

Decision Tree

Page No.

MilAn

Date

1) Decision Tree classifier [classification]

2) Decision Tree Regressor [Regression]

* Decision Tree classifier *

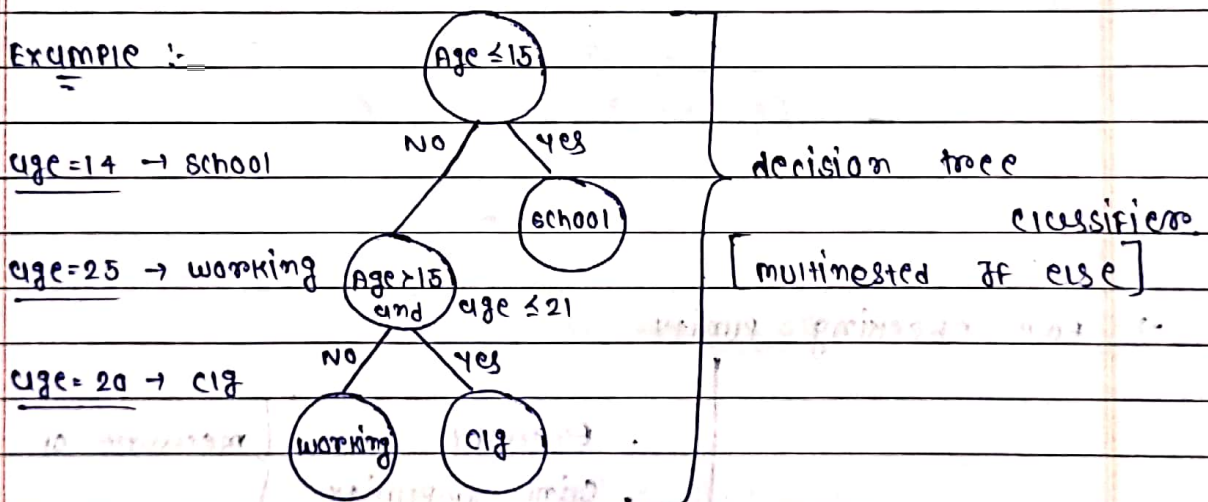
1) ID3 [Iterative dichotomiser 3]

2) CART [classification & Regressor Tree]

→ ID3 HAVE child more than two in tree

→ CART HAVE binary child.

Example :-



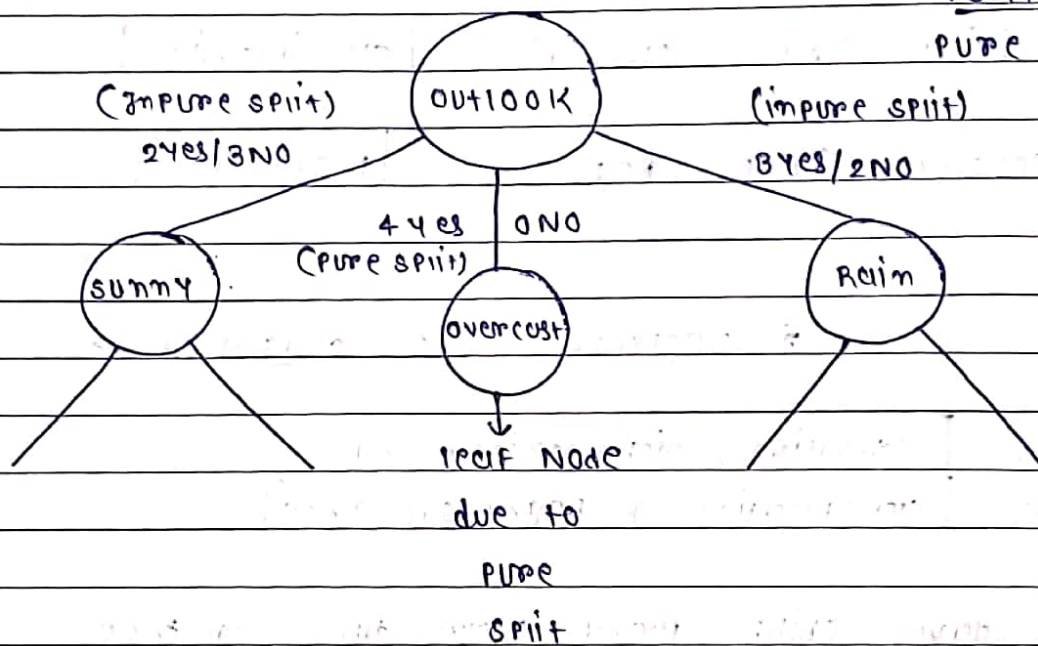
→ Example → Predict play tennis or not.

