



7F341 FEA (Ex 6A)

1) Are the formulae linear? (no exponents no multiplication of variables)

a) $x_1 + 5x_2 - \sqrt{2}x_3 = 1$ Linear

b) $x_1 + 3x_2 + x_1x_2 = 2$ Not Linear $\rightarrow (x_1, x_2)$

c) $x_1 = 7x_2 + 3x_3 \Rightarrow x_1 + 7x_2 - 3x_3 = 0$ Linear

d) $x_1^{-2} + x_2 + 8x_3 = 5$ Not Linear $\rightarrow x_1^{-2}$

e) $x_1^{3/5} - 2x_2 + x_3 = 4$ Not Linear \Rightarrow exponent

f) $\pi x_1 - \sqrt{2}x_3 = 7^{1/3}$ Linear

2. In each part, determine whether the equation is linear in x and y .

a. $2^{1/3}x + \sqrt{3}y = 1$ b. $2x^{1/3} + 3\sqrt{y} = 1$

c. $\cos\left(\frac{\pi}{7}\right)x - 4y = \log 3$ d. $\frac{\pi}{7} \cos x - 4y = 0$

e. $xy = 1$ f. $y + 7 = x$

a) Linear b) not Linear, variables have exponents

c) Linear d) Not Linear

e) Not Linear f) Linear

3. Using the notation of Formula (7), write down a general linear system of

- a. two equations in two unknowns.
- b. three equations in three unknowns.
- c. two equations in four unknowns.

a)

$$a_{11}x_1 + a_{12}x_2 = b_1$$

$$a_{21}x_1 + a_{22}x_2 = b_2$$

b) $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

c) $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$$

Augmented matrix

a) $\begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{bmatrix}$

b) $\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix}$

c) $\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & b_2 \end{bmatrix}$

In each part of Exercises 5–6, find a system of linear equations in the unknowns x_1, x_2, x_3, \dots , that corresponds to the given augmented matrix.

5. a. $\begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 0 & -2 & 5 \\ 7 & 1 & 4 & -3 \\ 0 & -2 & 1 & 7 \end{bmatrix}$

6. a. $\begin{bmatrix} 0 & 3 & -1 & -1 & -1 \\ 5 & 2 & 0 & -3 & -6 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 0 & 1 & -4 & 3 \\ -4 & 0 & 4 & 1 & -3 \\ -1 & 3 & 0 & -2 & -9 \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix}$

5-a) $2x_1 = 0$

$3x_1 - 4x_2 = 0$

$x_2 = 0$

b) $3x_1 + 0x_2 - 2x_3 = 5$

$7x_1 + x_2 + 4x_3 = -3$

$-2x_2 + x_3 = 7$

6-a) $0x_1 + 3x_2 - x_3 - x_4 = -1$

$5x_1 + 2x_2 + 0x_3 - 3x_4 = -6$

b) $3x_1 + 0x_2 + x_3 - 4x_4 = 3$

$-4x_1 + 0x_2 + 4x_3 + x_4 = -3$

$-x_1 + 3x_2 + 0x_3 - 2x_4 = -9$

$-x_4 = -2$

In each part of Exercises 7–8, find the augmented matrix for the linear system.

7. a. $-2x_1 = 6$
 $3x_1 = 8$
 $9x_1 = -3$

b. $6x_1 - x_2 + 3x_3 = 4$
 $5x_2 - x_3 = 1$

c. $2x_2 - 3x_4 + x_5 = 0$
 $-3x_1 - x_2 + x_3 = -1$
 $6x_1 + 2x_2 - x_3 + 2x_4 - 3x_5 = 6$

