evaluate the given line integral over the specified curve C.

$$\int_C (x,y)\,dS, \quad r = at\cdot i + bt\cdot j + ct\cdot k \qquad 0 \le t \le m$$

first we must get the parametric representation of the curve r. we get that by using the following.

$$dS = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

$$= \sqrt{a^2 + b^2 + c^2}$$

the integral can now be written as

$$\int_0^m (at + bt)\sqrt{a^2 + b^2 + c^2}\,dt =$$

$$= \int_0^m t(a+b)\cdot\sqrt{a^2 + b^2 + c^2}\,dt$$

$$= \left[\frac{t^2}{2}\cdot(a+b)\cdot\sqrt{a^2+b^2+c^2}\right]_0^m$$

$$= \frac{m^2}{2}\cdot(a+b)\cdot\sqrt{a^2+b^2+c^2}$$