Stanford University

Machine Learning

Padra Esfandiyar's ML Notes

machine learning

- Grew out of work in AI
- new capability for Computers

examples.

Database mining

large data sets from growth of automation/web

↳ ① · web click data ② · medical record ③ biology ④ · engineering

Applications can't program by hand

Autonomous helicopter, handwriting recognition

most of neural language Processing (NLP)

, Computer Vision

Self-Customizing Programs

Amazon, Netflix product recommendations

understanding human learning (brain, real AI)

machine learning Definition

checker game inventor ← **Arthur Samuel (1959):** machine learning: field of study that gives Computers the ability to learn without being explicity programmed.

having Program to Play & experiance tons of games

task for playing checkers

Probability for winning the game

tom mitchell (1998): A Computer Program is said to learn from experience E with respect to some task T and some Performance measure P, if its Performence on T as measure by P, improves with experience E.

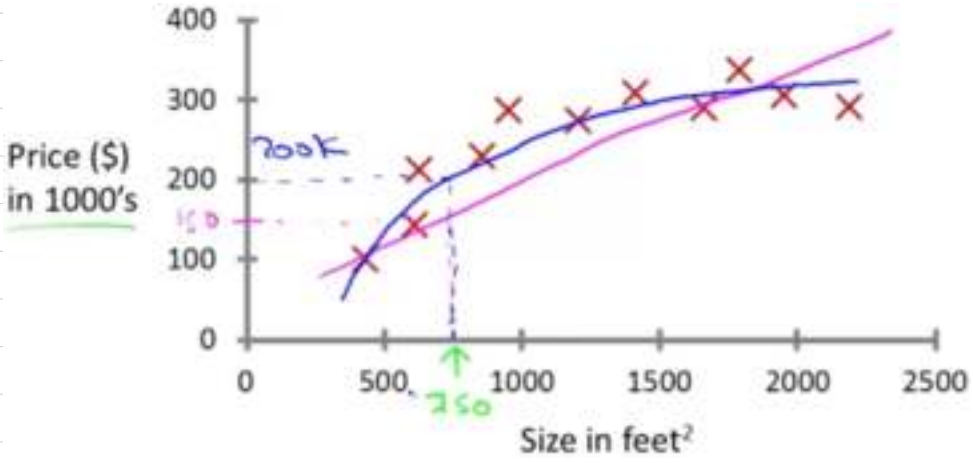
machine learning algorithms:

- Supervised learning → teach computers how to do sth
- un supervised learning → letting it learn by itself

others: reinforcement learning, recommender systems

Supervised Learning examples:

Housing Price Prediction



① Supervised learning ⇒ "right answers" given

② regression: (Supervised learning problem) → Predict Continuous valued output (Price)

Classification: Problem

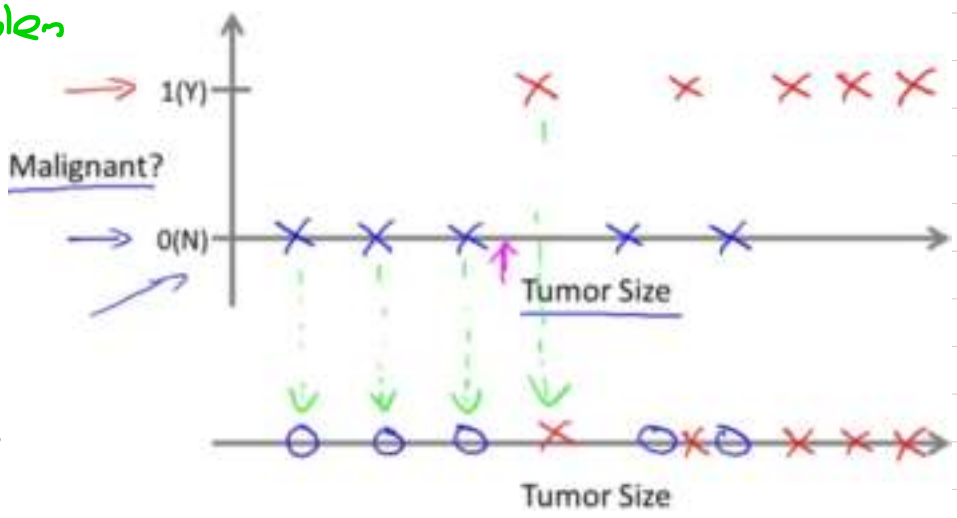
Discrete valued output (0 or 1)

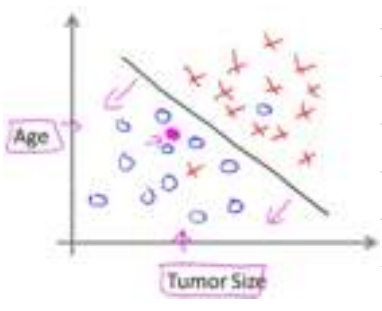
0 , 1 , 2 , 3

0 → benign

1 → malignant

2, 3 → more Attribute





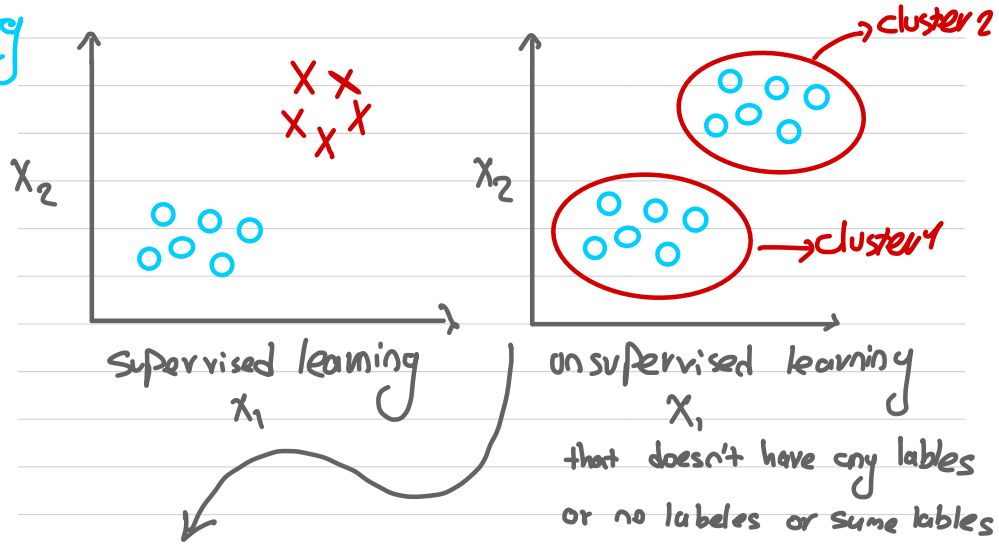
or even more features despit Age & Size like:

1- clump thickness
2- uniformity of cell size
3- uniformity of cell shape

but what about when we want use infinite number of features & attributes ? how we should deal with infinite number of features ?

It turns out that when we talk about an algorithm called the Support Vector machine there will be a neat mathematical trick that will allow a computer to deal with infinite number of features

unsupervised learning

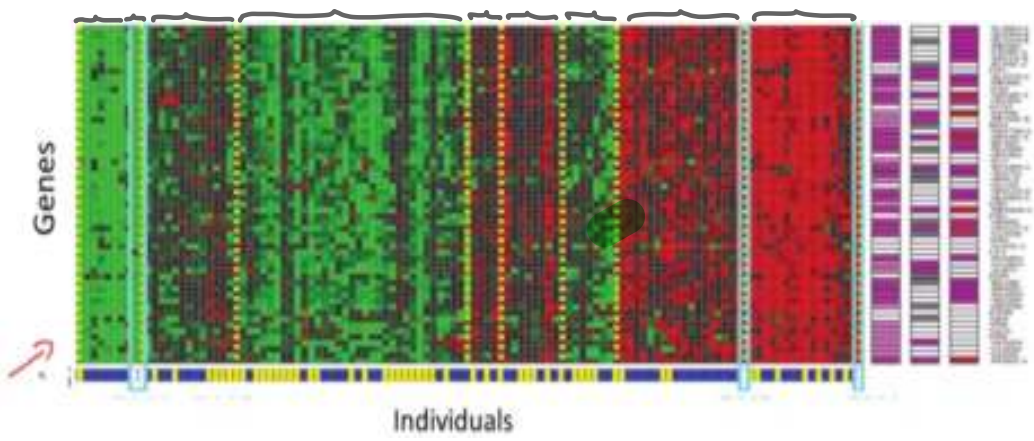


So in this unsupervised example, the Data lives in two different clusters

So the supervised learning algorithm may break these data into these two separate clusters & this is called a clustering algorithm.

one example of clustering in Google news

or another example:



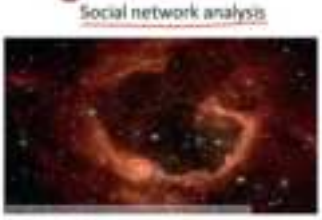
Here's an example of DNA microarray data.The idea is put a group of different individuals and for each of them, you measure how much they do or do not have a certain gene.
Technically you measure how much certain genes are expressed.
So these colors, red, green,gray and so on, they show the degree to which different individuals do or do not have a specific gene.
And what you can do is then run a clustering algorithm to group individuals into different categories or into different types of people.So this is Unsupervised Learning because we're not telling the algorithm in advance that these are type 1 people, those are type 2 persons, those are type 3 persons and so on and instead what were saying is yeah here's a bunch of data.
I don't know what's in this data.I don't know who's and what type.I don't even know what the different types of people are, but can you automatically find structure in the data from the you automatically cluster the individuals into these types that I don't know in advance?
Because we're not giving the algorithm the right answer for the examples in my data set, this is Unsupervised Learning.

unsupervised learning or clustering used in for bunch of other aplications → it's used to organise large Computer clusters like below

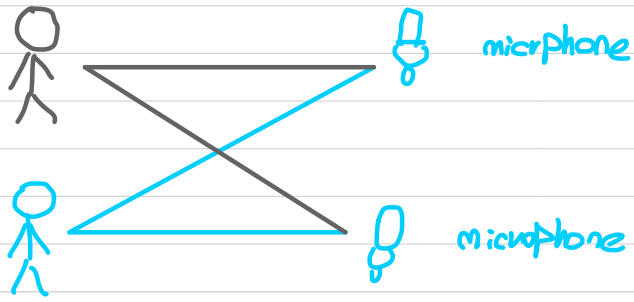

Organize computing clusters


Social network analysis


Market segmentation


Astronomical data analysis

Cocktail Party Problem



it's for those time that you record some different voice & you want separate the original voice from the other voice or background voices or noises

for doing this you have to wright lot's of Code in java or C++ and using their libraries but based on this Cocktail Party algorithm you can do that in just one line.

```
[W,S,V] = svd((repmat(sum(x.*x,1),size(x,1),1).*x)*x');
```

we will use octave → because is the best

=o= wraping_up =o=

Unsupervised Learning

Unsupervised learning allows us to approach problems with little or no idea what our results should look like. We can derive structure from data where we don't necessarily know the effect of the variables.

We can derive this structure by clustering the data based on relationships among the variables in the data.

With unsupervised learning there is no feedback based on the prediction results.

Example:

Clustering: Take a collection of 1,000,000 different genes, and find a way to automatically group these genes into groups that are somehow similar or related by different variables, such as lifespan, location, roles, and so on.

Non-clustering: The "Cocktail Party Algorithm", allows you to find structure in a chaotic environment. (i.e. identifying individual voices and music from a mesh of sounds at a cocktail party).

model Representation :

Dataset & model for housing Prices is like below:

Training set of housing Prices (Portland, OR)

Size in feet² (x)	Price ($) in 1000's (y)
i=1 2104	i=1 460
i=2 1416	i=2 232
i=3 1534	i=3 315
i=4 852	i=4 178
⋮	⋮

m = 47

i = row number in table above

Notation :

m = Number of training examples

x's = "input" Variable / features

y's = "output" Variable / "target" Variables

→ (x,y) = one training example

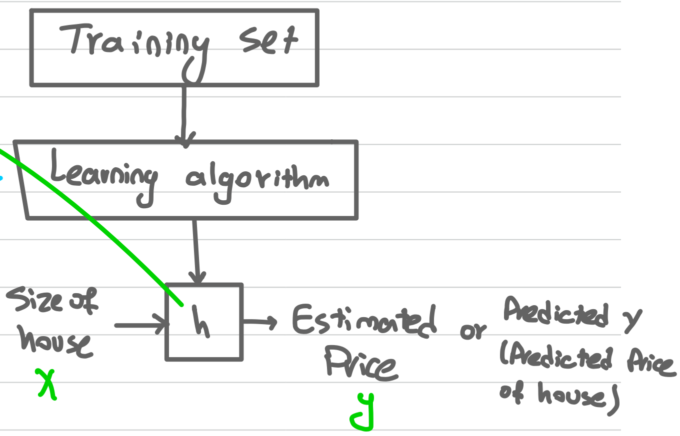
→ $(x^{(i)}, y^{(i)})$ = ith training example

$x^{(1)} = 2104$

$x^{(2)} = 1416$

$y^{(1)} = 460$

How supervised learning works:



The job of this learning algorithm to then output a function which by convention is usually denoted lowercase (h) and h stands for hypothesis

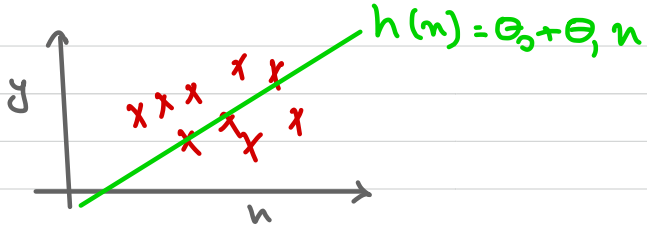
hypothesis job is that it is a function that takes as input the size of the house like maybe the size of the new house of your friend wanna sell & it try to output the estimated value of y for the corresponding house

حالا چرا اسمش hypothesis است؟ چون
همون اوایل علم این اسم انتخاب کردن
و همینجوری روش موند

Question: How do we represent h:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

shorthand: $h(x)$



This model name:

name1: Linear regression with one variable. (x)

name2: univariate linear regression.

a fancy way to say one variable

And why a linear function? Well, someti'mes we'll want to fit more complicated, perhaps non-linear functions as well. But since this linear case is the simple building block, we will start with this example first of fitting linear functions, and we will build on this to eventually have more complex models, and more complex learning algorithms.

Cost function

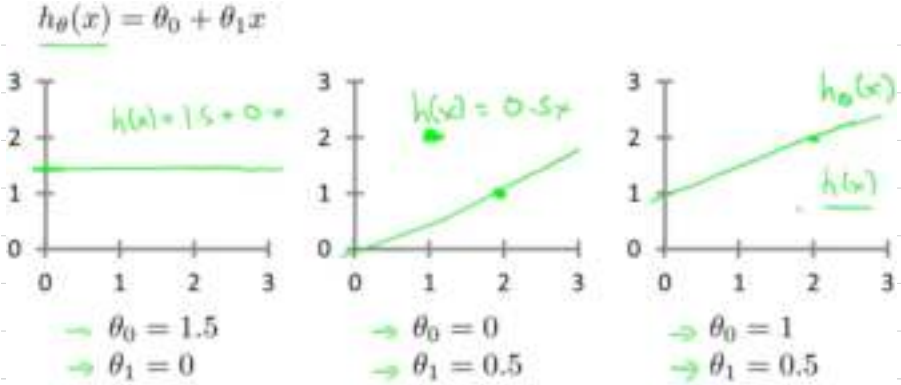
	Size in feet^2 (x)		Price ($) in 1000's ($y$)
$i=1$	2104	$i=1$	460
$i=2$	1416	$i=2$	232
$i=3$	1534	$i=3$	315
$i=4$	852	$i=4$	178
	⋮		⋮

$m = 47$

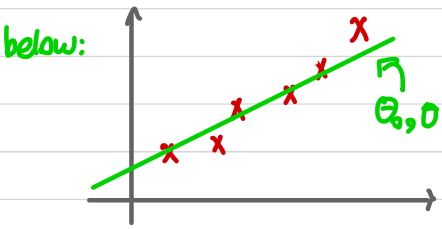
Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

θ_i's: Parameters

How to choose θ_i's ?



So how do we come up with values θ_0, θ_1 that corresponds to a good fit to the Data → for example for the training set below:



idea: choose θ_0, θ_1 so that $h_\theta(x)$ is close to y for our training example (x, y)

So linear regression, what we're going to do is,I'm going to want to solve a minimization problem.So I'll write minimize over theta0 theta1.
And I want this to be small, right?
I want the difference between h(x) and y to be small.And one thing I might do is try to minimize the square difference between the output of the hypothesis and the actual price of a house.
Okay. So lets find some details.You remember that I was using the notation (x(i),y(i)) to represent the i th training example.
So what I want really is to sum over my training set,something i = 1 to m,of the square difference between, this is the prediction of my hypothesis when it is input to size of house number i.
Right? Minus the actual price that house number
I was sold for, and I want to minimize the sum of my training set,
sum from I equals one through M, of the difference of this squared error,
the square difference between the predicted price of a house, and the price that it was actually sold for.

So just a recap.
We're closing this problem as, find me the values of theta zero and theta one so that the average, the 1 over the 2m,
times the sum of square errors between my predictions on the training set minus the actual values of the houses on the training set is minimized.
So this is going to be my overall objective function for linear regression.

training example ← number of

$$\underset{\theta_0,\theta_1}{\text{minimize}} \; \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$h_\theta(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$

So by Convention we usually define a Cost function

$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\underset{\theta_0,\theta_1}{\text{minimize}} \; J(\theta_0,\theta_1)$$

Cost function also called the Square error function

Please turn over (PTO)

When sometimes called the squared error cost function and it turns out that why do we take the squares of the erros.

It turns out that these squared error cost function is a reasonable choice and works well for problems for most regression programs.There are other cost functions that will work pretty well.But the square cost function is probably the most commonly used one forregression problems

= " = wrapping-up = " =

Cost Function

We can measure the accuracy of our hypothesis function by using a **cost function**. This takes an average difference (actually a fancier version of an average) of all the results of the hypothesis with inputs from x's and the actual output y's.

$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(\hat{y}_i - y_i)^2 = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x_i) - y_i)^2$$

To break it apart, it is $\frac{1}{2}\bar{x}$ where $\bar{x}$ is the mean of the squares of $h_\theta(x_i) - y_i$, or the difference between the predicted value and the actual value.

This function is otherwise called the "Squared error function", or "Mean squared error". The mean is halved $\left(\frac{1}{2}\right)$ as a convenience for the computation of the gradient descent, as the derivative term of the square function will cancel out the $\frac{1}{2}$ term. The following image summarizes what the cost function does:

Cost function — intuition 1 & 2

for example let's see the simpler version of last example

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters:

θ_0, θ_1

Cost function

$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Goal:

$$\underset{\theta_0,\theta_1}{\text{minimize}}\ J(\theta_0,\theta_1)$$

Simplified

$$h_\theta(x) = \theta_1 x \qquad \theta_0 = 0$$

θ_1

$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

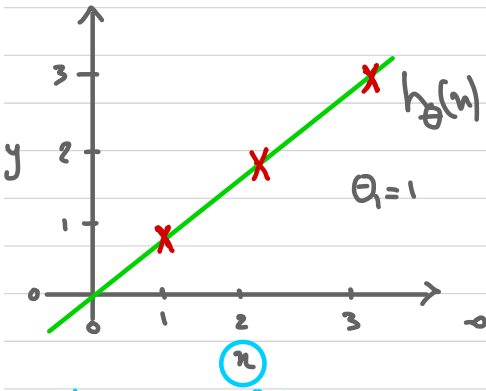
$h_\theta(x^{(i)}) \rightarrow \theta_1 x^{(i)}$

$$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$$

it turns out the two key functions we want to understand, The first is the hypothesis function and The second is a Cost function

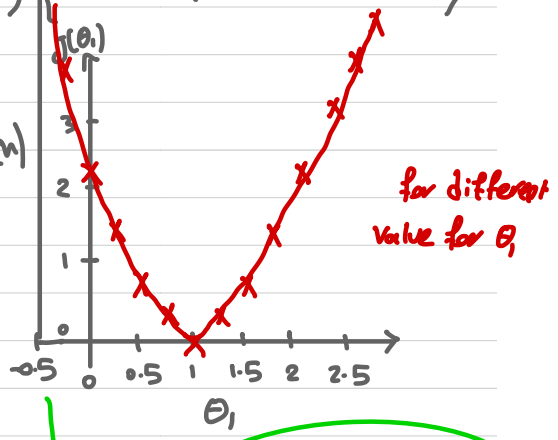
$h_\theta(x)$

(for fixed θ_1, This is the function of x)



$J(\theta_1)$

(function of the Parameter θ_1)



$h_\theta(x) = \theta_1 x \xrightarrow{\theta_1=1} h_\theta(x) = x$

so what is $J(\theta_1) = ?$

you can have negative value for θ_1

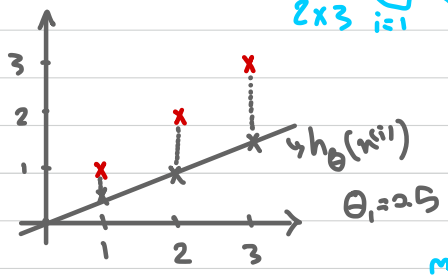
$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$= \frac{1}{2m}\sum_{i=1}^{m}\left(\theta_1 x^{(i)} - y^{(i)}\right)^2$$

1/ So if $\theta_1 = 1 \Rightarrow \frac{1}{2\times3}\sum_{i=1}^{3}\left(x^{(i)} - y^{(i)}\right)^2 \Rightarrow \frac{1}{6}\left(0^2+0^2+0^2\right) = 0$ $\quad J(1)=0$

2/ So if $\theta_1 = 0.5 \Rightarrow \frac{1}{2\times3}\sum_{i=1}^{3}\left(0.5\,x^{(i)} - y^{(i)}\right)^2 \Rightarrow \frac{1}{6}\left((0.5-1)^2+(1-2)^2+(1.5-3)^2\right)$

$= \frac{1}{6}\times(3.5) = 0.58 \Rightarrow J(0.5) = 0.58$



3/ So if $\theta_1 = 0 \Rightarrow \frac{1}{2\times3}\sum_{i=1}^{m=3}\left(0 - y^{(i)}\right)^2$

$= \frac{1}{6}\left((0-1)^2+(0-2)^2+(0-3)^2\right) \Rightarrow \frac{1}{6}\times14 = 2.3$

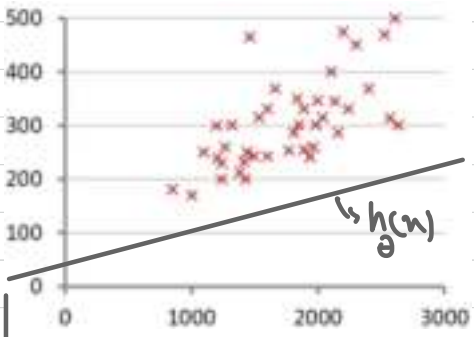
$$J(0) \simeq 2.3$$

So Each value of θ_1 corresponds to a different hypothesis

Cost function - intuition II 8 So this is like last time but this time we keep our both Parameter (θ_0, θ_1)

$h_\theta(x)$

(for fixed θ_0, θ_1 this a function of x)

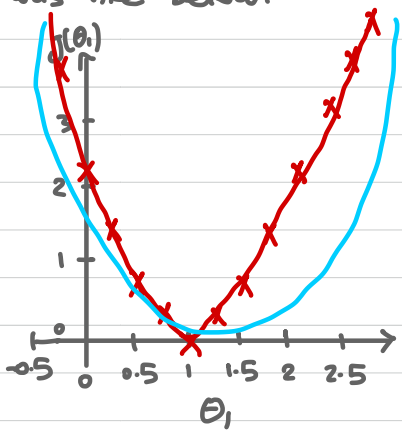


So for example if we have $\theta_0 = 50$, $\theta_1 = 0.06$ we have $h_\theta(x) = 50 + 0.06x$

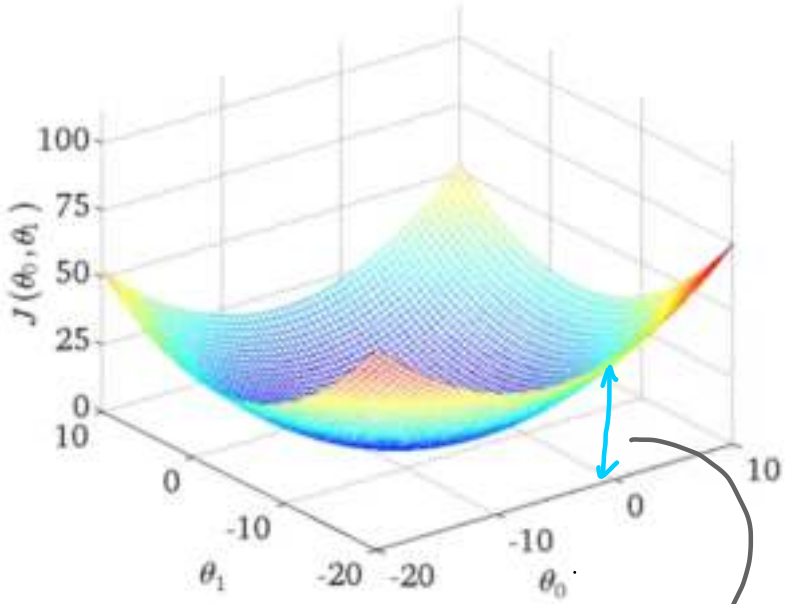
$J(\theta_0, \theta_1)$

(function of the Parameters θ_0, θ_1)

but in this case when we had only θ_1 The training set was like below:

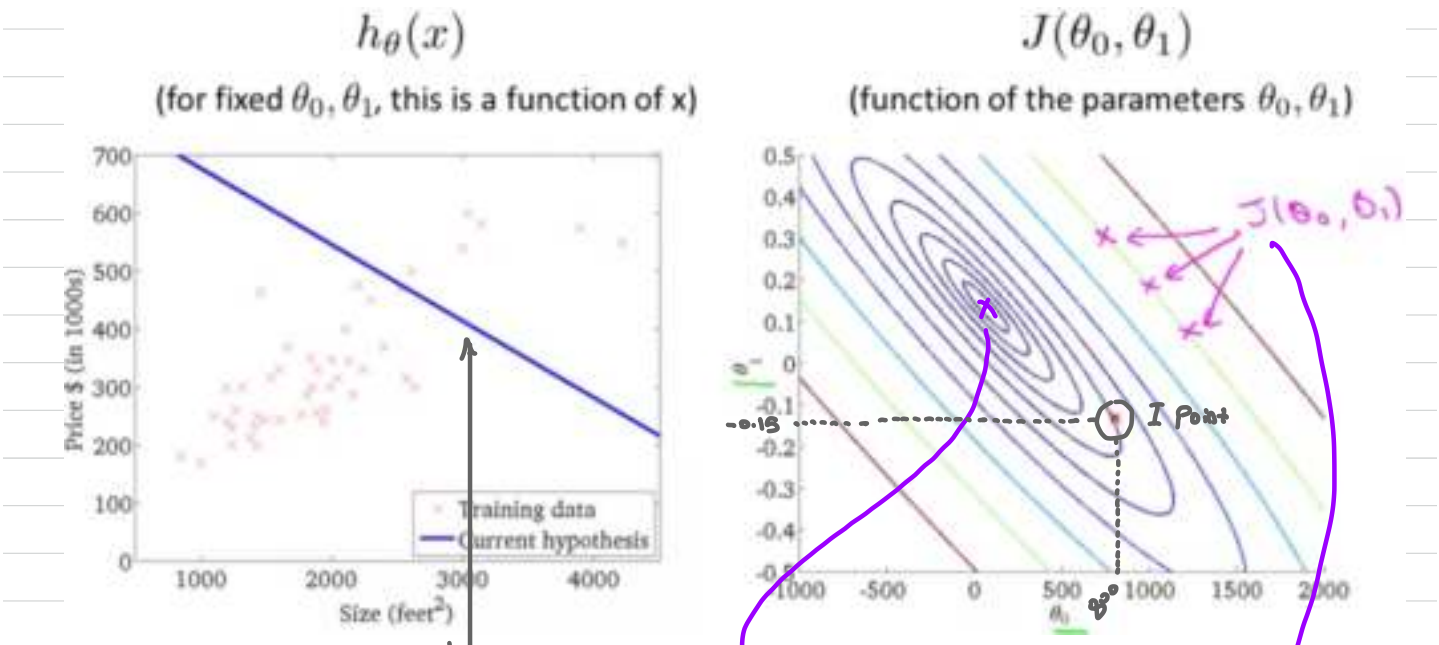


but now because we have both θ_0 , θ_1 it's going to be look like This



So, this is a 3-D surface plot, where the axes are labeled theta zero and theta one. So as you vary theta zero and theta one, the two parameters, you get different values of the cost function J (theta zero, theta one) and the height of this surface above a particular point of theta zero, theta one. Right, that's, that's the vertical axis. The height of the surface of the points indicates the value of J of theta zero, J of theta one. $\Rightarrow J(\theta_0, \theta_1)$

Example④ So finally it's like this8



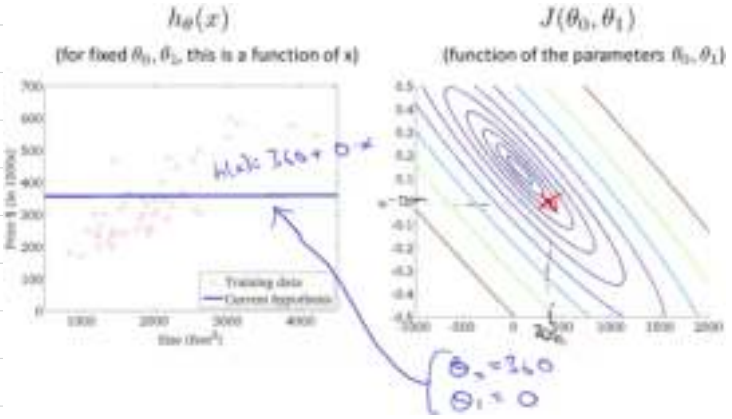
and I Point acully Corresponds to one set of Pair values of θ_0, θ_1 & they Corresponding in fact to that hypothesis

→ This is the bottom or lower point of that curv and these three Point for example, have same valu because they have same height on that line in the curve.

So now this line is really not such a good fit to the data. right. This hypothesis, h(x), with these values of θ_0, θ_1 it's really not such a good fit to the data. & so you find that it's Cost. is a value that's out here & that's you know it's pretty far from the minimum. IT's pretty far this is a pretty high Cost because this is just not that good a fit to the Data.

Example2:

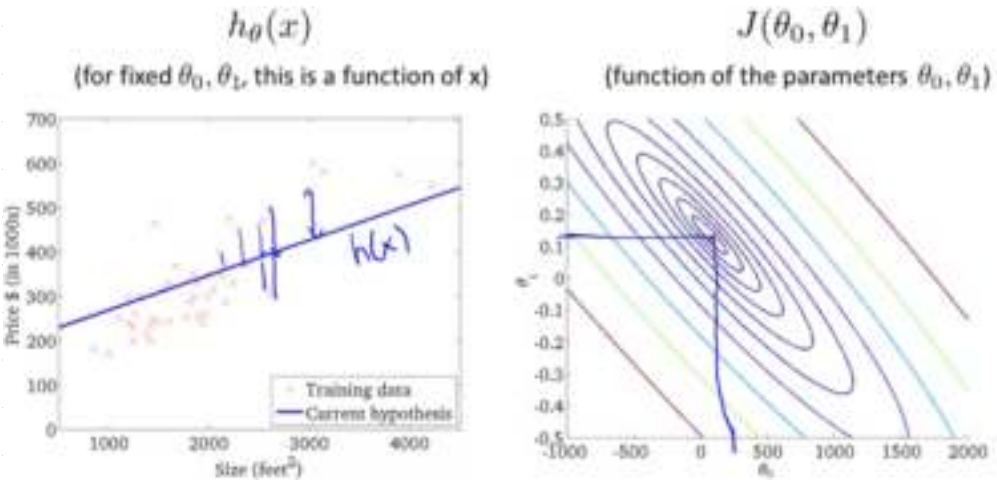
Now here's a different hypothesis that's you know still not a great fit for the data but may be slightly better



Now here's a different hypothesis that's you know still not a great fit for the data but may be slightly better

So, you know, let's break it out. Let's take theta zero equals 360 theta one equals zero. And this pair of parameters corresponds to that hypothesis, corresponds to flat line, that is, h(x) equals 360 plus zero times x. So that's the hypothesis. And this hypothesis again has some cost, and that cost is, you know, plotted as the height of the J function at that point.

example:



Last example, this is actually not quite at the minimum, but it's pretty close to the minimum. So this is not such a bad fit to the, to the data, where, for a particular value, of, theta zero. Which, one of them has value, as in for a particular value for theta one.We get a particular h(x). And this is, this is not quite at the minimum, but it's pretty close. And so the sum of squares errors is sum of squares distances between my, training samples and my hypothesis. Really, that's a sum of square distances,right? Of all of these errors. This is pretty close to the minimum even though it's not quite the minimum. So with these figures I hope that gives you a better understanding of what values of the cost function J, how they are and how that corresponds to different hypothesis and so as how better hypotheses may corresponds to points that are closer to the minimum of this cost function J.

Now of course what we really want is an efficient algorithm, right, a efficient piece of software for automatically finding The value of theta zero and theta one, that minimizes the cost function J, right? And what we, what we don't wanna do is to, you know, how to write software, to plot out this point, and then try to manually read off the numbers, that this is not a good way to do it. And, in fact, we'll see it later, that when we look at more complicated examples, we'll have high dimensional figures with more parameters, that, it turns out, we'll see in a few, we'll see later in this course, examples where this figure, you know, cannot really be plotted, and this becomes much harder to visualize. And so, what we want is to have software to find the value of theta zero, theta one that minimizes this function and in the next video we start to talk about an algorithm for automatically finding that value of theta zero and theta one that minimizes the cost function J.

Gradient Descent:

let's start to talk about an algorithm for automatically finding that value of θ_0, θ_1 that minimize the Cost function J.

Have some function $J(\theta_0, \theta_1)$

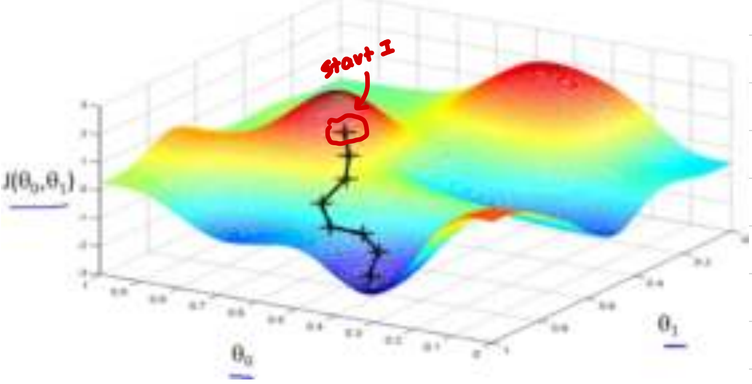
want $\min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$

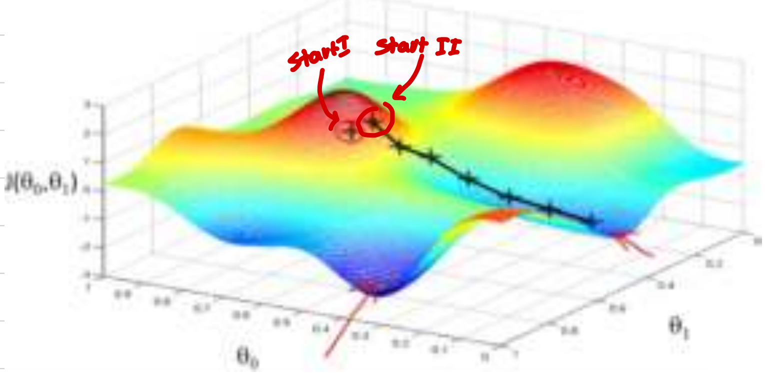
outline:

- Start with some θ_0, θ_1
- keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$ until we hopefully endup at a minimum

→ so we are going to start off with some initials guesses like we say $(\theta_0 = 0, \theta_1 = 1)$

so let's how it's look





assignment

$a := b$

$a := a + 1$

Truth assertion

$a = b$ → یعنی این دوتا مقدارشون برابره

$a = a + 1$ ✗

what is alpha? it's called learning rate and what alpha does is basically Controls how big a step we take downhill with creating descent. So if alpha is very large, then that corresponds to a very aggressive gradient descent procedure where we're trying take huge steps downhill and

So we use Gradient descent algorithm below:

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad (\text{for } j = 0 \text{ and } j = 1)$$

}

Correct: Simultaneous update

$$temp0 := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

→ derivative term & we will talk about this later

$$temp1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

$\theta_0 := temp0$

$\theta_1 := temp1$

if alpha is very small then we are taking little baby steps downhill.

So how you implement gradient descent is for

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad \text{(for } j = 0 \text{ and } j = 1\text{)}$$

this expression, for this update equation

you want to simultaneously update θ_0 & θ_1

and the way to implement is you should compute

the right hand side for θ_0 and θ_1 and then simultaneosly

at the same time update θ_0, θ_1

look at down below we store the right side to temp0 & temp1

and then update θ_0, θ_1

Correct: Simultaneous update

$$\begin{cases} \text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) \\ \text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) \end{cases}$$

derivative term & we will talk about this later

$$\begin{cases} \theta_0 := \text{temp0} \\ \theta_1 := \text{temp1} \end{cases}$$

and down below for example is a incorrect implementation:

$\rightarrow$ $\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$

$\rightarrow$ $\theta_0 := \text{temp0}$

$\rightarrow$ $\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$

$\rightarrow$ $\theta_1 := \text{temp1}$

θ_0 is change & temp1 is not right anymore

in the previous video we gave a mathematical definition of gradient descent. Now Let's delve deeper & in this video get better intuition about what the algorithm is doing and why the steps of the gradient descent algorithm might make sense.

let's review what we learnt so far:

Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

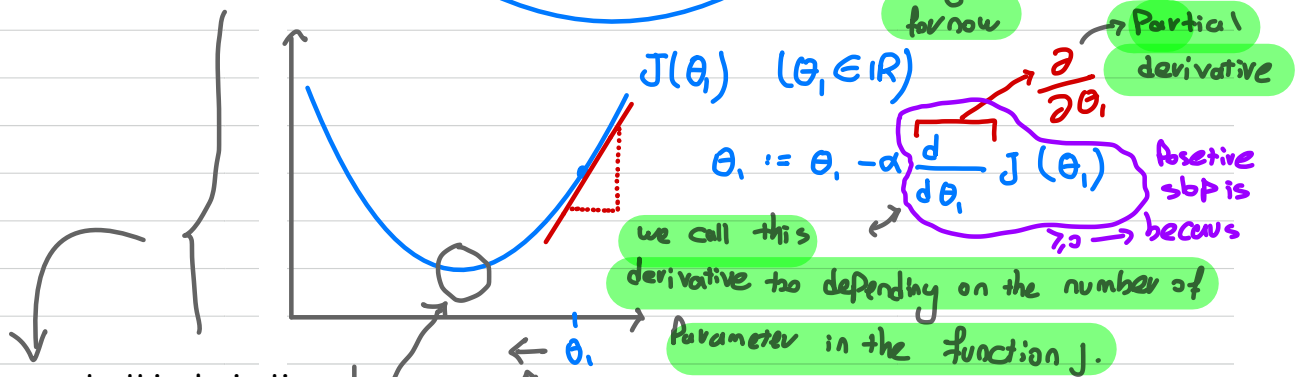
(simultaneously update $j = 0$ and $j = 1$)

}

learning rate

derivative term

So let's see an example where we want to minimize $J(\theta_1)$ when $(\theta_1 \in \mathbb{R})$ θ_1 is a real number.



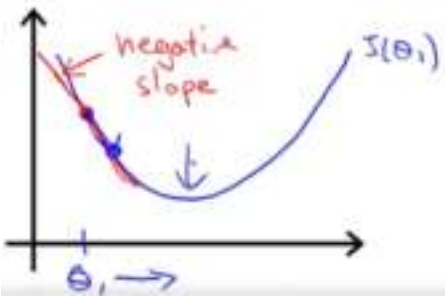
So we're going to compute this derivative, not sure if you've seen derivatives in calculus before, but what the derivative at this point does, is basically saying,now let's take the tangent to that point, like that straight line, that red line,is just touching this function, and let's look at the slope of this red line.
That's what the derivative is,it's saying what's the slope of the line that is just tangent to the function.
Okay, the slope of a line is just this height divided by this horizontal thing.Now, this line has a positive slope,so it has a positive derivative.
And so my update to theta is going to be theta 1,it gets updated as theta 1, minus alpha times some positive number.

$\theta_1 := \theta_1 - \alpha \cdot$ (Positive number)

always Positive number

So we are going update θ_1 as θ_1 minus smth. so i'm gonna end up moving θ_1 to the left & decrease θ_1 and we can see this is the right thing to do cuz I actually wanna head in this direction to get me close to the minimum over there.

let's see another example:



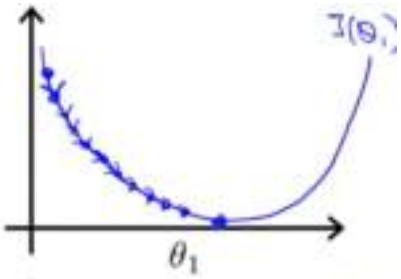
$$\frac{\partial}{\partial \theta_1} J(\theta_1) < 0$$

$\theta_1 := \theta_1 - \alpha$ (negative number)

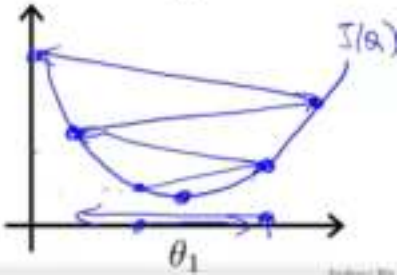
let's talk about a situation:

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.



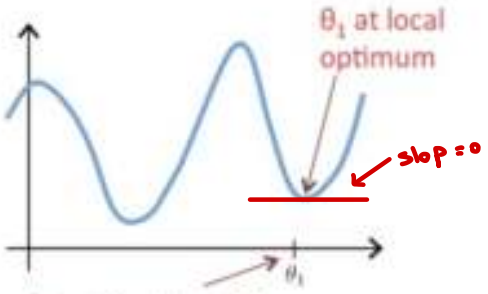
If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



let's see an example or a Tricky question

Suppose θ_1 is at a local optimum of $J(\theta_1)$, such as shown in the figure.

What will one step of gradient descent $\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$ do?



- Leave θ_1 unchanged
- Change θ_1 in a random direction
- Move θ_1 in the direction of the global minimum of $J(\theta_1)$
- Decrease θ_1

✓ Correct

It turns out the local optimum, your derivative will be equal to zero. So for that slope, that tangent point, so the slope of this line will be equal to zero and thus this derivative term is equal to zero.
And so your gradient descent update,you have theta one cuz I updated this theta one minus alpha times zero.
And so what this means is that if you're already at the local optimum it leaves theta 1 unchanged cause its updates as theta 1 equals theta 1.So if your parameters are already at a local minimum one step with gradient descent does absolutely nothing it doesn't your parameter which is what you want because it keeps your solution at the local optimum.

$$\theta_1 := \theta_1 - \alpha \underbrace{\frac{d}{d\theta_1} J(\theta_1)}_{=0}$$

$\theta_1 := \theta_1 - \alpha \cdot 0$

$\theta_1 := \theta_1$ → so it leaves θ_1 unchanged

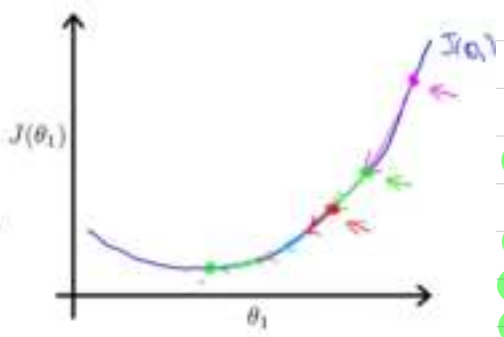
This also explains why gradient descent can converse the local minimum even with the learning rate alpha fixed

↳ turn over → in the next Page

Gradient descent can converge to a local minimum, even with the learning rate α fixed.

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.



So just to recap, in gradient descent as we approach a local minimum, gradient descent will automatically take smaller steps.And that's because as we approach the local minimum,by definition the local minimum is when the derivative is equal to zero.

As we approach local minimum, this derivative term will automatically get smaller, and so gradient descent will automatically take smaller steps.This is what so no need to decrease alpha or the time.

Gradient Descent for linear Regression

in this video we're gonna Put together, gradient descent with our Cost function, and that will give us an algorithm for linear regression or Putting a straight line to our data

Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

(for $j = 1$ and $j = 0$)

}

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$\min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$

so what we are going to do is apply gradient descent to minimize our squared error Cost function

So in order to apply gradient descent, in order to, you know write this piece of Code, the key term we need is this derivative term over here.

So we have:

$$\frac{\partial}{\partial \theta_j} J(\theta_0,\theta_1) = \frac{\partial}{\partial \theta_j} \cdot \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2 \Rightarrow$$

$$= \frac{\partial}{\partial \theta_j} \cdot \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)} \right)^2$$

$$\theta_0 \quad j=0 : \frac{\partial}{\partial \theta_0} J(\theta_0,\theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)$$

$$\theta_1 \quad j=1 : \frac{\partial}{\partial \theta_1} J(\theta_0,\theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)}$$

Gradient descent algorithm

repeat until convergence {

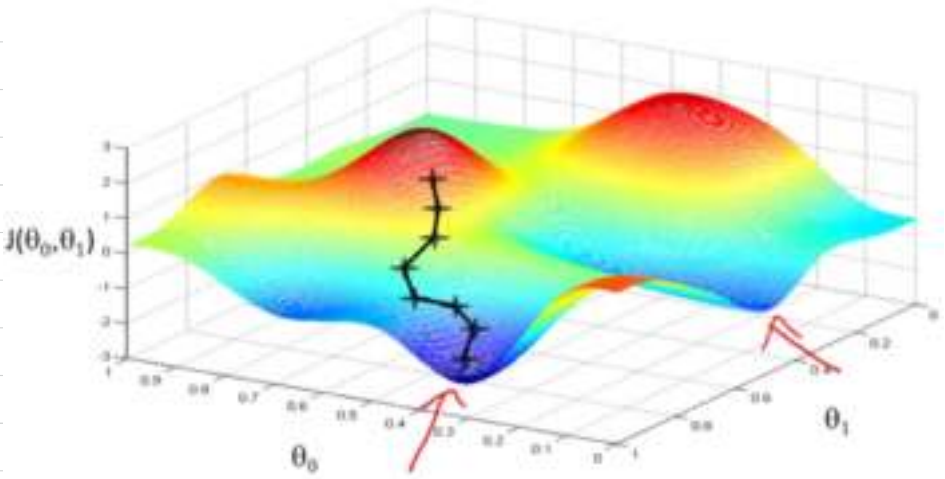
Update θ_0 & θ_1 simultaneously

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \quad \rightarrow \frac{\partial}{\partial \theta} J(\theta_0,\theta)$$

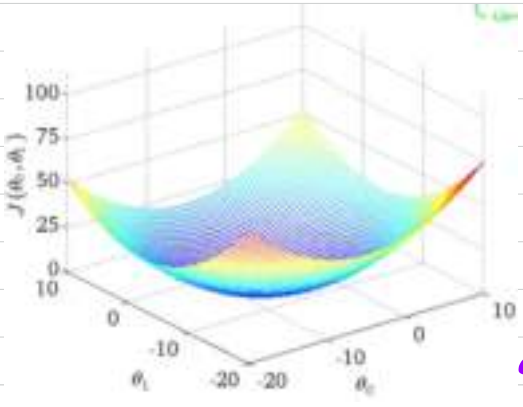
$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)} \quad \rightarrow \frac{\partial}{\partial \theta} J(\theta_0,\theta_1)$$

So let's see how gradient descent works.

Let's see how gradient descent works.One of the issues we saw with gradient descent is that it can be susceptible to local optima. So when I first explained gradient descent I showed you this picture of it going downhill on the surface, and we saw how depending on where you initialize it, you can end up at different local optima.You will either wind up here or here.



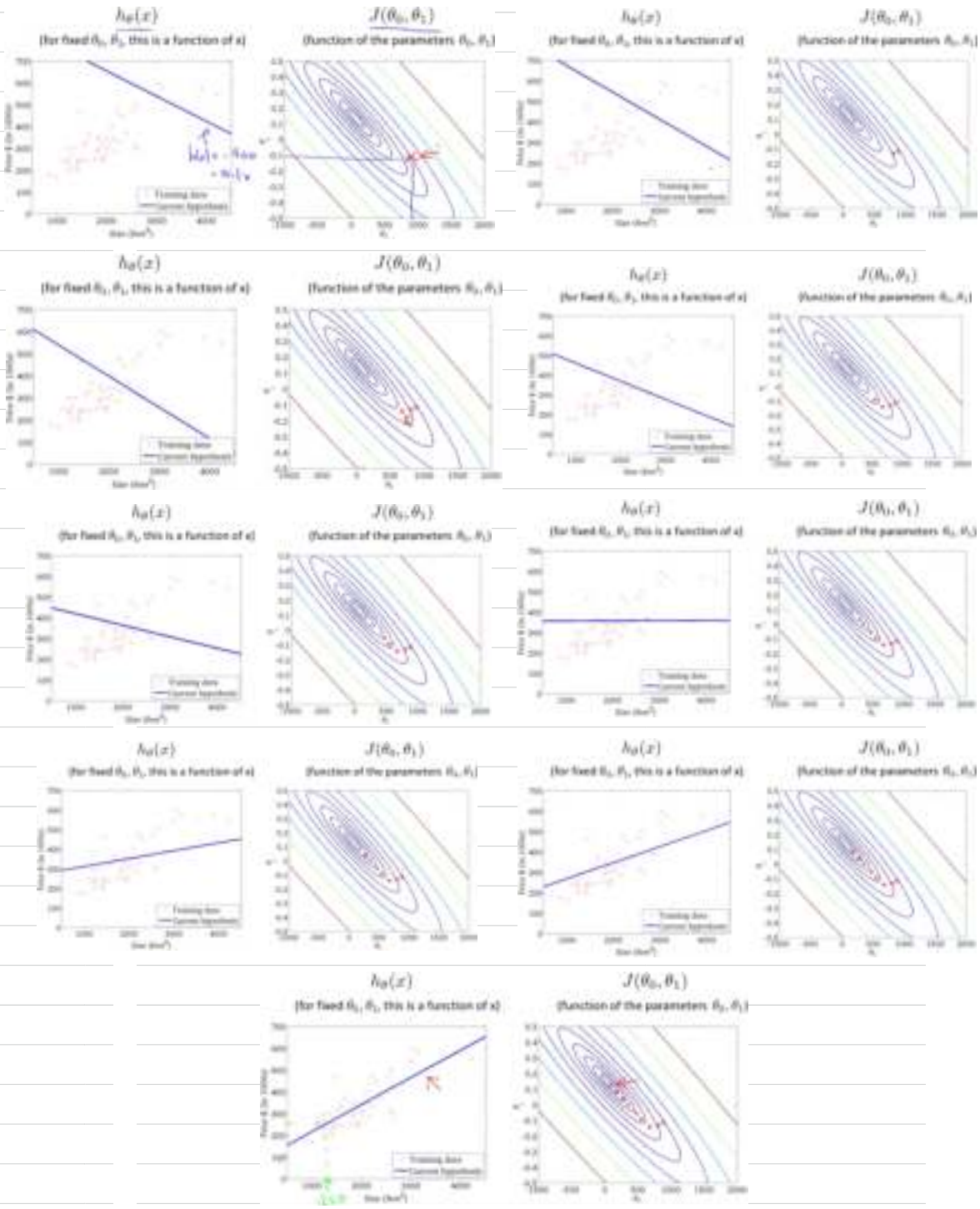
But, it turns out that that the cost function for linear regression is always going to be a bow shaped function like this.The technical term for this is that this is called a convex function.



informally a convex function means a bowl shaped function and so this function doesn't have any local optima except for the one global optimum

And does gradient descent on this type of cost function which you get whenever you're using linear regression it will always converge to the global optimum. Because there are no other local optimum, global optimum.

and now let's see this algorithm in action:



As usual, here are plots of the hypothesis function and of my cost function j.And so let's say I've initialized my parameters at this value.
Let's say, usually you initialize your parameters at zero, zero.
Theta zero and theta equals zero.
But for the demonstration, in this physical infrontation
I've initialized you know, theta zero at 900 and theta one at about -0.1 okay.And so this corresponds to h(x)=-900-0.1x, [the intercept should be +900] is this line, out here on the cost function.
Now, if we take one step in gradient descent,we end up going from this point out here, over to the down and left, to that second point over there.And you notice that my line changed a little bit, and as I take another step of gradient descent, my line on the left will change.

Right?And I've also moved to a new point on my cost function.

And as I take further steps of gradient descent, I'm going down in cost.
So my parameters and such are following this trajectory.

And if you look on the left, this corresponds with hypotheses.
That seem to be getting to be better and better fits to the data

until eventually I've now wound up at the global minimum and this global minimum corresponds to this hypothesis, which gets me a good fit to the data.

And so that's gradient descent, and we've just run it and
gotten a good fit to my data set of housing prices.
And you can now use it to predict, you know,
if your friend has a house size 1250 square feet,
you can now read off the value and tell them that I don't know maybe they could
get $250,000 for their house.

And it turns out that there are sometimes other versions of gradient descent that are not batch versions, but they are instead. Do not look at the entire training set but look at small subsets of the training sets at a time.

gradient Descent $\Longleftrightarrow$ "Batch" Gradient Descent

"Batch": Each step of gradient descent uses all the training examples.

So that's linear regression with gradient descent. If you've seen advanced linear algebra before, so some of you may have taken a class in advanced linear algebra. You might know that there exists a solution for numerically solving for the minimum of the cost function j without needing to use an iterative algorithm like gradient descent. Later in this course we'll talk about that method as well that just solves for the minimum of the cost function j without needing these multiple steps of gradient descent.
That other method is called the normal equations method.
But in case you've heard of that method it turns out that gradient descent will scale better to larger data sets than that normal equation method.
And now that we know about gradient descent we'll be able to use it in lots of different contexts and we'll use it in lots of different machine learning problems as well.

matrix and vectors 3

matrix: Rectangular array of numbers

$$\begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

4x2 matrix — $\mathbb{R}^{4\times 2}$

2x3 matrix — $\mathbb{R}^{2\times 3}$

Dimension of matrix: number of rows × number of columns

matrix Elements: (entries of matrix)

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

A_{ij} = "i, j entry" in the i^{th} row, j^{th} column

$A_{11} = 1402$

$A_{12} = 191$

$A_{32} = 1437$

$A_{41} = 147$

A_{13} = undefined (error)

hint: we use uppercase letter to refer to matrix like A, B, C, X and use lowercase like a, b, n, y to refer to either numbers, just raw numbers or scalars or to vectors.

