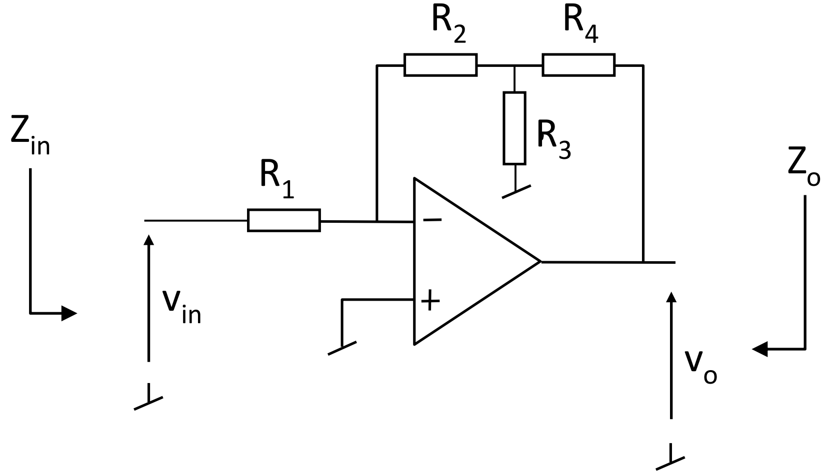
B1

Operationsforstærkeren kan regnes ideel i spørgsmål 1 til og med 4.

1. Find et bogstavudtryk og beregn en talværdi for α.
2. Find et bogstavudtryk og beregn en talværdi for β.
3. Find et bogstavudtryk og beregn en talværdi for $A_{SIGN} = v_o/v_{in}$.
4. Hvad kan fordelen være ved dette kredsløb fremfor en ganske almindelig inverterende kobling?

Operationsforstærkeren erstattes nu med µA741C, og i de følgende spørgsmål kan temperaturen regnes konstant lig 25 °C, og µA741C er forsynet med ±15 V.

5. Find den max. procentiske fejl på A_{SIGN} hidrørende fra ikke ideel åbensløjfeforstærkning.
6. Beregn en talværdi (typiske data) for Z_o.
7. Beregn minimumsværdien for Z_{in}.



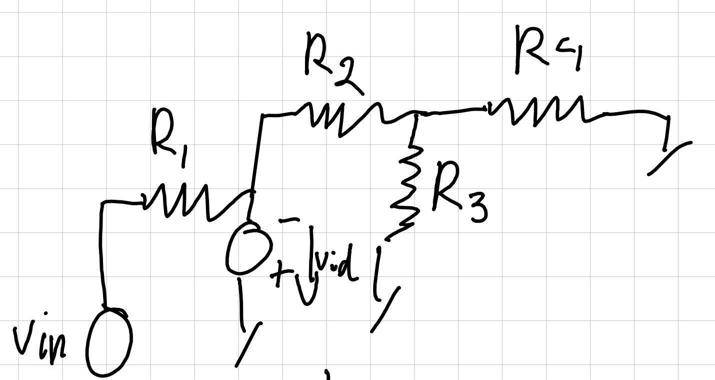
$R_1 = 1{,}0\ \text{M}\Omega$

$R_2 = 1{,}0\ \text{M}\Omega$

$R_3 = 10{,}2\ \text{k}\Omega$

$R_4 = 1{,}0\ \text{M}\Omega$

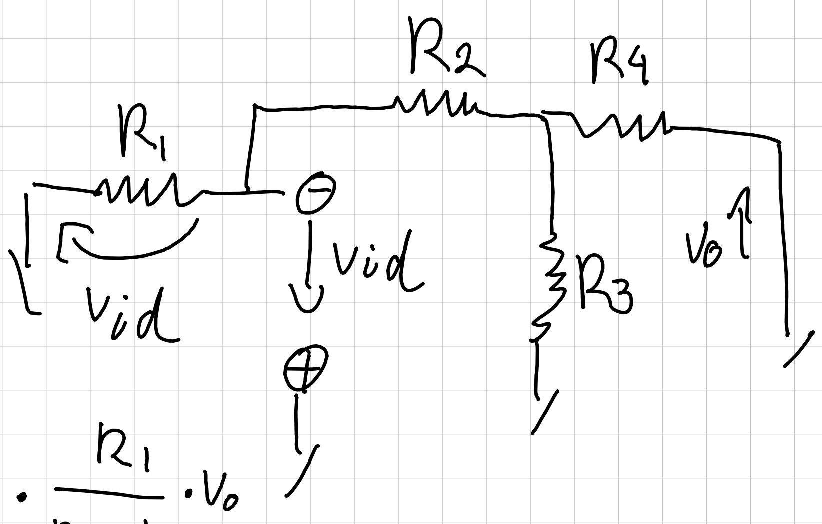
$$\alpha = \frac{V_{id}}{V_{in}}\Bigg|_{V_o=0}$$



