Лабораторная работа №4 Светлаковой Ульяны 1-ПМИ
Вариант №54

(1) $$\prod_{k=1}^{\infty}\left(1-\frac{1}{(3\ k)^2}\right) \to \frac{3 \cdot \sqrt{3}}{2 \cdot \pi} = 0.827$$

(2) $$\sum_{k=1}^{\infty} e^{-3\,k} \to \frac{1}{e^3-1} = 0.052$$

(3) $$\lim_{x \to 0} \frac{\sin(5\ x)}{\sin(9\ x)} \to \frac{5}{9} \qquad \lim_{x \to 7} \frac{2-\sqrt{x-3}}{x^2-49} \to -\frac{1}{56}$$

(4) $$\lim_{x \to 0^+} x \cdot \ln(x) \to 0 \qquad \lim_{x \to 0^+} (\tan(x))^x \to 1$$

(5) $$y(x) := 2^x\ \ln(x)$$

$$\frac{\mathrm{d}^1}{\mathrm{d}x^1} y(x) \to \frac{(x \cdot \ln(2) \cdot \ln(x)+1) \cdot 2^x}{x}$$

$$y(x) := \cot(x)\ \operatorname{atan}(x)$$

$$\frac{\mathrm{d}^1}{\mathrm{d}x^1} y(x) \to \frac{\cot(x)}{x^2+1} - \left(\operatorname{atan}(x) \cdot \cot(x)^2 + \operatorname{atan}(x)\right)$$

$$y(x) := x \cdot \sqrt{1-x^2} + \operatorname{asin}(x)$$

$$\frac{\mathrm{d}^1}{\mathrm{d}x^1} y(x) \to \frac{-\left(2 \cdot x^2\right)+2}{\sqrt{-x^2+1}}$$

(6) $f(x) := \dfrac{\cos(x)}{2-\sin(x)}$

$x := 0 \qquad \dfrac{d^1}{dx^1} f(x) \rightarrow \dfrac{1}{4}$

$x := 1 \qquad \dfrac{d^2}{dx^2} f(x) \rightarrow \dfrac{-\left(2 \cdot \cos(1) \cdot \sin(1)^2\right) + 2 \cdot \cos(1) \cdot \sin(1) + \left(4 \cdot \cos(1) - 2 \cdot \cos(1)^3\right)}{\sin(1)^3 - 6 \cdot \sin(1)^2 + 12 \cdot \sin(1) - 8}$

$x := 2 \qquad \dfrac{d^3}{dx^3} f(x) \rightarrow \dfrac{10 \cdot \sin(2)^3 + \left(8 \cdot \cos(2)^2 - 48\right) \cdot \sin(2)^2 + \left(72 - 8 \cdot \cos(2)^2\right) \cdot \sin(2) + \left(6 \cdot \cos(2)^4 - 16 \cdot \cos(2)^2 - 32\right)}{\sin(2)^4 - 8 \cdot \sin(2)^3 + 24 \cdot \sin(2)^2 - 32 \cdot \sin(2) + 16} + 2$

(7) $\int ctan^3\ (u)\, du \rightarrow \dfrac{ctan^3 \cdot u^2}{2}$

$\int (5\ u + 3) \cdot \sqrt{u^2 + 3\ u + 5}\, du \rightarrow \dfrac{\left(304128 \cdot u^4 + 1824768 \cdot u^3 + 4732992 \cdot u^2 + 5987520 \cdot u + 3094740\right) \cdot \ln\left(2 \cdot \sqrt{u^2 + 3 \cdot u + 5} - (2 \cdot u + 3)\right) - \left(81920 \cdot \right.}{49152 \cdot u^4 + 294912 \cdot u^3 + 764928} \ldots$

(8) $\int_0^{2\pi} \sin(3\ u)\, du \rightarrow 0 \qquad \int_0^{\infty} e^{-u} \cos(5\ u)\, du \rightarrow \dfrac{1}{26} \qquad \int_0^1 (\ln(u))^5\, du \rightarrow -120$

(9)

