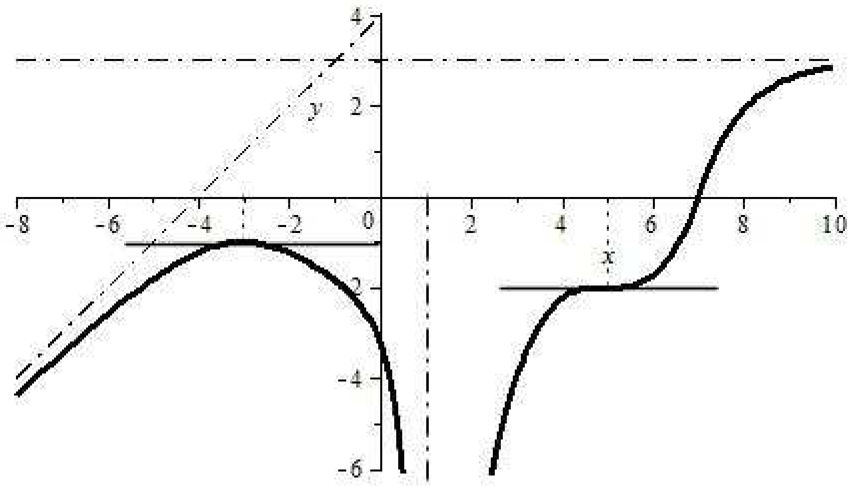
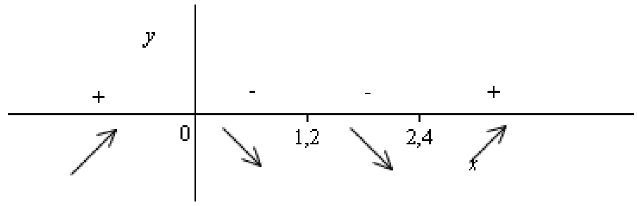
Řešení písemky IMA 7.5.2012 zadání C

1.



2. $f(x)=\dfrac{x^2}{5x-6},\quad f'(x)=\dfrac{x(5x-12)}{(5x-6)^2}$,

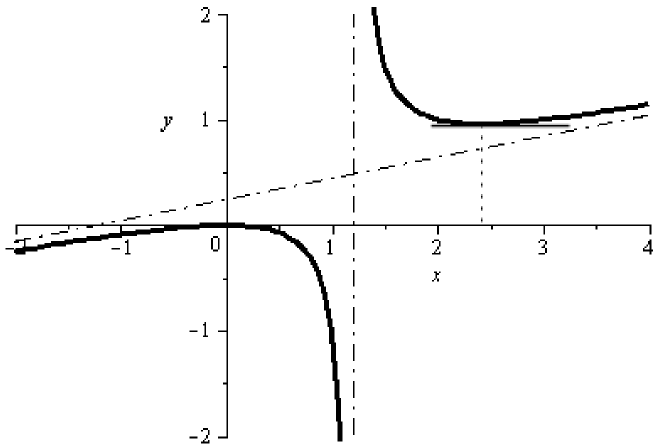


$f'(x)=0$ pro $x=0 \vee x=\dfrac{12}{5},\ f(0)=0,\ f\left(\dfrac{12}{5}\right)=\dfrac{24}{25}$

$$\lim_{x\to\frac{6}{5}^{\pm}} f(x)=\left\langle \begin{array}{l} \lim\limits_{x\to\frac{6}{5}^{+}} \dfrac{x^2}{5x-6}=+\infty \\ \lim\limits_{x\to\frac{6}{5}^{-}} \dfrac{x^2}{5x-6}=-\infty \end{array}\right.$$

- v $x=\dfrac{6}{5}$ je svislá asymptota , v $x=0$ je maximum, v $x=\dfrac{12}{5}$ minimum,

Asymptota se směrnicí:



$$a=\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{x}{5x-6}=\frac{1}{5},$$

$$b=\lim_{x\to\pm\infty}\left(f(x)-a\cdot x\right)=\lim_{x\to\pm\infty}\left(\frac{x^2}{5x-6}-\frac{x}{5}\right)=$$

$$=\lim_{x\to\pm\infty}\frac{5x^2-x(5x-6)}{5(5x-6)}=\lim_{x\to\pm\infty}\frac{6x}{25x-30}=\frac{6}{25}$$