$$\alpha = \frac{-R_2 + R_3 \| R_4}{R_1 + R_2 + R_3 \| R_4} = \frac{-1M\Omega + \frac{1}{\frac{1}{10{,}2k\Omega} + \frac{1}{1M\Omega}}}{1M\Omega + 1M\Omega + \frac{1}{\frac{1}{10{,}2k\Omega} + \frac{1}{1M\Omega}}} = -0{,}50$$

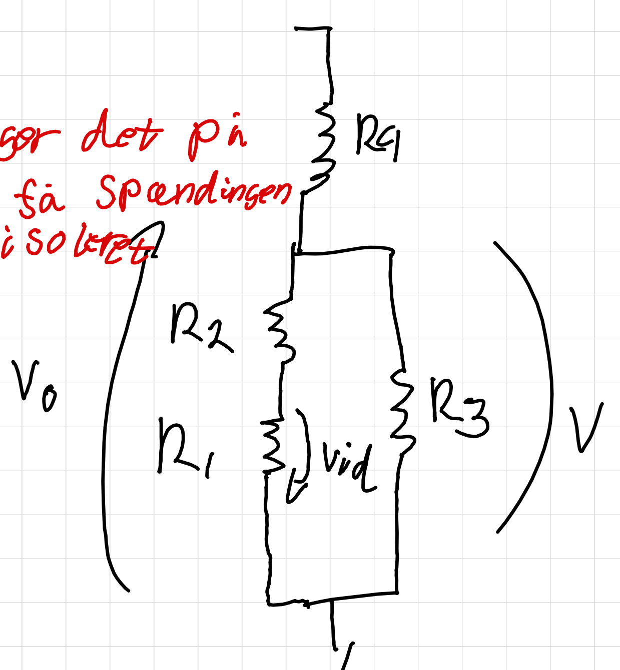
2.

$$\beta = \frac{-V_{id}}{V_o}\Bigg|_{V_{in}=0}$$



$$-V_{id} = \frac{R_3 \| (R_1+R_2)}{R_4 + R_3 \| (R_1+R_2)} \cdot \frac{R_1}{R_1+R_2} \cdot V_o$$

vi ganger det på for at få spændingen over R_1 i soleret



$$\beta = \frac{-V_{id}}{V_o} = \frac{R_3 \| (R_1+R_2)}{R_4 + R_3 \| (R_1+R_2)} =$$

$= 0{,}005$

opg 3. find bogstavsudtryk for $A_{Sign} = \frac{V_o}{V_{in}}$ altså, signal forstærkningen.

vi tager α og β fra tidligere.

$$\frac{\alpha}{\beta} = \frac{\frac{V_{id}}{V_{in}}}{\frac{-V_{id}}{V_o}} = \frac{\frac{R_3 \| (R_1+R_2)}{R_4 + R_3 \| (R_1+R_2)}}{\frac{-R_2 + R_3 \| R_4}{R_1 + R_2 + R_3 \| R_4}} = \frac{V_o}{V_{in}}$$

opg 4. hvad er fordelen ved dette kredsløb fremfor en ganske alm inverterende kobling.

fordelen er at man kun skal bruge en 1MΩ modstand, fremfor 100MΩ, hvilket er urealistisk.

opg 5 vi finder maksimal fejl med følgende formel.

$$K_f = \frac{1}{1 + \frac{1}{\beta A_{OL}}}$$

vi finder A_{OL} for μA741C i databladet. $A_{OL} = 20\frac{V}{mV} = 20 \cdot 10^3$

$$K_f = 1 - \frac{1}{1 + \frac{1}{0{,}005 \cdot 20 \cdot 10^3}} = 1 - 0{,}9901 = 0{,}9\ \%$$

opg 6. vi bruger feedback tabel til at finde z_o, altså output impedance. Og datablad ti at finde R_o, altså output modstand.

