Control of Autonomous Systems – Autumn 2025

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Exercise Session 7

1. **Heating system**. Consider the simplified dynamics of a heating system[1] consisting of $\dim(\mathbf{x}) = n$ coupled compartments and represented by the following linear state-space representation.

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 1 & 0 & 0 & \ddots & \ddots & \ddots \\ 1 & -2 & 1 & 0 & \ddots & \ddots & \ddots \\ 0 & 1 & -2 & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & \ddots & \ddots & \ddots & 0 & 0 \\ \ddots & \ddots & \ddots & \ddots & -2 & 1 & 0 \\ \ddots & \ddots & \ddots & 0 & 1 & -2 & 1 \\ \ddots & \ddots & \ddots & 0 & 0 & 1 & -1 \end{bmatrix} \mathbf{x} \ , \ \mathbf{x}(0) = \mathbf{x}_0 \qquad (1)$$

where component $x_i(t)$ is the temperature of compartment i. For simplicity, we assume that $n = 10$. For this system, we measure only the temperature of the first "boundary" compartment, i.e. we have $y = x_1$.

[1] These dynamics are also commonly found in dynamics of networks, and in particular consensus algorithms, hence the above illustration.

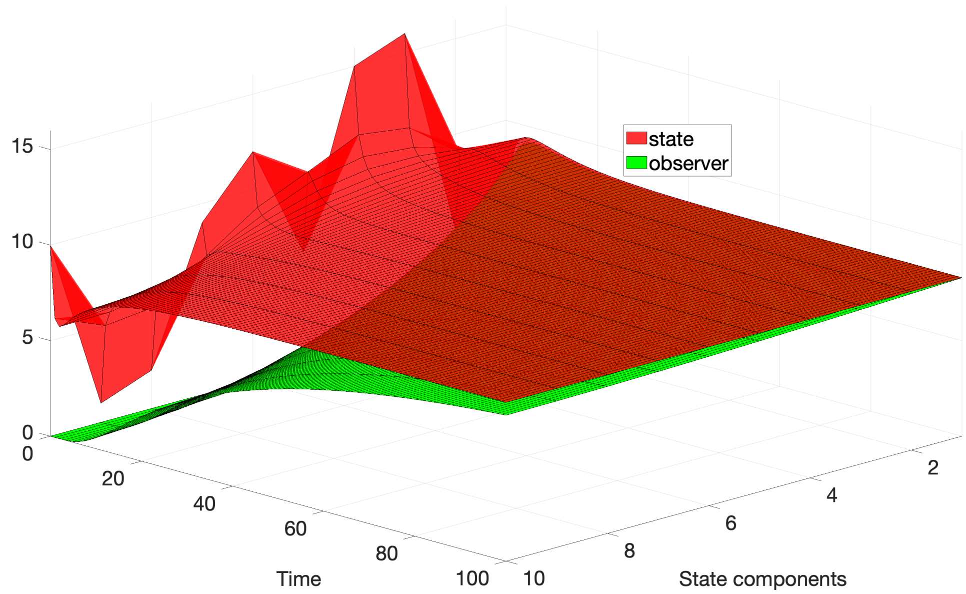

Figure 1: Evolution of the state of system (1) and an estimate.

1.1. In a new Simulink file, create a subsystem where you implement system (1) (remember to set the sampling time for the solver of your simulation to $T_s = 0.01s$). Simulate your plant from different initial conditions, where the vector of initial conditions contain random positive values. Comment on the behavior of the system at steady-state.

1.2. Compute the eigenvalues of matrix $\mathbf{A}$ of system (1). What can you infer from this result? What is one consequence?

1.3. Use a 'To Workspace' block to send the history of the system state to the work environment (feel free to 'decimate' to 100 not to have too many points). In a Matlab file, write a script to display this history similarly to what is diplayed in figure 1 (you can use the commands `surface` and `view` for that).

1.4. In your Simulink file, create a new subsystem where you will implement an observer. Tune your observer so that the state estimate converges to the actual state of the system, this without creating too large error in the transitory phase.

1.5. Add the history of the state estimate into your Matlab script so that it is superposed to the history of the plant state (see figure 1).

1.6. What is the simplest possible observer gain that can be used? Check whether this works.

2. **Adding a heat source**. We would like to add a simple heat source to system (1). To do so, add a n-by-1 B matrix full of zeros, except for $b_7 = 1$ to state-space representation (1) and, in matrix $\mathbf{A}$, set $a_{7,7}$=-3.

   2.1. In your Simulink file, set the heat source (ie the control input) to be a sine wave of amplitude 6, bias 3 and frequency 0.5. Observe the effect of this heat source by using your script diplaying the history of the state. See how the heat propagates in the whole system.

   2.2. What are the new eigenvalues of matrix $\mathbf{A}$. What can you deduce from that?

   2.3. Modify the observer you designed in the previous exercise to obtain the simplest possible observer. Check that it works in your script. What is its drawback?

   2.4. Change the tuning of the observer you designed so that the state estimate will converge faster to the state of the plant. Check the result in your script.