

# Fysik – lektion 1

Første lektions emner:

Kap. 2: Bevægelse langs en linje

Kap. 4: Bevægelse i 2 og 3 dimensioner.

- ✿ Videnskabelig metode
- ✿ SI enhedssystemet m.m.
- ✿ Middelhastighed og øjeblikshastighed
- ✿ Bevægelse med konstant acceleration – det frie fald
- ✿ Projektilbevægelsen
- ✿ Jævn cirkulær bevægelse
- ✿ Relative bevægelser
- ✿ Introduktion til lektion 2 (næste gang)
- ✿ Opgaveregning

# Fysik – lektion 1

## Robotingeniør og fysik ?

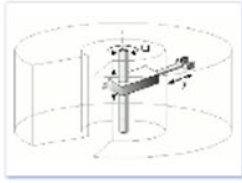


**STATIONARY  
ROBOTS**

**Cartesian Robots**



**Cylindrical**



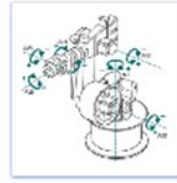
**Spherical**



**SCARA**



**Articulated**



**Parallel**

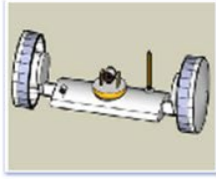


**WHEELED  
ROBOTS**

**Single Wheel**



**2 Wheeled**



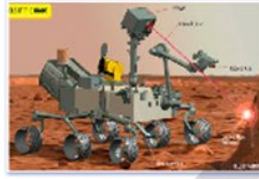
**3 Wheeled**



**4 Wheeled**



**6 Wheeled**



**Tracked Robots**



**LEGGED  
ROBOTS**

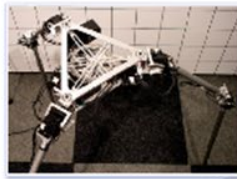
**One Leg**



**Bipedal**



**Tripedal**



**Quadrupedal**



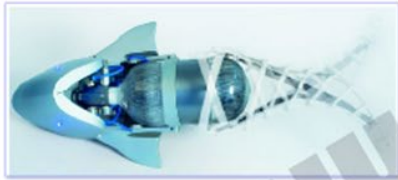
**Hexapod**



**Many Legs**



**SWIMMING ROBOTS**



**FLYING ROBOTS**



**Robotic Balls**



**SWARM ROBOTS**



**MODULAR ROBOTS**



**MICRO Robots**



**NANO Robots**



**SOFT ROBOTS**



**SNAKE Robots**



**CRAWLER Robots**



**HYBRID Robots**



## Robotic lab assistant is 1,000 times faster at conducting research

*Working 22 hours a day, seven days a week, in the dark*



*Uni. Liverpool*

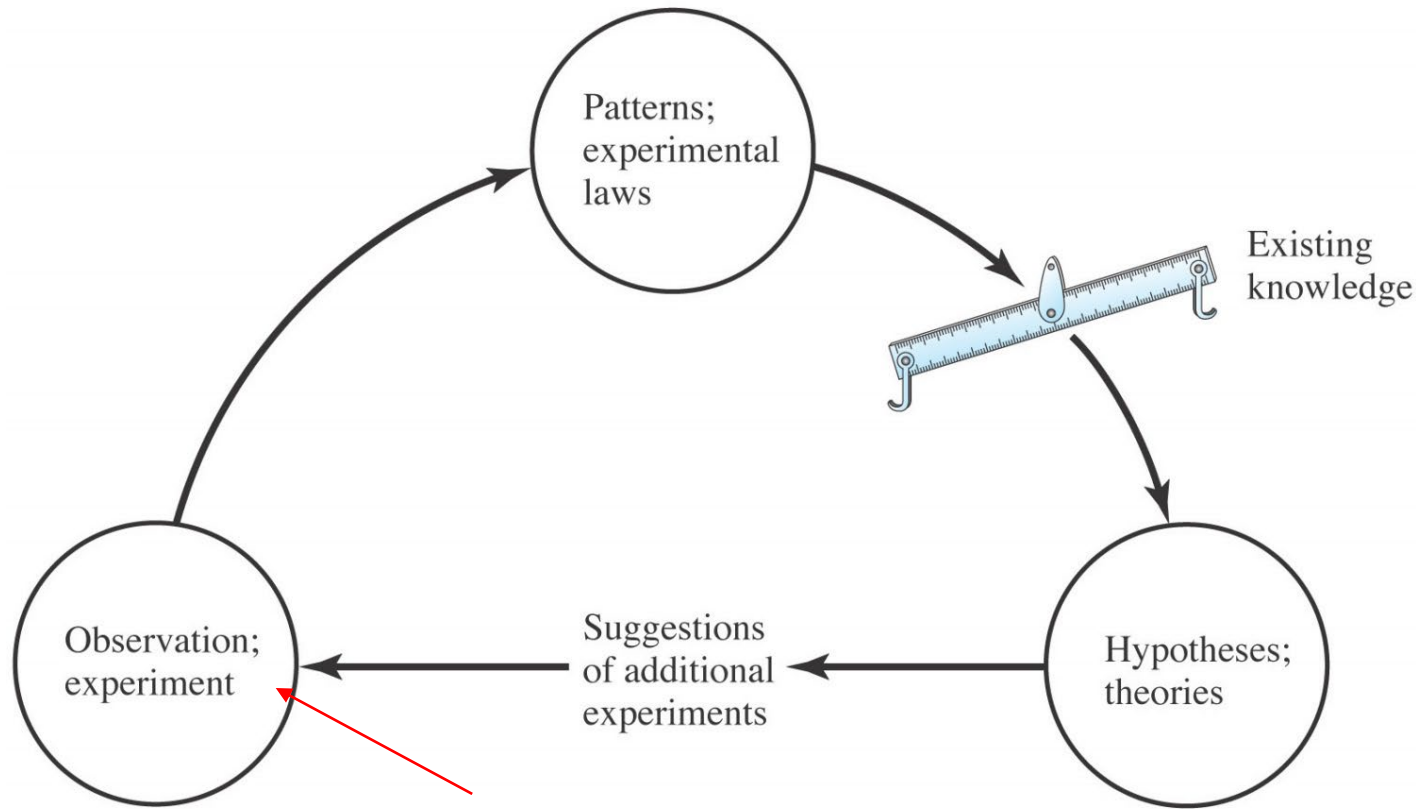
Robotic lab assistant, able to move around a laboratory and conduct scientific experiments just like a human

<https://www.theverge.com/21317052/mobile-autonomous-robot-lab-assistant-research-speed>

<https://youtu.be/dRT3tepdMyl>



# Videnskabelig metode i naturvidenskab



**Kræver målesystemer**

Galileo Galilei:

*Mål hvad der kan måles, og gør det måleligt, som endnu ikke er det.*

# SI-enhedssystemet

International standardisering af måleenheder under kontrol af:

[\*Système International d'Unités\*](#)

SI-enhedssystemet benyttes i det meste af verdenen undtagen USA.

