7F341FEA6F6A

1) Are the formula linear? (no exponents no multiplication of variables)

a) $x_1 + 5x_2 - \sqrt{2}x_3 = 1$ Linear

b) $x_1 + 3x_2 + x_1 x_2 = 2$ Not Linear $\rightarrow$ $(x_1 x_2)$

c) $x_1 = -7x_2 + 3x_3 \Rightarrow x_1 + 7x_2 - 3x_3 = 0$ Linear

d) $x_1^{-2} + x_2 + 8x_3 = 5$ Not Linear $\rightarrow$ x_1^{-2}

e) $x_1^{3/5} - 2x_2 + x_3 = 4$ Not Linear $\rightarrow$ exponent

f) $\pi x_1 - \sqrt{2}x_3 = 7^{1/3}$ Linear.

2. In each part, determine whether the equation is linear in x and y.

a. $2^{1/3}x + \sqrt{3}y = 1$ **b.** $2x^{1/3} + 3\sqrt{y} = 1$

c. $\cos\left(\frac{\pi}{7}\right)x - 4y = \log 3$ **d.** $\frac{\pi}{7}\cos x - 4y = 0$

e. $xy = 1$ **f.** $y + 7 = x$

a) Linear b) not linear, variables have exponents

c) Linear d) Not Linear

e) Not Linear f) Linear

3. Using the notation of Formula (7), write down a general linear system of

a. two equations in two unknowns.

b. three equations in three unknowns.

c. two equations in four unknowns.

a)

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

b)

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

c)

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$$

Augmented matrix

a) $\begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{bmatrix}$

b) $\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{bmatrix}$

c) $\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & b_2 \end{bmatrix}$

*In each part of Exercises **5–6**, find a system of linear equations in the unknowns $x_1, x_2, x_3, \ldots$, that corresponds to the given augmented matrix.*

5. a. $\begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 3 & 0 & -2 & 5 \\ 7 & 1 & 4 & -3 \\ 0 & -2 & 1 & 7 \end{bmatrix}$

6. a. $\begin{bmatrix} 0 & 3 & -1 & -1 & -1 \\ 5 & 2 & 0 & -3 & -6 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 0 & 1 & -4 & 3 \\ -4 & 0 & 4 & 1 & -3 \\ -1 & 3 & 0 & -2 & -9 \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix}$

5a) $2x_1 = 0$

$3x_1 - 4x_2 = 0$

$x_2 = 0$

b) $3x_1 + 0x_2 - 2x_3 = 5$

$7x_1 + x_2 + 4x_3 = -3$

$-2x_2 + x_3 = 7$

6a) $0x_1 + 3x_2 - x_3 - x_4 = -1$

$5x_1 + 2x_2 + 0x_3 - 3x_4 = -6$

b) $3x_1 + 0x_2 + x_3 - 4x_4 = 3$

$-4x_1 + 0x_2 + 4x_3 + x_4 = -3$

$-x_1 + 3x_2 + 0x_3 - 2x_4 = -9$

$-x_4 = -2$

*In each part of Exercises **7–8**, find the augmented matrix for the linear system.*

7. a. $-2x_1 = 6$
$3x_1 = 8$
$9x_1 = -3$

b. $6x_1 - x_2 + 3x_3 = 4$
$5x_2 - x_3 = 1$

c. $2x_2 - 3x_4 + x_5 = 0$
$-3x_1 - x_2 + x_3 = -1$
$6x_1 + 2x_2 - x_3 + 2x_4 - 3x_5 = 6$

8. a. $3x_1 - 2x_2 = -1$
$4x_1 + 5x_2 = 3$
$7x_1 + 3x_2 = 2$

b. $2x_1 + 2x_3 = 1$
$3x_1 - x_2 + 4x_3 = 7$
$6x_1 + x_2 - x_3 = 0$

c. $x_1 = 1$
$x_2 = 2$
$x_3 = 3$

hello my name is [illegible]

7a) $\begin{bmatrix} -2 & 6 \\ 3 & 8 \\ 9 & -3 \end{bmatrix}$ b) $\begin{bmatrix} 6 & -1 & 3 & 4 \\ 0 & 5 & -1 & 1 \end{bmatrix}$

8a) $\begin{bmatrix} 3 & -2 & -1 \\ 4 & 5 & 3 \\ 7 & 3 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 0 & 2 & 1 \\ 3 & -1 & 4 & 7 \\ 6 & 1 & -1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 2 & 0 & -3 & 1 & 0 \\ -3 & -1 & 1 & 0 & 0 & -1 \\ 6 & 2 & -1 & 2 & -3 & 6 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

In Exercises 1–2, determine whether the given matrix is elementary.

