Opg 1

$$u = L\dot{I} + R_m I + K_e \omega$$

$$K_m I = J\dot{\omega} + K_d \omega + F_c\, y(\omega)$$

$\downarrow$

$$\dot{I} = \frac{u}{L} - \frac{R_m I}{L} - \frac{K_e \omega}{L}$$

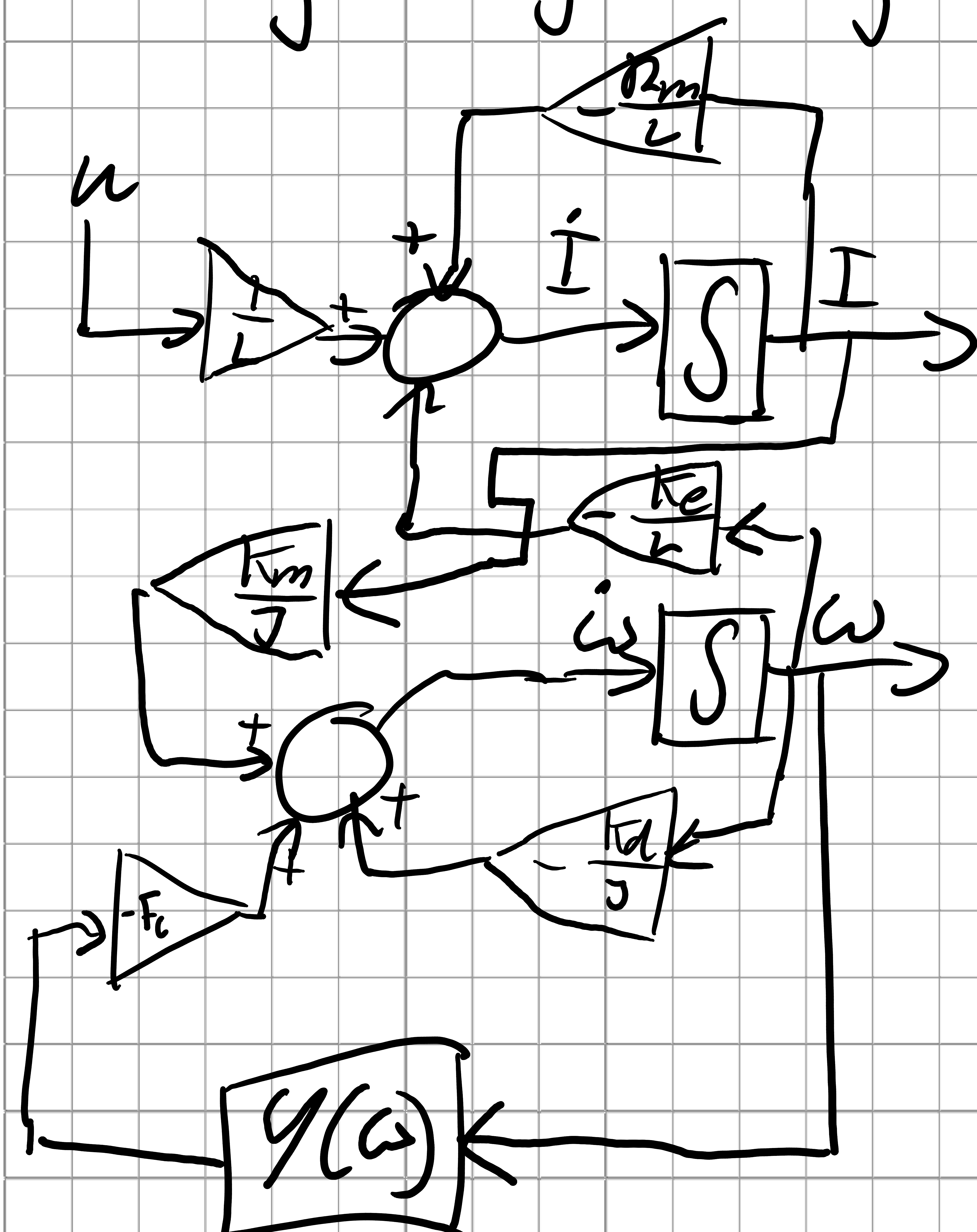
$$\dot{\omega} = \frac{K_m I}{J} - \frac{K_d \omega}{J} - \frac{F_c\, y(\omega)}{J}$$

$$\dot{x} = \begin{bmatrix} \dot{I} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{u}{L} & -\frac{R_m I}{L} & -\frac{K_e \omega}{L} \\ \frac{K_m I}{J} & -\frac{K_d \omega}{J} & -\frac{F_c\, y(\omega)}{J} \end{bmatrix}$$

Opg 2

$$\dot{I} = \frac{u}{L} - \frac{R_m I}{L} - \frac{K_e \omega}{L}$$

$$\dot{\omega} = \frac{K_m I}{J} - \frac{K_d \omega}{J} - \frac{F_c \, \gamma(\omega)}{J}$$



opg 3

with $F_e = 0$, we have

$$u = L\dot{I} + R_m I + K_e \omega \quad 1.$$

$$K_m I = J\dot{\omega} + K_d \omega + 0 \cdot y(\omega) \quad 2.$$

$\dot{I} = 0$, $\dot{\omega} = 0$

$$u = L \cdot 0 + R_m I + K_e \omega$$

$$\downarrow$$

$$u = R_m I + K_e \omega$$

eq 2.

$$K_m I = J \cdot 0 + K_d \omega \Rightarrow I = \frac{K_d \omega}{K_m}$$

we insert into eq 1.

$$u = R_m \cdot \left(\frac{K_d \omega}{K_m}\right) + K_e \omega \Rightarrow u = \left(\frac{K_d R_m}{K_m} + K_e\right)\omega$$

$$u = \left(\frac{K_d R_m}{K_m} + K_e\right)\omega \Rightarrow \omega = \frac{u}{\frac{K_d R_m}{K_m} + K_e}$$

We then see that $\omega \in \mathbb{R}$ when the input u changes.

$$\omega = \frac{u}{\frac{K_d R_m}{K_m} + K_e} \quad , \quad I = \frac{K_d \omega}{K_m}$$

there is exactly one equilibrium point to each input.
e.i. an input will never result in multiple eq points

Ex: $m = \sqrt{u}, \omega = c \cdot m$ has 3 eq points

$u < 0 \to m = \pm\sqrt{u}, \omega = c \cdot (\pm m)$.

$u = 0 \to m = 0, \omega = c \cdot 0$

OPS 4

$$\dot{I} = \frac{u}{L} - \frac{R_m I}{L} - \frac{K_e \omega}{L}$$

$$\dot{\omega} = \frac{K_m I}{J} - \frac{K_d \omega}{J} - \frac{F_c \, \gamma(\omega)}{J}$$

$$\downarrow$$

$$A = \begin{bmatrix} -\frac{R_m}{L} & \frac{K_e}{J} \\ -\frac{K_m}{J} & -\frac{K_d}{J} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{J} \\ 0 \end{bmatrix}$$

$C = [B \; AB]$, $\det(C) = -100$

as $\det(C) \neq 0$, the system is controllable.

opg 9

for the system to be observable, the rank of the observability matrix must equal the number of states.

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} \quad \text{rank}(O) = \underline{\underline{2}}$$

thus the system <u>is</u> observable.