OPG 1.

$$\dot{p}_1 = p_1 p_2 - p_1$$
$$\dot{p}_2 = -\frac{4}{3} p_1 p_2 + \frac{2}{3} p_2 + u \quad , \quad y = p_1$$

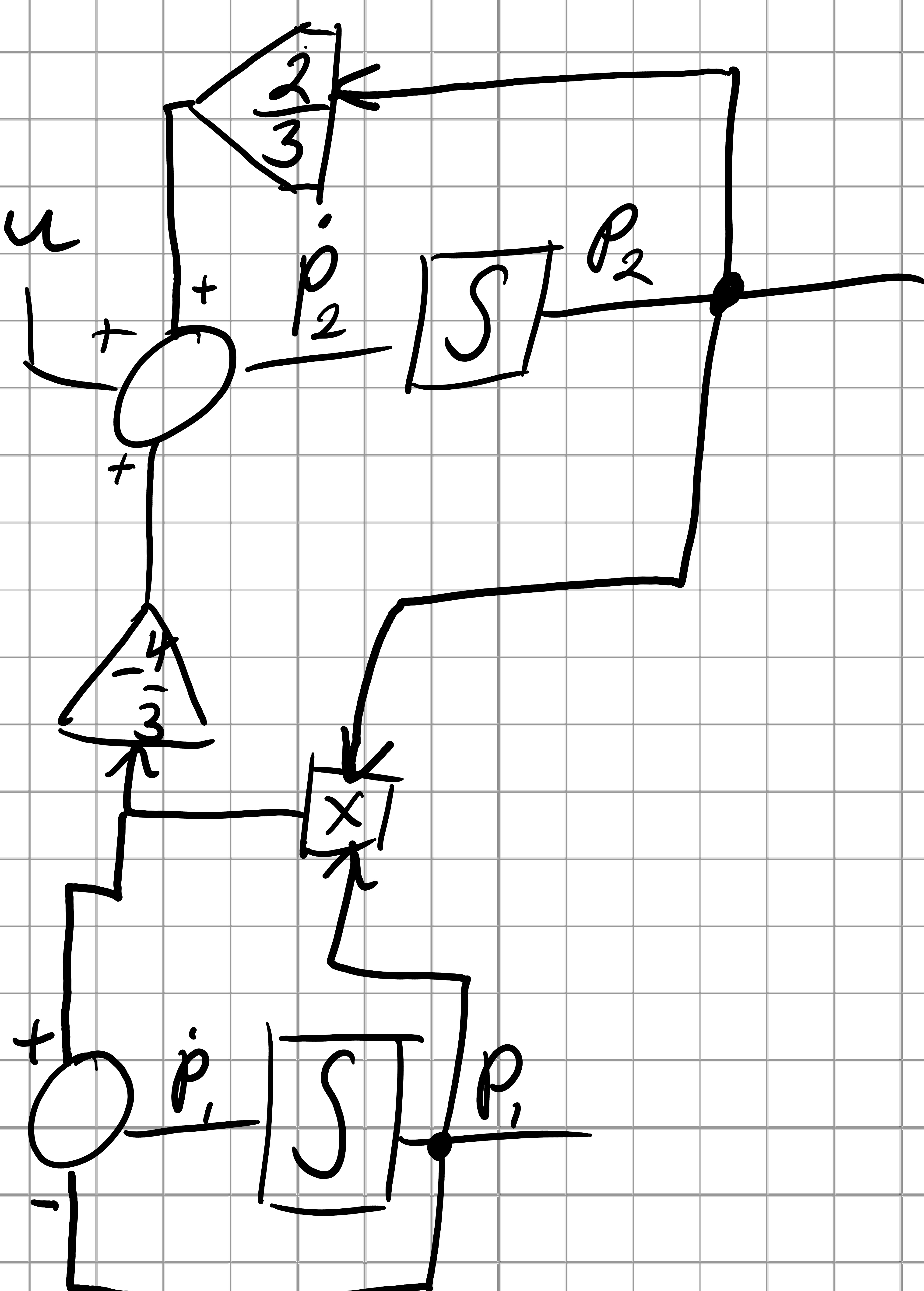
2 coupled first order diffs
nonlinear becaus of $p_1 p_2$

$$F(x,u) = \begin{bmatrix} x_1 x_2 - x_1 \\ -\frac{4}{3} x_1 x_2 + \frac{2}{3} x_2 + u \end{bmatrix}$$

Opg 2.

$$\dot{p}_1 = p_1 p_2 - p_1$$

$$\dot{p}_2 = -\frac{4}{3} p_1 p_2 + \frac{2}{3} p_2 + u$$



OPG 5.

$$\dot{p}_1 = p_1 p_2 - p_1 \quad 1 \qquad , u = 0$$

$$\dot{p}_2 = -\frac{4}{3} p_1 p_2 + \frac{2}{3} p_2 + u \quad 2$$

$$\Downarrow$$

$$0 = p_1 p_2 - p_1 \Rightarrow \frac{p_1}{p_1} = \frac{p_1 p_2}{p_1} \Rightarrow \underline{1 = p_2}$$

We Substitute $p_2 = 1$

$$0 = -\frac{4}{3} p_1 \cdot 1 + \frac{2}{3} \cdot 1 + 0 \Rightarrow 0 = -\frac{4}{3} p_1 + \frac{2}{3} \Rightarrow$$

$$p_1 = \frac{\frac{2}{3}}{-\frac{4}{3}} \Rightarrow -\frac{2 \cdot 3}{4 \cdot 3} = p_1 \Rightarrow \underline{\frac{1}{2} = p_1}$$

Opg 6.

$$\begin{cases} \dot{x} = x_1 x_2 - x_1 \\ \dot{x}_2 = -\frac{4}{3}x_1 x_2 + \frac{2}{3}x_2 + u \end{cases}$$

We use the Jacobian:

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} = x_2 - 1 & \frac{\partial f_1}{\partial x_2} = x_1 \\ \frac{\partial f_2}{\partial x_1} = -\frac{4}{3}x_2 & \frac{\partial f_2}{\partial x_2} = -\frac{3}{4} + \frac{2}{3} \end{bmatrix} \quad B = \begin{bmatrix} \frac{\partial f_1}{\partial u} = 0 \\ \frac{\partial f_2}{\partial u} = 1 \end{bmatrix}$$

$$A = \begin{bmatrix} x_2 - 1 & x_1 \\ -\frac{4}{3}x_2 & -\frac{3}{4} + \frac{2}{3} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

approx around $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\downarrow$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & \frac{2}{3} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

approx around $\begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{4}{3} & 0 \end{bmatrix}$$

OPS 9

$p_1 = 5$, $p_2 = 1$

$\dot{p}_1 = p_1 p_2 - p_1$

$\dot{p}_2 = -\frac{4}{3} p_1 p_2 + \frac{2}{3} p_2 + u$

$\Downarrow$

$0 = -\frac{4}{3} \cdot 5 \cdot 1 + \frac{2}{3} \cdot 1 + u$

$\Downarrow$

$\frac{18}{3} = u \Rightarrow \underline{\underline{6 = u}}$

to determine stability, we get the determinant of the Jacobian.

$$A = \frac{df}{dx} = \begin{bmatrix} x_2 - 1 & x_1 \\ -\frac{4}{3}x_2 & -\frac{4}{3}x_1 + \frac{2}{3} \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 5 \\ -\frac{4}{3} & -\frac{4}{3}\cdot 5 + \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ -\frac{4}{3} & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\mathrm{eig}(A) = -1.47, \; -4.53$$

as the eigenvalues are strictly negative, the System is Stable.