OUTLOOK

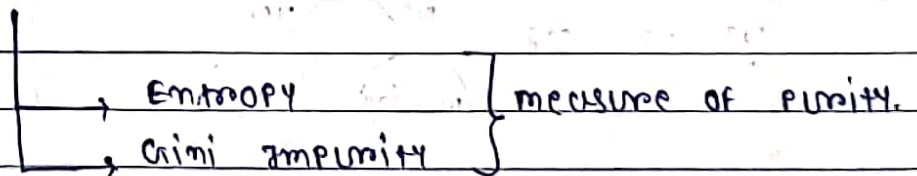
→ we select outlook from given example feature.

→ initial step we will select any one feature as root of the tree.

1) Purity Check:-
 pure split or impure



→ For checking purity



→ what feature you need to select as root?

We use "Information gain"

a) "Entropy"

b) "Gini impurity"

a) Entropy

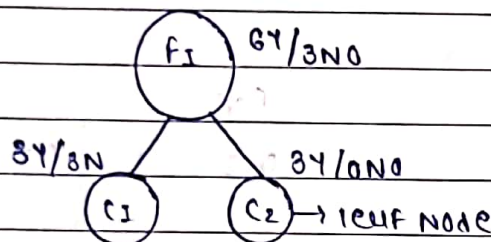
b) Gini Impurity

$$H(G) = -P+ \log_2 P+ -P- \log_2 P-$$

$$G.I = 1 - \sum_{i=1}^n (p_i)^2$$

$P+$ = probability of positive category

$P-$ = probability of negative category



Entropy of $C_1 = -P+ \log_2 P+ - P- \log_2 P-$

$$= -\frac{3}{6} \log_2 \left(\frac{3}{6}\right) - \frac{3}{6} \log_2 \left(\frac{3}{6}\right)$$

$$= -\frac{1}{2} \log_2 \left(\frac{1}{2}\right) - \frac{1}{2} \log_2 \left(\frac{1}{2}\right)$$

$= 1 \rightarrow$ Impure Split [50% Yes, & 50% No]

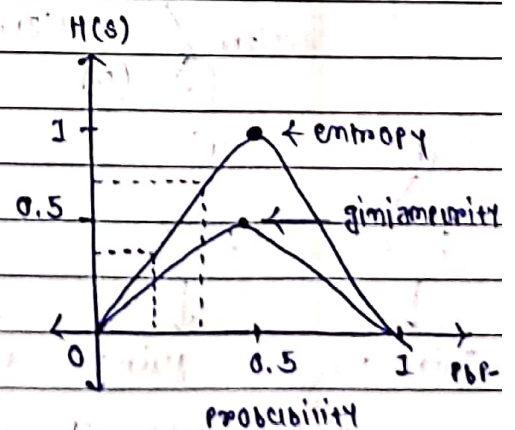
Entropy of $C_2 = -\frac{3}{6} \log_2 \left(\frac{3}{6}\right) - \frac{0}{6} \log_2 \left(\frac{0}{6}\right)$