7 grundlæggende SI-enheder for:

<i>Længde</i>	<i>[meter]</i>
<i>Masse</i>	<i>[kilogram]</i>
<i>Tid</i>	<i>[sekunder]</i>
<i>Elektrisk strøm</i>	<i>[Ampere]</i>
<i>Temperatur</i>	<i>[Kelvin]</i>
<i>Stofmængde</i>	<i>[mol]</i>
<i>Lysstyrke</i>	<i>[candela]</i>



Plus mange andre afledte af disse syv SI-enheder!

# SI-enhedssystemet

Definitionerne af enhederne i det nyligt vedtagende SI-enhedssystem er udtrykt ved

## 7 definerende konstanter

Konstanternes værdi er konsistent med de bedste eksperimentelle værdier til rådighed ved vedtagelsen af de nye definitioner

<i>Defining constant</i>	<i>Symbol</i>	<i>Numerical value</i>	<i>Unit</i>
hyperfine transition frequency of caesium	$\Delta\nu_{\text{Cs}}$	9 192 631 770	Hz
speed of light in vacuum	$c$	299 792 458	$\text{m s}^{-1}$
Planck constant	$h$	$6.626\,070\,15 \times 10^{-34}$	J s
elementary charge	$e$	$1.602\,176\,634 \times 10^{-19}$	C
Boltzmann constant	$k$	$1.380\,649 \times 10^{-23}$	$\text{J K}^{-1}$
Avogadro constant	$N_{\text{A}}$	$6.022\,140\,76 \times 10^{23}$	$\text{mol}^{-1}$
luminous efficacy	$K_{\text{cd}}$	683	$\text{lm W}^{-1}$

# SI-enhedsystemet

time	The <b>second</b> , symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{\text{Cs}}$ , the unperturbed ground-state hyperfine transition frequency of the caesium 133 atom, to be 9192 631 770 when expressed in the unit Hz, which is equal to $\text{s}^{-1}$ .
length	The <b>metre</b> , symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum $c$ to be 299 792 458 when expressed in the unit $\text{m s}^{-1}$ , where the second is defined in terms of the caesium frequency $\Delta\nu_{\text{Cs}}$ .
mass	The <b>kilogram</b> , symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant $h$ to be $6.626\,070\,15 \times 10^{-34}$ when expressed in the unit J s, which is equal to $\text{kg m}^2 \text{s}^{-1}$ , where the metre and the second are defined in terms of $c$ and $\Delta\nu_{\text{Cs}}$ .



# SI-enhedsystemet

electric current	The <b>ampere</b> , symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge $e$ to be $1.602\,176\,634 \times 10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{\text{CS}}$ .
thermodynamic temperature	The <b>kelvin</b> , symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant $k$ to be $1.380\,649 \times 10^{-23}$ when expressed in the unit J K <sup>-1</sup> , which is equal to kg m <sup>2</sup> s <sup>-2</sup> K <sup>-1</sup> , where the kilogram, metre and second are defined in terms of $h$ , $c$ and $\Delta\nu_{\text{CS}}$ .
amount of substance	The <b>mole</b> , symbol mol, is the SI unit of amount of substance. One mole contains exactly $6.022\,140\,76 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, $N_{\text{A}}$ , when expressed in the unit mol <sup>-1</sup> and is called the Avogadro number. The amount of substance, symbol $n$ , of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.
luminous intensity	The <b>candela</b> , symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency $540 \times 10^{12}$ Hz, $K_{\text{cd}}$ , to be 683 when expressed in the unit lm W <sup>-1</sup> , which is equal to cd sr W <sup>-1</sup> , or cd sr kg <sup>-1</sup> m <sup>-2</sup> s <sup>3</sup> , where the kilogram, metre and second are defined in terms of $h$ , $c$ and $\Delta\nu_{\text{CS}}$ .

# SI-enhedssystemet

Derived quantity	Name of derived unit	Symbol for unit	Expression in terms of other units
plane angle	radian	rad	$\text{m/m} = 1$
solid angle	steradian	sr	$\text{m}^2/\text{m}^2 = 1$
frequency	hertz	Hz	$\text{s}^{-1}$
force	newton	N	$\text{m kg s}^{-2}$
pressure, stress	pascal	Pa	$\text{N/m}^2 = \text{m}^{-1} \text{kg s}^{-2}$
energy, work, amount of heat	joule	J	$\text{N m} = \text{m}^2 \text{kg s}^{-2}$
power, radiant flux	watt	W	$\text{J/s} = \text{m}^2 \text{kg s}^{-3}$
electric charge	coulomb	C	$\text{s A}$
electric potential difference	volt	V	$\text{W/A} = \text{m}^2 \text{kg s}^{-3} \text{A}^{-1}$
capacitance	farad	F	$\text{C/V} = \text{m}^{-2} \text{kg}^{-1} \text{s}^4 \text{A}^2$
electric resistance	ohm	$\Omega$	$\text{V/A} = \text{m}^2 \text{kg s}^{-3} \text{A}^{-2}$

Derived quantity	Name of derived unit	Symbol for unit	Expression in terms of other units
electric conductance	siemens	S	$\text{A/V} = \text{m}^{-2} \text{kg}^{-1} \text{s}^3 \text{A}^2$
magnetic flux	weber	Wb	$\text{V s} = \text{m}^2 \text{kg s}^{-2} \text{A}^{-1}$
magnetic flux density	tesla	T	$\text{Wb/m}^2 = \text{kg s}^{-2} \text{A}^{-1}$
inductance	henry	H	$\text{Wb/A} = \text{m}^2 \text{kg s}^{-2} \text{A}^{-2}$
Celsius temperature	degree Celsius	$^{\circ}\text{C}$	K
luminous flux	lumen	lm	$\text{cd sr} = \text{cd}$
illuminance	lux	lx	$\text{lm/m}^2 = \text{m}^{-2} \text{cd}$
activity referred to a radionuclide	becquerel	Bq	$\text{s}^{-1}$
absorbed dose	gray	Gy	$\text{J/kg} = \text{m}^2 \text{s}^{-2}$
dose equivalent	sievert	Sv	$\text{J/kg} = \text{m}^2 \text{s}^{-2}$
catalytic activity	katal	kat	$\text{s}^{-1} \text{mol}$

