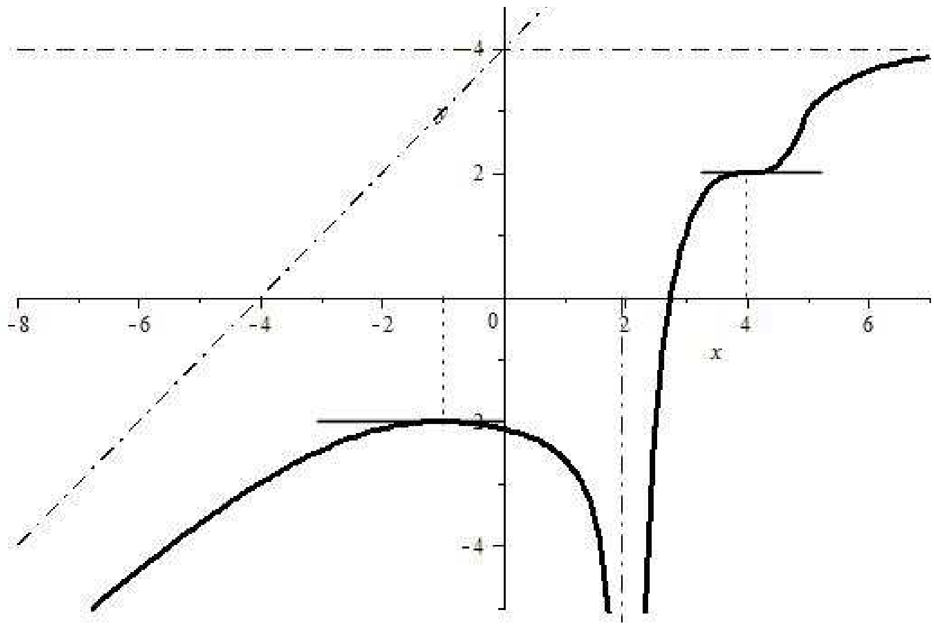
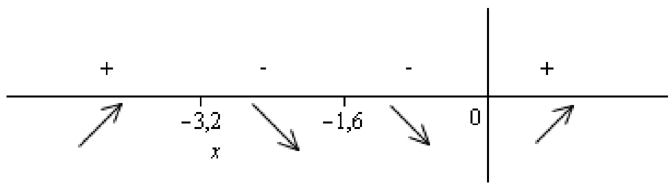
Řešení písemky IMA 7.5.2012 zadání B

1.



2. $f(x)=\dfrac{x^2}{5x+8}, \quad f'(x)=\dfrac{x(5x+16)}{(5x+8)^2}$,



$f'(x)=0$ pro $x=0 \vee x=-\dfrac{16}{5}$

$$\lim_{x\to-\frac{8}{5}^{\pm}} f(x)=\left\langle \begin{array}{l} \lim\limits_{x\to-\frac{8}{5}^{+}} \dfrac{x^2}{5x+8}=+\infty \\ \lim\limits_{x\to-\frac{8}{5}^{-}} \dfrac{x^2}{5x+8}=-\infty \end{array}\right.$$

- v $x=-\dfrac{8}{5}$ je svislá asymptota , v $x=0$ je minimum, v $x=-\dfrac{16}{5}$ maximum.

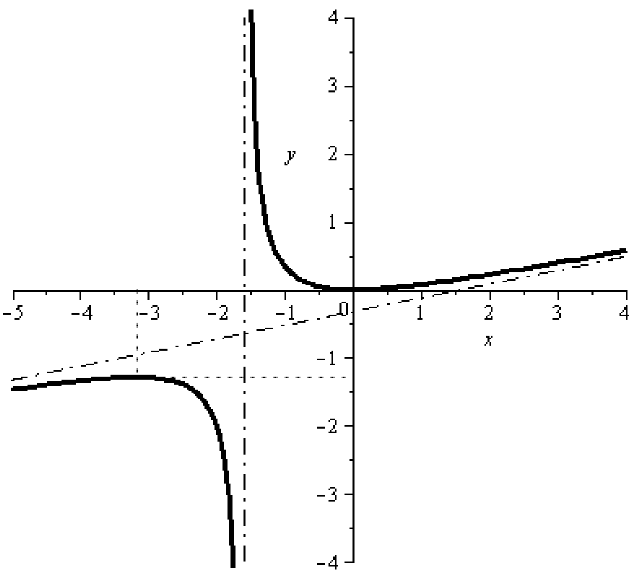
Asymptota se směrnicí:

$$a=\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{x}{5x+8}=\frac{1}{5},$$

$$b=\lim_{x\to\pm\infty}\left(f(x)-a\cdot x\right)=\lim_{x\to\pm\infty}\left(\frac{x^2}{5x+8}-\frac{x}{5}\right)=$$

