problem 1

non-Linear

$$\dot{m} = u - \boxed{q_1 \omega m}$$

$$J\dot{\omega} = q_2 m - q_3 \omega$$

$m(t)$ is mass

$\omega(t)$ is engine speed

$u(t)$ is air input

$J = 40$, $q_1 = 1$, $q_2 = 10$ and $q_3 = 0.1$

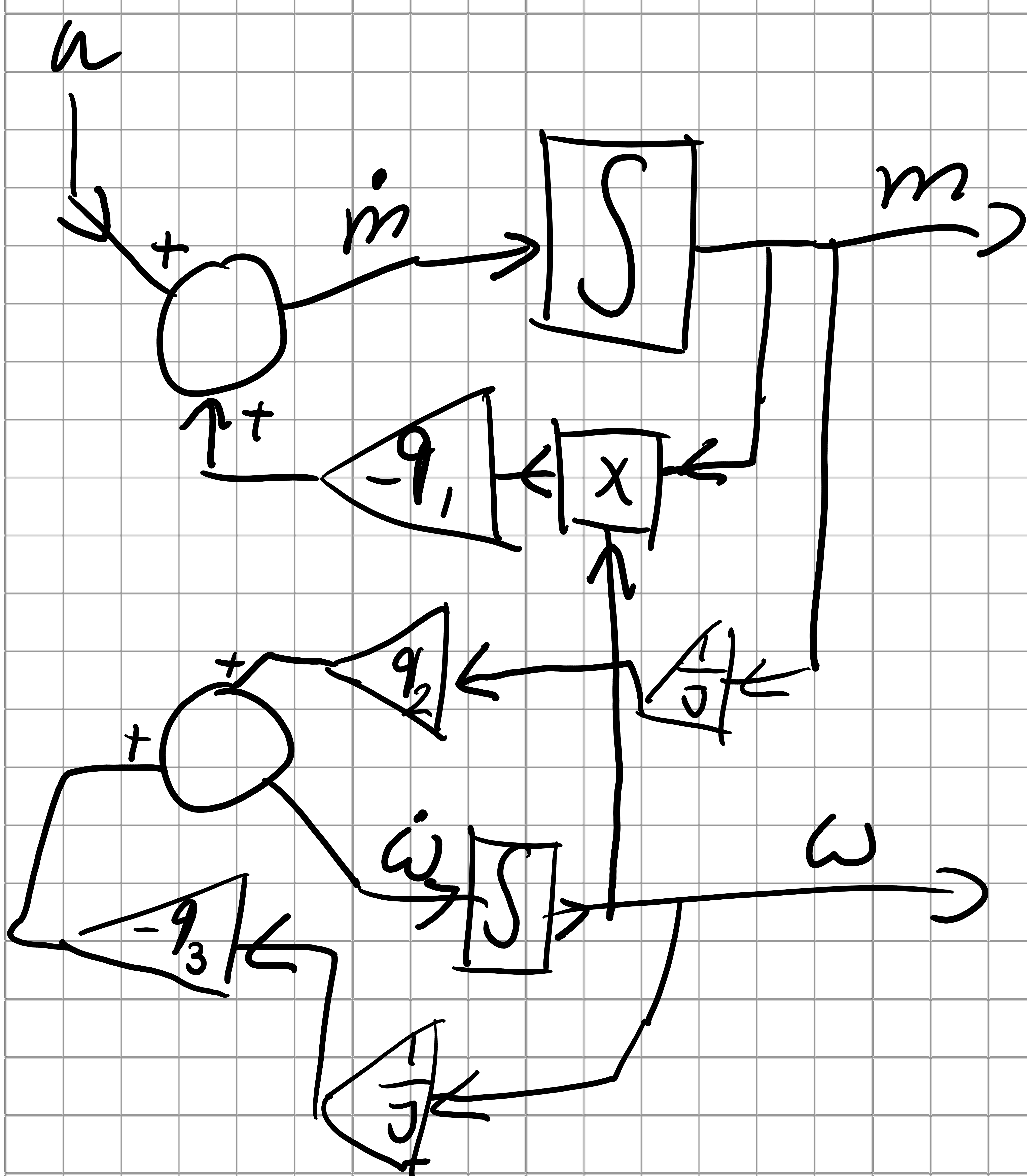
$$\dot{m} = u - q_1 \omega m \quad , \quad \dot{\omega} = \frac{q_2 m - q_3 \omega}{J}$$

As the system is nonLinear we write it in the following form

$$\dot{x} = \begin{bmatrix} \dot{m} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -q_1 \omega m \\ \frac{q_2 m}{J} - \frac{q_3 \omega}{J} \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

opgave 2

$$\dot{x} = \begin{bmatrix} \dot{m} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} u - q_1 \omega m \\ \frac{q_2 m}{J} - \frac{q_3 \omega}{J} \end{bmatrix}$$



