opg 2 find the fourier transform of

$$H(x)\quad \left(f(x) = \begin{cases} 1-x^2 & |x|<1 \\ 0 & |x|>1 \end{cases} \right.$$

$$F(\omega)=\int_{-\infty}^{\infty} f(x)\cdot e^{-j\omega x}\,dx$$

the function is nonzero from -1 to 1

$$F(\omega)=\int_{-1}^{1} f(x)\cdot e^{-j\omega x}\,dx$$

$$=\int_{-1}^{1}(1-x^2)\cdot e^{-j\omega x}\,dx \quad \text{partiel integrasion}$$

$$=(1-x^2)\cdot\frac{-1}{j\omega}e^{-j\omega x}-\int\frac{-1}{j\omega}e^{-j\omega x}\cdot(-2x)\,dx$$

$$=(1-x^2)\cdot\frac{-1}{j\omega}e^{-j\omega x}-\frac{1}{j\omega}\int e^{-j\omega x}\cdot 2x\,dx$$

$$=(1-x^2)\cdot\frac{-1}{j\omega}e^{-j\omega x}-\frac{1}{j\omega}\cdot\left(-\frac{1}{j\omega}e^{-j\omega x}\cdot 2x-\int-\frac{1}{j\omega}e^{-j\omega x}\cdot 2\,dx\right)$$

$$=\underbrace{(1-x^2)\cdot\frac{-1}{j\omega}e^{-j\omega x}}_{=0}+\frac{2x}{j^2\omega^2}e^{-j\omega x}+\frac{2}{j^3\omega^3}e^{-j\omega x}$$

$$=\frac{2}{j^2\omega^2}e^{-j\omega}+\frac{2}{j^3\omega^3}e^{-j\omega}-\left(\frac{2}{j^2\omega^2}e^{j\omega}+\frac{2}{j^3\omega^3}e^{j\omega}\right)$$

$$=-\frac{2}{\omega^2}e^{-j\omega}-\frac{2}{j\omega^3}e^{-j\omega}-\frac{2}{\omega^2}e^{j\omega}+\frac{2}{j\omega^3}e^{j\omega}$$

$$=-\frac{2}{\omega^2}\cdot\left(e^{j\omega}+e^{-j\omega}\right)-\frac{2}{j\omega^3}\cdot\left(e^{-j\omega}-e^{j\omega}\right)$$

$$=-\frac{4}{\omega^2}\cdot\left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right)-\frac{4}{\omega^3}\cdot\left(\frac{e^{-j\omega}-e^{j\omega}}{2j}\right)$$

$$=-\frac{4}{\omega^2}\cdot\cos(\omega)-\frac{4}{\omega^3}\cdot\sin(\omega)$$

$$=-\frac{4\omega}{\omega^3}\cdot\cos(\omega)-\frac{4}{\omega^3}\cdot\sin(\omega)$$

$$f(\omega)=-4\cdot\frac{\omega\cdot\cos(\omega)-\sin(\omega)}{\omega^3}$$

We use the inverse Laplace to evalnate the original functions.

$$f(x)=L^{-1}\{f(\omega)\}=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega x}\,d\omega=1-x^2$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}-4\,\frac{\omega\cos(\omega)-\sin(\omega)}{\omega^3}e^{j\omega x}\,d\omega$$

$$=-\frac{2}{\pi}\cdot\int_{-\infty}^{\infty}\frac{\omega\cdot\cos(\omega)-\sin(\omega)}{\omega^3}\cdot e^{j\omega x}\,d\omega$$

$$e^{j\omega x}=\cos(\omega x)+j\sin(\omega x)$$

but the function $1-x^2$ has no imaginary part

therefore $e^{j\omega x}=\cos(\omega x)$

$$=\frac{2}{\pi}\cdot\int_{0}^{\infty}\frac{\omega\cos(\omega)-\sin(\omega)}{\omega^3}\cdot\cos(\omega x)\,dx$$

$$f(x)=-\frac{4}{\pi}\cdot\int_{0}^{\infty}\frac{\omega\cos(\omega)-\sin(\omega)}{\omega^3}\cdot\cos(\omega x)\,dx=1-x^2$$

$$f\left(\frac{1}{2}\right)=-\frac{4}{\pi}\cdot\int_{0}^{\infty}\frac{\omega\cdot\cos(\omega)-\sin(\omega)}{\omega^3}\cdot\cos\left(\omega\cdot\frac{1}{2}\right)dx=1-\left(\frac{1}{2}\right)^2$$

$$f\left(\frac{1}{2}\right)=\int_{0}^{\infty}\frac{\omega\cdot\cos(\omega)-\sin(\omega)}{\omega^3}\cdot\cos\left(\frac{\omega}{2}\right)dx=\frac{1-\frac{1}{4}}{-\frac{4}{\pi}}$$

$$f\left(\frac{1}{2}\right)=\int_{0}^{\infty}\frac{\omega\cdot\cos(\omega)-\sin(\omega)}{\omega^3}\cdot\cos\left(\frac{\omega}{2}\right)dx=\underline{\underline{-\frac{3\pi}{16}}}$$