# SI præfixer

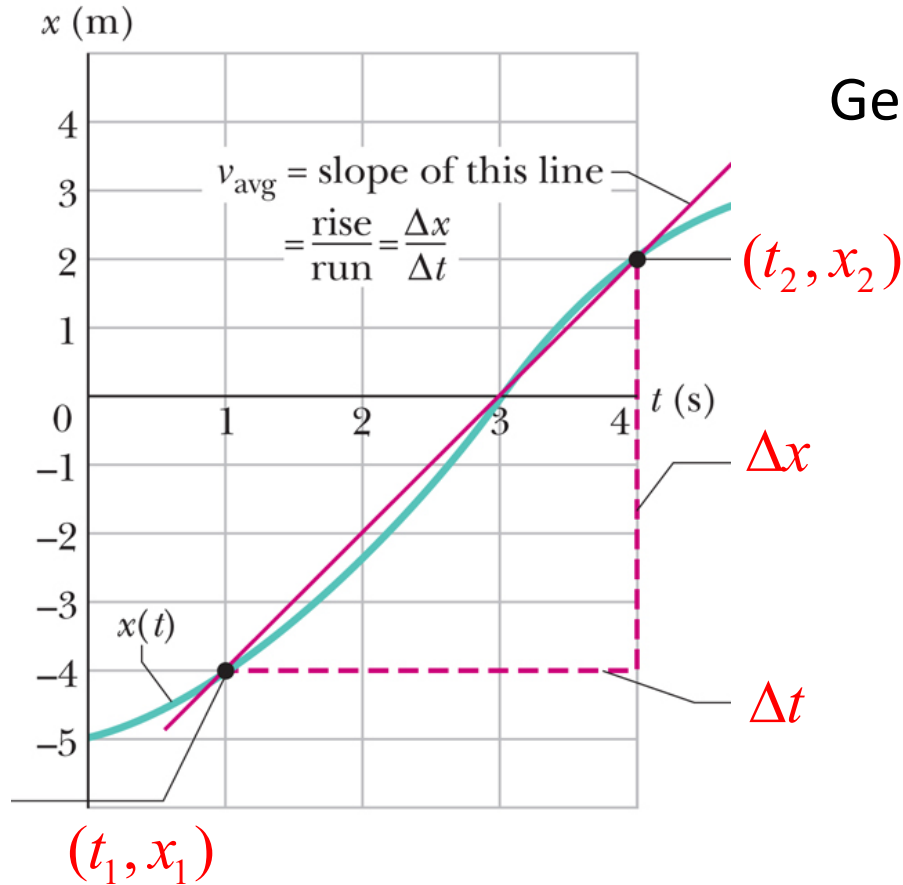
Factor	Name	Symbol	Factor	Name	Symbol
$10^1$	deca	da	$10^{-1}$	deci	d
$10^2$	hecto	h	$10^{-2}$	centi	c
$10^3$	kilo	k	$10^{-3}$	milli	m
$10^6$	mega	M	$10^{-6}$	micro	$\mu$
$10^9$	giga	G	$10^{-9}$	nano	n
$10^{12}$	tera	T	$10^{-12}$	pico	p
$10^{15}$	peta	P	$10^{-15}$	femto	f
$10^{18}$	exa	E	$10^{-18}$	atto	a
$10^{21}$	zetta	Z	$10^{-21}$	zepto	z
$10^{24}$	yotta	Y	$10^{-24}$	yocto	y

## Det græske alfabet

Stort	Lille	Navn
A	$\alpha$	alfa
B	$\beta$	beta
$\Gamma$	$\gamma$	gamma
$\Delta$	$\delta$	delta
E	$\varepsilon$	epsilon
Z	$\zeta$	zeta
H	$\eta$	eta
$\Theta$	$\theta$	theta
I	$\iota$	iota
K	$\kappa$	kappa
$\Lambda$	$\lambda$	lambda
M	$\mu$	my
N	$\nu$	ny
$\Xi$	$\xi$	ksi
O	$\omicron$	omicron
$\Pi$	$\pi$	pi
P	$\rho$	rho
$\Sigma$	$\sigma$	sigma
T	$\tau$	tau
Y	$\upsilon$	ypsilon
$\Phi$	$\varphi$	phi
X	$\chi$	chi
$\Psi$	$\psi$	psi
$\Omega$	$\omega$	omega

# Lineær bevægelse - hastighed

## Strækning vs. tid for en bevægelse



Gennemsnitsfart i tidsintervallet  $\Delta t$ :

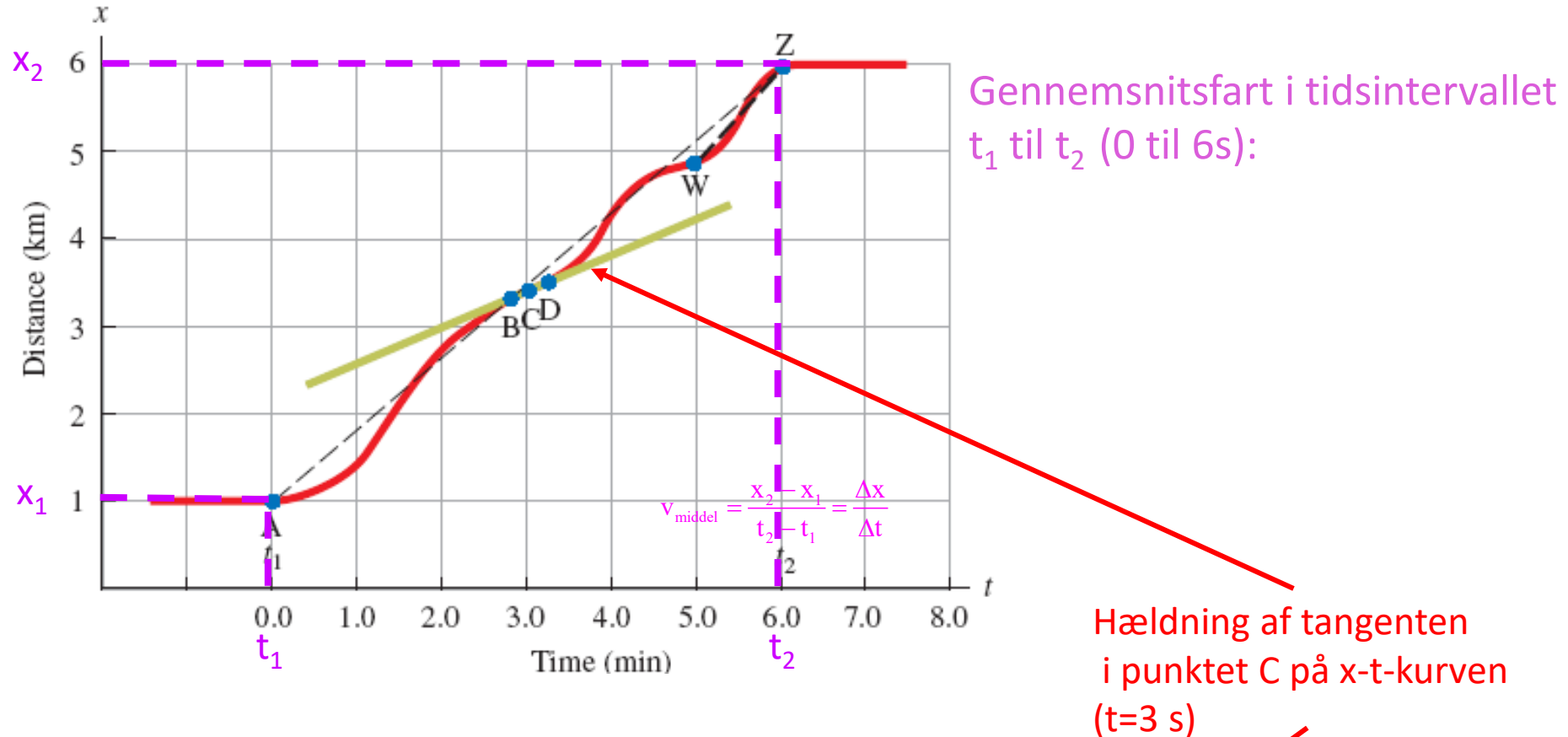
$$v_{\text{middel}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \quad (2-2)$$