$R_o = 75\Omega$

$A_{OL} = 200 \cdot 10^3$

$$z_o = \frac{R_o}{1 + \beta \cdot A_{OL}} = 75\,m\Omega$$

opg 7. vi finder så input impedance på samme måde.

$R_{in} = 0{,}3M\Omega$ } fra datablad.

$A_v = 20 \cdot 10^3$

$\beta = 0{,}05$ – fra tidligere.

når A_{OL} er meget større end 1, så bliver den inverteret $e^- = 0V$, altså der er virtuel stel

og så bliver $R_{in} = R_1 = 1M\Omega$

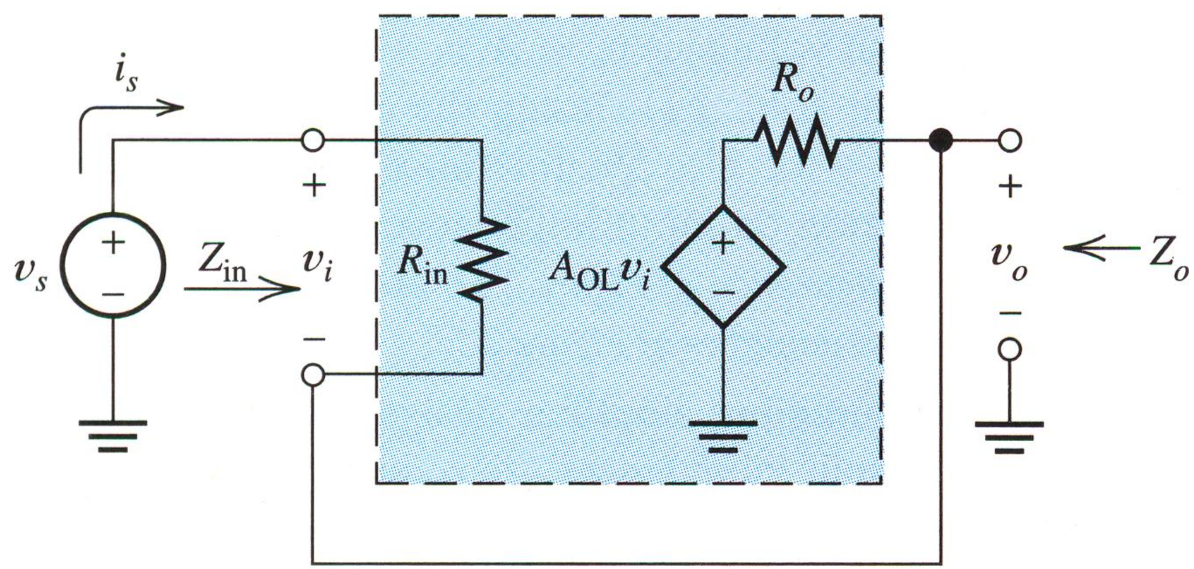
B2

The objective of this problem is to investigate the effects of finite gain, finite input impedance, and nonzero output impedance of the Op Amp on the voltage follower. Consider the circuit, including the op-amp model, shown in the Figure below.

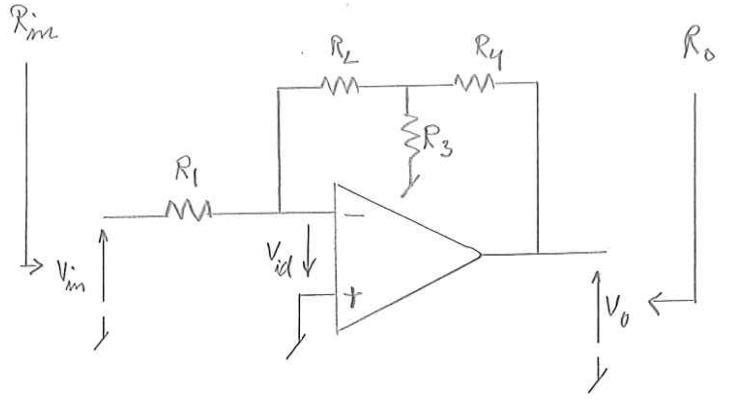
a) Derive an expression for the circuit voltage gain v_o/v_s.
Evaluate your expression for $A_{OL} = 10^5$, $R_{in} = 1\ M\Omega$, and $R_o = 25\ \Omega$. Compare this result with the circuit gain assuming an ideal Op Amp.