8. a. $3x_1 - 2x_2 = -1$
 $4x_1 + 5x_2 = 3$
 $7x_1 + 3x_2 = 2$

b. $2x_1 + 2x_3 = 1$
 $3x_1 - x_2 + 4x_3 = 7$
 $6x_1 + x_2 - x_3 = 0$

c. $x_1 = 1$
 $x_2 = 2$
 $x_3 = 3$

Written by
 Name of
 Author

7-a) $\begin{bmatrix} -2 & 6 \\ 3 & 8 \\ 9 & -3 \end{bmatrix}$

b) $\begin{bmatrix} 6 & -1 & 3 & 4 \\ 3 & 5 & -1 & 1 \end{bmatrix}$

8-a) $\begin{bmatrix} 3 & -2 & -1 \\ 4 & 5 & 3 \\ 7 & 3 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 0 & 2 & 1 \\ 3 & -1 & 4 & 7 \\ 6 & 1 & -1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 2 & 0 & -3 & 1 & 0 \\ -3 & -1 & 1 & 0 & 0 & -1 \\ 6 & 2 & -1 & 2 & -3 & 6 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

In Exercises 1-2, determine whether the given matrix is elementary.

1. a. $\begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$

b. $\begin{bmatrix} -5 & 1 \\ 1 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2. a. $\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{3} \end{bmatrix}$

b. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$

d. $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Set 1.5

A matrix is elementary if you can perform one elementary row operation to get an identity.

1. a) Elementary

$$\Leftrightarrow R_2 + (-5R_1)$$

b) Not elementary, needs row inversion and an elementary operation

c) Not elementary, can't get an identity

d) Elementary, $R_1/2$

Not needs two operations

2.)

a) elementary
 $\Leftrightarrow R_2/\sqrt{3}$

b) elementary
 \Leftrightarrow one row inversion

c) elementary
 $\Leftrightarrow R_2 - R_2 \cdot 9$

d) Not elementary

In Exercises 3–4, find a row operation and the corresponding elementary matrix that will restore the given elementary matrix to the identity matrix.

3. a. $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} -7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4. a. $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

c. $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 & -\frac{1}{7} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

In Exercises 5–6 an elementary matrix E and a matrix A are given. Identify the row operation corresponding to E and verify that the product EA results from applying the row operation to A .

5. a. $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}, A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

a) $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} + 3R_2$

positive $3R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X$

b) $\begin{bmatrix} -7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \frac{1}{7}$



$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix} \quad 5R_1$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^4 \quad \text{inversion}$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4) a) $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} + 3R_1$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \cdot \frac{1}{3}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \text{interchange}$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & -\frac{1}{7} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \frac{1}{7}R_3$

In Exercises 5–6 an elementary matrix E and a matrix A are given. Identify the row operation corresponding to E and verify that the product EA results from applying the row operation to A .

5. a. $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

c. $E = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

6. a. $E = \begin{bmatrix} -6 & 0 \\ 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

b)

$R_3 + 3R_2$

$$\left[\begin{array}{ccccc} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ -1 & 9 & 2 & 18 & 8 \end{array} \right] \xrightarrow{\text{result}} \left[\begin{array}{ccccc} 2 & -1 & 0 & 4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ -1 & 9 & 4 & -12 & -8 \end{array} \right]$$

Let's go from Δ to Δ'

a) Row inversion

$$\left[\begin{array}{ccccc} 3 & -6 & -6 & -6 \\ -1 & -2 & 5 & -1 \end{array} \right]$$

Row

$$\begin{array}{r} 0 \\ 1 \\ -1 \\ 3 \\ \hline 3 = 3 \end{array} \quad \begin{array}{r} 1 \\ 0 \\ -1 \\ 3 \\ \hline -1 \\ 0 = -1 \end{array}$$

Col 1

$$\begin{array}{r} -2 \\ -6 \\ 0 \\ -6 \\ \hline -2 \\ -6 \\ -2 + 0 = -2 \end{array}$$

Col 3

$$\begin{array}{r} 5 \\ -6 \\ 0 \\ -6 \\ \hline 5 \\ 6 \\ 5 \\ 0 = 5 \end{array}$$

Col 4

$$\begin{array}{r} -1 \\ -6 \\ 0 \\ -6 \\ \hline -1 \\ -6 \\ -1 \\ 0 = -1 \end{array}$$

Row 1

$$\begin{array}{r} 1 \\ 0 \\ 0 \\ 2 \\ 1 \\ 2 \\ \hline 2 \\ 0 \\ 0 = 2 \end{array}$$