$\tan(t) \xrightarrow{series,\, 10} t + \dfrac{1}{3} \cdot t^3 + \dfrac{2}{15} \cdot t^5 + \dfrac{17}{315} \cdot t^7 + \dfrac{62}{2835} \cdot t^9$

$\operatorname{acos}(t) \xrightarrow{series,\, 10} \dfrac{\pi}{2} - t - \dfrac{1}{6} \cdot t^3 - \dfrac{3}{40} \cdot t^5 - \dfrac{5}{112} \cdot t^7 - \dfrac{35}{1152} \cdot t^9$

(10) $g(q, b) := \dfrac{1-b}{\sqrt{b}}\ q^2 - 2\ q + \sqrt{b} \qquad q(b) := \dfrac{\sqrt{b}}{1-\sqrt{b}} \qquad g(q, b) \xrightarrow{simplify} \dfrac{\sqrt{b} \cdot (b-1)}{2 \cdot b \cdot \sqrt{\dfrac{-b+1}{\sqrt{b}}} + \left(b^{\frac{3}{2}} - \left(\sqrt{b} + b\right)\right)} + \dfrac{\left(\sqrt{2} - 1\right) \cdot \sqrt{b} + \sqrt{2}}{\sqrt{2} - 1}$

(11) (a)

$$n^5-3\,n^4+\left(a-b^3\right)\,n^3+3\,\left(b^3-a\right)\,n^2-ab^3\,\,n+3\,ab^3\xrightarrow{solve\,,n}\begin{bmatrix}3\\ \dfrac{\sqrt{2\cdot\sqrt{b^6-2\cdot a\cdot b^3+4\cdot ab^3+a^2}+\left(2\cdot b^3-2\cdot a\right)}}{2}\\ \dfrac{-\sqrt{2\cdot\sqrt{b^6-2\cdot a\cdot b^3+4\cdot ab^3+a^2}+\left(2\cdot b^3-2\cdot a\right)}}{2}\\ \dfrac{\sqrt{-\left(2\cdot\sqrt{b^6-2\cdot a\cdot b^3+4\cdot ab^3+a^2}\right)+\left(2\cdot b^3-2\cdot a\right)}}{2}\\ \dfrac{-\sqrt{-\left(2\cdot\sqrt{b^6-2\cdot a\cdot b^3+4\cdot ab^3+a^2}\right)+\left(2\cdot b^3-2\cdot a\right)}}{2}\end{bmatrix}$$

(b)

$$n^{10}-1\xrightarrow{solve}\begin{bmatrix}e^{\frac{4\cdot\pi\cdot 1i}{5}}\\ e^{\frac{3\cdot\pi\cdot 1i}{5}}\\ e^{\frac{2\cdot\pi\cdot 1i}{5}}\\ e^{\frac{\pi\cdot 1i}{5}}\\ 1\\ \dfrac{1}{e^{\frac{\pi\cdot 1i}{5}}}\\ \dfrac{1}{e^{\frac{2\cdot\pi\cdot 1i}{5}}}\\ \dfrac{1}{e^{\frac{3\cdot\pi\cdot 1i}{5}}}\\ \dfrac{1}{e^{\frac{4\cdot\pi\cdot 1i}{5}}}\\ -1\end{bmatrix}$$

(12)

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A(a,b,n) := for i ∈ 0..(n-1)
              A[i,i] ← -a
              if i < n-1
                A[i,i+1] ← 1
              if i > 0
                A[i,i-1] ← b
            A
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C(n) := for i ∈ 0..n-1
          if (mod(i,2) > 0)
            C[i] ← 2
          else
            C[i] ← 1
        C
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$$A(a,b,2) \rightarrow \begin{bmatrix} -a & 1 \\ b & -a \end{bmatrix} \qquad C(2) \rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\det(A(a,b,2)) \rightarrow -b+a^2 \qquad A(a,b,2)^{-1} \cdot C(2) \rightarrow \begin{bmatrix} \frac{a}{b-a^2}+\frac{2}{b-a^2} \\ \frac{b}{b-a^2}+\frac{2 \cdot a}{b-a^2} \end{bmatrix}$$