1. **a.** $\begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} -5 & 1 \\ 1 & 0 \end{bmatrix}$

   c. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ **d.** $\begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2. **a.** $\begin{bmatrix} 1 & 0 \\ 0 & \sqrt{3} \end{bmatrix}$ **b.** $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

   c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$ **d.** $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Set 1.5

A matrix is elementary if you can perform one elementary row operation to get an identity

1. a) Elementary
   ↳ $R_2 + (-5R_1)$

   b) Not elementary, needs row inversion and an elementary operation

   c) Not elementary, can't get an identity

   d) Elementary, $R_1/2$
   Not needs two operations

2)
   a) elementary
   ↳ $R_2/\sqrt{3}$

   b) elementary
   ↳ one row inversion

   c) elementary
   ↳ $R_2 - R_2 \cdot 9$

   d) Not elementary

In Exercises 3–4, find a row operation and the corresponding elementary matrix that will restore the given elementary matrix to the identity matrix.

3. **a.** $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} -7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$ **d.** $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4. **a.** $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

c. $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ **d.** $\begin{bmatrix} 1 & 0 & -\frac{1}{7} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

In Exercises 5–6 an elementary matrix E and a matrix A are given. Identify the row operation corresponding to E and verify that the product EA results from applying the row operation to A.

5. **a.** $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

a) $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$ $+3R_2$

positive $3R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✗

b) $\begin{bmatrix} -7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot -\frac{1}{7}$ ✓

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$ ✓ $5R_1$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ inversion ✓

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4) a) $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ $+3R_1$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \cdot \frac{1}{3}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ interchange

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & -\frac{1}{7} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\frac{1}{7}R_3$

*In Exercises **5–6** an elementary matrix E and a matrix A are given. Identify the row operation corresponding to E and verify that the product EA results from applying the row operation to A.*

5. **a.** $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ \cancel{2} & 0 & 1 & 3 & -1 \end{bmatrix}$

5 -9 -2 18 8

c. $E = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

6. **a.** $E = \begin{bmatrix} -6 & 0 \\ 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

That goes from I to E apply to A

a) Row inversion $\begin{bmatrix} 3 & -6 & -6 & -6 \\ -1 & -2 & 5 & -1 \end{bmatrix}$

Row

	0 1	1 0
Col1	-1 3 / 3 = 3	-1 3 / -1 0 = -1
Col2	-2 -6 / 0 -6 = -6	-2 -6 / -2 +0 = -2
Col3	5 -6 / 0 -6 = -6	5 -6 / 5 0 = 5
Col4	-1 -6 / 0 -6 = -6	-1 -6 / -1 0 = -1

b) $R_3 + 3R_2$

result

$\begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ -1 & 9 & -2 & 18 & 8 \end{bmatrix}$ $\begin{bmatrix} 2 & -1 & 0 & 4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ -1 & 9 & 4 & -12 & -8 \end{bmatrix}$

	Row 1	Row 2	Row 3
	1 0 0	0 1 0	0 -3 1
Col 1	2 1 2 / 2 0 0 = 2	2 1 2 / 0 1 0 = 1	2 1 2 / 0 -3 2 = -1
Col 2	-1 -3 0 / -1 0 0 = -1	-1 -3 0 / 0 -3 0 = -3	1 -3 0 / 0 9 0 = 9
Col 3	0 -1 1 / 0 0 0 = 0	0 -1 1 / 0 -1 0 = -1	0 -1 1 / 3 1 = 4
Col 4	-4 5 3 / -4 0 0 = -4	-4 5 3 / 0 5 0 = 5	-4 5 3 / -15 3 = -12
Col 5	-4 3 1 / -4 0 0 = -4	-4 3 1 / 0 3 0 = 3	-4 3 1 / -9 1 = -8

