## CSCB63

Design and Analysis of Data Structures

# **Topics**

- Worst Case Complexity
- Balanced Search Trees
- Graphs and Graph Traversals
- Priority Queues and Heaps
- Disjoint Sets
- Amortized Complexity
- Average Case Analysis
- Hashing

## **Term Work**

### **Assignments**

- 4 each worth 5% for a total of 20%
- Basic understanding questions
- More challenging questions
- Only some of the questions will be graded
- Applied question (programming in C)

## Term tests (3)

- 60min each held in Wednesday lecture
- Worth 15%, 15%, 15%
- Test 1 covers A1, Test 2 covers A2, Test 3 covers A3

#### **Final Exam**

– worth 35%

# **Textbook Options**

<u>Algorithm Design: Foundations, Analysis, and Internet Examples.</u>

Michael Goodrich and Roberto Tamassia, John Wiley & Sons (2002), ISBN:0471383651.

## Introduction to Algorithms (2nd edition).

Cormen, Leiserson, Rivest, Stein McGraw-Hill (2001),

ISBN:0070131511.

Free online access for U of T students:

http://main.library.utoronto.ca/eir/resources.cfm

# **Course Design**

- Each week there may be *pre-lecture preparation* and *pre-lecture exercises*.
- In lectures we will have a mixture of *slides* and *practice*.
- Completing the *pre-lecture work* will maximize your learning in this course.
- It will enable you to make the best use of lecture time.

## Resources

### **Communication**

- Piazza: you will get an invite all updates posted here.
- Website: mathlab.utsc.utoronto.ca/bretscher/b63
- Office Hours: Anna, TAs on calendar

- Calendar: google calendar has all dates
- U of T email: I will send you important updates to your email.

## **ADTs vs Data Structures**

What is a data structure? ADT?

Worst Case Complexity.

- Asymptotic Notation
  - Big "Oh"
  - Big "Omega"
  - Big "Theta"

## **ADTs and Data Structures**

What does *ADT* stand for?

Abstract Data Type

What is an *ADT*?

A set of *objects* together with a set of *operations*.

Examples

Objects: integers

Operations ADD(x,y), SUBTRACT(x,y), MULTIPLY(x,y), QUOTIENT(x,y), REMAINDER(x,y)

Objects: stacks

Operations: PUSH(S,x), POP(S), EMPTY(S)

## **Data Structures**

#### What is a data structure?

A data structure is an *implementation* of an *ADT*.

#### This includes

- a way to represent objects and
- algorithms for the operations

### Example

Objects: A stack could be implemented by either of

- a singly-linked list
- an array with a counter to keep track of the "top."

Operations: How would you implement PUSH, POP and EMPTY in each of these implementations?

## **Motivation**

### Why are *ADTs* important?

- Important for specification.
- Provides modularity:
  - Usage *depends* only on the *definition*, not on the *implementation*
  - implementation of the ADT can be changed (corrected or improved) without changing the rest of the program
- Reusability
  - an abstract data type can be implemented once, and used in many different programs

# **Key Points**

An *ADT* is a way to describe *WHAT* the data is and *WHAT* you can do with it.

A *data structure* is a way to describe *HOW* the objects are implemented and *HOW* the operations are performed.

## **Analysis of Data Structures and Algorithms**

The complexity of an algorithm is the amount of resources it uses.

How do we *quantify* this?

We express the resources as a function of the *size* of the input.

### Types of resources:

- Running time
- Space (memory)
- Number of logic gates (in a circuit)
- Area (in a VLSI) chip
- Messages or bits communicated (in a network)

# **Running Time**

*Input size* will depend on the *type* 

### **Examples**

Strings: number of characters

Lists: number of elements

Graphs: number of vertices and edges

The running time T(n) of an algorithm for input of size n is the number of primitive operations or "steps" executed.

What are some examples of steps?

The notion of "step" should be machine independent.

## **Worst-Case Complexity**

How do we *measure* the running time of an algorithm in terms of *input size* when there may be *many different inputs* of the same size?

- Average-Case Complexity
  We'll explore this later...
- 2. Worst-Case complexity

For an algorithm A, let t(x) be the number of steps A takes on input x.

Then, the *worst-case time complexity* of *A* on input of size *n* is

$$T_{wc}(n) = \max_{|x|=n} \{t(x)\}$$

#### *In words:*

Look at all inputs of size n and take the time of the one that is the worst (slowest).

## **Calculating the Worst Case**

How can we determine the number of steps of an algorithm?

We could count every computational step...

tiresome and machine dependent

What do we really care about?

- We care about when the input size is very big.
- We would like a measure that is *proportional to the input size* and not the machine.

#### Food for thought...

ullet Given an unsorted list of length  ${\cal H}$ , would you rather use an algorithm that takes a multiple of

$$n$$
 steps, or  $n^2$  steps, or  $n \log n$  steps, or  $n \log n^2$  steps?

## **For Tomorrow**

Remind yourself how insertion sort works.