Row 2

$$\begin{array}{r} 0 \\ 1 \\ 0 \\ 2 \\ 1 \\ 2 \\ \hline 0 \\ 1 \\ 0 = 1 \end{array}$$

Row 3

$$\begin{array}{r} 0 \\ -3 \\ 1 \\ 2 \\ 1 \\ 2 \\ \hline 0 \\ -3 \\ 2 = -1 \end{array}$$

Col 1

$$\begin{array}{r} -1 \\ 3 \\ 0 \\ -1 \\ 0 \\ 0 \\ \hline -1 \\ 0 \\ 0 = -1 \end{array}$$

Col 2

$$\begin{array}{r} 0 \\ -1 \\ 1 \\ 0 \\ -1 \\ 0 \\ \hline 0 \\ -1 \\ 0 = -1 \end{array}$$

Col 3

$$\begin{array}{r} 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ -1 \\ 1 = 0 \end{array}$$

Col 4

$$\begin{array}{r} -4 \\ 5 \\ 3 \\ -4 \\ 0 \\ 0 \\ \hline -4 \\ 5 \\ 3 = -4 \end{array}$$

Col 5

$$\begin{array}{r} -4 \\ 3 \\ 1 \\ -4 \\ 0 \\ 0 \\ \hline -4 \\ 3 \\ 1 = -4 \end{array}$$

$$\begin{array}{r} 0 \\ -1 \\ 1 \\ 3 \\ 1 \\ 0 \\ \hline 0 \\ -1 \\ 1 = 4 \end{array}$$

$$\begin{array}{r} -4 \\ 5 \\ 3 \\ -15 \\ 3 \\ 0 \\ \hline -4 \\ 5 \\ 3 = -12 \end{array}$$

$$\begin{array}{r} -4 \\ 3 \\ 1 \\ -9 \\ 1 \\ 0 \\ \hline -4 \\ 3 \\ 1 = 8 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To get from I_3 $R_1 + 4R_3$

$$\text{Row 1} \quad \begin{array}{c} 1 \\ 0 \\ 4 \\ \hline 1 - 3 \\ 1 - 0 = 13 \end{array} \quad \text{Row 2} \quad \begin{array}{c} 0 \\ 1 \\ 0 \\ \hline 1 - 3 \\ 0 - 0 = 2 \end{array} \quad R_3 \quad \begin{array}{c} 0 \\ 0 \\ 1 \\ \hline 1 - 3 \\ 0 - 3 = 3 \end{array}$$

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \quad 4R_3$$

$$\begin{bmatrix} 13 & 28 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \quad \text{result}$$

$$\text{Col 1} \quad \begin{array}{c} 45 \\ 40 \\ 24 \\ \hline 28 \end{array} \quad \text{Col 2} \quad \begin{array}{c} 45 \\ 05 \\ 0 \\ \hline 5 \end{array} \quad \text{Col 3} \quad \begin{array}{c} 45 \\ 0 \\ 0 \\ \hline 6 \end{array}$$

$$\begin{bmatrix} 13 & 28 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

In Exercises 5–6 an elementary matrix E and a matrix A are given. Identify the row operation corresponding to E and verify that the product EA results from applying the row operation to A .

5. a. $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & 6 & 6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 4 & 0 & 1 & 3 & -1 \\ -9 & -2 & 18 & 8 & 8 \end{bmatrix}$

c. $E = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

6. a. $E = \begin{bmatrix} -6 & 0 \\ 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

→ The operation that takes you from I_n to E .

$$\begin{bmatrix} 13 & 28 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

C) $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{Row } 2 \leftarrow R_2 + 4R_3} \text{ get } I_3 \quad R_2 + 4R_3 \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \quad 4R_3 \Rightarrow \begin{bmatrix} 13 & 28 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

Col 1 $\frac{R_2 \leftarrow 1}{\begin{array}{c} 1 \\ 1 \\ 1 \end{array}} \quad \frac{R_{3 \leftarrow 2}}{\begin{array}{c} 0 \\ 1 \\ 2 \end{array}} = 2 \quad \frac{R_{3 \leftarrow 3}}{\begin{array}{c} 0 \\ 0 \\ 1 \end{array}} = 3$

Col 2 $\frac{4 \ 5 \ 6}{4 \ 0 \ 24} = 28 \quad \frac{4 \ 5 \ 6}{0 \ 5 \ 0} = 5 \quad \frac{4 \ 5 \ 6}{0 \ 0 \ 6} = 6$

In Exercises 5–6 an elementary matrix E and a matrix A are given. Identify the row operation corresponding to E and verify that the product EA results from applying the row operation to A .