So it turns out that in most of math, the one index version is more common but for a lot of machine learning applications, zero index vectors gives us a more convenient notation → by default 1 index unless we want 0-index

Vector: An n x 1 matrix

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

$n = 4$

← 4-dimensional vector $\mathbb{R}^4$

$y_i = i^{th}$ element

$y_1 = 460$

$y_2 = 232$

$y_3 = 315$

1-index vs 0-indexed:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \qquad y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

1-indexed 0-indexed

Matrix Addition

$$\begin{bmatrix}1 & 0\\ 2 & 5\\ 3 & 1\end{bmatrix} + \begin{bmatrix}4 & 0.5\\ 2 & 5\\ 0 & 1\end{bmatrix} = \begin{bmatrix}5 & 0.5\\ 4 & 10\\ 3 & 2\end{bmatrix}$$

3×2 matrix + 3×2 = 3×2

$$\begin{bmatrix}4 & 0\\ 2 & 5\\ 3 & 1\end{bmatrix} + \begin{bmatrix}4 & 0.5\\ 2 & 5\end{bmatrix} = \text{error}$$

3×2 + 2×2

Scalar Multiplication

real number

$$3 \times \begin{bmatrix}1 & 0\\ 2 & 5\\ 3 & 1\end{bmatrix} = \begin{bmatrix}3 & 0\\ 6 & 15\\ 9 & 3\end{bmatrix} = \begin{bmatrix}1 & 0\\ 2 & 5\\ 3 & 1\end{bmatrix} \times 3$$

3×2

$$\begin{bmatrix}4 & 0\\ 6 & 3\end{bmatrix} /4 = \frac{1}{4}\begin{bmatrix}4 & 0\\ 6 & 3\end{bmatrix} = \begin{bmatrix}1 & 0\\ \frac{3}{2} & \frac{3}{4}\end{bmatrix}$$

Combination of Operands

$$3 \times \begin{bmatrix}1\\4\\2\end{bmatrix} + \begin{bmatrix}0\\0\\5\end{bmatrix} - \begin{bmatrix}3\\0\\2\end{bmatrix} /3$$

scalar multiplication — scalar division

$$= \begin{bmatrix}3\\12\\6\end{bmatrix} + \begin{bmatrix}0\\0\\5\end{bmatrix} - \begin{bmatrix}1\\0\\\frac{2}{3}\end{bmatrix}$$

matrix subtraction / vector subtraction

matrix addition / vector addition

$$= \begin{bmatrix}2\\12\\10\frac{1}{3}\end{bmatrix}$$

2×1 matrix
2-dimensional vector

Example

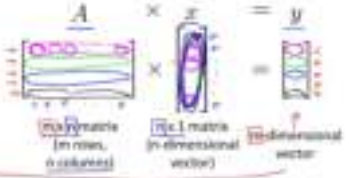
$$\begin{bmatrix}1 & 3\\ 4 & 0\\ 2 & 1\end{bmatrix} \begin{bmatrix}1\\5\end{bmatrix} = \begin{bmatrix}16\\4\\7\end{bmatrix}$$

3×2 · 2×1 = 3×1 matrix

1×1 + 3×5 = 16
4×1 + 0×5 = 4
2×1 + 1×5 = 7

⇒

Details:



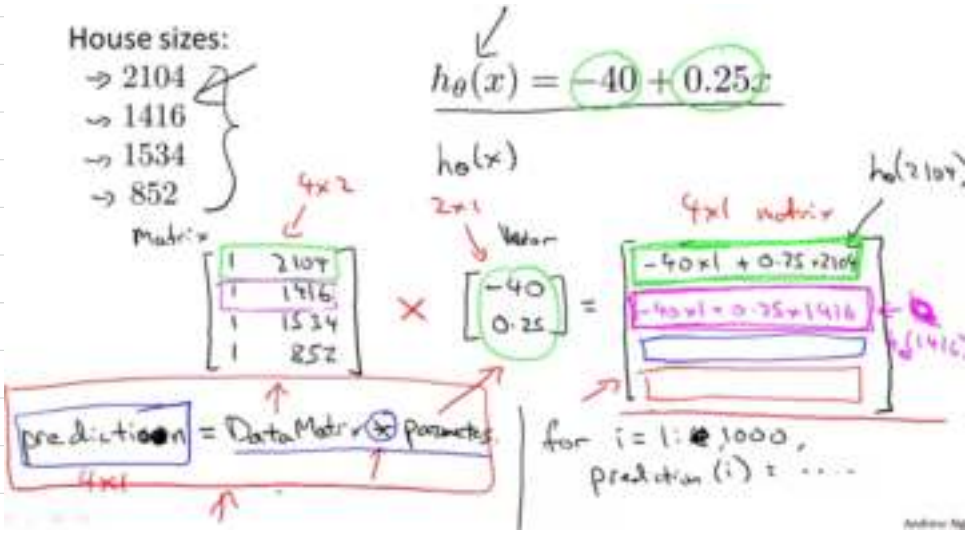
To get y_i, multiply A's i^{th} row with elements of vector x, and add them up.

Example

$$\begin{bmatrix}1 & 2 & 1 & 5\\ 0 & 4 & 0 & 4\\ -1 & -2 & 0 & 0\end{bmatrix} \begin{bmatrix}1\\3\\2\\1\end{bmatrix} = \begin{bmatrix}14\\12\\-7\end{bmatrix}$$

3×4 · 4×1 = 3×1

1×1 + 2×3 + 1×2 + 5×1 = 14
0×1 + 4×3 + 0×2 + 4×1 = 12
−1×1 + (−2)×3 + 0×2 + 0×1 = −7



if you're trying to predict the prices of not just four but maybe of a thousand houses then it turns out that when you implement this in the computer, implementing it like this, in any of the various languages.This is not only true for Octave, but for Supra Server Java or Python, other high-level, other languages as well.

It turns out, that, by writing code in this style on the left, it allows you to not only simplify the code, because, now, you're just

writing one line of code rather than the form of a bunch of things inside.But, for subtle reasons, that we will see later, it turns out to be much more computationally efficient to make predictions on all of the prices of all of your houses doing it the way on the left than the way on the right than if you were to write your own formula.

I'll say more about this later when we talk about vectorization, but, so, by posing a prediction this way, you get not only a simpler piece

of code, but a more efficient one.

```
% Initialize matrix A
A = [1, 2, 3; 4, 5, 6; 7, 8, 9]

% Initialize vector v
v = [1; 1; 1]

% Multiply A * v
Av = A * v
```

Run

```
A =
   1   2   3
   4   5   6
   7   8   9

v =
   1
   1
   1

Av =
   6
  15
  24
```

Matrix matrix multiplication:

Example

$$\begin{bmatrix}1 & 3 & 2\\ 4 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\ 0 & 1\\ 5 & 2\end{bmatrix} = \begin{bmatrix}11 & 10\\ 9 & 14\end{bmatrix}$$

$$\begin{bmatrix}1 & 3 & 2\\ 4 & 0 & 1\end{bmatrix} \times \begin{bmatrix}1\\ 0\\ 5\end{bmatrix} = \begin{bmatrix}11\\ 9\end{bmatrix}$$

$$\begin{bmatrix}1 & 3 & 2\\ 4 & 0 & 1\end{bmatrix} \times \begin{bmatrix}3\\ 1\\ 2\end{bmatrix} = \begin{bmatrix}10\\ 14\end{bmatrix}$$

Example

$$\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}\begin{bmatrix}0 & 1\\ 3 & 2\end{bmatrix} = \begin{bmatrix}9 & 4\\ 15 & 12\end{bmatrix}$$

$$\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}\begin{bmatrix}0\\ 3\end{bmatrix} = \begin{bmatrix}1\times 0 + 3\times 3\\ 2\times 0 + 5\times 3\end{bmatrix} = \begin{bmatrix}9\\ 15\end{bmatrix}$$

$$\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}\begin{bmatrix}1\\ 2\end{bmatrix} = \begin{bmatrix}1\times 1 + 3\times 2\\ 2\times 1 + 5\times 2\end{bmatrix} = \begin{bmatrix}7\\ 12\end{bmatrix}$$

Details:

$$A \times B = C$$

$$\begin{bmatrix}\quad\end{bmatrix} \times \begin{bmatrix}\quad\end{bmatrix} = \begin{bmatrix}\quad\end{bmatrix}$$

$m \times n$ matrix (m rows, n columns)

$n \times o$ matrix (n rows, o columns)

$m \times o$ matrix

The i^{th} column of the matrix C is obtained by multiplying A with the i^{th} column of B. (for $i = 1,2,\ldots,o$)

So with just one matrix multiplication step you to manage to make 12 Predictions and even better it turns out that in order to do that matrix multiplications, there are lots of good linear algebra libraries in order to do this multiplication step for you.

Example

$$\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}\begin{bmatrix}0 & 1\\ 3 & 2\end{bmatrix} = \begin{bmatrix}9 & 4\\ 15 & 12\end{bmatrix}$$

$$\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}\begin{bmatrix}0\\ 3\end{bmatrix} = \begin{bmatrix}1\times 0 + 3\times 3\\ 2\times 0 + 5\times 3\end{bmatrix} = \begin{bmatrix}9\\ 15\end{bmatrix}$$

$$\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}\begin{bmatrix}1\\ 2\end{bmatrix} = \begin{bmatrix}1\times 1 + 3\times 2\\ 2\times 1 + 5\times 2\end{bmatrix} = \begin{bmatrix}7\\ 12\end{bmatrix}$$

And so pretty much any reasonable programming language that you might be using. Certainly all the top ten most popular programming languages will have great linear algebra libraries.

And there'll be good linear algebra libraries that are highly optimized in order to do that matrix-matrix multiplication very efficiently.

Including taking advantage of any sort of parallel computation that your computer may be capable of, whether your computer has multiple cores or multiple processors.

Or within a processor sometimes there's parallelism as well called SIMD parallelism that your computer can take care of.

And there are very good free libraries that you can use to do this matrix-matrix multiplication very efficiently, so that you can very efficiently make lots of predictions with lots of hypotheses.

Matrix multiplication properties :

matrix multiplication is really useful, since you can pack a lot of computation in to just one matrix multiplication operation. but you should be careful how you use them

Commutative Property definition:

for example

$A \times B = B \times A$ → این ضرب دوسو قابل جابجایی است و یک زبان است یک گفته می‌شود Commutative Property.

Associative Property definition:

for example

$3 \times 5 \times 2 \Rightarrow \begin{cases} 15 \times 2 = 30 \\ 3 \times 10 = 30 \end{cases}$

خاصیت جابجایی در ضرب است

number one: Let A & B be matrices. Then in general

$$A \times B \neq B \times A \quad \text{(not Commutative)}$$

Eg. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

$\neq$

$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$

$A \times B$

$m \times n \quad n \times m$

$A \times B \Rightarrow m \times m$

$B \times A \Rightarrow n \times n$

number two:

$A \times (B \times C)$

$(A \times B) \times C$ → same answer

$A \times B \times C$

Let $D = B \times C$ Compute $A \times D$.

Let $E = A \times B$ Compute $E \times C$.

Identity Matrix

1 is identity $1 \times 7 = 7 \times 1 = 7$ for any 7

Denoted I (or $I_{n \times n}$).

Examples of identity matrices:

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 2×2 $\quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 3×3 $\quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 4×4

Informally: $\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$

For any matrix A,

$A \cdot I = I \cdot A = A$ $\quad I_{n \times n}$

$m \times n \quad n \times n \quad m \times m \quad m \times n \quad m \times n$

Note:

$AB \neq BA$ in general

$AI = IA$ ✓

inverse & transpose :

```
» A= [3 4; 2 16]

A =

    3    4
    2   16

» inverse of A = pinv(A)

inverse of A =

    0.400000   -0.100000
   -0.050000    0.075000

»» A * inverse of A

ans =

    1.0000e+000   -5.5511e-017
   -2.2204e-016    1.0000e+000
```

$1 =$ "identity" $\quad 3\,[3^{-1}] = 1 \quad 12\,(12^{-1}) = 1$

$0\,(0^{-1})$ undefined

Not all numbers have an inverse.

Matrix inverse: (Square matrix: # rows = # columns) A^{-1}

If A is an m x m matrix, and if it has an inverse,

$$A(A^{-1}) = A^{-1}A = I.$$

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ← no inverse

E.g. $\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}$ (A) $\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}$ (A^{-1}) $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$

$A^{-1}A$

Matrices that don't have an inverse are "singular" or "degenerate"

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$A^{-1}A$

Matrices that don't have an inverse are "singular" or "degenerate"

$$A_{ij} = A^T_{ji}$$

```
% Initialize matrix A
A = [1,2,0;0,5,6;7,0,9]

% Transpose A
A_trans = A'

% Take the inverse of A
A_inv = inv(A)

% What is A^(-1)*A?
A_invA = inv(A)*A
```

Run

Reset

```
A =

     1     2     0
     0     5     6
     7     0     9

A_trans =

     1     0     7
     2     5     0
     0     6     9

A_inv =

    0.348837   -0.139535    0.093023
    0.325581    0.069767   -0.046512
   -0.271318    0.108527    0.038760

A_invA =

    1.00000   -0.00000    0.00000
    0.00000    1.00000   -0.00000
   -0.00000    0.00000    1.00000
```

Linear Regression with multiple Variables

multiple features

let's see a more Powerful version of linear Regression

lately we had just the size of the house & we wanted to use that to Predict the Price of the house like below

Size (feet²) x	Price ($1000) y
2104	460
1416	232
1534	315
852	178
...	...

$$h_\theta(x) = \theta_0 + \theta_1 x$$

but now

Multiple features (variables).

number of rows

Size (feet²) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price ($1000) y
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178
...	...	...	...	...

$m = 47$

Notation:

- n = number of features $n = 4$
- $x^{(i)}$ = input (features) of i^{th} training example.
- $x_j^{(i)}$ = value of feature j in i^{th} training example.

$x^{(2)} = \begin{bmatrix} 1416 \\ 3 \\ 2 \\ 40 \end{bmatrix}$ ← $\mathbb{R}^4$

$x_3^{(2)} = 2$

it is a n dementional vector ← $\mathbb{R}^n$

2 dimentional vector ← $\mathbb{R}^2$

hypothesis:

Previously: $h_\theta(x) = \theta_0 + \theta_1 x$

new one: $h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$

E.g. $h_\theta(x) = 80 + 0.1 x_1 + 0.01 x_2 + 3 x_3 - 2 x_4$

x_1: # rooms; x_2: # of floors; x_4: age

$\rightarrow h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$

For convenience of notation, define $x_0 = 1$. $(x_0^{(i)} = 1)$

$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^{n+1}$ $\quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in \mathbb{R}^{n+1}$

$h_\theta(x) = \theta_0 x_0 + \theta_1 x_1 + \cdots + \theta_n x_n$ $\quad (x_0 = 1)$

$= \theta^T x$

θ^T is $(n+1) \times 1$ matrix.

$\theta^T x$

So that's the form of hypothesis when we have multiple features and just to give this another name - this is also called multivariate linear regression.

Linear regression with multiple variables

Gradient descent for multiple variables

Hypothesis: $h_\theta(x) = \theta^T x = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$ $\quad (x_0 = 1)$

Parameters: ~~$\theta_0, \theta_1, \ldots, \theta_n$~~ θ $\quad$ $n+1$-dimensional vector

Cost function:

$$~~J(\theta_0, \theta_1, \ldots, \theta_n)~~ = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

$J(\theta)$

Gradient descent:

Repeat {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} ~~J(\theta_0, \ldots, \theta_n)~~ J(\theta)$$

} (simultaneously update for every $j = 0, \ldots, n$)

let's explain this partial derivative of j function

Gradient Descent

Previously (n=1):

Repeat {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$$

(simultaneously update θ_0, θ_1)

}

$\frac{\partial}{\partial \theta_j} J(\theta)$

New algorithm $(n \geq 1)$:

Repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update θ_j for $j = 0, \ldots, n$)

}

$\frac{\partial}{\partial \theta_j} J(\theta)$

$x_0^{(i)} = 1$

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)}$$

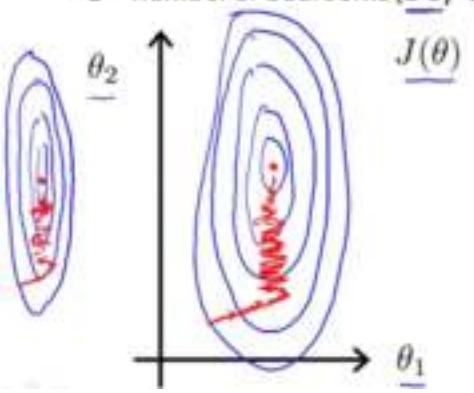
Gradient Descent in Practice I - Feature Scaling

Feature Scaling

Idea: Make sure features are on a similar scale.

E.g. x_1 = size (0-2000 feet^2)

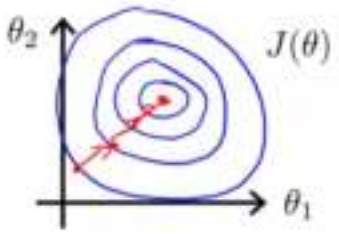
x_2 = number of bedrooms (1-5)



$$x_1 = \frac{\text{size (feet}^2)}{2000}$$

$$x_2 = \frac{\text{number of bedrooms}}{5}$$

$0 \le x_1 \le 1$ $\quad 0 \le x_2 \le 1$



Feature Scaling

Get every feature into approximately a $-1 \le x_i \le 1$ range.

$x_0 = 1$

$0 \le x_1 \le 3$ ✓

$-2 \le x_2 \le 0.5$ ✓

$-100 \le x_3 \le 100$ ✗

$-0.0001 \le x_4 \le 0.0001$ ✗

-3 to 3 ✓

$-\frac{1}{3}$ to $\frac{1}{3}$ ✓

Mean normalization

Replace x_i with $x_i - \mu_i$ to make features have approximately zero mean (Do not apply to $x_0 = 1$).

E.g. $x_1 = \frac{size - 1000}{2000}$

Average size = 1000

$x_2 = \frac{\#bedrooms - 2}{5}$

1-5 bedrooms

$-0.5 \le x_1 \le 0.5$ $\quad -0.5 \le x_2 \le 0.5$

$x_1 \leftarrow \frac{x_1 - \mu_1}{s_1}$ $\quad x_2 \leftarrow \frac{x_2 - \mu_1}{s_2}$

μ_1: avg value of x_1 in training set

s_1: range (max - min) (or standard deviation)

Gradient descent in Practice II : Learning rate

What we are going to Do?

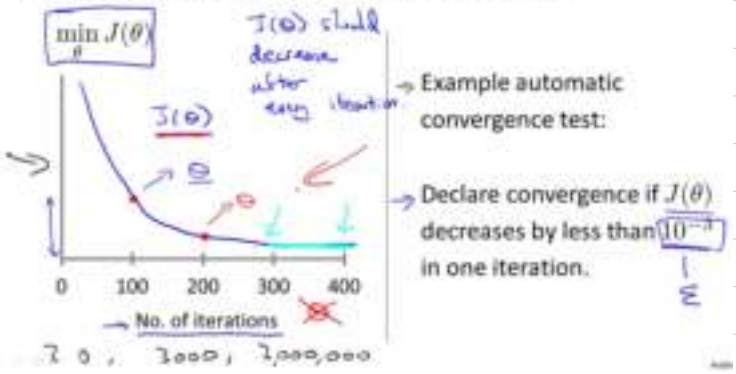
Gradient descent

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

1 - "Debugging": How to make sure Gradient descent is working Correctly

2- How to choose learning rate α.

Making sure gradient descent is working correctly.



working correctly. The job of gradient descent is to find the value of theta for you that hopefully minimizes the cost function J(theta). What I often do is therefore plot the cost function J(theta) as gradient descent runs. So the x axis here is a number of iterations of gradient descent and as gradient descent runs you hopefully get a plot that maybe looks like this.

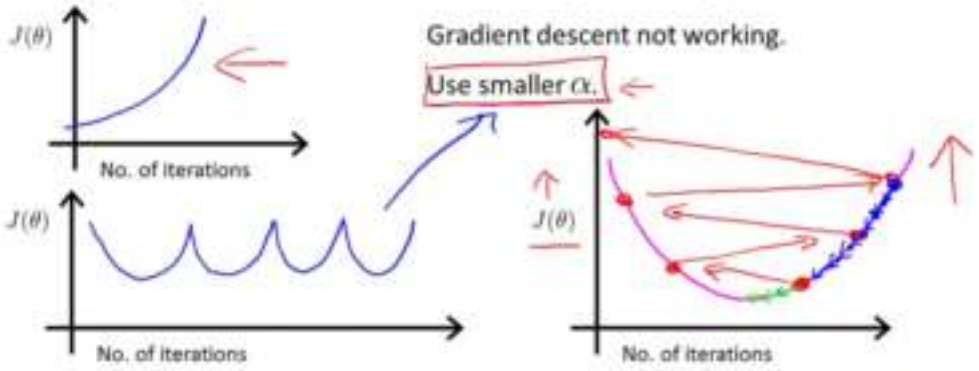
Notice that the x axis is number of iterations. Previously we where looking at plots of J(theta) where the x axis, where the horizontal axis, was the parameter vector theta but this is not what this is. Concretely, what this point is, is I'm going to run gradient descent for 100 iterations. And whatever value I get for theta after 100 iterations, I'm going to get some value of theta after 100 iterations. And I'm going to evaluate the cost function J(theta). For the value of theta I get after 100 iterations, and this vertical height is the value of J(theta). For the value of theta I got after 100 iterations of gradient descent. And this point here that corresponds to the value of J(theta) for the theta that I get after I've run gradient descent for 200 iterations.

So what this plot is showing is, is it's showing the value of your cost function after each iteration of gradient decent. And if gradient is working properly then J(theta) should decrease after every iteration.

And one useful thing that this sort of plot can tell you also is that if you look at the specific figure that I've drawn, it looks like by the time you've gotten out to maybe 300 iterations, between 300 and 400 iterations, in this segment it looks like J(theta) hasn't gone down much more. So by the time you get to 400 iterations, it looks like this curve has flattened out here. And so way out here 400 iterations, it looks like gradient descent has more or less converged because your cost function isn't going down much more. So looking at this figure can also help you judge whether or not gradient descent has converged.

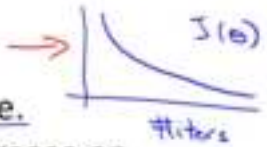
By the way, the number of iterations the gradient descent takes to converge for a physical application can vary a lot, so maybe for one application, gradient descent may converge after just thirty iterations. For a different application, gradient descent may take 3,000 iterations, for another learning algorithm, it may take 1 million iterations. It turns out to be very difficult to tell in advance how many iterations gradient descent needs to converge. And is usually by plotting this sort of plot, plotting the cost function as we increase in number in iterations, is usually by looking at these plots. But I try to tell if gradient descent has converged. It's also possible to come up with automatic convergence test, namely to have a algorithm try to tell you if gradient descent has converged. And here's maybe a pretty typical example of an automatic convergence test. And such a test may declare convergence if your cost function J(theta) decreases by less than some small value epsilon, some small value 10 to the minus 3 in one iteration. But I find that usually choosing what this threshold is is pretty difficult. And so in order to check your gradient descent's converge I actually tend to look at plots like these, like this figure on the left, rather than rely on an automatic convergence test. Looking at this sort of figure can also tell you, or give you an advance warning, if maybe gradient descent is not working correctly. Concretely, if you plot J(theta) as a function of the number of iterations. Then if you see a figure like this where J(theta) is actually increasing, then that gives you a clear sign that gradient descent is not working. And a theta like this usually means that you should be using learning rate alpha.

Making sure gradient descent is working correctly.



- For sufficiently small α, $J(\theta)$ should decrease on every iteration.
- But if α is too small, gradient descent can be slow to converge.

Summary:



- If α is too small: slow convergence.
- If α is too large: $J(\theta)$ may not decrease on every iteration; may not converge. (Slow converge also possible)

To choose α, try

$$\ldots, 0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1, \ldots$$

×3 ≈×3 ×3 ≈×3

And so on, where this is 0.001. I'll then increase the learning rate threefold to get 0.003. And then this step up, this is another roughly threefold increase from 0.003 to 0.01. And so these are, roughly, trying out gradient descents with each value I try being about 3x bigger than the previous value. So what I'll do is try a range of values until I've found one value that's too small and made sure that I've found one value that's too large. And then I'll sort of try to pick the largest possible value, or just something slightly smaller than the largest reasonable value that I found. And when I do that usually it just gives me a good learning rate for my problem. And if you do this too, maybe you'll be able to choose a good learning rate for your implementation of gradient descent.

Features and Polynomial Regression

Polynomial Regression allows you to use the machinery of Linear Regression to fit very Complicated, even very none-linear functions:

Sometimes by defining new features you might actually get a better model

Housing prices prediction

$h_\theta(x) = \theta_0 + \theta_1 \times \text{frontage} + \theta_2 \times \text{depth}$

x_1 x_2

Area

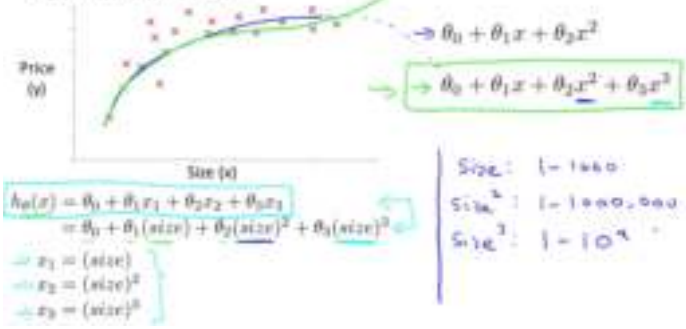
x = frontage * depth

$h_\theta(x) = \theta_0 + \theta_1 x$

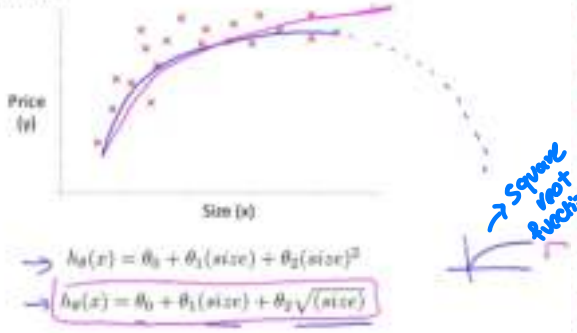
↑ land area

→ Then select my hypothesis as that using just one feature which is my land Area

Polynomial regression



Choice of features

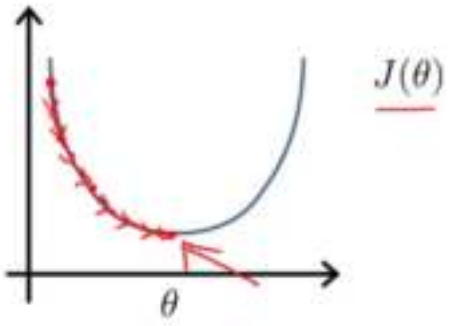


comparable ranges of values. Finally, here's one last example of how you really have broad choices in the features you use. Earlier we talked about how a quadratic model like this might not be ideal because, you know, maybe a quadratic model fits the data okay, but the quadratic function goes back down and we really don't want, right, housing prices that go down, to predict that, as the size of housing freezes. But rather than going to a cubic model there, you have, maybe, other choices of features and there are many possible choices. But just to give you another example of a reasonable choice, another reasonable choice might be to say that the price of a house is theta zero plus theta one times the size, and then plus theta two times the square root of the size, right? So the square root function is this sort of function, and maybe there will be some value of theta one, theta two, theta three, that will let you take this model and, for the curve that looks like that, and, you know, goes up, but sort of flattens out a bit and doesn't ever come back down. And, so, by having insight into, in this case, the shape of a square root function, and, into the shape of the data, by choosing different features, you can sometimes get better models. In this video, we talked about polynomial regression. That is, how to fit a

might actually get a better model. Closely related to the idea of choosing your features is this idea called polynomial regression. Let's say you have a housing price data set that looks like this. Then there are a few different models you might fit to this. One thing you could do is fit a quadratic model like this. It doesn't look like a straight line fits this data very well. So maybe you want to fit a quadratic model like this where you think the size, where you think the price is a quadratic function and maybe that'll give you, you know, a fit to the data that looks like that. But then you may decide that your quadratic model doesn't make sense because of a quadratic function, eventually this function comes back down and well, we don't think housing prices should go down when the size goes up too high. So then maybe we might choose a different polynomial model and choose to use instead a cubic function, and where we have now a third-order term and we fit that, maybe we get this sort of model, and maybe the green line is a somewhat better fit to the data cause it doesn't eventually come back down. So how do we actually fit a model like this to our data? Using the machinery of multivariant linear regression, we can do this with a pretty simple modification to our algorithm. The form of the hypothesis we, we know how the fit looks like this, where we say H of x is theta zero plus theta one x one plus x two theta X1. And if we want to fit this cubic model that I have boxed in green, what we're saying is that to predict the price of a house, it's theta 0 plus theta 1 times the size of the house plus theta 2 times the square size of the house. So this term is equal to that term. And then plus theta 3 times the cube of the size of the house raises that third term. In order to map these two definitions to each other, well, the natural way to do that is to set the first feature x one to be the size of the house, and set the second feature x two to be the square of the size of the house, and set the third feature x three to be the cube of the size of the house. And, just by choosing my three features this way and applying the machinery of linear regression, I can fit this model and end up with a cubic fit to my data. I just want to point out one more thing, which is that if you choose your features like this, then feature scaling becomes increasingly important. So if the size of the house ranges from one to a thousand, so, you know, from one to a thousand square feet, say, then the size squared of the house will range from one to one million, the square of a thousand, and your third feature x cubed, excuse me you, your third feature x three, which is the size cubed of the house, will range from one two ten to the nine, and so these three features take on very different ranges of values, and it's important to apply feature scaling if you're using gradient descent to get them into comparable ranges of values. Finally, here's one last example of how you really have broad

Normal equation

Gradient Descent



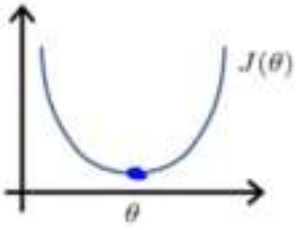
Normal equation: Method to solve for θ analytically.

Intuition: If 1D ($\theta \in \mathbb{R}$)

→ $J(\theta) = a\theta^2 + b\theta + c$

$\frac{d}{d\theta} J(\theta) = \dots \overset{\text{set}}{=} 0$

Solve for θ



$\theta \in \mathbb{R}^{n+1}$ $\quad J(\theta_0, \theta_1, \dots, \theta_m) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$

$\frac{\partial}{\partial \theta_j} J(\theta) = \dots \overset{\text{set}}{=} 0$ (for every j)

Solve for $\theta_0, \theta_1, \dots, \theta_n$ → it compute θ's that cause minimum cost function.

Examples: $m = 4$.

x_0	Size (feet²) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price ($1000) y
1	2104	5	1	45	460
1	1416	3	2	40	232
1	1534	3	2	30	315
1	852	2	1	36	178

$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix} \quad y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

$m \times (n+1)$ $\qquad$ m-dimensional vector

$\theta = (X^T X)^{-1} X^T y$ ←

m **examples** $(x^{(1)}, y^{(1)}), \dots, (x^{(m)}, y^{(m)})$; n **features.**

$x^{(i)} = \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ x_2^{(i)} \\ \vdots \\ x_n^{(i)} \end{bmatrix} \in \mathbb{R}^{n+1}$ $\quad X = \begin{bmatrix} — (x^{(1)})^T — \\ — (x^{(2)})^T — \\ \vdots \\ — (x^{(m)})^T — \end{bmatrix}$ (design matrix)

$m \times (n+1)$

E.g. If $x^{(i)} = \begin{bmatrix} 1 \\ x_1^{(i)} \end{bmatrix}$ $\quad X = \begin{bmatrix} 1 & x_1^{(1)} \\ 1 & x_1^{(2)} \\ \vdots & \vdots \\ 1 & x_1^{(m)} \end{bmatrix}$ $\quad y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}$

$m \times 2$

$\theta = (X^T X)^{-1} X^T y$

$(X^T X)^{-1}$ is inverse of matrix $X^T X$.

Set $A = X^T X$

$(X^T X)^{-1} = A^{-1}$

Octave: `pinv(X'*X)*X'*y`

`pinv(X'*X)*X'*y`

$\theta = (X^T X)^{-1} X^T y$ $\quad \min_\theta J(\theta)$

X' $\quad X^T$

$0 \le x_1 \le 1$

$0 \le x_2 \le 1000$

$0 \le x_3 \le 10^{-5}$ ✓

on similar ranges of Scales of similar ranges of values of each other. If you are using this normal equation method then feature scaling isn't actually necessary and is actually okay if, say, some feature X one is between zero and one, and some feature X two is between ranges from zero to one thousand and some feature x three ranges from zero to ten to the minus five and if you are using the normal equation method this is okay and there is no need to do features scaling, although of course if you are using gradient descent, then, features scaling is still important.

m training examples, n features.

Gradient Descent	Normal Equation
• Need to choose α.	• No need to choose α.
• Needs many iterations.	• Don't need to iterate.
• Works well even when n is large.	• Need to compute $(X^T X)^{-1}$ $\quad n \times n$ $\quad O(n^3)$
$n = 10^6$	• Slow if n is very large.
	$n = 100$
	$n = 10000$
	$n = 10000$

in normal equation method rather than, gradient descent. To preview some ideas that we'll talk about later in this course, as we get to the more complex learning algorithm, for example, when we talk about classification algorithm, like a logistic regression algorithm, We'll see that those algorithm actually... The normal equation method actually do not work for those more sophisticated learning algorithms, and, we will have to resort to gradient descent for those algorithms. So, gradient descent is a very useful algorithm to know. The linear regression will have a large number of features and for some of the other algorithms that we'll see in this course, because, for them, the normal equation method just doesn't apply and doesn't work. But for this specific model of linear regression, the normal equation can give you a alternative

Linear Regression with multiple variables

Normal equation and non-invertibility

more advance Concept & optional

Normal equation

$$\theta = (X^T X)^{-1} X^T y$$

$X^T X$

- What if $X^T X$ is non-invertible? (singular/ degenerate)
- Octave: `pinv(X'*X)*X'*y`

pinv
inv

What if $X^T X$ is non-invertible?

- Redundant features (linearly dependent).
  E.g. x_1 = size in feet^2
  ~~x_2 = size in m^2~~
  $1_m = 3.28$ feet
  $x_1 = (3.28)^2 x_2$
- Too many features (e.g. $m \leq n$).
  $m = 10$
  $n = 100$
  $\theta \in \mathbb{R}^{101}$
  - Delete some features, or use regularization.
    later

so long as you use the pinv function then this will actually compute the value of data that you want even if X transpose X is non-invertible.

The specific details between inv.What is the difference between pinv?

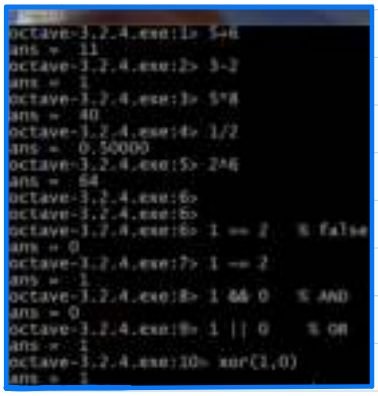
What is inv?That's somewhat advanced numerical computing concepts,I don't really want to get into.But I thought in this optional video, I'll try to give you little bit of intuition about what it means for X transpose X to be non-invertible.

small training set. But this regularization will be a later topic in this course. But to summarize if ever you find that x transpose x is singular or alternatively you find it non-invertable, what I would recommend you do is first look at your features and see if you have redundant features like this x1, x2. You're being linearly dependent or being a linear function of each other like so. And if you do have redundant features and if you just delete one of these features, you really don't need both of these features. If you just delete one of these features, that would solve your non-invertibility problem. And so I would first think through my features and check if any are redundant. And if so then keep deleting redundant features until they're no longer redundant. And if your features are not redundant, I would check if I may have too many features. And if that's the case, I would either delete some features if I can bear to use fewer features or else I would consider using regularization. Which is this topic that we'll talk about later.

So that's it for the normal equation and what it means for if the matrix X transpose X is non-invertable but this is a problem that you should run that hopefully you run into pretty rarely and if you just implement it in octave using P and using the P n function which is called a pseudo inverse function so you could use a different linear out your alive in Is called a pseudo-inverse but that implementation should just do the right thing, even if X transpose X is non-invertable, which should happen pretty rarely anyways, so this should not be a problem for most implementations of linear regression.

octave/matlab Tutorial

Basic operations



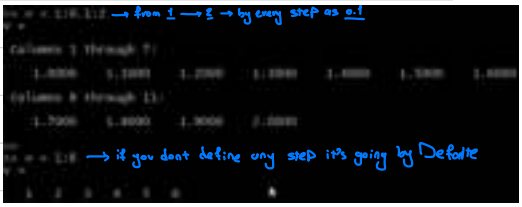

```
octave-3.2.4.exe:11> PS1('>> ');
>>
```

```
>>> a=3
a = 3
>>> a=3;
>>>
```
% semicolon supressing output

```
>>> b='Hi';
>>> b
b=Hi
```

```
>> c= (3>1);
>> c
c = 1  % True
```

```
>> a=pi;
>> a
a =  3.1416
>> disp(a);
 3.1416
>> disp(sprintf('2 decimals: %0.2f', a))
2 decimals: 3.14
>> disp(sprintf('6 decimals: %0.6f', a))
6 decimals: 3.141593
>> a
a =  3.1416
>> format long
>> a
a =  3.14159265358979
>> format short
>> a
a =  3.1416
```



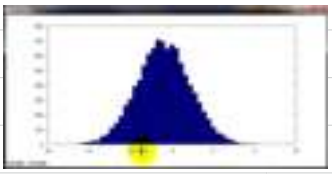

```
>>> ones (2,3)
ans =
   1  1  1
   1  1  1
>>> C = 2 * ones(2,3)
C =
   2  2  2
   2  2  2
```

```
>>> w = zeros (1,3)
w =
   0  0  0
>>> w = rand (1,3)   → This gives us a 1x3 matrix with random numbers
w =
   0.91477  0.14359  0.84860
```

rand by Default is between 0 & 1

but if you use randn you can have any random number or you can even Customize it → like below

```
>> w = randn(1,3)
w =
  -1.44264  -1.27860  -0.69640
>> w = randn(1,3)
w =
  -0.33517   1.26847  -0.28211
>>
>>
>>
>> w = -6 + sqrt(10)*(randn(1,10000));
>> hist(w)
>> hist(w,50)
```



```
>>> eye(3)
ans =
Diagonal matrix
   1  0  0
   0  1  0
   0  0  1
```

```
>>help eye
>>help rand
⋮
```

actave / matlab Tutorial

moving Data around

This session is about if you had data for machine learning Problem, how do you load that Data in octave or matlab? or how do you load that data in octave? how to put it in a matrix? how to manipulate these matrices? how to move Data around?

```
>>> A = [1 2; 3 4; 5 6]
A =
   1  2
   3  4
   5  6
>>> size(A)
ans =
   3  2
```

The size command itself is actually returning a one by two matrix.

```
>>> size(A,1)   → to turn back first dimension of A
ans = 3
>>> size(A,2)   → to turn back second dimension of A
ans = 2
```

```
V = [1 2 3 4]
V =
   1  2  3  4
>>> length(V)
ans = 4
```

→ turns back size of the longest dimension

```
>>> length(A)
ans = 3
```

but usually we apply length only to vectors

>>> pwd → The pwd Command shows the current directory, or the the current path that octave is in. So right now. we are in this maybe somewhat off scale directory

ans: c:\Octave\3.2.4_gcc-4.4.0\bin

>>> cd 'c:\Users\kang\Desktop' ← change Directory

>>> load featuresX.dat
>>> load priceY.dat

>>> load('featuresX.dat')
>>> load('priceY.dat')

work space

>>> who → shows us what variables I have in my octave
→ shows the variables that octave has in memory currently.

Variables in current scope:

A	I	ans	C	PriceY	V
C	a	b	featursX	sz	w

So if we type featuresX → it shows us the Content of featursX.dat

>>> size(featuresX)
ans:
47 2

>>> size(priceY)
ans:
47 1

>>> whos → This one gives you the detailed view of variables

```
>> whos
Variables in the current scope:

  Attr Name          Size                     Bytes  Class
  ==== ====          ====                     =====  =====
       A             3x2                         48  double
       C             2x3                         48  double
       I             6x6                         48  double
       a             1x1                          8  double
       ans           1x2                         16  double
       b             1x2                          2  char
       c             1x1                          1  logical
       featuresX    47x2                       752  double
       priceY       47x1                       376  double
       sz            1x2                         16  double
       v             1x4                         32  double
       w             1x10000                  80000  double

Total is 10201 elements using 81347 bytes
```

before

→ double means double position floating point so that just means that, these are real values, the floating point numbers

Clear all the variables.
if you type only clear it will

>>> clear featuresX → to delete a Variable

how to save data to a variable

>>> v = priceY(1:10) → This sets v to be the first 10 elements of vector Y.

v =

3999
3299
3690
2320
5399
2999
3149
1989
2120
2425

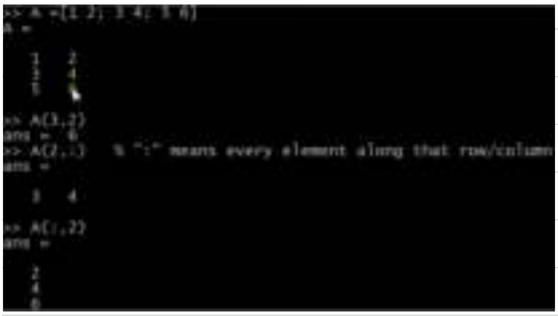
>>> save hello.mat v; → % create a hello.mat file with v Content

>>> save hello.txt v -ascii → % save as text (ascII)

octave / Matlab Tutorial

Computing on Data

now let's see how to manipulate data:




```
>> A([1 3],:)
ans =
   1  2
   5  6
```

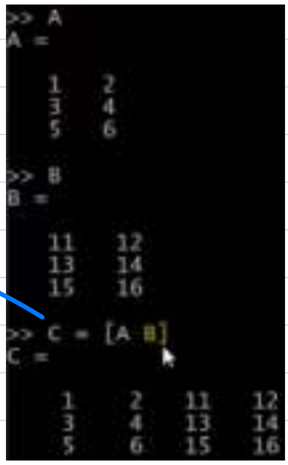
```
>> A(:,2) = [10;11;12]
A =
   1  10
   3  11
   5  12
```

```
>> A = [A, [100; 101; 102]];
>> A
A =
   1  10  100
   3  11  101
   5  12  102
```

% append another column vector to right

This is a somewhat special case syntax.

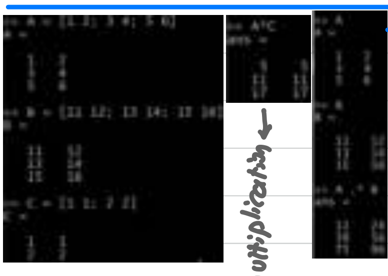
how to concatinate two matrices.



>>C = [A, B] ⇐ Same Result

```
>>C = [A; B]
C =
   1   2
   3   4
   5   6
  11  12
  13  14
  15  16
```

→ % Put them on top of eachother



multiplication

→ به ضرب نظیر به نظیر دو ماتریس با dimension های یکسان مثلاً دو ماتریس 3x2

```
>> A
A =
   1  2
   3  4
   5  6
```

```
>> A .^ 2
ans =
    1   4
    9  16
   25  36
```

→ square every element of A

```
>> log(v)
ans =
   0.00000
   0.69315
   1.09861
```

→ element-wise logarithm

v = 1, 2, 3

```
>> exp(v)
ans =
    2.7183
    7.3891
   20.0855
```

→ exp n → e^n → e=2.71828

```
>> v = [1; 2; 3]
v =
   1
   2
   3
```

```
>> 1 ./ v
ans =
   1.00000
   0.50000
   0.33333
```

```
>> abs(v)
ans =
   1
   2
   3
```

```
>> -v
ans =
   -1
   -2
   -3
```

```
>>v + ones(length(v),1)
ans =
   2
   3
   4
```

length(v)   v + [1; 1; 1]

increment

```
>> A
A =
   1   2
   3   4
   5   6
```

```
>> A'
ans =
   1   3   5
   2   4   6
```

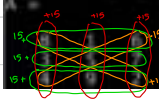
→ % transpose

$(A')' = A$

```
>> a = [1 15 2 0.5]
a =
    1.00000   15.00000    2.00000    0.50000
>> val = max(a)
val =  15
>> [val, ind] = max(a)
val =  15
ind =  2
>> max(A)
ans =
   5   6
>> A
A =
   1   2
   3   4
   5   6
```

→ if A is matrices it does Column-wise maximum

```
>> a
a =
    1.00000   15.00000    2.00000    0.50000
>> a < 3
ans =
   1   0   1   1
>> find(a < 3)
ans =
   1   3   4
>> A = magic(3)
A =
```



the magic function returns these matrices called magic squares They have mathematical Property that all of their rows & Columns and diagonals sum up to the same thing.

!!! note that 'magic(2)' is undefined since there is no 2-by-2 magic square.

```
>> A = magic(3)
A =
   8   1   6
   3   5   7
   4   9   2
>>
>> [r,c] = find(A >= 7)
r =
   1
   3
   2
c =
   1
   2
   3
```

Colum

rows

A(1,1) = 8
A(3,2) = 9
A(2,3) = 7
Y=7

```
>> a
a =
    1.00000   15.00000    2.00000    0.50000
>> sum(a)
ans =  18.500
>> prod(a)
ans =  15
>> floor(a)
ans =
    1   15    2    0
>> ceil(a)
ans =
    1   15    2    1
```

→ % sum all the elements

→ % multiply all the element

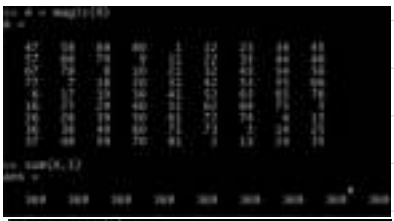
→ % Product

→ % round Down every element

→ % round up every elements

```
>> max(rand(3), rand(3))
ans =
   0.72763   0.78773   0.93872
   0.72363   0.83590   0.42763
   0.48315   0.41734   0.79961
```

what this does is it takes the element_wise maximum of 2 random 3by3 matrices.



```
>> sum(A,2)
ans =
   369
   369
   369
   369
   369
   369
   369
   369
   369
```

```
>> sum(sum(A.*eye(9)))
ans =  369
>> flipud(eye(9))
ans =
Permutation Matrix
     0     0     0     0     0     0     0     0     1
     0     0     0     0     0     0     0     1     0
     0     0     0     0     0     0     1     0     0
     0     0     0     0     0     1     0     0     0
     0     0     0     0     1     0     0     0     0
     0     0     0     1     0     0     0     0     0
     0     0     1     0     0     0     0     0     0
     0     1     0     0     0     0     0     0     0
     1     0     0     0     0     0     0     0     0
```

```
>> A
A =
   8   1   6
   3   5   7
   4   9   2
>> max(A,[],1)
ans =
   8   9   7
```

what this does is this texts the column wise maximum. So the max of the first column is 8, max of second column is 9, the max of the third column is 7. This 1 means to take the max among the first dimension of 8.

```
>> max(A,[],2)
ans =
   8
   7
   9
```

→ exactly like the left one but row-wise

max method by default is by Column wise

There is two way to find the maximum element in entire matrix A.

①

Turning it to vector ← ②

then find the maximum elem

```
>> max(A)
ans =
   8   9   7
>> max(max(A))
ans =  9
>> A(:)
ans =
   8
   3
   4
   1
   5
   9
   6
   7
   2
>> max(A(:))
ans =  9
```

مراحل اثبات اینکه جمع عناصر diagonal های سطر ها، ستون های magic square با یک مقدار است

```
>> A = magic(3)
A =

   8   1   6
   3   5   7
   4   9   2
```

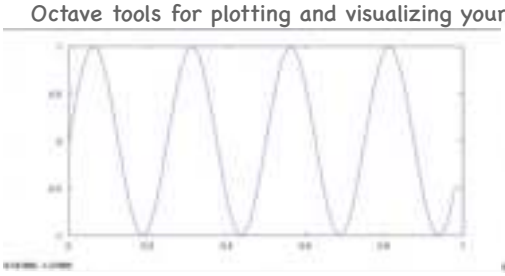
```
>> pinv(A)
ans =

   0.147222  -0.144444   0.063889
  -0.061111   0.022222   0.105556
  -0.019444   0.188889  -0.102778

>> temp = pinv(A)
temp =

   0.147222  -0.144444   0.063889
  -0.061111   0.022222   0.105556
  -0.019444   0.188889  -0.102778

>> temp * A
ans =

   1.0000e+000   1.5266e-016  -2.8588e-015
  -6.1236e-015   1.0000e+000   6.2277e-015
   3.1364e-015  -3.6429e-016   1.0000e+000
```

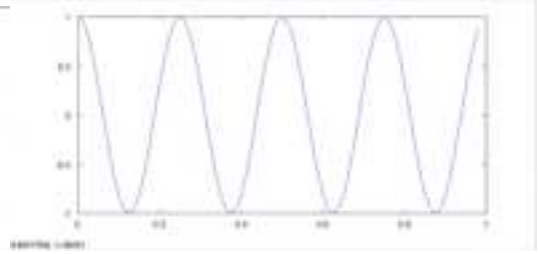
Octave / Matlab Tutorial

Plotting Data

When developing learning algorithms, very often a few simple plots can give you a better sense of what the algorithm is doing and just sanity check that everything is going okay and the algorithms doing what is supposed to.
For example, in an earlier video, I talked about how plotting the cost function J of theta can help you make sure that gradient descent is converging.
Often, plots of the data or of all the learning algorithm outputs will also give you ideas for how to improve your learning algorithm.
Fortunately, Octave has very simple tools to generate lots of different plots and when I use learning algorithms, I find that plotting the data, plotting the learning algorithm and so on are often an important part of how I get ideas for improving the algorithms and in this video, I'd like to show you some of these Octave tools for plotting and visualizing your data.

```
>> t=[0:0.01:0.98];
>> t
>> y1 = sin(2*pi*4*t);
>> plot(t,y1);
>> y2 = cos(2*pi*4*t);
>> plot(t,y2);
```

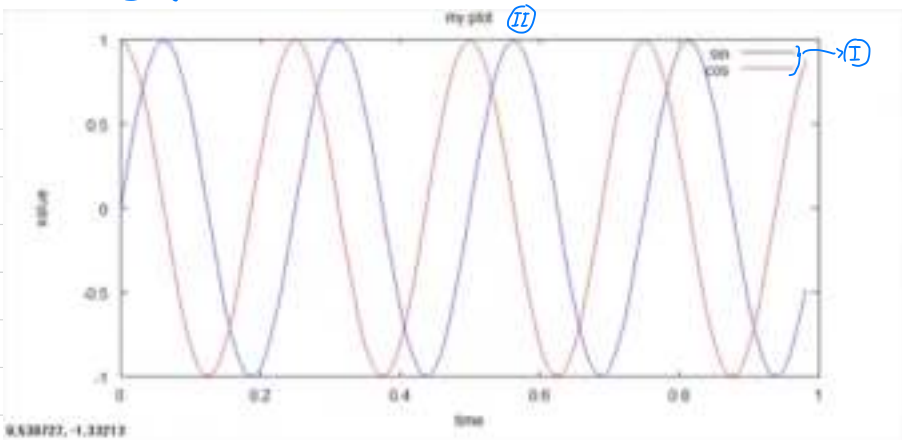




OR

```
>> plot(t,y1);
>> hold on;
>> plot(t,y2,'r');
>> xlabel('time')
>> ylabel('value')
>> legend('sin','cos')
>> title('my plot')
>> cd 'C:\Users\ang\Desktop'; print -dpng 'myPlot.png'
warning: implicit conversion from matrix to string
>> close
```

to close the figure or plot window



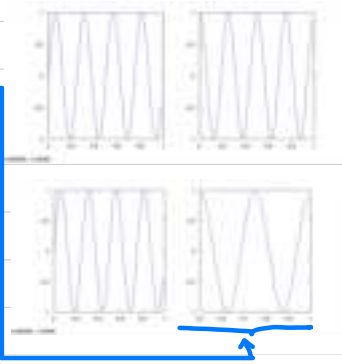
if you wanted to show every plot in it's own window use:

```
>> figure(1); plot(t,y1);
>> figure(2); plot(t,y2);
```

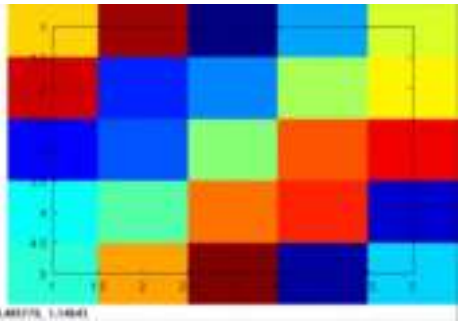
Shows plot(t,y1) in windows figure(1)

Shows plot(t,y2) in windows figure(2)

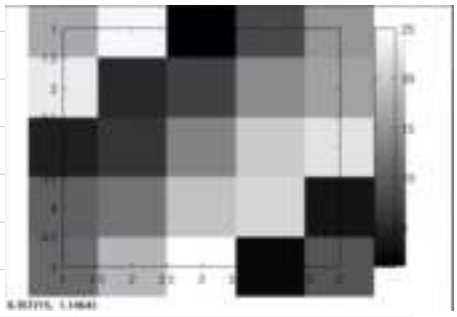
```
clear figure
```



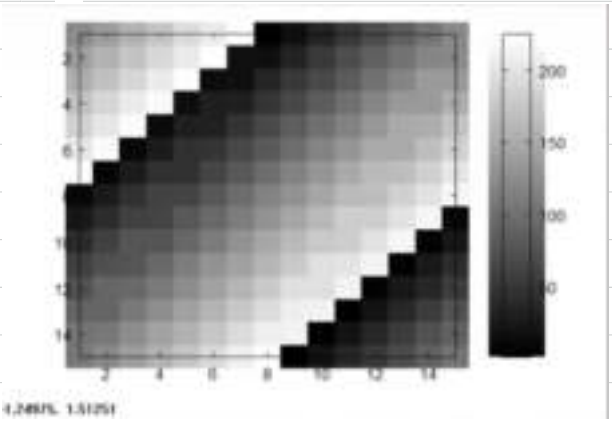
```
>> A = magic(5)
A =

   17   24    1    8   15
   23    5    7   14   16
    4    6   13   20   22
   10   12   19   21    3
   11   18   25    2    9

>> imagesc(A)
```



```
>> imagesc(A), colorbar, colormap gray;
>> A(1,2)
ans =  24
>> A(4,5)
ans =  3
```



```
>> imagesc(magic(15)), colorbar, colormap gray;
```



```
>> a=1, b=2, c=3
a =  1
b =  2
c =  3
>> a=1; b=2; c=3;
>> a=1, b=2, c=3
a =  1
b =  2
c =  3
>>
```

how to put multiple command in one line.