Forskydning:  $\Delta x \equiv x_2 - x_1 \quad (2-1)$

Tidsinterval:  $\Delta t = t_2 - t_1$

# Lineær bevægelse - hastighed

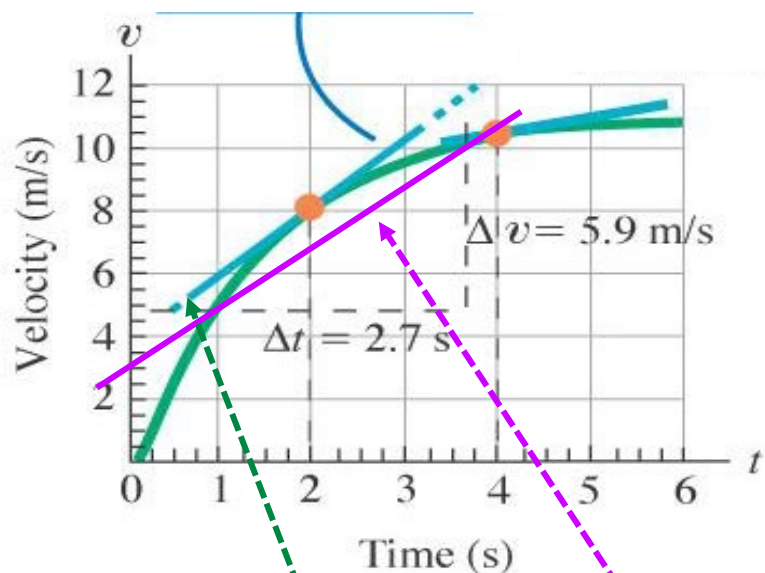
## Strækning vs. tid for en bevægelse



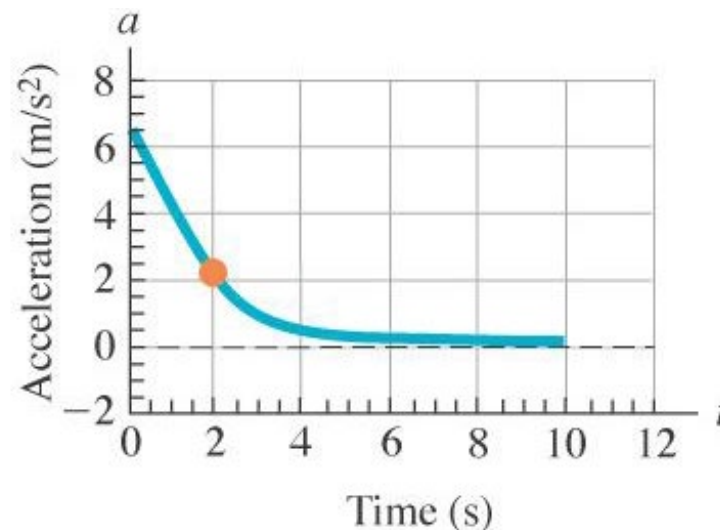


# Lineær bevægelse - acceleration

## Hastighed vs. tid



## Acceleration vs. tid



Gennemsnits-  
acceleration:

$$a_{av} \equiv \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \quad (2-7)$$

Øjeblikkelig  
acceleration:  
(for t= 2s)

$$a \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \quad (2-8)$$

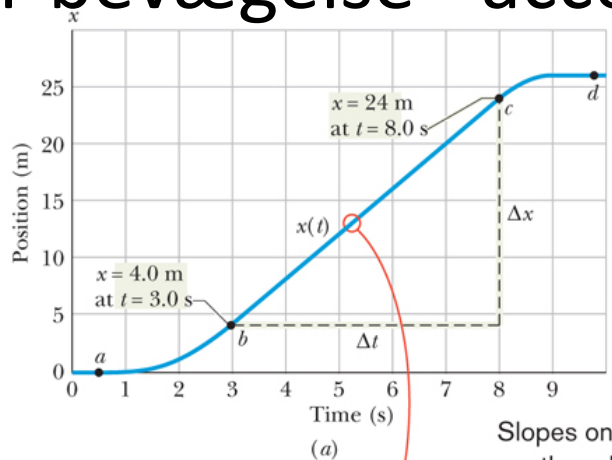
$$a = \frac{dv}{dt} = \frac{d}{dt} \left( \frac{dx}{dt} \right) = \frac{d^2 x}{dt^2} \quad (2-9)$$

# Lineær bevægelse - acceleration

## Sample problem 2.02

Elevator i opad bevægelse

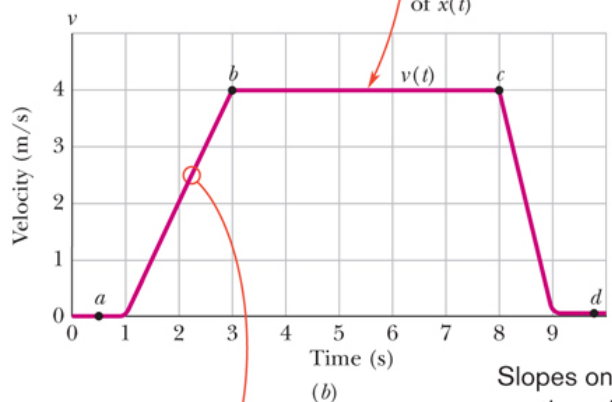
$x(t)$



Slopes on the  $x$  versus  $t$  graph are the values on the  $v$  versus  $t$  graph.

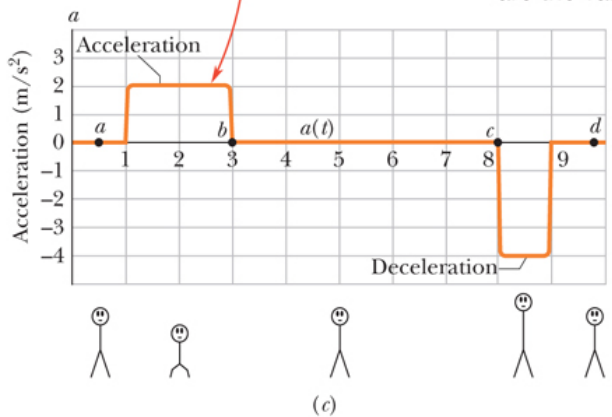
Slope of  $x(t)$

$$v(t) = \frac{dx}{dt}$$



Slopes on the  $v$  versus  $t$  graph are the values on the  $a$  versus  $t$  graph.

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$



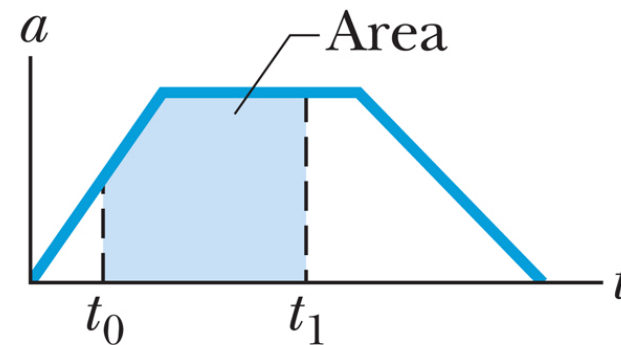
What you would feel.