5. a. $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

c. $E = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

6. a. $E = \begin{bmatrix} -6 & 0 \\ 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

6a) result

$$\begin{bmatrix} 3 & -6 & -6 & -6 \\ -1 & -2 & 5 & -1 \end{bmatrix} \checkmark$$

6b) result

$$\begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ -6 & 1 & -1 & 21 & 19 \\ 2 & 0 & 1 & 3 & 1 \end{bmatrix} \checkmark$$

$$E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ Row interchange } I \xrightarrow{\quad} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} -4R_1 \text{ From } I_3$$

$$A = \begin{bmatrix} -1 & -2 & 5 & 1 \\ 3 & -6 & -6 & -6 \end{bmatrix} \xrightarrow{\quad} \begin{bmatrix} 3 & -6 & -6 & -6 \\ -1 & -2 & 5 & 1 \end{bmatrix} \checkmark$$

$$A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & 1 \end{bmatrix} -4R_1 \Rightarrow \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ -8 & 1 & 0 & 21 & 19 \\ 2 & 0 & 1 & 3 & 1 \end{bmatrix}$$

$$c_1 \quad \begin{array}{r} R_1 \\ 0 & 1 \\ \hline -1 & 3 \\ \hline 0 & 3 \end{array} \quad \begin{array}{r} R_2 \\ 1 & 0 \\ \hline -1 & 0 \\ \hline -1 & 0 \end{array} = -1$$

$$c_2 \quad \begin{array}{r} R_1 \\ -2 & 6 \\ \hline 0 & -6 \end{array} = 6 \quad \begin{array}{r} R_2 \\ -2 & 6 \\ \hline -2 & 0 \\ \hline -2 & 0 \end{array} = -2$$

$$c_3 \quad \begin{array}{r} R_1 \\ 5 & -6 \\ \hline 0 & -6 \end{array} = -6 \quad \begin{array}{r} R_2 \\ 5 & -6 \\ \hline 5 & 0 \\ \hline 0 & 0 \end{array} = 5$$

$$c_4 \quad \begin{array}{r} R_1 \\ -1 & -6 \\ \hline 0 & -6 \end{array} \quad \begin{array}{r} R_2 \\ -1 & -6 \\ \hline -1 & 0 \\ \hline -1 & 0 \end{array} = -1$$

R_1	R_2	R_3
$\begin{array}{ccc} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & 0 \end{array}$	$\begin{array}{ccc} -4 & 1 & 6 \\ 2 & -1 & 2 \\ -8 & 1 & 0 \end{array}$	$\begin{array}{ccc} 0 & 0 & 1 \\ 2 & 1 & 2 \\ 0 & 0 & 2 \end{array}$
$\begin{array}{ccc} -1 & -3 & 0 \\ -10 & 0 & 0 \\ 0 & 0 & 0 \end{array}$	$\begin{array}{ccc} -1 & -3 & 0 \\ 4 & -3 & 0 \\ 0 & -1 & 1 \end{array}$	$\begin{array}{ccc} -1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}$
$\begin{array}{ccc} 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}$	$\begin{array}{ccc} 0 & -1 & 0 \\ 4 & 5 & 3 \\ 16 & 5 & 0 \end{array}$	$\begin{array}{ccc} 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{array}$
$\begin{array}{ccc} -4 & 5 & 3 \\ -4 & 0 & 0 \\ -4 & 0 & 0 \end{array}$	$\begin{array}{ccc} -4 & 5 & 3 \\ 16 & 5 & 0 \\ 16 & 3 & 0 \end{array}$	$\begin{array}{ccc} -4 & 5 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{array}$
$\begin{array}{ccc} -4 & 3 & 1 \\ -4 & 0 & 0 \\ -4 & 0 & 0 \end{array}$	$\begin{array}{ccc} -4 & 3 & 1 \\ 16 & 3 & 0 \\ 16 & 3 & 0 \end{array}$	$\begin{array}{ccc} -4 & 3 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{array}$

