

$$3) T(n) = 2T\left(\frac{n}{2}\right) + 1, T(1) = 1$$

$$T(n) = 2T\left(\frac{n}{2}\right) + 1, T(1) = 1 \quad \sum_{i=0}^{k-1} 2^i = 2^k - 1$$

$$T(n) = 2(2T\left(\frac{n}{4}\right) + 1) + 1$$

$$T(n) = 4T\left(\frac{n}{4}\right) + 2 + 1$$

$$T(n) = 4(2T\left(\frac{n}{8}\right) + 1) + 2 + 1$$

$$T(n) = 8T\left(\frac{n}{8}\right) + 4 + 2 + 1$$

$$T(n) = 8(2T\left(\frac{n}{16}\right) + 1) + 4 + 2 + 1$$

$$T(n) = 16T\left(\frac{n}{16}\right) + 8 + 4 + 2 + 1$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + \left(\sum_{i=0}^{k-1} 2^i\right)$$

$$\frac{n}{2^k} = 1, n = 2^k, k = \log_2 n$$

$$T(n) = 2^{\log_2 n} (T\left(\frac{n}{2^{\log_2 n}}\right)) + (2^{\log_2 n} - 1)$$

$$= n(T(1)) + (n - 1)$$

$$= n + n - 1$$

$$= 2n - 1$$

$$\boxed{O(n)}$$

Substituting Equations

$$n \rightarrow n/2$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n/2}{2}\right) + 1$$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n/4}{2}\right) + 1$$

$$T\left(\frac{n}{8}\right) = 2T\left(\frac{n/8}{2}\right) + 1$$