Q9. Ex Nb 007

Is $x^* = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ an equilibrium point?

$$\begin{cases} 0 = 5 - 5 \\ 0 = -\frac{4}{3}5 + \frac{2}{3}1 + u^* \end{cases}$$

$$\Leftrightarrow u^* = \frac{20}{3} - \frac{2}{3} = 6$$

$$\Rightarrow \boxed{x^* = \begin{bmatrix} 5 \\ 1 \end{bmatrix}} \text{ is an eq. point with associated } \boxed{u^* = 6}$$

Linear approximation around $x^* = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

Using result from Q6, we have

$$\boxed{A = \frac{\partial f}{\partial x}(x)\Big|_{x=x^*=\begin{bmatrix} 5 \\ 1 \end{bmatrix}} = \begin{bmatrix} 0 & 5 \\ -\frac{4}{3} & -6 \end{bmatrix}} \qquad \boxed{B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

Matlab command eig(A) gives 2 eigenvalues:

$$\lambda_1 = -1.4725 \quad , \quad \lambda_2 = -4.5275$$

$\Rightarrow$ the linear approximation is stable

$\Rightarrow$ the simplest feedback controller is therefore NO feedback:

$$K = 0$$

(see simulation for illustration)