$$\text{c. } E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

I_3 R_2

$$R \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \cdot 5 \Rightarrow \begin{bmatrix} 1 & 4 \\ 10 & 25 \\ 3 & 6 \end{bmatrix} \quad R_{\text{new}} H \begin{bmatrix} 1 & 4 \\ 10 & 25 \\ 3 & 6 \end{bmatrix}$$

$$R_1 \quad R_2 \quad R_3$$

$$\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

$$C_1 \quad \frac{1 \ 2 \ 3}{1 \ 0 \ 0} = 1 \quad \frac{1 \ 2 \ 3}{0 \ 10 \ 0} = 10 \quad \frac{1 \ 2 \ 3}{0 \ 0 \ 3} = 3$$

$$C_2 \quad \frac{4 \ 5 \ 6}{4 \ 0 \ 0} = 4 \quad \frac{4 \ 5 \ 6}{0 \ 25 \ 0} = 25 \quad \frac{4 \ 5 \ 6}{0 \ 0 \ 6} = 6$$

1.4

These notes maybe less clear than my others
 → it is hard to write steps & do the math
 to keep up w/ Alberto.

45)

a) if $A, B \nexists A+B$ are invertible

Show that $A(A^{-1} + B^{-1})B(A+B)^{-1} = I$

doesn't work

$$(A+B)^{-1} \neq A^{-1} + B^{-1}$$

$$A(A^{-1} + B^{-1})B(A+B)^{-1} = I$$

→ Multiply $A(A^{-1} + B^{-1})$

$$(AA^{-1} + AB^{-1})B(A+B)^{-1} = I$$

→ Reduce

$$(I + AB^{-1})B(A+B)^{-1} = I$$

→ Multiply

$$(IB + AB^{-1}B)(A+B)^{-1} = I$$

Reduce

$$(B+A)(A+B)^{-1} = I \quad \leftarrow \text{This is true.}$$

$$(AB)^{-1} = A^{-1}B^{-1}$$

$$(AB)^T = B^T A^T$$

$$(A+B)^T = A^T + B^T$$

$$(A+B)(B+A)^{-1} = I$$

O^{-1} = inverse of O

$$\triangle O = I$$

\triangle is the inverse of O
 O is the inverse of \triangle

if asked about a definition
 you use the definition in the question

Def: B is the inverse of A if $AB = I$

$$AA^{-1} = I$$

Definition

A is idempotent if $A^2 = A$ (Assume $A^2 = A$)

Show that if A is idempotent then $I-A$ is also idempotent.

$$(I-A)^2 = I-A \quad \leftarrow \text{need to prove}$$

$$(I-A)(I-A) = I-A$$

$$II - IA - IA + A^2 \quad \cancel{IA} = A$$

$$I - A - A + A^2 = I - A$$

simplify

$$I - 2A + A^2 = I - A$$

$$-I + 2A \quad -I + 2A \quad \leftarrow \text{isolate } A^2$$

$$A^2 = A$$

Definition

A matrix A is symmetric if, and only if, the transpose is the matrix $A^T = A$

Question using the definition

$$\begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}$$

matrix must look like this.

Show that if B is symmetric, B^{-1} is also symmetric

$$(B^{-1})^T = B^{-1} \quad \text{Apply the rule to the current problem.}$$

$$(B^T)^{-1} = B^{-1}$$

$$B^{-1} = B^{-1}$$

Q

Show that $A+B$ is symmetric if A & B are symmetric

$$(A+B)^T = A+B$$

$$(A+B)^T = A^T + B^T$$

$$A^T + B^T = A+B$$

$$\downarrow$$

$$A+B = A+B \quad \checkmark$$

$$A^T = A$$

$$B^T = B$$

$A-B$ is also symmetric.

