

# Finding a s estimationer

$$\hat{\beta}: \hat{\beta}_1 \sim N(?, ?)$$

$$E[\hat{\beta}_1] = \frac{\sum (x_i - \bar{x})(\beta_0 + \beta_1 x_i)}{\sum (x_i - \bar{x})^2}$$

$$= \beta_0$$

$$V[\hat{\beta}_1] = \frac{\sum (x_i - \bar{x}) V[Y_i]}{(\sum (x_i - \bar{x})^2)^2}$$

$$= \frac{\sigma^2}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

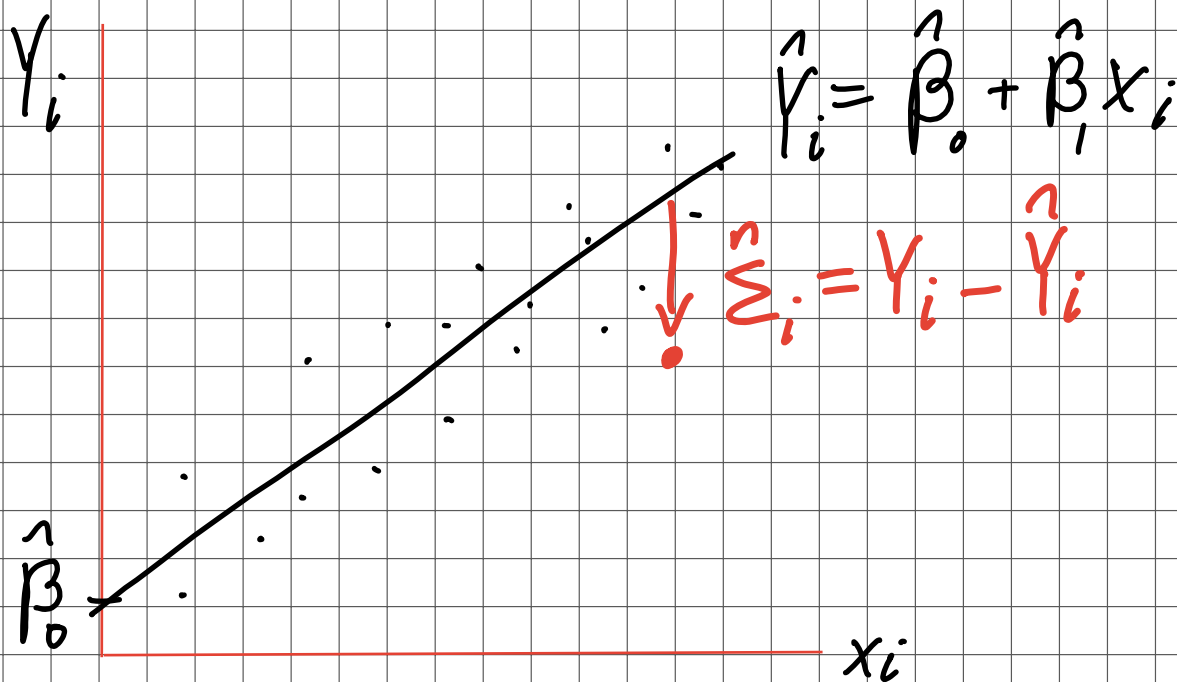
$$= \frac{\sigma^2}{S_{xx}}$$

$$\hat{\beta}_1 \sim N(\beta_1, \frac{\sigma^2}{S_{yx}})$$

Så vi vil gerne have så lille varians som muligt

$$\hat{\beta}_0 \sim N(\beta_0, \sigma^2 \left( \frac{1}{n} + \frac{\sigma^2}{S_{yx}} \right))$$

$$\hat{\sigma}^2 \sim \frac{\sigma^2}{n-2} \chi_{n-2}^2$$



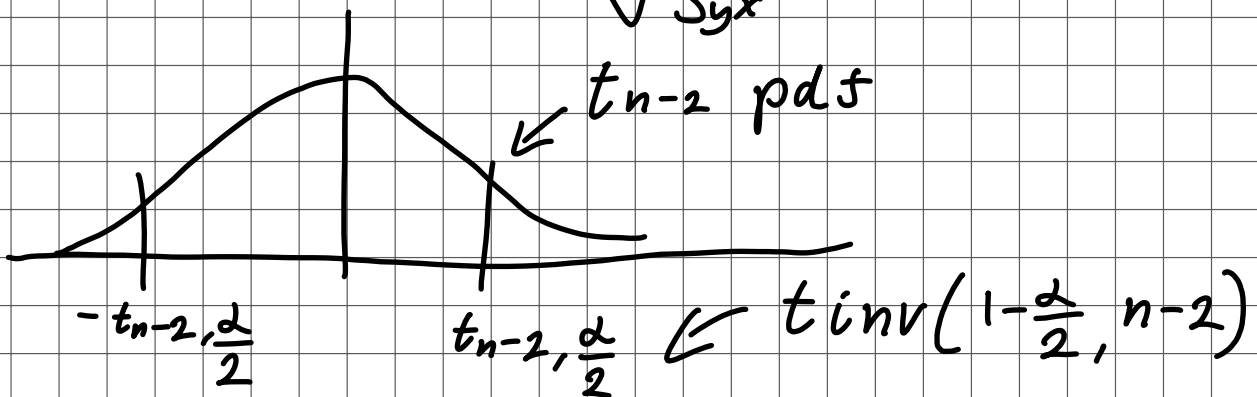
# Simple Linear regression

test statistic

$$T_1 = \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}} \sim t_{n-2}$$

$H_0: \beta_1 = \beta_1^*$  indsæt data i  $H_0$

$$H_1: \beta_1 \neq \beta_1^* \quad T_1 = \frac{\hat{\beta}_1 - \beta_1^*}{\sqrt{\frac{\sigma^2}{S_{yx}}}} = t_1$$



Forcast  $H_0 \rightarrow |t_1| > t_{n-2, \frac{\alpha}{2}}$

$$P\left(-t_{n-2, \frac{\alpha}{2}} < \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}} < t_{n-2, \frac{\alpha}{2}}\right) = 1 - \alpha$$

$100(1-\alpha)\% \text{ CI for } \beta_1$

$$\left[ \hat{\beta}_1 \pm t_{n-2, \frac{\alpha}{2}} \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}} \right]$$