$$= -\frac{1}{2} \log_2 \left(\frac{3}{3}\right)$$

$$= -\frac{3}{3} \log_2 \left(\frac{3}{3}\right)$$

$$= -1 \log_2 (1)$$

$= 0 \rightarrow$ Pure Split



* For multiclass category

Ex → yes/no/maybe

$$H(S) = -P_1 \log_2 P_1 - P_2 \log_2 P_2 - P_3 \log_2 P_3$$

b) Gini Impurity

(C₁)

$$G.I. = 1 - \sum_{i=1}^n (P_i)^2$$

$$= 1 - \left[\left(\frac{8}{14}\right)^2 + \left(\frac{6}{14}\right)^2 \right]$$

$$= 1 - \left[\left(\frac{8}{14}\right)^2 + \left(\frac{6}{14}\right)^2 \right]$$

$$= \frac{5}{7} = 0.714 \rightarrow \text{impure}$$

(C₂)

$$G.I. = 1 - \left[\left(\frac{3}{3}\right)^2 + \left(\frac{0}{3}\right)^2 \right]$$

$$= 1 - 1$$

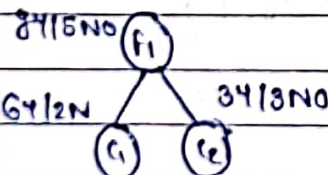
$$= 0 \rightarrow \text{pure split.}$$

→ Information gain

Entropy of Root

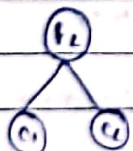
f₁ f₂ f₃ O/P

$$\text{Gain}(S, f_2) = H(S) - \sum \frac{|S_v|}{|S|} H(S_v)$$



$$H(S) = -P_1 \log_2 P_1 - P_2 \log_2 P_2$$

$$H(S) = 0.94$$



$$H(C_1) = 0.81$$

$$H(C_2) = 1$$

$$\text{Gain} = 0.081$$

$$\text{Gain} = 0.048$$

$$\text{Gain}(S, f_2) = 0.94 - \left[\frac{8}{14} \times 0.81 + \frac{6}{14} \times 1 \right]$$

* we use (f₁) as root node.

$$\text{Gain}(S, f_1) = 0.049 < \text{Gain}(S, f_2)$$

* Information gain is more when we split using f₁.

Decision Tree for Numeric Feature

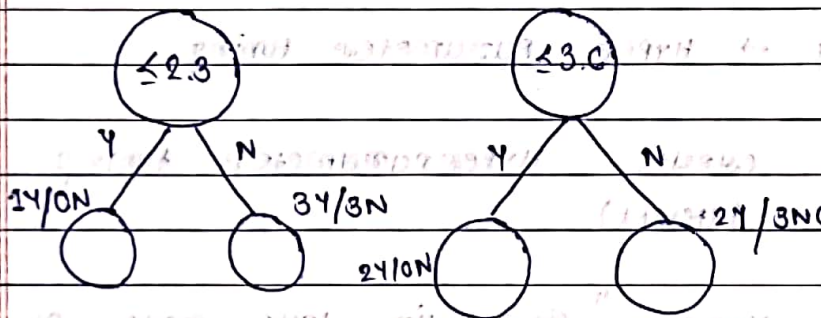
Page No.

MilAm

Date

→ dataset

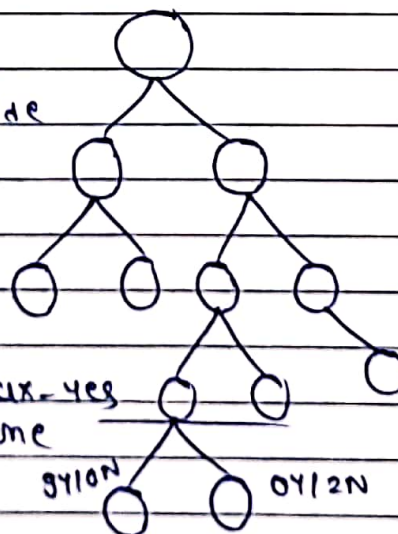
f_1	O/P	if sorting the feature.
2.3	Y	
3.6	Y	* Threshold value = 2.3 [first feature]
4	N	
5.2	N	Next Threshold value = 3.6 [next feature]
6.7	Y	
7.8	N	
9.0	Y	



→ disadvantage is time complexity.

