opg 2.

find the total charge on the surface
$r = e^u \cos v i + e^u \sin v j + uk \quad (0 \le u \le 1, 0 \le v \le \pi)$
if the charge density on the surface
is $\delta = \sqrt{1+e^{2u}}$

We use $\iint_D f(r(u,v)) \left| \frac{dr}{du} \times \frac{dr}{dv} \right| du\, dv$ to find dS

$$dS = \left| \frac{dr}{du} \times \frac{dr}{dv} \right| = \left| \begin{pmatrix} e^u \cdot \cos v \\ e^u \cdot -\sin v \\ 1 \end{pmatrix} \times \begin{pmatrix} e^u \cdot (-\sin v) \\ e^u \cdot \cos v \\ 0 \end{pmatrix} \right| du\, dv$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ e^u \cdot \cos v & e^u \cdot -\sin v & 1 \\ e^u \cdot -\sin v & e^u \cdot \cos v & 0 \end{vmatrix} =$$

$\cos^2 + \sin^2 = 1$

$$= i \cdot (0 - e^u \cdot \cos v) - j \cdot (0 - e^u \cdot -\sin v) + k \cdot (e^{2u} \cdot \cos^2 v + e^{2u} \cdot \sin^2 v)$$

$$= -e^u \cdot \cos v \cdot i - e^u \cdot \sin v \cdot j + e^{2u} \cdot k$$

We find the length to get dS

$$\left| \frac{dr}{du} \times \frac{dr}{dv} \right| = \sqrt{(-e^u \cdot \cos v)^2 + (e^u \cdot \sin v)^2 + (e^{2u})^2}$$

$$= \sqrt{e^{2u} \cdot (\cos^2 v + \sin^2 v)} + \sqrt{e^{4u}}$$

$$= \sqrt{e^{2u} + e^{4u}}\, du\, dv$$

now we have dS.

the limits are given as

$0 \le u \le 1$ and $0 \le v \le \pi$.

We can now integrate the density formula

$$\iint_S \delta \, dS =$$

$$= \int_0^\pi \int_0^1 \sqrt{1+e^{2u}} \cdot \sqrt{e^{2u} + e^{4u}}\, du\, dv$$

$$= \int_0^\pi \left[\frac{(3+e^{2u}) \cdot \sqrt{e^{2u} + e^{4u}}}{3 \cdot \sqrt{1+e^{2u}}} \right]_0^1 dv$$

$$= \int_0^\pi \frac{1}{3} \cdot (-4 + 3 \cdot e + e^3)\, dv$$

$$= \left[\frac{v}{3} \cdot (-4 + 3 \cdot e + e^3) \right]_0^\pi$$

$$= \frac{\pi}{3} \cdot (-4 + 3 \cdot e + e^3$$