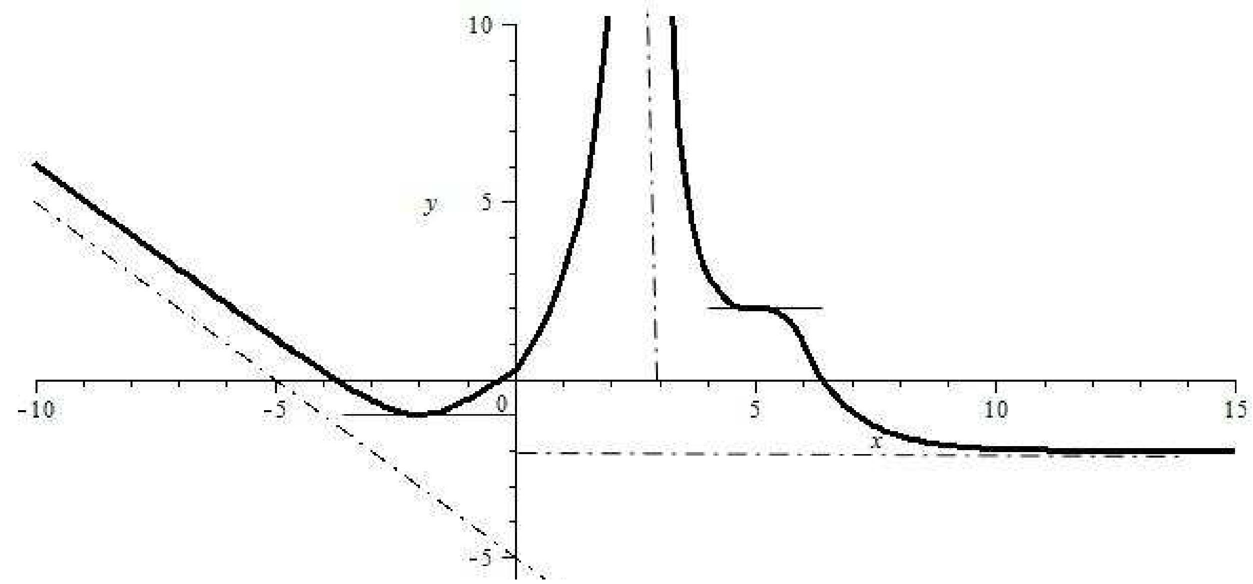
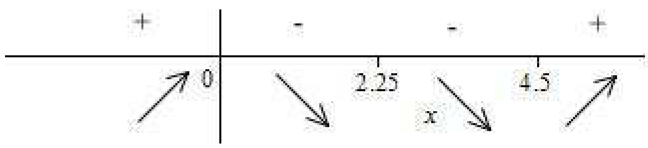
Řešení písemky IMA 7.5.2012 zadání A

1.



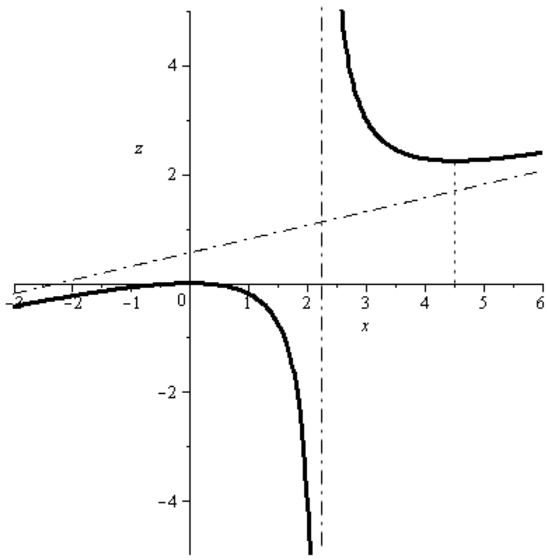
2. $f(x)=\dfrac{x^2}{4x-9}$, $\quad f'(x)=\dfrac{2x(2x-9)}{(4x-9)^2}$,



$f'(x)=0$ pro $x=0 \vee x=\dfrac{9}{2}$

$$\lim_{x\to\frac{9}{4}^{\pm}} f(x)=\left\langle \begin{array}{l} \lim\limits_{x\to\frac{9}{4}^{+}} \dfrac{x^2}{4x-9}=+\infty \\ \\ \lim\limits_{x\to\frac{9}{4}^{-}} \dfrac{x^2}{4x-9}=-\infty \end{array}\right.$$

- v $x=\dfrac{9}{4}$ je svislá asymptota , v $x=0$ je maximum, v $x=\dfrac{9}{2}$ minimum.