, → show answer

; → doesn't show the answer

octave/matlab Tutorial

Control statements: for, while, if statements

for statement

```
>> for i=1:10,
>     v(i) = 2^i;
>   end;
>> v
v =

      2
      4
      8
     16
     32
     64
    128
    256
    512
   1024
```

```
>> indices=1:10;
>> indices
indices =

   1   2   3   4   5   6   7   8   9  10

>> for i=indices,
>   disp(i);
>  end;
 1
 2
 3
 4
 5
 6
 7
 8
 9
 10
```

how to use a break statement

```
>> i=1;
>> while true,
>     v(i) = 999;
>     i = i+1;
>     if i == 6,
>       break;
>     end;
>   end;
>> v
v =

   999
   999
   999
   999
   999
    64
   128
   256
   512
  1024
```

So now let's use while

```
>> i = 1;
>> while i <= 5,
>     v(i) = 100;
>     i = i+1;
>   end;
>> v
v =

   100
   100
   100
   100
   100
    64
   128
   256
   512
  1024
```

>> v → v before while loop

```
v =
   2
   4
   8
  16
  32
  :
  :
```

if/else statement

```
>> v(1)
ans =  999
>> v(1) = 2;
>> if v(1)==1,
>    disp('The value is one');
>  elseif v(1) == 2,
>    disp('The value is two');
>  else
>    disp('The value is not one or two.');
>  end;
The value is two
```

how to define a function in octave / matlab ?

You create a file called, you know,with your function name and then ending in .m,and when Octave finds this file, it knows that this where it should look for the definition of the function "squarethisnumber.m".

The first line says function Y equals square root number of X, this tells Octave that I'm gonna return the value Y, I'm gonna return one value and that the value is going to be saved in the variable Y and moreover, it tells Octave that this function has one argument, which is the argument X, and the way the function body is defined, if Y equals X squared.

```
>> pwd
ans = C:\Octave\3.2.4_gcc-4.4.0\bin
>> cd 'C:\User\ang\Desktop'
error: C:\User\ang\Desktop: No such file or directory
>> cd 'C:\Users\ang\Desktop'
>> squareThisNumber(5)
ans =  25
>>
```

how to modify the octave/matlab search path

```
>> % Octave search path (advanced/optional)
>> addpath('C:\Users\ang\Desktop')
>> cd 'C:\'
>> squareThisNumber(5)
ans =  25
>> pwd
ans = C:\
```

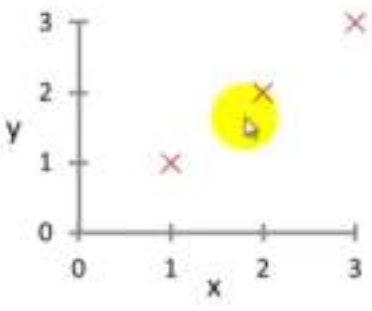
but you can use the term addpath, safety colon,slash users/ANG/ desktop to add that directory to the Octave search path so that even if you know, go to some other directory I can still, Octave still knows to look in the users ANG desktop directory for functions so that even though I'm in a different directory now, it still knows where to find the square this number function.

One concept that Octave has that many other programming languages don't is that it can also let you define functions that return multiple values or multiple arguments. So here's an example of that. Define the function called square and cube this number X and what this says is this function returns 2 values, y1 and y2. When I set down, this follows, y1 is squared, y2 is execute. And what this does is this really returns 2 numbers. So, some of you depending on what programming language you use,if you're familiar with, you know, CC++ your offer.
Often, we think of the function as return in just one value.But just so the syntax in Octave that should return multiple values.

```
>> [a,b] = squareAndCubeThisNumber(5);
>> a
a =  25
>> b
b =  125
```

Define a function to Compute the Cost function J(θ)

Data set is like below!




```
>> X = [1 1; 1 2; 1 3]
X =

   1   1
   1   2
   1   3

>> y = [1; 2; 3]
y =

   1
   2
   3

>> theta = [0;1];
```

```
>> j = costFunctionJ(X,y,theta)
j = 0
>> theta = [0;0];
>> j = costFunctionJ(X,y,theta)
j =  2.3333
>> (1^2 + 2^2 + 3^2)/ (2*m)
error:  m' undefined near line 34 column 23
>> (1^2 + 2^2 + 3^2)/ (2*3)
ans =  2.3333
>>
```

octave / matlab Tutorials

Vectorization

let's see some vectorized example:

Vectorization example.

$$h_\theta(x) = \sum_{j=0}^{n} \theta_j x_j = \theta^T x$$

$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix}$ theta(1) theta(2) theta(3) $x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}$

Unvectorized implementation

```
prediction = 0.0;
for j = 1:n+1,
  prediction = prediction +
               theta(j) * x(j)
end;
```

Vectorized implementation

```
prediction = theta' * x;
```

the issue of vectorization applies to other Programming language as well.

Let's look on the example in C++.

Vectorization example.

$$h_\theta(x) = \sum_{j=0}^{n} \theta_j x_j$$

$$= \theta^T x$$

Unvectorized implementation

```
double prediction = 0.0;
for (int j = 0; j <= n; j++)
  prediction += theta[j] * x[j];
```

Vectorized implementation

```
double prediction
  = theta.transpose() * x;
```

let' see can we vectorized gridient descent algorithm 8

Gradient descent

$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$ (for all j)

Simultaneous update

$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$

$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$

$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)}$

$(n = 2)$

$u(j) = 2v(j) + 5w(j)$ (for all j)

$u = 2v + 5w$

Vectorized implementation:

$\theta := \theta - \alpha \delta$

Subtraction of two vector

$\theta \in \mathbb{R}^{n+1}$, α real number, $\delta \in \mathbb{R}^{n+1}$

where $\delta = \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$

$\delta = \begin{bmatrix} \delta_0 \\ \delta_1 \\ \delta_2 \end{bmatrix}$ $\rightarrow \delta_0 = \frac{1}{m} \sum (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$

$x^{(i)} \in \mathbb{R}^{n+1}$ $x^{(i)} = \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ x_2^{(i)} \end{bmatrix}$

$(h_\theta(x^{(1)}) - y^{(1)}) x^{(1)} + (h_\theta(x^{(2)}) - y^{(2)}) x^{(2)} + \dots$

$\mathbb{R}$ → real number

so sometimes we use linear regression with 10's or 100's or 1,000's of features. But if you use the vectorized implementation of linear regression,you'll see that will run much faster than if you had, say, your old for loop that was updating theta zero, then theta one, then theta two yourself. So, using a vectorized implementation, you should be able to get a much more efficient implementation of linear regression. And when you vectorize later algorithms that we'll see in this class,there's good trick, whether in Octave or some other language like C++, Java,for getting your code to run more efficiently.

Classification and Representation — Some Classification example

Classification

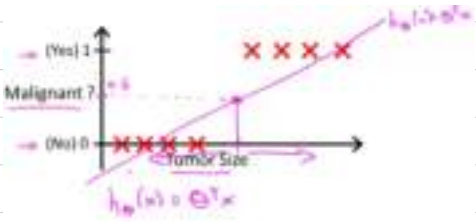
Email: Spam / Not spam ?
Online Transactions: Fraudulent (Yes / No) ?
Tumor: Malignant / Benign ?

$y \in \{0, 1\}$
- 0: "Negative Class" (e.g., benign tumor)
- 1: "Positive Class" (e.g., malignant tumor)

!! a negative class is conveying the absence of something like the absence of a malignant tumor whereas one the positive class is conveying the presence of something

$y \in \{0, 1, 2, 3\}$

For now we're going to start with classification problems with just two classes zero and one.Later one we'll talk about multi class problems as well where therefore y may take on four values zero, one, two, and three. This is called a multiclass classification problem.But for the next few videos, let's start with the two class or the binary classification problem and we'll worry about the multiclass setting later.



- Threshold classifier output $h_\theta(x)$ at 0.5:
  - If $h_\theta(x) \geq 0.5$, predict "y = 1"
  - If $h_\theta(x) < 0.5$, predict "y = 0"

multiclass setting later. So how do we develop a classification algorithm? Here's an example of a training set for a classification task for classifying a tumor as malignant or benign. And notice that malignancy takes on only two values, zero or no, one or yes. So one thing we could do given this training set is to apply the algorithm that we already know.

Linear regression to this data set and just try to fit the straight line to the data. So if you take this training set and fit a straight line to it, maybe you get a hypothesis that looks like that, right. So that's my hypothesis. H(x) equals theta transpose x. If you want to make predictions one thing you could try doing is then threshold the classifier outputs at 0.5 that is at a vertical axis value 0.5 and if the hypothesis outputs a value that is greater than equal to 0.5 you can take y = 1. If it's less than 0.5 you can take y=0. Let's see what happens if we do that. So 0.5 and so that's where the threshold is and that's using linear regression this way. Everything to the right of this point we will end up predicting as the positive cross. Because the output values is greater than 0.5 on the vertical axis and everything to the left of that point we will end up predicting as a negative value.

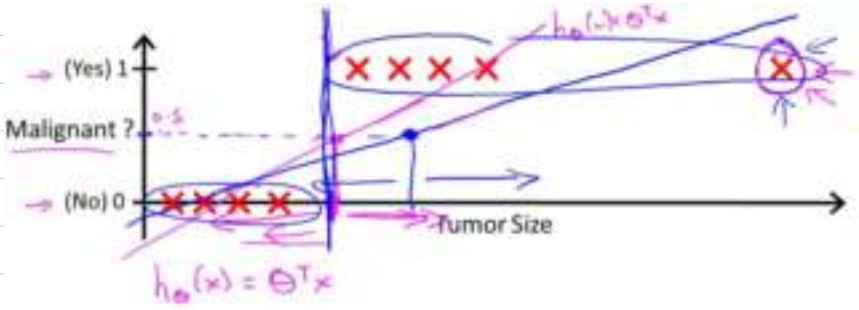
In this particular example, it looks like linear regression is actually doing something reasonable. Even though this is a classification toss we're interested in. But now let's try changing the problem a bit. Let me extend out the horizontal access a little bit and let's say we got one more training example way out there on the right. Notice that that additional training example, this one out here, it doesn't actually change anything, right. Looking at the training set it's pretty clear what a good hypothesis is. Is that well everything to the right of somewhere around here, to the right of this we should predict this positive. Everything to the left we should probably predict as negative because from this training set, it looks like all the tumors larger than a certain value around here are malignant, and all the tumors smaller than that are not malignant, at least for this training set.

But once we've added that extra example over here, if you now run linear regression, you instead get a straight line fit to the data. That might maybe look like this. blue line

And if you know threshold hypothesis at 0.5, you end up with a threshold that's around here, so that everything to the right of this point you predict as positive and everything to the left of that point you predict as negative.

And this seems a pretty bad thing for linear regression to have done, right, because you know these are our positive examples, these are our negative examples. It's pretty clear we really should be separating the two somewhere around there, but somehow by adding one example way out here to the right, this example really isn't giving us any new information. I mean, there should be no surprise to the learning algorithm. That the example way out here turns out to be malignant. But somehow having that example out there caused linear regression to change its straight line fit to the data from this magenta line out here to this blue line over here, and caused it to give us a worse hypothesis.

So, applying linear regression to a classification problem often isn't a great idea. In the first example, before I added this extra training example, previously linear regression was just getting lucky and it got us a hypothesis that worked well for that particular example, but usually applying linear regression to a data set, you might get lucky but often it isn't a good idea. So I wouldn't use linear regression for classification problems.



- Threshold classifier output $h_\theta(x)$ at 0.5:
  - If $h_\theta(x) \geq 0.5$, predict "y = 1"
  - If $h_\theta(x) < 0.5$, predict "y = 0"

Classification: y = 0 or 1

$h_\theta(x)$ can be > 1 or < 0

Logistic Regression: $0 \leq h_\theta(x) \leq 1$

(Classification)

Here's one other funny thing about what would happen if we were to use linear regression for a classification problem. For classification we know that y is either zero or one. But if you are using linear regression where the hypothesis can output values that are much larger than one or less than zero, even if all of your training examples have labels y equals zero or one.

And it seems kind of strange that even though we know that the labels should be zero, one it seems kind of strange if the algorithm can output values much larger than one or much smaller than zero.

So what we'll do in the next few videos is develop an algorithm called logistic regression, which has the property that the output, the predictions of logistic regression are always between zero and one, and doesn't become bigger than one or become less than zero.

And by the way, logistic regression is, and we will use it as a classification algorithm, is some, maybe sometimes confusing that the term regression appears in this name even though logistic regression is actually a classification algorithm. But that's just a name it was given for historical reasons. So don't be confused by that logistic regression is actually a classification algorithm that we apply to settings where the label y is discrete value, when it's either zero or one. So hopefully you now know why, if you have a classification problem, using linear regression isn't a good idea. In the next video, we'll start working out the details of the logistic regression algorithm.

Classification and Representation

Hypothesis Representation

what is the function that we are going to use to represent our hypothesis when we have a classification problem?

Logistic Regression Model

Want $0 \leq h_\theta(x) \leq 1$

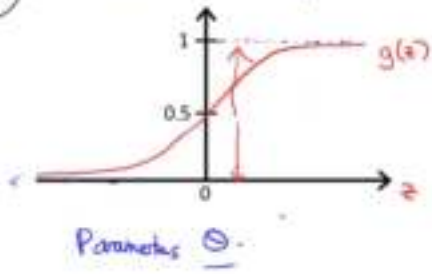
$h_\theta(x) = g(\theta^T x)$

$g(z) = \frac{1}{1+e^{-z}}$

$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$

Sigmoid function

Logistic function

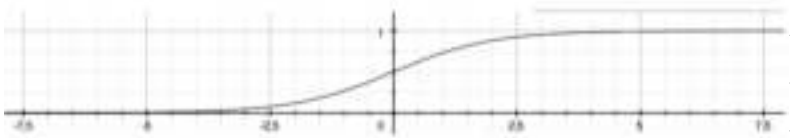


Parameters θ

Earlier, we said that we would like our classifier to output values that are between 0 and 1. So we'd like to come up with a hypothesis that satisfies this property, that is, predictions are maybe between 0 and 1. When we were using linear regression, this was the form of a hypothesis, where h(x) is theta transpose x. For logistic regression, I'm going to modify this a little bit and make the hypothesis g of theta transpose x. Where I'm going to define the function g as follows. G(z), z is a real number, is equal to one over one plus e to the negative z.

This is called the sigmoid function, or the logistic function, and the term logistic function, that's what gives rise to the name logistic regression. And by the way, the terms sigmoid function and logistic function are basically synonyms and mean the same thing. So the two terms are basically interchangeable, and either term can be used to refer to this function g. And if we take these two equations and put them together, then here's just an alternative way of writing out the form of my hypothesis. I'm saying that h(x) is 1 over 1 plus e to the negative theta transpose x. And all I've do is I've taken this variable z, z here is a real number, and plugged in theta transpose x. So I end up with theta transpose x in place of z there. Lastly, let me show you what the sigmoid function looks like. We're gonna plot it on this figure here. The sigmoid function, g(z), also called the logistic function, it looks like this. It starts off near 0 and then it rises until it crosses 0.5 and the origin, and then it flattens out again like so. So that's what the sigmoid function looks like. And you notice that the sigmoid function, while it asymptotes at one and asymptotes at zero, as a z axis, the horizontal axis is z. As z goes to minus infinity, g(z) approaches zero. And as g(z) approaches infinity, g(z) approaches one. And so because g(z) upwards values are between zero and one, we also have that h(x) must be between zero and one. Finally, given this hypothesis representation, what we need to do, as before, is fit the parameters theta to our data. So given a training set we need to a pick a value for the parameters theta and this hypothesis will then let us make predictions. We'll talk about a learning algorithm later for fitting the parameters theta, and-

The following image shows us what the sigmoid function looks like:



Let's talk a bit about the interpretation of this model.

Interpretation of Hypothesis Output $h_\theta(x)$

$h_\theta(x)$ = estimated probability that y = 1 on input x

Example: If $x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{tumorSize} \end{bmatrix}$

$h_\theta(x) = 0.7$ $\quad y = 1$

Tell patient that 70% chance of tumor being malignant

$h_\theta(x) = P(y=1|x;\theta)$ "probability that y = 1, given x, parameterized by θ"

$y = 0$ or 1

$\rightarrow P(y=0|x;\theta) + P(y=1|x;\theta) = 1$

$P(y=0|x;\theta) = 1 - P(y=1|x;\theta)$

Classification and Representation

Decision Boundary

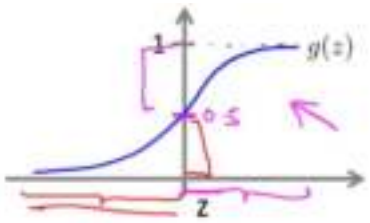
Decision boundary will give us a better sense of what the logistic regressions hypothesis function is computing.

Let's use this to better understanding about how the hypothesis of logistic regression makes those predictions.

Logistic regression

$\rightarrow h_\theta(x) = g(\theta^T x) = P(y=1|x;\theta)$

$\rightarrow g(z) = \frac{1}{1+e^{-z}}$



Suppose predict "$y = 1$" if $h_\theta(x) \geq 0.5$

$\rightarrow \theta^T x \geq 0$

$g(z) \geq 0.5$ when $z \geq 0$

$h_\theta(x) = g(\theta^T x) \geq 0.5$ whenever $\theta^T x \geq 0$

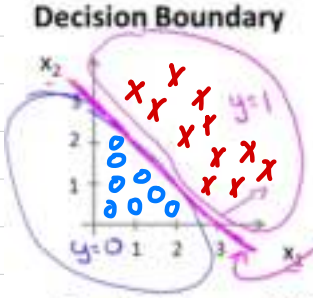
predict "$y = 0$" if $h_\theta(x) < 0.5$

$h_\theta(x) = g(\theta^T x)$ $\quad g(z) < 0.5$

$\rightarrow \theta^T x < 0$

for example

Decision Boundary



$\theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$

$\rightarrow h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$

-3 $\quad$ 1 $\quad$ 1

Decision boundary

Predict "$y = 1$" if $-3 + x_1 + x_2 \geq 0$ $\quad \theta^T x$

$\rightarrow x_1 + x_2 \geq 3$

x_1, x_2 Decision boundary

$\rightarrow h_\theta(x) = 0.5$

$x_1 + x_2 = 3$

$x_1 + x_2 < 3$

$y = 0$

The Decision boundary is a Property of the hypothesis and of the Parameters of the hypothesis and not a Property of the Data set.

once we have particular values for the Parameters $\theta_0, \theta_1, \theta_2$ then that completely defines the Decision boundary & we don't actually need to plot a training set in order to plot a

Now let's look at a more complex example:

①

Let's now look at a more complex example where as usual, I have crosses to denote my positive examples and Os to denote my negative examples. Given a training set like this, how can I get logistic regression to fit the sort of data?

Earlier when we were talking about polynomial regression or when we're talking about linear regression, we talked about how we could add extra higher order polynomial terms to the features. And we can do the same for logistic regression. Concretely, let's say my hypothesis looks like this where I've added two extra features, x1 squared and x2 squared, to my features. So that I now have five parameters, theta zero through theta four.

As before, we'll defer to the next video, our discussion on how to automatically choose values for the parameters theta zero through theta four. But let's say that varied procedure to be specified, I end up choosing theta zero equals minus one, theta one equals zero, theta two equals zero, theta three equals one and theta four equals one.

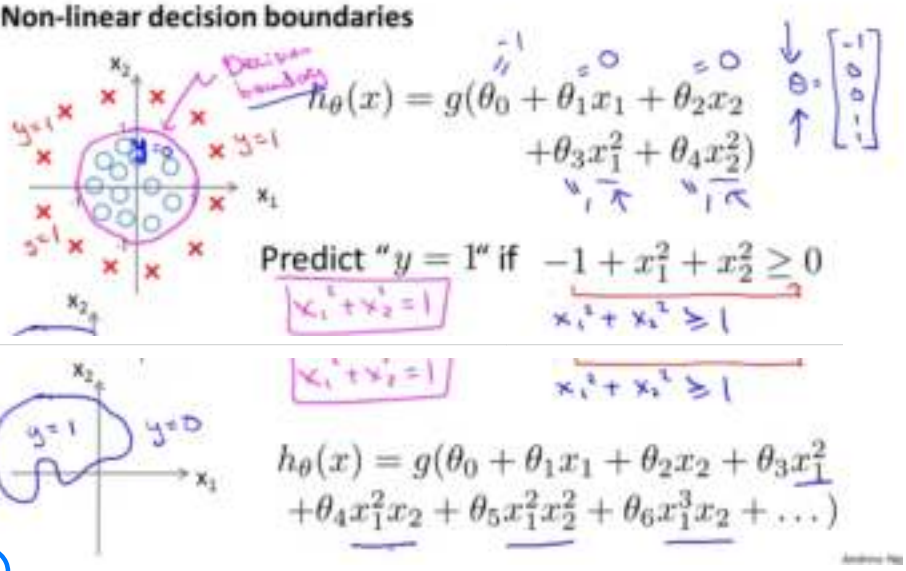
What this means is that with this particular choice of parameters, my parameter effect theta theta looks like minus one, zero, zero, one, one.

Following our earlier discussion, this means that my hypothesis will predict that y=1 whenever -1 + x1 squared + x2 squared is greater than or equal to 0. This is whenever theta transpose times my theta transfers, my features is greater than or equal to zero. And if I take minus 1 and just bring this to the right, I'm saying that my hypothesis will predict that y is equal to 1 whenever x1 squared plus x2 squared is greater than or equal to 1. So what does this decision boundary look like? Well, if you were to plot the curve for x1 squared plus x2 squared equals 1. Some of you will recognize that, that is the equation for circle of radius one, centered around the origin. So that is my decision boundary.

And everything outside the circle, I'm going to predict as y=1. So out here is my y equals 1 region, we'll predict y equals 1 out here and inside the circle is where I'll predict y is equal to 0. So by adding these more complex, or these polynomial terms to my features as well, I can get more complex decision boundaries that don't just try to separate the positive and negative examples in a straight line that I can get in this example, a decision boundary that's a circle.

Once again, the decision boundary is a property, not of the training set, but of the hypothesis under the parameters. So, so long as we're given my parameter vector theta, that defines the decision boundary, which is the circle. But the training set is not what we use to define the decision boundary. The training set may be used to fit the parameters theta. We'll talk about how to do that later. But, once you have the parameters theta, that is what defines the decision boundary.

Let me put back the training set just for visualization.



②

And finally let's look at a more complex example.

So can we come up with even more complex decision boundaries then this? If I have even higher polynomial terms so things like

X1 squared, X1 squared X2, X1 squared squared and so on. And have much higher polynomials, then it's possible to show that you can get even more complex decision boundaries and the regression can be used to find decision boundaries that may, for example, be an ellipse like that or maybe a little bit different setting of the parameters maybe you can get instead a different decision boundary which may even look like some funny shape like that.

Or for even more complete examples maybe you can also get this decision boundaries that could look like more complex shapes like that where everything in here you predict y = 1 and everything outside you predict y = 0. So this higher autopolynomial features you can a very complex decision boundaries. So, with these visualizations, I hope that gives you a sense of what's the range of hypothesis functions we can represent using the representation that we have for logistic regression.

Now that we know what h(x) can represent, what I'd like to do next in the following video is talk about how to automatically choose the parameters theta so that given a training set we can automatically fit the parameters to our data.

logistic Regression model

Cost function

In this video, we'll talk about how to fit the parameters of theta for the logistic compression.In particular, I'd like to define the optimization objective, or the cost function that we'll use to fit the parameters.

Here's the supervised learning Problem of fiting logistic regression model.

Training set: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \cdots, (x^{(m)}, y^{(m)})\}$

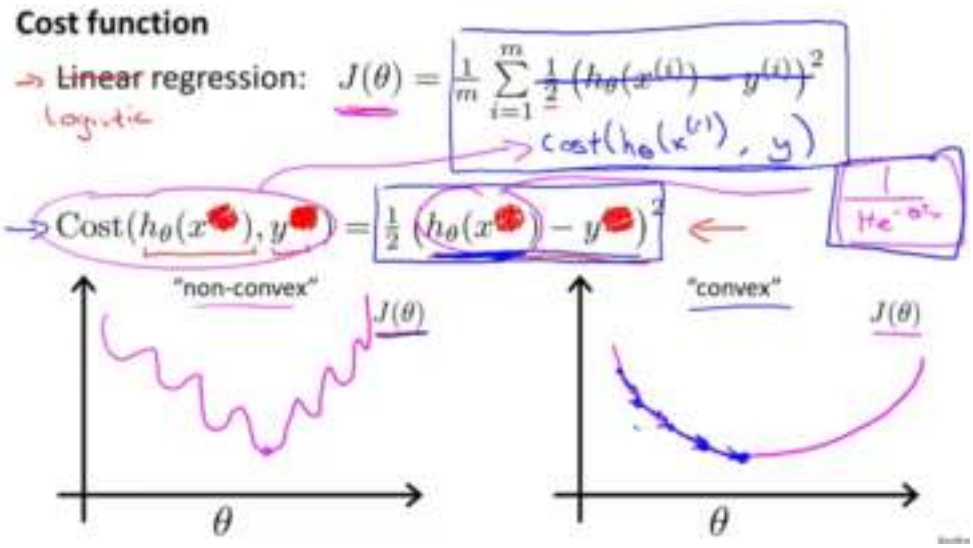
m examples $\quad x \in \begin{bmatrix} x_0 \\ x_1 \\ \cdots \\ x_n \end{bmatrix} \in \mathbb{R}^{n+1}$ $\quad x_0 = 1, y \in \{0, 1\}$

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

How to choose parameters θ ?

Here's the supervised learning problem of fitting logistic regression model. We have a training set of m training examples and as usual, each of our examples is represented by a that's n plus one dimensional,

and as usual we have x 0 equals one. First feature or a zero feature is always equal to one. And because this is a computation problem, our training set has the property that every label y is either 0 or 1. This is a hypothesis, and the parameters of a hypothesis is this theta over here. And the question that I want to talk about is given this training set, how do we choose, or how do we fit the parameter's theta? ~~Back when we were developing the linear regression model, we used~~



fit the parameter's theta? Back when we were developing the linear regression model, we used the following cost function. I've written this slightly differently where instead of 1 over 2m, I've taken a one-half and put it inside the summation instead. Now I want to use an alternative way of writing out this cost function. Which is that instead of writing out this square of return here, let's write in here costs of h of x, y and I'm going to define that total cost of h of x, y to be equal to this. Just equal to this one-half of the squared error. So now we can see more clearly that the cost function is a sum over my training set, which is 1 over n times the sum of my training set of this cost term here.

And to simplify this equation a little bit more, it's going to be convenient to get rid of those superscripts. So just define cost of h of x comma y to be equal to one half of this squared error. And interpretation of this cost function is that, this is the cost I want my learning algorithm to have to pay if it outputs that value, if its prediction is h of x, and the actual label was y. So just cross off the superscripts, right, and no surprise for linear regression the cost we've defined is that or the cost of this is that is one-half times the square difference between what I predicted and the actual value that we have, 0 for y. Now this cost function worked fine for linear regression. But here, we're interested in logistic regression. If we could minimize this cost function that is plugged into J here, that will work okay. But it turns out that if we use this particular cost function, this would be a non-convex function of the parameter's data. Here's what I mean by non-convex. Have some cross function j of theta and for logistic regression, this function h here

has a nonlinearity that is one over one plus e to the negative theta transpose. So this is a pretty complicated nonlinear function. And if you take the function, plug it in here. And then take this cost function and plug it in there and then plot what j of theta looks like. You find that j of theta can look like a function that's like this

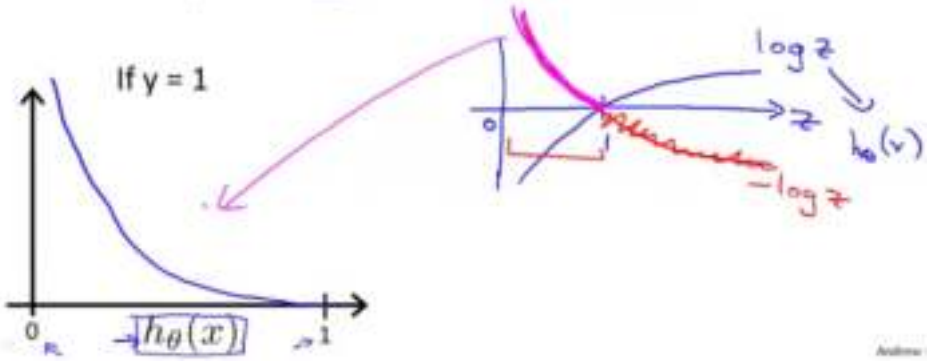
with many local optima. And the formal term for this is that this is a non-convex function. And you can kind of tell, if you were to run gradient descent on this sort of function It is not guaranteed to converge to the global minimum. Whereas in contrast what we would like is to have a cost function j of theta that is convex, that is a single bow-shaped function that looks like this so that if you run theta in the we would be guaranteed that

would converge to the global minimum. And the problem with using this great cost function is that because of this very nonlinear function that appears in the middle here, J of theta ends up being a nonconvex function if you were to define it as a square cost function. So what we'd like to do is, instead of come up with a different cost function, that is convex, and so that we can apply a great algorithm, like gradient descent and be guaranteed to find the global minimum. Here's the

Here is the Cost function that we're going to use for logistic regression.

Logistic regression cost function

$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



~~great algorithm, like gradient descent and be guaranteed to find the global minimum~~. Here's the cost function that we're going to use for logistic regression. We're going to say that the cost, or the penalty that the algorithm pays, if it upwards the value of h(x), so if this is some number like 0.7, it predicts the value h of x. And the actual cost label turns out to be y. The cost is going to be -log(h(x)) if y = 1 and -log(1- h(x)) if y = 0. This looks like a pretty complicated function, but let's plot this function to gain some intuition about what it's doing. Let's start off with the case of y = 1. If y = 1, then the cost function is -log(h(x)). And if we plot that, so let's say that the horizontal axis is h(x), so we know that a hypothesis is going to output a value between 0 and 1. Right, so h(x), that varies between 0 and 1. If you plot what this cost function looks like, you find that it looks like this. One way to see why the plot looks like this is because if you were to plot log z

with z on the horizontal axis, then that looks like that. And it approaches minus infinity, right? So this is what the log function looks like. And this is 0, this is 1. Here, z is of course playing the role of h of x. And so -log z will look like this.

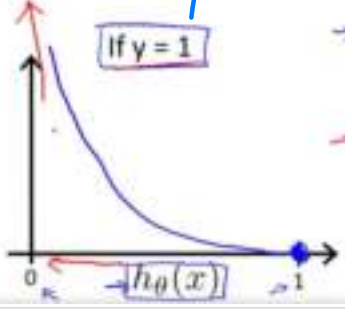
Just flipping the sign, minus log z, and we're interested only in the range of when this function goes between zero and one, so get rid of that. And so we're just left with, you know, this part of the curve, and that's what this curve on the left looks like. ~~Now, this cost function has a few~~

~~the curve, and that's what this curve on the left looks like~~. Now, this cost function has a few interesting and desirable properties. First, you notice that if y is equal to 1 and h(x) is equal to 1, in other words, if the hypothesis exactly predicts h equals 1 and y is exactly equal to what it predicted, then the cost = 0 right? That corresponds to the curve doesn't actually flatten out. The curve is still going. First, notice that if h(x) = 1, if that hypothesis predicts that y = 1 and if indeed y = 1 then the cost = 0. That corresponds to this point down here, right? If h(x) = 1 and we're only considering the case of y = 1 here. But if h(x) = 1 then the cost is down here, is equal to 0. And that's where we'd like it to be because if we correctly predict the output y, then the cost is 0. But now notice also that as h(x) approaches 0, so as the output of a hypothesis approaches 0, the cost blows up and it goes to infinity. And what this does is this captures the intuition that if a hypothesis of 0, that's like saying a hypothesis saying the chance of y equals 1 is equal to 0. It's kinda like our going to our medical patients and saying the probability that you have a malignant tumor, the probability that y=1, is zero. So, it's like absolutely impossible that your tumor is malignant.

But if it turns out that the tumor, the patient's tumor, actually is malignant, so if y is equal to one, even after we told them, that the probability of it happening is zero. So it's absolutely impossible for it to be malignant. But if we told them this with that level of certainty and we turn out to be wrong, then we penalize the learning algorithm by a very, very large cost. And that's captured by having this cost go to infinity if y equals 1 and h(x) approaches 0 ~~this slide consider the case of y~~

Logistic regression cost function

$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



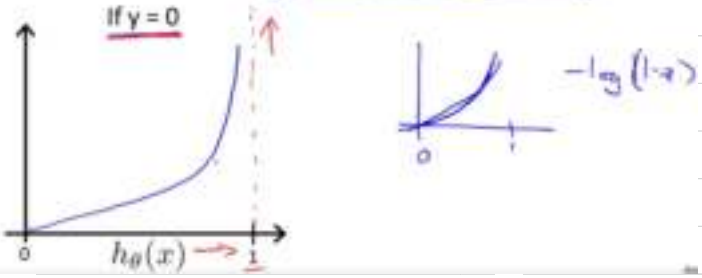
→ Cost = 0 if $y = 1, h_\theta(x) = 1$
But as $h_\theta(x) \to 0$
$Cost \to \infty$

→ Captures intuition that if $h_\theta(x) = 0$, (predict $P(y = 1|x; \theta) = 0$), but $y = 1$, we'll penalize learning algorithm by a very large cost.

Priviouse slide Consider the case of $y=1$, and now let's look at what the Cost function looks like for $y=0$ &

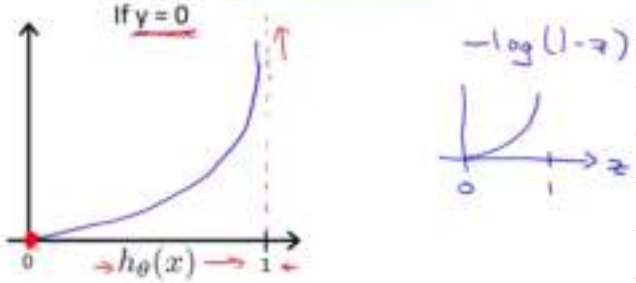
Logistic regression cost function

$$\text{Cost}(h_\theta(x^{(i)}), y^{(i)}) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



Logistic regression cost function

$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



If y is equal to 0, then the cost looks like this, it looks like this expression over here, and if you plot the function, -log(1-z), what you get is the cost function actually looks like this. So it goes from 0 to 1, something like that and so if you plot the cost function for the case of y equals 0, you find that it looks like this. And what this curve does is it now goes up and it goes to plus infinity as h of x goes to 1 because as I was saying, that if y turns out to be equal to 0. But we predicted that y is equal to 1 with almost certainly, probably 1, then we end up paying a very large cost. And conversely, if h of x is equal to 0 and y equals 0, then the hypothesis melted. The protected y of z is equal to 0, and it turns out y is equal to 0, so at this point, the cost function is going to be 0. In this video, we will define the cost function for a single train example. The topic of convexity analysis is now beyond the scope of this course, but it is possible to show that with a particular choice of cost function, this will give a convex optimization problem. Overall cost function j of theta will be convex and local optima free. In the next video we're gonna take these ideas of the cost function for a single training example and develop that further, and define the cost function for the entire training set. And we'll also figure out a simpler way to write it than we have been using so far, and based on that we'll work out grading descent, and that will give us logistic compression algorithm.

Some conclusion & hint of this section.

- ✓ - if $h_\theta(x) = y$, then $\text{Cost}(h_\theta(x), y) = 0$ (for $y=0$ and $y=1$)
- ✓ - if $y=0$, then $\text{Cost}(h_\theta(x), y) \to \infty$ as $h_\theta(x) \to 1$
- ✓ - Regardless of whether $y=0$ or $y=1$, if $h_\theta(x) = 0.5$, then $\text{Cost}(h_\theta(x), y) > 0$

Logistic Regression model

Simplified Cost Function and gradient Descent

Logistic regression cost function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \text{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

Note: $y = 0$ or 1 always

$$\text{Cost}(h_\theta(x), y) = -y \log(h_\theta(x)) - (1-y)\log(1 - h_\theta(x))$$

If $y=1$: $\text{Cost}(h_\theta(x), y) = -\log h_\theta(x)$

If $y=0$: $\text{Cost}(h_\theta(x), y) = -\log(1 - h_\theta(x))$

Putting both formulas in to the one

Logistic regression cost function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \text{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

$$= -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))\right]$$

To fit parameters θ:

$$\min_\theta J(\theta)$$ Get θ

To make a prediction given new x:

Output $h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$ $p(y=1 \mid x; \theta)$

Gradient Descent

$$J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))\right]$$

Want $\min_\theta J(\theta)$: $\frac{\partial}{\partial \theta_j} J(\theta)$ $\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$ for i=0 to n

Repeat {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update all θ_j)

}

$h_\theta(x) = \theta^T x$

$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$

$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m}\sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$

Algorithm looks identical to linear regression!

Gradient descent درشیب اینبرای یادسازی logistic

When implementing logistic regression with gradient descent, we have all of these different parameter values, theta zero down to theta n, that we need to update using this expression. And one thing we could do is have a for loop. So for i equals zero to n, or for i equals one to n plus one. So update each of these parameter values in turn. But of course rather than using a for loop, ideally we would also use a vector rise implementation. So that a vector rise implementation can update all of these n plus one parameters all in one fell swoop. And to check your own understanding, you might see if you can figure out how to do this vector-rise implementation with this algorithm as well.

So, now you you know how to implement gradient descents for logistic regression. There was one last idea that we had talked about earlier, for linear regression, which was feature scaling. We saw how feature scaling can help gradient descent converge faster for linear regression. The idea of feature scaling also applies to gradient descent for logistic regression. And yet we have features that are on very different scale, then applying feature scaling can also make grading descent run faster for logistic regression.

So that's it, you now know how to implement logistic regression and this is a very powerful, and probably the most widely used, classification algorithm in the world. And you now know how we get it to work for yourself.

A Victorized implementation is:

$$\theta := \theta - \frac{\alpha}{m} X^T (g(X\theta) - \vec{y})$$

Logistic Regression model

Advance optimization

in this video I'd like to tell you about some advanced optimization algorithms and some advanced optimization Concepts. & using some of these ideas, we will be able to get logistic Regression to Run much more quickly than it's possible with gradient Descent. and this will also let the algorithm Scale much better to very large machine Learning Problems. Such as if we had very large number of features

Optimization algorithm

Cost function $J(\theta)$. Want $\min_\theta J(\theta)$.

Given θ, we have code that can compute

- $J(\theta)$
- $\frac{\partial}{\partial \theta_j} J(\theta)$ (for $j = 0, 1, \ldots, n$)

Gradient descent:

Repeat {

$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$

}

~~large number of features~~ Here's an alternative view of what gradient descent is doing. We have some cost function J and we want to minimize it. So what we need to is, we need to write code that can take as input the parameters theta and they can compute two things: J of theta and these partial derivative terms for, you know, J equals 0, 1 up to N. Given code that can do these two things, what gradient descent does is it repeatedly performs the following update. Right? So given the code that we wrote to compute these partial derivatives, gradient descent plugs in here and uses that to update our parameters theta.

So another way of thinking about gradient descent is that we need to supply code to compute J of theta and these derivatives, and then these get plugged into gradient descents, which can then try to minimize the function for us. For gradient descent, I guess technically you don't actually need code to compute the cost function J of theta. You only need code to compute the derivative terms. But if you think of your code as also monitoring convergence of some such, we'll just think of ourselves as providing code to compute both the cost function and the derivative terms.

Optimization algorithm

Given θ, we have code that can compute

- $J(\theta)$
- $\frac{\partial}{\partial \theta_j} J(\theta)$ (for $j = 0, 1, \ldots, n$)

Optimization algorithms:
- Gradient descent
- Conjugate gradient
- BFGS
- L-BFGS

Advantages:
- No need to manually pick α
- Often faster than gradient descent.

Disadvantages:
- More complex

(1)

So, having written code to compute these two things, one algorithm we can use is gradient descent.

But gradient descent isn't the only algorithm we can use. And there are other algorithms, more advanced, more sophisticated ones, that, if we only provide them a way to compute these two things, then these are different approaches to optimize the cost function for us. So conjugate gradient BFGS and L-BFGS are examples of more sophisticated optimization algorithms that need a way to compute J of theta, and need a way to compute the derivatives, and can then use more sophisticated strategies than gradient descent to minimize the cost function.

The details of exactly what these three algorithms is well beyond the scope of this course. And in fact you often end up spending, you know, many days, or a small number of weeks studying these algorithms. If you take a class and advance the numerical computing.

But let me just tell you about some of their properties.

These three algorithms have a number of advantages. One is that, with any of this algorithms you usually do not need to manually pick the learning rate alpha.

So one way to think of these algorithms is that given is the way to compute the derivative and a cost function. You can think of these algorithms as having a clever inter-loop. And, in fact, they have a clever

inter-loop called a line search algorithm that automatically tries out different values for the learning rate alpha and automatically picks a good learning rate alpha so that it can even pick a different learning rate for every iteration. And so then you don't need to choose it yourself.

(2)

These algorithms actually do more sophisticated things than just pick a good learning rate, and so they often end up converging much faster than gradient descent, but detailed discussion of exactly what they do is beyond the scope of this course.

In fact, I actually used to have used these algorithms for a long time, like maybe over a decade, quite frequently, and it was only, you know, a few years ago that I actually figured out for myself the details of what conjugate gradient, BFGS and O-BFGS do. So it is actually entirely possible to use these algorithms successfully and apply to lots of different learning problems without actually understanding the inter-loop of what these algorithms do.

If these algorithms have a disadvantage, I'd say that the main disadvantage is that they're quite a lot more complex than gradient descent. And in particular, you probably should not implement these algorithms - conjugate gradient, L-BGFS, BFGS - yourself unless you're an expert in numerical computing.

Instead, just as I wouldn't recommend that you write your own code to compute square roots of numbers or to compute inverses of matrices, for these algorithms also what I would recommend you do is just use a software library. So, you know, to take a square root what all of us do is use some function that someone else has written to compute the square roots of our numbers.

And fortunately, Octave and the closely related language MATLAB - we'll be using that - Octave has a very good. Has a pretty reasonable library implementing some of these advanced optimization algorithms. And so if you just use the built-in library, you know, you get pretty good results.

I should say that there is a difference between good and bad implementations of these algorithms. And so, if you're using a different language for your machine learning application, if you're using C, C++, Java, and so on, you might want to try out a couple of different libraries to make sure that you find a good library for implementing these algorithms. Because there is a difference in performance between a good implementation of, you know, contour gradient or LPFGS versus less good implementation of contour gradient or LPFGS.

So now let's explain how to use these algorithms, I'm going to do so with an example. Let's say that you have a problem with two parameters equals theta zero and theta one. And let's say your cost function is J of theta equals theta one minus five squared, plus theta two minus five squared.So with this cost function.You know the value for theta 1 and theta 2.If you want to minimize J of theta as a function of theta.The value that minimizes it is going to be theta 1 equals 5, theta 2 equals equals five.Now, again, I know some of you know more calculus than others,but the derivatives of the cost function J turn out to be these two expressions.I've done the calculus.So if you want to apply one of the advanced optimization algorithms to minimize cost function J.

Now let's explain how to use these algorithms

Example: $\min_\theta J(\theta)$

$\theta = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}$ $\theta_1=5, \theta_2=5.$

$J(\theta) = (\theta_1 - 5)^2 + (\theta_2 - 5)^2$

$\frac{\partial}{\partial\theta_1} J(\theta) = 2(\theta_1 - 5)$

$\frac{\partial}{\partial\theta_2} J(\theta) = 2(\theta_2 - 5)$

```
function [jVal, gradient]
        = costFunction(theta)
  jVal = (theta(1)-5)^2 + ...
         (theta(2)-5)^2;
  gradient = zeros(2,1);
  gradient(1) = 2*(theta(1)-5);
  gradient(2) = 2*(theta(2)-5);
```

```
options = optimset('GradObj', 'on', 'MaxIter', '100');
initialTheta = zeros(2,1);
[optTheta, functionVal, exitFlag] ...
    = fminunc(@costFunction, initialTheta, options);
```

$\theta \in \mathbb{R}^d$ $d \geq 2$.

So, you know, if we didn't know the minimum was at 5, 5, but if you want to have a cost function 5 the minimum numerically using something like gradient descent but preferably more advanced than gradient descent, what you would do is implement an octave function like this, so we implement a cost function,cost function theta function like that,and what this does is that it returns two arguments, the first J-val, is how we would compute the cost function J. And so this says J-val equals, you know, theta one minus five squared plus theta two minus five squared.

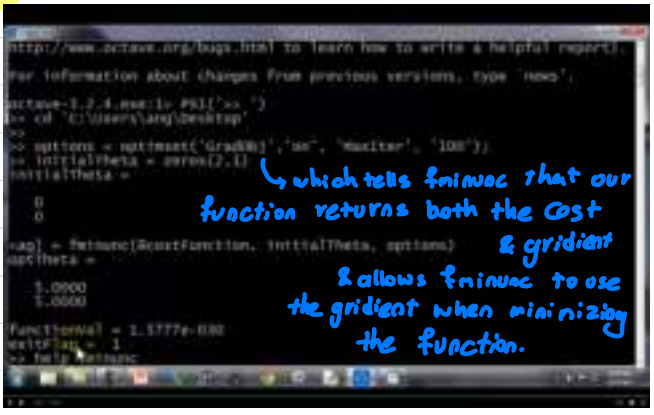
So it's just computing this cost function over here.And the second argument that this function returns is gradient.So gradient is going to be a two by one vector,and the two elements of the gradient vector correspond to the two partial derivative terms over here.Having implemented this cost function,you would, you can then call the advanced optimization

function called the fminunc- it stands for function minimization unconstrained in Octave -and the way you call this is as follows.You set a few options.This is a options as a data structure that stores the options you want.So grant up on,this sets the gradient objective parameter to on.It just means you are indeed going to provide a gradient to this algorithm.I'm going to set the maximum number of iterations to, let's say, one hundred.We're going give it an initial guess for theta.There's a 2 by 1 vector.And then this command calls fminunc.This at symbol presents a pointer to the cost function that we just defined up there.And if you call this,this will compute, you know, will use one of the more advanced optimization algorithms.And if you want to think it as just like gradient descent.But automatically choosing the learning rate alpha for so you don't have to do so yourself.But it will then attempt to use the sort of advanced optimization algorithms.Like gradient descent on steroids.To try to find the optimal value of theta for you.Let me actually show you what this looks like in Octave.

So I've written this cost function of theta function exactly as we had it on the previous line.It computes J-val which is the cost function.And it computes the gradient with the two elements being the partial derivatives of the cost function with respect to, you know,the two parameters, theta one and theta two.Now let's switch to my Octave window.I'm gonna type in those commands I had just now.So, options equals optimset. This is the notation for setting my parameters on my options, for my optimization algorithm. Grant option on, maxIter, 100 so that says 100 iterations, and I am going to provide the gradient to my algorithm.Let's say initial theta equals zero's two by one.So that's my initial guess for theta.And now I have of theta,function val exit flag equals fminunc constraint.A pointer to the cost function.and provide my initial guess.

And the options like so.And if I hit enter this will run the optimization algorithm.And it returns pretty quickly.This funny formatting that's because my line, you know, my code wrapped around.So, this funny thing is just because my command line had wrapped around.But what this says is that numerically renders, you know, think of it as gradient descent on steroids, they found the optimal value of a theta is theta 1 equals 5, theta 2 equals 5, exactly as we're hoping for.

The function value at the optimum is essentially 10 to the minus 30.So that's essentially zero, which is also what we're hoping for.And the exit flag is 1, and this shows what the convergence status of this.And if you want you can do help fminunc to read the documentation for how to interpret the exit flag.But the exit flag let's you verify whether or not this algorithm thing has converged.



So that how you Run these algorithm in Octave

hint:

I should mention, by the way, that for the Octave implementation, this value of theta, your parameter vector of theta, must be in rd for d greater than or equal to 2. So if theta is just a real number. So, if it is not at least a two-dimensional vector or some higher than two-dimensional vector, this fminunc may not work, so and if in case you have a one-dimensional function that you use to optimize, you can look in the octave documentation for fminunc for additional details.

So, that's how we optimize our trial example of this simple quick driving cost function. However, we apply this to let's just say progression.

But the main concept I hope you take away from this slide is, that what you need to do, is write a function that returns

the cost function and returns the gradient.

And so in order to apply this to logistic regression or even to linear regression, if you want to use these optimization algorithms for linear regression.

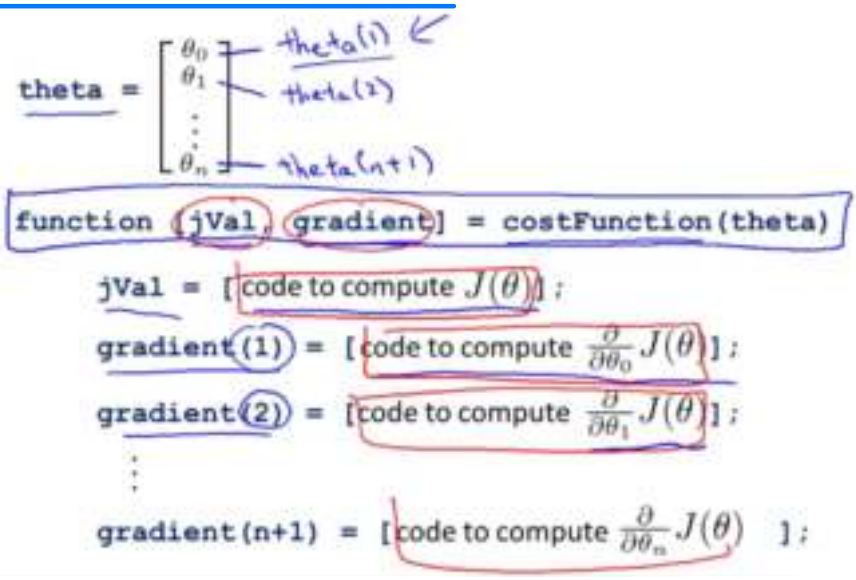
What you need to do is plug in the appropriate code to compute these things over here.

So, now you know how to use these advanced optimization algorithms.

Because, using, because for these algorithms, you're using a sophisticated optimization library, it makes the just a little bit more opaque and so just maybe a little bit harder to debug. But because these algorithms often run much faster than gradient descent, often quite typically whenever I have a large machine learning problem, I will use these algorithms instead of using gradient descent.

And with these ideas, hopefully, you'll be able to get logistic progression and also linear regression to work on much larger problems. So, that's it for advanced optimization concepts.

And in the next and final video on Logistic Regression, I want to tell you how to take the logistic regression algorithm that you already know about and make it work also on multi-class classification problems.



= 0" = wrapping-up = 0" =

Advanced Optimization

Note: [7:35 - '100' should be 100 instead. The value provided should be an integer and not a character string.]

"Conjugate gradient", "BFGS", and "L-BFGS" are more sophisticated, faster ways to optimize θ that can be used instead of gradient descent. We suggest that you should not write these more sophisticated algorithms yourself (unless you are an expert in numerical computing) but use the libraries instead, as they're already tested and highly optimized. Octave provides them.

We first need to provide a function that evaluates the following two functions for a given input value θ:

$J(\theta)$

$\frac{\partial}{\partial \theta_j} J(\theta)$

We can write a single function that returns both of these:

```
function [jVal, gradient] = costFunction(theta)
  jVal = [...code to compute J(theta)...];
  gradient = [...code to compute derivative of J(theta)...];
end
```

Then we can use octave's "fminunc()" optimization algorithm along with the "optimset()" function that creates an object containing the options we want to send to "fminunc()". (Note: the value for MaxIter should be an integer, not a character string - errata in the video at 7:30)

```
options = optimset('GradObj', 'on', 'MaxIter', 100);
initialTheta = zeros(2,1);
   [optTheta, functionVal, exitFlag] = fminunc(@costFunction, initialTheta, options);

```

We give to the function "fminunc()" our cost function, our initial vector of theta values, and the "options" object that we created beforehand.

Multi class Classification

Multi class classification: One-vs-all

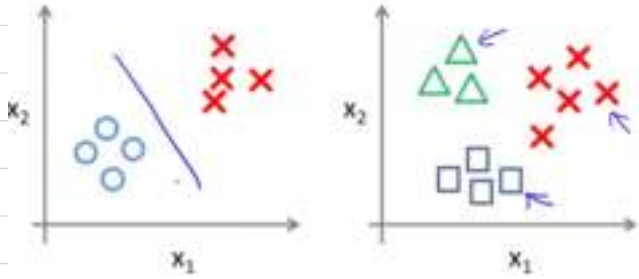
In this video we'll talk about how to get logistic regression to work for multiclass classification problems.And in particular I want to tell you about an algorithm called one-versus-all classification.

Multiclass classification

Email foldering/tagging: Work, Friends, Family, Hobby

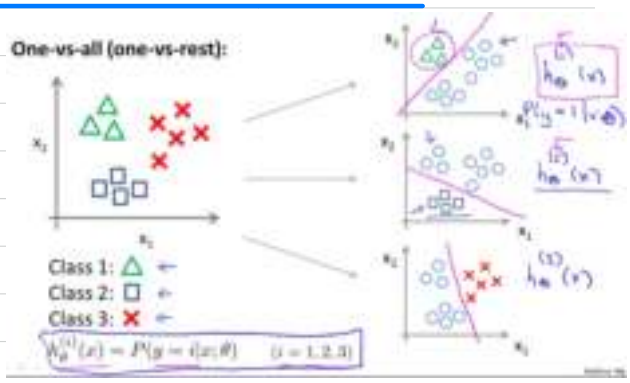
Medical diagrams: Not ill, Cold, Flu

Weather: Sunny, Cloudy, Rain, Snow



Whereas previously for a binary classification problem, our data sets look like this. For a multi-class classification problem our data sets may look like this where here I'm using three different symbols to represent our three classes. So the question is given the data set with three classes where this is an example of one class, that's an example of a different class, and that's an example of yet a third class. How do we get a learning algorithm to work for the setting? We already know how to do binary classification using a regression. We know how to you know maybe fit a straight line to set for the positive and negative classes. You see an idea called one-vs-all classification. We can then take this and make it work for multi-class classification as well.

Here's how a one-vs-all classification works. And this is also sometimes called one-vs-rest. Let's say we have a training set like that shown on the left, where we have three classes of y equals 1, we denote that with a triangle, if y equals 2, the square, and if y equals three, then the cross. What we're going to do is take our training set and turn this into three separate binary classification problems. I'll turn this into three separate two class classification problems. So let's start with class one which is the triangle. We're gonna essentially create a new sort of fake training set where classes two and three get assigned to the negative class. And class one gets assigned to the positive class. You want to create a new training set like that shown on the right, and we're going to fit a classifier which I'm going to call h subscript theta superscript one of x where here the triangles are the positive examples and the circles are the negative examples. So think of the triangles being assigned the value of one and the circles assigned the value of zero. And we're just going to train a standard logistic regression classifier and maybe that will give us a position boundary that looks like that. Okay?



This superscript one here stands for class one, so we're doing this for the triangles of class one. Next we do the same thing for class two. Gonna take the squares and assign the squares as the positive class, and assign everything else, the triangles and the crosses, as a negative class. And then we fit a second logistic regression classifier and call this h of x superscript two, where the superscript two denotes that we're now doing this, treating the square class as the positive class. And maybe we get classified like that. And finally, we do the same thing for the third class and fit a third classifier h super script three of x, and maybe this will give us a decision bounty of the visible cross fire. This separates the positive and negative examples like that.