Q

Show that AA^T is symmetrical if A is symmetric

$$A^T = A$$

$$AA^T = (AA^T)^T$$

$$AA = (AA)^T \leftarrow \text{replaced all } A^T \text{ w/ } A$$

$$AA = A^TA^T$$

$$AA = AA \quad \leftarrow \text{The math is screaming}$$

$$(AB)^T = B^T A^T$$

Same question
written differently

Show that A^TA is symmetrical

$$ATA = (A^TA)^T$$

$$= A^T(A^T)^T$$

$$= A^TA$$

Q

Prove that $A \cdot B$ is symmetric given that $AB = BA$

$$AB = (AB)^T$$

$$\text{AND } A^T = A \quad B^T = B$$

$$AB = (BA)^T$$

$$AB = A^T B^T \quad A^T = A$$

$$AB = AB \quad B^T = B$$

Q

If A is invertible $(AA^{-1} = I)$ show that AA^T is also invertible

$$AA^T (AA^T)^{-1} = I$$

$$(AA^T)^{-1}$$

$$(A^T)^{-1} A^{-1}$$

\leftarrow No clue

$$(A^{-1})^T A^{-1}$$

$(A^{-1})^T A^{-1}$ is the inverse of AA^T

$$((A^{-1})^T A^{-1})(AA^T) = I$$

~~$$(A^{-1})^T A^{-1} A^T$$~~

$$(A^T)^{-1} (A^T) = I$$

Q

Show that $2A^2 - 3A + I$ is symmetric given A is symmetric

$$(2A^2 - 3A + I)^T$$

$$= (2A^2)^T - (3A)^T + I^T$$

 $A^T - A$

$$2A^TA - 3A^T + I \leftarrow \text{Replace all } A^T \text{ w/ } A$$

$$2AA - 3A + I$$

$$2A^2 - 3A + I$$

A lot of expansion

Q

Prove that if $A^T A = A$ then A is symmetric. $A^T = A$

$$\text{if } A^T A = A$$

then doesn't A need to be an identity

$$I^T = I$$

$$(A^T A) = (A^T A)^T$$

$$= A^T (A^T)$$

$$= A^T A$$

What the fuck?

$$A = A^T A$$

$$A^T = A^T A$$

$$A^T = (A^T A)^T$$

$$A = A^T A$$

$$A^T = (A^T A)^T = A^T (A^T)^T = A^T A \therefore A^T = A$$

Q

Find the values of a, b, c

such that

$$\begin{pmatrix} 4 & -3 \\ a+5 & -1 \end{pmatrix} \text{ is symmetric } A^T = A$$

must be equal for it to be symmetrical

$$a+5 = -3 \quad a = -8$$

$$\begin{pmatrix} 4 & -3 \\ -8 & -1 \end{pmatrix}^T = \begin{pmatrix} 4 & -8 \\ -3 & -1 \end{pmatrix}$$

same as last page

Q

$$\begin{pmatrix} 2 & a-2b+2c & 2a+b+c \\ 3 & 5 & a+c \\ 0 & -2 & 7 \end{pmatrix}$$

$$a - 2b + 2c = 3$$

$$2a + b + c = 0$$

$$a + c = -2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & -2 \end{array} \right] \xrightarrow{2R_1 - R_2} \left[\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 0 & 3 & -1 & -6 \\ 1 & 0 & 1 & -2 \end{array} \right] \xrightarrow{-R_1} \left[\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 0 & 3 & -1 & -6 \\ 0 & 2 & -1 & -5 \end{array} \right]$$

$$8 \left[\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 0 & 3 & -1 & -6 \\ 0 & 2 & -1 & -5 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 0 & 3 & -1 & -6 \\ 0 & 10 & -5 & -25 \end{array} \right] \xrightarrow{-2R_2 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 0 & 3 & -1 & -6 \\ 0 & 0 & 1 & 12 \end{array} \right]$$

$$2 \left[\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 0 & 3 & -1 & -6 \\ 0 & 0 & 1 & 12 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 0 & 10 & -6 & -12 \\ 0 & 0 & 1 & -13 \end{array} \right]$$

Continue w/ gauss-jordan

Everything taught today is
on the exam.

for Which condition of b_1, b_2, b_3
is the following system consistent?

