Auxiliary Conditions (boundary)

1) initial condition.

e.g. $u(x, t_0) = f(x)$

$$\left.\frac{du}{dt}\right|_{t=t_0} = g(x)$$

the order of derivatives dictate the needed number of initial conditions.
e.g. 3. order needs 3 initial conditions.

2) boundary condition.

specify solution properties at particular spatial positions, typically on the edges of the spatial domain where the PDE applies.

3. types

a) Dirichlet: Function u (to be solved for) is specified at the boundary.

e.g

$u(x=0, t) = h_1(t)$ if $u(x,t)$

but if

$u = u(x, y, t)$ then $u(x=0, y, t) = m_1(y, t)$

b) Neumann: spatial derivative of u (e.g. $\frac{du}{dx}$) is specified at the boundary:

e.g. $\left.\frac{du}{dx}\right|_{x=0} = h_2(t)$ if $u = u(x,t)$

c) Robin: A linear combination of the derivative of a function and the function itself is specified at the boundary

e.g. $\left.\frac{du}{dx}\right|_{x=0} + \left.ku\right|_{x=0} = g(t)$ if $u = u(x,t)$

In general number of boundary conditions needed = sum of orders of highest partial derivative in each spatial variable

e.g. $\frac{du}{dt} = \frac{d^2u}{dx^2}$ → to solve 'integrate' twice in x. → 2 arbitrary constants → 2 boundary condi.

$\frac{du}{dt} = \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2}$ → 4 boundary conditions

$\frac{du}{dt} = \frac{d^6u}{dx^6} + \frac{d^8u}{dy^8}$ → 14 boundary conditions