ops 5.1.2

Show that the following function satisfies the properties of a joint probability mass function.

x	y	$f_{XY}(x,y)$
-1	-2	$\frac{1}{8}$
-0,5	-1	$\frac{1}{4}$
0,5	1	$\frac{1}{2}$
1	2	$\frac{1}{8}$

$$\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \frac{1}{8} = \frac{2}{8} + \frac{3}{4} = \frac{8}{8} = 1$$

a) $P(X < 0{,}5, Y < 1{,}5)$

$f_{XY}(-1,-2) + f_{XY}(-0{,}5,-1) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

b) $P(X < 0{,}5)$

$f_X(-0{,}5) + f_X(-1) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

c) $P(Y < 1{,}5)$

$f_Y(1) + f_Y(-1) + f_Y(-2) = \frac{1}{8} + \frac{1}{4} + \frac{1}{2} =$

$= \frac{7}{8}$

d) $P(X > 0{,}25, Y < 4{,}5)$

$f_{XY}(0{,}5, 1) + f_{XY}(1, 2) = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$

e)

$E[X], E[Y], V[X], V[Y]$

$$E[X] = -1 \cdot \frac{1}{8} - \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2} + 1 \cdot \frac{1}{8}$$

$$= -\frac{1}{8} - \frac{1}{8} + \frac{1}{4} + \frac{1}{8}$$

$$= \underline{\underline{\frac{1}{8}}}$$

$$E[Y] = -2 \cdot \frac{1}{8} - 1 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{8}$$

$$= -\frac{1}{4} - \frac{1}{4} + \frac{1}{2} + \frac{1}{4}$$

$$= \underline{\underline{\frac{1}{4}}}$$

$$V[X] = \left(-1 - \frac{1}{8}\right)^2 \cdot \frac{1}{8} + \left(-\frac{1}{2} - \frac{1}{8}\right)^2 \cdot \frac{1}{4} + \left(\frac{1}{2} - \frac{1}{8}\right)^2 \cdot \frac{1}{2}$$

$$+ \left(1 - \frac{1}{8}\right)^2 \cdot \frac{1}{8}$$

$$= -\frac{2^2}{8^2} \cdot \frac{1}{8} + \frac{5^2}{8^2} \cdot \frac{1}{4} + \frac{4^2}{8^2} \cdot \frac{1}{2} + \frac{2^2}{8^2} \cdot \frac{1}{8}$$

$= 0,4219$