# Lineær bevægelse - acceleration

$$a(t) = \frac{dv}{dt} \Rightarrow dv = a(t)dt \rightarrow$$

$$\int_{v_0}^v dv = \int_{t_0}^t a(t)dt \Rightarrow \Delta v = v - v_0 = \int_{t_0}^t a(t)dt$$

(2-27)

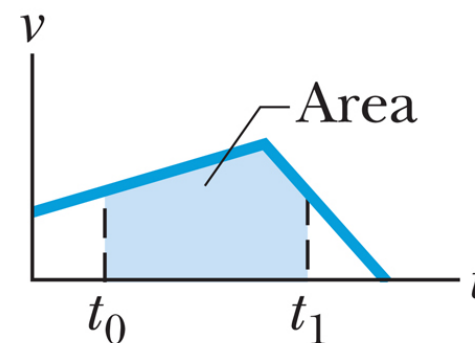


(a)

$$v(t) = \frac{dx}{dt} \Rightarrow dx = v(t)dt \rightarrow$$

$$\int_{x_0}^x dx = \int_{t_0}^t v(t)dt \Rightarrow \Delta x = x - x_0 = \int_{t_0}^t v(t)dt$$

(2-29)



(b)

# Lineær bevægelse – konstant acceleration

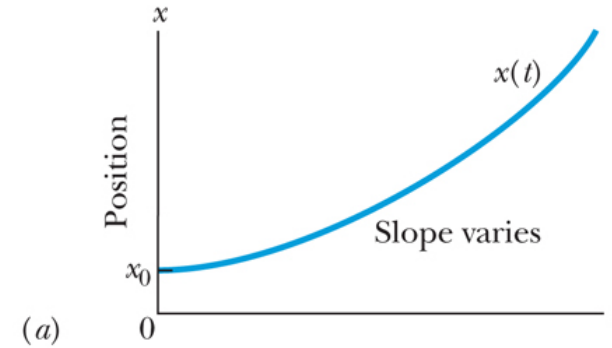
**Table 2-1** Equations for Motion with  
**Constant Acceleration<sup>a</sup>**

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0t + \frac{1}{2}at^2$	$v$
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	$t$
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	$a$
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	$v_0$

<sup>a</sup>Make sure that the acceleration is indeed constant before using the equations in this table.

# Lineær bevægelse – konstant acceleration

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \quad (2-15)$$

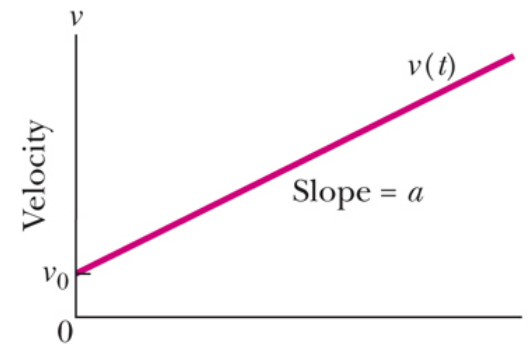


Slopes of the position graph are plotted on the velocity graph.

$$v_{avg} = \frac{x - x_0}{t - 0} \Rightarrow x = x_0 + v_{avg} t$$

$$v_{avg} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(v_0 + v_0 + at) = v_0 + \frac{1}{2}at$$

(b)



Slope of the velocity graph is plotted on the acceleration graph.

$$a = a_{avg} = \frac{v - v_0}{t - 0} \Rightarrow v = v_0 + at \quad (2-11)$$



# Lineær bevægelse – konstant acceleration

$$(2-11) \quad v = v_0 + at \Rightarrow t = \underbrace{\frac{v - v_0}{a}}$$

$$(2-15) \quad x - x_0 = v_0 t + \frac{1}{2} at^2 = v_0 \frac{v - v_0}{a} + \frac{1}{2} a \left[ \frac{v - v_0}{a} \right]^2 \Leftrightarrow$$

$$2a(x - x_0) = 2v_0 v - 2v_0^2 + v^2 + v_0^2 - 2v_0 v = v^2 - v_0^2 \Leftrightarrow$$

$$(2-16) \quad \boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

$$(2-11) \quad v = v_0 + at \Rightarrow a = \underbrace{\frac{v - v_0}{t}} \quad \text{eller} \quad v_0 = \underbrace{v - at}$$

$$(2-15) \quad x - x_0 = v_0 t + \frac{1}{2} at^2 = v_0 t + \frac{1}{2} \left[ \frac{v - v_0}{t} \right] t^2 = \frac{1}{2} (v_0 + v) t$$

$$(2-18) \quad x - x_0 = \frac{1}{2} [v - at + v] t = vt - \frac{1}{2} at^2 \Rightarrow \boxed{x = x_0 + vt - \frac{1}{2} at^2}$$



# Bevægelse i 3 dimensioner - position

## Position

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad (4-1)$$

## Forskydning

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 \quad (4-2)$$

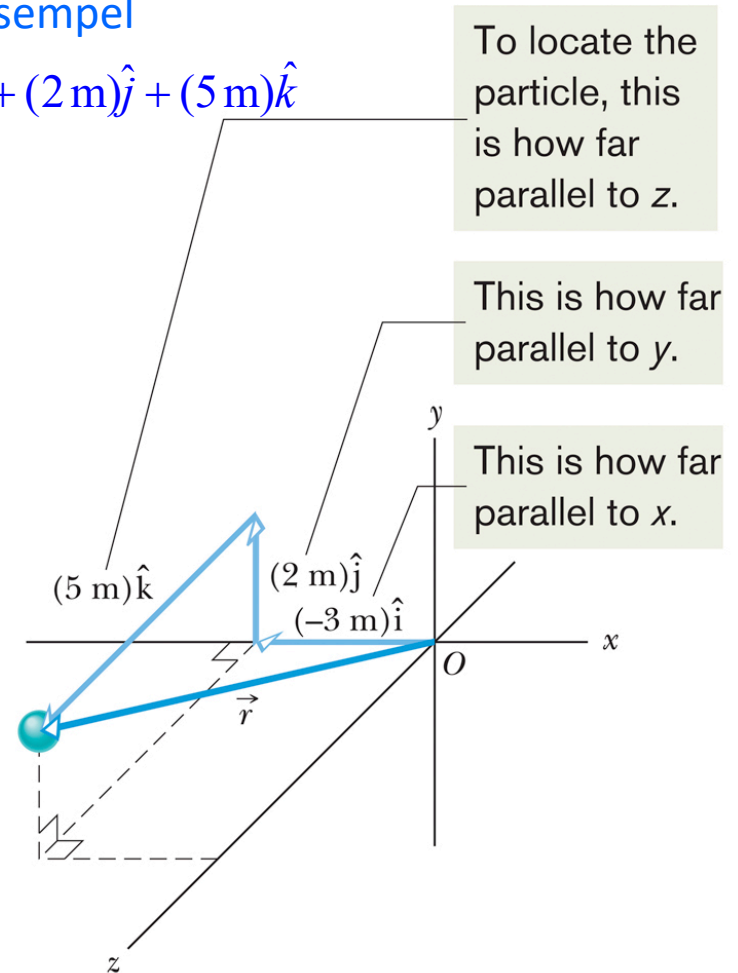
$$\Delta\vec{r} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k} \quad (4-4)$$