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To get from I_3 $\quad R_1 + 4R_3$

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \; 4R_3$$

$$\begin{bmatrix} 13 & 28 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \text{ result}$$

	Row 1	Row 2	R3
	1 0 4	0 1 0	0 0 1
col 1	1 2 3	1 2 3	1 2 3
	1 0 12 = 13	0 2 0 = 2	0 0 3 = 3
col 2	4 5 6	4 5 6	4 5 6
	4 0 24 = 28	0 5 0 = 5	0 0 6 = 6

$$\begin{bmatrix} 13 & 28 \\ & 5 \\ 2 & \\ 3 & 6 \end{bmatrix}$$

In Exercises 5–6 an elementary matrix E and a matrix A are given. Identify the row operation corresponding to E and verify that the product EA results from applying the row operation to A.

→ The operation that takes you from I_n to E.

5\. **a.** $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

5 -9 -2 18 8

c. $E = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

6\. **a.** $E = \begin{bmatrix} -6 & 0 \\ 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

b. $E = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

$$\begin{bmatrix} 13 & 28 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

C) $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ To get I_3 $R_2 + 4R_3$ $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ $4R_3$ $\Rightarrow \begin{bmatrix} 13 & 28 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

	Row 1	Row 2	Row 3
col 1	1 0 4 1 2 3 1 0 12 = 13	0 1 0 1 2 3 0 2 0 = 2	0 0 1 1 2 3 0 0 3 = 3
Col 2	4 5 6 4 0 24 = 28	4 5 6 0 5 0 = 5	4 5 6 0 0 6 = 6

*In Exercises **5–6** an elementary matrix E and a matrix A are given. Identify the row operation corresponding to E and verify that the product EA results from applying the row operation to A.*

5. **a.** $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

   b. $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

   c. $E = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

6. **a.** $E = \begin{bmatrix} -6 & 0 \\ 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} -1 & -2 & 5 & -1 \\ 3 & -6 & -6 & -6 \end{bmatrix}$

   b. $E = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$

6a result

$$\begin{bmatrix} 3 & -6 & -6 & -6 \\ -1 & -2 & 5 & -1 \end{bmatrix} \checkmark$$

6b) result

$$\begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ -6 & 1 & -1 & 21 & 19 \\ 2 & 0 & 1 & 3 & 1 \end{bmatrix} \checkmark$$

$E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Row interchange $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} -1 & -2 & 5 & 1 \\ 3 & -6 & -6 & -6 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -6 & -6 & -6 \\ -1 & -2 & 5 & 1 \end{bmatrix} \checkmark$

	R_1	R_2
	0 1	1 0
C_1	-1 3 / 0 3 = 3	-1 3 / -1 0 = -1
C_2	-2 -6 / 0 -6 = -6	-2 -6 / -2 0 = -2
C_3	5 -6 / 0 -6 = -6	5 -6 / 5 0 = 5
C_4	-1 -6 / 0 -6	-1 -6 / -1 + 0 = -1

$E = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $-4R_1$ From I_3

$A = \begin{bmatrix} 2 & -1 & 0 & 4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & 1 \end{bmatrix} -4R_1 \Rightarrow \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ -7 & 1 & -1 & 21 & 19 \\ 2 & 0 & 1 & 3 & 1 \end{bmatrix}$

	R_1	R_2	R_3
	1 0 0	-4 1 0	0 0 1
C_1	2 1 2 / 2 0 0 = 2	2 1 2 / -8 1 0 = -7	2 1 2 / 0 0 2 = 2
C_2	-1 -3 0 / -1 0 0 = -1	-1 -3 0 / +4 -3 0 = 1	-1 -3 0 / 0 0 0 = 0
C_3	0 -1 1 / 0 0 0 = 0	0 -1 1 / 0 -1 0 = -1	0 -1 1 / 0 0 1 = 1
C_4	-4 5 3 / -4 0 0 = -4	-4 5 3 / 16 5 0 = 21	-4 5 3 / 0 0 3 = 3
C_5	-4 3 1 / -4 0 0 = -4	-4 3 1 / 16+3+0 = 19	-4 3 1 / 0 0 1 = 1

c. $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

I_3 $\quad 5 \cdot R_2$

$A \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \cdot 5 \Rightarrow \begin{bmatrix} 1 & 4 \\ 10 & 25 \\ 3 & 6 \end{bmatrix}$ $\quad$ Result $\begin{bmatrix} 1 & 4 \\ 10 & 25 \\ 3 & 6 \end{bmatrix}$