b) Derive an expression for the circuit input impedance $Z_{in} = v_s/i_s$. Evaluate your expression for $A_{OL} = 10^5$, $R_{in} = 1\ M\Omega$, and $R_o = 25\ \Omega$. Compare this result to the input impedance with an ideal Op Amp.

c) Derive an expression for the circuit output impedance Z_o.
Evaluate your expression for $A_{OL} = 10^5$, $R_{in} = 1\ M\Omega$, and $R_o = 25\ \Omega$. Compare this result with the output impedance of the circuit assuming an ideal Op Amp.



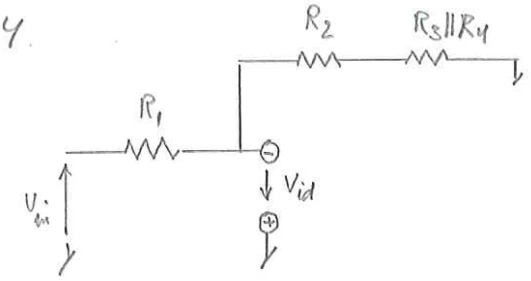
B1



$R_1 = R_2 = R_4 = 1{,}0\ M\Omega$

$R_3 = 10{,}2\ k\Omega$

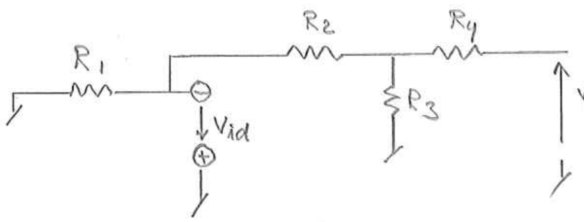
Op Amp ideel i sp. 1–4.



Sp. 1 $\quad \alpha = \left.\dfrac{V_{id}}{V_{in}}\right|_{V_o=0}$

$$\alpha = -\frac{R_2 + R_3 \| R_4}{R_1 + R_2 + R_3 \| R_4} = -\frac{10^6 + 10{,}2\cdot 10^3 \| 10^6}{10^6 + 10^6 + 10{,}2\cdot 10^3 \| 10^6} = -0{,}5025 \cong -0{,}5$$

Sp. 2 $\quad \beta = -\left.\dfrac{V_{id}}{V_o}\right|_{V_{in}=0}$



$$\beta = \frac{R_3 \| (R_1 + R_2)}{R_4 + R_3 \| (R_1 + R_2)} \cdot \frac{R_1}{R_1 + R_2} = \frac{10{,}2\cdot 10^3 \| (10^6 + 10^6)}{10^6 + 10{,}2\cdot 10^3 \| (10^6 + 10^6)} \cdot \frac{10^6}{10^6 + 10^6} = 5{,}0 \cdot 10^{-3}$$

Sp. 3 $\quad$ Op. Amp ideel $\Rightarrow \quad A_{SIGN} = \dfrac{V_o}{V_{in}} = \dfrac{\alpha}{\beta} = -\dfrac{0{,}5025}{5{,}0\cdot 10^{-3}} = -100$

Sp. 4 $\quad$ En traditionel inverterende kobling ville kræve en $100\ M\Omega$ modstand i feedback, såfremt indg. modstand på $1 M\Omega$ og $A_{SIGN} = -100$ skulle bibeholdes.

En modstand på $100 M\Omega$ er i praksis urealistisk!

Sp. 5 $\mu A741C$: $A_{OL\,min} = 20\ V/mV = 20 \cdot 10^3$

$$K_f = \frac{1}{1+\frac{1}{\beta A_{OL}}} = \frac{1}{1+\frac{1}{5{,}0\cdot 10^{-3}\cdot 20\cdot 10^3}} = 0{,}9901 \quad {}^{*)}$$

Fejl : $1 - |K_f| = 1 - 0{,}9901 = 0{,}0099 \cong \underline{\underline{1\%}}$

Sp. 6 $\mu A741C$ (typiske data): $Z_o = 75\ \Omega$; $A_{OL} = 200 \cdot 10^3$

$$R_o = \frac{Z_o}{1+\beta \cdot A_{OL}} = \frac{75}{1+5{,}0\cdot 10^{-3}\cdot 200\cdot 10^3} = 74{,}93 \cdot 10^{-3} \cong \underline{\underline{75\ m\Omega}}$$

