1.4

These notes maybe less clear than my others as it is hard to write steps & do the math to keep up w/ Alberto.

Sept 13

Exam next Wednesday 21st

Tomorrow we get the exam subjects.

45)

a) if A, B & $A+B$ are invertible

Show that $A(A^{-1}+B^{-1})B(A+B)^{-1}=I$

doesn't work → $(A+B)^{-1} \neq A^{-1}+B^{-1}$

$(AB)^{-1} = A^{-1}B^{-1}$

$(AB)^{T} = B^{T}A^{T}$

$(A+B)^{T} = A^{T}+B^{T}$

$(A+B)(B+A)^{-1} = I$

$\bigcirc^{-1} \equiv$ inverse of $\bigcirc$

$\triangle\bigcirc = I$

$\triangle$ is the inverse of $\bigcirc$

$\bigcirc$ is the inverse of $\triangle$

$A(A^{-1}+B^{-1})B(A+B)^{-1}=I$

→ Multiply $A(A^{-1}+B^{-1})$

$(AA^{-1}+AB^{-1})B(A+B)^{-1}=I$

→ Reduce

$\underbrace{(I+AB^{-1})B}(A+B)^{-1}=I$

→ Multiply

$(IB+AB^{-1}B)(A+B)^{-1}=I$

Reduce

$(B+A)(A+B)^{-1}=I$ ← This is true.

b)

if asked about a definition you use the definition in the question

Def: B is the inverse of A if $AB=I$

$AA^{-1}=I$

Definition

A is idempotent if $A^2 = A$ (Assume $A^2 = A$)

Show that if A is idempotent then $I - A$ is Also indempotent.

$(I-A)^2 = I - A$ ← need to prove

$(I-A)(I-A) = I-A$

Expand

$I \cdot A = A$

$II - IA - IA + A^2$

$I - A - A + A^2 = I - A$

Simplify

$I - 2A + A^2 = I - A$ ← isolate A^2

$-I + 2A \qquad -I + 2A$

$A^2 = A$

Definition

A matrix A is symmetric if, and only if, the transpose is the matrix $A^T = A$

$$\begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}$$

Matrix must look like this.

Question using the definition

Show that if B is symmetric, B^{-1} is also symmetric

$(B^{-1})^T = B^{-1}$ Apply the rule to the current problem.

$(B^T)^{-1} = B^{-1}$

$B^{-1} = B^{-1}$

Q.

Show that $A+B$ is symmetric if $\underline{A \,\&\, B}$ are symmetric

$A^T = A$
$B^T = B$

$(A+B)^T = A+B$ $\quad$ $(A+B)^T = A^T + B^T$

$\downarrow$

$A^T + B^T = A+B$

$\downarrow$

$A+B = A+B$ ✓

$A \cdot B$ is also symmetric.

Q

Show That AA^T is symmetrical if A is symmetric

$A^T = A$

$AA^T = (AA^T)^T$

$AA = (AA)^T$ ← replaced all A^T w/ A

$AA = A^T A^T$ ← $(AB)^T = B^T A^T$

$AA = AA$ ← The math is screaming

Same question written differently

Show that $A^T A$ is symmetrical

$A^T A = (A^T A)^T$

$= A^T (A^T)^T$

$= A^T A$

Q

Prove that $A \cdot B$ is symmetric given that $AB = BA$

AND $A^T = A$ & $B^T = B$

$$AB = (AB)^T$$

$$AB = (BA)^T$$

$$AB = A^T B^T \qquad A^T = A$$

$$AB = AB \qquad B^T = B$$

Q

If A is invertible $(AA^{-1} = I)$ Show that AA^T is also invertible

$AA^T (AA^T)^{-1} = I$

$$(AA^T)^{-1}$$

$$(A^T)^{-1} A^{-1} \qquad \text{No clue}$$

$$(A^{-1})^T A^{-1}$$

$(A^{-1})^T A^{-1}$ is the inverse of AA^T

$$\left[(A^{-1})^T A^{-1} \right] (AA^T) = I$$

$$(A^{-1})^T \cancel{A^{-1} A} A^T$$

$$(A^T)^{-1} (A^T) = I$$

Q

Show that $2A^2 - 3A + I$ is symmetric given A is symmetric

$A^T = A$

$(2A^2 - 3A + I)^T$

$= (2A^2)^T - (3A)^T + I^T$

$2A^TA^T - 3A^T + I$ ← Replace all A^T w/ A

$2AA - 3A + I$

A lot of expansion

$2A^2 - 3A + I$

Q

Prove that if $A^TA = A$ then A is symmetric $A^T = A$

if $A^TA = A$

Then doesn't A need to be an identity.

$I^T = I$

$(A^TA) = (A^TA)^T$

$= A^T(A^T)^T$

$= A^TA$

what the fuck?

$A^T = A$

$A = A^TA$

$A^T = (A^TA)^T$

$A = A^TA$

$A^T = A^TA$

$A = A^TA$

$A^T = (A^TA)^T = A^T(A^T)^T = A^TA \therefore A^T = A$.

Q

Find the values of a, b, c

Such that

$\begin{pmatrix} 4 & -3 \\ a+5 & -1 \end{pmatrix}$ is symmetric $\quad A^T = A$

must be equal for it to be symmetrical

$a+5=-3 \quad a=-8$

$$\begin{pmatrix} 4 & -3 \\ a+5 & -1 \end{pmatrix}^T = \begin{pmatrix} 4 & a+5 \\ -3 & -1 \end{pmatrix}$$