Problem 1

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$$F(s) = \frac{s+3}{(s+5)(s^2+4s+5)} = \frac{A}{s+5} + \frac{Bs+C}{s^2+4s+5}$$

$$= \frac{A}{s+5} + \frac{B}{s+(2-i)} + \frac{C}{s+(s+i)}$$

$$s+3 = A(s^2+4s+5) + (s+5)(Bs+c)$$

$s=-3$

$$0 = A(\overbrace{9-12+5}^{14-12=2}) + (2)(-3B+c)$$

$$\boxed{0 = 2A - 6B + 2C}$$

$s=-5$

$$-2 = A(\overbrace{25-20+5}^{30}) + \cancel{(0)(-5B+C)} \rightarrow 0$$

$$-2 = A(10)$$

$$\boxed{A = -1/5}$$

$s=0$

$$3 = A(0+0+5) + (5)(C)$$

$$3 = \left(-\frac{1}{5}\right)(5) + 5C$$

$$3 = -1 + 5C$$

$$4 = 5C \qquad \boxed{C = \frac{4}{5}}$$

$$\left(0 = 2\left(-\frac{1}{\cancel{5}}\right) - \overset{-30}{6B} + 2\left(\frac{4}{\cancel{5}}\right)\right)(5)$$

$$0 = 2(-1) - 30B + 8$$

$$0 = -2 + 8 - 30B$$

$$0 = 6 - 30B$$

$$30B = 6$$

$$B = 1/5$$

$$\boxed{B = \frac{1}{5}}$$

$$Im(F_2(s)) = 1$$

$$Re(F_2(s) = -2$$

$$F(s) = -\frac{1}{5}\left(\frac{1}{s-(-5)}\right) + \frac{1}{5}\left(\frac{s+4}{s^2+4s+5}\right)$$

$$+ \frac{1}{}\Big(\frac{s+2}{} + 2(1^2)$$

$$+\frac{1}{5}\left(\frac{s+2}{(s-(-2))^2+1^2}+\frac{2(1^2)}{(s-(-2))^2+1^2}\right.$$

$$\boxed{f(t)=-\frac{1}{5}e^{-5t}+\frac{1}{5}e^{-2t}\left(2\sin(t)+\cos(t)\right)}$$

Problem 2:

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$\ddot{x} + 4\dot{x} + 3x = 1^{-}$ $\quad 1/s$

$\mathcal{L}\{f^{(2)}\} = s^2 F(s) - s^1 f^{(0)}(0) - s^0 \dot{f}^{(1)}(0)$

$\mathcal{L}\{f^{(1)}\} = s^1 F(s) - f(0)$

$\mathcal{L}\{x\} = F(s)$

$\underline{s^2F(s)} - \overline{sf(0)} - \dot{f}(0) + \underline{4sF(s)} - \overline{4f(0)} + \underline{3F(s)} = \frac{1}{s}$

$F(s)(s^2+4s+3) - \underset{\downarrow}{f(0)}(s+4) - \underset{\downarrow}{\dot{f}(0)} = \frac{1}{s}$

$\qquad\qquad\qquad\qquad\quad 2 \qquad\qquad\qquad 1$

$3 \underset{4}{\overset{3}{\times}} 1$

$(s+1)(s+3)$

$F(s)(s+1)(s+3) - 2(s+4) - 1 = \frac{1}{s}$

$F(s)(s+1)(s+3) = \frac{1}{s} + 2(s+4) + 1$

$= \frac{1}{s} + \frac{2s^2+8s}{s} + \frac{s}{s}$

$F(s)(s+1)(s+3) = \frac{2s^2+9s+1}{s}$

$F(s) \quad \frac{2s^2+9s+1}{} \quad A \quad B \quad C$

$$F(s) = \frac{2s^2+9s+1}{(s)(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

$$2s^2+9s+1 = A(s+1)(s+3) + B(s)(s+3) + C(s)(s+1)$$

$\underline{s=0}$

$$1 = A(1)(3) + 0 + 0$$

$$\boxed{A = 1/3}$$

$\underline{s=-1}$

$$\underbrace{2-9+1}_{3-9=-6} = \cancel{A(0)(2)} + B(-1)(2) + \cancel{C(-1)(0)}$$

$$-6 = -2B \Rightarrow \boxed{B = 3}$$

$\underline{s=-3}$

$$2(9) - 27 + 1 = A\cancel{(-2)(0)} + B\cancel{(-3)(0)} + C(-3)(-2)$$

$$19 - 27$$

$$-8 = 6C$$

$$\boxed{-\frac{4}{3} = C}$$

$$F(s) = 3\left(\frac{1}{s-(-1)}\right) - \frac{4}{3}\left(\frac{1}{s-(-3)}\right) + \frac{1}{3}\left(\frac{1}{s}\right)$$

$$\boxed{x(t) = 3e^{-t} - \frac{4}{3}e^{-3t} + \frac{1}{3}}$$

Problem 3:

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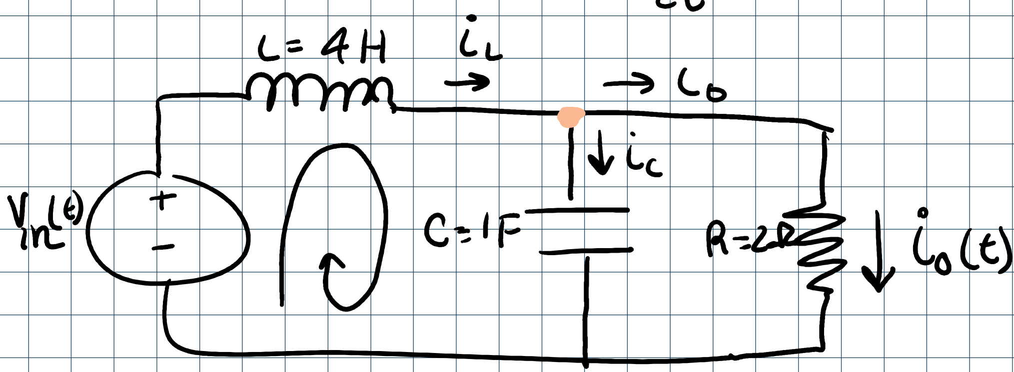
Let $i' := \frac{di}{dt}$



$$i_L = i_c + i_o$$

$$(1)\quad -V_{in}(t) + V_L(t) + V_c(t) = 0$$

$$(2)\quad V_c(t) = V_R(t) = I(t)R$$

$$(3)\quad I_L = \frac{1}{L}\int v_L(t)\,dt$$

$$v_c = v_o = I_o(t)R$$

$$\frac{dV}{dt} = \frac{di_o}{dt}R$$

$$(4)\quad I_c = C\frac{dV}{dt}$$

$$I_c = CR\frac{di_o}{dt}$$

$$\frac{1}{L}\int v_L\,dt = CR\frac{di_o}{dt} + i_o(t)$$

$$\frac{v_L(t)}{L} = CR\frac{d^2 i_o}{dt^2} + \frac{di_o}{dt}$$

$$v_L(t) = LRC\frac{d^2 i_o}{dt^2} + L\frac{di_o}{dt}$$

$$v_{in}(t) = v_L(t) + v_c(t)$$

$$v_{in}(t) = v_L(t) + i_o(t)R$$

$$v_{in}(t) = LRC\frac{d^2 i_o}{dt^2} + L\frac{di_o}{dt} + i_o(t)R$$

$$\boxed{8\frac{d^2 i_o}{dt^2} + 4\frac{di_o}{dt} + 2i_o(t) = v_{in}(t)}$$

Problem 4:

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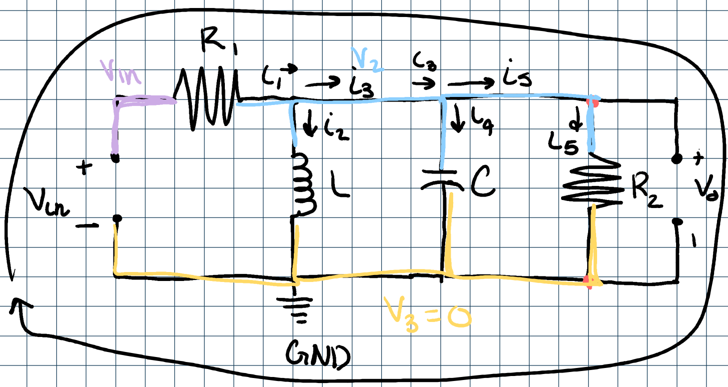
$v_{in}-v_2$

$v_{in}-v_2$



$v_{in}-v_2 = R_1 I(t)$

$v_2 - 0 = v_o$

$v_{in} - v_o = v_R$

v_{in}

$v_2 = \frac{1}{C}\int I(t)dt$

$\dot{v}_2 = I$

$v_{in} = v_{in} - v_2 + L\frac{dI_2}{dt}$

$v_o(t) = L\frac{dI}{dt}$

$-V_{in}(t) + V_{R_1} + V_o = 0$

$R i_1(t) + V_o(t) = v_{in}(t)$

$R_1 \dot{i}_1(t) + \dot{v}_o(t) = \dot{v}_{in}(t)$

$v_R = v_{in} - v_o$

$i_1 = i_2 + i_3$

$i_3 = i_4 + i_5$

$> i_1 = i_2 + i_4 + i_5$

$\frac{di_1}{dt} = (\dot{v}_{in} - \dot{v}_o)\frac{1}{R_1}$

$V(t) = I(t)R$

$\dot{V}(t) = \dot{I}(t)R$

$\frac{V_R(t)}{R_1} = \frac{1}{L}\int v_o(t)dt + C\frac{dV_o}{dt} + \frac{V_o(t)}{R_L}$

$\frac{1}{R}\dot{V}_R = \frac{v_o(t)}{L} + C\ddot{v}_o + \frac{\dot{v}_o}{R_L} \rightarrow L\frac{di_1}{dt} = v_o(t) + LC\ddot{v}_o + \frac{L}{R_L}\dot{v}_o$

$i_1 = \frac{v_R(t)}{R_1}$ $\quad i^{(1)} = \frac{1}{R_1}\dot{v}_R$

$R_1\left(\frac{1}{L}\int v_o(t)dt + C\dot{v}_o + \frac{1}{R_L}v_o(t)\right) + v_o(t) = v_{in}(t)$

$v_R = v_{in} - v_2$

$$\boxed{\frac{R_1}{L}v_o(t) + R_1C\ddot{v}_o + \frac{R_1}{R_L}\dot{v}_o(t) + \dot{v}_o(t) = \dot{v}_{in}(t)}$$

let $x = v_{in}$

$\dot{x} = v_{in}$

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Problem 5:

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$$y(t) = t - (2)(H(t-5))$$

$$\boxed{Y(s) = \frac{1}{s^2} - 2\frac{e^{-5s}}{s}}$$