ops3 using green's theorem evaluate

$\oint_C (x^2y)\,dx + (xy^2)\,dy$, clockwise boundary of the region of the region $0 \le y \le \sqrt{9-x^2}$

$$\iint_R \frac{df_2}{dx} - \frac{df_1}{dy}\, dA =$$

$$= \iint_R 2xy - xy\, dA$$

We convert to polar.

$$x^2 + y^2 = r^2$$

$$0 \le y \le \sqrt{9-x^2} \Rightarrow x^2 + y^2 = 9 \Rightarrow r = 3$$

We then have our boundaries.

$0 \le r \le 3$ and $0 \le \theta \le \pi$

because clockwise

$$\iint_A 2xy - x^2\, dA$$

$$\downarrow$$

$$\int_0^{\pi}\int_0^{3} 2(r\cdot\cos\theta\cdot r\sin\theta) - r^2\cos^2\theta\, dr\, d\theta =$$

$$= \int_0^{\pi}\left[r^2\cos\theta\cdot r^2\sin\theta - \frac{r^3}{3}\cos^2\theta\right]_0^3 d\theta$$

$$= \int_0^{\pi}\left(9^2\cos\theta\cdot\sin\theta - \frac{3^3}{3}\cos^2\theta\right) d\theta$$

$$= \int_0^{\pi} 81\cdot\cos\theta\cdot\sin\theta - \cos^2\theta\, d\theta$$

$$= \int_0^{\pi} \frac{81}{2}\cdot 2\cos\theta\cdot\sin\theta - \cos^2\theta\, d\theta$$

$$= \left[\frac{81}{2}\sin(2\theta) - \frac{1}{2}(\theta + \sin\theta\cos\theta)\right]_0^{\pi}$$

$$= \left(\frac{81}{2}\cdot 0 - \frac{1}{2}\cdot(\pi + 0\cdot 1)\right) - \left(\frac{81}{2}\cdot 0 - \frac{1}{2}(0 + 0\cdot 1)\right)$$

$$= -\frac{\pi}{2}$$