So to summarize, what we've done is, we've fit three classifiers. So, for i = 1, 2, 3, we'll fit a classifier x super script i subscript theta of x. Thus trying to estimate what is the probability that y is equal to class i, given x and parametrized by theta. Right? So in the first instance for this first one up here, this classifier was learning to recognize the triangles. So it's thinking of the triangles as a positive clause, so x superscript one is essentially trying to estimate what is the probability that the y is equal to one, given that x is parametrized by theta. And similarly, this is treating the square class as a positive class and so it's trying to estimate the probability that y = 2 and so on. So we now have three classifiers, each of which was trained to recognize one of the three classes.

=0= Summarize =0=

One-vs-all

Train a logistic regression classifier $h_\theta^{(i)}(x)$ for each class i to predict the probability that $y = i$.

On a new input x, to make a prediction, pick the class i that maximizes

$$\max_i h_\theta^{(i)}(x)$$

Just to summarize, what we've done is we want to train a logistic regression classifier h superscript i of x for each class i to predict the probability that y is equal to i. Finally to make a prediction, when we're given a new input x, and we want to make a prediction. What we do is we just run all three of our classifiers on the input x and we then pick the class i that maximizes the three. So we just basically pick the classifier, I think whichever one of the three classifiers is most confident and so the most enthusiastically says that it thinks it has the right clause. So whichever value of i gives us the highest probability we then predict y to be that value.

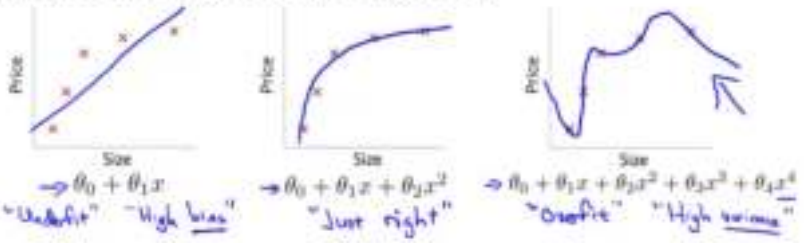
So that's it for multi-class classification and one-vs-all method. And with this little method you can now take the logistic regression classifier and make it work on multi-class classification problems as well

Solving the Problem of overfitting

The Problem of overfitting

By now, you've seen a couple different learning algorithms, linear regression and logistic regression. They work well for many problems, but when you apply them to certain machine learning applications, they can run into a problem called overfitting that can cause them to perform very poorly. What I'd like to do in this video is explain to you what is this overfitting problem, and in the next few videos after this, we'll talk about a technique called regularization, that will allow us to ameliorate or to reduce this overfitting problem and get these learning algorithms to maybe work much better. So what is overfitting? Let's keep using our running example of predicting housing prices with linear regression where we want to predict the price as a function of the size of the house. One thing we could do is fit a linear function to this data, and if we do that, maybe we get that sort of straight line fit to the data. But this isn't a very good model. Looking at the data, it seems pretty clear that as the size of the housing increases, the housing prices plateau, or kind of flattens out as we move to the right and so this algorithm does not fit the training and we call this problem underfitting, and another term for this is that this algorithm has high bias. Both of these roughly mean that it's just not even fitting the training data very well. The term is kind of a historical or technical one, but the idea is that if a fitting a straight line to the data, then, it's as if the algorithm has a very strong preconception, or a very strong bias that housing prices are going to vary linearly with their size and despite the data to the contrary. Despite the evidence of the contrary is preconceptions still are bias, still closes it to fit a straight line and this ends up being a poor fit to the data. Now, in the middle, we could fit a quadratic functions enter and, with this data set, we fit the quadratic function, maybe, we get that kind of curve and, that works pretty well. And, at the other extreme, would be if we were to fit, say a fourth other polynomial to the data. So, here we have five parameters, theta zero through theta four, and, with that, we can actually fit a curve that process through all five of our training examples. You might get a curve that looks like this.

Example: Linear regression (housing prices)



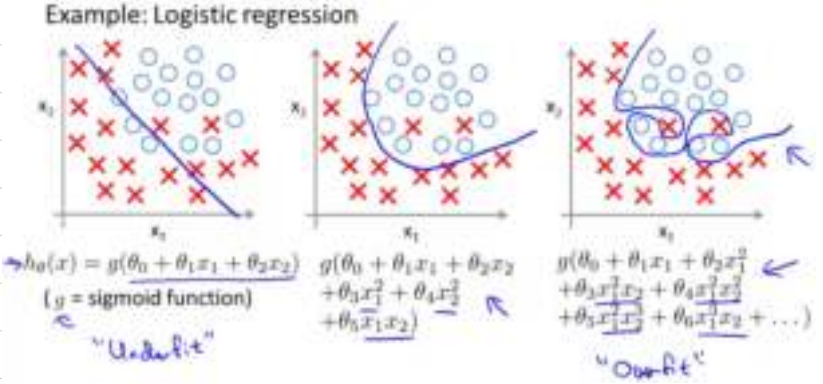
Overfitting: If we have too many features, the learned hypothesis may fit the training set very well ($J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 = 0$), but fail to generalize to new examples (predict prices on new examples).

That, on the one hand, seems to do a very good job fitting the training set and, that is processed through all of my data, at least. But, this is still a very wiggly curve, right? So, it's going up and down all over the place, and, we don't actually think that's such a good model for predicting housing prices. So, this problem we call overfitting, and, another term for this is that this algorithm has high variance. The term high variance is another historical or technical one. But, the intuition is that, if we're fitting such a high order polynomial, then, the hypothesis can fit, you know, it's almost as if it can fit almost any function and this face of possible hypothesis is just too large, it's too variable. And we don't have enough data to constrain it to give us a good hypothesis so that's called overfitting. And in the middle, there isn't really a name but I'm just going to write, you know, just right. Where a second degree polynomial, quadratic function seems to be just right for fitting this data. To recap a bit the problem of over fitting comes when if we have too many features, then to learn hypothesis may fit the training side very well. So, your cost function may actually be very close to zero or may be even zero exactly, but you may then end up with a curve like this that, you know tries too hard to fit the training set, so that it even fails to generalize to new examples and fails to predict prices on new examples as well, and here the term generalized refers to how well a hypothesis applies even to new examples. That is to data to houses that it has not seen in the training set. On this slide, ~~we looked at over fitting for the case~~

So we looked at overfitting for the case of linear Regression. A similar thing can apply to logistic Regression as well.

Here is a logistic Regression example with two features X_1, X_2.

Example: Logistic regression



Later, in this course, when we talk about debugging and diagnosing things that can go wrong with learning algorithms, we'll give you specific tools to recognize when overfitting and, also,when underfitting may be occurring.But, for now, lets talk about the problem of, if we think overfitting is occurring, what can we do to address it?

regression example with two features X1 and x2. One thing we could do, is fit logistic regression with just a simple hypothesis like this, where, as usual, G is my sigmoid function. And if you do that, you end up with a hypothesis, trying to use, maybe, just a straight line to separate the positive and the negative examples. And this doesn't look like a very good fit to the hypothesis. So, once again, this is an example of underfitting or of the hypothesis having high bias. In contrast, if you were to add to your features these quadratic terms, then, you could get a decision boundary that might look more like this. And, you know, that's a pretty good fit to the data. Probably, about as good as we could get, on this training set. And, finally, at the other extreme, if you were to fit a very high-order polynomial, if you were to generate lots of high-order polynomial terms of speeches, then, logistical regression may contort itself, may try really hard to find a decision boundary that fits your training data or go to great lengths to contort itself, to fit every single training example well. And, you know, if the features X1 and X2 offer predicting, maybe, the cancer to the, you know, cancer is a malignant, benign breast tumors. This doesn't, this really doesn't look like a very good hypothesis, for making predictions. And so, once again, this is an instance of overfitting and, of a hypothesis having high variance and not really, and, being unlikely to generalize well to new examples. ~~Later, in this course, when we talk about~~

Addressing overfitting:

x_1 = size of house
x_2 = no. of bedrooms
x_3 = no. of floors
x_4 = age of house
x_5 = average income in neighborhood
x_6 = kitchen size
$\vdots$
x_{100}

Addressing overfitting:

Options:
1. Reduce number of features.
   - Manually select which features to keep.
   - Model selection algorithm (later in course).
2. Regularization.
   - Keep all the features, but reduce magnitude/values of parameters θ_j.
   - Works well when we have a lot of features, each of which contributes a bit to predicting y.

address it? In the previous examples, we had one or two dimensional data so, we could just plot the hypothesis and see what was going on and select the appropriate degree polynomial. So, earlier for the housing prices example, we could just plot the hypothesis and, you know, maybe see that it was fitting the sort of very wiggly function that goes all over the place to predict housing prices. And we could then use figures like these to select an appropriate degree polynomial. So plotting the hypothesis, could be one way to try to decide what degree polynomial to use. But that doesn't always work. And, in fact more often we may have learning problems that where we just have a lot of features. And there is not just a matter of selecting what degree polynomial. And, in fact, when we have so many features, it also becomes much harder to plot the data and it becomes much harder to visualize it, to decide what features to keep or not. So concretely, if we're trying predict housing prices sometimes we can just have a lot of different features. And all of these features seem, you know, maybe they seem kind of useful. But, if we have a lot of features, and, very little training data, then, over fitting can become a problem. In order to address over fitting, there are two main options for things that we can do. The first option is, to try to reduce the number of features. Concretely, one thing we could do is manually look through the list of features, and, use that to try to decide which are the more important features, and, therefore, which are the features we should keep, and, which are the features we should throw out. Later in this course, where also talk about model selection algorithms. Which are algorithms for automatically deciding which features to keep and, which features to throw out. This idea of reducing the number of features can work well, and, can reduce over fitting. And, when we talk about model selection, we'll go into this in much greater depth. But, the disadvantage is that, by throwing away some of the features, is also throwing away some of the information you have about the problem. For example, maybe, all of those features are actually useful for predicting the price of a house, so, maybe, we don't actually want to throw some of our information or throw some of our features away. The second option, which we'll talk about in the next few videos, is regularization. Here, we're going to keep all the features, but we're going to reduce the magnitude or the values of the parameters theta J. And, this method works well, we'll see, when we have a lot of features, each of which contributes a little bit to predicting the value of Y, like we saw in the housing price prediction example. Where we could have a lot of features, each of which are, you know, somewhat useful, so, maybe, we don't want to throw them away. So, this subscribes the idea of regularization at a very high level. And, I realize that, all of these details probably don't make sense to you yet. But, in the next video, we'll start to formulate exactly how to apply regularization and, exactly what regularization means. And, then we'll start to figure out, how to use this, to make how learning algorithms work well and avoid overfitting.

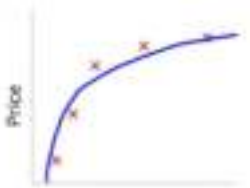
Solving the Problem of overfitting

Cost function

In this video, I'd like to convey to you, the main intuitions behind how regularization works.And, we'll also write down the cost function that we'll use, when we were using regularization.With the hand drawn examples that we have on these slides, I think I'll be able to convey part of the intuition.But, an even better way to see for yourself, how regularization works, is if you implement it, and, see it work for yourself. And, if you do the appropriate exercises after this, you get the chance to self see regularization in action for yourself.

get the chance to self see regularization in action for yourself. So, here is the intuition. In the previous video, we saw that, if we were to fit a quadratic function to this data, it gives us a pretty good fit to the data. Whereas, if we were to fit an overly high order degree polynomial, we end up with a curve that may fit the training set very well, but, really not be a, but overfit the data poorly, and, not generalize well. Consider the following, suppose we were to penalize, and, make the parameters theta 3 and theta 4 really small. Here's what I mean, here is our optimization objective, or here is our optimization problem, where we minimize our usual squared error cause function. Let's say I take this objective and modify it and add to it, plus 1000 theta 3 squared, plus 1000 theta 4 squared. 1000 I am just writing down as some huge number. Now, if we were to minimize this function, the only way to make this new cost function small is if theta 3 and data 4 are small, right? Because otherwise, if you have a thousand times theta 3, this new cost functions gonna be big. So when we minimize this new function we are going to end up with theta 3 close to 0 and theta 4 close to 0, and as if we're getting rid of these two terms over there. And if we do that, well then, if theta 3 and theta 4 close to 0 then we are being left with a quadratic function, and, so, we end up with a fit to the data, that's, you know, quadratic function plus maybe, tiny contributions from small terms, theta 3, theta 4, that they may be very close to 0. And, so, we end up with essentially, a quadratic function, which is good. Because this is a much better hypothesis. In this particular example, we looked at the effect of penalizing two of the parameter values being large. More generally, here is the idea behind regularization. The idea is that, if we have

Intuition



$\theta_0 + \theta_1 x + \theta_2 x^2$

$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

Suppose we penalize and make θ_3, θ_4 really small.

$$\min_\theta \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2 + 1000\,\theta_3^2 + 1000\,\theta_4^2$$

$\theta_3 \approx 0 \quad \theta_4 \approx 0$

Regularization.

Small values for parameters $\theta_0, \theta_1, \ldots, \theta_n$
- "Simpler" hypothesis
- Less prone to overfitting

Housing:
- Features: $x_1, x_2, \ldots, x_{100}$
- Parameters: $\theta_0, \theta_1, \theta_2, \ldots, \theta_{100}$

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^{100} \theta_j^2\right]$$

By the way, by convention the summation here starts from one so I am not actually going penalize theta zero being large. That sort of the convention that, the sum I equals one through N, rather than I equals zero through N. But in practice, it makes very little difference, and, whether you include, you know, theta zero or not, in practice, make very little difference to the results. But by convention, usually, we regularize only theta through theta 100. Writing down our regularized

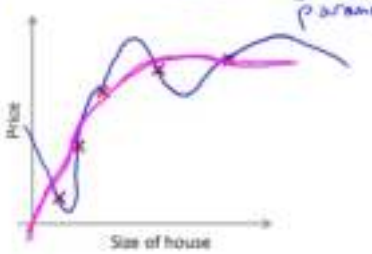
being large. More generally, here is the idea behind regularization. The idea is that, if we have small values for the parameters, then, having small values for the parameters, will somehow, will usually correspond to having a simpler hypothesis. So, for our last example, we penalize just theta 3 and theta 4 and when both of these were close to zero, we wound up with a much simpler hypothesis that was essentially a quadratic function. But more broadly, if we penalize all the parameters usually that, we can think of that, as trying to give us a simpler hypothesis as well because when, you know, these parameters are as close as you in this example, that gave us a quadratic function. But more generally, it is possible to show that having smaller values of the parameters corresponds to usually smoother functions as well for the simpler. And which are therefore, also, less prone to overfitting. I realize that the reasoning for why having all the parameters be small. Why that corresponds to a simpler hypothesis; I realize that reasoning may not be entirely clear to you right now. And it is kind of hard to explain unless you implement yourself and see it for yourself. But I hope that the example of having theta 3 and theta 4 be small and how that gave us a simpler hypothesis, I hope that helps explain why, at least give some intuition as to why this might be true. Lets look at the specific example. For housing price prediction we may have our hundred features that we talked about where may be x1 is the size, x2 is the number of bedrooms, x3 is the number of floors and so on. And we may we may have a hundred features. And unlike the polynomial example, we don't know, right, we don't know that theta 3, theta 4, are the high order polynomial terms. So, if we have just a bag, if we have just a set of a hundred features, it's hard to pick in advance which are the ones that are less likely to be relevant. So we have a hundred or a hundred one parameters. And we don't know which ones to pick, we don't know which parameters to try to pick, to try to shrink. So, in regularization, what we're going to do, is take our cost function, here's my cost function for linear regression. And what I'm going to do is, modify this cost function to shrink all of my parameters, because, you know, I don't know which one or two to try to shrink. So I am going to modify my cost function to add a term at the end. Like so we have square brackets here as well. When I add an extra regularization term at the end to shrink every single parameter and so this term we tend to shrink all of my parameters theta 1, theta 2, theta 3 up to theta 100.

Regularization.

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^{n} \theta_j^2\right]$$

regularization parameter

$$\min_\theta J(\theta)$$

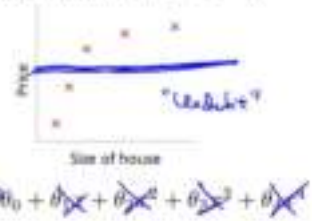


by convention, usually, we regularize only theta through theta 100. Writing down our regularized optimization objective, our regularized cost function again. Here it is. Here's J of theta where, this term on the right is a regularization term and lambda here is called the regularization parameter and what lambda does, is it controls a trade off between two different goals. The first goal, capture it by the first goal objective, is that we would like to train, is that we would like to fit the training data well. We would like to fit the training set well. And the second goal is, we want to keep the parameters small, and that's captured by the second term, by the regularization objective. And by the regularization term. And what lambda, the regularization parameter does is the controls the trade of between these two goals, between the goal of fitting the training set well and the goal of keeping the parameter plan small and therefore keeping the hypothesis relatively simple to avoid overfitting. For our housing price prediction example, whereas, previously, if we had fit a very high order polynomial, we may have wound up with a very, sort of wiggly or curvy function like this. If you still fit a high order polynomial with all the polynomial features in there, but instead, you just make sure, to use this sole of regularized objective, then what you can get out is in fact a curve that isn't quite a quadratic function, but is much smoother and much simpler and maybe a curve like the magenta line that, you know, gives a much better hypothesis for this data. Once again, I realize it can be a bit difficult to see why strengthening the parameters can have this effect, but if you implement yourselves with regularization you will be able to see this effect firsthand.

In regularized linear regression, we choose θ to minimize

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^{n} \theta_j^2\right]$$

What if λ is set to an extremely large value (perhaps for too large for our problem, say $\lambda = 10^{10}$)?



$h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

$\theta_1, \theta_2, \theta_3, \theta_4 \approx 0$

$h_\theta(x) = \theta_0$

In regularized linear regression, if the regularization parameter monitor is set to be very large, then what will happen is we will end up penalizing the parameters theta 1, theta 2, theta 3, theta 4 very highly. That is, if our hypothesis is this is one down at the bottom. And if we end up penalizing theta 1, theta 2, theta 3, theta 4 very heavily, then we end up with all of these parameters close to zero, right? Theta 1 will be close to zero; theta 2 will be close to zero. Theta three and theta four will end up being close to zero. And if we do that, it's as if we're getting rid of these terms in the hypothesis so that we're just left with a hypothesis that will say that. It says that, well, housing prices are equal to theta zero, and that is akin to fitting a flat horizontal straight line to the data. And this is an example of underfitting, and in particular this hypothesis, this straight line it just fails to fit the training set well. It's just a flat straight line, it doesn't go, you know, go near. It doesn't go anywhere near most of the training examples. And another way of saying this is that this hypothesis has too strong a preconception or too high bias that housing prices are just equal to theta zero, and despite the clear data to the contrary, you know chooses to fit a sort of, flat line, just a flat horizontal line. I didn't draw that very well. This just a horizontal flat line to the data. So for regularization to work well, some care should be taken, to choose a good choice for the regularization parameter lambda as well. And when we talk about multi-selection later in this course, we'll talk about a way, a variety of ways for automatically choosing the regularization parameter lambda as well. So, that's the idea of the high regularization and the cost function reviews in order to use regularization In the next two videos, lets take these ideas and apply them to linear regression and to logistic regression, so that we can then get them to avoid overfitting.

Solving the Problem of overfitting

Regularized Linear Regression

For linear regression, we have previously worked out two learning algorithms. One based on gradient descent and one based on the normal equation. In this video, we'll take those two algorithms and generalize them to the case of regularized linear regression.

Regularized linear regression

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda\sum_{j=1}^{n}\theta_j^2\right]$$

$$\min_\theta J(\theta)$$

①

Here's the optimization objective that we came up with last time for regularized linear regression. This first part is our usual objective for linear regression. And we now have this additional regularization term, where lambda is our regularization parameter, and we like to find parameters theta that minimizes this cost function, this regularized cost function, J of theta. Previously, we were using gradient descent for the original cost function without the regularization term. And we had the following algorithm, for regular linear regression, without regularization, we would repeatedly update the parameters theta J as follows for J equals 0, 1, 2, up through n.

Let me take this and just write the case for theta 0 separately. So I'm just going to write the update for theta 0 separately than for the update for the parameters 1, 2, 3, and so on up to n. And so this is, I haven't changed anything yet, right. This is just writing the update for theta 0 separately from the updates for theta 1, theta 2, theta 3, up to theta n. And the reason I want to do this is you may remember that for our regularized linear regression, we penalize the parameters theta 1, theta 2, and so on up to theta n. But we don't penalize theta 0. So, when we modify this algorithm for regularized linear regression, we're going to end up treating theta zero slightly differently.

Concretely, if we want to take this algorithm and modify it to use the regular rise objective, all we need to do is take this term at the bottom and modify it as follows. We'll take this term and add minus lambda over m times theta j. And if you implement this, then you have gradient descent for trying to minimize the regularized cost function, j of theta. And concretely, I'm not gonna do the calculus to prove it, but concretely if you look at this term, this term hat I've written in square brackets, if you know calculus it's possible to prove that that term is the partial derivative with respect to J of theta using the new definition of J of theta with the regularization term. And similarly, this term up on top which I'm drawing the cyan box, that's still the partial derivative respect of theta zero of J of theta. If you look at the update for theta j, it's possible to show something very interesting. Concretely, theta j gets updated as theta j minus alpha times, and then you have this other term here that depends on theta J. So if you group all the terms together that depend on theta j, you can show that this update can be written equivalently as follows. And all I did was add theta j here is, so theta j times 1. And this term is, right, lambda over m, there's also an alpha here, so you end up with alpha lambda over m multiplied into theta j.

Gradient descent

Repeat {

$$\theta_0 := \theta_0 - \alpha\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_0^{(i)}$$

$$\theta_j := \theta_j - \alpha\left[\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)} + \frac{\lambda}{m}\theta_j\right]$$

$$(j = 1, 2, 3, \ldots, n)$$

}

$$\theta_j := \theta_j(1 - \alpha\frac{\lambda}{m}) - \alpha\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$$

$1 - \alpha\frac{\lambda}{m} < 1$

②

And this term here, 1 minus alpha times lambda m, is a pretty interesting term. It has a pretty interesting effect.

Concretely this term, 1 minus alpha times lambda over m, is going to be a number that is usually a little bit less than one, because alpha times lambda over m is going to be positive, and usually if your learning rate is small and if m is large, this is usually pretty small. So this term here is gonna be a number that's usually a little bit less than 1, so think of it as a number like 0.99, let's say. And so the effect of our update to theta j is, we're going to say that theta j gets replaced by theta j times 0.99, right?

So theta j times 0.99 has the effect of shrinking theta j a little bit towards zero. So this makes theta j a bit smaller. And more formally, this makes the square norm of theta j a little bit smaller. And then after that, the second term here, that's actually exactly the same as the original gradient descent update that we had, before we added all this regularization stuff. So, hopefully this gradient descent, hopefully this update makes sense. When we're using a regularized linear regression and what we're doing is on every iteration we're multiplying theta j by a number that's a little bit less then one, so its shrinking the parameter a little bit, and then we're performing a similar update as before. Of course that's just the intuition behind what this particular update is doing. Mathematically what it's doing is it's exactly gradient descent on the cost function J of theta that we defined on the previous slide that uses the regularization term. Gradient descent was just one of our two algorithms for fitting a linear regression model. The second algorithm was the one based on the normal equation, where what we did was we created the design matrix X where each row corresponded to a separate training example. And we created a vector y, so this is a vector, that's an m dimensional vector. And that contained the labels from my training set. So whereas X is an m by (n+1) dimensional matrix, y is an m dimensional vector. And in order to minimize the cost function J, we found that one way to do so is to set theta to be equal to this. Right, you have X transpose X, inverse, X transpose Y. I'm leaving room here to fill in stuff of course. And what this value for theta does is this minimizes the cost function J of theta, when we were not using regularization.

Now that we are using regularization, if you were to derive what the minimum is, and just to give you a sense of how to derive the minimum, the way you derive it is you take partial derivatives with respect to each parameter. Set this to zero, and then do a bunch of math and you can then show that it's a formula like this that minimizes the cost function. And concretely, if you are using regularization, then this formula changes as follows. Inside this parenthesis, you end up with a matrix like this. 0, 1, 1, 1, and so on, 1, until the bottom. So this thing over here is a matrix whose upper left-most entry is 0. There are ones on the diagonals, and then zeros everywhere else in this matrix. Because I'm drawing this rather sloppily. But as a example, if n = 2, then this matrix is going to be a three by three matrix. More generally, this matrix is an (n+1) by (n+1) dimensional matrix. So if n = 2, then that matrix becomes something that looks like this. It would be 0, and then 1s on the diagonals, and then 0s on the rest of the diagonals. And once again, I'm not going to show this derivation, which is frankly somewhat long and involved, but it is possible to prove that if you are using the new definition of J of theta, with the regularization objective, then this new formula for theta is the one that we give you, the global minimum of J of theta.

Normal equation

$$X = \begin{bmatrix} (x^{(1)})^T \\ \vdots \\ (x^{(m)})^T \end{bmatrix} \quad m \times (n+1)$$

$$y = \begin{bmatrix} y^{(1)} \\ \vdots \\ y^{(m)} \end{bmatrix} \quad \mathbb{R}^m$$

$$\min_\theta J(\theta)$$

$$\theta = \left(X^TX + \lambda\begin{bmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & \ddots \\ & & & & 1 \end{bmatrix}\right)^{-1}X^Ty$$

$(n+1)\times(n+1)$

e.g. $n=2$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\frac{\partial}{\partial\theta_j}J(\theta) \overset{\text{set}}{=} 0$ for all j

gives you the global minimum of $J(\theta)$

Non-invertibility (optional/advanced).

Suppose $m \leq n$,
(#examples) (#features)

$$\theta = (X^T X)^{-1} X^T y$$

non-invertible / singular

pinv inv

If $\lambda > 0$,

$$\theta = \left(X^T X + \lambda \begin{bmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & \ddots \\ & & & & 1 \end{bmatrix} \right)^{-1} X^T y$$

invertible.

So finally I want to just quickly describe the issue of non-invertibility. This is relatively advanced material, so you should consider this as optional. And feel free to skip it, or if you listen to it and positive it doesn't really make sense, don't worry about it either. But earlier when I talked about the normal equation method, we also had an optional video on the non-invertibility issue. So this is another optional part to this, sort of an add-on to that earlier optional video on non-invertibility. Now, consider a setting where m, the number of examples, is less than or equal to n, the number of features. If you have fewer examples than features, than this matrix, X transpose X will be non-invertible, or singular. Or the other term for this is the matrix will be degenerate. And if you implement this in Octave anyway and you use the pinv function to take the pseudo inverse, it will kind of do the right thing, but it's not clear that it would give you a very good hypothesis, even though numerically the Octave pinv function will give you a result that kinda makes sense.

But if you were doing this in a different language, and if you were taking just the regular inverse, which in Octave denoted with the function inv, we're trying to take the regular inverse of X transpose X. Then in this setting, you find that X transpose X is singular, is non-invertible, and if you're doing this in different program language and using some linear algebra library to try to take the inverse of this matrix, it just might not work because that matrix is non-invertible or singular. Fortunately, regularization also takes care of this for us. And concretely, so long as the regularization parameter lambda is strictly greater than 0, it is actually possible to prove that this matrix, X transpose X plus lambda times this funny matrix here, it is possible to prove that this matrix will not be singular and that this matrix will be invertible. So using regularization also takes care of any non-invertibility issues of the X transpose X matrix as well. So you now know how to implement regularized linear regression. Using this you'll be able to avoid overfitting even if you have lots of features in a relatively small training set. And this should let you get linear regression to work much better for many problems. In the next video we'll take this regularization idea and apply it to logistic regression. So that you'd be able to get logistic regression to avoid overfitting and perform much better as well.

Note 8 [So important] ***

Note: [8:43 - It is said that X is non-invertible if m ≤ n. The correct statement should be that X is non-invertible if m < n, and may be non-invertible if m = n.

Solving the Problem of Overfitting

Regularized Logistic Regression

So, here's the idea. We saw earlier that Logistic Regression can also be prone to overfitting if you fit it with a very, sort of, high order polynomial features like this. Where G is the sigmoid function and in particular you end up with a hypothesis, you know, whose decision bound to be just sort of an overly complex and extremely contortive function that really isn't such a great hypothesis for this training set, and more generally if you have logistic regression with a lot of features. Not necessarily polynomial ones, but just with a lot of features you can end up with overfitting.

This was our cost function for logistic regression. And if we want to modify it to use regularization, all we need to do is add to it the following term plus lambda over 2M, sum from J equals 1, and as usual sum from J equals 1. Rather than the sum from J equals 0, of theta J squared. And this has to effect therefore, of penalizing the parameters theta 1 theta 2 and so on up to theta N from being too large.

And if you do this,

then it will the have the effect that even though you're fitting a very high order polynomial with a lot of parameters. So long as you apply regularization and keep the parameters small you're more likely to get a decision boundary

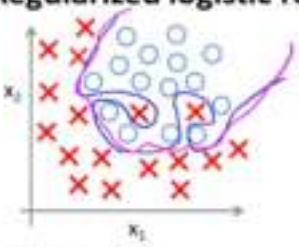
You know, that maybe looks more like this. It looks more reasonable for separating

the positive and the negative examples.

So, when using regularization

even when you have a lot of features, the regularization can help take care of the overfitting problem.

Regularized logistic regression.



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_1^2 + \theta_3 x_1^2 x_2 + \theta_4 x_1^2 x_2^2 + \theta_5 x_1^2 x_2^3 + \dots)$$

Cost function:

$$J(\theta) = -\left[\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)}))\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} \theta_j^2$$

$\theta_1, \theta_2, \dots \theta_n$

How do we actually implement this ?

Gradient descent

Repeat {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m}\sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_j := \theta_j - \alpha \left[\frac{1}{m}\sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)} + \frac{\lambda}{m}\theta_j\right] \quad (j = 1, 2, 3, \dots, n)$$

$\theta_1, \dots \theta_n$

}

$\frac{\partial}{\partial \theta_j} J(\theta)$

$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$

How do we actually implement this? Well, for the original gradient descent algorithm, this was the update we had. We will repeatedly perform the following update to theta J. This slide looks a lot like the previous one for linear regression. But what I'm going to do is write the update for theta 0 separately. So, the first line is for update for theta 0 and a second line is now my update for theta 1 up to theta N. Because I'm going to treat theta 0 separately. And in order to modify this algorithm, to use

a regularized cos function, all I need to do is pretty similar to what we did for linear regression is actually to just modify this second update rule as follows.

And, once again, this, you know, cosmetically looks identical what we had for linear regression. But of course is not the same algorithm as we had, because now the hypothesis is defined using this. So this is not the same algorithm as regularized linear regression. Because the hypothesis is different. Even though this update that I wrote down, it actually looks cosmetically the same as what we had earlier. We're working out gradient descent for regularized linear regression.

And of course, just to wrap up this discussion, this term here in the square brackets, so this term here, this term is, of course, the new partial derivative for respect of theta J of the new cost function J of theta. Where J of theta here is the cost function we defined on a previous slide that does use regularization.

So, that's gradient descent for regularized linear regression.

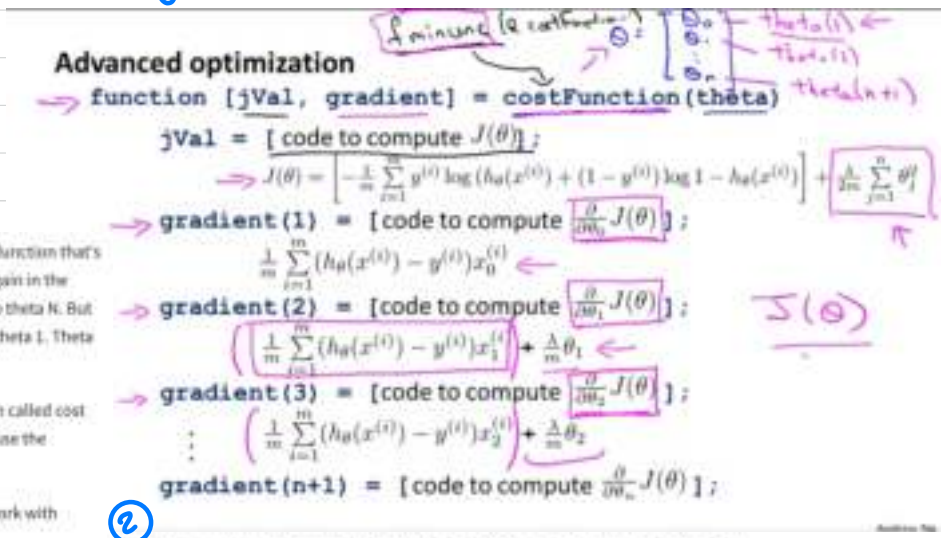
Question

When using regularized logistic regression, which of these is the best way to monitor whether gradient descent is working correctly?

- Plot $-\frac{1}{m}\sum_{i=1}^{m} [y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))]$ as a function of the number of iterations and make sure it's decreasing
- Plot $-\frac{1}{m}\sum_{i=1}^{m} [y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))] - \frac{\lambda}{2m}\sum_{j=1}^{n} \theta_j^2$ as a function of the number of iterations and make sure it's decreasing
- Plot $-\frac{1}{m}\sum_{i=1}^{m} [y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))] + \frac{\lambda}{2m}\sum_{j=1}^{n} \theta_j^2$ as a function of the number of iterations and make sure it's decreasing
- Plot $\sum_{j=1}^{n} \theta_j^2$ as a function of the number of iterations and make sure it's decreasing

✓ Correct

Let's talk about how to get regularized linear regression to work using the more advanced optimization methods.



① And just to remind you for those methods what we needed to do was to define the function that's called the cost function, that takes as input the parameter vector theta and once again in the equations we've been writing here we used 0 index vectors. So we had theta 0 up to theta N. But because Octave indexes the vectors starting from 1. Theta 0 is written in Octave as theta 1. Theta 1 is written in Octave as theta 2, and so on down to theta

N plus 1. And what we needed to do was provide a function. Let's provide a function called cost function that we would then pass in to what we have, what we saw earlier. We will use the fminunc and then you know at cost function,

and so on, right. But the F min, u and c was the F min unconstrained and this will work with fminunc was what will take the cost function and minimize it for us.

So the two main things that the cost function needed to return were first J-val. And for that, we need to write code to compute the cost function J of theta.

Now, when we're using regularized logistic regression, of course the cost function j of theta changes and, in particular,

now a cost function needs to include this additional regularization term at the end as well. So, when you compute j of theta be sure to include that term at the end.

And then, the other thing that this cost function thing needs to derive with a gradient. So gradient one needs to be set to the partial derivative of J of theta with respect to theta zero, gradient two needs to be set to that, and so on. Once again, the index is off by one. Right, because of the indexing from one Octave users.

And looking at these terms.

This term over here. We actually worked this out on a previous slide is actually equal to this. It doesn't change. Because the derivative for theta zero doesn't change. Compared to the version without regularization.

And the other terms do change. And in particular the derivative respect to theta one. We worked this out on the previous slide as well. Is equal to, you know, the original term and then minus lander M times theta 1. Just so we make sure we pass this correctly. And we can add parentheses here. Right, so the summation doesn't extend. And similarly, you know, this other term here looks like this, with this additional term that we had on the previous slide, that corresponds to the gradient from their regularization objective. So if you implement this cost function and pass this into fminunc or to one of those advanced optimization techniques, that will minimize the new regularized cost function J of theta.

And the parameters you get out

② will be the ones that correspond to logistic regression with regularization.

So, now you know how to implement regularized logistic regression.

When I walk around Silicon Valley, I live here in Silicon Valley, there are a lot of engineers that are frankly, making a ton of money for their companies using machine learning algorithms.

And I know we've only been, you know, studying this stuff for a little while. But if you understand linear

regression, the advanced optimization algorithms and regularization, by now, frankly, you probably know quite a lot more machine learning than many, certainly now, but you probably know quite a lot more machine learning right now than frankly, many of the Silicon Valley engineers out there having very successful careers. You know, making tons of money for the companies. Or building products using machine learning algorithms.

So, congratulations.

You've actually come a long ways. And you can actually, you actually know enough to apply this stuff and get to work for many problems.

So congratulations for that. But of course, there's still a lot more that we want to teach you, and in the next set of videos after this, we'll start to talk about a very powerful cause of non-linear classifier. So whereas linear regression, logistic regression, you know, you can form polynomial terms, but it turns out that there are much more powerful nonlinear quantifiers that can then sort of polynomial regression. And in the next set of videos after this one, I'll start telling you about them. So that you have even more powerful learning algorithms than you have now to apply to different problems.

Algorithem	Hypothesis Function	Cost Function	Gradient Descent
Linear Regression	$\hat{y} = h_\theta(x) = \theta_0 + \theta_1 x$	$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(\hat{y}_i - y_i)^2 = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x_i) - y_i)^2$	$\theta_j := \theta_j - \alpha\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$ (simultaneously update θ_j for all j)
Linear Regression with Multiple variables	$\hat{y} = h_\theta(x) = \theta_0 + \theta_1 x$	$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(\hat{y}_i - y_i)^2 = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x_i) - y_i)^2$	repeat until convergence: { $\theta_j := \theta_j - \alpha\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$ for $j := 0...n$ }
Logistic Regression	$h_\theta(x) = g(\theta^T x)$	$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}[y^{(i)}\log(h_\theta(x^{(i)})) + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))]$ matlab ← $J(\theta) = \frac{1}{m}\sum_{i=1}^{m}[-y^{(i)}\log(h_\theta(x^{(i)})) - (1-y^{(i)})\log(1-h_\theta(x^{(i)}))]$	repeat : { $\theta_j := \theta_j - \frac{\alpha}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$ } $\frac{\partial J(\theta)}{\partial \theta_j} = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$ → matlab
Logistic Regression with Multiple Variable		$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}[y^{(i)}\log(h_\theta(x^{(i)})) + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$	Repeat : { $\theta_0 := \theta_0 - \alpha\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_0^{(i)}$ $\theta_j := \theta_j - \alpha\left[\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)} + \frac{\lambda}{m}\theta_j\right]$ $j = 1,2,...,n$ }
Nural Networks		$-\frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{K}[y_k^{(i)}\log((h_\Theta(x^{(i)}))_k) + (1-y_k^{(i)})\log(1-(h_\Theta(x^{(i)}))_k)] + \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(\Theta_{ji}^{(l)})^2$	

Regularized logistic Regression8

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left[-y^{(i)}\log(h_\theta(x^{(i)})) - (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)} \qquad \text{for } j = 0,$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = \left(\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}\right) + \frac{\lambda}{m}\theta_j \qquad \text{for } j \geq 1,$$

نکات جمع بندی logistic Regression

- ☐ The one-vs-all technique allows you to use logistic regression for problems in which each $y^{(i)}$ comes from a fixed, discrete set of values.

  If each $y^{(i)}$ is one of k different values, we can give a label to each $y^{(i)} \in \{1, 2, \ldots, k\}$ and use one-vs-all as described in the lecture.

- ☐ The cost function $J(\theta)$ for logistic regression trained with $m \geq 1$ examples is always greater than or equal to zero.

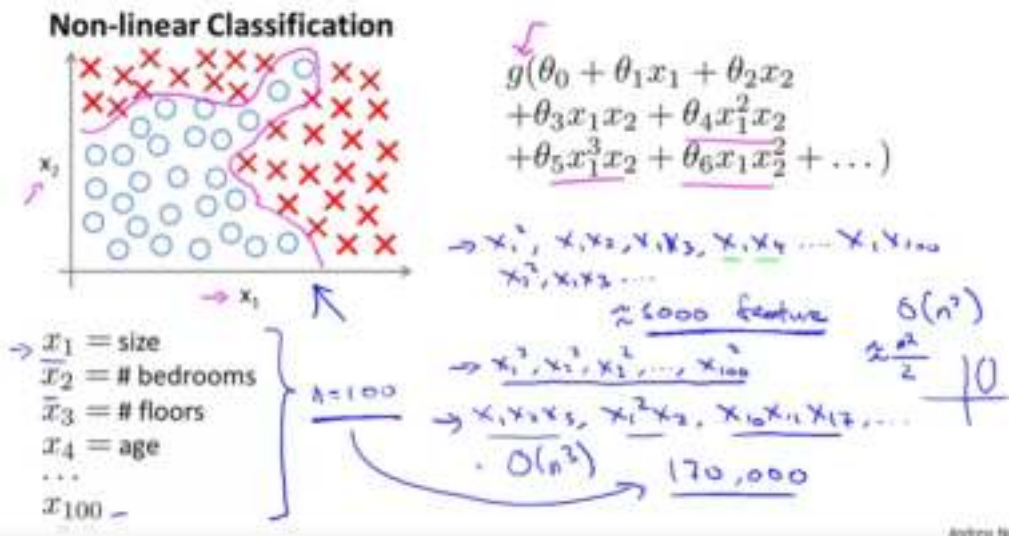
  The cost for any example $x^{(i)}$ is always ≥ 0 since it is the negative log of a quantity less than one. The cost function $J(\theta)$ is a summation over the cost for each sample, so the cost function itself must be greater than or equal to zero.

Week 4 8

Motivations

Non-Linear Hypotheses

In this and in the next set of videos, I'd like to tell you about a learning algorithm called a Neural Network.We're going to first talk about the representation and then in the next set of videos talk about learning algorithms for it.Neutral networks is actually a pretty old idea, but had fallen out of favor for a while.But today, it is the state of the art technique for many different machine learning problems.So why do we need yet another learning algorithm? We already have linear regression and we have logistic regression, so why do we need, you know, neural networks?
In order to motivate the discussion of neural networks, let me start by showing you a few examples of machine learning problems where we need to learn complex non-linear hypotheses.



Consider a supervised learning classification problem where you have a training set like this. If you want to apply logistic regression to this problem, one thing you could do is apply logistic regression with a lot of nonlinear features like that. So here, g as usual is the sigmoid function, and we can include lots of polynomial terms like these.And, if you include enough polynomial terms then, you know, maybe you can get a hypotheses that separates the positive and negative examples. This particular method works well when you have only, say, two features - x1 and x2- because you can then include all those polynomial terms of x1 and x2. But for many interesting machine learning problems would have a lot more features than just two.

We've been talking for a while about housing prediction, and suppose you have a housing classification problem rather than a regression problem, like maybe if you have different features of a house, and you want to predict what are the odds that your house will be sold within the next six months, so that will be a classification problem.
And as we saw we can come up with quite a lot of features, maybe a hundred different features of different houses.For a problem like this, if you were to include all the quadratic terms, all of these, even all of the quadratic that is the second or the polynomial terms, there would be a lot of them.There would be terms like x1 squared,x1x2, x1x3, you know, x1x4 up to x1x100 and then you have x2 squared, x2x3 and so on.And if you include just the second order terms, that is, the terms that are a product of, you know, two of these terms, x1 times x1 and so on, then, for the case of n equals 100, you end up with about five thousand features.

And, asymptotically, the number of quadratic features grows roughly as order n squared, where n is the number of the original features,like x1 through x100 that we had.And its actually closer to n squared over two.So including all the quadratic features doesn't seem like it's maybe a good idea, because that is a lot of features and you might up overfitting the training set, and it can also be computationally expensive, you know, to be working with that many features.

One thing you could do is include only a subset of these, so if you include only the features x1 squared, x2 squared, x3 squared, up to maybe x100 squared, then the number of features is much smaller.Here you have only 100 such quadratic features, but this is not enough features and certainly won't let you fit the data set like that on the upper left. In fact, if you include only these quadratic features together with the original x1, and so on, up to x100 features, then you can actually fit very interesting hypotheses. So, you can fit things like, you know, access a line of the ellipses like these, but you certainly cannot fit a more complex data set like that shown here. So 5000 features seems like a lot, if you were to include the cubic, or third order known of each others,the x1, x2, x3. You know, x1 squared,x2, x10 and x11, x17 and so on.You can imagine there are gonna be a lot of these features.In fact, they are going to be order and cube such features and if any is 100 you can compute that, you end up with on the order of about 170,000 such cubic features and so including these higher auto-polynomial features when your original feature set end is large this really dramatically blows up your feature space and this doesn't seem like a good way to come up with additional features with which to build none many classifiers when n is large.

For many machine learning problems, n will be pretty large. Here's an example.

Let's consider the problem of computer vision.

And suppose you want to use machine learning to train a classifier to examine an image and tell us whether or not the image is a car.

Many people wonder why computer vision could be difficult. I mean when you and I look at this picture it is so obvious what this is. You wonder how is it that a learning algorithm could possibly fail to know what this picture is.

To understand why computer vision is hard let's zoom into a small part of the image like that area where the little red rectangle is. It turns out that where you and I see a car, the computer sees that. What it sees is this matrix, or this grid, of pixel intensity values that tells us the brightness of each pixel in the image. So the computer vision problem is to look at this matrix of pixel intensity values, and tell us that these numbers represent the door handle of a car.

Concretely, when we use machine learning to build a car detector, what we do is we come up with a label training set, with, let's say, a few label examples of cars and a few label examples of things that are not cars, then we give our training set to the learning algorithm trained a classifier and then, you know, we may test it and show the new image and ask, "What is this new thing?".

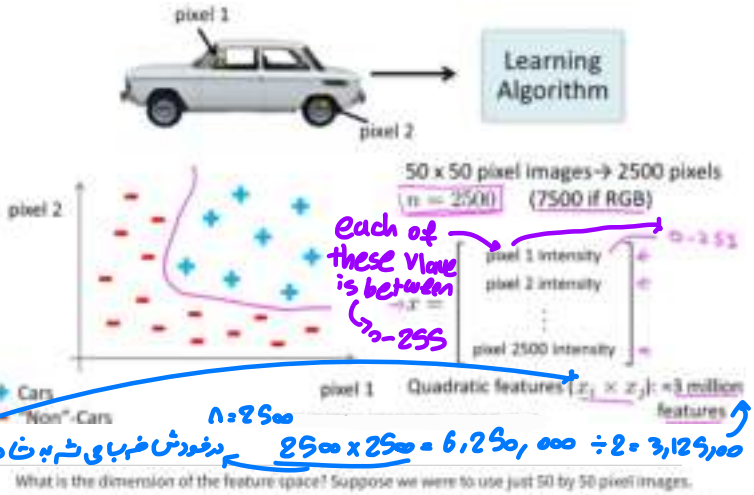
And hopefully it will recognize that that is a car.

To understand why we need nonlinear hypotheses, let's take a look at some of the images of cars and maybe non-cars that we might feed to our learning algorithm.

Let's pick a couple of pixel locations in our images, so that's pixel one location and pixel two location, and let's plot this car, you know, at the location, at a certain point, depending on the intensities of pixel one and pixel two.

And let's do this with a few other images. So let's take a different example of the car and you know, look at the same two pixel locations

and that image has a different intensity for pixel one and a different intensity for pixel two. So, it ends up at a different location on the figure. And then let's plot some negative examples as well. That's a non-car, that's a non-car . And if we do this for more and more examples using the pluses to denote cars and minuses to denote non-cars, what we'll find is that the cars and non-cars end up lying in different regions of the space, and what we need therefore is some sort of non-linear hypotheses to try to separate out the two classes.



What is the dimension of the feature space? Suppose we were to use just 50 by 50 pixel images. Now that suppose our images were pretty small ones, just 50 pixels on the side. Then we would have 2500 pixels,

and so the dimension of our feature size will be N equals 2500 where our feature vector x is a list of all the pixel testings, you know, the pixel brightness of pixel one, the brightness of pixel two, and so on down to the pixel brightness of the last pixel where, you know, in a typical computer representation, each of these may be values between say 0 to 255 if it gives us the grayscale value. So we have n equals 2500, and that's if we were using grayscale images. If we were using RGB images with separate red, green and blue values, we would have n equals 7500.

So, if we were to try to learn a nonlinear hypothesis by including all the quadratic features, that is all the terms of the form, you know, Xi times Xj, while with the 2500 pixels we would end up with a total of three million features. And that's just too large to be reasonable; the computation would be very expensive to find and to represent all of these three million features per training example.

So, simple logistic regression together with adding in maybe the quadratic or the cubic features - that's just not a good way to learn complex nonlinear hypotheses when n is large because you just end up with too many features. In the next few videos, I would like to tell you about Neural Networks, which turns out to be a much better way to learn complex hypotheses, complex nonlinear hypotheses even when your input feature space, even when n is large. And along the way I'll also get to show you a couple of fun videos of historically important applications

of Neural networks as well that I hope these videos that we'll see later will be fun for you to watch as well.

Motivations

Neurons and the Brain

Neural Networks are a pretty old algorithm that was originally motivated by the goal of having machines that can mimic the brain.Now in this class, of course I'm teaching Neural Networks to you because they work really well for different machine learning problems and not, certainly not because they're logically motivated.In this video, I'd like to give you some of the background on Neural Networks.So that we can get a sense of what we can expect them to do.Both in the sense of applying them to modern day machinery problems, as well as for those of you that might be interested in maybe the big AI dream of someday building truly intelligent machines.
Also, how Neural Networks might pertain to that.

The origins of Neural Networks was as algorithms that try to mimic the brain and those a sense that if we want to build learning systems while why not mimic perhaps the most amazing learning machine we know about, which is perhaps the brain. Neural Networks came to be very widely used throughout the 1980's and 1990's and for various reasons as popularity diminished in the late 90's. But more recently, Neural Networks have had a major recent resurgence.

One of the reasons for this resurgence is that Neural Networks are computationally some what more expensive algorithm and so, it was only, you know, maybe somewhat more recently that computers became fast enough to really run large scale Neural Networks and because of that as well as a few other technical reasons which we'll talk about later, modern Neural Networks today are the state of the art technique for many applications.

Neural Networks

- Origins: Algorithms that try to mimic the brain.
- Was very widely used in 80s and early 90s; popularity diminished in late 90s.
- Recent resurgence: State-of-the-art technique for many applications

So, when you think about mimicking the brain while one of the human brain does tell me same things, right? The brain can learn to see process images than to hear, learn to process our sense of touch. We can, you know, learn to do math, learn to do calculus, and the brain does so many different and amazing things. It seems like if you want to mimic the brain it seems like you have to write lots of different pieces of software to mimic all of these different fascinating, amazing things that the brain tell us, but does is this fascinating hypothesis that the way the brain does all of these different things is not worth like a thousand different programs, but instead, the way the brain does it is worth just a single learning algorithm. This is just a hypothesis but let me share with you some of the evidence for this. This part of the brain, that little red part of the brain, is your auditory cortex and the way you're understanding my voice now is your ear is taking the sound signal and routing the sound signal to your auditory cortex and that's what's allowing you to understand my words.

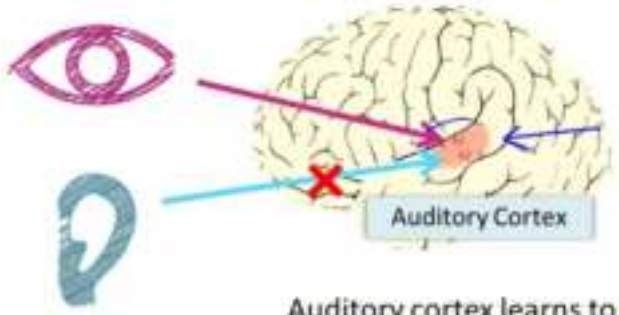
Neuroscientists have done the following fascinating experiments where you cut the wire from the ears to the auditory cortex and you re-wire,

in this case an animal's brain, so that the signal from the eyes to the optic nerve eventually gets routed to the auditory cortex.

If you do this it turns out, the auditory cortex will learn

to see. And this is in every single sense of the word see as we know it. So, if you do this to the animals, the animals can perform visual discrimination task and as they can look at images and make appropriate decisions based on the images and they're doing it with that piece of brain tissue.

The "one learning algorithm" hypothesis



Auditory cortex learns to see

Here's another example.

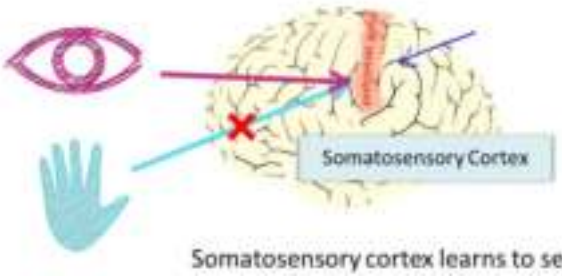
That red piece of brain tissue is your somatosensory cortex. That's how you process your sense of touch. If you do a similar re-wiring process

then the somatosensory cortex will learn to see. Because of this and other similar experiments, these are called neuro-rewiring experiments.

There's this sense that if the same piece of physical brain tissue can process sight or sound or touch then maybe there is one learning algorithm that can process sight or sound or touch. And instead of needing to implement a thousand different programs or a thousand different algorithms to do, you know, the thousand wonderful things that the brain does, maybe what we need to do is figure out some approximation or to whatever the brain's learning algorithm is and implement that and that the brain learned by itself how to process these different types of data.

To a surprisingly large extent, it seems as if we can plug in almost any sensor to almost any part of the brain and so, within the reason, the brain will learn to deal with it.

The "one learning algorithm" hypothesis



Somatosensory cortex learns to see

① Here are a few more examples. On the upper left is an example of learning to see with your tongue. The way it works is--this is actually a system called BrainPort undergoing FDA trials now to help blind people see--but the way it works is, you strap a grayscale camera to your forehead, facing forward, that takes the low resolution grayscale image of what's in front of you and you then run a wire

to an array of electrodes that you place on your tongue so that each pixel gets mapped to a location on your tongue where maybe a high voltage corresponds to a dark pixel and a low voltage corresponds to a bright pixel and, even as it does today, with this sort of system you and I will be able to learn to see, you know, in tens of minutes with our tongues. Here's a second example of human echo location or human sonar.

So there are two ways you can do this. You can either snap your fingers,

or click your tongue. I can't do that very well. But there are blind people today that are actually being trained in schools to do this and learn to interpret the pattern of sounds bouncing off your environment - that's sonar. So, if after you search on YouTube, there are actually videos of this amazing kid who tragically because of cancer had his eyeballs removed, so this is a kid with no eyeballs. But by snapping his fingers, he can walk around and never hit anything. He can ride a skateboard. He can shoot a basketball into a hoop and this is a kid with no eyeballs.

② Third example is the Haptic Belt where if you have a strap around your waist, ring up buzzers and always have the northmost one buzzing. You can give a human a direction sense similar to maybe how birds can, you know, sense where north is. And, some of the bizarre example, but if you plug a third eye into a frog, the frog will learn to use that eye as well.

So, it's pretty amazing to what extent is as if you can plug in almost any sensor to the brain and the brain's learning algorithm will just figure out how to learn from that data and deal with that data.

And there's a sense that if we can figure out what the brain's learning algorithm is, and, you know, implement it or implement some approximation to that algorithm on a computer, maybe that would be our best shot at, you know, making real progress towards the AI, the artificial intelligence dream of someday building truly intelligent machines.

