

5.

Use the iteration technique to find a Big-Oh bound for the recurrence relation below.

Note you may find the following mathematical result helpful:  $2^{\log_3 n} = n^{\log_3 2}$  ,

$$\sum_{i=0}^{\infty} (2/3)^i = 3$$

$$T(n) = 2T(n/3) + cn \quad T(1) = 1$$

$$T(\frac{n}{3}) = 2T(\frac{n}{9}) + c(\frac{n}{3})$$

$$T(\frac{n}{9}) = 2T(\frac{n}{27}) + c(\frac{n}{9})$$

$$T(\frac{n}{27}) = 2T(\frac{n}{81}) + c(\frac{n}{27})$$

$$T(n) = 2T(\frac{n}{3}) + cn, T(1) = 1$$

$$T(n) = 2^{\log_3 n} + 3cn$$

$$T(n) = 2(2T(\frac{n}{9}) + c(\frac{n}{3})) + cn$$

$$T(n) = 4T(\frac{n}{9}) + c(\frac{2n}{3}) + cn$$

$$T(n) = 4(2T(\frac{n}{27}) + c(\frac{n}{9})) + c(\frac{2n}{3}) + cn$$

$$T(n) = 8T(\frac{n}{27}) + c(\frac{4n}{9}) + c(\frac{2n}{3}) + cn$$

$$T(n) = 2^k T(\frac{n}{3^k}) + cn(\sum_{i=0}^{k-1} (\frac{2}{3})^i)$$

$$k = \log_3 n$$

$$O(n)$$