R_1 $\quad 1\ 0\ 0$ $\qquad R_2$ $\quad 0\ 5\ 0$ $\qquad R_3$ $\quad 0\ 0\ 1$

C_1 $\quad \dfrac{1\ 2\ 3}{1\ 0\ 0} = 1 \qquad \dfrac{1\ 2\ 3}{0\ 10\ 0} = 10 \qquad \dfrac{1\ 2\ 3}{0\ 0\ 3} = 3$

C_2 $\quad \dfrac{4\ 5\ 6}{4\ 0\ 0} = 4 \qquad \dfrac{4\ 5\ 0}{0\ 25\ 0} = 25 \qquad \dfrac{4\ 5\ 6}{0\ 0\ 6} = 6$

1.4

These notes maybe less clear than my others as it is hard to write steps & do the math to keep up w/ Alberto.

Sept 13

45)

a) if A, B & $A+B$ are invertible

Show that $A(A^{-1}+B^{-1})B(A+B)^{-1} = I$

$$A(A^{-1}+B^{-1})B(A+B)^{-1} = I$$

→ Multiply $A(A^{-1}+B^{-1})$

$$(AA^{-1}+AB^{-1})B(A+B)^{-1} = I$$

→ Reduce

$$\underbrace{(I+AB^{-1})B}(A+B)^{-1} = I$$

→ Multiply

$$(IB+AB^{-1}B)(A+B)^{-1} = I$$

Reduce

$$(B+A)(A+B)^{-1} = I$$ ← This is true.

b)

Exam next Wednesday 21st

Tomorrow we get the exam subjects.

doesn't work → $(A+B)^{-1} \neq A^{-1}+B^{-1}$

$$(AB)^{-1} = A^{-1}B^{-1}$$

$$(AB)^T = B^TA^T$$

$$(A+B)^T = A^T+B^T$$

$$(A+B)(B+A)^{-1} = I$$

$\bigcirc^{-1} \equiv$ inverse of $\bigcirc$

$\triangle\bigcirc = I$

$\triangle$ is the inverse of $\bigcirc$

$\bigcirc$ is the inverse of $\triangle$

if asked about a definition you use the definition in the question

Def: B is the inverse of A if $AB = I$

$$AA^{-1} = I$$

Definition

A is idempotent if $A^2 = A$ (Assume $A^2 = A$)

Show that if A is idempotent then $I - A$ is Also indempotent.

$(I-A)^2 = I - A$ ← need to prove

$(I-A)(I-A) = I - A$

Expand

$I \cdot A = A$

$II - IA - IA + A^2$

$I - A - A + A^2 = I - A$

Simplify

$I - 2A + A^2 = I - A$ ← isolate A^2

$-I + 2A \qquad -I + 2A$

$A^2 = A$

Definition

A matrix A is symmetric if, and only if, the transpose is the matrix $A^T = A$

$$\begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix}$$

Matrix must look like this.

Question using the definition

Show that if B is symmetric, B^{-1} is also symmetric

$(B^{-1})^T = B^{-1}$ Apply the rule to the current problem.

$(B^T)^{-1} = B^{-1}$

$B^{-1} = B^{-1}$

Q.

Show that $A+B$ is symmetric if A \$ B are symmetric

$A^T = A$
$B^T = B$

$(A+B)^T = A+B$

$\downarrow$

$A^T + B^T = A+B$

$\downarrow$

$A+B = A+B$ ✓

$(A+B)^T = A^T + B^T$

$A \cdot B$ is also symmetric.