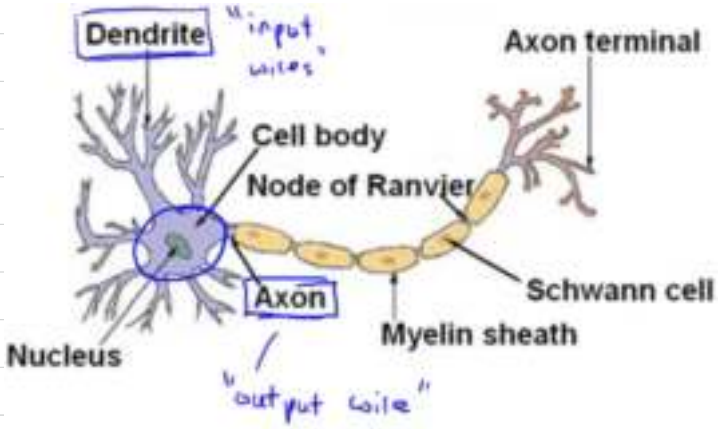
Now, of course, I'm not teaching Neural Networks, you know, just because they might give us a window into this far-off AI dream, even though I'm personally, that's one of the things that I personally work on in my research life. But the main reason I'm teaching Neural Networks in this class is because it's actually a very effective state of the art technique for modern day machine learning applications. So, in the next few videos, we'll start diving into the technical details of Neural Networks so that you can apply them to modern-day machine learning applications and get them to work well on problems. But for me, you know, one of the reasons the excite me is that maybe they give us this window into what we might do if we're also thinking of what algorithms might someday be able to learn in a manner similar to humankind.

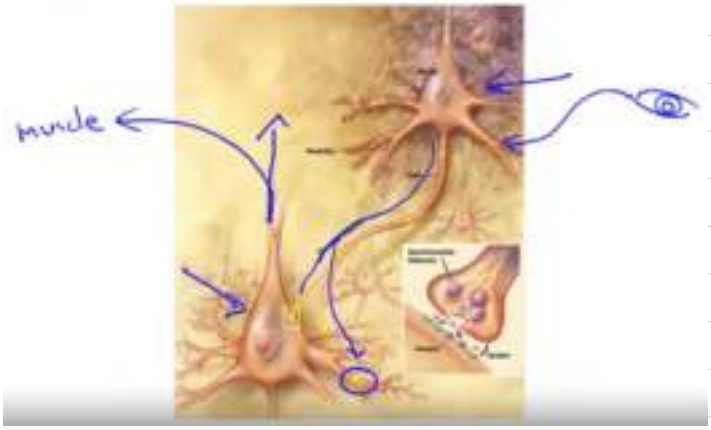
Neural Network

Model Representation I

Neurons in the brain

In this video, I want to start telling you about how we represent neural networks. In other words, how we represent our hypothesis or how we represent our model when using neural networks. Neural networks were developed as simulating neurons or networks of neurons in the brain. So, to explain the hypothesis representation let's start by looking at what a single neuron in the brain looks like. Your brain and mine is jam packed full of neurons like these and neurons are cells in the brain. And two things to draw attention to are that first, the neuron has a cell body, like so, and moreover, the neuron has a number of input wires, and these are called the dendrites. You think of them as input wires, and these receive inputs from other locations. And a neuron also has an output wire called an Axon, and this output wire is what it uses to send signals to other neurons, so to send messages to other neurons. So, at a simplistic level what a neuron is, is a computational unit that gets a number of inputs through it input wires and does some computation and then it says outputs via its axon to other nodes or to other neurons in the brain.



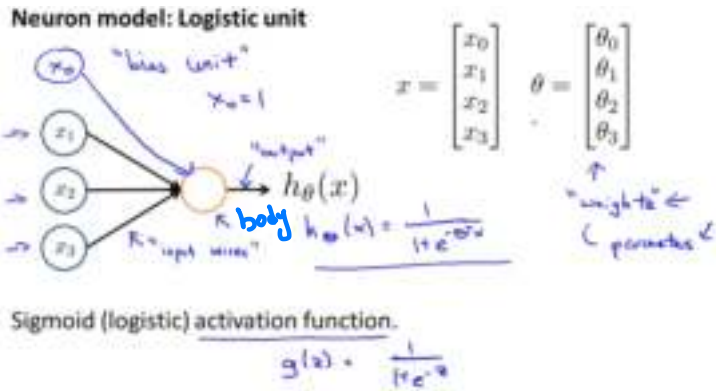


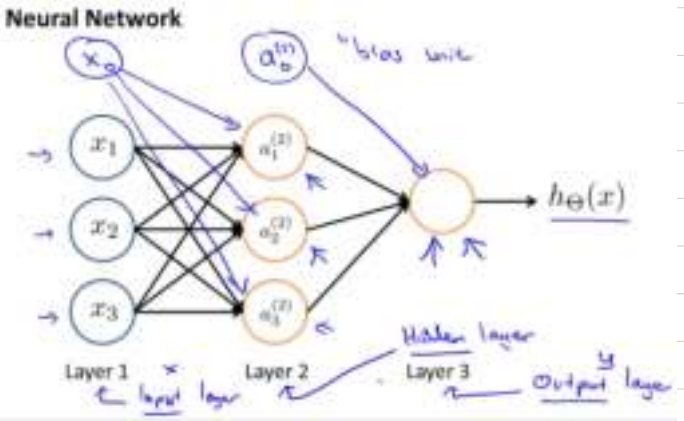
Here's a illustration of a group of neurons. The way that neurons communicate with each other is with little pulses of electricity, they are also called spikes but that just means pulses of electricity. So here is one neuron and what it does is if it wants a send a message what it does is sends a little pulse of electricity. Varis axon to some different neuron and here, this axon that is this open wire, connects to the dendrites of this second neuron over here, which then accepts this incoming message that some computation. And they, in turn, decide to send out this message on this axon to other neurons, and this is the process by which all human thought happens. It's these Neurons doing computations and passing messages to other neurons as a result of what other inputs they've got. And, by the way, this is how our senses and our muscles work as well. If you want to move one of your muscles the way that where else in your neuron may send this electricity to your muscle and that causes your muscles to contract and your eyes, some senses like your eye must send a message to your brain while it does it senses hosts electricity entity to a neuron in your brain like so. In a neuro network, or rather, in an artificial neuron network that we've

In a neuro network, or rather, in an artificial neuron network that we've implemented on the computer, we're going to use a very simple model of what a neuron does we're going to model a neuron as just a logistic unit.So, when I draw a yellow circle like that, you should think of that as a playing a role analysis, who's maybe the body of a neuron, and we then feed the neuron a few inputs who's various dendrites or input wiles.And the neuron does some computation.And output some value on this output wire, or in the biological neuron, this is an axon.And whenever I draw a diagram like this,what this means is that this represents a computation of h of x equals one over one plus e to the negative theta transpose x,where as usual, x and theta are our parameter vectors, like so.So this is a very simple, maybe a vastly oversimplified model,of the computations that the neuron does, where it gets a number of inputs, x1,x2, x3 and it outputs some value computed like so.When I draw a neural network, usually I draw only the input nodes x1, x2, x3.Sometimes when it's useful to do so, I'll draw an extra node for x0.This x0 now that's sometimes called the bias unit or the bias neuron, but because x0 is already equal to 1, sometimes, I draw this, sometimes I won't just depending on whatever is more notationally convenient for that example. Finally, one last bit of terminology when we talk about neural networks,sometimes we'll say that this is a neuron or an artificial neuron with a Sigmoid or logistic activation function.So this activation function in the neural network terminology.This is just another term for that function for that non-linearity g(z) = 1 over 1+e to the -z.And whereas so far I've been calling theta the parameters of the model,I'll mostly continue to use that terminology.

Here, it's a copy to the parameters, but in neural networks, in the neural network literature sometimes you might hear people talk about weights of a model and weights just means exactly the same thing as parameters of a model.But I'll mostly continue to use the terminology parameters in these videos,but sometimes, you might hear others use the weights terminology.

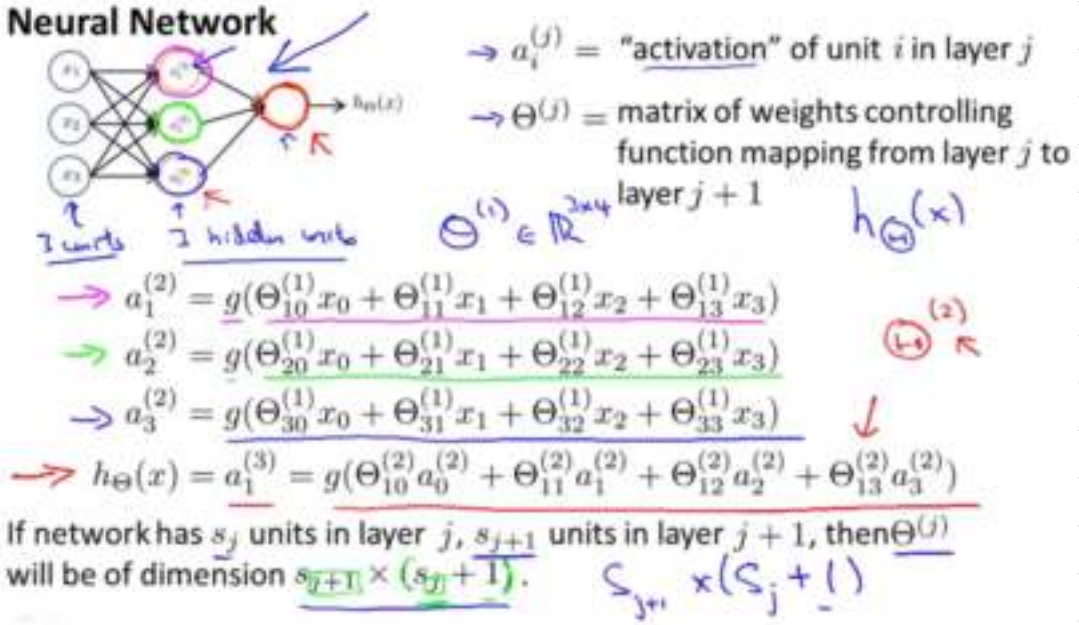
So, this little diagram represents a single neuron.





What a neural network is, is just a group of this different neurons strong together. Completely, here we have input units x1, x2, x3 and once again, sometimes you can draw this extra note x0 and Sometimes not, just flow that in here. And here we have three neurons which have written 81, 82, 83. I'll talk about those indices later. And once again we can if we want add in just a0 and add the mixture bias unit there. There's always a value of 1. And then finally we have this third node and the final layer, and there's this third node that outputs the value that the hypothesis h(x) computes. To introduce a bit more terminology, in a neural network, the first layer, this is also called the input layer because this is where we input our features, x1, x2, x3. The final layer is also called the output layer because that layer has a neuron, this one over here, that outputs the final value computed by a hypothesis. And then, layer 2 in between, this is called the hidden layer. The term hidden layer isn't a great terminology, but this ideation is that, you know, you supervised early, where you get to see the inputs and get to see the correct outputs, where there's a hidden layer of values you don't get to observe in the training setup. It's not x, and it's not y, and so we call those hidden. And they try to see neural nets with more than one hidden layer but in this example, we have one input layer, Layer 1, one hidden layer, Layer 2, and one output layer, Layer 3. But basically, anything that isn't an input layer and isn't an output layer is called a hidden layer.

So I want to be really clear about what this neural network is doing. Let's step through the computational steps that are and body represented by this diagram. ~~To explain these specific~~



computational steps that are and body represented by this diagram. To explain these specific computations represented by a neural network, here's a little bit more notation. I'm going to use a superscript j subscript i to denote the activation of neuron i or of unit i in layer j. So completely this gave superscript to sub group one, that's the activation of the first unit in layer two, in our hidden layer. And by activation I just mean the value that's computed by and as output by a specific. In addition, new network is parametrize by these matrixes, theta super script j Where theta j is going to be a matrix of weights controlling the function mapping form one layer, maybe the first layer to the second layer, or from the second layer to the third layer.

So here are the computations that are represented by this diagram.

This first hidden unit here has it's value computed as follows, there's a is a21 is equal to the sigma function of the sigma activation function, also called the logistics activation function, apply to this sort of linear combination of these inputs. And then this second hidden unit has this activation value computer as sigmoid of this. And similarly for this third hidden unit is computed by that formula. So here we have 3 theta 1 which is matrix of parameters governing our mapping from our three different units, our hidden units. Theta 1 is going to be a 3.

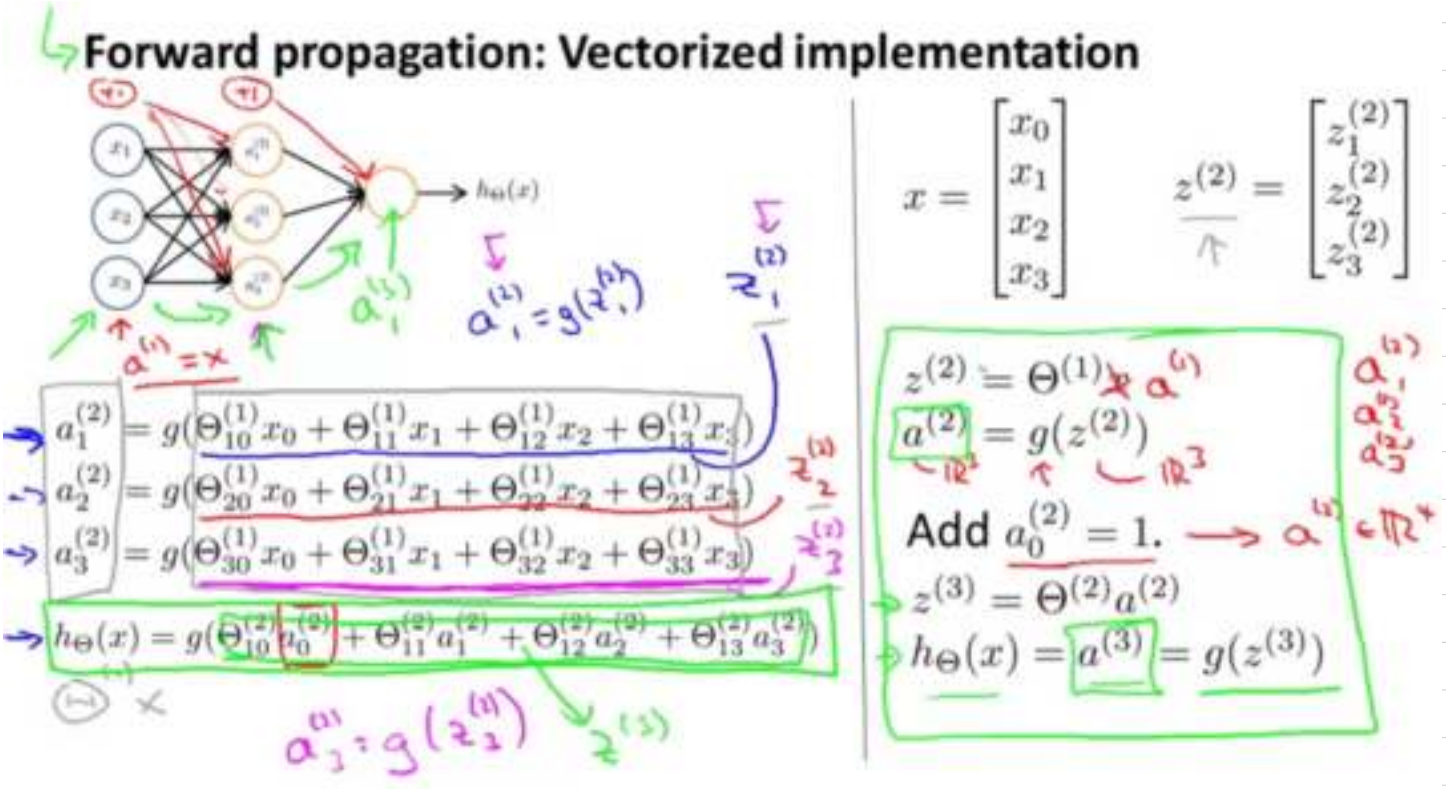
Theta 1 is going to be a 3x4-dimensional matrix. And more generally, if a network has SJU units in there j and sj + 1 units and sj + 1 then the matrix theta j which governs the function mapping from there sj + 1. That will have to mention sj +1 by sj + 1 I'll just be clear about this notation right. This is Subscript j + 1 and that's s subscript j, and then this whole thing, plus 1, this whole thing (sj + 1), okay? So that's s subscript j + 1 by,

So that's s subscript j + 1 by sj + 1 where this plus one is not part of the subscript. Okay, so we talked about what the three hidden units do to compute their values. Finally, there's a loss of this final and after that we have one more unit which computer h of x and that's equal can also be written as a(3)1 and that's equal to this. And you notice that I've written this with a superscript two here, because theta of superscript two is the matrix of parameters, or the matrix of weights that controls the function that maps from the hidden units, that is the layer two units to the one layer three unit, that is the output unit. To summarize, what we've done is shown how a picture like this over here defines an artificial neural network which defines a function h that maps with x's input values to hopefully to some space that provisions y. And these hypothesis are parameterized by parameters denoting with a capital theta so that, as we vary theta, we get different hypothesis and we get different functions. Mapping say from x to y.

So this gives us a mathematical definition of how to represent the hypothesis in the neural network. In the next few videos what I would like to do is give you more intuition about what these hypothesis representations do, as well as go through a few examples and talk about how to compute them efficiently.

Neural network

Model Representaion II



①

In the last video, we gave a mathematical definition of how to represent or how to compute the hypotheses used by Neural Network.

In this video, I like show you how to actually carry out that computation efficiently, and that is show you a vector rise implementation.

And second, and more importantly, I want to start giving you intuition about why these neural network representations might be a good idea and how they can help us to learn complex nonlinear hypotheses.

Consider this neural network. Previously we said that the sequence of steps that we need in order to compute the output of a hypotheses is these equations given on the left where we compute the activation values of the three hidden uses and then we use those to compute the final output of our hypotheses h of x. Now, I'm going to define a few extra terms. So, this term that I'm underlining here, I'm going to define that to be z superscript 2 subscript 1. So that we have that a(2)1, which is this term is equal to g of z to 1. And by the way, these superscript 2, you know, what that means is that the z2 and this a2 as well, the superscript 2 in parentheses means that these are values associated with layer 2, that is with the hidden layer in the neural network.

Now this term here I'm going to similarly define as

z(2)2. And finally, this last term here that I'm underlining,

let me define that as z(2)3. So that similarly we have a(2)3 equals g of

z(2)3. So these z values are just a linear combination, a weighted linear combination, of the input values x0, x1, x2, x3 that go into a particular neuron.

Now if you look at this block of numbers,

you may notice that that block of numbers corresponds suspiciously similar

to the matrix vector operation, matrix vector multiplication of x1 times the vector x. Using this observation we're going to be able to vectorize this computation of the neural network.

Concretely, let's define the feature vector x as usual to be the vector of x0, x1, x2, x3 where x0 as usual is always equal 1 and that defines z2 to be the vector of these z-values, you know, of z(2)1 z(2)2, z(2)3.

②

And notice that, there, z2 this is a three dimensional vector.

We can now vectorize the computation

of a(2)1, a(2)2, a(2)3 as follows. We can just write this in two steps. We can compute z2 as theta 1 times x and that would give us this vector z2; and then a2 is g of z2 and just to be clear z2 here, This is a three-dimensional vector and a2 is also a three-dimensional vector and thus this activation g. This applies the sigmoid function element-wise to each of the z2's elements. And by the way, to make our notation a little more consistent with what we'll do later, in this input layer we have the inputs x, but we can also thing it is as in activations of the first layers. So, if I defined a1 to be equal to x. So, the a1 is vector, I can now take this x here and replace this with z2 equals theta1 times a1 just by defining a1 to be activations in my input layer.

Now, with what I've written so far I've now gotten myself the values for a1, a2, a3, and really I should put the superscripts there as well. But I need one more value, which is I also want this a(0)2 and that corresponds to a bias unit in the hidden layer that goes to the output there. Of course, there was a bias unit here too that, you know, it just didn't draw under here but to take care of this extra bias unit, what we're going to do is add an extra a0 superscript 2, that's equal to one, and after taking this step we now have that a2 is going to be a four dimensional feature vector because we just added this extra, you know, a0 which is equal to 1 corresponding to the bias unit in the hidden layer. And finally,

to compute the actual value output of our hypotheses, we then simply need to compute

z3. So z3 is equal to this term here that I'm just underlining. This inner term there is z3.

And z3 is stated 2 times a2 and finally my hypotheses output h of x which is a3 that is the activation of my one and only unit in the output layer. So, that's just the real number. You can write it as a3 or as a(3)1 and that's g of z3. This process of computing h of x is also called forward propagation

and is called that because we start of with the activations of the input-units and then we sort of forward-propagate that to the hidden layer and compute the activations of the hidden layer and then we sort of forward propagate that and compute the activations of

the output layer, but this process of computing the activations from the input then the hidden then the output layer, and that's also called forward propagation

and what we just did is we just worked out a vector wise implementation of this procedure. So, if you implement it using these equations that we have on the right, these would give you an efficient way or both of the efficient way of computing h of x.

This forward propagation view also

helps us to understand what Neural Networks might be doing and why they might help us to learn interesting nonlinear hypotheses.

Consider the following neural network and let's say I cover up the left path of this picture for now. If you look at what's left in this picture. This looks a lot like logistic regression where what we're doing is we're using that note, that's just the logistic regression unit and we're using that to make a prediction h of x. And concretely, what the hypotheses is outputting is h of x is going to be equal to g which is my sigmoid activation function times theta 0 times a0 is equal to 1 plus theta 1

plus theta 2 times a2 plus theta 3 times a3 whether values a1, a2, a3 are those given by these three given units.

Now, to be actually consistent to my early notation. Actually, we need to, you know, fill in these superscript 2's here everywhere

and I also have these indices 1 there because I have only one output unit, but if you focus on the blue parts of the notation. This is, you know, this looks awfully like the standard logistic regression model, except that I now have a capital theta instead of lower case theta.

And what this is doing is just logistic regression.

But where the features fed into logistic regression are these values computed by the hidden layer.

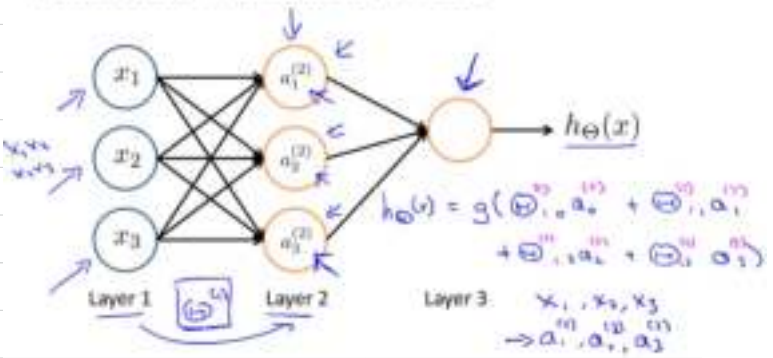
Just to say that again, what this neural network is doing is just like logistic regression, except that rather than using the original features x1, x2, x3,

is using these new features a1, a2, a3. Again, we'll put the superscripts

there, you know, to be consistent with the notation.

And the cool thing about this, is that the features a1, a2, a3, they themselves are learned as functions of the input.

Neural Network learning its own features

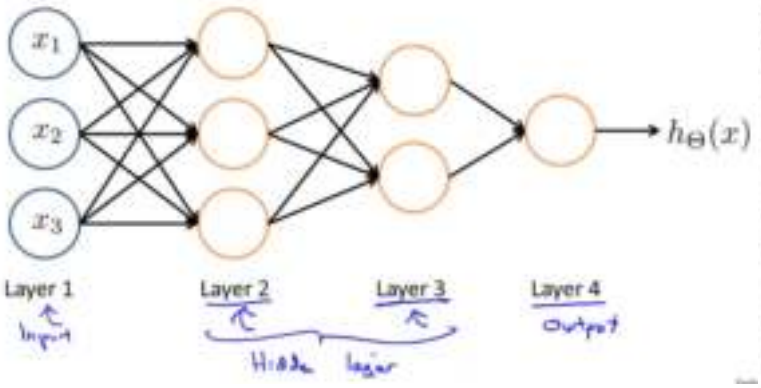


Concretely, the function mapping from layer 1 to layer 2, that is determined by some other set of parameters, theta 1. So it's as if the neural network, instead of being constrained to feed the features x1, x2, x3 to logistic regression. It gets to learn its own features, a1, a2, a3, to feed into the logistic regression and as you can imagine depending on what parameters it chooses for theta 1. You can learn some pretty interesting and complex features and therefore

you can end up with a better hypotheses than if you were constrained to use the raw features x1, x2 or x3 or if you will constrain to say choose the polynomial terms, you know, x1, x2, x3, and so on. But instead, this algorithm has the flexibility to try to learn whatever features at once, using these a1, a2, a3 in order to feed into this last unit that's essentially

a logistic regression here. I realized this example is described as a somewhat high level and so I'm not sure if this intuition of the neural network, you know, having more complex features will quite make sense yet, but if it doesn't yet in the next two videos I'm going to go through a specific example of how a neural network can use this hidden there to compute more complex features to feed into this final output layer and how that can learn more complex hypotheses. So, in case what I'm saying here doesn't quite make sense, stick with me for the next two videos and hopefully out there working through those examples this explanation will make a little bit more sense. But just the point O. You can have neural networks with other types of diagrams as well,

Other network architectures



sense. But just the point O. You can have neural networks with other types of diagrams as well, and the way that neural networks are connected, that's called the architecture. So the term architecture refers to how the different neurons are connected to each other. This is an example of a different neural network architecture

and once again you may be able to get this intuition of how the second layer, here we have three heading units that are computing some complex function maybe of the input layer, and then the third layer can take the second layer's features and compute even more complex features in layer three so that by the time you get to the output layer, layer four, you can have even more complex features of what you are able to compute in layer three and so get very interesting nonlinear hypotheses.

By the way, in a network like this, layer one, this is called an input layer. Layer four is still our output layer, and this network has two hidden layers. So anything that's not an input layer or an output layer is called a hidden layer.

So, hopefully from this video you've gotten a sense of how the feed forward propagation step in a neural network works where you start from the activations of the input layer and forward propagate that to the first hidden layer, then the second hidden layer, and then finally the output layer. And you also saw how we can vectorize that computation.

In the next, I realized that some of the intuitions in this video of how, you know, other certain layers are computing complex features of the early layers. I realized some of that intuition may be still slightly abstract and kind of a high level. And so what I would like to do in the two videos is work through a detailed example of how a neural network can be used to compute nonlinear functions of the input and hope that will give you a good sense of the sorts of complex nonlinear hypotheses we can get out of Neural Networks.

wrapping-up

Model Representation II

To re-iterate, the following is an example of a neural network:

$$a_1^{(2)} = g(\Theta_{10}^{(1)} x_0 + \Theta_{11}^{(1)} x_1 + \Theta_{12}^{(1)} x_2 + \Theta_{13}^{(1)} x_3)$$
$$a_2^{(2)} = g(\Theta_{20}^{(1)} x_0 + \Theta_{21}^{(1)} x_1 + \Theta_{22}^{(1)} x_2 + \Theta_{23}^{(1)} x_3)$$
$$a_3^{(2)} = g(\Theta_{30}^{(1)} x_0 + \Theta_{31}^{(1)} x_1 + \Theta_{32}^{(1)} x_2 + \Theta_{33}^{(1)} x_3)$$
$$h_\Theta(x) = a_1^{(3)} = g(\Theta_{10}^{(2)} a_0^{(2)} + \Theta_{11}^{(2)} a_1^{(2)} + \Theta_{12}^{(2)} a_2^{(2)} + \Theta_{13}^{(2)} a_3^{(2)})$$

In this section we'll do a vectorized implementation of the above functions. We're going to define a new variable $z_k^{(j)}$ that encompasses the parameters inside our g function. In our previous example if we replaced by the variable z for all the parameters we would get:

$$a_1^{(2)} = g(z_1^{(2)})$$
$$a_2^{(2)} = g(z_2^{(2)})$$
$$a_3^{(2)} = g(z_3^{(2)})$$

In other words, for layer j=2 and node k, the variable z will be:

$$z_k^{(2)} = \Theta_{k,0}^{(1)} x_0 + \Theta_{k,1}^{(1)} x_1 + \cdots + \Theta_{k,n}^{(1)} x_n$$

The vector representation of x and z^j is:

$$x = \begin{matrix} x_0 \\ x_1 \\ \cdots \\ x_n \end{matrix} \quad z^{(j)} = \begin{matrix} z_1^{(j)} \\ z_2^{(j)} \\ \cdots \\ z_n^{(j)} \end{matrix}$$

Setting $x = a^{(1)}$, we can rewrite the equation as:

$$z^{(j)} = \Theta^{(j-1)} a^{(j-1)}$$

We are multiplying our matrix $\Theta^{(j-1)}$ with dimensions $s_j \times (n+1)$ (where s_j is the number of our activation nodes) by our vector $a^{(j-1)}$ with height (n+1). This gives us our vector $z^{(j)}$ with height s_j. Now we can get a vector of our activation nodes for layer j as follows:

$$a^{(j)} = g(z^{(j)})$$

Where our function g can be applied element-wise to our vector $z^{(j)}$.

We can then add a bias unit (equal to 1) to layer j after we have computed $a^{(j)}$. This will be element $a_0^{(j)}$ and will be equal to 1. To compute our final hypothesis, let's first compute another z vector:

$$z^{(j+1)} = \Theta^{(j)} a^{(j)}$$

We get this final z vector by multiplying the next theta matrix after $\Theta^{(j-1)}$ with the values of all the activation nodes we just got. This last theta matrix $\Theta^{(j)}$ will have only **one row** which is multiplied by one column $a^{(j)}$ so that our result is a single number. We then get our final result with:

$$h_\Theta(x) = a^{(j+1)} = g(z^{(j+1)})$$

Notice that in this **last step**, between layer j and layer j+1, we are doing **exactly the same thing** as we did in logistic regression. Adding all these intermediate layers in neural networks allows us to more elegantly produce interesting and more complex non-linear hypotheses.

Applications

Neural Networks: Representation

Examples and intuitions I

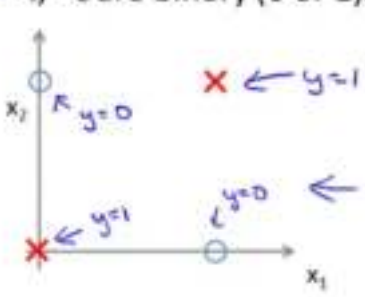
In this and the next video I want to work through a detailed example showing how a neural network can compute a complex non-linear function of the input. And hopefully this will give you a good sense of why neural networks can be used to learn complex non-linear hypotheses. Consider the following problem where we have features x1 and x2 that are binary values. So, either 0 or 1. So, X1 and X2 can each take on only one of two possible values. In this example, I've drawn only two positive examples and two negative examples. That you can think of this as a simplified version of a more complex learning problem where we may have a bunch of positive examples in the upper right and lower left and a bunch of negative examples denoted by the circles. And what we'd like to do is learn a non-linear division of boundary that may need to separate the positive and negative examples.

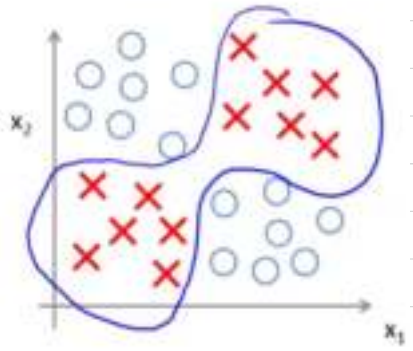
So, how can a neural network do this and rather than using the example and the variable to use this maybe easier to examine example on the left. Concretely what this is, is really computing the type of label y equals x 1 x or x 2. Or actually this is actually the x 1 x nor x 2 function where x nor is the alternative notation for not x 1 or x 2. So, x 1 x or x 2 that's true only if exactly 1 of x 1 or x 2 is equal to 1. It turns out that these specific examples in the works out a little bit better if we use the XNOR example instead. These two are the same of course. This means not x1 or x2 and so, we're going to have positive examples of either both are true or both are false and what have an y equals 1, y equals 1. And we're going to have y equals 0 if only one of them is true and we're going to figure out if we can get a neural network to fit to this sort of training set.

In order to build up to a network that fits the XNOR example we're going to start with a slightly simpler one and show a network that fits the AND function. ~~Concretely, let's say we have inputs x1 and x2 that are again binaries so, it's either 0 or 1 and let's say our target labels y = x1 AND x2. This is a logical AND.~~

Non-linear classification example: XOR/XNOR

→ x_1, x_2 are binary (0 or 1).





$y = x_1 \text{ XOR } x_2$

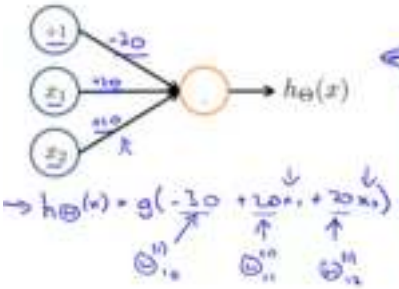
$x_1 \text{ XNOR } x_2$

$\text{NOT } (x_1 \text{ XOR } x_2)$

Simple example: AND

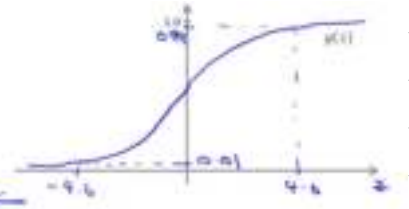
→ $x_1, x_2 \in \{0, 1\}$

→ $y = x_1 \text{ AND } x_2$



→ $h_\Theta(x) = g(-30 + 20x_1 + 20x_2)$

$\Theta^{(1)}_{10}$ $\Theta^{(1)}_{11}$ $\Theta^{(1)}_{12}$



x_1	x_2	$h_\Theta(x)$
0	0	$g(-30) \approx 0$
→ 0	1	$g(-10) \approx 0$
1	0	$g(-10) \approx 0$
1	1	$g(10) \approx 1$

$h_\Theta(x) \approx x_1 \text{ AND } x_2$

simpler one and show a network that fits the AND function. Concretely, let's say we have input x1 and x2 that are again binaries so, it's either 0 or 1 and let's say our target labels y = x1 AND x2. This is a logical AND.

So, can we get a one-unit network to compute this logical AND function? In order to do so, I'm going to actually draw in the bias unit as well the plus one unit.

Now let me just assign some values to the weights or parameters of this network. I'm gonna write down the parameters on this diagram here, -30 here. +20 and + 20. And what this mean is just that I'm assigning a value of -30 to the value associated with X0 this +1 going into this unit and a parameter value of +20 that multiplies to X1 a value of +20 for the parameter that multiplies into x 2. So, concretely it's the same that the hypothesis h(x)=g(-30+20 X1 plus 20 X2. So, sometimes it's just convenient to draw these weights. Draw these parameters up here in the diagram within and of course this- 30. This is actually theta 1 of 1 0. This is theta 1 of 1 1 and that's theta 1 of 1 2 but it's just easier to think about it as associating these parameters with the edges of the network.

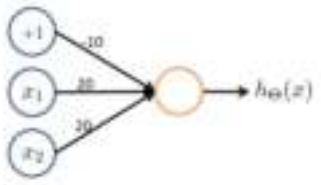
Let's look at what this little single neuron network will compute. Just to remind you the sigmoid activation function g(z) looks like this. It starts from 0 rises smoothly crosses 0.5 and then it asymptotic as 1 and to give you some landmarks, if the horizontal axis value z is equal to 4.6 then the sigmoid function is equal to 0.99. This is very close to 1 and kind of symmetrically, if it's -4.6 then the sigmoid function there is 0.01 which is very close to 0. Let's look at the four possible input values for x1 and x2 and look at what the hypotheses will output in that case. If x1 and x2 are both equal to 0. If you look at this, if x1 x2 are both equal to 0 then the hypothesis of g of -30. So, this is a very far to the left of this diagram so it will be very close to 0. If x 1 equals 0 and x equals 1, then this formula here evaluates the g that is the sigma function applied to -10, and again that's you know to the far left of this plot and so, that's again very close to 0.

This is also g of minus 10 that is if x 1 is equal to 1 and x 2 0, this minus 30 plus 20 which is minus 10 and finally if x 1 equals 1 x 2 equals 1 then you have g of minus 30 plus 20 plus 20. So, that's g of positive 10 which is there for very close to 1.

And if you look in this column this is exactly the logical and function. So, this is computing h of x is approximately x 1 and x 2. In other words it outputs one if and only if x2, x1 and x2, are both equal to 1. So, by writing out our little truth table like this we manage to figure what's the logical function that our neural network computes. This network showed here computes the OR

OR example →

Example: OR function



$g(-10 + 20x_1 + 20x_2)$

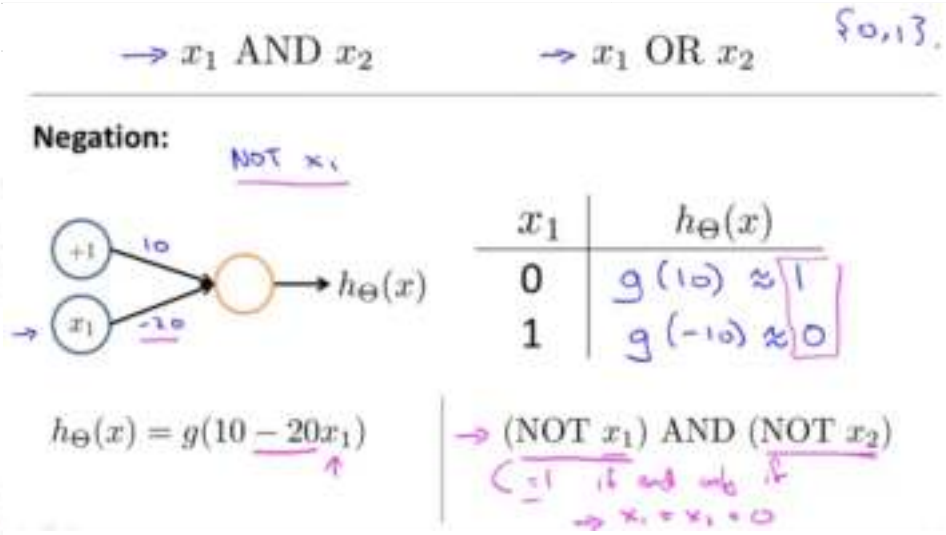
x_1	x_2	$h_\Theta(x)$
0	0	$g(-10) \approx 0$
0	1	$g(10) \approx 1$
1	0	≈ 1
1	1	≈ 1

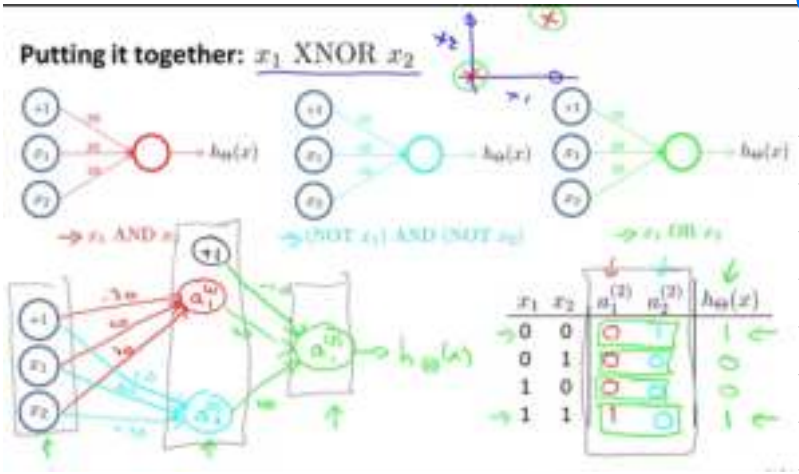
Example and Intuitions II

In this video I'd like to keep working through our example to show how a Neural Network can compute complex non linear hypothesis.

In the last video we saw how a Neural Network can be used to compute the functions x1 AND x2, and the function x1 OR x2 when x1 and x2 are binary, that is when they take on values 0,1. We can also have a network to compute negation, that is to compute the function not x1. Let me just write down the ways associated with this network. We have only one input feature x1 in this case and the bias unit +1. And if I associate this with the weights plus 10 and -20, then my hypothesis is computing this h(x) equals sigmoid (10 -20 x1). So when x1 is equal to 0, my hypothesis would be computing g(10 -20 x 0) is just 10. And so that's approximately 1, and when x is equal to 1, this will be g(10) which is approximately equal to 0. And if you look at what these values are, that's essentially like not x1 function. Cells include negations, the general idea is to put that large negative weight in front of the variable you want to negate. Minus 20 multiplied by x1 and that's the general idea of how you end up negating x1. And so in an example that I hope that you can figure out yourself. If you want to compute a function like this NOT x1 AND NOT x2, part of that will probably be putting large negative weights in front of x1 and x2, but it should be feasible. So you get a neural network with just one output unit to compute this as well. All right, so this logical function, NOT x1 AND NOT x2, is going to be equal to 1 if and only if

x1 equals x2 equals 0. All right since this is a logical function, this says NOT x1 means x1 must be 0 and NOT x2, that means x2 must be equal to 0 as well. So this logical function is equal to 1 if and only if both x1 and x2 are equal to 0 and hopefully you should be able to figure out how to make a small neural network to compute this logical function as well.





Now, taking the three pieces that we have put together as the network for computing x1 AND x2, and the network computing for computing NOT x1 AND NOT x2. And one last network computing for computing x1 OR x2, we should be able to put these three pieces together to compute this x1 XNOR x2 function. And just to remind you if this is x1, x2, this function that we want to compute would have negative examples here and here, and we'd have positive examples there and there. And so clearly this will need a non linear decision boundary in order to separate the positive and negative examples.

Let's draw the network. I'm going to take my input +1, x1, x2 and create my first hidden unit here. I'm gonna call this a 21 cuz that's my first hidden unit. And I'm gonna copy the weight over from the red network, the x1 and x2. As well so then -30, 20, 20. Next let me create a second hidden unit which I'm going to call a 2 2. That is the second hidden unit of layer two. I'm going to copy over the cyan that's work in the middle, so I'm gonna have the weights 10 -20 -20. And so, let's pull some of the truth table values. For the red network, we know that was computing the x1 and x2, and so this will be approximately 0 0 0 1, depending on the values of x1 and x2, and for a 2 2, the cyan network. What do we know? The function NOT x1 AND NOT x2, that outputs 1 0 0 0, for the 4 values of x1 and x2.

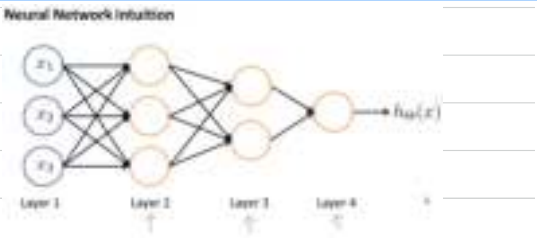
Finally, I'm going to create my output node, my output unit that is a 3 1. This is one more output h(x) and I'm going to copy over the old network for that. And I'm going to need a +1 bias unit here, so you draw that in, And I'm going to copy over the weights from the green networks. So that's -10, 20, 20 and we know earlier that this computes the OR function.

So let's fill in the truth table entries.

So the first entry is 0 OR 1 which can be 1 that makes 0 OR 0 which is 0, 0 OR 0 which is 0, 1 OR 0 and that falls to 1. And thus h(x) is equal to 1 when either both x1 and x2 are zero or when x1 and x2 are both 1 and concretely h(x) outputs 1 exactly at these two locations and then outputs 0 otherwise.

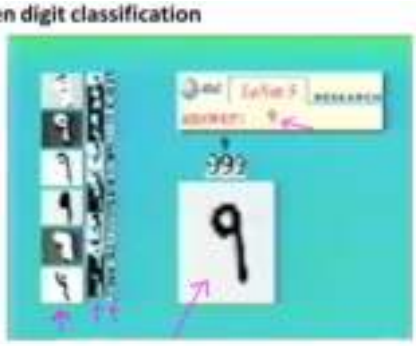
And thus will this neural network, which has a input layer, one hidden layer, and one output layer, we end up with a nonlinear decision boundary that computes this XNOR function. And the more general intuition is that in the input layer, we just have our four inputs. Then we have a hidden layer, which computed some slightly more complex functions of the inputs that its shown here this is slightly more complex functions. And then by adding yet another layer we end up with an even more complex non linear function.

And this is a sort of intuition about why neural networks can compute pretty complicated functions. That when you have multiple layers you have relatively simple function of the inputs of



functions. That when you have multiple layers you have relatively simple function of the inputs of the second layer. But the third layer I can build on that to complete even more complex functions, and then the layer after that can compute even more complex functions.

Handwritten digit classification



To wrap up this video, I want to show you a fun example of an application of a this Neural Network that captures this intuition of the deeper layers computing more complex features. I want to show you a video of Yann LeCun. Yann is a professor at New York University, NYU and he was one of the early pioneers of Neural Network research and is sort of a legend in the field now and his ideas are used in all sorts of products and applications throughout the world now.

So I want to show you a video from some of his early work in which he was using a neural network to recognize handwriting, to do handwritten digit recognition. You might remember early in this class, at the start of this class I said that one of the earliest successes of neural networks was trying to use it to read zip codes in help USPS Laos and read postal codes. So this is one of the attempts, this is one of the algorithms used to try to address that problem. In the video that I'll show you this area here is the input area that shows a canvassing character shown to the network. This column here shows a visualization of the features computed by each of the first hidden layer of the network. So that the first hidden layer of the network and so the first hidden layer, this visualization shows different features. Different edges and lines and so on detected. This is a visualization of the next hidden layer. It's kinda harder to see, harder to understand the deeper hidden layers, and that's a visualization of why the next hidden layer is confusing. You probably have a hard time seeing what's going on much beyond the first hidden layer, but then finally, all of these learned features get fed to the upper layer. And shown over here is the final answer, it's the final prediction value for what handwritten digit the neural network thinks it is being shown. So let's take a look at the video. [BEEP]

So I hope you enjoyed the video and that this hopefully gave you some intuition about the source of pretty complicated functions neural networks can learn: in which it takes its input this image, just takes this input, the raw pixels and the first hidden layer computes some set of features. The next hidden layer computes even more complex features and even more complex features. And those features can then be used by essentially the final layer of the logistic classifiers to make accurate predictions without the numbers that the network sees.

wrapping up

Examples and Intuitions II

The $\Theta^{(1)}$ matrices for AND, NOR, and OR are:

$$AND:$$
$$\Theta^{(1)} = \begin{bmatrix} -30 & 20 & 20 \end{bmatrix}$$
$$NOR:$$
$$\Theta^{(1)} = \begin{bmatrix} 10 & -20 & -20 \end{bmatrix}$$
$$OR:$$
$$\Theta^{(1)} = \begin{bmatrix} -10 & 20 & 20 \end{bmatrix}$$

We can combine these to get the XNOR logical operator (which gives 1 if x_1 and x_2 are both 0 or both 1).

$$\begin{matrix} x_0 & & \\ x_1 & \rightarrow & a_1^{(2)} \\ x_2 & & a_2^{(2)} \end{matrix} \rightarrow a^{(3)} \rightarrow h_\Theta(x)$$

For the transition between the first and second layer, we'll use a $\Theta^{(1)}$ matrix that combines the values for AND and NOR:

$$\Theta^{(1)} = \begin{bmatrix} -30 & 20 & 20 \\ 10 & -20 & -20 \end{bmatrix}$$

For the transition between the second and third layer, we'll use a $\Theta^{(2)}$ matrix that uses the value for OR:

$$\Theta^{(2)} = \begin{bmatrix} -10 & 20 & 20 \end{bmatrix}$$

Let's write out the values for all our nodes:

$$a^{(2)} = g(\Theta^{(1)} \cdot x)$$
$$a^{(3)} = g(\Theta^{(2)} \cdot a^{(2)})$$
$$h_\Theta(x) = a^{(3)}$$

And there we have the XNOR operator using a hidden layer with two nodes! The following summarizes the above algorithm:

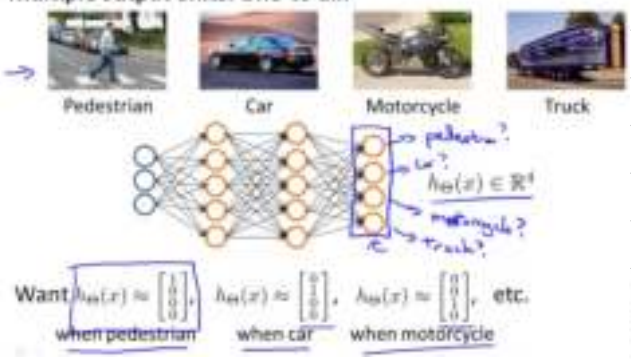
in the Previous Page

Applications

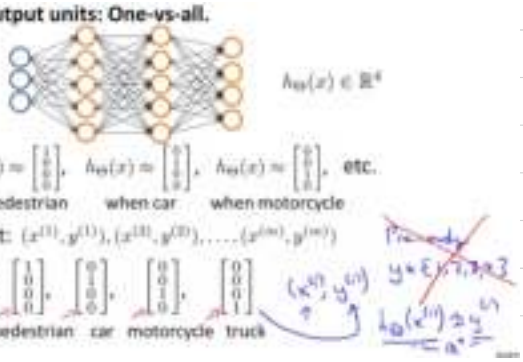
Neural Networks: Representation

Multiclass Classification

Multiple output units: One-vs-all.



Multiple output units: One-vs-all.



Cost function and Back Propagation

Cost function

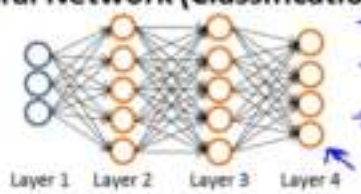
① Neural networks are one of the most powerful learning algorithms that we have today. In this and in the next few videos, I'd like to start talking about a learning algorithm for fitting the parameters of a neural network given a training set. As with the discussion of most of our learning algorithms, we're going to begin by talking about the cost function for fitting the parameters of the network.

I'm going to focus on the application of neural networks to classification problems. So suppose we have a network like that shown on the left. And suppose we have a training set like this to x(1), y(1) pairs of M training example.

I'm going to use upper case L to denote the total number of layers in this network. So for the network shown on the left we would have capital L equals 4. I'm going to use S subscript L to denote the number of units, that is the number of neurons. Not counting the bias unit in their L of the network. So for example, we would have a 1 one, which is equal there, equals 5 three unit, 5 two in my example is five units. And the output layer 5 four, which is also equal to 5 L because capital L is equal to four. The output layer in my example under that has four units.

We're going to consider two types of classification problems. The first is Binary classification, where the labels y are either 0 or 1. In this case, we will have 1 output unit, so this Neural Network unit on top has 4 output units, but if we had binary classification we would have only one output unit that computes h(x). And the output of the neural network would be h(x) is going to be a real number. And in this case the number of output units, S L, where L is again the index of the final layer. Cuz that's the number of layers we have in the network so the number of units we have in the output layer is going to be equal to 1. In this case to simplify notation later, I'm also going to set K=1 so you can think of K as also denoting the number of units in the output layer. The second type of classification problem we'll consider will be multi class classification problem where we may have K distinct classes. So our early example had this representation for y if we have 4 classes, and in this case we will have capital K output units and our hypothesis or output vectors that are K dimensional. And the number of output units will be equal to K. And usually we would have K greater than or equal to 3 in this case, because if we had two classes, then we don't need to use the one versus all method. We use the one versus all method only if we have K greater than or equals 3 classes, so having only two classes we will need to use only one upper unit. Now let's define the cost function for our neural network.

Neural Network (Classification)



$\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$

L = total no. of layers in network $L = 4$

s_l = no. of units (not counting bias unit) in layer l $s_1 = 3$, $s_2 = 5$, $s_4 = s_L = 4$

Binary classification

$y = 0$ or 1

1 output unit

$h_\Theta(x) \in \mathbb{R}$

$s_L = 1$, $K = 1$

Multi-class classification (K classes)

$y \in \mathbb{R}^K$ e.g. $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

pedestrian car motorcycle truck

K output units

$h_\Theta(x) \in \mathbb{R}^K$

$s_L = K$ $(K \geq 3)$

Cost function

Logistic regression:

$$J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} \theta_j^2$$

Neural network:

$h_\Theta(x) \in \mathbb{R}^K$ $(h_\Theta(x))_i = i^{th}$ output

$$J(\Theta) = -\frac{1}{m}\left[\sum_{i=1}^{m}\sum_{k=1}^{K} y_k^{(i)} \log(h_\Theta(x^{(i)}))_k + (1 - y_k^{(i)}) \log(1 - (h_\Theta(x^{(i)}))_k)\right]$$

$$+ \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}} (\Theta_{ji}^{(l)})^2$$

$i = 1 \ldots s_l$

The cost function we use for the neural network is going to be a generalization of the one that we use for logistic regression. For logistic regression we used to minimize the cost function J(theta) that was minus 1/m of this cost function and then plus this extra regularization term here, where this was a sum from J=1 through n, because we did not regularize the bias term theta0.

For a neural network, our cost function is going to be a generalization of this. Where instead of having basically just one, which is the compression output unit, we may instead have K of them. So here's our cost function. Our new network now outputs vectors in R K where R might be equal to 1 if we have a binary classification problem. I'm going to use this notation h(x) subscript i to denote the ith output. That is, h(x) is a k-dimensional vector and so this subscript i just selects out the ith element of the vector that is output by my neural network.

My cost function J(theta) is now going to be the following. is - 1 over M of a sum of a similar term to what we have for logistic regression, except that we have the sum from K equals 1 through K. This summation is basically a sum over my K output. A unit. So if i have four output units, that is if the final layer of my neural network has four output units, then this is a sum from k equals one through four of basically the logistic regression algorithm's cost function but summing that cost function over each of my four output units in turn. And so you notice in particular that this applies to Yk Hk, because we're basically taking the K upper units, and comparing that to the value of Yk which is that one of those vectors saying what cost it should be. And finally, the second term here is the regularization term, similar to what we had for the logistic regression. This summation term looks really complicated, but all it's doing is it's summing over these terms theta j i l for all values of i j and l. Except that we don't sum over the terms corresponding to these bias values like we have for logistic progression. Completely, we don't sum over the terms responding to where i is equal to 0. So that is because when we're computing the activation of a neuron, we have terms like these. Theta i 0. Plus theta i1, x1 plus and so on. Where i guess put in a two there, this is the first hit in there. And so the values with a zero there, that corresponds to something that multiplies into an x0 or an a0. And so this is kinda like a bias unit and by analogy to what we were doing for logistic progression, we won't sum over those terms in our regularization term because we don't want to regularize them and string their values as zero. But this is just one possible convention, and even if you were to sum over i equals 0 up to Sl, it would work about the same and doesn't make a big difference. But maybe this convention of not regularizing the bias term is just slightly more common.

So that's the cost function we're going to use for our neural network. In the next video we'll start to talk about an algorithm for trying to optimize the cost function.

Costfunction and Back Propagation

Back propagation Algorithm

In the previous video, we talked about a cost function for the neural network. In this video, let's start to talk about an algorithm, for trying to minimize the cost function. In particular, we'll talk about the back propagation algorithm.

