Home work # 3

Problem 1:

* Recall: Laplace integral: $F(s) = \int_0^\infty f(t)e^{-st}\,dt$

* Write out integral in terms of bounds

$$\int_0^1 t e^{-st}\,dt + \int_1^2 (-t+2)e^{-st}\,dt + \int_2^\infty \varnothing\,dt$$

* evaluate each integral

$$\int_2^\infty \varnothing\,dt = \varnothing$$

* evaluating other values with Integration by parts (recall L.I.A.T.E)

$$\int_0^1 te^{-st}\,dt \qquad \text{* let: } u = t \quad dv = e^{-st} \quad du = 1\,dt \quad v = -\frac{e^{-st}}{s}$$

* put in form $uv - \int v\,du$

$$-\frac{te^{-st}}{s} + \int \frac{e^{-st}}{s}\,dt = \left[-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_0^1$$

* plug in bounds

$$\left(\frac{(-1)e^{-s}}{s} - \frac{e^{-(1)s}}{s^2}\right) - \left(\frac{\varnothing}{s} - \frac{1}{s^2}\right) = \frac{-e^{-s}}{s} - \frac{e^{-s}+1}{s^2}$$

* evaluate integral $\int_1^2 (-t+2)e^{-st}\,dt$

* let: $u = -t+2 \quad dv = e^{-st}$

$du = -dt \quad v = -\frac{e^{-st}}{s}$

* put in form $uv - \int v\,du$: evaluate : plug in bounds

$$-\frac{(-t+2)e^{-st}}{s} + \frac{e^{-st}}{s^2} = \left[-\frac{s(-t+2)e^{-st} + e^{-st}}{s^2}\right]_1^2$$

$$-\frac{s(-2+2)e^{-2s} + e^{-2s}}{s^2} + \frac{s(-1+2)e^{-s} + e^{-s}}{s^2} = \frac{e^{-2s} + se^{-s} - e^{-s}}{s^2}$$

Continue →

* simplify equation

$$F(s) = \frac{-\cancel{se^{-s}} - e^{-s} + 1 + e^{-2s} + \cancel{se^{-s}} - e^{-s} + 0}{s^2}$$

Solution:

$$\boxed{\mathcal{L}\{f(t)\} = \frac{e^{-2s} - 2e^{-s} + 1}{s^2}}$$