

Determine whether the given vector field is conservative, and find a potential function if it is.

$$F(x, y, z) = y\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$$

We check with:

$$\frac{df_1}{dy} = \frac{df_2}{dx}, \frac{df_2}{dz} = \frac{df_3}{dy}, \frac{df_1}{dz} = \frac{df_3}{dx}$$

$$\frac{df_1}{dy} = \frac{df_2}{dx} \rightarrow 1 = 1$$

$$\frac{df_2}{dz} = \frac{df_3}{dy} \rightarrow 0 = 0$$

$$\frac{df_1}{dz} = \frac{df_3}{dx} \rightarrow 0 = 0$$

therefore the field is conservative.

we then find the potential function.

$$\int f_1 dx = xy$$

$$\int f_2 dy = xy$$

$$\int f_3 dz = \frac{z^3}{3}$$

duplicates can be removed.

$$\underline{\underline{\phi(x, y, z) = xy + \frac{z^3}{3}}}$$