$$=\lim_{x\to\pm\infty}\frac{5x^2-x(5x+8)}{5(5x+8)}=\lim_{x\to\pm\infty}\frac{-8x}{25x+40}=\frac{-8}{25}$$

asymptota: $y=\dfrac{x}{5}-\dfrac{8}{25}$

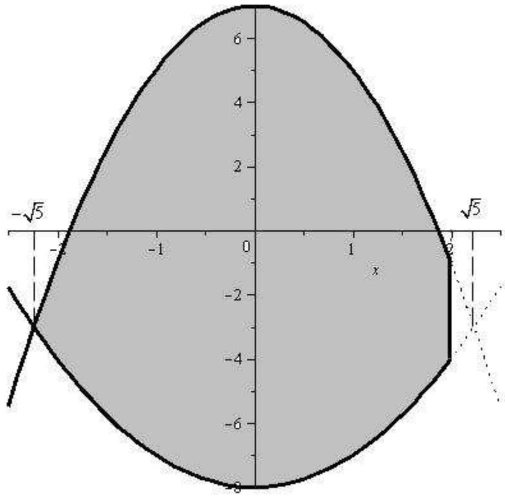


3.

$$\int (4x-3)\sin\frac{x}{3}\,dx = \begin{vmatrix} u=4x-3 & u'=4 \\ v'=\sin\frac{x}{3} & v=-3\cos\frac{x}{3} \end{vmatrix} = -3(4x-3)\cos\frac{x}{3} + 12\int \cos\frac{x}{3}\,dx =$$

$$= -3(4x-3)\cos\frac{x}{3} + 36\sin\frac{x}{3} + c$$

4. Obsah plochy $x^2-8 \le y \le 7-2x^2,\ x \le 2$



Průsečíky: $x^2-8 = 7-2x^2 \Rightarrow 3x^2 = 15 \Rightarrow x = \pm\sqrt{5}$

$$\int_{-\sqrt{5}}^{2} \left(7-2x^2-(x^2-8)\right)dx = \int_{-\sqrt{5}}^{2} \left(15-3x^2\right)dx =$$

$$= \left[15x - x^3\right]_{-\sqrt{5}}^{2} = 22+10\sqrt{5}$$

Pomocí dvojného integrálu:

$$\int_M 1\,dxdy = \int_{-\sqrt{5}}^{2} dx \int_{x^2-8}^{7-2x^2} dy = \int_{-\sqrt{5}}^{2} \left[y\right]_{x^2-8}^{7-2x^2} dx = \int_{-\sqrt{5}}^{2} \left(7-2x^2-(x^2-8)\right)dx$$ a dále jako nahoře.

5. Lokální extrémy $z(x,y) = x^2+5x+4y^2e^{-3x}$

$$z'(x,y) = \left(2x+5-12y^2e^{-3x}, 8ye^{-3x}\right) = \mathbf{0}$$

$8ye^{-3x} = 0 \Rightarrow y = 0,\quad 2x+5-12\cdot 0\cdot e^{-3x} = 0 \Rightarrow x = -\dfrac{5}{2}$ - stacionární bod $A = \left[-\dfrac{5}{2}, 0\right]$.

$$z''(x,y) = \begin{bmatrix} 2+36y^2e^{-3y} & -24ye^{-3x} \\ -24ye^{-3x} & 8e^{-3x} \end{bmatrix},\ z''(A) = \begin{bmatrix} 2 & 0 \\ 0 & 8e^{\frac{15}{2}} \end{bmatrix} \quad D_1(A) = 2 > 0,\ D_2(A) = 16e^{\frac{15}{2}} > 0$$

V bodě *A* nastane minimum, $f_{\min} = f(A) = -\dfrac{25}{4}$.

6. Řešte $\displaystyle\sum_{n=2}^{\infty}\left(\frac{-2}{x}\right)^n = 6$ Geometrická řada, kvocient $\dfrac{-2}{x}$, první člen $\left(\dfrac{-2}{x}\right)^2$.

Podmínka konvergence $\left|\dfrac{-2}{x}\right| < 1 \Leftrightarrow \dfrac{2}{|x|} < 1 \Rightarrow |x| > 2$; součet $s = \left(\dfrac{-2}{x}\right)^2 \cdot \dfrac{1}{1-\dfrac{-2}{x}} = \dfrac{4}{x^2}\cdot\dfrac{x}{x+2}$

$$\frac{4}{x(x+2)} = 6 \Rightarrow 2 = 3x(x+2) \Rightarrow 3x^2+6x-2 = 0,\quad x_{1,2} = \frac{-3\pm\sqrt{9+6}}{3} = -1\pm\frac{\sqrt{15}}{3}$$

Podmínce konvergence vyhovuje $x = -1-\dfrac{\sqrt{15}}{3}$