Eksempel

$$\vec{r} = (-3\text{ m})\hat{i} + (2\text{ m})\hat{j} + (5\text{ m})\hat{k}$$

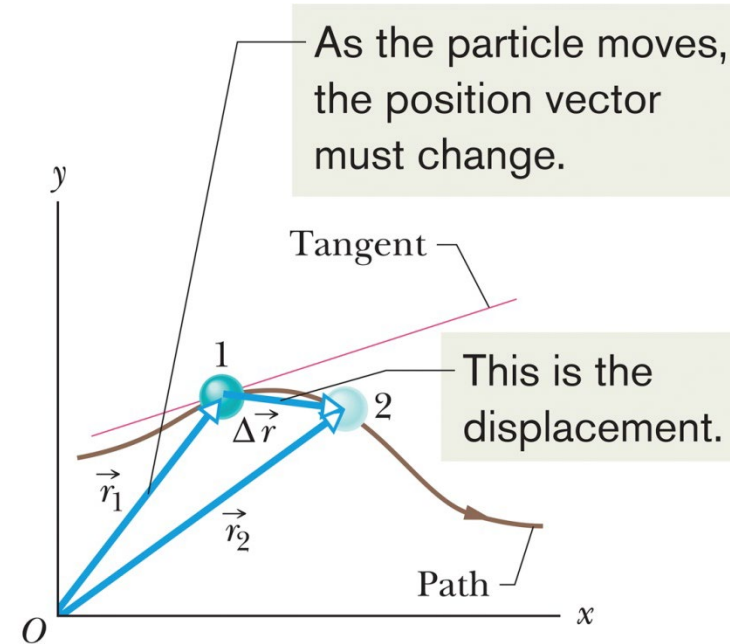


# Bevægelse i 3 dimensioner - hastighed

## Gennemsnitshastighed

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} \quad (4-8)$$

$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k} \quad (4-9)$$



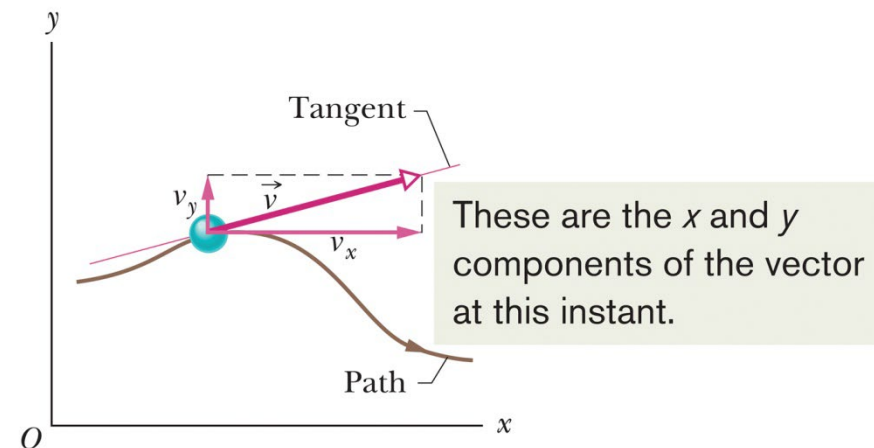
## Øjeblikshastighed

$$\vec{v}(t) = \lim_{\Delta t \rightarrow 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k} \quad (4-11)$$

The velocity vector is always tangent to the path.



# Bevægelse i 3 dimensioner - acceleration

## Gennemsnitsacceleration

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} \quad (4-15)$$

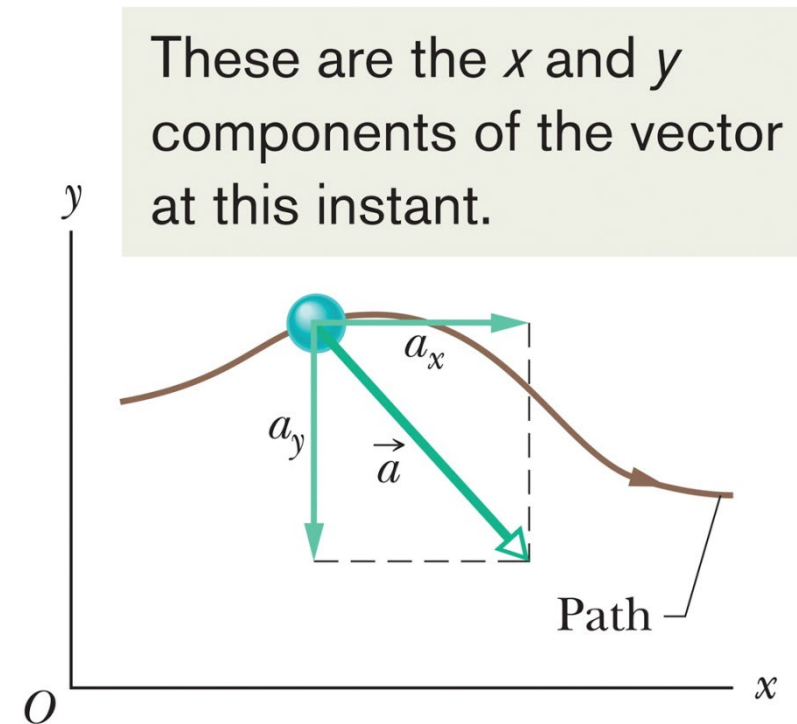
$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k}$$

## Øjeblikksacceleration

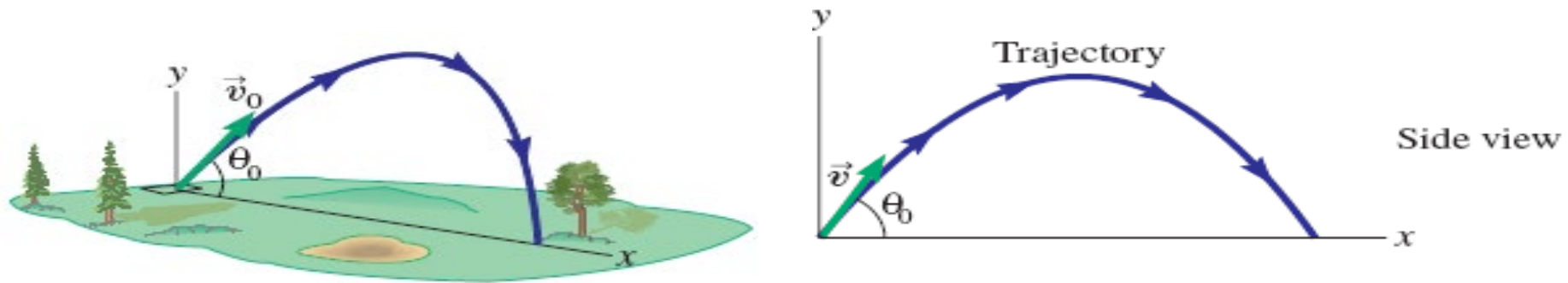
$$\vec{a}(t) = \lim_{\Delta t \rightarrow 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k} \quad (4-18)$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$



