Don't forget to go through the more recurrence relation examples pdf file to see more examples. That may help you with some answers.

Practice with Recurrence Relations

Solve the following recurrence relations using the iteration technique:

1) $T(n) = T(n-1) + 2, \quad T(1) = 1$

$T(n) = T(n-1) + 2, \quad T(1) = 1$

$T(n) = T(n-2) + 2 + 2$

$T(n) = T(n-3) + 2 + 2 + 2$

$T(n) = T(n-k) + 2k$

$n - k = 1, \quad k = n - 1$

$T(n) = T(n-(n-1)) + 2(n-1)$

$= T(1) + 2(n-1)$

$= 1 + 2n - 2$

$= 2n - 1$

$\boxed{O(n)}$

Substituting Equations

$T(n-1) = T(n-2) + 2$

$T(n-2) = T(n-3) + 2$

2) $T(n) = 2T(n/2) + n, \quad T(1) = 1$

$T(n)=2T(\frac{n}{2})+n, \quad T(1)=1$

$T(n)=2(2T(\frac{n}{4})+\frac{n}{2})+n$

$T(n)=4T(\frac{n}{4})+n+n$

$T(n)=4(2T(\frac{n}{8})+\frac{n}{4})+n+n$

$T(n)=8T(\frac{n}{8})+n+n+n$

$T(n)=8(2T(\frac{n}{16})+\frac{n}{8})+n+n+n$

$T(n)=16T(\frac{n}{16})+n+n+n+n$

$T(n)=2^kT(\frac{n}{2^k})+kn$

$\frac{n}{2^k}=1, \quad n=2^k, \quad k=\log_2 n$

$T(n)=2^{\log_2 n}(T(\frac{n}{2^{\log_2 n}}))+n\log_2 n$

$T(n)=n(T(\frac{n}{n}))+n\log_2 n$

$=n(T(1))+n\log_2 n$

$=n+n\log_2 n$

$$\boxed{O(n\log_2 n)}$$

<u>Substituting Equations</u>

$T(\frac{n}{2})=2T(\frac{n/2}{2})+\frac{n}{2}$

$T(\frac{n}{4})=2T(\frac{n/4}{2})+\frac{n}{4}$

$T(\frac{n}{8})=2T(\frac{n/8}{2})+\frac{n}{8}$

3) $T(n) = 2T\left(\frac{n}{2}\right) + 1, T(1) = 1$

$T(n) = 2T\left(\frac{n}{2}\right) + 1, \quad T(1) = 1$

$T(n) = 2\left(2T\left(\frac{n}{4}\right) + 1\right) + 1$

$T(n) = 4T\left(\frac{n}{4}\right) + 2 + 1$

$T(n) = 4\left(2T\left(\frac{n}{8}\right) + 1\right) + 2 + 1$

$T(n) = 8T\left(\frac{n}{8}\right) + 4 + 2 + 1$

$T(n) = 8\left(2T\left(\frac{n}{16}\right) + 1\right) + 4 + 2 + 1$

$T(n) = 16T\left(\frac{n}{16}\right) + 8 + 4 + 2 + 1$

$T(n) = 2^k T\left(\frac{n}{2^k}\right) + \left(\sum_{i=0}^{k-1} 2^i\right)$

$\sum_{i=0}^{k-1} 2^i = 2^k - 1$

$\frac{n}{2^k} = 1, \quad n = 2^k, \quad k = \log_2 n$

$T(n) = 2^{\log_2 n}\left(T\left(\frac{n}{2^{\log_2 n}}\right)\right) + \left(2^{\log_2 n} - 1\right)$

$= n(T(1)) + (n - 1)$

$= n + n - 1$

$= 2n - 1$

$\boxed{O(n)}$

Substituting Equations

$n \rightarrow n/2$

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n/2}{2}\right) + 1$

$T\left(\frac{n}{4}\right) = 2T\left(\frac{n/4}{2}\right) + 1$

$T\left(\frac{n}{8}\right) = 2T\left(\frac{n/8}{2}\right) + 1$

4) $T(n) = T(n-1) + n, T(1) = 1$

$T(n)=T(n-1)+n, \quad T(1)=1$

$T(n)=T(n-2)+(n-1)+n$

$T(n)=T(n-3)+(n-2)+(n-1)+n$

$T(n)=T(n-4)+(n-3)+(n-2)+(n-1)+n$

$T(n)=T(n-k)+(n-k+1)+(n-k+2)+\dots+n$

$T(n)=T(n-k)+\left(\sum_{i=0}^{k-1}(n-k+1+i)\right)$

$k=n-1$

$T(n)=T(1)+\left(\sum_{i=0}^{n-2}(2+i)\right)$

$2\sum_{i=0}^{n-2}1 + \sum_{i=0}^{n-2} i = 2(n-1)+\frac{(n-2)(n-1)}{2}$

$=\frac{(n-2)(n-1)+4(n-1)}{2}$

$=\frac{(n-1)(n+2)}{2}$

$T(n)=1+\frac{(n-1)(n+2)}{2}$

$T(n)=\frac{n^2+n}{2}$

$\boxed{O(n^2)}$

Substituting Equations

$n \rightarrow n-1$

$T(n-1)=T(n-2)+(n-1)$

$T(n-2)=T(n-3)+(n-2)$

$T(n-3)=T(n-4)+(n-3)$

5.

Use the iteration technique to find a Big-Oh bound for the recurrence relation below.

Note you may find the following mathematical result helpful: $2^{\log_3 n} = n^{\log_3 2}$,

$\sum_{i=0}^{\infty} (2/3)^i = 3$

$T(n) = 2T(n/3) + cn$ $\quad T(1) = 1$

$T(\frac{n}{3}) = 2T(\frac{n/3}{3}) + c(\frac{n}{3})$

$T(\frac{n}{9}) = 2T(\frac{n/9}{3}) + c(\frac{n}{9})$

$T(\frac{n}{27}) = 2T(\frac{n/27}{3}) + c(\frac{n}{27})$

$T(n) = 2T(\frac{n}{3}) + cn, \; T(1) = 1$

$T(n) = 2(2T(\frac{n}{9}) + c(\frac{n}{3})) + cn$

$T(n) = 4T(\frac{n}{9}) + c(\frac{2n}{3}) + cn$

$T(n) = 4(2T(\frac{n}{27}) + c(\frac{n}{9})) + c(\frac{2n}{3}) + cn$

$T(n) = 8T(\frac{n}{27}) + c(\frac{4n}{9}) + c(\frac{2n}{3}) + cn$

$T(n) = 2^k T(\frac{n}{3^k}) + cn(\sum_{i=0}^{k-1} (\frac{2}{3})^i)$

$k = \log_3 n$

$T(n) = 2^{\log_3 n} + 3cn$

$\boxed{O(n)}$