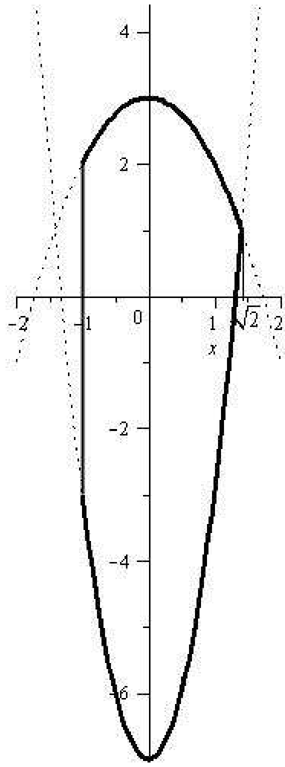
asymptota: $y=\dfrac{x}{5}+\dfrac{6}{25}$

3.

$$\int(3x-2)\cos\frac{x}{4}\,dx=\left|\begin{array}{ll} u=3x-2 & u'=3 \\ v'=\cos\dfrac{x}{4} & v=4\sin\dfrac{x}{4}\end{array}\right|=4(3x-2)\sin\frac{x}{4}-12\int\sin\frac{x}{4}\,dx=$$

$$=4(3x-2)\sin\frac{x}{4}-48\cos\frac{x}{4}+c$$

4. Obsah $4x^2-7\le y\le 3-x^2,\quad x\ge -1$



Průsečíky: $4x^2-7=3-x^2\Rightarrow 5x^2=10\Rightarrow x=\pm\sqrt{2}$

$$\int_{-1}^{\sqrt{2}}\left(3-x^2-(4x^2-7)\right)dx=\int_{-1}^{\sqrt{2}}(10-5x^2)\,dx=$$

$$=\left[10x-\tfrac{5}{3}x^3\right]_{-1}^{\sqrt{2}}=\frac{20}{3}\sqrt{2}+\frac{25}{3}$$

Pomocí dvojného integrálu:

$$\int_M 1\,dx\,dy=\int_{-1}^{\sqrt{2}}dx\int_{4x^2-7}^{3-x^2}dy=\int_{-1}^{\sqrt{2}}\left[y\right]_{4x^2-7}^{3-x^2}dx=\int_{-1}^{\sqrt{2}}\left(3-x^2-(4x^2-7)\right)dx$$

a dále jako nahoře.

5. Lokální extrémy $z(z,y)=x^2-7x+5y^2e^{-2x}$

$$z'(x,y)=\left(2x-7-10y^2e^{-2x},10ye^{-2x}\right)=\mathbf{0}$$

$$10ye^{-2y}=0\Rightarrow y=0,\quad 2x-7-10\cdot 0\cdot e^{-2x}=0\Rightarrow y=\frac{7}{2}$$

stacionární bod $A=\left[\frac{7}{2},0\right]$.

$$z''(x,y)=\begin{bmatrix}2+20y^2e^{-2x} & -20ye^{-2x}\\ -20ye^{-2x} & 10e^{-2x}\end{bmatrix},\ z''(A)=\begin{bmatrix}2 & 0\\ 0 & 10e^{-7}\end{bmatrix}\quad D_1(A)=2>0,\ D_2(A)=20e^{-7}>0$$

V bodě *A* nastane minimum, $f_{\min}=f(A)=-\dfrac{49}{4}$.

6. Řešte $\displaystyle\sum_{n=2}^{\infty}\left(\frac{-3}{x}\right)^n=3$

Geometrická řada, kvocient $\dfrac{-3}{x}$, první člen $\left(\dfrac{-3}{x}\right)^2$.

Podmínka konvergence $\left|\dfrac{-3}{x}\right|<1\Leftrightarrow\dfrac{3}{|x|}<1\Rightarrow|x|>3$; součet $s=\left(\dfrac{-6}{x}\right)^2\cdot\dfrac{1}{1-\dfrac{-3}{x}}=\dfrac{9}{x^2}\cdot\dfrac{x}{x+3}$

$$\frac{9}{x(x+3)}=3\Rightarrow 3=x(x+3)\Rightarrow x^2+3x-3=0,\quad x_{1,2}=-\frac{3}{2}\pm\frac{1}{2}\sqrt{21}$$

Podmínce konvergence vyhovuje $x=-\dfrac{3}{2}-\dfrac{\sqrt{21}}{2}$