Asymptota se směrnicí:

$$a=\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{x}{4x-9}=\frac{1}{4},$$

$$b=\lim_{x\to\pm\infty}\left(f(x)-a\cdot x\right)=\lim_{x\to\pm\infty}\left(\frac{x^2}{4x-9}-\frac{x}{4}\right)=$$

$$=\lim_{x\to\pm\infty}\frac{4x^2-x(4x-9)}{4(4x-9)}=\lim_{x\to\pm\infty}\frac{9x}{16x-36}=\frac{9}{16}$$

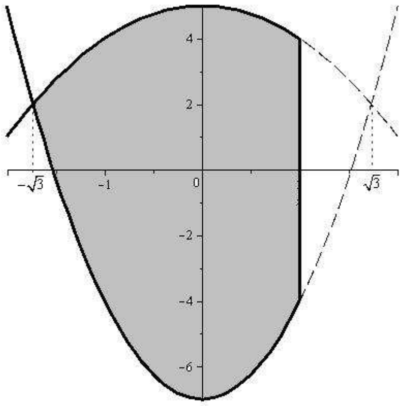
asymptota: $y=\dfrac{x}{4}+\dfrac{9}{16}$

3.

$$\int (7x-1)\sin 4x\,dx = \begin{vmatrix} u = 7x-1 & u' = 7 \\ v' = \sin 4x & v = -\frac{1}{4}\cos 4x \end{vmatrix} = -\tfrac{1}{4}(7x-1)\cos 4x + \tfrac{7}{4}\int \cos 4x\,dx =$$

$$= -\tfrac{1}{4}(7x-1)\cos 4x + \tfrac{7}{16}\sin 4x + c$$

4. Obsah $\quad 3x^2 - 7 \le y \le 5 - x^2, \quad x \le 1$



$3x^2 - 7 = 5 - x^2 \quad 4x^2 = 12 \quad x = \pm\sqrt{3}$

$$\int_{-\sqrt{3}}^{1} \left(5 - x^2 - (3x^2 - 7)\right)dx = \int_{-\sqrt{3}}^{1} \left(12 - 4x^2\right)dx = \left[12x - \tfrac{4}{3}x^3\right]_{-\sqrt{3}}^{1} = \frac{32}{3} + 8\sqrt{3}$$

Pomocí dvojného integrálu:

$$\int_M 1\,dxdy = \int_{-\sqrt{3}}^{1} dx \int_{3x^2-7}^{5-x^2} 1\,dy = \int_{-\sqrt{3}}^{1} [y]_{3x^2-7}^{5-x^2}\,dx = \int_{-\sqrt{3}}^{1} \left(5 - x^2 - (3x^2 - 7)\right)dx$$

a dále jako nahoře.

5. Lokální extrémy funkce $z(x, y) = 4x^2 e^{-3y} + y^2 - 5y$

$$z'(x, y) = \left(8xe^{-3y}, -12x^2 e^{-3y} + 2y - 5\right) = \mathbf{0}$$

$$8xe^{-3y} = 0 \Rightarrow x = 0, \quad -12\cdot 0 \cdot e^{-3y} + 2y - 5 = 0 \Rightarrow y = \frac{5}{2}$$

- stacionární bod $\quad A = \left[0, \dfrac{5}{2}\right]$.

$$z''(x, y) = \begin{bmatrix} 8e^{-3y} & -24xe^{-3y} \\ -24xe^{-3y} & 36x^2 e^{-3y} + 2 \end{bmatrix}, \; z''(A) = \begin{bmatrix} 8e^{-\frac{15}{2}} & 0 \\ 0 & 2 \end{bmatrix} \quad D_1(A) = 8e^{-\frac{15}{2}} > 0, \; D_2(A) = 16e^{-\frac{15}{2}} > 0$$

V bodě A nastane minimum, $\quad f_{\min} = f(A) = -\dfrac{25}{4}$.

6. Řešte $\displaystyle\sum_{n=2}^{\infty} \left(\frac{-6}{x}\right)^n = 4$

Geometrická řada, kvocient $\dfrac{-6}{x}$, první člen $\left(\dfrac{-6}{x}\right)^2$.

Podmínka konvergence $\left|\dfrac{-6}{x}\right| < 1 \Leftrightarrow \dfrac{6}{|x|} < 1 \Rightarrow |x| > 6$; součet $\; s = \left(\dfrac{-6}{x}\right)^2 \cdot \dfrac{1}{1 - \frac{-6}{x}} = \dfrac{36}{x^2} \cdot \dfrac{x}{x+6}$

$$\frac{36}{x(x+6)} = 4 \Rightarrow 9 = x(x+6) \Rightarrow x^2 + 6x - 9 = 0, \quad x_{1,2} = -3 \pm \sqrt{18} = -3\left(1 \mp \sqrt{2}\right)$$

Podmínce vyhovuje $x = -3\left(1 + \sqrt{2}\right)$