use the divergenc theorem to calculate the flux of the given vector field out of the sphere S with equation: $x^2+y^2+z^2=a^2$, where $a>0$

$F = xi - 2yj + 4zk$

we use:

$$\oint\oint (\vec{F}\cdot\vec{n})\,dS = \iiint div(\vec{F})\,dV$$

$$div(\vec{F}) = \frac{df_1}{dx} + \frac{df_2}{dy} + \frac{df_3}{dz} =$$

$$= 1-2+4$$

$$\iiint 1-2+4\,dV = \iiint 3\,dV$$

we then convert to spherical.

$x = \rho \cdot \sin\phi \cdot \cos\theta$

$y = \rho \cdot \sin\phi \cdot \sin\theta$

$z = \rho \cdot \cos\phi$

$dV = \rho \cdot \sin\phi \; d\rho\, d\phi\, d\theta$

$x^2+y^2+z^2 = a^2 \Rightarrow \sqrt{x^2+y^2+z^2} = \sqrt{a^2} \Rightarrow \rho = a$

the limits are then:

$0 \le \theta \le 2\pi, \quad 0 \le \phi \le \pi, \quad 0 \le \rho \le a$

we now have our integral.

$$\int_0^a\int_0^\pi\int_0^{2\pi} 3\cdot\rho\cdot\rho\sin(\phi)\,d\theta\,d\phi\,d\rho =$$

$$= 3\int_0^a \rho^2 \int_0^\pi \sin\phi \int_0^{2\pi} 1\,d\theta\,d\phi\,d\rho$$

$$= 3\int_0^a \rho^2 \int_0^\pi \sin\phi \cdot \Big[\theta\Big]_0^{2\pi}$$

$$= 3\cdot 2\pi \int_0^a \rho^2 \int_0^\pi \sin\phi\,d\phi\,d\rho$$

$$= 6\pi \cdot \int_0^a \rho^2 \cdot \Big[-\cos\phi\Big]_0^\pi\,d\rho$$

$$= 6\pi \cdot \int_0^a \rho^2 \cdot (-1-(1))\,d\rho$$

$$= 6\pi \cdot 2 \cdot \int_0^a \rho^2\,d\rho$$

$$= 12\pi \cdot \left[\frac{\rho^3}{3}\right]_0^a$$

$$= 12\pi \cdot \frac{a^3}{3}$$

$$= \underline{\underline{4\pi a^3}}$$