opg 6

find volume of solid bounded by

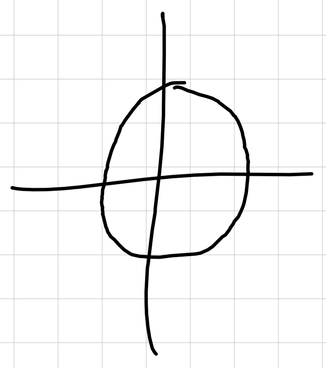
$x^2+y^2+z^2=9$ and $x^2+y^2=8z$

surface   cylinder

find where they meet

$z=0$

$x^2+y^2=9 \longrightarrow$



vi sætter den lig med vores cylinder

$8z+z^2=9$

$z=1 \wedge -9$

use only positive root

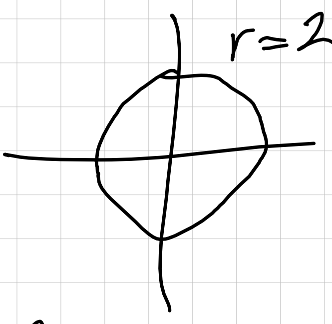
insert into surface $x^2+y^2+z^2=9$

$x^2+y^2+1=9$

$x^2+y^2=8$

$x^2+y^2=2\sqrt{2} \longrightarrow$

$r=2\sqrt{2}$



Lower or higher function

$z=0$

$x^2+y^2+1^2=9=x^2+y^2=8$

$x^2+y^2=8\cdot 0=x^2+y^2=0$ Larest

$z=\frac{x^2+y^2}{8}$

$=\sqrt{9-(x^2+y^2)}$

Konverterer til cylindriske koordinater med $x^2+y^2=r^2$

$x^2+y^2+z^2=9$

$r^2+z^2=9$

$z=\sqrt{9-r^2}$

$x^2+y^2=8z$

$r^2=8z$

$z=\frac{r^2}{8}$ vi ved så at $\frac{r^2}{8}\leq z\leq\sqrt{9-r^2}$

from earlier when we found the lower function by inserting the roots

We can now integrat r with the limits.

$$\int_0^{2\pi}\int_0^{2\sqrt{2}}\int_{\frac{r^2}{8}}^{\sqrt{9-r^2}} r\,dz\,dr\,d\theta$$

$0\leq r\leq 2\sqrt{2}$

$0\leq\theta\leq 2\pi$

$\frac{r^2}{8}\leq z\leq\sqrt{9-r^2}$

$$\int_0^{2\pi}\int_0^{2\sqrt{2}} r\int_{\frac{r^2}{8}}^{\sqrt{9-r^2}} 1\,dz\,dr\,d\theta=$$

$$=\underline{\underline{\frac{90\pi}{3}}}$$