Problem 5

Thursday, February 8, 2024 6:36 PM

$$m\ddot{y} + b\dot{y} + ky = b\dot{x} + kx$$

$$x(t) = tH(t) - 2(t-1)H(t-1) + tH(t-2)$$

$$m(s^2Y(s) - \cancel{sy(0)} - \cancel{\dot{y}(0)})$$
$$+ b\left(sY(s) - \cancel{y(0)}\right) + kY(s) = b(sX(s) - \cancel{x(0)}) + kX(s)$$

$$ms^2Y(s) + bsY(s) + kY(s) = bsX(s) + kX(s)$$

$$Y(s)\left(ms^2 + bs + k\right) = X(s)\left(bs + k\right)$$

$$X(s) = \mathcal{L}\{tH(t) - 2(t-1)H(t-1) + (t-2)H(t-2)\}$$

$$= \cancel{e^{0s}}^{1} \cdot \frac{1}{s^2} - 2\left(e^{-s}\frac{1}{s^2}\right) + e^{-2s}\left(\frac{1}{s^2}\right)$$

$$Y(s) = \frac{s+1}{s^2+s+1}\left(\frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}\right)$$

$$= s^{-2}\frac{s+1}{s^2+s+1}\left(1 - 2e^{-s} - e^{-2s}\right)$$

$$= \left(\frac{1}{s^2} - \frac{1}{(s+1/2)^2 + 3/4}\right)\left(1 - 2e^{-s} - e^{-2s}\right)$$

Thus let $\omega = \sqrt{3}/2 ; \omega^2 = 3/4$

$$Y_1(s) = \frac{1}{s^2}, \quad \mathcal{L}^{-1}\{Y_1(s)\} = t$$

$$\omega^{-1} Y_2(s) = \frac{-\omega}{\omega\left((s+1/2)^2+\omega^2\right)}, \quad \omega^{-1}\mathcal{L}^{-1}\{Y_2(s)\} = -\omega^{-1}e^{-1/2t}\sin\omega t$$

So,

$$\underline{Y}(s) = \left(Y_1(s) + \omega^{-1}Y_2(s)\right)\left(1-2e^{-s}-e^{-2s}\right)$$

$$= \left(Y_1(s) + \omega^{-1}Y_2(s)\right)\left(1-2e^{-s}-e^{-2s}\right)$$

$$= \Big(Y_1(s) - 2Y_1(s)e^{-(1)s} + Y_1(s)e^{-(2)s}$$

$$+ \omega^{-1}Y_2(s) - 2Y_2(s)e_{\omega^{-1}}^{-(1)s} + Y_2(s)e_{\omega^{-1}}^{-(2)s}\Big)$$