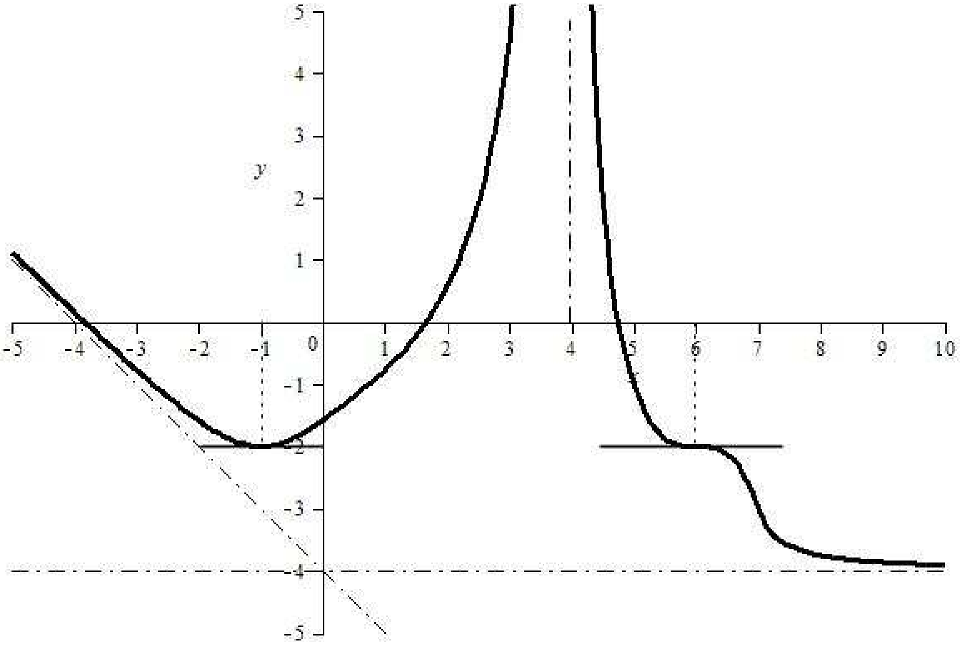
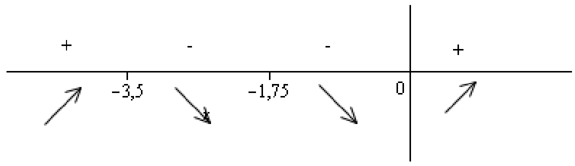
Řešení písemky IMA 7.5.2012 zadání D

1.



2. $f(x)=\dfrac{x^2}{4x+7}, \quad f'(x)=\dfrac{2x(2x+7)}{(4x+7)^2}$,



$f'(x)=0$ pro $x=0 \ \vee \ x=-\dfrac{7}{2}, \quad f(0)=0, f\left(-\dfrac{7}{2}\right)=\dfrac{49}{28}$

$$\lim_{x\to-\frac{7}{4}^{\pm}} f(x)=\left\langle \begin{array}{l} \displaystyle\lim_{x\to-\frac{7}{4}^{+}} \frac{x^2}{4x+7}=+\infty \\ \displaystyle\lim_{x\to-\frac{7}{4}^{-}} \frac{x^2}{4x+7}=-\infty \end{array} \right.$$

- v $x=-\dfrac{7}{4}$ je svislá asymptota , v $x=0$ je maximum, v $x=-\dfrac{7}{2}$ minimum.

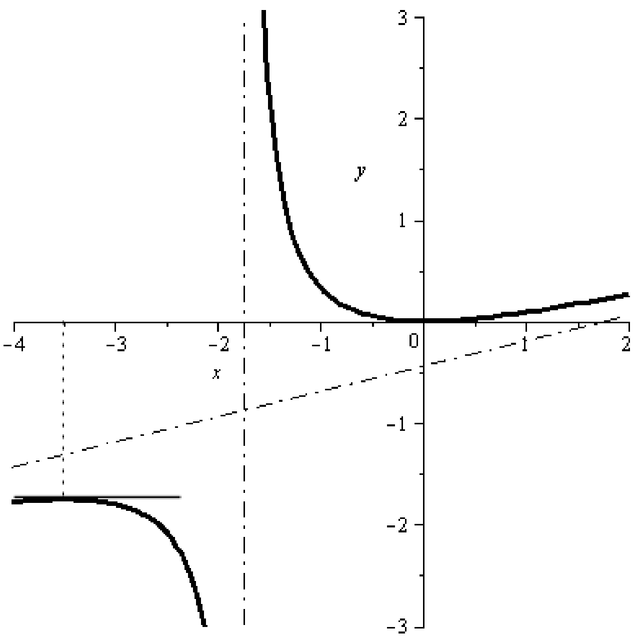
Asymptota se směrnicí:

$$a=\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{x}{4x+7}=\frac{1}{4},$$

$$b=\lim_{x\to\pm\infty}\left(f(x)-a\cdot x\right)=\lim_{x\to\pm\infty}\left(\frac{x^2}{4x+7}-\frac{x}{4}\right)=$$

$$=\lim_{x\to\pm\infty}\frac{4x^2-x(4x+7)}{4(4x+7)}=\lim_{x\to\pm\infty}\frac{-7x}{4(4x+7)}=\frac{-7}{16}$$

asymptota: $y=\dfrac{x}{4}-\dfrac{7}{16}$

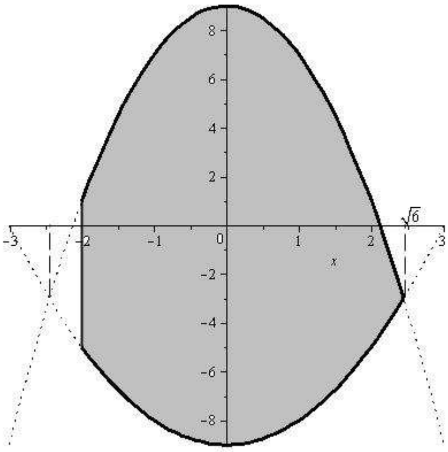


3.

$$\int (5x-4)\cos 3x\,dx = \left| \begin{array}{ll} u=5x-4 & u'=5 \\ v'=\cos 3x & v=\frac{1}{3}\sin 3x \end{array} \right| = \tfrac{1}{3}(5x-4)\sin 3x - \tfrac{5}{3}\int \sin 3x\,dx =$$

$$= \tfrac{1}{3}(5x-4)\sin 3x + \tfrac{5}{9}\cos 3x + c$$

4. Obsah $x^2-9 \le y \le 9-2x^2, \quad x \ge -2$



Průsečíky: $x^2-9 = 9-2x^2 \Leftrightarrow 3x^2 = 18 \Leftrightarrow x = \pm 6$

$$\int_{-2}^{\sqrt{6}} \left(9-2x^2-(x^2-9)\right)dx = \int_{-2}^{\sqrt{6}} (18-3x^2)\,dx =$$

$$= \left[18x - x^3\right]_{-2}^{\sqrt{6}} = 12\sqrt{6}+28$$

Pomocí dvojného integrálu:

$$\int_M 1\,dx\,dy = \int_{-2}^{\sqrt{6}} dx \int_{x^2-9}^{9-2x^2} dy = \int_{-2}^{\sqrt{6}} \left[y\right]_{x^2-9}^{9-2x^2} dx = \int_{-2}^{\sqrt{6}} \left(9-2x^2-(x^2-9)\right)dx$$ dále jako nahoře.

5. Lokální extrémy $z(x,y) = 5x^2e^{-2y} + y^2 + 7y$

$$z'(x,y) = \left(10xe^{-2y}, -10x^2e^{-2y} + 2y + 7\right) = \mathbf{0}$$

$10xe^{-3y} = 0 \Rightarrow x = 0, \quad -10\cdot 0\cdot e^{-2y} + 2y + 7 = 0 \Rightarrow y = -\dfrac{7}{2}$ - stacionární bod $A = \left[0, -\dfrac{7}{2}\right]$.

$$z''(x,y) = \begin{bmatrix} 10e^{-2y} & -20xe^{-2y} \\ -20xe^{-2y} & 20x^2e^{-2y}+2 \end{bmatrix}, \; z''(A) = \begin{bmatrix} 10e^7 & 0 \\ 0 & 2 \end{bmatrix} \quad D_1(A) = 10e^7 > 0,\; D_2(A) = 20e^7 > 0$$

V bodě A nastane minimum, $\quad f_{\min} = f(A) = -\dfrac{49}{4}$.

6. Řešte $\displaystyle\sum_{n=2}^{\infty} \left(\frac{-8}{x}\right)^n = 2$

Geometrická řada, kvocient $\dfrac{-8}{x}$, první člen $\left(\dfrac{-8}{x}\right)^2$.

Podmínka konvergence $\left|\dfrac{-8}{x}\right| < 1 \Leftrightarrow \dfrac{8}{|x|} < 1 \Rightarrow |x| > 8$; součet $s = \left(\dfrac{-8}{x}\right)^2 \cdot \dfrac{1}{1-\frac{-8}{x}} = \dfrac{64}{x^2}\cdot\dfrac{x}{x+8}$

$$\frac{32}{x(x+8)} = 1 \Rightarrow x^2 + 8x - 32 = 0, \quad x_{1,2} = -4 \pm \sqrt{48}$$

Podmínce vyhovuje $x = -4\left(1+\sqrt{3}\right)$