Opgave 3

To find the equilibrium points of the system we set the change to zero.

$$\dot{m} = u - q_1 \omega m \quad \rightarrow \quad 0 = u - q_1 \omega m \qquad 1$$

$$J\dot{\omega} = q_2 m - q_3 \omega \qquad J \cdot 0 = q_2 m - q_3 \omega \qquad 2$$

$$q_2 m = q_3 \omega \Rightarrow \frac{q_2 m}{q_3} = \omega$$

We then substitute ω into eq 1

$$0 = u - q_1 \left(\frac{q_2 m}{q_3}\right) \cdot m \Rightarrow 0 = u - \frac{q_1 q_2 m^2}{q_3}$$

$$u \cdot q_3 = q_1 q_2 m^2 \Rightarrow \sqrt{\frac{u q_3}{q_1 q_2}} = m$$

we then have the 2 equili points

$$\sqrt{\frac{u q_3}{q_1 q_2}} = m \quad \text{and} \quad \frac{q_2}{q_3} m = w$$

$$m^2 = \frac{u\, q_3}{q_1\, q_2} \rightarrow \frac{m^2 q_1 q_2}{q_3} = u$$

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To lineariz, we insert the parameters.

$$\sqrt{\frac{u q_3}{q_1 q_2}} = m \quad \text{and} \quad \frac{q_2}{q_3} m = \omega$$

$$300 = \frac{10}{0{,}1} m \rightarrow 300 = 100m \rightarrow \underline{m = 3}$$

$$3 = \sqrt{\frac{u\ 0{,}1}{1 \cdot 10}} \rightarrow 3 = \sqrt{u \cdot 0{,}01}$$

$$\frac{3^2}{0{,}01} = u \rightarrow \underline{\underline{u = 900}}$$

To determine the stability of we insert the points into the Jacobian.

$$\begin{bmatrix} u - q_1 \omega m \\ \dfrac{q_2 m}{J} - \dfrac{q_3 \omega}{J} \end{bmatrix}$$

$$\downarrow$$

$$A = \begin{bmatrix} \dfrac{\partial f_1}{\partial m} = -q_1 \omega & \dfrac{\partial f_1}{\partial \omega} = -q_1 m \\ \dfrac{\partial f_2}{\partial m} = \dfrac{q_2}{J} & \dfrac{\partial f_2}{\partial \omega} = \dfrac{q_3}{J} \end{bmatrix} \quad B = \begin{bmatrix} \dfrac{\partial f_1}{\partial u} = 1 \\ \dfrac{\partial f_2}{\partial u} = 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -q_1 \omega & -q_1 m \\ \dfrac{q_2}{J} & -\dfrac{q_3}{J} \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

We then insert the parameters and find the eigenvalues.

$$\begin{bmatrix} -q_1 \omega & -q_1 m \\ \frac{q_2}{J} & -\frac{q_3}{J} \end{bmatrix} \rightarrow \begin{bmatrix} -300 & -3 \\ 0{,}25 & -0{,}0025 \end{bmatrix}$$

$$\underline{\underline{\lambda_1 = -299{,}9975}} \qquad \underline{\underline{\lambda_2 = -0{,}005}}$$

As the eigenvalues are negative we can determine that the system is stable.

opgave 7

We want to make the system linear, so we will cancel out the non linear part:

$\dot{m} = u - q_1 \omega m \qquad J\dot{\omega} = q_2 m - q_3 \omega$

non-Linear

$u = q_1 \omega m + v$ v is the new virtual input

$\dot{m} = (q_1 \omega m + v) - q_1 \omega m \Rightarrow \dot{m} = v$

The closed loop equatio is thus: $\dot{m} = v$

$$\dot{\omega} = \frac{q_2}{J} m - \frac{q_3}{J} \omega$$

now the state space model

$$\begin{bmatrix}\dot{m}\\ \dot{\omega}\end{bmatrix} = \begin{bmatrix}0 & 0\\ \frac{a_2}{J} & \frac{a_3}{J}\end{bmatrix}\begin{bmatrix}m\\ \omega\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix} v$$

virtual input

we check for stability.

$$\underline{\underline{\lambda_1 = -0{,}0025}} \qquad \underline{\underline{\lambda_2 = 0}}$$

The system is not asymptotically stable as the eigenvalues are <u>not</u> strictly negative.

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$\dot{m} = v$

$\dot{\omega} = \frac{q_2}{J} m - \frac{q_3}{J} \omega$

to cancel out the non-Linear part;

$v = \dot{m} \Rightarrow v = u - q_1 m \cdot \omega$

to cancel out:

$u = (-P_{v0} m - P_{v1} \omega) + q_1 m \omega$