Sp. 7 $A_{OL} >> 1 \Rightarrow e^- \cong 0\,V$ (virtuel stel) $\Rightarrow$

$R_{in} \cong R_1 = \underline{\underline{1\ M\Omega}}$

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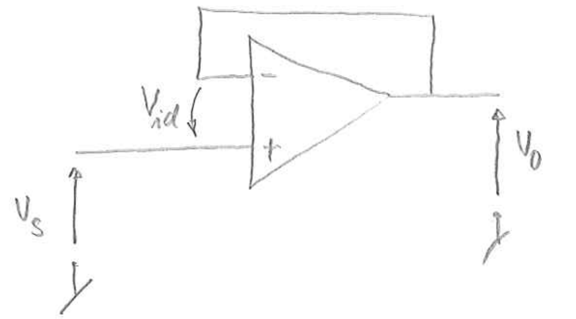
*) Her er ikke taget hensyn til fasedrejning i A_{OL}. Dette kommer senere.

(B2) $A_{OL} = 10^5$; $R_{in} = 1M\Omega$; $R_o = 25\Omega$

Op Amp koblet som spændingsfølger:



Ideelt:

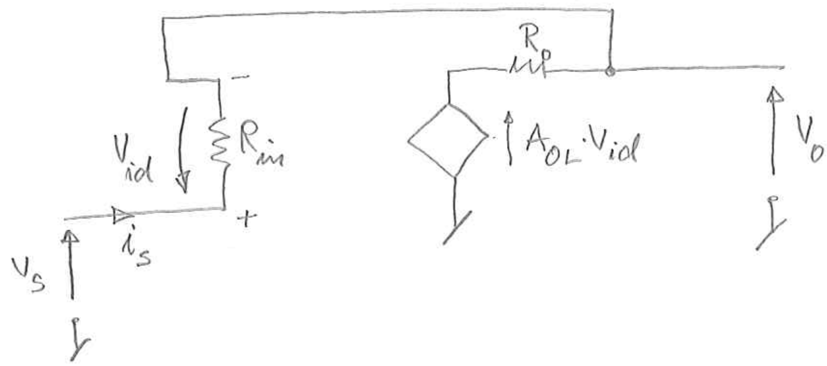
$V_o = V_s \Rightarrow$

$A_v = \frac{V_o}{V_s} = 1$

a)

Forstærkning A_v

Ækv. diagram:



$$\alpha = \left.\frac{V_{id}}{V_{in}}\right|_{V_o=0} = 1; \qquad \beta = -\left.\frac{V_{id}}{V_o}\right|_{V_s=0} = 1$$

$$A_v = \frac{\alpha}{\beta} \cdot K_f = 1 \cdot K_f \leftarrow \text{Fejl ligger i } K_f$$

$$K_f = \frac{1}{1 + \frac{1}{\beta A_{OL}}} = \frac{1}{1 + \frac{1}{1 \cdot 10^5}} = 0{,}99999000 1 \Rightarrow$$

$$A_v = \frac{\alpha}{\beta} \cdot K_f = 1 \cdot 0{,}999990001 \simeq \underline{\underline{0{,}999990}}$$

Fejl på A_v: $1 - 0{,}999990001 \simeq 10\,ppm$

Indgangsimpedans Z_{in} –

b)

$$Z_{in} = \frac{V_s}{i_s}$$

$\Sigma V = 0$ benyttes på ovr. diagram:

$$V_s = i_s \cdot R_{in} + i_s \cdot R_O + A_{OL} \cdot \underbrace{V_{id}}_{V_{id} = i_s \cdot R_{in}} = i_s (R_{in} + R_O + A_{OL} \cdot R_{in}) \Rightarrow$$

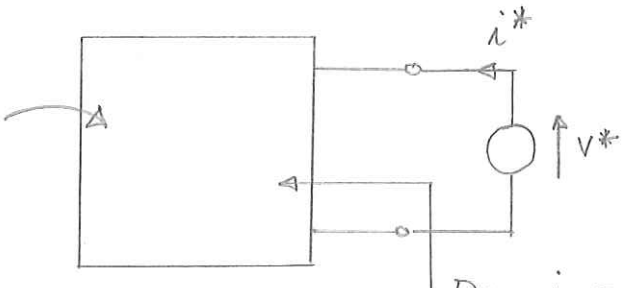
$$Z_{in} = \frac{V_s}{i_s} = R_{in} + R_o + A_{OL} \cdot R_{in} = R_o + R_{in}(1 + A_{OL}) = 25 + 10^6(1 + 10^5) = \underline{\underline{10^{11}\,\Omega}}$$