→ Decision tree post pruning and pre pruning.

→ we find leaf node
in general
decision tree.



overfitting

TRAINING ACC ↑
TEST DATA ACC ↓

max = yes
post prune

3Y/10N 0Y/2N

* Two method are use for Reduce overfitting.

1) Pre-pruning

2) Post pruning

1) Post pruning → 1) construct decision tree
2) Prune the tree (cut the tree)

Hyper-parameter → max-depth

2) Pre-pruning → Hyper-parameter tuning

First create Hyperparameter tuning
(For small dataset)

→ max_depth = None "Find till leaf node of decision tree"

* Decision Tree Regressor

Dataset

O/P

Exp

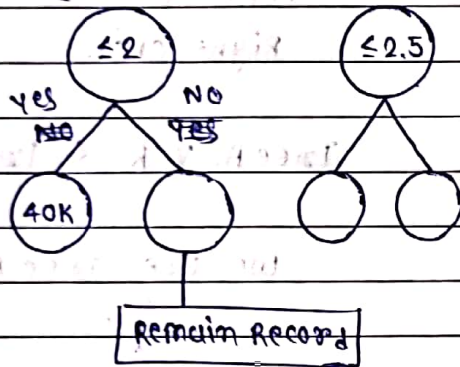
covered gap

Salary

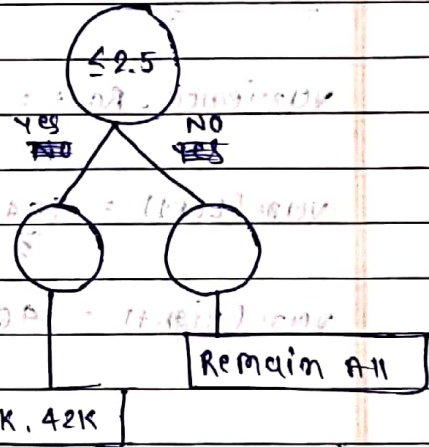
2	yes	40K
2.5	yes	42K
3	No	52K
4	No	60K
4.5	yes	50K

$$\text{Avg} = 50K = \hat{y}$$

(60.8) var



Tree - A



Tree - B

1) we use "variation"
"variance) Reduction"

$$\text{variance} = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y})^2 \quad [\text{mean squared error}]$$

Tree-A {

$$\begin{aligned} \text{variance (Root)} &= \frac{1}{5} [(40-50)^2 + (42-50)^2 + (52-50)^2 + (60-50)^2 + (50-50)^2] \\ &= 60.8 \\ \text{variance (Left)} &= 100 \\ \text{variance (Right)} &= 51 \end{aligned}$$

$$\text{Variance-Reduction} = \text{var}(\text{Root}) - \sum w_i \text{var}(\text{child})$$

For Tree A (L) $w_i = \frac{1}{5}$ $w_i(R) = \frac{4}{5}$ CU2 OF Elements.

$$= 60.8 - \left[\frac{1}{5} * 100 + \frac{4}{5} * 51 \right]$$

$$\text{Variance-Reduction} = 0$$

Variance-Reduction of

For Tree-B

Left split is less than
Right split.

$$\text{variance-Root} = 60.8$$

$$\text{Tree A-V-R} < \text{Tree B-V-R}$$

$$\text{var}(\text{Left}) = \frac{164}{2} = 82$$

We use Tree B

$$\text{var}(\text{Right}) = 46.66$$

$$\text{Variance-Reduction} = 0.004$$