opg 1

4th order

$$\epsilon \ddot{x} = F - \theta$$

$$\epsilon \ddot{\theta} = (1+\epsilon)\theta - F$$

with $u = F(t)$

$\Downarrow$

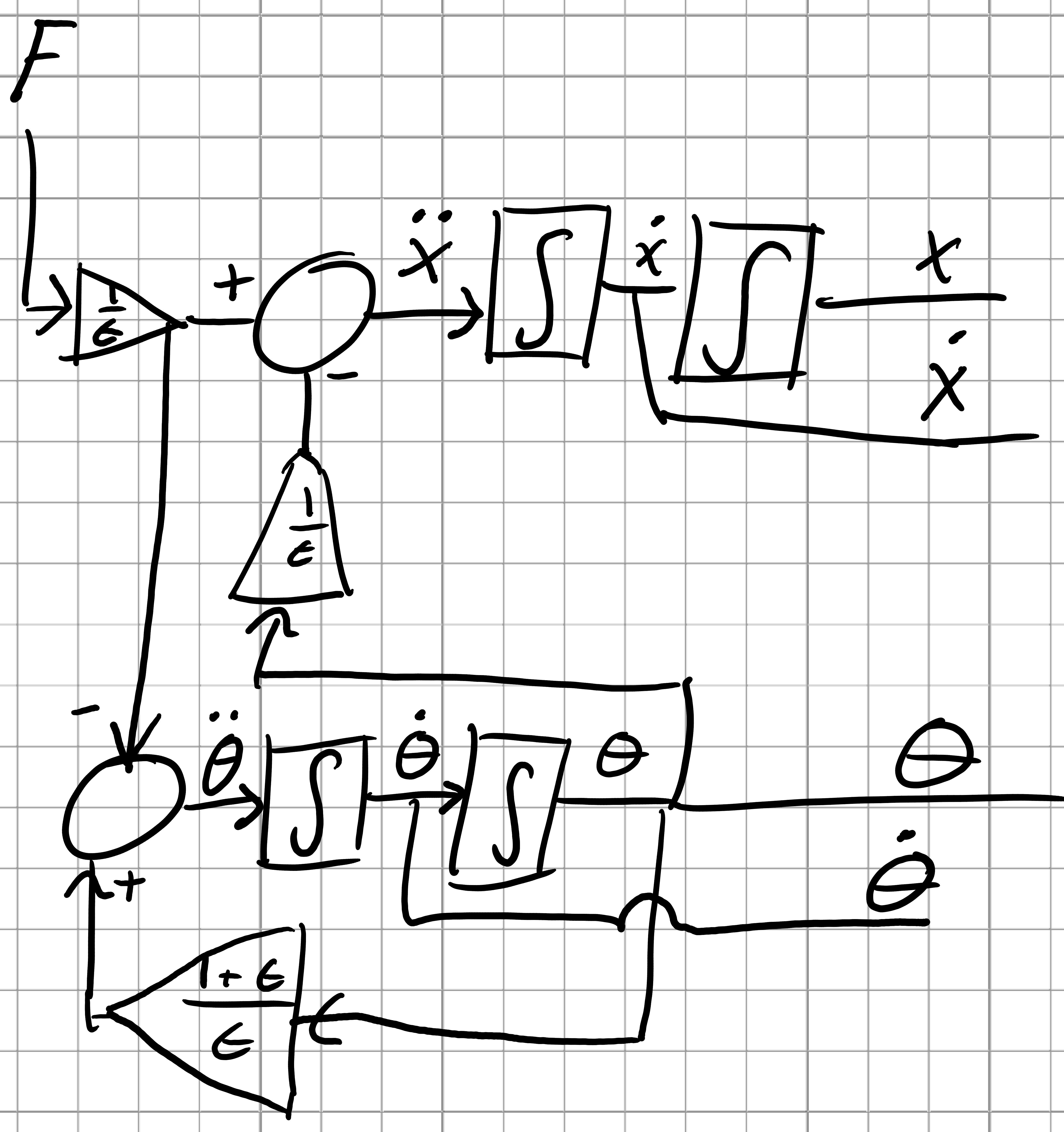
$$\ddot{x} = \frac{F}{\epsilon} - \frac{\theta}{\epsilon} \quad , \quad \ddot{\theta} = \frac{(1+\epsilon)\theta}{\epsilon} - \frac{F}{\epsilon}$$

$$\dot{x} = \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-1}{\epsilon} & 0 & 0 \\ 0 & \frac{(1+\epsilon)}{\epsilon} & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}}_{x}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\epsilon} \\ -\frac{1}{\epsilon} \end{bmatrix} u$$

opg 2

$$\ddot{x} = \frac{F}{\epsilon} - \frac{\theta}{\epsilon} \quad , \quad \ddot{\theta} = \frac{(1+\epsilon)\theta}{\epsilon} - \frac{F}{\epsilon}$$



opg 3

$\ddot{x} = \frac{F}{\epsilon} - \frac{\theta}{\epsilon}$, $\ddot{\theta} = \frac{(1+\epsilon)\theta}{\epsilon} - \frac{F}{\epsilon}$

we insert params and set change to zero

$0 = \frac{(1+\epsilon)}{\epsilon}\theta - \frac{F}{\epsilon} \Rightarrow F = (1+\epsilon)\theta$

$\frac{F}{(1+\epsilon)} = \theta$

as seen we have an infinite amount of equilibrium points. Thus $F \in \mathbb{R}$

opg 7

cart position $= x(t)$

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} \quad \text{where } C = [1\ 0\ 0\ 0]$$

The system is observable only if the rank of O is equal to the number of states.

Here $\text{rank}(O) = 4$

thus the system <u>is</u> <u>observable</u>

opg 8

we use the Jacobian

$$\ddot{x} = \frac{F}{\epsilon} - \frac{\theta}{\epsilon}, \quad \ddot{\theta} = \frac{(1+\epsilon)\theta}{\epsilon} - \frac{F}{\epsilon}$$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial \theta} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial \theta} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{\epsilon} \\ 0 & \frac{(1+\epsilon)}{\epsilon} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\epsilon} \\ -\frac{1}{\epsilon} \end{bmatrix}$$