$$A(a,b,4) \rightarrow \begin{bmatrix} -a & 1 & 0 & 0 \\ b & -a & 1 & 0 \\ 0 & b & -a & 1 \\ 0 & 0 & b & -a \end{bmatrix} \qquad C(4) \rightarrow \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

$$\det(A(a,b,4)) \rightarrow b^2-3 \cdot a^2 \cdot b+a^4$$

$$A(a,b,4)^{-1} \cdot C(4) \rightarrow \begin{bmatrix} \frac{2 \cdot a \cdot b-a^3}{b^2-3 \cdot a^2 \cdot b+a^4}+\left(\frac{2 \cdot b-2 \cdot a^2}{b^2-3 \cdot a^2 \cdot b+a^4}-\left(\frac{a}{b^2-3 \cdot a^2 \cdot b+a^4}+\frac{2}{b^2-3 \cdot a^2 \cdot b+a^4}\right)\right) \\ \frac{b^2-a^2 \cdot b}{b^2-3 \cdot a^2 \cdot b+a^4}+\left(\frac{2 \cdot a \cdot b-2 \cdot a^3}{b^2-3 \cdot a^2 \cdot b+a^4}-\left(\frac{a^2}{b^2-3 \cdot a^2 \cdot b+a^4}+\frac{2 \cdot a}{b^2-3 \cdot a^2 \cdot b+a^4}\right)\right) \\ -\frac{a \cdot b^2}{b^2-3 \cdot a^2 \cdot b+a^4}+\left(\frac{a \cdot b-a^3}{b^2-3 \cdot a^2 \cdot b+a^4}+\frac{2 \cdot b-2 \cdot a^2}{b^2-3 \cdot a^2 \cdot b+a^4}-\frac{2 \cdot a^2 \cdot b}{b^2-3 \cdot a^2 \cdot b+a^4}\right) \\ -\frac{b^3}{b^2-3 \cdot a^2 \cdot b+a^4}+\left(\frac{b^2-a^2 \cdot b}{b^2-3 \cdot a^2 \cdot b+a^4}+\frac{4 \cdot a \cdot b-2 \cdot a^3}{b^2-3 \cdot a^2 \cdot b+a^4}-\frac{2 \cdot a \cdot b^2}{b^2-3 \cdot a^2 \cdot b+a^4}\right) \end{bmatrix}$$

$$A(a,b,8) \rightarrow \begin{bmatrix} -a & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ b & -a & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & b & -a & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & b & -a & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & b & -a & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & b & -a & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & b & -a & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & b & -a \end{bmatrix} \qquad C(8) \rightarrow \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

$$\det(A(a,b,8)) \rightarrow b^4 - 10 \cdot a^2 \cdot b^3 + 15 \cdot a^4 \cdot b^2 - 7 \cdot a^6 \cdot b + a^8 \qquad A(a,b,8)^{1} \cdot C(8) \rightarrow \begin{bmatrix} -a+2 \\ b+(1-2 \cdot a) \\ 2 \cdot b+(2-a) \\ b+(1-2 \cdot a) \\ 2 \cdot b+(2-a) \\ b+(1-2 \cdot a) \\ 2 \cdot b+(2-a) \\ b-2 \cdot a \end{bmatrix}$$

$$A(a,b,16) \rightarrow \begin{bmatrix} -a & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b & -a & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b & -a & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b & -a & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b & -a & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b & -a & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b & -a & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b & -a & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & -a & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & -a & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & -a & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & -a & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & -a & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & -a & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & -a & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b & -a \end{bmatrix} \qquad C(16) \rightarrow \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

$$\det(A(a,b,16)) \rightarrow b^8 - 36 \cdot a^2 \cdot b^7 + 210 \cdot a^4 \cdot b^6 - 462 \cdot a^6 \cdot b^5 + 495 \cdot a^8 \cdot b^4 - 286 \cdot a^{10} \cdot b^3 + 91 \cdot a^{12} \cdot b^2 - 15 \cdot a^{14} \cdot b + a^{16}$$

и так далее ...