Q

Show That AA^T is symmetrical if A is symmetric

$A^T = A$

$AA^T = (AA^T)^T$

$AA = (AA)^T$ ← repleced all A^T w/ A

$AA = A^T A^T$ ← $(AB)^T = B^T A^T$

$AA = AA$ ← The math is screaming

Same question written differently

Show that $A^T A$ is symmetrical

$A^T A = (A^T A)^T$

$= A^T (A^T)^T$

$= A^T A$

Q

Prove that $A \cdot B$ is symmetric given that $AB = BA$

AND $A^T = A$ & $B^T = B$

$AB = (AB)^T$

$AB = (BA)^T$

$AB = A^T B^T$ $\quad A^T = A$

$AB = AB$ $\quad B^T = B$

Q

If A is invertible $(AA^{-1} = I)$ Show that AA^T is also invertible

$AA^T (AA^T)^{-1} = I$

$(AA^T)^{-1}$

$(A^T)^{-1} A^{-1}$ ← No clue

$(A^{-1})^T A^{-1}$

$(A^{-1})^T A^{-1}$ is the inverse of AA^T

$$\left[(A^{-1})^T A^{-1} \right) \left(AA^T \right) = I$$

$(A^{-1})^T \cancel{A^{-1} A} A^T$

$(A^T)^{-1} (A^T) = I$

Q

Show that $2A^2 - 3A + I$ is symmetric given A is symmetric

$A^T = A$

$(2A^2 - 3A + I)^T$

$= (2A^2)^T - (3A)^T + I^T$

$2A^TA^T - 3A^T + I$ ← Replace all A^T w/ A

$2AA - 3A + I$

A lot of expansion

$2A^2 - 3A + I$

Q

Prove that if $A^TA = A$ then A is symmetric $A^T = A$

if $A^TA = A$

Then doesn't A need to be an identity.

$I^T = I$

$(A^TA) = (A^TA)^T$

$= A^T(A^T)^T$

$\neq A^TA$

what the fuck?

$A^T = A$

$A = A^TA$

$A^T = (A^TA)^T$

$A = A^TA$

$A^T = A^TA$

$A = A^TA$

$A^T = (A^TA)^T = A^T(A^T)^T = A^TA \;\therefore\; A^T = A.$

Q

Find the values of a, b, c

Such that

$$\begin{pmatrix} 4 & -3 \\ a+5 & -1 \end{pmatrix} \text{ is symmetric} \qquad A^T = A$$

must be equal for it to be symmetrical

$a+5=-3 \qquad a=-8$

$$\begin{pmatrix} 4 & -3 \\ a+5 & -1 \end{pmatrix}^T = \begin{pmatrix} 4 & a+5 \\ -3 & -1 \end{pmatrix}$$

Same as last page

Q $\begin{pmatrix} 2 & a-2b+2c & 2a+b+c \\ 3 & 5 & a+c \\ 0 & -2 & 7 \end{pmatrix}$

$$\begin{aligned} a \quad -2b \quad +2c &= 3 \\ 2a \quad +b \quad +c &= 0 \\ a \quad\quad\quad +c &= -2 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & -2 \end{array}\right] \begin{array}{lllll} -2R_1 & -2 & +4 & -4 & -6 \\ -R_1 & -1 & +2 & -2 & -3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 3 \\ 0 & 5 & -3 & -6 \\ 0 & 2 & -1 & -5 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & -2 & 2 & 3 \\ 0 & 5 & -3 & -6 \\ 0 & 10 & -5 & -25 \end{array}\right] \quad -2R_2 \quad 0 \quad -10 \quad +6 \quad +12$$

$$2\left[\begin{array}{cccc} 1 & -2 & 2 & 3 \\ 0 & 5 & -3 & -6 \\ 0 & 0 & 1 & -13 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & -2 & 2 & 3 \\ 0 & 10 & -6 & -12 \\ 0 & 0 & 1 & -13 \end{array}\right]$$

Continue w/ gauss-jordan

Everything before today is on the exam.

for which condition of b_1, b_2, b_3 is the following system consistent.

Consistent means it has atleast one solution

↳ inconsistent = no solutions.