# Plan bevægelse - projektilbevægelsen



- Objekts bevægelse under indflydelse af tyngdekraften:

$$a_y = -g \quad (g = 9.82 \text{ m/s}^2 \text{ (dk)})$$

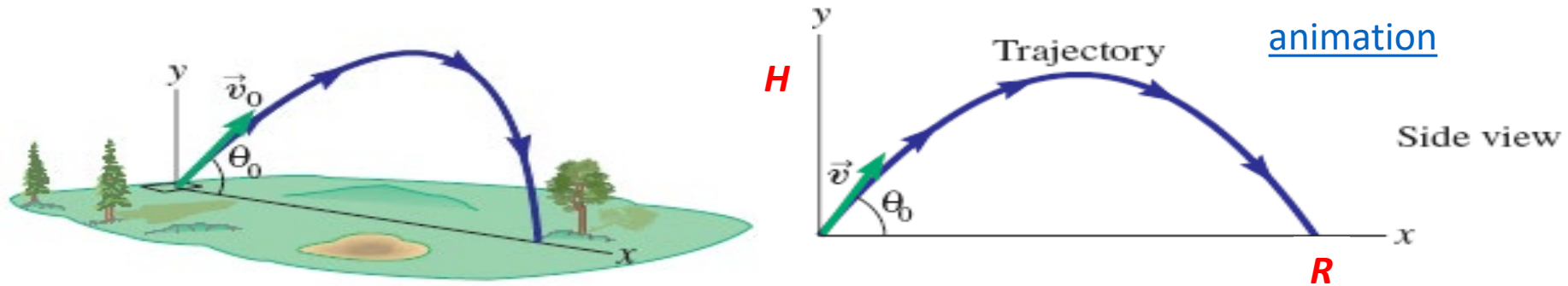
- Hastighedens x-komponent er konstant (*ikke påvirket af g*):

$$v_x = v_{0x} = v_0 \cos \theta_0 \Rightarrow x = x_0 + (v_0 \cos \theta_0)t \quad (4-21)$$

- Hastighedens y-komponent findes ved at benytte ligninger for konstant acceleration:

$$v_y = v_{0y} - gt = v_0 \sin \theta_0 - gt \Rightarrow y = y_0 + (v_0 \sin \theta_0)t - \frac{1}{2}gt^2 \quad (4-22)$$

# Plan bevægelse - projektilbevægelsen



Antager vi, at objektet kastes fra positionen  $(x_0, y_0) = (0, 0)$  fås:

$$\begin{aligned}x &= (v_0 \cos \theta_0)t \Rightarrow t = \frac{x}{v_0 \cos \theta_0} \\y &= (v_0 \sin \theta_0)t - \frac{1}{2}gt^2 = (v_0 \sin \theta_0) \left[ \frac{x}{v_0 \cos \theta_0} \right] - \frac{1}{2}g \left[ \frac{x}{v_0 \cos \theta_0} \right]^2 \Rightarrow \\y &= x \tan \theta_0 - \frac{1}{2} \left[ \frac{g}{(v_0 \cos \theta_0)^2} \right] x^2 \quad (4-25) \quad (\text{ligningen for en parabel})\end{aligned}$$

Kastelængde:  $R = \frac{v_0^2}{g} \sin 2\theta_0$       Max højde:  $H = \frac{v_0^2}{2g} (\sin \theta_0)^2$

# Jævn cirkulær bevægelse

Beskrives ved polære koordinater:

Radius:  $R$

Vinkel:  $\phi$

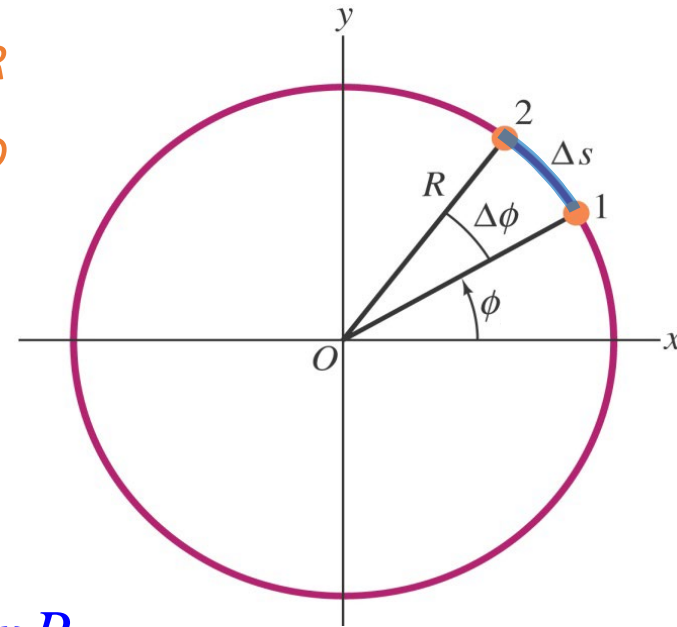
Buelængde:  $\Delta s = R\Delta\phi$

Øjeblikkelig hastighed:  $v = \frac{ds}{dt} = R \frac{d\phi}{dt}$

Vinkelhastighed:  $\omega \equiv \frac{d\phi}{dt} \quad v = \omega R$

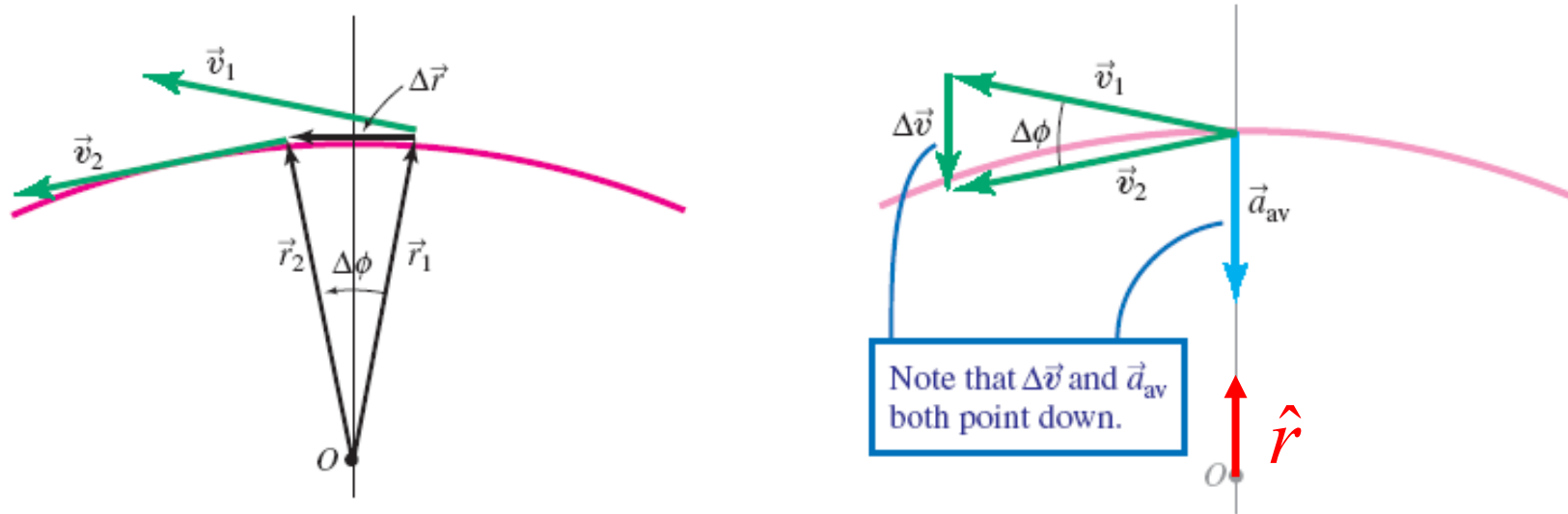
Periodetid:  $T = \frac{2\pi R}{v} = \frac{2\pi R}{\omega R} = \frac{2\pi}{\omega}$

