

find the mass of the wire along a curve:

$$r = 3ti + 3t^2j + 2t^3k, (0 \leq t \leq 1)$$

if the density at $r(t)$ is $1+t$ ^{grams} / _{unit length}

We have the limit $0 \leq t \leq 1$ and the function $f(r(t)) = 1+t$

We use:

$$\int_0^1 f(r(t)) ds = \int_0^1 1+t ds$$

we then find ds with:

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt =$$

$$\frac{dx}{dt} = 3 \text{ and } \frac{dy}{dt} = 6t \text{ and } \frac{dz}{dt} = 6t^2$$

$$ds = \sqrt{3^2 + (6t)^2 + (6t^2)^2} = \sqrt{9 + 36t^2 + 36t^4}$$

we insert into the formula.

$$\begin{aligned} & \int_0^1 (1+t) \cdot \sqrt{9 + 36t^2 + 36t^4} dt \\ &= \left[\frac{3t \sqrt{(1+2t^2)^2} \cdot (6 + 3t + 4t^2 + 3t^3)}{6 + 12t^2} \right]_0^1 \\ &= \underline{\underline{8}} \text{ g} \end{aligned}$$