$Z_{in_{ideel}} \to \infty$

Udgangsimpedans Z_o –

c) Samme princip som når man finder Thevenin imp.:

Nulstil alle uafhængige kilder

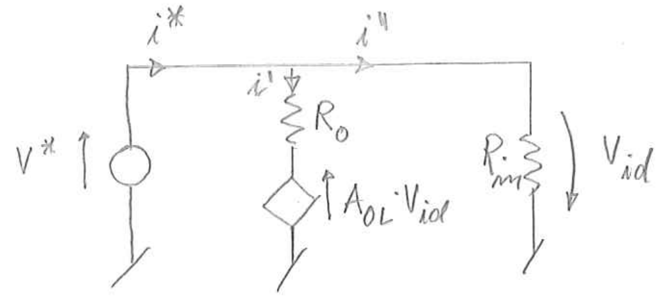


Den impedans, man her kigger ind i, er således udgangs-impedansen (Thevenin), dvs.:

$$Z_o = \left.\frac{V^*}{i^*}\right|_{V_s = 0}$$

V_s er den afhængige kilde

Ekv. diagram med $V_s = 0$:



NB! $V_{id} = -V^*$

$$① \quad i^* = i' + i''$$

$$\left.\begin{array}{l} i' = \dfrac{V^* - A_{OL} \cdot V_i}{R_0} = \dfrac{V^* + A_{OL} \cdot V^*}{R_0} \\[2ex] i'' = \dfrac{V^*}{R_{in}} \end{array}\right\} \rightarrow ①$$

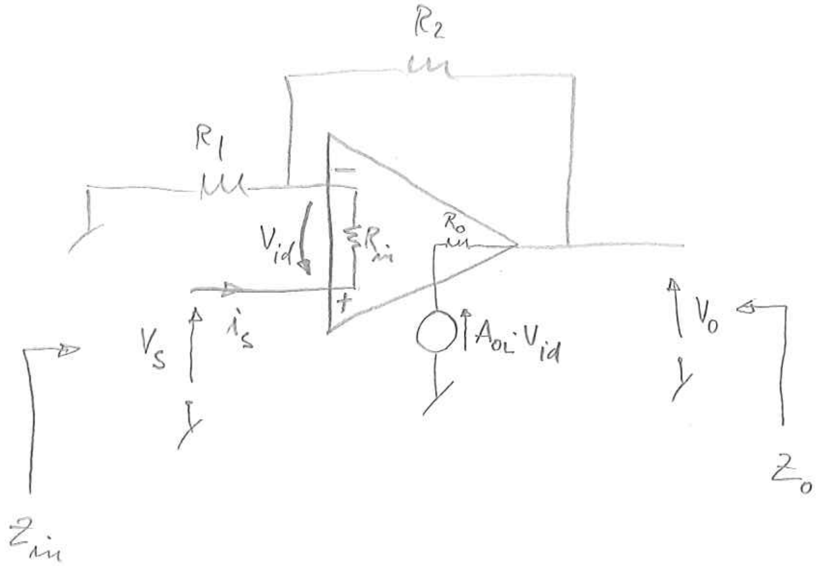
$$i^* = \frac{V^* + A_{OL} \cdot V^*}{R_0} + \frac{V^*}{R_{in}} \Rightarrow$$

$$\frac{i^*}{V^*} = \frac{1}{z_0} = \frac{1 + A_{OL}}{R_0} + \frac{1}{R_{in}} \Rightarrow$$

$$z_0 = \frac{R_0}{1 + A_{OL}} \,\Big\|\, R_{in} \simeq \left.\frac{R_0}{1 + A_{OL}}\right|_{\frac{R_0}{1 + A_{OL}} \ll R_{in}} = \frac{25}{1 + 10^5} = \underline{\underline{250\ \mu\Omega}}$$