Frekvens:  $f = \frac{1}{T} = \frac{\omega}{2\pi} \quad \omega = 2\pi f$





# Jævn cirkulær bevægelse



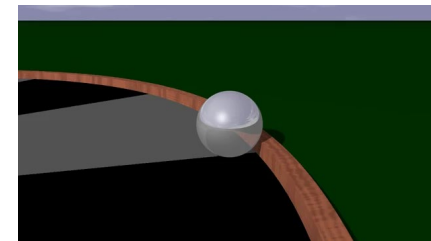
Ændringen i hastighedsvektoren angiver retningen af :  $\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$

Ud fra de ligedannede trekanter i figurerne ses:

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \Leftrightarrow \Delta v = \frac{v}{r} \Delta r \Rightarrow \frac{\Delta v}{\Delta t} = \frac{v}{r} \frac{\Delta r}{\Delta t}$$

Den øjeblikkelige acceleration findes som:

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{v}{r} \lim_{\Delta t \rightarrow 0} \frac{\Delta r}{\Delta t} = \frac{v}{r} v = \frac{v^2}{r} = r\omega^2 \quad (4-34)$$

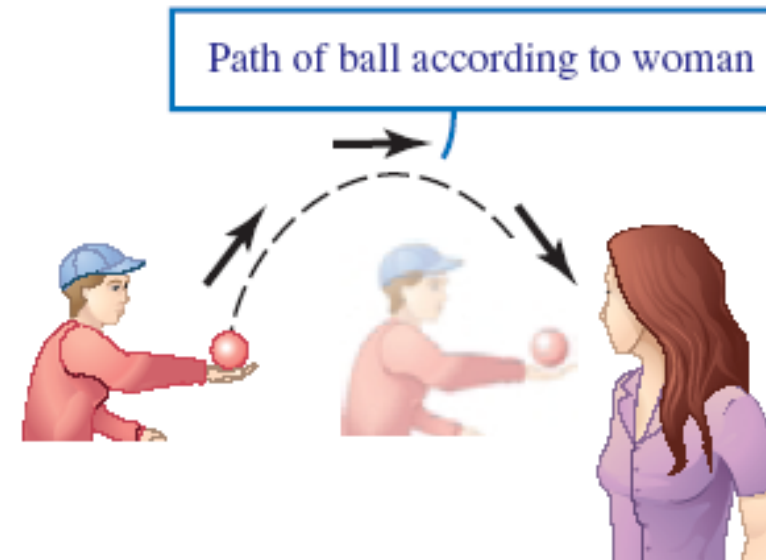
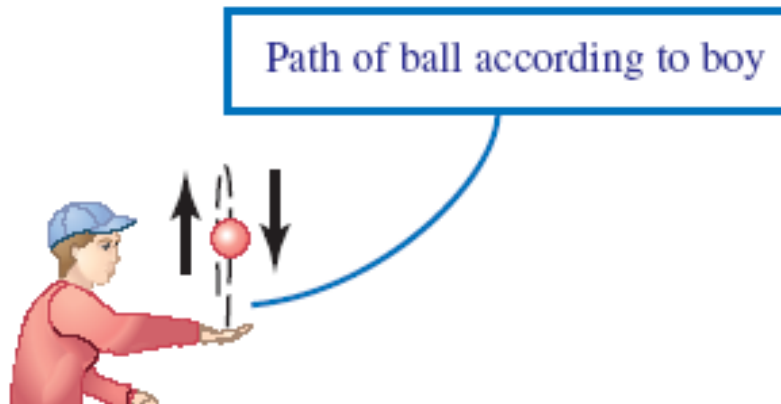


$$\vec{a} = -\frac{v^2}{r} \hat{r}$$

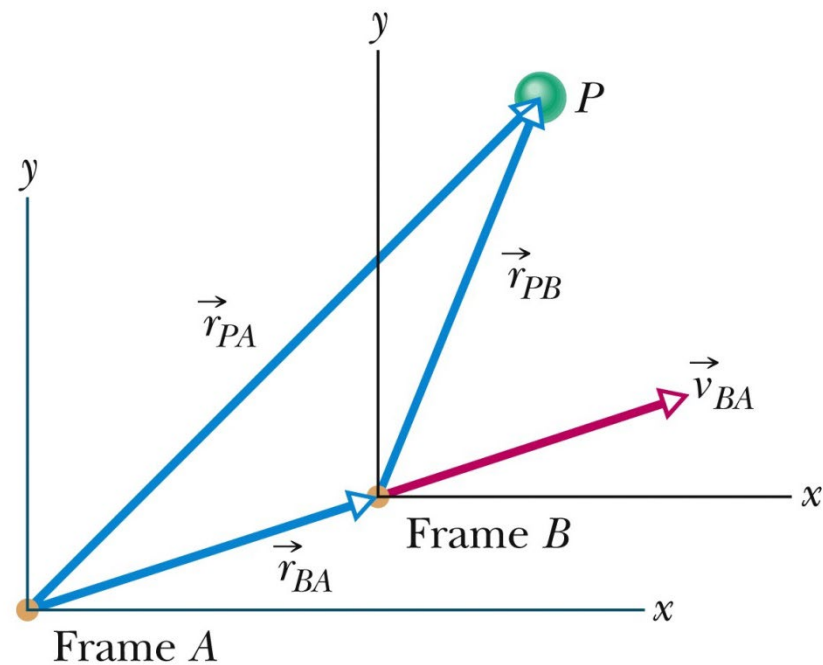
# Relativ bevægelse



En dreng står stille en bus og kaster en bold op. Observatører som bevæger sig i forhold til hinanden vil beskrive bevægelsen forskelligt.



# Relativ bevægelse



$$\vec{r}_{PB} = \vec{r}_{PB} + \vec{r}_{BA} \quad (4-43)$$

$$\vec{v}_{PB} = \vec{v}_{PB} + \vec{v}_{BA} \quad (4-44)$$

$$\vec{a}_{PA} = \vec{a}_{PB} \quad (4-45)$$

# Næste gang: Newton's Love

## Newton's Første Lov

Et legeme som ikke er påvirket af en kraft, eller af kræfter der ophæver hinandens virkning, vil enten være i hvile eller foretage en jævn retlinet bevægelse.

## Newton's Anden Lov

Et legeme med massen  $m$ , der påvirkes af en resulterende kraft  $F$ , vil have en acceleration  $a$ , som opfylder:  $\vec{F} = m\vec{a}$

## Newton's Tredie Lov

Et legeme A der påvirker et legeme B med en kraft, vil blive påvirket med en lige stor modsat rettet (Lov om aktion og reaktion)