Here's the cost function that we wrote down in the previous video. What we'd like to do is try to find parameters theta to try to minimize j of theta. In order to use either gradient descent or one of the advance optimization algorithms. What we need to do therefore is to write code that takes this input the parameters theta and computes j of theta and these partial derivative terms. Remember, that the parameters in the the neural network of these things, theta superscript l subscript ij, that's the real number and so, these are the partial derivative terms we need to compute. In order to compute the cost function j of theta, we just use this formula up here and so, what I want to do for the most of this video is focus on talking about how we can compute these partial derivative terms. Let's start by talking about the case of when we have only one training example, so imagine, if you will that our entire training set comprises only one training example which is a pair xy. I'm not going to write x1y1 just write this. Write a one training example as xy and let's tap through the sequence of calculations we would do with this one training example.

The first thing we do is we apply forward propagation in order to compute whether a hypotheses actually outputs given the input. Concretely, the called the a(1) is the activation values of this first layer that was the input there. So, I'm going to set that to x and then we're going to compute z(2) equals theta(1) a(1) and a(2) equals g, the sigmoid activation function applied to z(2) and this would give us our activations for the first middle layer. That is for layer two of the network and we also add those bias terms. Next we apply 2 more steps of this four and propagation to compute a(3) and a(4) which is also the upwards of a hypotheses h of x. So this is our vectorized implementation of forward propagation and it allows us to compute the activation values for all of the neurons in our neural network.

Next, in order to compute the derivatives, we're going to use an algorithm called back propagation.

Gradient computation

$$J(\Theta) = -\frac{1}{m}\left[\sum_{i=1}^{m}\sum_{k=1}^{K} y_k^{(i)} \log h_\Theta(x^{(i)})_k + (1 - y_k^{(i)}) \log(1 - h_\Theta(x^{(i)})_k)\right] + \frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(\Theta_{ji}^{(l)})^2$$

$$\min_\Theta J(\Theta)$$

Need code to compute:

- $J(\Theta)$
- $\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta)$

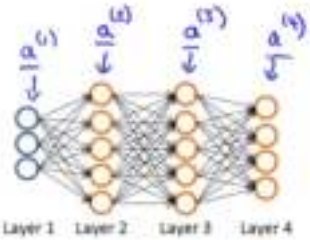
$\Theta_{ij}^{(l)} \in \mathbb{R}$

Gradient computation

Given one training example (x, y):

Forward propagation:

$$a^{(1)} = x$$
$$z^{(2)} = \Theta^{(1)} a^{(1)}$$
$$a^{(2)} = g(z^{(2)}) \quad (\text{add } a_0^{(2)})$$
$$z^{(3)} = \Theta^{(2)} a^{(2)}$$
$$a^{(3)} = g(z^{(3)}) \quad (\text{add } a_0^{(3)})$$
$$z^{(4)} = \Theta^{(3)} a^{(3)}$$
$$a^{(4)} = h_\Theta(x) = g(z^{(4)})$$



Gradient computation: Backpropagation algorithm

Intuition: $\delta_j^{(l)}$ = "error" of node j in layer l.

For each output unit (layer L = 4)

$$\delta_j^{(4)} = a_j^{(4)} - y_j$$

$$\delta^{(3)} = (\Theta^{(3)})^T \delta^{(4)} .* g'(z^{(3)})$$

$$\delta^{(2)} = (\Theta^{(2)})^T \delta^{(3)} .* g'(z^{(2)})$$

$$g'(z^{(3)}) = a^{(3)} .* (1 - a^{(3)})$$

$$g'(z^{(2)}) = a^{(2)} .* (1 - a^{(2)})$$

$$\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta) = a_j^{(l)} \delta_i^{(l+1)} \quad (\text{ignoring } \lambda; \text{ if } \lambda = 0)$$

The intuition of the back propagation algorithm is that for each note we're going to compute the term delta superscript l subscript j that's going to somehow represent the error of note j in the layer l. So, recall that a superscript l subscript j that does the activation of the j of unit in layer l and so, this delta term is in some sense going to capture our error in the activation of that neural duo. So, how we might wish the activation of that note is slightly different. Concretely, taking the example neural network that we have on the right which has four layers. And so capital L is equal to 4. For each output unit, we're going to compute this delta term. So, delta for the j of unit in the fourth layer is equal to

just the activation of that unit minus what was the actual value of 0 in our training example.

So, this term here can also be written h of x subscript j, right. So this delta term is just the difference between when a hypotheses output and what was the value of y in our training set whereas y subscript j is the j of element of the vector value y in our labeled training set.

And by the way, if you think of delta a and y as vectors then you can also take those and come up with a vectorized implementation of it, which is just delta 4 gets set as a4 minus y. Where here, each of these delta 4 a4 and y, each of these is a vector whose dimension is equal to the number of output units in our network.

So we've now computed the era term's delta 4 for our network.

What we do next is compute the delta terms for the earlier layers in our network. Here's a formula for computing delta 3 is delta 3 is equal to theta 3 transpose times delta 4. And this dot times, this is the element y's multiplication operation

that we know from MATLAB. So delta 3 transpose delta 4, that's a vector, g prime z3 that's also a vector and so dot times is in element y's multiplication between these two vectors.

This term g prime of z3, that formally is actually the derivative of the activation function g evaluated at the input values given by z3. If you know calculus, you can try to work it out yourself and see that you can simplify it to the same answer that I get. But I'll just tell you pragmatically what that means. What you do to compute this g prime, these derivative terms is just a3 dot times1 minus A3 where A3 is the vector of activations. 1 is the vector of ones and A3 is again the activation the vector of activation values for that layer. Next you apply a similar formula to compute delta 2 where again that can be computed using a similar formula.

Only now it is a2 like so and I then prove it here but you can actually, it's possible to prove it if you know calculus that this expression is equal to mathematically, the derivative of the g function of the activation function, which I'm denoting by g prime. And finally, that's it and there is no delta1 term, because the first layer corresponds to the input layer and that's just the feature we observed in our training sets, so that doesn't have any error associated with that. It's not like, you know, we don't really want to try to change those values. And so we have delta terms only for layers 2, 3 and for this example.

The name back propagation comes from the fact that we start by computing the delta term for the output layer and then we go back a layer and compute the delta terms for the third hidden layer and then we go back another step to compute delta 2 and so, we're sort of back propagating the errors from the output layer to layer 3 to their to hence the name back complication.

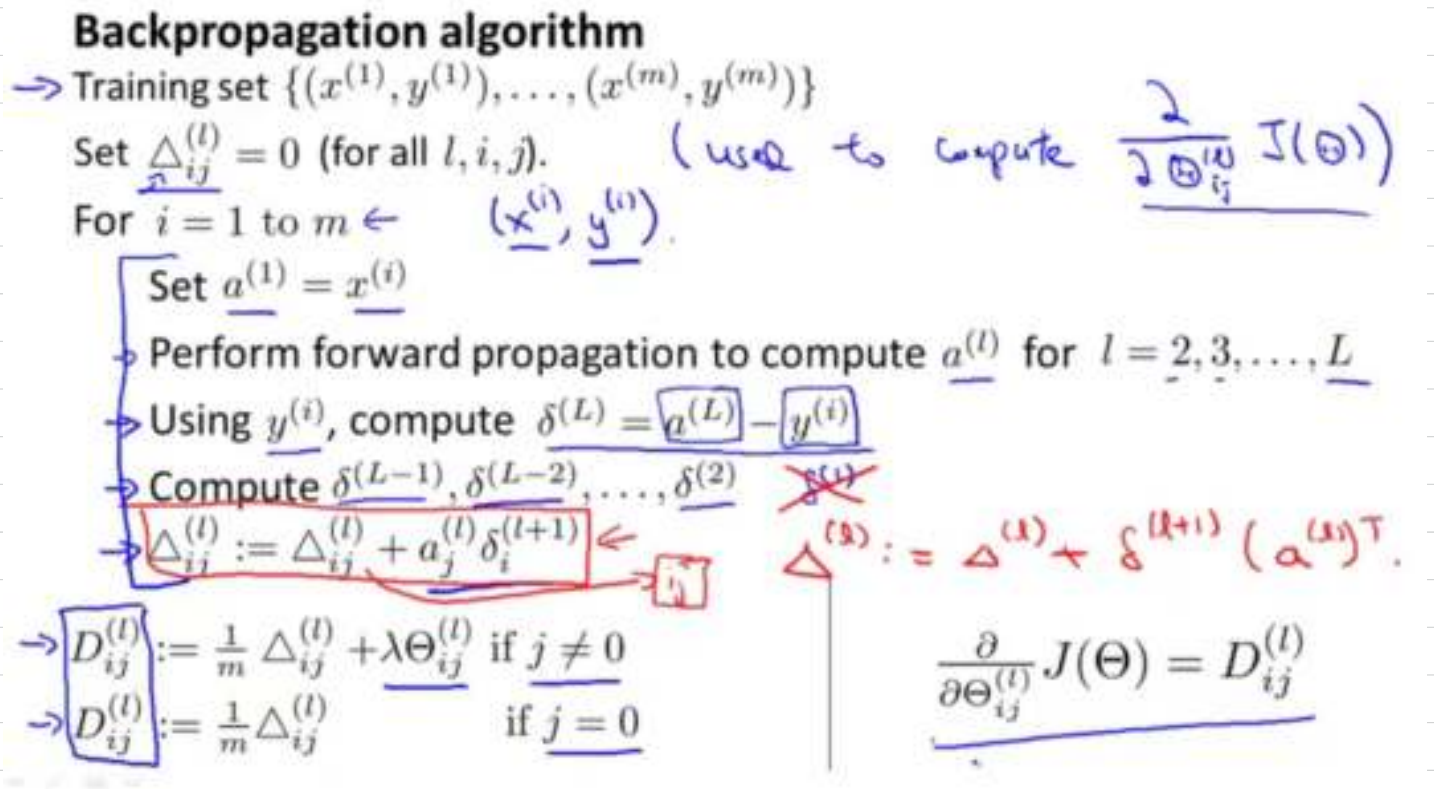
Finally, the derivation is surprisingly complicated, surprisingly involved but if you just do this few steps steps of computation it is possible to prove viral frankly some what complicated mathematical proof. It's possible to prove that if you ignore authorization then the partial derivative terms you want

are exactly given by the activations and these delta terms. This is ignoring lambda or alternatively the regularization

term lambda will equal to 0. We'll fix this detail later about the regularization term, but so by performing back propagation and computing these delta terms, you can, you know, pretty quickly compute these partial derivative terms for all of your parameters. So this is a lot of detail. Let's take everything and put it all together to talk about how to implement back propagation

to compute derivatives with respect to your parameters.

And for the case of when we have a large training set, not just a training set of one example, here's what we do. Suppose we have a training set of m examples like that shown here. The first



here's what we do. Suppose we have a training set of m examples like that shown here. The first thing we're going to do is we're going to set these delta l subscript i j. So this triangular symbol? That's actually the capital Greek alphabet delta . The symbol we had on the previous slide was the lower case delta. So the triangle is capital delta. We're gonna set this equal to zero for all values of l i j. Eventually, this capital delta l i j will be used

to compute the partial derivative term, partial derivative respect to theta l i j of J of theta.

So as we'll see in a second, these deltas are going to be used as accumulators that will slowly add things in order to compute these partial derivatives.

Next, we're going to loop through our training set. So, we'll say for i equals 1 through m and so for the i iteration, we're going to working with the training example xi, yi.

So the first thing we're going to do is set a1 which is the activations of the input layer, set that to be equal to xi is the inputs for our i training example, and then we're going to perform forward propagation to compute the activations for layer two, layer three and so on up to the final layer, layer capital L. Next, we're going to use the output label yi from this specific example we're looking at to compute the error term for delta L for the output there. So delta L is what a hypotheses output minus what the target label was?

And then we're going to use the back propagation algorithm to compute delta L minus 1, delta L minus 2, and so on down to delta 2 and once again there is now delta 1 because we don't associate an error term with the input layer.

And finally, we're going to use these capital delta terms to accumulate these partial derivative terms that we wrote down on the previous line.

And by the way, if you look at this expression, it's possible to vectorize this too. Concretely, if you think of delta ij as a matrix, indexed by subscript ij.

Then, if delta L is a matrix we can rewrite this as delta L, gets updated as delta L plus

lower case delta L plus one times ai, transpose. So that's a vectorized implementation of this that automatically does an update for all values of i and j. Finally, after executing the body of the four-loop we then go outside the four-loop and we compute the following. We compute capital D as follows and we have two separate cases for j equals zero and j not equals zero.

The case of j equals zero corresponds to the bias term so when j equals zero that's why we're missing is an extra regularization term.

Finally, while the formal proof is pretty complicated what you can show is that once you've computed these D terms, that is exactly the partial derivative of the cost function with respect to each of your perimeters and so you can use those in either gradient descent or in one of the advanced authorization

algorithms.

So that's the back propagation algorithm and how you compute derivatives of your cost function for a neural network. I know this looks like this was a lot of details and this was a lot of steps strung together. But both in the programming assignments write out and later in this video, we'll give you a summary of this so we can have all the pieces of the algorithm together so that you know exactly what you need to implement if you want to implement back propagation to compute the derivatives of your neural network's cost function with respect to those parameters.

Suppose you have two training examples $(x^{(1)}, y^{(1)})$ and $(x^{(2)}, y^{(2)})$. Which of the following is a correct sequence of operations for computing the gradient? (Below, FP = forward propagation, BP = back propagation).

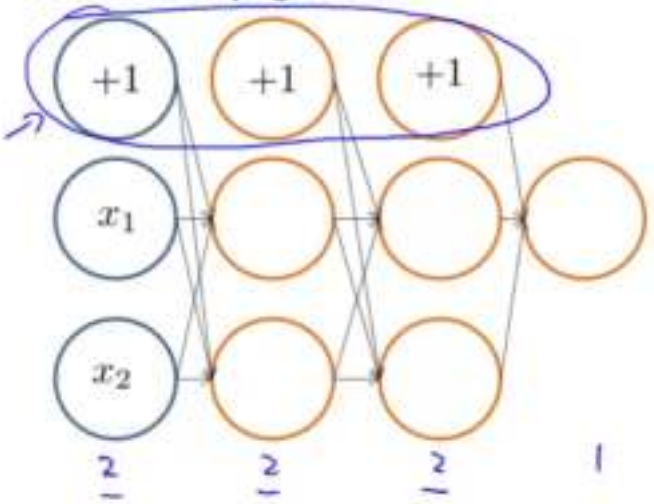
- [ ] FP using $x^{(1)}$ followed by FP using $x^{(2)}$. Then BP using $y^{(1)}$ followed by BP using $y^{(2)}$.
- [ ] FP using $x^{(1)}$ followed by BP using $y^{(2)}$. Then FP using $x^{(2)}$ followed by BP using $y^{(1)}$.
- [ ] BP using $y^{(1)}$ followed by FP using $x^{(1)}$. Then BP using $y^{(2)}$ followed by FP using $x^{(2)}$.
- [x] FP using $x^{(1)}$ followed by BP using $y^{(1)}$. Then FP using $x^{(2)}$ followed by BP using $y^{(2)}$.

✓ Correct

Cost function and Back proPagation

Back pro Pagation Intuition

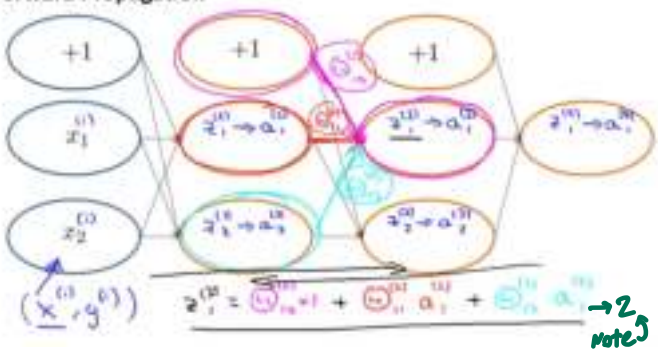
Forward Propagation



In the previous video, we talked about the backpropagation algorithm. To a lot of people seeing it for the first time, their first impression is often that wow this is a really complicated algorithm, and there are all these different steps, and I'm not sure how they fit together. And it's kinda this black box of all these complicated steps. In case that's how you're feeling about backpropagation, that's actually okay. Backpropagation maybe unfortunately is a less mathematically clean, or less mathematically simple algorithm, compared to linear regression or logistic regression. And I've actually used backpropagation, you know, pretty successfully for many years. And even today I still don't sometimes feel like I have a very good sense of just what it's doing, or intuition about what back propagation is doing. If, for those of you that are doing the programming exercises, that will at least mechanically step you through the different steps of how to implement back prop. So you'll be able to get it to work for yourself. And what I want to do in this video is look a little bit more at the mechanical steps of backpropagation, and try to give you a little more intuition about what the mechanical steps the back prop is doing to hopefully convince you that, you know, it's at least a reasonable algorithm.

In case even after this video in case back propagation still seems very black box and kind of like a, too many complicated steps and a little bit magical to you, that's actually okay. And Even though I've used back prop for many years, sometimes this is a difficult algorithm to understand, but hopefully this video will help a little bit. In order to better understand backpropagation, let's take another closer look at what forward propagation is doing. Here's a neural network with two input units that is not counting the bias unit, and two hidden units in this layer, and two hidden units in the next layer. And then, finally, one output unit. Again, these counts two, two, two, are not counting these bias units on top. In order to illustrate forward propagation, I'm going to draw this network a little bit differently.

Forward Propagation



And in particular I'm going to draw this neuro-network with the nodes drawn as these very fat ellipsis, so that I can write text in them. When performing forward propagation, we might have some particular example. Say some example x i comma y i. And it'll be this x i that we feed into the input layer. So this maybe x i 2 and x i 2 are the values we set the input layer to. And when we forward propagated to the first hidden layer here, what we do is compute z (2) 1 and z (2) 2. So these are the weighted sum of inputs of the input units. And then we apply the sigmoid of the logistic function, and the sigmoid activation function applied to the z value. Here's are the activation values. So that gives us a (2) 1 and a (2) 2. And then we forward propagate again to get here z (3) 1. Apply the sigmoid of the logistic function, the activation function to that to get a (3) 1. And similarly, like so until we get z (4) 1. Apply the activation function. This gives us a (4)1, which is the final output value of the neural network.

Let's erase this arrow to give myself some more space. And if you look at what this computation really is doing, focusing on this hidden unit, let's say. We have to add this weight. Shown in magenta there is my weight theta (2) 1 0, the indexing is not important. And this way here, which I'm highlighting in red, that is theta (2) 1 1 and this weight here, which I'm drawing in cyan, is theta (2) 1 2. So the way we compute this value, z(3)1 is, z(3)1 is as equal to this magenta weight times this value. So that's theta (2) 10 x 1. And then plus this red weight times this value, so that's theta(2) 11 times a(2)1. And finally this cyan weight times this value, which is therefore plus theta(2)12 times a(2)1. And so that's forward propagation. And it turns out that as we'll see later in this video, what backpropagation is doing is doing a process very similar to this. Except that instead of the computations flowing from the left to the right of this network, the computations since their flow from the right to the left of the network. And using a very similar computation as this. And I'll say in two slides exactly what I mean by that. To better understand what

this. And I'll say in two slides exactly what I mean by that. To better understand what backpropagation is doing, let's look at the cost function. It's just the cost function that we had for when we have only one output unit. If we have more than one output unit, we just have a summation you know over the output units indexed by k there. If you have only one output unit then this is a cost function. And we do forward propagation and backpropagation on one example at a time. So let's just focus on the single example, x (i) y (i) and focus on the case of having one output unit. So y (i) here is just a real number. And let's ignore regularization, so lambda equals 0. And this final term, that regularization term, goes away. Now if you look inside the summation, you find that the cost term associated with the training example, that is the cost associated with the training example x(i), y(i). That's going to be given by this expression. So, the cost is live off example i is written as follows. And what this cost function does is it plays a role similar to the squared arrow. So, rather than looking at this complicated expression, if you want you can think of cost of i being approximately the square difference between what the neural network outputs, versus what is the actual value. Just as in logistic repression, we actually prefer to use the slightly more complicated cost function using the log. But for the purpose of intuition, feel free to think of the cost function as being the sort of the squared error cost function. And so this cost(i) measures how well is the network doing on correctly predicting example i. How close is the output to the actual observed label y(i)? Now let's look at what backpropagation is doing.

What is backpropagation doing?

$$J(\Theta) = -\frac{1}{m}\left[\sum_{i=1}^{m} \underbrace{y^{(i)} \log(h_\Theta(x^{(i)})) + (1-y^{(i)}) \log(1-(h_\Theta(x^{(i)})))}_{(x^{(i)}, y^{(i)})}\right] + \cancel{\frac{\lambda}{2m}\sum_{l=1}^{L-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}(\Theta_{ji}^{(l)})^2}$$

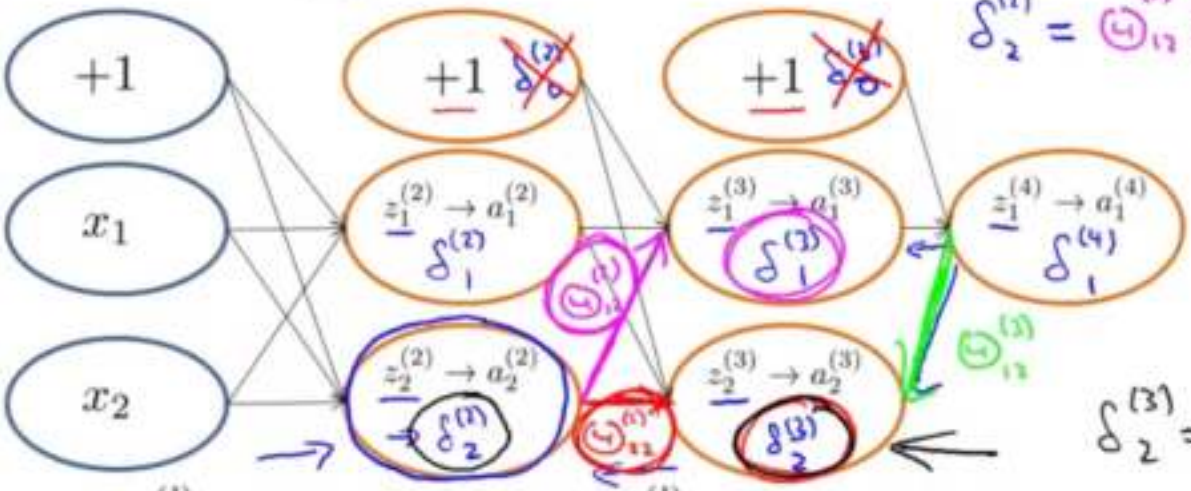
Focusing on a single example $x^{(i)}$, $y^{(i)}$, the case of 1 output unit, and ignoring regularization ($\lambda = 0$),

$$\text{cost}(i) = y^{(i)} \log h_\Theta(x^{(i)}) + (1-y^{(i)}) \log h_\Theta(x^{(i)})$$

(Think of $\text{cost}(i) \approx (h_\Theta(x^{(i)}) - y^{(i)})^2$)

I.e. how well is the network doing on example i?

Forward Propagation



$\delta_j^{(l)}$ = "error" of cost for $a_j^{(l)}$ (unit j in layer l).

Formally, $\delta_j^{(l)} = \frac{\partial}{\partial z_j^{(l)}} \text{cost}(i)$ (for $j \geq 0$), where

$$\text{cost}(i) = y^{(i)} \log h_\Theta(x^{(i)}) + (1 - y^{(i)}) \log h_\Theta(x^{(i)})$$

Andrew Ng

Note: [4:39, the last term for the calculation for z_1^3 (three-color handwritten formula) should be a_2^2 instead of a_1^2. 6:08 - the equation for cost(i) is incorrect. The first term is missing parentheses for the log() function, and the second term should be $(1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))$. 8:50 - $\delta^{(4)} = y - a^{(4)}$ is incorrect and should be $\delta^{(4)} = a^{(4)} - y$.]

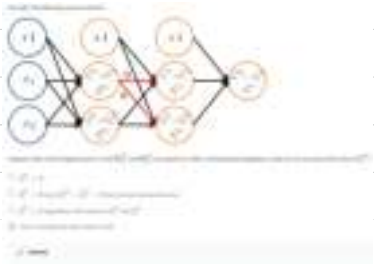
① ~~If the output is the actual observed label y(i)?~~ Now let's look at what backpropagation is doing. One useful intuition is that backpropagation is computing these delta superscript l subscript j terms. And we can think of these as the quote error of the activation value that we got for unit j in the layer, in the lth layer.

More formally, for, and this is maybe only for those of you who are familiar with calculus. More formally, what the delta terms actually are is this, they're the partial derivative with respect to z,l,j, that is this weighted sum of inputs that were confusing these z terms. Partial derivatives with respect to these things of the cost function.

So concretely, the cost function is a function of the label y and of the value, this h of x output value neural network. And if we could go inside the neural network and just change those z l j values a little bit, then that will affect these values that the neural network is outputting. And that will end up changing the cost function. And again really, this is only for those of you who are expert in Calculus. If you're comfortable with partial derivatives, what these delta terms are is they turn out to be the partial derivative of the cost function, with respect to these intermediate terms that were confusing.

And so they're a measure of how much would we like to change the neural network's weights, in order to affect these intermediate values of the computation. So as to affect the final output of the neural network h(x) and therefore affect the overall cost. In case this last part of this partial derivative intuition, in case that doesn't make sense. Don't worry about the rest of this, we can do without really talking about partial derivatives. But let's look in more detail about what backpropagation is doing. For the output layer, the first set's this delta term, delta (4) 1, as y (i) if we're doing forward propagation and back propagation on this training example i. That says y(i) minus a(4)1. So this is really the error, right? It's the difference between the actual value of y minus what was the value predicted, and so we're gonna compute delta(4)1 like so. Next we're gonna do, propagate these values backwards. I'll explain this in a second, and end up computing the delta terms for the previous layer. We're gonna end up with delta(3)1. Delta(3)2. And then we're gonna propagate this further backward, and end up computing delta(2)1 and delta(2)2.

② Now the backpropagation calculation is a lot like running the forward propagation algorithm, but doing it backwards. So here's what I mean. Let's look at how we end up with this value of delta(2)2. So we have delta(2)2. And similar to forward propagation, let me label a couple of the weights. So this weight, which I'm going to draw in cyan. Let's say that weight is theta(2)1 2, and this one down here when we highlight this in red. That is going to be let's say theta(2) of 2 2. So if we look at how delta(2)2, is computed, how it's computed with this note. It turns out that what we're going to do, is gonna take this value and multiply it by this weight, and add it to this value multiplied by that weight. So it's really a weighted sum of these delta values, weighted by the corresponding edge strength. So completely, let me fill this in, this delta(2)2 is going to be equal to, Theta(2)1 2 is that magenta lay times delta(3)1. Plus, and the thing I had in red, that's theta (2)2 times delta (3)2. So it's really literally this red wave times this value, plus this magenta weight times this value. And that's how we wind up with that value of delta. And just as another example, let's look at this value. How do we get that value? Well it's a similar process. If this weight, which I'm gonna highlight in green, if this weight is equal to, say, delta (3) 1 2. Then we have that delta (3) 2 is going to be equal to that green weight, theta (3) 12 times delta (4) 1. And by the way, so far I've been writing the delta values only for the hidden units, but excluding the bias units. Depending on how you define the backpropagation algorithm, or depending on how you implement it, you know, you may end up implementing something that computes delta values for these bias units as well. The bias units always output the value of plus one, and they are just what they are, and there's no way for us to change the value. And so, depending on your implementation of back prop, the way I usually implement it. I do end up computing these delta values, but we just discard them, we don't use them. Because they don't end up being part of the calculation needed to compute a derivative. So hopefully that gives you a little better intuition about what back propagation is doing. In case of all of this still seems sort of magical, sort of black box, in a later video, in the putting it together video, I'll try to get a little bit more intuition about what backpropagation is doing. But unfortunately this is a difficult algorithm to try to visualize and understand what it is really doing. But fortunately I've been, I guess many people have been using very successfully for many years. And if you implement the algorithm you can have a very effective learning algorithm. Even though the inner workings of exactly how it works can be harder to visualize.



Back Propagation in Practice

Implementation Note: Unrolling Parameters

In the previous video, we talked about how to use back propagation

to compute the derivatives of your cost function. In this video, I want to quickly tell you about one implementational detail of unrolling your parameters from matrices into vectors, which we need in order to use the advanced optimization routines.

Concretely, let's say you've implemented a cost function that takes this input, you know, parameters theta and returns the cost function and returns derivatives.

Then you can pass this to an advanced authorization algorithm by fminunc and fminunc isn't the only one by the way. There are also other advanced authorization algorithms.

But what all of them do is take those input pointedly the cost function, and some initial value of theta.

And both, and these routines assume that theta and the initial value of theta, that these are parameter vectors, maybe Rn or Rn plus 1. But these are vectors and it also assumes that, you know, your cost function will return as a second return value this gradient which is also Rn and Rn plus 1. So also a vector. This worked fine when we were using logistic progression but now that we're using a neural network our parameters are no longer vectors, but instead they are these matrices where for a full neural network we would have parameter matrices theta 1, theta 2, theta 3 that we might represent in Octave as these matrices theta 1, theta 2, theta 3. And similarly these gradient terms that were expected to return. Well, in the previous video we showed how to compute these gradient matrices, which was capital D1, capital D2, capital D3, which we might represent as octave as matrices D1, D2, D3.

In this video I want to quickly tell you about the idea of how to take these matrices and unroll them into vectors. So that they end up being in a format suitable for passing into an theta here off for getting out for a gradient there.

Advanced optimization

```
function [jVal, gradient] = costFunction(theta)
  ...
optTheta = fminunc(@costFunction, initialTheta, options)
```

Neural Network (L=4):

→ $\Theta^{(1)}, \Theta^{(2)}, \Theta^{(3)}$ - matrices (`Theta1, Theta2, Theta3`)

→ $D^{(1)}, D^{(2)}, D^{(3)}$ - matrices (`D1, D2, D3`)

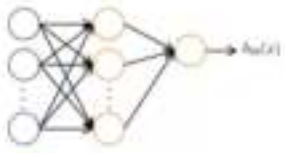
"Unroll" into vectors

Example

$s_1 = 10, s_2 = 10, s_3 = 1$

→ $\Theta^{(1)} \in \mathbb{R}^{10\times 11}, \Theta^{(2)} \in \mathbb{R}^{10\times 11}, \Theta^{(3)} \in \mathbb{R}^{1\times 11}$

→ $D^{(1)} \in \mathbb{R}^{10\times 11}, D^{(2)} \in \mathbb{R}^{10\times 11}, D^{(3)} \in \mathbb{R}^{1\times 11}$



```
→ thetaVec = [ Theta1(:); Theta2(:); Theta3(:)];
→ DVec = [D1(:); D2(:); D3(:)];

  Theta1 = reshape(thetaVec(1:110),10,11);
→ Theta2 = reshape(thetaVec(111:220),10,11);
→ Theta3 = reshape(thetaVec(221:231),1,11);
```

Concretely, let's say we have a neural network with one input layer with ten units, hidden layer with ten units and one output layer with just one unit, so s1 is the number of units in layer one and s2 is the number of units in layer two, and s3 is a number of units in layer three. In this case, the dimension of your matrices theta and D are going to be given by these expressions. For example, theta one is going to a 10 by 11 matrix and so on.

So in if you want to convert between these matrices. vectors. What you can do is take your theta 1, theta 2, theta 3, and write this piece of code and this will take all the elements of your three theta matrices and take all the elements of theta one, all the elements of theta 2, all the elements of theta 3, and unroll them and put all the elements into a big long vector.

Which is thetaVec and similarly

the second command would take all of your D matrices and unroll them into a big long vector and call them DVec. And finally if you want to go back from the vector representations to the matrix representations.

What you do to get back to theta one say is take thetaVec and pull out the first 110 elements. So theta 1 has 110 elements because it's a 10 by 11 matrix so that pulls out the first 110 elements and then you can use the reshape command to reshape those back into theta 1. And similarly, to get back theta 2 you pull out the next 110 elements and reshape it. And for theta 3, you pull out the final eleven elements and run reshape to get back the theta 3.

Suppose D1 is a 10x6 matrix and D2 is a 1x11 matrix. You set:

DVec = [D1(:); D2(:)];

Which of the following would get D2 back from DVec?

- reshape(DVec(60:71), 1, 11)
- reshape(DVec(61:72), 1, 11)
- reshape(DVec(61:71), 1, 11)
- reshape(DVec(60:70), 11, 1)

✓ Correct

To make this process really concrete, here's how we use the unrolling idea to implement our learning algorithm.

Let's say that you have some initial value of the parameters theta 1, theta 2, theta 3. What we're going to do is take these and unroll them into a long vector we're gonna call initial theta to pass in to fminunc as this initial setting of the parameters theta.

The other thing we need to do is implement the cost function.

Here's my implementation of the cost function.

The cost function is going to give us input, thetaVec, which is going to be all of my parameters vectors that in the form that's been unrolled into a vector.

So the first thing I'm going to do is I'm going to use thetaVec and I'm going to use the reshape functions. So I'll pull out elements from thetaVec and use reshape to get back my original parameter matrices, theta 1, theta 2, theta 3. So these are going to be matrices that I'm going to get. So that gives me a more convenient form in which to use these matrices so that I can run forward propagation and back propagation to compute my derivatives, and to compute my cost function j of theta.

And finally, I can then take my derivatives and unroll them, to keeping the elements in the same ordering as I did when I unroll my thetas. But I'm gonna unroll D1, D2, D3, to get gradientVec which is now what my cost function can return. It can return a vector of these derivatives.

So, hopefully, you now have a good sense of how to convert back and forth between the matrix representation of the parameters versus the vector representation of the parameters.

The advantage of the matrix representation is that when your parameters are stored as matrices it's more convenient when you're doing forward propagation and back propagation and it's easier when your parameters are stored as matrices to take advantage of the, sort of, vectorized implementations.

Whereas in contrast the advantage of the vector representation, when you have like thetaVec or DVec, is that when you are using the advanced optimization algorithms. Those algorithms tend to assume that you have all of your parameters unrolled into a big long vector. And so with what we just went through, hopefully you can now quickly convert between the two as needed.

Learning Algorithm

→ Have initial parameters $\Theta^{(1)}, \Theta^{(2)}, \Theta^{(3)}$.

→ Unroll to get `initialTheta` to pass to

→ `fminunc(@costFunction, initialTheta, options)`

`function [jval, gradientVec] = costFunction(thetaVec)`

→ From `thetaVec`, get $\Theta^{(1)}, \Theta^{(2)}, \Theta^{(3)}$. reshape.

→ Use forward prop/back prop to compute $D^{(1)}, D^{(2)}, D^{(3)}$ and $J(\Theta)$.

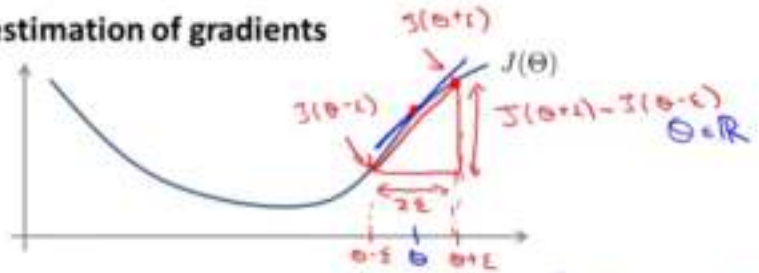
Unroll $D^{(1)}, D^{(2)}, D^{(3)}$ to get `gradientVec`.

Back Propagation in Practice

Gradient checking

In the last few videos we talked about how to do forward propagation and back propagation in a neural network in order to compute derivatives. But back prop as an algorithm has a lot of details and can be a little bit tricky to implement. And one unfortunate property is that there are many ways to have subtle bugs in back prop. So that if you run it with gradient descent or some other optimizational algorithm, it could actually look like it's working. And your cost function, J of theta may end up decreasing on every iteration of gradient descent. But this could prove true even though there might be some bug in your implementation of back prop. So that it looks J of theta is decreasing, but you might just wind up with a neural network that has a higher level of error than you would with a bug free implementation. And you might just not know that there was this subtle bug that was giving you worse performance. So, what can we do about this? There's an idea called gradient checking that eliminates almost all of these problems. So, today every time I implement back propagation or a similar gradient to a [INAUDIBLE] on a neural network or any other reasonably complex model, I always implement gradient checking. And if you do this, it will help you make sure and sort of gain high confidence that your implementation of four prop and back prop or whatever is 100% correct. And from what I've seen this pretty much eliminates all the problems associated with a sort of a buggy implementation as a back prop. And in the previous videos I asked you to take on faith that the formulas I gave for computing the deltas and the vs and so on, I asked you to take on faith that those actually do compute the gradients of the cost function. But once you implement numerical gradient checking, which is the topic of this video, you'll be able to absolute verify for yourself that the code you're writing does indeed, is indeed computing the derivative of the cross function J.

Numerical estimation of gradients



$$\frac{d}{d\theta} J(\theta) \approx \frac{J(\theta+\varepsilon) - J(\theta-\varepsilon)}{2\varepsilon} \qquad \varepsilon = 10^{-4}$$

$$\frac{J(\theta+\varepsilon) - J(\theta)}{\varepsilon}$$

Implement: `gradApprox = (J(theta + EPSILON) - J(theta - EPSILON)) /(2*EPSILON)`

(1)

So here's the idea, consider the following example. Suppose that I have the function J of theta and I have some value theta and for this example gonna assume that theta is just a real number. And let's say that I want to estimate the derivative of this function at this point and so the derivative is equal to the slope of that tangent one.

Here's how I'm going to numerically approximate the derivative, or rather here's a procedure for numerically approximating the derivative. I'm going to compute theta plus epsilon, so now we move it to the right. And I'm gonna compute theta minus epsilon and I'm going to look at those two points, And connect them by a straight line

And I'm gonna connect these two points by a straight line, and I'm gonna use the slope of that little red line as my approximation to the derivative. Which is, the true derivative is the slope of that blue line over there. So, you know it seems like it would be a pretty good approximation.

Mathematically, the slope of this red line is this vertical height divided by this horizontal width. So this point on top is the J of (Theta plus Epsilon). This point here is J (Theta minus Epsilon), so this vertical difference is J (Theta plus Epsilon) minus J of theta minus epsilon and this horizontal distance is just 2 epsilon.

So my approximation is going to be that the derivative respect of theta of J of theta at this value of theta, that that's approximately J of theta plus epsilon minus J of theta minus epsilon over 2 epsilon.

(2)

Usually, I use a pretty small value for epsilon, expect epsilon to be maybe on the order of 10 to the minus 4. There's usually a large range of different values for epsilon that work just fine. And in fact, if you let epsilon become really small, then mathematically this term here, actually mathematically, it becomes the derivative. It becomes exactly the slope of the function at this point. It's just that we don't want to use epsilon that's too, too small, because then you might run into numerical problems. So I usually use epsilon around ten to the minus four. And by the way some of you may have seen an alternative formula for s meeting the derivative which is this formula.

This one on the right is called a one-sided difference, whereas the formula on the left, that's called a two-sided difference. The two sided difference gives us a slightly more accurate estimate, so I usually use that, rather than this one sided difference estimate.

So, concretely, when you implement an octave, is you implemented the following, you implement call to compute gradApprox, which is going to be our approximation derivative as just here this formula, J of theta plus epsilon minus J of theta minus epsilon divided by 2 times epsilon. And this will give you a numerical estimate of the gradient at that point. And in this example it seems like it's a pretty good estimate.

Parameter vector θ

- $\theta \in \mathbb{R}^n$ (E.g. θ is "unrolled" version of $\Theta^{(1)}, \Theta^{(2)}, \Theta^{(3)}$)
- $\theta = [\theta_1, \theta_2, \theta_3, \ldots, \theta_n]$
- $\frac{\partial}{\partial \theta_1} J(\theta) \approx \frac{J(\theta_1+\epsilon, \theta_2, \theta_3, \ldots, \theta_n) - J(\theta_1-\epsilon, \theta_2, \theta_3, \ldots, \theta_n)}{2\epsilon}$
- $\frac{\partial}{\partial \theta_2} J(\theta) \approx \frac{J(\theta_1, \theta_2+\epsilon, \theta_3, \ldots, \theta_n) - J(\theta_1, \theta_2-\epsilon, \theta_3, \ldots, \theta_n)}{2\epsilon}$
- $\vdots$
- $\frac{\partial}{\partial \theta_n} J(\theta) \approx \frac{J(\theta_1, \theta_2, \theta_3, \ldots, \theta_n+\epsilon) - J(\theta_1, \theta_2, \theta_3, \ldots, \theta_n-\epsilon)}{2\epsilon}$

Now on the previous slide, we considered the case of when theta was a rolled number. Now let's look at a more general case of when theta is a vector parameter, so let's say theta is an R n. And it might be an unrolled version of the parameters of our neural network. So theta is a vector that has n elements, theta 1 up to theta n. We can then use a similar idea to approximate all the partial derivative terms. Concretely the partial derivative of a cost function with respect to the first parameter, theta one, that can be obtained by taking J and increasing theta one. So you have J of theta one plus epsilon and so on. Minus J of this theta one minus epsilon and divide it by two epsilon. The partial derivative respect to the second parameter theta two, is again this thing except that you would take J of here you're increasing theta two by epsilon, and here you're decreasing theta two by epsilon and so on down to the derivative. With respect of theta n would give you increase and decrease theta and by epsilon over there.

So, these equations give you a way to numerically approximate the partial derivative of J with respect to any one of your parameters theta i.

Completely, what you implement is therefore the following

```
for i = 1:n,
  thetaPlus = theta;
  thetaPlus(i) = thetaPlus(i) + EPSILON;
  thetaMinus = theta;
  thetaMinus(i) = thetaMinus(i) - EPSILON;
  gradApprox(i) = (J(thetaPlus) - J(thetaMinus))
                  /(2*EPSILON);
end;
```

Check that `gradApprox` ≈ `DVec`

From backprop.

We implement the following in octave to numerically compute the derivatives. We say, for i = 1:n, where n is the dimension of our parameter of vector theta. And I usually do this with the unrolled version of the parameter. So theta is just a long list of all of my parameters in my neural network, say. I'm gonna set thetaPlus = theta, then increase thetaPlus of the (i) element by epsilon. And so this is basically thetaPlus is equal to theta except for thetaPlus(i) which is now incremented by epsilon. Epsilon, so theta plus is equal to, write theta 1, theta 2 and so on. Then theta I has epsilon added to it and then we go down to theta N. So this is what theta plus is. And similar these two lines set theta minus to something similar except that this instead of theta I plus Epsilon, this now becomes theta I minus Epsilon.

And then finally you implement this gradApprox (i) and this would give you your approximation to the partial derivative respect of theta i of J of theta.

And the way we use this in our neural network implementation is, we would implement this four loop to compute the top partial derivative of the cost function for respect to every parameter in that network, and we can then take the gradient that we got from backprop. So DVec was the derivative we got from backprop. All right, so backprop, backpropogation, was a relatively efficient way to compute a derivative or a partial derivative. Of a cost function with respect to all our parameters. And what I usually do is then, take my numerically computed derivative that is this gradApprox that we just had from up here. And make sure that that is equal or approximately equal up to small values of numerical round up, that it's pretty close. So the DVec that I got from backprop. And if these two ways of computing the derivative give me the same answer, or give me any similar answers, up to a few decimal places, then I'm much more confident that my implementation of backprop is correct. And when I plug these DVec vectors into gradient assent or some advanced optimization algorithm, I can then be much more confident that I'm computing the derivatives correctly, and therefore that hopefully my code will run correctly and do a good job optimizing J of theta.

Implementation Note:

- Implement backprop to compute `DVec` (unrolled $D^{(1)}, D^{(2)}, D^{(3)}$).
- Implement numerical gradient check to compute `gradApprox`.
- Make sure they give similar values.
- Turn off gradient checking. Using backprop code for learning.

Important:

- Be sure to disable your gradient checking code before training your classifier. If you run numerical gradient computation on every iteration of gradient descent (or in the inner loop of `costFunction(...)`) your code will be very slow.

Finally, I wanna put everything together and tell you how to implement this numerical gradient checking. Here's what I usually do. First thing I do is implement back propagation to compute DVec. So there's a procedure we talked about in the earlier video to compute DVec which may be our unrolled version of these matrices. So then what I do, is implement a numerical gradient checking to compute gradApprox. So this is what I described earlier in this video and in the previous slide.

Then should make sure that DVec and gradApprox give similar values, you know let's say up to a few decimal places.

And finally and this is the important step, before you start to use your code for learning, for seriously training your network, it's important to turn off gradient checking and to no longer compute this gradApprox thing using the numerical derivative formulas that we talked about earlier in this video.

And the reason for that is the numeric code gradient checking code, the stuff we talked about in this video, that's a very computationally expensive, that's a very slow way to try to approximate the derivative. Whereas In contrast, the back propagation algorithm that we talked about earlier, that is the thing we talked about earlier for computing. You know, D1, D2, D3 for Dvec. Backprop is much more computationally efficient way of computing for derivatives.

So once you've verified that your implementation of back propagation is correct, you should turn off gradient checking and just stop using that. So just to reiterate, you should be sure to disable your gradient checking code before running your algorithm for many iterations of gradient descent or for many iterations of the advanced optimization algorithms, in order to train your classifier. Concretely, if you were to run the numerical gradient checking on every single iteration of gradient descent. Or if you were in the inner loop of your costFunction, then your code would be very slow. Because the numerical gradient checking code is much slower than the backpropagation algorithm, than the backpropagation method where, you remember, we were computing delta(4), delta(3), delta(2), and so on. That was the backpropagation algorithm. That is a much faster way to compute derivates than gradient checking. So when you're ready, once you've verified the implementation of back propagation is correct, make sure you turn off or you disable your gradient checking code while you train your algorithm, or else you code could run very slowly.

So, that's how you take gradients numericaly, and that's how you can verify tha implementation of back propagation is correct. Whenever I implement back propagation or similar gradient discerning algorithm for a complicated mode,l I always use gradient checking and this really helps me make sure that my code is correct.

What is the main reason that we use the backpropagation algorithm rather than the numerical gradient computation method during learning?

- [ ] The numerical gradient computation method is much harder to implement.
- [x] The numerical gradient algorithm is very slow.
- [ ] Backpropagation does not require setting the parameter EPSILON.
- [ ] None of the above.

✓ Correct

BackPropagation in Practice

Random initialization

In the previous video, we've put together almost all the pieces you need in order to implement and train in your network. There's just one last idea I need to share with you, which is the idea of random initialization.

When you're running an algorithm of gradient descent, or also the advanced optimization algorithms, we need to pick some initial value for the parameters theta. So for the advanced optimization algorithm, it assumes you will pass it some initial value for the parameters theta.

Now let's consider a gradient descent. For that, we'll also need to initialize theta to something, and then we can slowly take steps to go downhill using gradient descent. To go downhill, to minimize the function j of theta. So what can we set the initial value of theta to? Is it possible to set the initial value of theta to the vector of all zeros? Whereas this worked okay when we were using logistic regression, initializing all of your parameters to zero actually does not work when you are trading on your own network. ~~Consider trading the follow Neural network, and let's say~~

Initial value of Θ

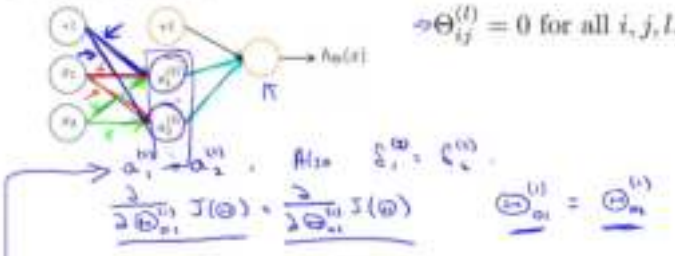
For gradient descent and advanced optimization method, need initial value for Θ.

```
optTheta = fminunc(@costFunction,
           initialTheta, options)
```

Consider gradient descent

Set `initialTheta = zeros(n,1)` ?

Zero initialization



After each update, parameters corresponding to inputs going into each of two hidden units are identical.

~~you are trading on your own network~~. Consider trading the follow Neural network, and let's say we initialize all the parameters of the network to 0. And if you do that, then what you, what that means is that at the initialization, this blue weight, colored in blue is gonna equal to that weight, so they're both 0. And this weight that I'm coloring in in red, is equal to that weight, colored in red, and also this weight, which I'm coloring in green is going to equal to the value of that weight. And what that means is that both of your hidden units, A1 and A2, are going to be computing the same function of your inputs. And thus you end up with for every one of your training examples, you end up with A 2 1 equals A 2 2.

And moreover because I'm not going to show this in too much detail, but because these outgoing weights are the same you can also show that the delta values are also gonna be the same. So concretely you end up with delta 1 1, delta 2 1 equals delta 2 2, and if you work through the map further, what you can show is that the partial derivatives with respect to your parameters will satisfy the following, that the partial derivative of the cost function with respected to breaking out the derivatives respect to these two blue waves in your network. You find that these two partial derivatives are going to be equal to each other.

And so what this means is that even after say one greater descent update, you're going to update, say, this first blue rate was learning rate times this, and you're gonna update the second blue rate with some learning rate times this. And what this means is that even after one created the descent update, those two blue rates, those two blue color parameters will end up the same as each other. So there'll be some nonzero value, but this value would equal to that value. And similarly, even after one gradient descent update, this value would equal to that value. There'll still be some non-zero values, just that the two red values are equal to each other. And similarly, the two green ways. Well, they'll both change values, but they'll both end up with the same value as each other. So after each update, the parameters corresponding to the inputs going into each of the two hidden units are identical. That's just saying that the two green weights are still the same, the two red weights are still the same, the two blue weights are still the same, and what that means is that even after one iteration of say, gradient descent and descent. You find that your two headed units are still computing exactly the same functions of the inputs. You still have the a1(2) = a2(2). And so you're back to this case. And as you keep running greater descent, the blue waves,, the two blue waves, will stay the same as each other. The two red waves will stay the same as each other and the two green waves will stay the same as each other.

And what this means is that your neural network really can compute very interesting functions, right? Imagine that you had not only two hidden units, but imagine that you had many, many hidden units. Then what this is saying is that all of your headed units are computing the exact same feature. All of your hidden units are computing the exact same function of the input. And this is a highly redundant representation because you find the logistic progression unit. It really has to see only one feature because all of these are the same. And this prevents you and your network from doing something interesting.

In order to get around this problem, the way we initialize the parameters of a neural network therefore is with random initialization.

Random initialization: Symmetry breaking

→ Initialize each $\Theta_{ij}^{(l)}$ to a random value in $[-\epsilon, \epsilon]$
(i.e. $-\epsilon \leq \Theta_{ij}^{(l)} \leq \epsilon$)

E.g.

random 10×11 matrix (betw. 0 and 1)

```
→ Theta1 = rand(10,11)*(2*INIT_EPSILON)
           - INIT_EPSILON;
```

$[-\epsilon, \epsilon]$

```
→ Theta2 = rand(1,11)*(2*INIT_EPSILON)
           - INIT_EPSILON;
```

Concretely, the problem was saw on the previous slide is something called the problem of symmetric ways, that's the ways are being the same. So this random initialization is how we perform symmetry breaking. So what we do is we initialize each value of theta to a random number between minus epsilon and epsilon. So this is a notation to b numbers between minus epsilon and plus epsilon. So my weight for my parameters are all going to be randomly initialized between minus epsilon and plus epsilon. The way I write code to do this in octave is I've said Theta1 should be equal to this. So this rand 10 by 11, that's how you compute a random 10 by 11 dimensional matrix. All the values are between 0 and 1, so these are going to be raw numbers that take on any continuous values between 0 and 1. And so if you take a number between zero and one, multiply it by two times INIT_EPSILON then minus INIT_EPSILON, then you end up with a number that's between minus epsilon and plus epsilon.

And the so that leads us, this epsilon here has nothing to do with the epsilon that we were using when we were doing gradient checking. So when numerical gradient checking, there we were adding some values of epsilon and theta. This is your unrelated value of epsilon. We just wanted to notate init epsilon just to distinguish it from the value of epsilon we were using in gradient checking. And similarly if you want to initialize theta2 to a random 1 by 11 matrix you can do so using this piece of code here.

So to summarize, to create a neural network what you should do is randomly initialize the waves to small values close to zero, between -epsilon and +epsilon say. And then implement back propagation, do great in checking, and use either great in descent or 1b advanced optimization algorithms to try to minimize j(theta) as a function of the parameters theta starting from just randomly chosen initial value for the parameters. And by doing symmetry breaking, which is this process, hopefully great gradient descent or the advanced optimization algorithms will be able to find a good value of theta.

Consider this procedure for initializing the parameters of a neural network:

1. Pick a random number r = rand(1,1) * (2 * INIT_EPSILON) - INIT_EPSILON;
2. Set $\Theta_{ij}^{(l)} = r$ for all i, j, l.

Does this work?

- Yes, because the parameters are chosen randomly.
- Yes, unless we are unlucky and get r=0 (up to numerical precision).
- Maybe, depending on the training set inputs x(i).
- (●) No, because this fails to break symmetry.

✓ Correct

Hence, we initialize each $\Theta_{ij}^{(l)}$ to a random value between $[-\epsilon, \epsilon]$. Using the above formula guarantees that we get the desired bound. The same procedure applies to all the Θ's. Below is some working code you could use to experiment.

```
1  If the dimensions of Theta1 is 10x11, Theta2 is 10x11 and Theta3 is 1x11.
2
3  Theta1 = rand(10,11) * (2 * INIT_EPSILON) - INIT_EPSILON;
4  Theta2 = rand(10,11) * (2 * INIT_EPSILON) - INIT_EPSILON;
5  Theta3 = rand(1,11) * (2 * INIT_EPSILON) - INIT_EPSILON;
6
```

rand(x,y) is just a function in octave that will initialize a matrix of random real numbers between 0 and 1.

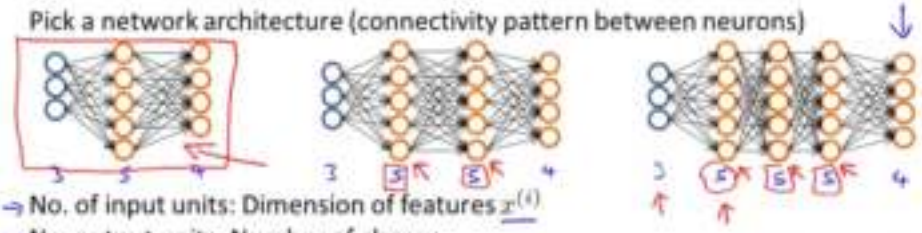
(Note: the epsilon used above is unrelated to the epsilon from Gradient Checking)

Back Propagation in Practice

Putting it together

Training a neural network

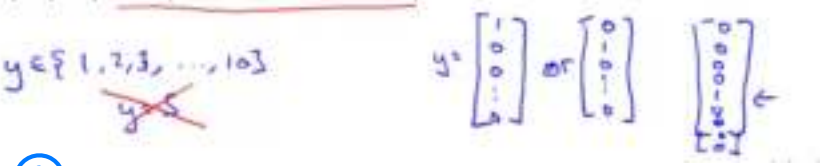
Pick a network architecture (connectivity pattern between neurons)



→ No. of input units: Dimension of features $x^{(i)}$

→ No. output units: Number of classes

Reasonable default: 1 hidden layer, or if >1 hidden layer, have same no. of hidden units in every layer (usually the more the better)



(1)

In this video, what I'd like to do is try to put all the pieces together, to give a overall summary or a bigger picture view, of how all the pieces fit together and of the overall process of how to implement a neural network learning algorithm.

