opg 1

find $\iint_S x\, ds$ over the part of the parabolic cylinder $z=\frac{x^2}{2}$ that lies inside the first octant part of the cylinder $x^2+y^2=1$

convert to cylindrical

vi take $0 \le \frac{\pi}{2}$ since we work in the first octant.

$0 \le \theta \le \frac{\pi}{2}$

$0 \le r \le 1$

fra formel

Where $ds = \sqrt{1+(g_1(x,y))^2+(g_2(x,y))^2}\, dx\, dy$

and $ds = |n|$

$g_1 = \left(\frac{dz}{dx}\right)^2 = x \qquad g_2 = \left(\frac{dz}{dy}\right)^2 = 0$

$ds = \sqrt{1+x^2}\, dx\, dy = x \cdot 1 \cdot \sqrt{1+x^2}$

Limits.

cylinder $= x^2+y^2=1$ radius of cylinder

therefore $0 \le x \le 1$

and $y^2 = 1-x^2 = y = \sqrt{1-x^2} \qquad 0 \le y \le \sqrt{1-x^2}$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} x \cdot 1 \cdot \sqrt{1+x^2}\, dy\, dx =$$

$$= \int_0^1 x\sqrt{1+x^2} \cdot \Big[y \Big]_0^{\sqrt{1-x^2}} dx$$

$$= \int_0^1 x\sqrt{1+x^2} \cdot \sqrt{1-x^2}\, dx$$

$$= \int_0^1 x\sqrt{1-x^4}\, dx$$

$$= \int_0^1 x \cdot \sqrt{1-t^2} \qquad t=x^2, \quad \frac{dt}{dx}=2x, \quad \frac{1}{2}dt = xdx$$

$$= \int_0^1 \sqrt{1-t^2} \cdot \frac{1}{2} dt$$

the x in the original function then goes away

$\int \sqrt{1-y^2}\, dt = \sin^{-1}(t) + u \cdot \sqrt{1-t^2}$

$$= \int_0^1 \sqrt{1-t^2} \cdot \frac{1}{2} dt$$

$$= \frac{1}{2} \cdot \left[\sin^{-1}(t) + u \cdot \sqrt{1-t^2} \right]_0^1$$

$$= \frac{1}{2} \cdot \left[\sin^{-1}(x^2) + x^2 \cdot \sqrt{1-x^2} \right]_0^1$$

$$= \frac{1}{2} \cdot \left(\sin^{-1}(1^2) + 1^2 \cdot \sqrt{1-(1^2)} - (\sin^{-1}(0)) \right)$$

$$= \frac{1}{2} \cdot \left(\frac{\pi}{2} + 1 \cdot \sqrt{0} - 0 \right)$$

$$= \frac{\pi}{4}$$