$z_{0_{ideal}} = 0\ \Omega$

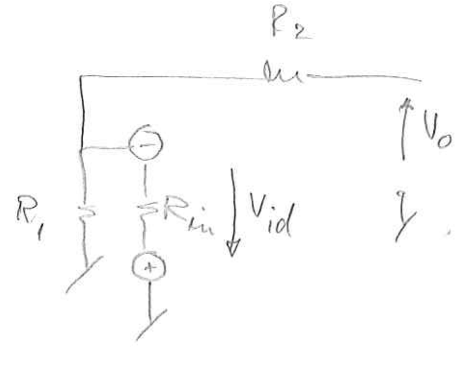
Effekt af negativ feedback på Z_o og Z_{in}



Spændingsfølgeren er blot et specialtilfælde af ovenstående ikke-inverterende kobling med $R_2 = 0\,\Omega$ og $R_1 \to \infty$.

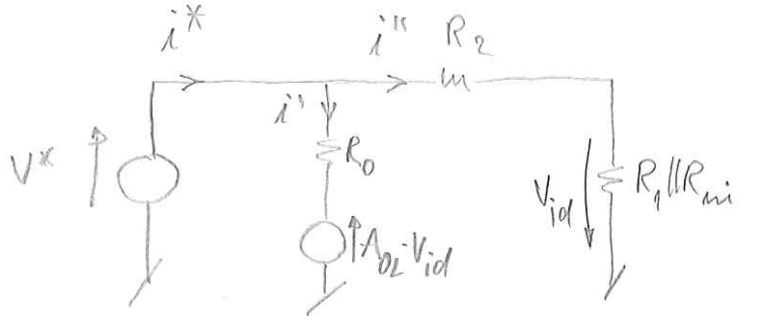
Hvad er β for ovenstående?

$$\beta = -\frac{V_{id}}{V_o}\Big|_{V_S=0}$$



$$\beta = \frac{R_1 \| R_{in}}{R_2 + R_1 \| R_{in}}$$

Z_o ~ Theveninimpedansen



$$Z_o = \frac{v^*}{i^*}\Big|_{V_S=0}$$

$$① \quad i^* = i' + i''$$

$$\left.\begin{aligned} i' &= \frac{V^* - A_{OL} \cdot V_{id}}{R_0} \\ V_{id} &= -\frac{R_1 \| R_{in}}{R_2 + R_1 \| R_{in}} \cdot V^* = -\beta \cdot V^* \end{aligned}\right\} \rightarrow i' = \frac{V^* + A_{OL} \cdot \beta \cdot V^*}{R_0} \quad ②$$

$$i'' = \frac{V^*}{R_2 + R_1 \| R_{in}} \quad ③$$

② og ③ indsat i ① $\rightarrow$

$$i^* = \frac{V^* + \beta A_{OL} \cdot V^*}{R_0} + \frac{V^*}{R_2 + R_1 \| R_{in}} \Rightarrow$$

$$\frac{i^*}{V^*} = \frac{1}{Z_0} = \frac{1 + \beta A_{OL}}{R_0} + \frac{1}{R_2 + R_1 \| R_{in}} \Rightarrow$$

$$\left\{ \begin{aligned} &\frac{1}{R_X} = \frac{1}{R_A} + \frac{1}{R_B} \Rightarrow \\ &R_X = R_A \| R_B \end{aligned} \right.$$

$$\underline{\underline{Z_0 = \frac{R_0}{1 + \beta A_{OL}} \Big\| \left(R_2 + R_1 \| R_{in}\right) \cong \frac{R_0}{1 + \beta A_{OL}}}} \Bigg|_{(R_2 + R_1 \| R_{in}) \gg \frac{R_0}{1 + \beta A_{OL}}}$$

$$Z_{in} = \frac{V_s}{i_s}$$

(Her ses bort fra R_0)

$$V_{id} = e^+ - e^- = V_s - \beta V_0 \cong V_s - \beta \cdot A_{OL} \cdot V_{id} \Rightarrow$$

$$\left.\begin{aligned} &V_{id}(1 + \beta A_{OL}) = V_s \\ &\uparrow \\ &V_{id} = i_s \cdot R_{in} \end{aligned}\right\} \rightarrow i_s \cdot R_{in}(1 + \beta A_{OL}) = V_s \Rightarrow$$

$$Z_{in} = \frac{V_s}{i_s} = \underline{\underline{R_{in}(1 + \beta A_{OL})}}$$