When training a neural network, the first thing you need to do is pick some network architecture and by architecture I just mean connectivity pattern between the neurons. So, you know, we might choose between say, a neural network with three input units and five hidden units and four output units versus one of 3, 5 hidden, 5 hidden, 4 output and here are 3, 5, 5, 5 units in each of three hidden layers and four open units, and so these choices of how many hidden units in each layer and how many hidden layers, those are architecture choices. So, how do you make these choices?

Well first, the number of input units well that's pretty well defined. And once you decides on the fix set of features x the number of input units will just be, you know, the dimension of your features x(i) would be determined by that. And if you are doing multiclass classifications the number of output of this will be determined by the number of classes in your classification problem. And just a reminder if you have a multiclass classification where y takes on say values between

1 and 10, so that you have ten possible classes.

Then remember to right, your output y as these were the vectors. So instead of clause one, you recode it as a vector like that, or for the second class you recode it as a vector like that. So if one of these apples takes on the fifth class, you know, y equals 5, then what you're showing to your neural network is not actually a value of y equals 5, instead here at the upper layer which would have ten output units, you will instead feed to the vector which you know

with one in the fifth position and a bunch of zeros down here. So the choice of number of input units and number of output units is maybe somewhat reasonably straightforward.

And as for the number of hidden units and the number of hidden layers, a reasonable default is to use a single hidden layer and so this type of neural network shown on the left with just one hidden layer is probably the most common.

Or if you use more than one hidden layer, again the reasonable default will be to have the same number of hidden units in every single layer. So here we have two hidden layers and each of these hidden layers have the same number five of hidden units and here we have, you know, three hidden layers and each of them has the same number, that is five hidden units.

Rather than doing this sort of network architecture on the left would be a perfect ably reasonable default.

(2)

And as for the number of hidden units - usually, the more hidden units the better; it's just that if you have a lot of hidden units, it can become more computationally expensive, but very often, having more hidden units is a good thing.

And usually the number of hidden units in each layer will be maybe comparable to the dimension of x, comparable to the number of features, or it could be any where from same number of hidden units of input features to maybe so that three or four times of that. So having the number of hidden units is comparable. You know, several times, or some what bigger than the number of input features is often a useful thing to do So, hopefully this gives you one reasonable set of default choices for neural architecture and and if you follow these guidelines, you will probably get something that works well, but in a later set of videos where I will talk specifically about advice for how to apply algorithms, I will actually say a lot more about how to choose a neural network architecture. Or actually have quite a lot I want to say later to make good choices for the number of hidden units, the number of hidden layers, and so on.

Training a neural network

→ 1. Randomly initialize weights

→ 2. Implement forward propagation to get $h_\Theta(x^{(i)})$ for any $x^{(i)}$

→ 3. Implement code to compute cost function $J(\Theta)$

→ 4. Implement backprop to compute partial derivatives $\frac{\partial}{\partial \Theta_{jk}^{(l)}} J(\Theta)$

→ for i = 1:m {

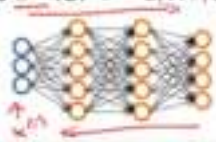
→ Perform forward propagation and backpropagation using example $(x^{(i)}, y^{(i)})$

(Get activations $a^{(l)}$ and delta terms $\delta^{(l)}$ for $l = 2, \ldots, L$).

→ $\Delta^{(l)} := \Delta^{(l)} + \delta^{(l+1)} (a^{(l)})^T$

}

Compute $\frac{\partial}{\partial \Theta_{jk}^{(l)}} J(\Theta)$.



(1)

Next, here's what we need to implement in order to trade in neural network, there are actually six steps that I have; I have four on this slide and two more steps on the next slide. First step is to set up the neural network and to randomly initialize the values of the weights. And we usually initialize the weights to small values near zero.

Then we implement forward propagation so that we can input any excellent neural network and compute h of x which is this output vector of the y values.

We thee also implement code to compute this cost function j of theta.

And next we implement back prop, or the back propagation

algorithm, to compute these partial derivatives terms, partial derivatives of j of theta with respect to the parameters. Concretely, to implement back prop. Usually we will do that with a fore loop over the training examples.

Some of you may have heard of advanced, and frankly very advanced factorization methods where you don't have a four-loop over the m-training examples, that the first time you're implementing back prop there should almost certainly the four loop in your code, where you're iterating over the examples, you know, x1, y1, then so you do forward prop and back prop on the first example, and then in the second iteration of the four loop, you do forward propagation and back propagation on the second example, and so on. Until you get through the final example. So there should be a four-loop in your implementation of back prop, at least the first time implementing it. And then there are frankly somewhat complicated ways to do this without a four-loop, but I definitely do not recommend trying to do that much more complicated version the first time you try to implement back prop.

(2)

So concretely, we have a four-loop over my m-training examples

and inside the four-loop we're going to perform fore prop and back prop using just this one example.

And what that means is that we're going to take x(i), and feed that to my input layer, perform forward-prop, perform back-prop

and that will if all of these activations and all of these delta terms for all of the layers of all my units in the neural network then still inside this four-loop, let me draw some curly braces just to show the scope with the four loop, this is in

octave code of course, but it's more a sequence Java code, and a four-loop encompasses all this. We're going to compute those delta terms, which are is the formula that we gave earlier.

Plus, you know, delta l plus one times

a, l transpose of the code. And then finally, outside the having computed these delta terms, these accumulation terms, we would then have some other code and then that will allow us to compute these partial derivative terms. Right and these partial derivative terms have to take into account the regularization term lambda as well. And so, those formulas were given in the earlier video.

So, how do you done that you now hopefully have code to compute these partial derivative terms.

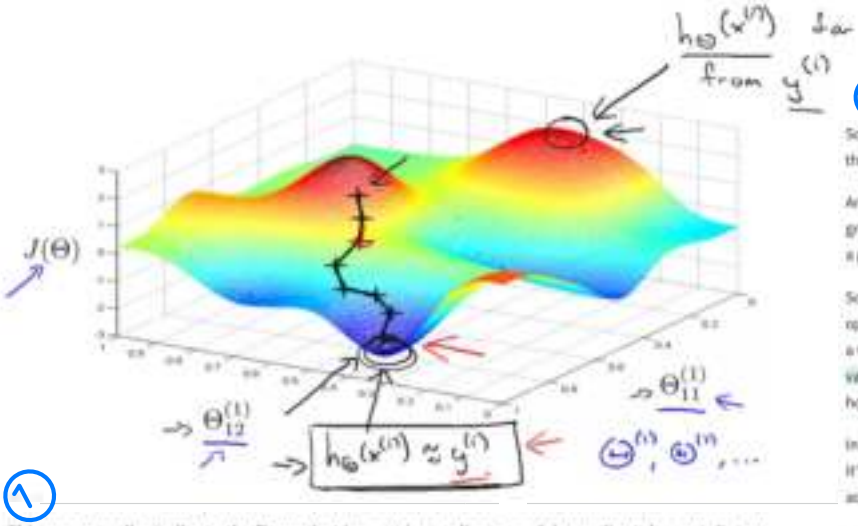
Training a neural network

5. Use gradient checking to compare $\frac{\partial}{\partial \Theta_{jk}^{(l)}} J(\Theta)$ computed using backpropagation vs. using numerical estimate of gradient of $J(\Theta)$.
   - Then disable gradient checking code.
6. Use gradient descent or advanced optimization method with backpropagation to try to minimize $J(\Theta)$ as a function of parameters Θ

$\frac{\partial}{\partial \Theta_{jk}^{(l)}} J(\Theta)$

$J(\Theta)$ — non-convex.

Finally, gradient descents for a neural network might still seem a little bit magical. So, let me just show one more figure to try to get that intuition about what gradient descent for a neural network is doing.



This was actually similar to the figure that I was using earlier to explain gradient descent. So, we have some cost function, and we have a number of parameters in our neural network. Right here I've just written down two of the parameter values. In reality, of course, in the neural network, we can have lots of parameters with these. Theta one, theta two—all of these are matrices, right? So we can have very high dimensional parameters but because of the limitations the source of parts we can draw. I'm pretending that we have only two parameters in this neural network. Although obviously we have a lot more in practice.

Now, this cost function j of theta measures how well the neural network fits the training data.

So, if you take a point like this one, down here,

that's a point where j of theta is pretty low, and so this corresponds to a setting of the parameters. There's a setting of the parameters theta, where, you know, for most of the training examples, the output

of my hypothesis, that may be pretty close to y(i) and if this is true than that's what causes my cost function to be pretty low.

Whereas in contrast, if you were to take a value like that, a point like that corresponds to, where for many training examples, the output of my neural network is far from the actual value y(i) that was observed in the training set. So points like this on the line correspond to where the hypothesis, where the neural network is outputting values on the training set that are far from y(i). So, it's not fitting the training set well, whereas points like this with low values of the cost function corresponds to where j of theta is low, and therefore corresponds to where the neural network happens to be fitting my training set well, because I mean this is what's needed to be true in order for j of theta to be small.

Next is step five, what I do is then use gradient checking to compare these partial derivative terms that were computed. So, I've compared the versions computed using back propagation versus the partial derivatives computed using the numerical

estimates as using numerical estimates of the derivatives. So, I do gradient checking to make sure that both of these give you very similar values.

Having done gradient checking just now reassures us that our implementation of back propagation is correct, and is then very important that we disable gradient checking, because the gradient checking code is computationally very slow.

And finally, we then use an optimization algorithm such as gradient descent, or one of the advanced optimization methods such as LBFGS, conjugate gradient has embodied into fminunc or other optimization methods. We use these together with back propagation, so back propagation is the thing that computes these partial derivatives for us.

And so, we know how to compute the cost function, we know how to compute the partial derivatives using back propagation, so we can use one of these optimization methods to try to minimize j of theta as a function of the parameters theta. And by the way, for neural networks, this cost function j of theta is non-convex, or is not convex and so it can theoretically be susceptible to local minima, and in fact algorithms like gradient descent and the advance optimization methods can, in theory, get stuck in local

optima, but it turns out that in practice this is not usually a huge problem and even though we can't guarantee that these algorithms will find a global optimum, usually algorithms like gradient descent will do a very good job minimizing this cost function j of theta and get a very good local minimum, even if it doesn't get to the global optimum. ~~Finally, gradient descents for a neural~~

So what gradient descent does is we'll start from some random initial point like that one over there, and it will repeatedly go downhill.

And so what back propagation is doing is computing the direction of the gradient, and what gradient descent is doing is it's taking little steps downhill until hopefully it gets to, in this case, a pretty good local optimum.

So, when you implement back propagation and use gradient descent or one of the advanced optimization methods, this picture sort of explains what the algorithm is doing. It's trying to find a value of the parameters where the output values in the neural network closely matches the values of the y(i)'s observed in your training set. So, hopefully this gives you a better sense of how the many different pieces of neural network learning fit together.

In case even after this video, in case you still feel like there are, like, a lot of different pieces and it's not entirely clear what some of them do or how all of these pieces come together, that's actually okay.

Neural network learning and back propagation is a complicated algorithm.

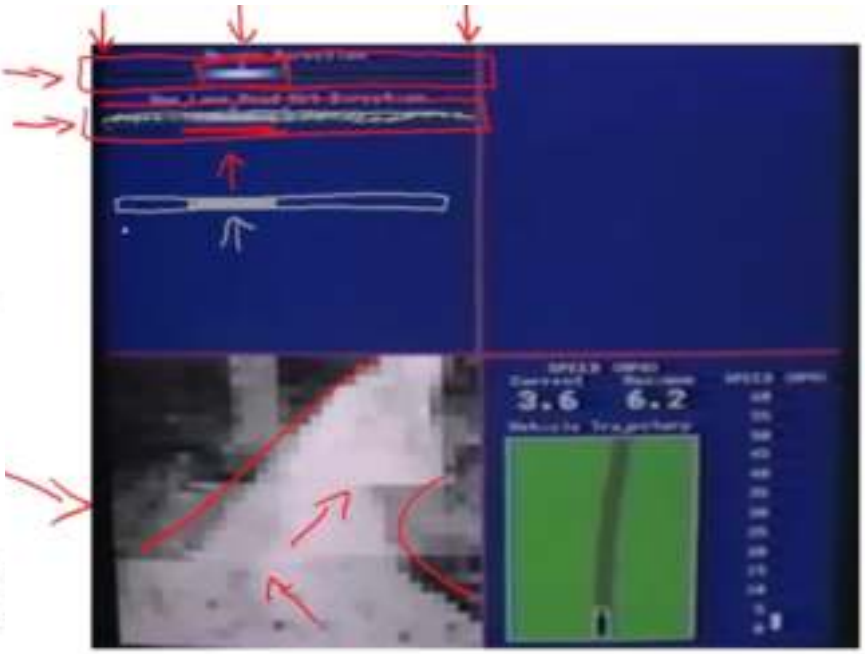
And even though I've seen the math behind back propagation for many years and I've used back propagation, I think very successfully, for many years, even today I still feel like I don't always have a great grasp of exactly what back propagation is doing sometimes. And what the optimization process looks like of minimizing j if theta. Much this is a much harder algorithm to feel like I have a much less good handle on exactly what this is doing compared to say, linear regression or logistic regression.

Which were mathematically and conceptually much simpler and much cleaner algorithms.

But so in case if you feel the same way, you know, that's actually perfectly okay, but if you do implement back propagation, hopefully what you find is that this is one of the most powerful learning algorithms and if you implement this algorithm, implement back propagation, implement one of these optimization methods, you find that back propagation will be able to fit very complex, powerful, non-linear functions to your data, and this is one of the most effective learning algorithms we have today.

Application of Neural networks

Autonomous Driving



In this video, I'd like to show you a fun and historically important example of neural networks learning of using a neural network for autonomous driving. That is getting a car to learn to drive itself.

The video that I'll showed a minute was something that I'd gotten from Dean Pomerleau, who was a colleague who works out in Carnegie Mellon University out on the east coast of the United States. And in part of the video you see visualizations like this. And I want to tell what a visualization looks like before starting the video.

Down here on the lower left is the view seen by the car of what's in front of it. And so here you kinda see a road that's maybe going a bit to the left, and then going a little bit to the right.

And up here on top, this first horizontal bar shows the direction selected by the human driver. And this location of this bright white band that shows the steering direction selected by the human driver where you know here far to the left corresponds to steering hard left, here corresponds to steering hard to the right. And so this location which is a little bit to the left, a little bit left of center means that the human driver at this point was steering slightly to the left. And this second but here corresponds to the steering direction selected by the learning algorithm and again the location of this sort of white band means that the neural network was here selecting a steering direction that's slightly to the left. And in fact before the neural network starts learning initially, you see that the network outputs a grey band, like a grey, like a uniform grey band throughout this region and sort of a uniform gray fuzz corresponds to the neural network having been randomly initialized. And initially having no idea how to drive the car. Or initially having no idea of what direction to steer in. And it only after it has learned for a while, that will then start to output like a solid white band in just a small part of the region corresponding to choosing a particular steering direction. And that corresponds to when the neural network becomes more confident in selecting a band in one particular location, rather than outputting a sort of light gray fuzz, but instead outputting a white band that's more constantly selecting one's steering direction. >> ALVINN is a system of artificial neural networks that learns to steer by

Evaluating a learning Algorithm

Deciding what to try next

Debugging a learning algorithm:
Suppose you have implemented regularized linear regression to predict housing prices.

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda\sum_{j=1}^{n}\theta_j^2\right]$$

However, when you test your hypothesis on a new set of houses, you find that it makes unacceptably large errors in its predictions. What should you try next?

- Get more training examples
- Try smaller sets of features
- Try getting additional features
- Try adding polynomial features $(x_1^2, x_2^2, x_1x_2, \text{etc.})$
- Try decreasing λ
- Try increasing λ

① By now you have seen a lot of different learning algorithms.

And if you've been following along these videos you should consider yourself an expert on many state of the art machine learning techniques. But even among people that know a certain learning algorithm. There's often a huge difference between someone that really knows how to powerfully and effectively apply that algorithm, versus someone that's less familiar with some of the material that I'm about to teach and who doesn't really understand how to apply these algorithms and can end up wasting a lot of their time trying things out that don't really make sense.

What I would like to do is make sure that if you are developing machine learning systems, that you know how to choose one of the most promising avenues to spend your time pursuing. And so this and the next few videos I'm going to give a number of practical suggestions, advice, guidelines on how to do that. And concretely what we'll focus on is the problem of, suppose you are developing a machine learning system or trying to improve the performance of a machine learning system, how do you go about deciding what are the promising avenues to try next?

② To explain this, let's continue using our example of learning to predict housing prices. And let's say you've implement your regularized linear regression. Thus minimizing that cost function J. Now suppose that after you take your learn parameters, if you test your hypothesis on the new set of houses, suppose you find that this is making huge errors in this prediction of the housing prices.

The question is what should you then try running in order to improve the learning algorithm?

There are many things that one can think of that could improve the performance of the learning algorithm.

One thing they could try, is to get more training examples. And concretely, you can imagine, maybe, you know, setting up phone surveys, going door to door, to try to get more data on how much different houses sell for.

And the sad thing is I've seen a lot of people spend a lot of time collecting more training examples, thinking etc. If we have twice as much or ten times as much training data, that is certainly going to help, right? but sometimes getting more training data doesn't actually help and in the next few videos we will see why, and we will see how you can avoid spending a lot of time collecting more training data in settings where it is just not going to help.

Other things you might try are to well maybe try a smaller set of features. So if you have some set of features such as x_1, x_2, x_3 and so on, maybe a large number of features. Maybe you want to spend time carefully selecting some small subset of them to prevent overfitting.

Or maybe you need to get additional features. Maybe the current set of features aren't informative enough and you want to collect more data in the sense of getting more features.

③ And once again this is the sort of project that can scale up the huge projects can you imagine getting phone surveys to find out more houses, or extra hired surveys to find out more about the places of land and so on, as a huge project. And once again it would be nice to know in advance if this is going to help before we spend a lot of time doing something like this. We can also try adding polynomial features things like x_1^2, x_2^2, x_1x_2, and product features x_1, x_2^3. We can still spend quite a lot of time thinking about that and we can also try other things like decreasing lambda, the regularization parameter or increasing lambda.

Given a menu of options like these, some of which can easily scale up to six month or longer projects.

Unfortunately, the most common method that people use to pick one of these is to go by gut feeling. In which what many people will do is sort of randomly pick one of these options and maybe say, "Oh, lets go and get more training data." And easily spend six months collecting more training data or maybe someone else would rather be saying, "Well, let's go collect a lot more features on these houses in our data set." And I have a lot of times, sadly sees people spend, you know, literally 6 months doing one of these avenues that they have sort of at random only to discover six months later that that really wasn't a promising avenue to pursue.

Fortunately, there is a pretty simple technique that can let you very quickly rule out half of the things on this list as being potentially promising things to pursue. And there is a very simple technique, that if you run, can easily rule out many of these options,

and potentially save you a lot of time pursuing something that's just is not going to work.

④ **Machine learning diagnostic:**

Diagnostic: A test that you can run to gain insight what is/isn't working with a learning algorithm, and gain guidance as to how best to improve its performance.

Diagnostics can take time to implement, but doing so can be a very good use of your time.

In the next two videos after this, I'm going to first talk about how to evaluate learning algorithms.

And in the next few videos after that, I'm going to talk about these techniques,

which are called the machine learning diagnostics.

And what a diagnostic is, is a test you can run, to get insight into what is or isn't working with an algorithm, and which will often give you insight as to what are promising things to try to improve a learning algorithm's

performance. We'll talk about specific diagnostics later in this video sequence. But I should mention in advance that diagnostics can take time to implement and can sometimes, you know, take quite a lot of time to implement and understand but doing so can be a very good use of your time when you are developing learning algorithms because they can often save you from spending many months pursuing an avenue that you could have found out much earlier just was not going to be fruitful.

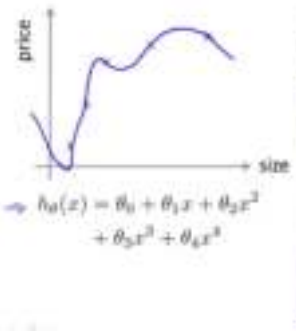
So in the next few videos, I'm going to first talk about how evaluate your learning algorithms and after that I'm going to talk about some of these diagnostics which will hopefully let you much more effectively select some of the useful things to try among if your goal to improve the machine learning system.

Evaluating a learning algorithm

Evaluating a Hypothesis

In this video I would like to talk about how to Evaluate a hypothesis that has been learned by your algorithm. In later videos, we will build on this to talk about how to Prevent in the Problems of overfitting and underfitting as well.

Evaluating your hypothesis



Fails to generalize to new examples not in training set.

x_1 = size of house
x_2 = no. of bedrooms
x_3 = no. of floors
x_4 = age of house
x_5 = average income in neighborhood
x_6 = kitchen size
⋮
x_{100}

~~of overfitting and underfitting as well.~~ When we fit the parameters of our learning algorithm we think about choosing the parameters to minimize the training error. One might think that getting a really low value of training error might be a good thing, but we have already seen that just because a hypothesis has low training error, that doesn't mean it is necessarily a good hypothesis. And we've already seen the example of how a hypothesis can overfit. And therefore fail to generalize the new examples not in the training set. So how do you tell if the hypothesis might be overfitting. In this simple example we could plot the hypothesis h of x and just see what was going on. But in general for problems with more features than just one feature, for problems with a large number of features like these it becomes hard or may be impossible to plot what the hypothesis looks like and so we need some other way to evaluate our hypothesis. The standard way to evaluate a learned hypothesis is as follows. Suppose we have a data set like this. Here I have just shown 10 training examples, but of course usually we may have dozens or hundreds or maybe thousands of training examples. In order to make sure we can evaluate our hypothesis, what we are going to do is split the data we have into two portions. The first portion is going to be our usual training set

and the second portion is going to be our test set, and a pretty typical split of this all the data we have into a training set and test set might be around say a 70%, 30% split. Worth more today to grade the training set and relatively less to the test set. And so now, if we have some data set, we run a sine of say 70% of the data to be our training set where here "m" is as usual our number of training examples and the remainder of our data might then be assigned to become our test set. And here, I'm going to use the notation m subscript test to denote the number of test examples. And so in general, this subscript test is going to denote examples that come from a test set so that x1 subscript test, y1 subscript test is my first test example which I guess in this example might be this example over here. Finally, one last detail whereas here I've drawn this as though the first 70% goes to the training set and the last 30% to the test set. If there is any sort of ordinary to the data. That should be better to send a random 70% of your data to the training set and a random 30% of your data to the test set. So if your data were already randomly sorted, you could just take the first 70% and last 30% that if your data were not randomly ordered, it would be better to randomly shuffle or to randomly reorder the examples in your training set. Before you know sending the first 70% in the training set and the last 30% of the test set. ~~Here then is a fairly~~

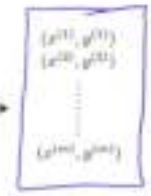
Evaluating your hypothesis

Dataset:

Size	Price
2104	400
1600	330
2400	369
1416	232
3000	540
1985	300
1534	315
1427	199
1380	212
1494	243

70% Training set

30% Test set



Training/testing procedure for linear regression

- Learn parameter θ from training data (minimizing training error $J(\theta)$) — 70%
- Compute test set error:

$$J_{test}(\theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} \left(h_\theta(x^{(i)}_{test}) - y^{(i)}_{test} \right)^2$$

~~sending the first 70% in the training set and the last 30% of the test set.~~ Here then is a fairly typical procedure for how you would train and test the learning algorithm and the learning regression. First, you learn the parameters theta from the training set so you minimize the usual training error objective j of theta, where j of theta here was defined using that 70% of all the data you have. There is only the training data. And then you would compute the test error. And I am going to denote the test error as j subscript test. And so what you do is take your parameter theta that you have learned from the training set, and plug it in here and compute your test set error. Which I am going to write as follows. So this is basically the average squared error as measured on your test set. It's pretty much what you'd expect. So if we run every test example through your hypothesis with parameter theta and just measure the squared error that your hypothesis has on your m subscript test, test examples. And of course, this is the definition of the test set error if we are using linear regression and using the squared error metric. How about if we were doing a classification problem and say using logistic regression instead. In that case, ~~the procedure for~~

PTO (Please turn over)

Training/testing procedure for logistic regression

- Learn parameter θ from training data
- Compute test set error:

$$J_{test}(\theta) = -\frac{1}{m_{test}} \sum_{i=1}^{m_{test}} y_{test}^{(i)} \log h_\theta(x_{test}^{(i)}) + (1 - y_{test}^{(i)}) \log h_\theta(x_{test}^{(i)})$$

- Misclassification error (0/1 misclassification error):

$$err(h_\theta(x), y) = \begin{cases} 1 & \text{if } h_\theta(x) \geq 0.5, \ y = 0 \text{ or if } h_\theta(x) < 0.5, \ y = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Test error} = \frac{1}{m_{test}} \sum_{i=1}^{m_{test}} err(h_\theta(x_{test}^{(i)}), y_{test}^{(i)})$$

classification problem and say using logistic regression instead. In that case, the procedure for training and testing say logistic regression is pretty similar first we will do the parameters from the training data, that first 70% of the data. And it will compute the test error as follows. It's the same objective function as we always use but we just logistic regression, except that now is define using our m subscript test, test examples. While this definition of the test set error j subscript test is perfectly reasonable. Sometimes there is an alternative test sets metric that might be easier to interpret, and that's the misclassification error. It's also called the zero one misclassification error, with zero one denoting that you either get an example right or you get an example wrong. Here's what I mean. Let me define the error of a prediction. That is h of x. And given the label y as equal to one if my hypothesis outputs the value greater than equal to five and Y is equal to zero or if my hypothesis outputs a value of less than 0.5 and y is equal to one, right, so both of these cases basic respond to if your hypothesis mislabeled the example assuming your threshold at an 0.5. So either thought it was more likely to be 1, but it was actually 0, or your hypothesis stored was more likely to be 0, but the label was actually 1. And otherwise, we define this error function to be zero. If your hypothesis basically classified the example y correctly. We could then define the test error, using the misclassification error metric to be one of the m tests of sum from i equals one to m subscript test of the error of h of x(i) test comma y(i). And so that's just my way of writing out that this is exactly the fraction of the examples in my test set that my hypothesis has mislabeled. And so that's the definition of the test set error using the misclassification error of the 0 1 misclassification metric. So that's the standard technique for evaluating how good a learned hypothesis is. In the next video, we will adapt these ideas to helping us do things like choose what features like the degree polynomial to use with the learning algorithm or choose the regularization parameter for learning algorithm.

Evaluating a learning algorithm

Model selection and train/validating /test sets

Problems

Suppose you're left to decide what degree of polynomial to fit to a data set. So that what features to include that gives you a learning algorithm. Or suppose you'd like to choose the regularization parameter longer for learning algorithm. How do you do that? This account model selection process Browsers, and in our discussion of how to do this, we'll talk about not just how to split your data into the train and test sets, but how to switch data into what we discover is called the train, validation, and test sets. We'll see in this video just what these things are, and how to use them to do model selection. We've already seen a lot of times the problem of overfitting, in which just because a learning algorithm fits a training set well, that doesn't mean it's a good hypothesis. More generally, this is why the training set's error is not a good predictor for how well the hypothesis will do on new example. Concretely, if you fit some set of parameters. Theta0, theta1, theta2, and so on, to your training set. Then the fact that your hypothesis does well on the training set. Well, this doesn't mean much in terms of predicting how well your hypothesis will generalize to new examples not seen in the training set. And a more general principle is that once your parameter is what fit to some set of data. Maybe the training set, maybe something else. Then the error of your hypothesis as measured on that same data set, such as the training error, that's unlikely to be a good estimate of your actual generalization error. That is how well the hypothesis will generalize to new examples. Now let's consider the model selection problem.

Overfitting example

$$h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

Once parameters $\theta_0, \theta_1, \ldots, \theta_4$ were fit to some set of data (training set), the error of the parameters as measured on that data (the training error $J(\theta)$) is likely to be lower than the actual generalization error.

hypothesis will generalize to new examples. Now let's consider the model selection problem. Let's say you're trying to choose what degree polynomial to fit to data. So, should you choose a linear function, a quadratic function, a cubic function? All the way up to a 10th order polynomial.

So it's as if there's one extra parameter in this algorithm, which I'm going to denote d, which is, what degree of polynomial. Do you want to pick. So it's as if, in addition to the theta parameters, it's as if there's one more parameter, d, that you're trying to determine using your data set. So, the first option is d equals one, if you fit a linear function. We can choose d equals two, d equals three, all the way up to d equals 10. So, we'd like to fit this extra sort of parameter which I'm denoting by d. And concretely let's say that you want to choose a model, that is choose a degree of polynomial, choose one of these 10 models. And fit that model and also get some estimate of how well your fitted hypothesis was generalize to new examples. Here's one thing you could do. What you could, first take your first model and minimize the training error. And this would give you some parameter vector theta. And you could then take your second model, the quadratic function, and fit that to your training set and this will give you some other. Parameter vector theta. In order to distinguish between these different parameter vectors, I'm going to use a superscript one superscript two there where theta superscript one just means the parameters I get by fitting this model to my training data. And theta superscript two just means the parameters I get by fitting this quadratic function to my training data and so on. By fitting a cubic model I get parenthesis three up to, well, say theta 10. And one thing we could do is that take those parameters and look at test error. So I can compute on my test set J test of one, J test of theta two, and so on.

J test of theta three, and so on.

So I'm going to take each of my hypotheses with the corresponding parameters and just measure the performance of on the test set. Now, one thing I could do then is, in order to select one of these models, I could then see which model has the lowest test set error. And let's just say for this example that I ended up choosing the fifth order polynomial. So, this seems reasonable so far. But now let's say I want to take my fifth hypothesis, this, this, fifth order model, and let's say I want to ask, how well does this model generalize?

One thing I could do is look at how well my fifth order polynomial hypothesis had done on my test set. But the problem is this will not be a fair estimate of how well my hypothesis generalizes. And the reason is what we've done is we've fit this extra parameter d, that is this degree of polynomial. And what fits that parameter d, using the test set, namely, we chose the value of d that gave us the best possible performance on the test set. And so, the performance of my parameter vector theta5, on the test set, that's likely to be an overly optimistic estimate of generalization error. Right, so, that because I had fit this parameter d to my test set is no longer fair to evaluate my hypothesis on this test set, because I fit my parameters to this test set. I've chose the degree d of polynomial using the test set. And so my hypothesis is likely to do better on this test set than it would on new examples that it hasn't seen before, and that's which is, which is what I really care about. So just to reiterate, on the previous slide, we saw that if we fit some set of parameters, you know, say theta0, theta1, and so on, to some training set, then the performance of the fitted model on the training set is not predictive of how well the hypothesis will generalize to new examples. Is because those parameters were fit to the training set, so they're likely to do well on the training set, even if the parameters don't do well on other examples. And, in the procedure I just described on this line, we just did the same thing. And specifically, what we did was, we fit this parameter d to the test set. And by having fit the parameter to the test set, this means that the performance of the hypothesis on that test set may not be a fair estimate of how well the hypothesis is, is likely to do on examples we haven't seen before. To address this problem, in a model selection setting, if we want to evaluate a hypothesis,

Model selection

d = degree of polynomial

1. $h_\theta(x) = \theta_0 + \theta_1 x$ → $\Theta^{(1)}$ → $J_{test}(\Theta^{(1)})$
2. $h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2$ → $\Theta^{(2)}$ → $J_{test}(\Theta^{(2)})$
3. $h_\theta(x) = \theta_0 + \theta_1 x + \cdots + \theta_3 x^3$ → $\Theta^{(3)}$ → $J_{test}(\Theta^{(3)})$

⋮

10. $h_\theta(x) = \theta_0 + \theta_1 x + \cdots + \theta_{10} x^{10}$ → $\Theta^{(10)}$ → $J_{test}(\Theta^{(10)})$

Choose $\theta_0 + \ldots \theta_5 x^5$

How well does the model generalize? Report test set error $J_{test}(\theta^{(5)})$.

Problem: $J_{test}(\theta^{(5)})$ is likely to be an optimistic estimate of generalization error. I.e. our extra parameter (d = degree of polynomial) is fit to test set.

Evaluating your hypothesis

Dataset:

	Size	Price
60%	2104	400
	1600	330
	2400	369
	1416	232
	3000	540
	1985	300
20%	1534	315
	1427	199
20%	1380	212
	1494	243

Training set

Cross validation (CV) set

Test set

$(x^{(1)}, y^{(1)})$, $(x^{(2)}, y^{(2)})$, …, $(x^{(m)}, y^{(m)})$

$(x_{cv}^{(1)}, y_{cv}^{(1)})$, $(x_{cv}^{(2)}, y_{cv}^{(2)})$, …, $(x_{cv}^{(m_{cv})}, y_{cv}^{(m_{cv})})$ — m_{cv} = no. of cv examples

$(x_{test}^{(1)}, y_{test}^{(1)})$, $(x_{test}^{(2)}, y_{test}^{(2)})$, …, $(x_{test}^{(m_{test})}, y_{test}^{(m_{test})})$ — m_{test}

validation test

~~before~~. To address this problem, in a model selection setting, if we want to evaluate a hypothesis, this is what we usually do instead. Given the data set, instead of just splitting into a training test set, what we're going to do is then split it into three pieces. And the first piece is going to be called the training set as usual.

So let me call this first part the training set.

And the second piece of this data, I'm going to call the cross validation set. [SOUND] Cross validation. And the cross validation, as V-D. Sometimes it's also called the validation set instead of cross validation set. And then the loss can be to call the usual test set. And the pretty, pretty typical ratio at which to split these things will be to send 60% of your data's, your training set, maybe 20% to your cross validation set, and 20% to your test set. And these numbers can vary a little bit but this integration be pretty typical. And so our training sets will now be only maybe 60% of the data, and our cross-validation set, or our validation set, will have some number of examples. I'm going to denote that m subscript cv. So that's the number of cross-validation examples.

Following our early notational convention I'm going to use xi cv comma y i cv, to denote the i cross validation example. And finally we also have a test set over here with our m subscript test being the number of test examples. ~~So, now that we've defined the training validation or cross~~

Train/validation/test error

Training error:

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2 \qquad J(\theta)$$

Cross Validation error:

$$J_{cv}(\theta) = \frac{1}{2m_{cv}} \sum_{i=1}^{m_{cv}} (h_\theta(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$

Test error:

$$J_{test}(\theta) = \frac{1}{2m_{test}} \sum_{i=1}^{m_{test}} (h_\theta(x_{test}^{(i)}) - y_{test}^{(i)})^2$$

being the number of test examples. So, now that we've defined the training validation or cross validation and test sets. We can also define the training error, cross validation error, and test error. So here's my training error, and I'm just writing this as J subscript train of theta. This is pretty much the same things. These are the same thing as the J of theta that I've been writing so far, this is just a training set error you know, as measuring a training set and then J subscript cv my cross validation error, this is pretty much what you'd expect, just like the training error you've set measure it on a cross validation data set, and here's my test set error same as before.

Model selection

1. $h_\theta(x) = \theta_0 + \theta_1 x$ → $\min_\theta J(\theta)$ → $\theta^{(1)}$ → $J_{cv}(\theta^{(1)})$
2. $h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2$ → $\theta^{(2)}$ → $J_{cv}(\theta^{(2)})$
3. $h_\theta(x) = \theta_0 + \theta_1 x + \cdots + \theta_3 x^3$ → $\theta^{(3)}$ → $J_{cv}(\theta^{(3)})$

⋮

10. $h_\theta(x) = \theta_0 + \theta_1 x + \cdots + \theta_{10} x^{10}$ → $\theta^{(10)}$ → $J_{cv}(\theta^{(10)})$

$d = 4$ →

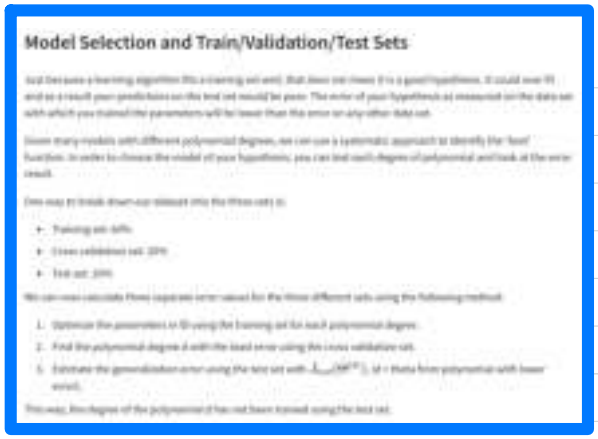
Pick $\theta_0 + \theta_1 x_1 + \cdots + \theta_4 x^4$ ←

Estimate generalization error for test set $J_{test}(\theta^{(4)})$ ←

So when faced with a model selection problem like this, what we're going to do is, instead of using the test set to select the model, we're instead going to use the validation set, or the cross validation set, to select the model. Concretely, we're going to first take our first hypothesis, take this first model, and say, minimize the cross function, and this would give me some parameter vector theta for the new model. And, as before, I'm going to put a superscript 1, just to denote that this is the parameter for the new model. We do the same thing for the quadratic model. Get some parameter vector theta two. Get some para, parameter vector theta three, and so on, down to theta ten for the polynomial. And what I'm going to do is, instead of testing these hypotheses on the test set, I'm instead going to test them on the cross validation set. And measure J subscript cv, to see how well each of these hypotheses do on my cross validation set.

And then I'm going to pick the hypothesis with the lowest cross validation error. So for this example, let's say for the sake of argument, that it was my 4th order polynomial, that had the lowest cross validation error. So in that case I'm going to pick this fourth order polynomial model. And finally, what this means is that that parameter d, remember d was the degree of polynomial, right? So d equals two, d equals three, all the way up to d equals 10. What we've done is we'll fit that parameter d and we'll say d equals four. And we did so using the cross-validation set. And so this degree of polynomial, so the parameter, is no longer fit to the test set, and so we've not saved away the test set, and we can use the test set to measure, or to estimate the generalization error of the model that was selected. By the of them. So, that was model selection and how you can take your data, split it into a training, validation, and test set. And use your cross validation data to select the model and evaluate it on the test set.

One final note, I should say that in. The machine learning, as of this practice today, there aren't many people that will do that early thing that I talked about, and said that, you know, it isn't such a good idea, of selecting your model using this test set. And then using the same test set to report the error as though selecting your degree of polynomial on the test set, and then reporting the error on the test set as though that were a good estimate of generalization error. That sort of practice is unfortunately many, many people do do it. If you have a massive, massive test that is maybe not a terrible thing to do, but many practitioners, most practitioners that machine learnimg tend to advise against that. And it's considered better practice to have separate train validation and test sets. I just warned you to sometimes people to do, you know, use the same data for the purpose of the validation set, and for the purpose of the test set. You need a training set and a test set, and that's good, that's practice, though you will see some people do it. But, if possible, I would recommend against doing that yourself.

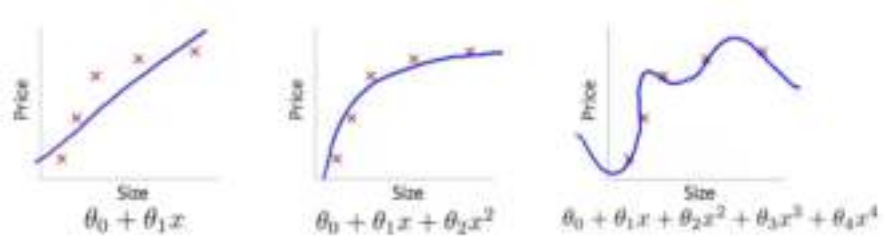

Model Selection and Train/Validation/Test Sets

Bias vs. Variance

Diagnosing Bias vs. Variance

If you run a learning algorithm and it doesn't do as long as you are hoping, almost all the time, it will be because you have either a high bias problem or a high variance problem, in other words, either an underfitting problem or an overfitting problem. In this case, it's very important to figure out which of these two problems is bias or variance or a bit of both that you actually have. Because knowing which of these two things is happening would give a very strong indicator for whether the useful and promising ways to try to improve your algorithm. In this video, I'd like to delve more deeply into this bias and variance issue and understand them better as was figure out how to look in a learning algorithm and evaluate or diagnose whether we might have a bias problem or a variance problem since this will be critical to figuring out how to improve the performance of a learning algorithm that you will implement. So, you've already seen this figure

Bias/variance



High bias (underfit) $d=1$

"Just right" $d=2$

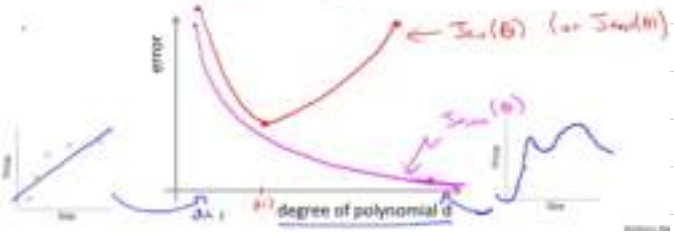
High variance (overfit) $d=4$

performance of a learning algorithm that you will implement. So, you've already seen this figure a few times where if you fit two simple hypothesis like a straight line that underfits the data, if you fit a two complex hypothesis, then that might fit the training set perfectly but overfit the data and this may be hypothesis of some intermediate level of complexities of some maybe degree two polynomials or not too low and not too high degree that's like just right and gives you the best generalization error over these options. Now that we're armed with the notion of chain training and validation in test sets, we can understand the concepts of bias and variance a little bit better. Concretely, let's let our training error and cross validation error be defined as in the

Bias/variance

Training error: $J_{train}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$

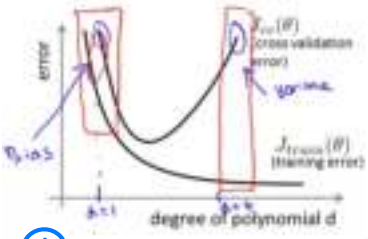
Cross validation error: $J_{cv}(\theta) = \frac{1}{2m_{cv}}\sum_{i=1}^{m_{cv}}(h_\theta(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$ ($\leftarrow J_{train}(\theta)$)



bit better. Concretely, let's let our training error and cross validation error be defined as in the previous videos. Just say the squared error, the average squared error, as measured on the training sets or as measured on the cross validation set. Now, let's plot the following figure. On the horizontal axis I'm going to plot the degree of polynomial. So, as I go to the right I'm going to be fitting higher and higher order polynomials. So where the left of this figure where maybe d equals one, we're going to be fitting very simple functions whereas we're here on the right of the horizontal axis, I have much larger values of ds, of a much higher degree polynomial. So here, that's going to correspond to fitting much more complex functions to your training set. Let's look at the training error and the cross validation error and plot them on this figure. Let's start with the training error. As we increase the degree of the polynomial, we're going to be able to fit our training set better and better and so if d equals one, then there is high training error, if we have a very high degree of polynomial our training error is going to be really low, maybe even 0 because will fit the training set really well. So, as we increase the degree of polynomial, we find typically that the training error decreases. So I'm going to write J subscript train of theta there, because our training error tends to decrease with the degree of the polynomial that we fit to the data. Next, let's look at the cross-validation error or for that matter, if we look at the test set error, we'll get a pretty similar result as if we were to plot the cross validation error. So, we know that if d equals one, we're fitting a very simple function and so we may be underfitting the training set and so it's going to be very high cross-validation error. If we fit an intermediate degree polynomial, we had d equals two in our example in the previous slide, we're going to have a much lower cross-validation error because we're finding a much better fit to the data. Conversely, if d were too high. So if d took on say a value of four, then we're again overfitting, and so we end up with a high value for cross-validation error. So, if you were to vary this smoothly and plot a curve, you might end up with a curve like that where that's JCV of theta. Again, if you plot J test of theta you get something very similar. So, this sort of plot also helps us to better understand the notions of bias and variance. Concretely, suppose you have applied a learning

Diagnosing bias vs. variance

Suppose your learning algorithm is performing less well than you were hoping. ($J_{cv}(\theta)$ or $J_{test}(\theta)$ is high.) Is it a bias problem or a variance problem?



Bias (underfit):
→ $J_{train}(\Theta)$ will be high
$J_{cv}(\Theta) \approx J_{train}(\Theta)$

Variance (overfit):
→ $J_{train}(\Theta)$ will be low
$J_{cv}(\Theta) \gg J_{train}(\Theta)$

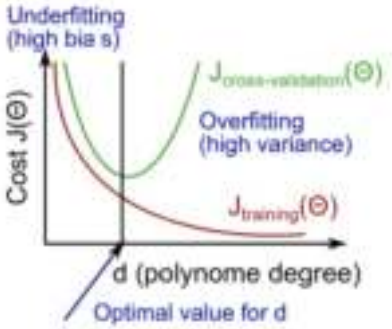
①

~~understand the notions of bias and variance.~~ Concretely, suppose you have applied a learning algorithm and it's not performing as well as you are hoping, so if your cross-validation set error or your test set error is high, how can we figure out if the learning algorithm is suffering from high bias or suffering from high variance? So, the setting of a cross-validation error being high corresponds to either this regime or this regime. So, this regime on the left corresponds to a high bias problem. That is, if you are fitting a overly low order polynomial such as a d equals one when we really needed a higher order polynomial to fit to data, whereas in contrast this regime corresponds to a high variance problem. That is, if d the degree of polynomial was too large for the data set that we have, and this figure gives us a clue for how to distinguish between these two cases. Concretely, for the high bias case, that is the case of underfitting, what we find is that both the cross validation error and the training error are going to be high. So, if your algorithm is

②

suffering from a bias problem, the training set error will be high and you might find that the cross validation error will also be high. It might be close, maybe just slightly higher, than the training error. So, if you see this combination, that's a sign that your algorithm may be suffering from high bias. In contrast, if your algorithm is suffering from high variance, then if you look here, we'll notice that J train, that is the training error, is going to be low. That is, you're fitting the training set very well, whereas your cross validation error assuming that this is, say, the squared error which we're trying to minimize say, whereas in contrast your error on a cross validation set or your cross function or cross validation set will be much bigger than your training set error. So, this is a double greater than sign. That's the map symbol for much greater thans, denoted by two greater than signs. So if you see this combination of values, then that's a clue that your learning algorithm may be suffering from high variance and might be overfitting. The key that distinguishes these two cases is, if you have a high bias problem, your training set error will also be high is your hypothesis just not fitting the training set well. If you have a high variance problem, your training set error will usually be low, that is much lower than your cross-validation error. So hopefully that gives you a somewhat better understanding of the two problems of bias and variance. I still have a lot more to say about bias and variance in the next few videos, but what we'll see later is that by diagnosing whether a learning algorithm may be suffering from high bias or high variance, I'll show you even more details on how to do that in later videos. But we'll see that by figuring out whether a learning algorithm may be suffering from high bias or high variance or combination of both, that that would give us much better guidance for what might be promising things to try in order to improve the performance of a learning algorithm.

The is summarized in the figure below:



Bias vs. Variance

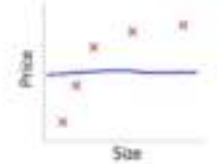
Regularization and Bias / Variance

You've seen how regularization can help prevent over-fitting. But how does it affect the bias and variances of a learning algorithm? In this video I'd like to go deeper into the issue of bias and variances and talk about how it interacts with and is affected by the regularization of your learning algorithm.

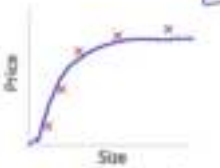
Linear regression with regularization

Model: $h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$ ←

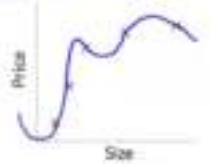
$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2$$ ←



Large λ ←
→ High bias (underfit)
→ $\lambda = 10000$. $\theta_1 \approx 0, \theta_2 \approx 0, \ldots$
$h_\theta(x) \approx \theta_0$



Intermediate λ ←
"Just right"



→ Small λ
High variance (overfit)
→ $\lambda = 0$

Suppose we're fitting a high auto polynomial, like that showed here, but to prevent over fitting we need to use regularization, like that shown here. So we have this regularization term to try to keep the values of the peem to small. And as usual, the regularizations comes from J = 1 to m, rather than j = 0 to m. Let's consider three cases. The first is the case of the very large value of the regularization parameter lambda, such as if lambda were equal to 10,000. Some huge value.

In this case, all of these parameters, theta 1, theta 2, theta 3, and so on would be heavily penalized and so we end up with most of these parameter values being closer to zero. And the hypothesis will be roughly h of x, just equal or approximately equal to theta zero. So we end up with a hypothesis that more or less looks like that, more or less a flat, constant straight line. And so this hypothesis has high bias and it badly under fits this data set, so the horizontal straight line is just not a very good model for this data set. At the other extreme is if we have a very small value of lambda, such as if lambda were equal to zero. In that case, given that we're fitting a high order polynomial, this is a usual over fitting setting. In that case, given that we're fitting a high-order polynomial, basically, without regularization or with very minimal regularization, we end up with our usual high-variance, over fitting setting. This is basically if lambda is equal to zero, we're just fitting with our regularization, so that over fits the hypothesis. And it's only if we have some intermediate value of longer that is neither too large nor too small that we end up with parameters data that give us a reasonable fit to this data. So, how can we automatically choose a good value for the regularization parameter?

Choosing the regularization parameter λ

$$h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2$$

$$J_{train}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$

$$J_{cv}(\theta) = \frac{1}{2m_{cv}}\sum_{i=1}^{m_{cv}}(h_\theta(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$

$$J_{test}(\theta) = \frac{1}{2m_{test}}\sum_{i=1}^{m_{test}}(h_\theta(x_{test}^{(i)}) - y_{test}^{(i)})^2$$

Just to reiterate, here's our model, and here's our learning algorithm's objective. For the setting where we're using regularization, let me define J train(theta) to be something different, to be the optimization objective, but without the regularization term. Previously, in an earlier video, when we were not using regularization I define J train of data to be the same as J of theta as the cause function but when we're using regularization when the six well under term we're going to define J train my training set to be just my sum of squared errors on the training set or my average squared error on the training set without taking into account that regularization. And similarly I'm then also going to define the cross validation sets error and to test that error as before to be the average sum of squared errors on the cross validation in the test sets so just to summarize my definitions of J train J CU and J test are just the average square there one half of the other square record on the training validation of the test set without the extra regularization term. So, this is how we can automatically choose the regularization parameter lambda. So what I usually do is maybe have some range of values of lambda I want to try out. So I might be considering not using regularization or here are a few values I might try lambda considering lambda = 0.01, 0.02, 0.04, and so on. And I usually set these up in multiples of two, until some maybe larger value if I were to do these in multiples of 2 I'd end up with a 10.24. It's 10 exactly, but this is close enough. And the three to four decimal places won't effect your result that much. So, this gives me maybe 12 different models. And I'm trying to select a month corresponding to 12 different values of the regularization of the parameter lambda. And of course you can also go to values less than 0.01 or values larger than 10 but I've just truncated it here for convenience. Given the issue of these 12 models, what we can do is then the following, we can take this first model with lambda equals zero and minimize my cost function J of data and this will give me some parameter of active data. And similar to the earlier video, let me just denote this as theta super script one.

Choosing the regularization parameter λ

Model: $h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2$$

1. Try $\lambda = 0$ → $\min_\theta J(\theta)$ → $\theta^{(1)}$ → $J_{cv}(\theta^{(1)})$
2. Try $\lambda = 0.01$ → $\min_\theta J(\theta)$ → $\theta^{(2)}$ → $J_{cv}(\theta^{(2)})$
3. Try $\lambda = 0.02$ → $\theta^{(3)}$ → $J_{cv}(\theta^{(3)})$
4. Try $\lambda = 0.04$
5. Try $\lambda = 0.08$ → $\theta^{(5)}$ → $J_{cv}(\theta^{(5)})$

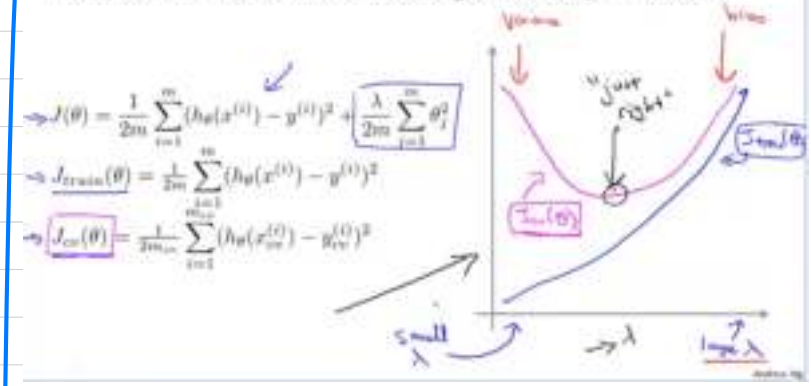
⋮

12. Try $\lambda = 10$ → $\theta^{(12)}$ → $J_{cv}(\theta^{(12)})$

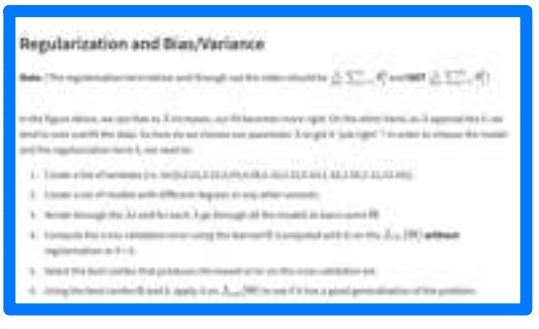
Pick (say) $\theta^{(5)}$. Test error: $J_{test}(\theta^{(5)})$

And then I can take my second model with lambda set to 0.01 and minimize my cost function now using lambda equals 0.01 of course. To get some different parameter vector theta. Let me denote that theta(2). And for that I end up with theta(3). So if part for my third model. And so on until for my final model with lambda set to 10 or 10.24, I end up with this theta(12). Next, I can talk all of these hypotheses, all of these parameters and use my cross validation set to validate them so I can look at my first model, my second model, fit to these different values of the regularization parameter, and evaluate them with my cross validation set based in measure the average square error of each of these square vector parameters theta on my cross validation sets. And I would then pick whichever one of these 12 models gives me the lowest error on the trans validation set. And let's say, for the sake of this example, that I end up picking theta 5, the 5th order polynomial, because that has the lowest cause validation error. Having done that, finally what I would do if I wanted to report each test set error, is to take the parameter theta 5 that I've selected, and look at how well it does on my test set. So once again, here is as if we've fit this parameter, theta, to my cross-validation set, which is why I'm setting aside a separate test set that I'm going to use to get a better estimate of how well my parameter vector, theta, will generalize to previously unseen examples. So that's model selection applied to selecting the regularization parameter lambda.