$$\left\{ \begin{array}{l} x_1 - 2x_2 + 5x_3 = b_1 \\ 4x_1 - 5x_2 + 8x_3 = b_2 \\ -3x_1 + 2x_2 - 3x_3 = b_3 \end{array} \right.$$

Consistent means it has

at least one solution

↳ inconsistent = no solutions

$0 | 0$ int. solutions

$\neq 0 | \neq 0$ unique solution

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 4 & -5 & 8 & b_2 \\ -3 & 3 & -3 & b_3 \end{array} \right] \xrightarrow{-4R_1} \left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & b_2 - 4b_1 \\ -3 & 3 & -3 & b_3 \end{array} \right] \xrightarrow{3R_1} \left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & b_2 - 4b_1 \\ 0 & -3 & 12 & b_3 + 3b_1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & b_2 - 4b_1 \\ 0 & -3 & 12 & b_3 + 3b_1 \end{array} \right] \xrightarrow{R_2} \left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & b_2 - 4b_1 \\ 0 & 0 & 0 & b_3 - b_1 + b_2 \end{array} \right]$$

$0 | 0$ int. solutions

$b_2 - b_1 + b_3 = 0$

← This has to be 0

$b_2 - b_1 + b_3 = 0$ for it to be consistent (int. solutions)

Def A diagonal Matrix

must be 0. Like reduce row echelon.

$$A = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{bmatrix}$$

These can also be \emptyset

if $(d_1, d_2, d_3, d_4, \dots, d_n) = \emptyset$

Then $A' = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{bmatrix}$

Elementary Row operations

practice inverting matrices

"gau-jordan"

"multiplying matrices"

$$A | I$$

$$I | A^{-1}$$

1) Swap Rows

2) Multiply a row by a number (scalar)

3) add a multiple of a row to another row.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

① Swap R_1, R_2 $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

② Multiply a row by a scalar $R_i(c)$
where c is a number $\begin{bmatrix} c & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

③ Add multiple of a row
 $R_2 + CR_1$ $\begin{bmatrix} 1 & 0 & 0 \\ 0+c \cdot 1 & 1+c \cdot 0 & 0+c \cdot 0 \\ 0 & 0 & 1 \end{bmatrix}$

An elementary matrix is a matrix that is ONE elementary row operation away from the Identity matrix

I didn't pay any attention. I don't think we did much, mostly just explained these concepts for 30 minutes.

$$5R_2 \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} 21 & 27 & 33 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 21 & 27 & 33 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Linear Algebra

Specifics

Parametrized sol sets

Sept 16th

List of Exam subjects

↳ Linear Systems (solving)

We will specify 12 methods to use

↳ Gaussian Elimination (back substitution)

↳ Gauss-Jordan (rref)

↳ Inverse Matrix $\bar{x} = A^{-1}\bar{b}$

$A \mid I$

gauss-jordan \rightarrow $I \mid A^{-1}$ do row operations to both sides w/ the goal of making $A \rightarrow I$

$A\bar{x} = \bar{b}$

No. of solutions

inf solutions $0 \mid 0$ need to calculate the parameters.

one solution $\neq 0 \mid \square$ can be anything

no solution $0 \mid \neq 0$

$$2x + 2y = 3$$

$$x - y = 1$$

$$\begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

general solution \rightarrow solution set.

Multiplication

$$A_{(k \times m)} \cdot B_{(m \times r)} = C_{(k \times r)}$$

Properties of matrices

$$AB^T = B^T A^T$$

$$(KA)^T = \frac{1}{k} A^T$$

$$(A^T)^T = A$$

$$(A^{-1})^n = (A^n)^{-1}$$

$$(A^{-1})^T = (A^T)^{-1}$$

$$(A+B)^T = A^T + B^T$$

$$C(A+B) = CA + CB$$

$$C(A+B) \neq AC+CB \quad \text{not the same}$$

$$A^T = A \Rightarrow A \text{ is symmetric}$$

$$AA^{-1} = I$$

$$AB = I \Rightarrow A \text{ is the inverse of } B$$

$$\Delta^{-1} = \Delta \quad B \text{ is the inverse of } A$$

$$\Delta \Delta^{-1} = I$$

1

Examples

Isolation

$$(7A^{-1}) = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

$$EDAE = B$$

$$EDA = BE^{-1}$$

$$DA = E^{-1}BE^{-1}$$

$$A = D^{-1}E^{-1}BE^{-1}$$

$$\frac{1}{7}(A^{-1}) = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 7 & 14 \\ 21 & 0 \end{pmatrix}$$

