$
$a+b=\left[\frac{\mathrm{cm}}{\mathrm{s}}\right]$
0
a
$b+a=\left[\frac{\mathrm{cm}}{\mathrm{s}}\right]$
b
x

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