$0 \mid 0$ inf solutions

$\neq 0 \mid \neq 0$ unique solution

$$\begin{cases} x_1 - 2x_2 + 5x_3 = b_1 \\ 4x_1 - 5x_2 + 8x_3 = b_2 \\ -3x_1 + 3x_2 - 3x_3 = b_3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & +5 & b_1 \\ 4 & -5 & 8 & b_2 \\ -3 & 3 & -3 & b_3 \end{array}\right]$$

$-4R_1 \quad [-4 \quad +8 \quad -20 \mid -4b_1$

$3R_1 \quad [-3 \quad -6 \quad 15 \mid 3b_1$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & b_2 - 4b_1 \\ 0 & -3 & +12 & b_3 + 3b_1 \end{array}\right] \quad R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & b_1 \\ 0 & 3 & -12 & b_2 - 4b_1 \\ 0 & 0 & 0 & b_2 - 4b_1 + b_3 + 3b_1 \\ & & & b_2 - b_1 + b_3 \end{array}\right]$$

← This has to be 0

$0 \mid 0$ inf solutions

$b_2 - b_1 + b_3 = 0$ for it to be consistent (inf solutions)

Def A diagonal Matrix

must be 0. Like reduce row echelon.

$$A = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{bmatrix}$$

These can also be $\emptyset$

if $(d_1, d_2, d_3, d_4, \dots d_n \neq \emptyset)$

Then $A^{-1} = \begin{bmatrix} \frac{1}{d_1} & 0 & 0 & 0 \\ 0 & \frac{1}{d_2} & 0 & 0 \\ 0 & 0 & \frac{1}{d_3} & 0 \\ 0 & 0 & 0 & \frac{1}{d_4} \end{bmatrix}$

Elementary Row operations

practice inverting matrices
↳ gauss-jordan
↳ multiplying matrices

$$\begin{array}{c|c} A & I \\ I & A^{-1} \end{array}$$

1) Swap Rows
2) Multiply a row by a number (scalar)
3) add a multiple of a row to another row.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1 Swap R_1, R_2 $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

An elementary matrix is a matrix that is **ONE** elementary row operation away from the Identity matrix

2 Multiply a row by a scalar $R_1(c)$ where c is a number $\begin{bmatrix} c & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3 Add multiple of a row

$R_2 + cR_1$ $\begin{bmatrix} 1 & 0 & 0 \\ 0+c\cdot 1 & 1+c\cdot 0 & 0+c\cdot 0 \\ 0 & 0 & 1 \end{bmatrix}$

I didnt pay any attention. I dont think we did much, mostly just explained these concepts for 30 minutes.

$5R_2$ $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $\begin{bmatrix} 21 & 27 & 33 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$$\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 21 & 27 & 33 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Specifics

Parameters/sol sets

Linear Algebra

Sept 16^{th}

List of Exam subjects

↳ Linear Systems (solving) — He will specify the method to use

↳ Gaussian Elimination (back substitution)

↳ Gauss-Jordan (rre)

↳ Inverse Matrix $\bar{x} = A^{-1}\bar{b}$

$$\begin{array}{c|c} A & I \\ \downarrow & \downarrow \\ I & A^{-1} \end{array}$$

gauss-jordan → ← do row operations to both sides w/ the goal of making $A \to I$

$A\bar{x} = \bar{b}$

$2x + 2y = 3$

$x - y = 1$

$$\begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Num of Solutions

inf solutions $0 \mid 0$ ← need to calculate the parameters.

one solution $\neq 0 \mid \square$ ← can be anything

no solution $0 \mid \neq 0$

general solution → solution set.

Multiplication

Homogenous Systems ?