Determine whether the following homogeneous system has non-trivial solutions.

$$2x_1 - 3x_2 + 4x_3 - x_4 = 0$$

$$7x_1 + x_2 - 8x_3 + 7x_4 = 0$$

$$2x_1 + 8x_2 + x_3 - x_4 = 0$$

Trivial solution $x_1 = x_2 = x_3 = x_4 = 0$ either this is the only solution or it has inf solutions

$$\left[\begin{array}{cccc} 2 & -3 & 4 & -1 \\ 7 & 1 & -8 & 9 \\ 2 & 8 & 1 & -1 \end{array} \right]$$

When we do gauss - your bottom row will

x_4 will not be a leading entry, and is therefore a parameter

less equations than variables means there must be a parameter \therefore infinite solutions for homogeneous systems

$$\begin{cases} 2x + 3y - z = 1 \\ 4x + 6y - 2z = 3 \end{cases} \leftarrow \text{no solutions, they are parallel inconsistent.}$$

homogeneous solutions

Homogeneous solutions are always consistent

Find the value of a such that the system has:

$$x + 2y + 3z = 4$$

$$3x - y + 5z = 2$$

$$4x + y + (a^2 - 14)z = a + 2$$

no solution
inf solution
one solution

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & (a^2 - 14) & a + 2 \end{array} \right] \xrightarrow{-3R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -7 & -14 & -10 \\ 4 & 1 & (a^2 - 14) & a + 2 \end{array} \right] \xrightarrow{-4R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -7 & -14 & -10 \\ 0 & -7 & a^2 - 14 - 12 & a + 2 - 16 \end{array} \right] \xrightarrow{-1R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -7 & -14 & -10 \\ 0 & 0 & a^2 - 26 & a - 14 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -7 & -14 & -10 \\ 0 & 0 & a^2 - 26 & a - 14 \end{array} \right]$$

a) $a^2 - 22 = 0 \Rightarrow a = \pm\sqrt{22}$
 b) impossible
 c) $a^2 - 22 \neq 0$

$$a \neq -\sqrt{22} \text{ and } a \neq \sqrt{22}$$

Gaussian elimination

$$\left\{ \begin{array}{l} x - y + 2z - w = -1 \\ 2x + y - 2z - 2w = -2 \\ -x + 2y - 4z + w = 1 \end{array} \right.$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & 4 & 1 & 1 \end{array} \right] \xrightarrow{-2R_1} \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \end{array} \right] \xrightarrow{\text{swap}}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array} \right] \xrightarrow{3R_2} \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad \text{inf solutions}$$

$$x = -1 - 2z + 2z + w = -1 + w$$

$$y = 2z$$

ZER

WTR

ZER WGR

Because no leading entry

Theoretical Questions

$$\begin{cases} x - y + 2z - w = -1 \\ 2x + y - 2z - 2w = -2 \\ -x + 2y - 4z + w = 1 \end{cases}$$

expressed in matrix form using matrix multiplication

$$\begin{pmatrix} 1 & -1 & 2 & -1 \\ 2 & 1 & -2 & -2 \\ -1 & 2 & -4 & 1 \end{pmatrix}_{3 \times 4} \cdot \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}_{(4 \times 1)} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} a & 3 \\ -1 & (a+b) \end{pmatrix} = \begin{pmatrix} 4 & d-2c \\ d+2c & -2 \end{pmatrix}$$

$$c = 4 \quad 3 = d-2c$$

$$-1 = d+2c \quad -2 = a+b$$

$$\left\{ \begin{array}{l} a = 4 \\ d-2c = 3 \\ d+2c = -1 \\ a+b = -2 \end{array} \right.$$