Control of Autonomous Systems – Autumn 2025

Jerome Jouffroy

Assignment

Preliminary instructions for coding and writing your assignment: The answer to each question should be in a separate file or two if needed (`.pdf` for (pen and paper), `.slx` for (Simulink), `.m` for (Matlab). The name of the file should be `exnbexamnumber_Qi.ext` (example: `exnb007_Q2.pdf`). All your files should be gathered in a single zip file (no “rar” file!) under the name `exnbexamnumber.zip` (example: `exnb007.zip`). Regarding programming, subsystems should be used, scopes should have proper labels such as `x(t)`, for example, and not have generic and uninformative name like `Scope6`, inputs and outputs of block should have proper labels as well, etc. All Matlab and Simulink files should be compatible with (or anterior to) version R2024b. If any data file is necessary to run the programs, include it in your zip file. If any associated Matlab file is necessary to the initialization of a Simulink file `exnbexamnumber_Qi.slx`, name it `exnbexamnumber_Qi_par.m`. *Any lack of compliance with the above instructions will mean that your answers will NOT be considered*.

Problem 1

Consider the so-called predator-prey system modelled by the following set of differential equations.

$$\begin{cases} \dot{p}_1(t) = p_1(t)p_2(t) - p_1(t) \\ \dot{p}_2(t) = -\dfrac{4}{3}p_1(t)p_2(t) + \dfrac{2}{3}p_2(t) + u(t) \end{cases} \tag{1}$$

where $p_1(t)$ and $p_2(t)$ represent the (positive) mass of preys and predators, respectively. Control input $u(t)$ represents an external food supply. We are interested in monitoring the mass/quantity of preys, ie we set the output to $y = p_1$.

1 (pen and paper) Characterize (linear/nonlinear, order, control input vector dimension) and give a state-space representation of system (1).

2 (pen and paper) Make a block diagram of system (1) (a block diagram means *not* a Simulink diagram).

3 (Simulink) In Simulink, implement the block diagram you obtained in question 2, with initial conditions $p_1(0) = p_2(0) = 1$, $u = 0$, sampling period $T_s = 0.01$ and a simulation time of 100.

4 (Simulink, pen and paper) Use the Euler method to implement a discrete-time version of (1) in your previous Simulink file, in parallel with the continuous-time one. For this example, is the Euler method a good approximation of the continuous-time system? Why/why not?

5 (pen and paper) What are the equilibrium points of the state-space representation of system (1) when $u = 0$?

6 (pen and paper) Compute the linear approximation of (1) around the set of equilibrium points you obtained in question 5.

7 (Matlab, Simulink) From the set of equilibrium points you found in question 5, choose one that is not at the origin. Check 'experimentally' that it works on your continuous-time implementation and give the corresponding linear approximation. Is it stable? (prove your statement).

8 (Simulink, Matlab) Implement a state-feedback controller and use it to stabilize your system around the equilibrium point studied in question 7.

9 (Simulink, Matlab, pen and paper) We now wish to stabilize our predator-prey system around the values $p_1 = 5$ and $p_2 = 1$. After prooving that these values are an equilibrium point (what is the corresponding u^*?), study the corresponding linear approximation and modify your feedback controller to stabilize your system around this new equilibrium point. Following your analysis, what is the simplest feedback controller that can be proposed?

10 (Simulink, Matlab) To your input, add a disturbance term $d = -2$ to represent a part of your external food supply being stolen. Assuming you do not know the amplitude of this disturbance, modify your state-feedback controller with an integral term to

compensate for the disturbance. Show that it works by changing the disturbance from $d = -2$ to $d = -3$.