$$A_{(k \times m)} B_{(m \times r)} = C_{(k \times r)}$$

Properties of matrices

$AB^{-1} = B^{-1}A^{-1}$

$(kA)^{-1} = \frac{1}{k}A^{-1}$

$(A^{-1})^{-1} = A$

$(A^{-1})^n = (A^n)^{-1}$

$(A^{-1})^T = (A^T)^{-1}$

$(A+B)^T = A^T + B^T$

$C(A+B) = CA + CB$

$C(A+B) \neq AC + CB$ not the same

$A^T = A \Rightarrow A$ is symmetric

$AA^{-1} = I$

$AB = I \Rightarrow$ A is the invers of B
B is the inverse of A

$\bigcirc^{-1} = \triangle$

$\bigcirc \triangle = I$

Examples

Isolation

$$(7A^{-1}) = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

$$\frac{1}{7}(A^{-1}) = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 7 & 14 \\ 21 & 0 \end{pmatrix}$$

$$EDAE = B$$

$$EDA = BE^{-1}$$

$$DA = E^{-1}BE^{-1}$$

$$A = D^{-1}E^{-1}BE^{-1}$$

Determine wheter the following homogeneous system has non-trivial solutions?

$$2x_1 - 3x_2 + 4x_3 - x_4 = 0$$
$$7x_1 + x_2 - 8x_3 + 9x_4 = 0$$

$$2x_1 + 8x_2 + x_3 - x_4 = 0$$

trivial solution $x_1 = x_2 = x_3 = x_4 = 0$ either this is the only solution or it has inf solutions

$$\begin{bmatrix} 2 & -3 & 4 & -1 \\ 7 & 1 & -8 & 9 \\ 2 & 8 & 1 & -1 \end{bmatrix}$$

When we do gauss - your bottom row will

x_4 will not be a leading entry, and is therefore a parameter

less equations than variables means there must be a parameter ∴ infinite solutions for homogenous systems

$$2x + 3y - z = 1$$
$$4x + 6y - 2z = 3$$

← no solutions, they are parallel

inconsistent.

Homogenous solutions

Homogenous solutions are always consistent

Find the value of a such that the system has:

no solution
inf solution
one solution

$x + 2y + 3z = 4$

$3x - y + 5z = 2$

$4x + y + (a^2-14)z = a+2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & (a^2-14) & a+2 \end{array}\right] \begin{array}{l} \\ -3R_1 \\ -4R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -7 & -4 & -10 \\ 0 & -7 & \begin{matrix} a^2-14-12 \\ a^2-26 \end{matrix} & \begin{matrix} a+2-16 \\ a-14 \end{matrix} \end{array}\right] \begin{array}{l} \\ \\ -1R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -7 & -4 & -10 \\ 0 & 0 & a^2-22 & a-4 \end{array}\right]$$

a) $a^2-22=0$ && $a-4 \neq 0$

$a = \pm\sqrt{22}$

b) impossible

c) $a^2-22 \neq 0$

$a \neq -\sqrt{22}$ & $a \neq \sqrt{22}$

Gaussian elimination

$$\begin{cases} x - y + 2z - w = -1 \\ 2x + y - 2z - 2w = -2 \\ -x + 2y - 4z + w = 1 \end{cases}$$

$$\begin{array}{cl} \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \end{array}\right] & \begin{array}{l} \\ -2R_1 \\ R_1 \end{array} \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \end{array}\right]$$

$$\begin{array}{cl} \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right] & \begin{array}{l} \\ \\ -3R_2 \end{array} \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

inf solutions

$\underbrace{z \in \mathbb{R} \quad w \in \mathbb{R}}$ → Because no leading entry

$x = -1 - \cancel{2z + 2z + w} = -1 + w$

$y = 2z$

$z \in \mathbb{R}$

$w \in \mathbb{R}$

Theoretical Questions

$$\begin{cases} x - y + 2z - w = -1 \\ 2x + y - 2z - 2w = -2 \\ -x + 2y - 4z + w = 1 \end{cases}$$

expressed in matrix form using matrix multiplication

$$\underbrace{\begin{pmatrix} 1 & -1 & 2 & -1 \\ 2 & 1 & -2 & -2 \\ -1 & 2 & -4 & 1 \end{pmatrix}}_{5x4} \underbrace{\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}}_{(4x1)} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} a & 3 \\ -1 & (a+b) \end{pmatrix} = \begin{pmatrix} 4 & d-2c \\ d+2c & -2 \end{pmatrix}$$

$a = 4 \qquad 3 = d-2c$

$-1 = d+2c \qquad -2 = a+b$

$$\begin{cases} a = 4 \\ d-2c = 3 \\ d+2c = -1 \\ a+b = -2 \end{cases}$$