Bias/variance as a function of the regularization parameter λ



Regularization and Bias/Variance



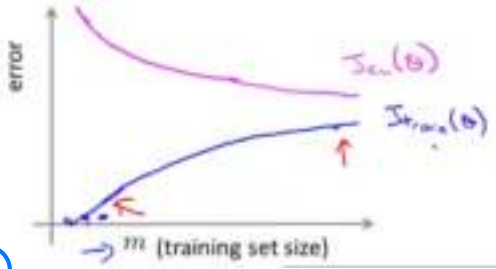
Bias Vs. Variance

Learning Curves

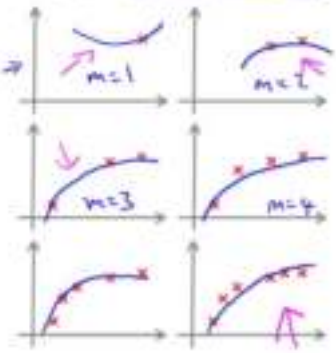
Learning curves

$$J_{train}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$

$$J_{cv}(\theta) = \frac{1}{2m_{cv}}\sum_{i=1}^{m_{cv}}(h_\theta(x_{cv}^{(i)}) - y_{cv}^{(i)})^2$$



$h_\theta(x) = \theta_0 + \theta_1 x + \theta_2 x^2$



(1)

In this video, I'd like to tell you about learning curves.

Learning curves is often a very useful thing to plot. If either you wanted to sanity check that your algorithm is working correctly, or if you want to improve the performance of the algorithm.

And learning curves is a tool that I actually use very often to try to diagnose if a physical learning algorithm may be suffering from bias, sort of variance problem or a bit of both.

Here's what a learning curve is. To plot a learning curve, what I usually do is plot j train which is, say,

average squared error on my training set or Jcv which is the average squared error on my cross validation set. And I'm going to plot that as a function of m, that is as a function of the number of training examples I have. And so m is usually a constant like maybe I just have, you know, a 100 training examples but what I'm going to do is artificially with use my training set exercise. So, I deliberately limit myself to using only, say, 10 or 20 or 30 or 40 training examples and plot what the training error is and what the cross validation is for this smallest training set exercises. So let's see what these plots may look like. Suppose I have only one training example like that shown in this this first example here and let's say I'm fitting a quadratic function. Well, I

have only one training example. I'm going to be able to fit it perfectly right? You know, just fit the quadratic function. I'm going to have 0 error on the one training example. If I have two training examples. Well the quadratic function can also fit that very well. So,

even if I am using regularization, I can probably fit this quite well. And if I am using no neural regularization, I'm going to fit this perfectly and if I have three training examples again. Yeah, I can fit a quadratic function perfectly so if m equals 1 or m equals 2 or m equals 3,

my training error on my training set is going to be 0 assuming I'm not using regularization or it may slightly large in 0 if I'm using regularization and by the way if I have a large training set and I'm artificially restricting the size of my training set in order to J train. Here if I set M equals 3, say, and I train on only three examples, then, for this figure I am going to measure my training error only on the three examples that actually fit my data too

and so even I have to say a 100 training examples but if I want to plot what my training error is the m equals 3. What I'm going to do

is to measure the training error on the three examples that I've actually fit to my hypothesis 2.

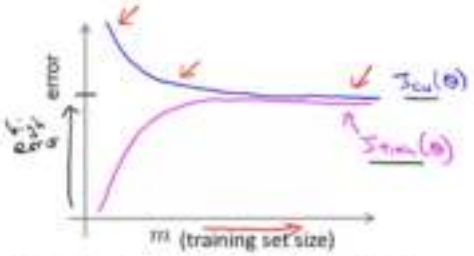
(2)

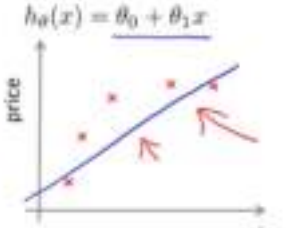
And not all the other examples that I have deliberately omitted from the training process. So just to summarize what we've seen is that if the training set size is small then the training error is going to be small as well. Because you know, we have a small training set is going to be very easy to fit your training set very well may be even perfectly now say we have m equals 4 for example. Well then a quadratic function can be a longer fit this data set perfectly and if I have m equals 5 then you know, maybe quadratic function will fit to stay there so on, then as my training set gets large

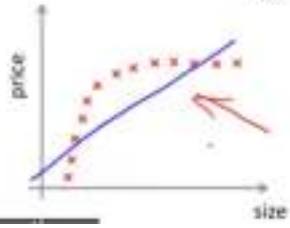
it becomes harder and harder to ensure that I can find the quadratic function that process through all my examples perfectly. So in fact as the training set size grows what you find is that my average training error actually increases and so if you plot this figure what you find is that the training set error that is the average error on your hypothesis grows as m grows and just to repeat when the intuition is that when m is small when you have very few training examples. It's pretty easy to fit every single one of your training examples perfectly and so your error is going to be

small whereas when m is larger then gets harder all the training examples perfectly and so your training set error becomes more larger now, how about the cross validation error. Well, the cross validation is my error on this cross validation set that I haven't seen and so, you know, when I have a very small training set, I'm not going to generalize well, just not going to do well on that. So, right, this hypothesis here doesn't look like a good one, and it's only when I get a larger training set that, you know, I'm starting to get hypotheses that maybe fit the data somewhat better. So your cross validation error and your test set error will tend to decrease as your training set size increases because the more data you have, the better you do at generalizing to new examples. So, just the more data you have, the better the hypothesis you fit. So if you plot j train, and Jcv this is the sort of thing that you get.

High bias







If a learning algorithm is suffering from high bias, getting more training data will not (by itself) help much.

(1)

well, Jcv this is the sort of thing that you get. Now let's look at what the learning curves may look like if we have either high bias or high variance problems. Suppose your hypothesis has high bias and to explain this I'm going to use a, set an example, of fitting a straight line to data that, you know, can't really be fit well by a straight line.

So we end up with a hypotheses that maybe looks like that.

Now let's think what would happen if we were to increase the training set size. So if instead of five examples like what I've drawn there, imagine that we have a lot more training examples.

Well what happens, if you fit a straight line to this. What you find is that, you end up with you know, pretty much the same straight line. I mean a straight line that just cannot fit this data and getting a ton more data, well the straight line isn't going to change that much. This is the best possible straight line fit to this data, but the straight line just can't fit this data set that well. So, if you plot across validation error,

this is what it will look like.

Option on the left, if you have already a miniscule training set size like you know, maybe just one training example and is not going to do well. But by the time you have reached a certain number of training examples, you have almost fit the best possible straight line, and even if you end up with a much larger training set size, a much larger value of m, you know, you're basically getting the same straight line, and so, the cross validation error - let me label that - or test set error or plateau out, or flatten out pretty soon, once you reached beyond a certain the number of training examples, unless you pretty much fit the best possible straight line. And how about training error? Well, the training error will again be small.

(2)

And what you find in the high bias case is that the training error will end up close to the cross validation error, because you have so few parameters and so much data, at least when m is large. The performance on the training set and the cross validation set will be very similar.

And so, this is what your learning curves will look like, if you have an algorithm that has high bias.

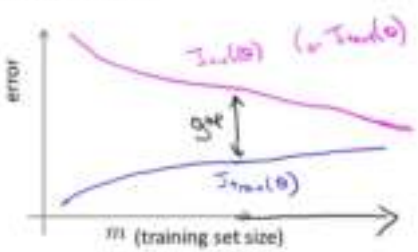
And finally, the problem with high bias is reflected in the fact that both the cross validation error and the training error are high, and so you end up with a relatively high value of both Jcv and the j train.

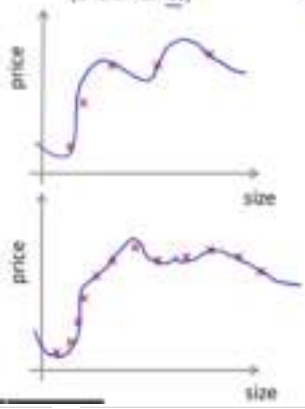
This also implies something very interesting, which is that, if a learning algorithm has high bias, as we get more and more training examples, that is, as we move to the right of this figure, we'll notice that the cross validation error isn't going down much, it's basically flattened up, and so if learning algorithms are really suffering from high bias.

Getting more training data by itself will actually not help that much,and as our figure example in the figure on the right, here we had only five training examples, and we fit certain straight line. And when we had a ton more training data, we still end up with roughly the same straight line. And so if the learning algorithm has high bias give me a lot more training data. That doesn't actually help you get a much lower cross validation error or test set error. So knowing if your learning algorithm is suffering from high bias seems like a useful thing to know because this can prevent you from wasting a lot of time collecting more training data where it might just not end up being helpful. Next let us look at the setting of a learning algorithm that may have high variance.

High variance

$h_\theta(x) = \theta_0 + \theta_1 x + \cdots + \theta_{100} x^{100}$
(and small λ)





If a learning algorithm is suffering from high variance, getting more training data is likely to help.

(1) Let us just look at the training error in a around if you have very smart training set like five training examples shown on the figure on the right and if we're fitting say a very high order polynomial,

and I've written a hundredth degree polynomial which really no one uses, but just an illustration.

And if we're using a fairly small value of lambda, maybe not zero, but a fairly small value of lambda, then we'll end up, you know, fitting this data very well that with a function that overfits this. So, if the training set size is small, our training error, that is, j train of theta will be small.

And as this training set size increases a bit, you know, we may still be overfitting this data a little bit but it also becomes slightly harder to fit this data set perfectly, and so, as the training set size increases, we'll find that j train increases, because it is just a little harder to fit the training set perfectly when we have more examples, but the training set error will still be pretty low. Now, how about the cross validation error? Well, in high variance setting, a hypothesis is overfitting and so the cross validation error will remain high, even as we got you know, a moderate number of training examples and, so maybe, the cross validation error may look like that. And the indicative diagnostic that we have a high variance problem,

is the fact that there's this large gap between the training error and the cross validation error.

(2) And looking at this figure. If we think about adding more training data, that is, taking this figure and extrapolating to the right, we can kind of tell that, you know the two curves, the blue curve and the magenta curve, are converging to each other. And so, if we were to extrapolate this figure to the right, then it seems it likely that the training error will keep on going up and the

cross-validation error would keep on going down. And the thing we really care about is the cross-validation error or the test set error, right? So in this sort of figure, we can tell that if we keep on adding training examples and extrapolate to the right, well our cross validation error will keep on coming down. And, so, in the high variance setting, getting more training data is, indeed, likely to help. And so again, this seems like a useful thing to know if your learning algorithm is suffering from a high variance problem, because that tells you, for example that it may be be worth your while to see if you can go and get some more training data.

(3) Now, on the previous slide and this slide, I've drawn fairly clean fairly idealized curves. If you plot these curves for an actual learning algorithm, sometimes you will actually see, you know, pretty much curves, like what I've drawn here. Although, sometimes you see curves that are a little bit noisier and a little bit messier than this. But plotting learning curves like these can often tell you, can often help you figure out if your learning algorithm is suffering from bias, or variance or even a little bit of both. So when I'm trying to improve the performance of a learning algorithm, one thing that I'll almost always do is plot these learning curves, and usually this will give you a better sense of whether there is a bias or variance problem.

And in the next video we'll see how this can help suggest specific actions is to take, or to not take, in order to try to improve the performance of your learning algorithm.

In which of the following circumstances is getting more training data likely to significantly help a learning algorithm's performance?

- [ ] Algorithm is suffering from high bias.
- [x] Algorithm is suffering from high variance.

✓ Correct

- [x] $J_{CV}(\theta)$ (cross validation error) is much larger than $J_{train}(\theta)$ (training error).

✓ Correct

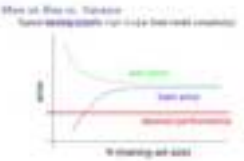
- [ ] $J_{CV}(\theta)$ (cross validation error) is about the same as $J_{train}(\theta)$ (training error).

Experiencing high bias:

Low training set size: causes $J_{train}(\Theta)$ to be low and $J_{CV}(\Theta)$ to be high.

Large training set size: causes both $J_{train}(\Theta)$ and $J_{CV}(\Theta)$ to be high with $J_{train}(\Theta) \approx J_{CV}(\Theta)$.

If a learning algorithm is suffering from **high bias**, getting more training data will not **(by itself)** help much.

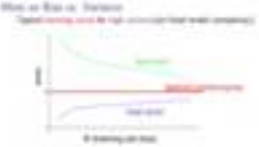


Experiencing high variance:

Low training set size: $J_{train}(\Theta)$ will be low and $J_{CV}(\Theta)$ will be high.

Large training set size: $J_{train}(\Theta)$ increases with training set size and $J_{CV}(\Theta)$ continues to decrease without leveling off. Also, $J_{train}(\Theta) < J_{CV}(\Theta)$ but the difference between them remains significant.

If a learning algorithm is suffering from **high variance**, getting more training data is likely to help.



Bias Vs. Variance

Deciding what to do next (Revisited)

Debugging a learning algorithm:
Suppose you have implemented regularized linear regression to predict housing prices. However, when you test your hypothesis in a new set of houses, you find that it makes unacceptably large errors in its prediction. What should you try next?

- Get more training examples → fixes high variance
- Try smaller sets of features → fixes high variance
- Try getting additional features → fixes high bias
- Try adding polynomial features $(x_1^2, x_2^2, x_1x_2, \text{etc})$ → fixes high bias.
- Try decreasing λ → fixes high bias
- Try increasing λ → fixes high variance

①

We've talked about how to evaluate learning algorithms, talked about model selection, talked a bit about bias and variance. So how does this help us figure out what are potentially fruitful, potentially not fruitful things to try to do to improve the performance of a learning algorithm.

Let's go back to our original motivating example and go for the result.

So here is our earlier example of maybe having fit regularized linear regression and finding that it doesn't work as well as we're hoping. We said that we had this menu of options. So is there some way to figure out which of these might be fruitful options? The first thing all of this was getting more training examples. What this is good for, is this helps to fix high variance.

And concretely, if you instead have a high bias problem and don't have any variance problem, then we saw in the previous video that getting more training examples,

while maybe just isn't going to help much at all. So the first option is useful only if you, say, plot the learning curves and figure out that you have at least a bit of a variance, meaning that the cross-validation error is, you know, quite a bit bigger than your training set error. How about trying a smaller set of features? Well, trying a smaller set of features, that's again something that fixes high variance.

And in other words, if you figure out, by looking at learning curves or something else that you used, that have a high bias problem; then for goodness sakes, don't waste your time trying to carefully select out a smaller set of features to use. Because if you have a high bias problem, using fewer features is not going to help. Whereas in contrast, if you look at the learning curves or something else you figure out that you have a high variance problem, then, indeed trying to select out a smaller set of features, that might indeed be a very good use of your time. How about trying to get additional features, adding features, usually, not always, but usually we think of this as a solution

for fixing high bias problems. So if you are adding extra features it's usually because

your current hypothesis is too simple, and so we want to try to get additional features to make our hypothesis better able to fit the training set. And similarly, adding polynomial features, this is another way of adding features and so there is another way to try to fix the high bias problem.

And, if concretely if your learning curves show you that you still have a high variance problem, then, you know, again this is maybe a less good use of your time.

②

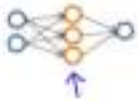
And finally, decreasing and increasing lambda. This are quick and easy to try, I guess these are less likely to be a waste of, you know, many months of your life. But decreasing lambda, you already know fixes high bias.

In case this isn't clear to you, you know, I do encourage you to pause the video and think through this that convince yourself that decreasing lambda helps fix high bias, whereas increasing lambda fixes high variance.

And if you aren't sure why this is the case, do pause the video and make sure you can convince yourself that this is the case. Or take a look at the curves that we were plotting at the end of the previous video and try to make sure you understand why these are the case.

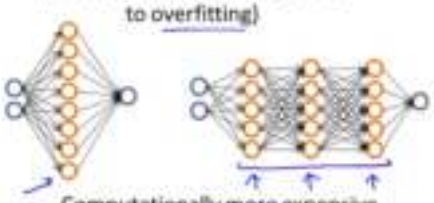
Neural networks and overfitting

→ "Small" neural network (fewer parameters; more prone to underfitting)



Computationally cheaper

→ "Large" neural network (more parameters; more prone to overfitting)



Computationally more expensive.

Use regularization (λ) to address overfitting.

$J_{cv}(\theta)$

①

Finally, let us take everything we have learned and relate it back to neural networks and so, here is some practical advice for how I usually choose the architecture or the connectivity pattern of the neural networks I use.

So, if you are fitting a neural network, one option would be to fit, say, a pretty small neural network with you know, relatively few hidden units, maybe just one hidden unit. If you're fitting a neural network, one option would be to fit a relatively small neural network with, say,

relatively few, maybe only one hidden layer and maybe only a relatively few number of hidden units. So, a network like this might have relatively few parameters and be more prone to underfitting.

The main advantage of these small neural networks is that the computation will be cheaper.

An alternative would be to fit a, maybe relatively large neural network with either more hidden units--there's a lot of hidden in one there--or with more hidden layers.

And so these neural networks tend to have more parameters and therefore be more prone to overfitting.

One disadvantage, often not a major one but something to think about, is that if you have a large number of neurons in your network, then it can be more computationally expensive.

Although within reason, this is often hopefully not a huge problem.

The main potential problem of these much larger neural networks is that it could be more prone to overfitting and it turns out if you're applying neural network very often using a large neural network often it's actually the larger, the better

②

but if it's overfitting, you can then use regularization to address overfitting, usually using a larger neural network by using regularization to address is overfitting that's often more effective than using a smaller neural network. And the main possible disadvantage is that it can be more computationally expensive.

And finally, one of the other decisions is, say, the number of hidden layers you want to have, right? So, do you want one hidden layer or do you want three hidden layers, as we've shown here, or do you want two hidden layers?

And usually, as I think I said in the previous video, using a single hidden layer is a reasonable default, but if you want to choose the number of hidden layers, one other thing you can try is find yourself a training cross-validation, and test set split and try training neural networks with one hidden layer or two hidden layers or three hidden layers and see which of those neural networks performs best on the cross-validation sets. You take your three neural networks with one, two and three hidden layers, and compute the cross validation error at Jcv and all of them and use that to select which of these is you think the best neural network.

So, that's it for bias and variance and ways like learning curves, who tried to diagnose these problems. As far as what you think is implied, for one might be truthful or not truthful things to try to improve the performance of a learning algorithm.

If you understood the contents of the last few videos and if you apply them you actually be much more effective already and getting learning algorithms to work on problems and even a large fraction, maybe the majority of practitioners of machine learning here in Silicon Valley today doing these things as their full-time jobs.

So I hope that these pieces of advice on by experience in diagnostics

will help you to much effectively and powerfully apply learning and get them to work very well.

Building a spam classifier

Prioritizing what to work on

In the next few videos I'd like to talk about machine learning system design.

These videos will touch on the main issues that you may face when designing a complex machine learning system,

and will actually try to give advice on how to strategize putting together a complex machine learning system.

In case this next set of videos seems a little disjointed that's because these videos will touch on a range of the different issues that you may come across when designing complex learning systems.

And even though the next set of videos may seem somewhat less mathematical, I think that this material may turn out to be very useful, and potentially huge time savers when you're building big machine learning systems.

Concretely, I'd like to begin with the issue of prioritizing how to spend your time on what to work on, and I'll begin with an example on spam classification.

Let's say you want to build a spam classifier.

Here are a couple of examples of obvious spam and non-spam emails.

If like you on the left tried to sell things. And notice how spammers will deliberately misspell words, like Vincent with a 1 there, and mortgages.

And on the right as maybe an obvious example of non-stamp email, actually email from my younger brother.

Let's say we have a labeled training set of some number of spam emails and some non-spam emails denoted with labels y equals 1 or 0. How do we build a classifier using supervised learning to distinguish between spam and non-spam?

Building a spam classifier

```
From: cheapsales@buystufffromme.com
To: ang@cs.stanford.edu
Subject: Buy now!

Deal of the week! Buy now!
Rolex w4tchs - $100
Medicine (any kind) - $50
Also low cost M0rgages
available.
```

Spam (1)

```
From: Alfred Ng
To: ang@cs.stanford.edu
Subject: Christmas dates?

Hey Andrew,
Was talking to Mom about plans
for Xmas. When do you get off
work. Meet Dec 22?
Alf
```

Non-spam (0)

Building a spam classifier

Supervised learning. x = features of email. y = spam (1) or not spam (0).

Features x: Choose 100 words indicative of spam/not spam.

E.g. deal, buy, discount, andrew, now, ...

$$x = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ \vdots \\ 1 \\ \vdots \end{bmatrix} \begin{matrix} \text{andrew} \\ \text{buy} \\ \text{deal} \\ \text{discount} \\ \\ \text{now} \\ \end{matrix} \quad x \in \mathbb{R}^{100}$$

$x_j = \begin{cases} 1 & \text{if word } j \text{ appears in email} \\ 0 & \text{otherwise.} \end{cases}$

```
From: cheapsales@buystufffromme.com
To: ang@cs.stanford.edu
Subject: Buy now!

Deal of the week! Buy now!
```

Note: In practice, take most frequently occurring n words (10,000 to 50,000) in training set, rather than manually pick 100 words.

In order to apply supervised learning, the first decision we must make is how do we want to represent x, that is the features of the email. Given the features x and the labels y in our training set, we can then train a classifier, for example using logistic regression.

Here's one way to choose a set of features for our emails.

We could come up with, say, a list of maybe a hundred words that we think are indicative of whether e-mail is spam or non-spam, for example, if a piece of e-mail contains the word 'deal' maybe it's more likely to be spam if it contains the word 'buy' maybe more likely to be spam, a word like 'discount' is more likely to be spam, whereas if a piece of email contains my name,

Andrew, maybe that means the person actually knows who I am and that might mean it's less likely to be spam.

And maybe for some reason I think the word "now" may be indicative of non spam because I get a lot of urgent emails, and so-on, and maybe we choose a hundred words or so.

Given a piece of email, we can then take this piece of email and encode it into a feature vector as follows. I'm going to take my list of a hundred words and sort them in alphabetical order say. It doesn't have to be sorted. But, you know, here's a, here's my list of words, just count and so on, until eventually I'll get down to now, and so on and given a piece of e-mail like that shown on the right, I'm going to check and see whether or not each of these words appears in the e-mail and then I'm going to define a feature vector x where in this piece of an email on the right, my name doesn't appear so I'm gonna put a zero there. The word "by" does appear,

so I'm gonna put a one there and I'm just gonna put one's or zeroes. I'm gonna put a one even though the word "by" occurs twice. I'm not gonna recount how many times the word occurs.

The word "due" appears, I put a one there. The word "discount" doesn't appear, at least not in this this little short email, and so on. The word "now" does appear and so on. So I put ones and zeroes in this feature vector depending on whether or not a particular word appears. And in this example my feature vector would have to mention one hundred,

if I have a hundred, if if I chose a hundred words to use for this representation and each of my features x_j will basically be 1 if

you have a particular word that, we'll call this word j, appears in the email and x_j

would be zero otherwise.

Okay. So that gives me a feature representation of a piece of email. By the way, even though I've described this process as manually picking a hundred words, in practice what's most commonly done is to look through a training set, and in the training set depict the most frequently occurring n words where n is usually between ten thousand and fifty thousand, and use those as your features. So rather than manually picking a hundred words, here you look through the training examples and pick the most frequently occurring words like ten thousand to fifty thousand words, and those form the features that you are going to use to represent your email for spam classification.

Building a spam classifier

How to spend your time to make it have low error?

- Collect lots of data
  - E.g. “honeypot” project.
- Develop sophisticated features based on email routing information (from email header).
- Develop sophisticated features for message body, e.g. should “discount” and “discounts” be treated as the same word? How about “deal” and “Dealer”? Features about punctuation?
- Develop sophisticated algorithm to detect misspellings (e.g. m0rtgage, med1cine, w4tches.)

①

Now, if you're building a spam classifier one question that you may face is, what's the best use of your time in order to make your spam classifier have higher accuracy, you have lower error.

One natural inclination is going to collect lots of data. Right? And in fact there's this tendency to think that, well the more data we have the better the algorithm will do. And in fact, in the email spam domain, there are actually pretty serious projects called Honey Pot Projects, which create fake email addresses and try to get these fake email addresses into the hands of spammers and use that to try to collect tons of spam email, and therefore you know, get a lot of spam data to train learning algorithms. But we've already seen in the previous sets of videos that getting lots of data will often help, but not all the time.

But for most machine learning problems, there are a lot of other things you could usually imagine doing to improve performance.

For spam, one thing you might think of is to develop more sophisticated features on the email, maybe based on the email routing information.

And this would be information contained in the email header.

So, when spammers send email, very often they will try to obscure the origins of the email, and maybe use fake email headers.

Or send email through very unusual sets of computer service. Through very unusual routes, in order to get the spam to you. And some of this information will be reflected in the email header.

And so one can imagine,

looking at the email headers and trying to develop more sophisticated features to capture this sort of email routing information to identify if something is spam. Something else you might consider doing is to look at the email message body, that is the email text, and try to develop more sophisticated features. For example, should the word 'discount' and the word 'discounts' be treated as the same words or should we have treat the words 'deal' and 'dealer' as the same word? Maybe even though one is lower case and one in capitalized in this example. Or do we want more complex features about punctuation because maybe spam

is using exclamation marks a lot more. I don't know. And along the same lines, maybe we also want to develop more sophisticated algorithms to detect and maybe to correct to deliberate misspellings, like mortgage, medicine, watches.

Because spammers actually do this, because if you have watches

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with a 4 in there then well, with the simple technique that we talked about just now, the spam classifier might not equate this as the same thing as the word "watches," and so it may have a harder time realizing that something is spam with these deliberate misspellings. And this is why spammers do it.

While working on a machine learning problem, very often you can brainstorm lists of different things to try, like these. By the way, I've actually worked on the spam problem myself for a while. And I actually spent quite some time on it. And even though I kind of understand the spam problem, I actually know a bit about it, I would actually have a very hard time telling you of these four options which is the best use of your time so what happens, frankly what happens far too often is that a research group or product group will randomly fixate on one of these options. And sometimes that turns out not to be the most fruitful way to spend your time depending, you know, on which of these options someone ends up randomly fixating on. By the way, in fact, if you even get to the stage where you brainstorm a list of different options to try, you're probably already ahead of the curve. Sadly, what most people do is instead of trying to list out the options of things you might try, what far too many people do is wake up one morning and, for some reason, just, you know, have a weird gut feeling that, "Oh let's have a huge honeypot project to go and collect tons more data" and for whatever strange reason just sort of wake up one morning and randomly fixate on one thing and just work on that for six months.

But I think we can do better. And in particular what I'd like to do in the next video is tell you about the concept of error analysis

and talk about the way where you can try to have a more systematic way to choose amongst the options of the many different things you might work, and therefore be more likely to select what is actually a good way to spend your time, you know for the next few weeks, or next few days or the next few months.

Building a spam classifier

Error Analysis

In the last video I talked about how, when faced with a machine learning problem, there are often lots of different ideas for how to improve the algorithm. In this video, let's talk about the concept of error analysis. Which will hopefully give you a way to more systematically make some of these decisions.

(1) Recommended approach

- Start with a simple algorithm that you can implement quickly. Implement it and test it on your cross-validation data.
- Plot learning curves to decide if more data, more features, etc. are likely to help.
- Error analysis: Manually examine the examples (in cross validation set) that your algorithm made errors on. See if you spot any systematic trend in what type of examples it is making errors on.

If you're starting work on a machine learning problem, or building a machine learning application. It's often considered very good practice to start, not by building a very complicated system with lots of complex features and so on. But to instead start by building a very simple algorithm that you can implement quickly. And when I start with a learning problem what I usually do is spend at most one day, like literally at most 24 hours, To try to get something really quick and dirty. Frankly not at all sophisticated system but get something really quick and dirty running, and implement it and then test it on my cross-validation data. Once you've done that you can then plot learning curves, this is what we talked about in the previous set of videos. But plot learning curves of the training and test errors to try to figure out if you're learning algorithm maybe suffering from high bias or high variance, or something else. And use that to try to decide if having more data, more features, and so on are likely to help. And the reason that this is a good approach is often, when you're just starting out on a learning problem, there's really no way to tell in advance. Whether you need more complex features, or whether you need more data, or something else. And it's just very hard to tell in advance, that is, in the absence of evidence, in the absence of seeing a learning curve. It's just incredibly difficult to figure out where you should be spending your time. And it's often by implementing even a very, very quick and dirty implementation. And by plotting learning curves, that helps you make these decisions. So if you like you can to think of this as a way of avoiding what's sometimes called premature optimization in computer programming. And this idea that says we should let evidence guide our decisions on where to spend our time rather than our gut feeling, which is often wrong. In addition to plotting learning curves, one other thing that's often very useful to do is what's called error analysis. And what I mean by that is that when building say a spam classifier. I will often look at my cross validation set and manually look at the emails that my algorithm is making errors on. So look at the spam e-mails and non-spam e-mails that the algorithm is misclassifying and see if you can spot any systematic patterns in what type of examples it is misclassifying. And often, by doing that, this is the process that will inspire you to design new features. Or they'll tell you what are the current things or current shortcomings of the system. And give you the inspiration you need to come up with improvements to it. Concretely, here's a specific example. ~~Let's say you've built~~ a

(2) Error Analysis

m_{CV} = 500 examples in cross validation set
Algorithm misclassifies 100 emails.
Manually examine the 100 errors, and categorize them based on:
- (i) What type of email it is — pharma, replica, steal passwords, ...
- (ii) What cues (features) you think would have helped the algorithm classify them correctly.

Pharma: 12
Replica/fake: 4
Steal passwords: 53
Other: 31

- Deliberate misspellings: 5 (m0rgage, med1cine, etc.)
- Unusual email routing: 16
- Unusual (spamming) punctuation: 32

~~to come up with improvements to it. Concretely, here's a specific example~~ Let's say you've built a spam classifier and you have 500 examples in your cross validation set. And let's say in this example that the algorithm has a very high error rate. And this classifies 100 of these cross validation examples.

So what I do is manually examine these 100 errors and manually categorize them. Based on things like what type of email it is, what cues or what features you think might have helped the algorithm classify them correctly. So, specifically, by what type of email it is, if I look through these 100 errors, I might find that maybe the most common types of spam emails in these classifies are maybe emails on pharma or pharmacies, trying to sell drugs. Maybe emails that are trying to sell replicas such as fake watches, fake random things, maybe some emails trying to steal passwords,. These are also called phishing emails, that's another big category of emails, and maybe other categories. So in terms of classify what type of email it is, I would actually go through and count up my hundred emails. Maybe I find that 12 of them is label emails, or pharma emails, and maybe 4 of them are emails trying to sell replicas, that sell fake watches or something. And maybe I find that 53 of them are these what's called phishing emails, basically emails trying to persuade you to give them your password. And 31 emails are other types of emails. And it's by counting up the number of emails in these different categories that you might discover, for example. That the algorithm is doing really, particularly poorly on emails trying to steal passwords. And that may suggest that it might be worth your effort to look more carefully at that type of email and see if you can come up with better features to categorize them correctly.

And, also what I might do is look at what cues or what additional features might have helped the algorithm classify the emails. So let's say that some of our hypotheses about things or features that might help us classify emails better are. Trying to detect deliberate misspellings versus unusual email routing versus unusual spamming punctuation. Such as if people use a lot of exclamation marks. And once again I would manually go through and let's say I find five cases of this and 16 of this and 32 of this and a bunch of other types of emails as well. And if this is what you get on your cross validation set, then it really tells you that maybe deliberate spellings is a sufficiently rare phenomenon that maybe it's not worth all the time trying to write algorithms that detect that. But if you find that a lot of spammers are using, you know, unusual punctuation, then maybe that's a strong sign that it might actually be worth your while to spend the time to develop more sophisticated features based on the punctuation. So this sort of error analysis, which is really the process of manually examining the mistakes that the algorithm makes, can often help guide you to the most fruitful avenues to pursue. And this also explains why I often recommend implementing a quick and dirty implementation of an algorithm. What we really want to do is figure out what are the most difficult examples for an algorithm to classify. And very often for different algorithms, for different learning algorithms they'll often find similar categories of examples difficult. And by having a quick and dirty implementation, that's often a quick way to let you identify some errors and quickly identify what are the hard examples. So that you can focus your effort on those.

Why is the recommended approach to perform error analysis using the cross validation data used to compute $J_{cv}(\theta)$ rather than the test data used to compute $J_{test}(\theta)$?

- [ ] The cross validation data set is usually large.
- [ ] This process will give a lower error on the test set.
- [x] If we develop new features by examining the test set, then we may end up choosing features that work well specifically for the test set, so $J_{test}(\theta)$ is no longer a good estimate of how well we generalize to new examples.
- [ ] Doing so is less likely to lead to choosing an excessive number of features.

✓ Correct

The importance of numerical evaluation

Should discount/discounts/discounted/discounting be treated as the same word?

Can use "stemming" software (E.g. "Porter stemmer")

universe/university.

Error analysis may not be helpful for deciding if this is likely to improve performance. Only solution is to try it and see if it works.

Need numerical evaluation (e.g., cross validation error) of algorithm's performance with and without stemming.

Without stemming: 5% error With stemming: 3% error

Distinguish upper vs. lower case (Mom/mom): 3.2%

~~recommended way to, thing to do than to do error analysis on your cross validation set.~~ Set to wrap up this video, when starting on a new machine learning problem, what I almost always recommend is to implement a quick and dirty implementation of your learning out of them. And I've almost never seen anyone spend too little time on this quick and dirty implementation. I've pretty much only ever seen people spend much too much time building their first, supposedly, quick and dirty implementation. So really, don't worry about it being too quick, or don't worry about it being too dirty. But really, implement something as quickly as you can. And once you have the initial implementation, this is then a powerful tool for deciding where to spend your time next. Because first you can look at the errors it makes, and do this sort of error analysis to see what other mistakes it makes, and use that to inspire further development. And second, assuming your quick and dirty implementation incorporated a single real number evaluation metric. This can then be a vehicle for you to try out different ideas and quickly see if the different ideas you're trying out are improving the performance of your algorithm. And therefore let you, maybe much more quickly make decisions about what things to fold in and what things to incorporate into your learning algorithm.

Handling skewed Data

Error metrics for skewed classes

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In the previous video, I talked about error analysis and the importance of having error metrics, that is of having a single real number evaluation metric for your learning algorithm to tell how well it's doing.

In the context of evaluation

and of error metrics, there is one important case, where it's particularly tricky to come up with an appropriate error metric, or evaluation metric, for your learning algorithm.

That case is the case of what's called skewed classes.

Let me tell you what that means.

Consider the problem of cancer classification, where we have features of medical patients and we want to decide whether or not they have cancer. So this is like the malignant versus benign tumor classification example that we had earlier.

So let's say y equals 1 if the patient has cancer and y equals 0 if they do not. We have trained the progression classifier and let's say we test our classifier on a test set and find that we get 1 percent error. So, we're making 99% correct diagnosis. Seems like a really impressive result, right. We're correct 99% percent of the time.

But now, let's say we find out that only 0.5 percent of patients in our training test sets actually have cancer. So only half a percent of the patients that come through our screening process have cancer.

In this case, the 1% error no longer looks so impressive.

And in particular, here's a piece of code, here's actually a piece of non learning code that takes this input of features x and it ignores it. It just sets y equals 0 and always predicts, you know, nobody has cancer and this algorithm would actually get 0.5 percent error. So this is even better than the 1% error that we were getting just now and this is a non learning algorithm that you know, it is just predicting y equals 0 all the time.

So this setting of when the ratio of positive to negative examples is very close to one of two extremes, where, in this case, the number of positive examples is much, much smaller than the number of negative examples because y equals one so rarely, this is what we call the case of skewed classes.

We just have a lot more of examples from one class than from the other class. And by just predicting y equals 0 all the time, or maybe our predicting y equals 1 all the time, an algorithm can do pretty well. So the problem with using classification error or classification accuracy as our evaluation metric is the following.

Cancer classification example

Train logistic regression model $h_\theta(x)$. ($y = 1$ if cancer, $y = 0$ otherwise)

Find that you got 1% error on test set.

(99% correct diagnoses)

Only 0.50% of patients have cancer.

→ skewed classes.

```
function y = predictCancer(x)
  → y = 0; %ignore x!
return
```

→ 0.5% error

→ 99.2% accuracy (0.8% error)

→ 99.5% accuracy (0.5% error)

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Let's say you have one joining algorithm that's getting 99.2% accuracy.

So, that's a 0.8% error. Let's say you make a change to your algorithm and you now are getting 99.5% accuracy.

That is 0.5% error.

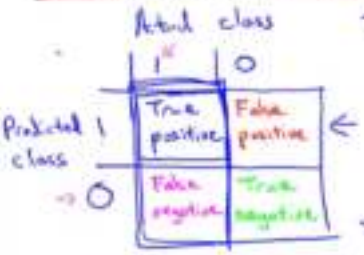
So, is this an improvement to the algorithm or not? One of the nice things about having a single real number evaluation metric is this helps us to quickly decide if we just need a good change to the algorithm or not. By going from 99.2% accuracy to 99.5% accuracy.

You know, did we just do something useful or did we just replace our code with something that just predicts y equals zero more often? So, if you have very skewed classes it becomes much harder to use just classification accuracy, because you can get very high classification accuracies or very low errors, and it's not always clear if doing so is really improving the quality of your classifier because predicting y equals 0 all the time doesn't seem like a particularly good classifier.

But just predicting y equals 0 more often can bring your error down to, you know, maybe as low as 0.5%. When we're faced with such a skewed classes therefore we would want to come up with a different error metric or a different evaluation metric. One such evaluation metric are what's called precision recall.

Precision/Recall

$y = 1$ in presence of rare class that we want to detect



→ **Precision** → Higher better

(Of all patients where we predicted $y = 1$, what fraction actually has cancer?)

$$\frac{\text{True positives}}{\#\text{predicted positive}} = \frac{\text{True positive}}{\text{True pos + False pos}}$$

→ **Recall** → lower better

(Of all patients that actually have cancer, what fraction did we correctly detect as having cancer?)

$$\frac{\text{True positives}}{\#\text{actual positives}} = \frac{\text{True positives}}{\text{True pos + False neg}}$$

$y = 0$

recall = 0

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Let me explain what that is.

Let's say we are evaluating a classifier on the test set. For the examples in the test set the actual class of that example in the test set is going to be either one or zero, right, if there is a binary classification problem.

And what our learning algorithm will do is it will, you know, predict some value for the class and our learning algorithm will predict the value for each example in my test set and the predicted value will also be either one or zero.

So let me draw a two by two table as follows, depending on a full of these entries depending on what was the actual class and what was the predicted class. If we have an example where the actual class is one and the predicted class is one then that's called

an example that's a true positive, meaning our algorithm predicted that it's positive and it really the example is positive. If our learning algorithm predicted that something is negative, class zero, and the actual class is also class zero then that's what's called a true negative. We predicted zero and it actually is zero.

To find the other two boxes, if our learning algorithm predicts that the class is one but the actual class is zero, then that's called a false positive.

So that means our algorithm for the patient is cancelled out in reality if the patient does not.

And finally, the last box is a zero, one. That's called a false negative because our algorithm predicted zero, but the actual class was one.

And so, we have this little sort of two by two table based on what was the actual class and what was the predicted class.

So here's a different way of evaluating the performance of our algorithm. We're going to compute two numbers. The first is called precision - and what that says is,

of all the patients where we've predicted that they have cancer,

what fraction of them actually have cancer?

So let me write this down, the precision of a classifier is the number of true positives divided by

the number that we predicted

as positive, right?

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And in particular if we have a learning algorithm that predicts y equals zero all the time, if it predicts no one has cancer, then this classifier will have a recall equal to zero, because there won't be any true positives and so that's a quick way for us to recognize that, you know, a classifier that predicts y equals 0 all the time, just isn't a very good classifier. And more generally, even for settings where we have very skewed classes, it's not possible for an algorithm to sort of "cheat" and somehow get a very high precision and a very high recall by doing some simple thing like predicting y equals 0 all the time or predicting y equals 1 all the time. And so we're much more sure that a classifier of a high precision or high recall actually is a good classifier, and this gives us a more useful evaluation metric that is a more direct way to actually understand whether, you know, our algorithm may be doing well.

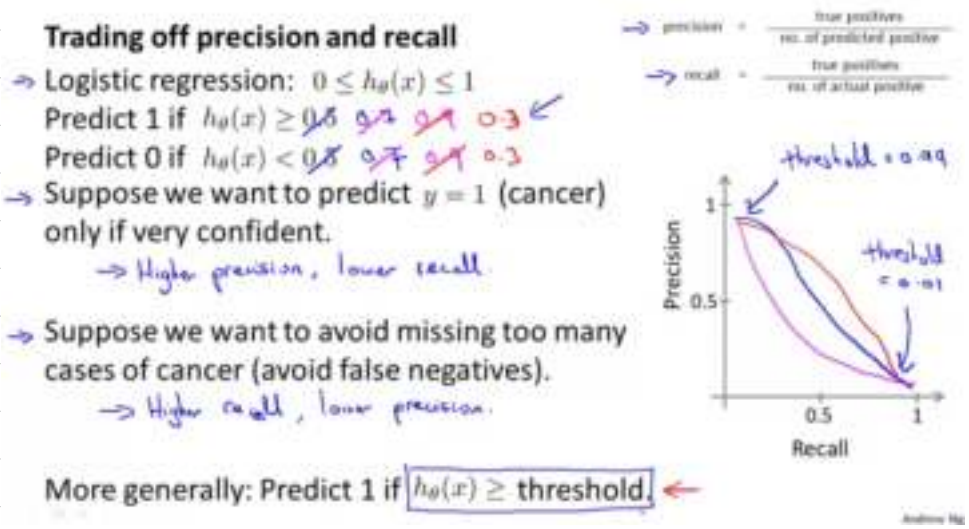
So one final note is the definition of precision and recall. And we would define precision and recall, usually we use the convention that y is equal to 1, in the presence of the more rare class. So if we are trying to detect, rare conditions such as cancer, hopefully that's a rare condition, precision and recall are defined setting y equals 1, rather than y equals 0, to be sort of that the presence of that rare class that we're trying to detect. And by using precision and recall, we find, what happens is that even if we have very skewed classes, it's not possible for an algorithm to, you know, "cheat" and predict y equals 1 all the time, or predict y equals 0 all the time, and get high precision and recall. And in particular, if a classifier is getting high precision and high recall, then we are actually confident that the algorithm has to be doing well, even if we have very skewed classes.

So for this problem of skewed classes precision recall gives us more direct insight into how the learning algorithm is doing and this is often a much better way to evaluate our learning algorithms, than looking at classification error or classification accuracy, when the classes are very skewed.

Handling skewed Data

Trading off Precision & Recall

In the last video, we talked about precision and recall as an evaluation metric for classification problems with skewed constants. For many applications, we'll want to somehow control the trade-off between precision and recall. Let me tell you how to do that and also show you some even more effective ways to use precision and recall as an evaluation metric for learning algorithms.



But I hope the details of the algorithm is true and the more general principle is depending on where you want, whether you want higher precision- lower recall, or higher recall- lower precision. You can end up predicting y=1 when h(x) is greater than some threshold. And so in general, for most classifiers there is going to be a trade off between precision and recall, and as you vary the value of this threshold that we join here, you can actually plot out some curve that trades off precision and recall. Where a value up here, this would correspond to a very high value of the threshold, maybe threshold equals 0.99. So that's saying, predict y=1 only if we're more than 99% confident, at least 99% probability this one. So that would be a high precision, relatively low recall. Where as the point down here, will correspond to a value of the threshold that's much lower, maybe equal 0.01, meaning, when in doubt at all, predict y=1, and if you do that, you end up with a much lower precision, higher recall classifier. And as you vary the threshold, if you want you can actually trace of a curve for your classifier to see the range of different values you can get for precision recall. And by the way, the precision-recall curve can look like many different shapes. Sometimes it will look like this, sometimes it will look like that. Now there are many different possible shapes for the precision-recall curve, depending on the details of the classifier. So, this raises another interesting question which is, is there a way to choose this threshold automatically? Or more generally, if we have a few different algorithms or a few different ideas for algorithms, how do we compare different precision recall numbers?

F_1 Score (F score)

How to compare precision/recall numbers?

	Precision(P)	Recall (R)	Average	F_1 Score
Algorithm 1	0.5	0.4	0.45	0.444
Algorithm 2	0.7	0.1	0.4	0.175
Algorithm 3	0.02	1.0	0.51	0.0392

Predict y=1 all the time

Average: $\frac{P+R}{2}$

F_1 Score: $2\frac{PR}{P+R}$

P=0 or R=0 $\Rightarrow$ F-score = 0.

P=1 and R=1 $\Rightarrow$ F-score = 1

Concretely, suppose we have three different learning algorithms. So actually, maybe these are three different learning algorithms, maybe these are the same algorithm but just with different values for the threshold. How do we decide which of these algorithms is best? One of the things we talked about earlier is the importance of a single real number evaluation metric. And that is the idea of having a number that just tells you how well is your classifier doing. But by switching to the precision recall metric we've actually lost that. We now have two real numbers. And so we often, we end up face the situations like if we trying to compare Algorithm 1 and Algorithm 2, we end up asking ourselves, is the precision of 0.5 and a recall of 0.4, was that better or worse than a precision of 0.7 and recall of 0.1? And, if every time you try out a new algorithm you end up having to sit around and think, well, maybe 0.5/0.4 is better than 0.7/0.1, or maybe not, I don't know. If you end up having to sit around and think and make these decisions, that really slows down your decision making process for what changes are useful to incorporate into your algorithm.

Whereas in contrast, if we have a single real number evaluation metric like a number that just tells us is algorithm 1 or is algorithm 2 better, then that helps us to much more quickly decide which algorithm to go with. It helps us as well to much more quickly evaluate different changes that we may be contemplating for an algorithm. So how can we get a single real number evaluation metric?

One natural thing that you might try is to look at the average precision and recall. So, using P and R to denote precision and recall, what you could do is just compute the average and look at what classifier has the highest average value.

But this turns out not to be such a good solution, because similar to the example we had earlier it turns out that if we have a classifier that predicts y=1 all the time, then if you do that you can get a very high recall, but you end up with a very low value of precision. Conversely, if you have a classifier that predicts y equals zero, almost all the time, that is that it predicts y=1 very sparingly, this corresponds to setting a very high threshold using the notation of the previous y. Then you can actually end up with a very high precision with a very low recall. So, the two extremes of either a very high threshold or a very low threshold, neither of that will give a particularly good classifier. And the way we recognize that is by seeing that we end up with a very low precision or a very low recall. And if you just take the average of (P+R)/2 from this example, the average is actually highest for Algorithm 3, even though you can get that sort of performance by predicting y=1 all the time and that's just not a very good classifier, right? You predict y=1 all the time, just normal useful classifier, but all it does is prints out y=1. And so Algorithm 1 or Algorithm 2 would be more useful than Algorithm 3. But in this example, Algorithm 3 has a higher average value of precision recall than Algorithms 1 and 2. So we usually think of this average of precision and recall as not a particularly good way to evaluate our learning algorithm.

In contrast, there's a different way for combining precision and recall. This is called the F Score and it uses that formula. And so in this example, here are the F Scores. And so we would tell from these F Scores, it looks like Algorithm 1 has the highest F Score, Algorithm 2 has the second highest, and Algorithm 3 has the lowest. And so, if we go by the F Score we would pick probably Algorithm 1 over the others.

The F Score, which is also called the F1 Score, is usually written F1 Score that I have here, but often people will just say F Score, either term is used. Is a little bit like taking the average of precision and recall, but it gives the lower value of precision and recall, whichever it is, it gives it a higher weight. And so, you see in the numerator here that the F Score takes a product of precision and recall. And so if either precision is 0 or recall is equal to 0, the F Score will be equal to 0. So in that sense, it kind of combines precision and recall, but for the F Score to be large, both precision and recall have to be pretty large. I should say that there are many different possible formulas for combing precision and recall. This F Score formula is really maybe a, just one out of a much larger number of possibilities, but historically or traditionally this is what people in Machine Learning seem to use. And the term F Score, it doesn't really mean anything, so don't worry about why it's called F Score or F1 Score.

But this usually gives you the effect that you want because if either a precision is zero or recall is zero, this gives you a very low F Score, and so to have a high F Score, you kind of need a precision or recall to be one. And concretely, if P=0 or R=0, then this gives you that the F Score = 0. Whereas a perfect F Score, so if precision equals one and recall equals 1, that will give you an F Score,

that's equal to 1 times 1 over 2 times 2, so the F Score will be equal to 1, if you have perfect precision and perfect recall. And intermediate values between 0 and 1, this usually gives a reasonable rank ordering of different classifiers.

So in this video, we talked about the notion of trading off between precision and recall, and how we can vary the threshold that we use to decide whether to predict y=1 or y=0. So it's the threshold that says, do we need to be at least 70% confident or 90% confident, or whatever before we predict y=1. And by varying the threshold, you can control a trade off between precision and recall. We also talked about the F Score, which takes precision and recall, and again, gives you a single real number evaluation metric. And of course, if your goal is to automatically set that threshold to decide what's really y=1 and y=0, one pretty reasonable way to do that would also be to try a range of different values of thresholds. So you try a range of values of thresholds and evaluate these different thresholds on, say, your cross-validation set and then to pick whatever value of threshold gives you the highest F Score on your cross[validation [INAUDIBLE]. And that be a pretty reasonable way to automatically choose the threshold for your classifier as well.

You have trained a logistic regression classifier and plan to make predictions according to:

- Predict $y = 1$ if $h_\theta(x) \geq$ threshold
- Predict $y = 0$ if $h_\theta(x) <$ threshold

For different values of the threshold parameter, you get different values of precision (P) and recall (R). Which of the following would be a reasonable way to pick the value to use for the threshold?

- [ ] Measure precision (P) and recall (R) on the **test set** and choose the value of threshold which maximizes $P+R$
- [ ] Measure precision (P) and recall (R) on the **test set** and choose the value of threshold which maximizes $2\frac{PR}{P+R}$
- [ ] Measure precision (P) and recall (R) on the **cross validation set** and choose the value of threshold which maximizes $P+R$
- [x] Measure precision (P) and recall (R) on the **cross validation set** and choose the value of threshold which maximizes $2\frac{PR}{P+R}$

✓ Correct

Using Large Data Sets

Data for machine Learning

In the previous video, we talked about evaluation metrics.

In this video, I'd like to switch tracks a bit and touch on another important aspect of machine learning system design, which will often come up, which is the issue of how much data to train on. Now, in some earlier videos, I had cautioned against blindly going out and just spending lots of time collecting lots of data, because it's only sometimes that that would actually help.

But it turns out that under certain conditions, and I will say in this video what those conditions are, getting a lot of data and training on a certain type of learning algorithm, can be a very effective way to get a learning algorithm to do very good performance.

And this arises often enough that if those conditions hold true for your problem and if you're able to get a lot of data, this could be a very good way to get a very high performance learning algorithm.

So in this video, let's talk more about that. Let me start with a story.

Many, many years ago, two researchers that I know, Michelle Banko and Eric Broule ran the following fascinating study.

They were interested in studying the effect of using different learning algorithms versus trying them out on different training set sizes, they were considering the problem of classifying between confusable words, so for example, in the sentence: for breakfast I ate, should it be to, two or too? Well, for this example, for breakfast I ate two, 2 eggs.

So, this is one example of a set of confusable words and that's a different set. So they took machine learning problems like these, sort of supervised learning problems to try to categorize what is the appropriate word to go into a certain position in an English sentence.

They took a few different learning algorithms which were, you know, sort of considered state of the art back in the day, when they ran the study in 2001, so they took a variance, roughly a variance on logistic regression called the Perceptron. They also took some of their algorithms that were fairly out back then but somewhat less used now so when the algorithm also very similar to which is a regression but different in some ways, much used somewhat less, used not too much right now took what's called a memory based learning algorithm again used somewhat less now. But I'll talk a little bit about that later. And they used a naive based algorithm, which is something they'll actually talk about in this course. The exact algorithms of these details aren't important. Think of this as, you know, just picking four different classification algorithms and really the exact algorithms aren't important.

Designing a high accuracy learning system

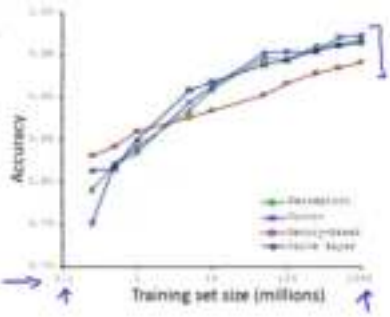
E.g. Classify between confusable words.

{to, two, too} {then, than}

→ For breakfast I ate _____ eggs.

Algorithms

- Perceptron (Logistic regression)
- Winnow
- Memory-based
- Naïve Bayes



"It's not who has the best algorithm that wins. It's who has the most data."

[Banko and Brill, 2001]

But what they did was they varied the training set size and tried out these learning algorithms on the range of training set sizes and that's the result they got. And the trends are very clear right first most of those outer rooms give remarkably similar performance. And second, as the training set size increases, on the horizontal axis is the training set size in millions

go from you know a hundred thousand up to a thousand million that is a billion training examples. The performance of the algorithms

all pretty much monotonically increase and the fact that if you pick any algorithm may be pick a "inferior algorithm" but if you give that "inferior algorithm" more data, then from these examples, it looks like it will most likely beat even a "superior algorithm".

So since this original study which is very influential, there's been a range of many different studies showing similar results that show that many different learning algorithms you know tend to, can sometimes, depending on details, can give pretty similar ranges of performance, but what can really drive performance is you can give the algorithm a ton of training data.

And this is, results like these has led to a saying in machine learning that often in machine learning it's not who has the best algorithm that wins, it's who has the

most data So when is this true and when is this not true? Because we have a learning algorithm for which this is true then getting a lot of data is often maybe the best way to ensure that we have an algorithm with very high performance rather than you know, debating worrying about exactly which of these items to use.

Large data rationale

Assume feature $x \in \mathbb{R}^{n+1}$ has sufficient information to predict y accurately.

Example: For breakfast I ate _____ eggs.

Counterexample: Predict housing price from only size ($feet^2$) and no other features.

Useful test: Given the input x, can a human expert confidently predict y?

Let's try to lay out a set of assumptions under which having a massive training set we think will be able to help.

Let's assume that in our machine learning problem, the features x have sufficient information with which we can use to predict y accurately.

For example, if we take the confusable words all of them that we had on the previous slide. Let's say that it features x capture what are the surrounding words around the blank that we're trying to fill in. So the features capture then we want to have, sometimes for breakfast I have black eggs. Then yeah that is pretty much information to tell me that the word I want in the middle is TWO and that is not word TO and its not the word TOO. So

the features capture, you know, one of these surrounding words then that gives me enough information to pretty unambiguously decide what is the label y or in other words what is the word that I should be using to fill in that blank out of this set of three confusable words.

So that's an example what the future ex has sufficient information for specific y. For a counter example.

Consider a problem of predicting the price of a house from only the size of the house and from no other

features. So if you imagine I tell you that a house is, you know, 500 square feet but I don't give you any other features. I don't tell you that the house is in an expensive part of the city. Or it's don't tell you that the house, the number of rooms in the house, or how nicely furnished the house is, or whether the house is new or old. If I don't tell you anything other than that this is a 500 square foot house, well there's so many other factors that would affect the price of a house other than just the size of a house that if all you know is the size, it's actually very difficult to predict the price accurately.

So that would be a counter example to this assumption that the features have sufficient information to predict the price to the desired level of accuracy. The way I think about testing this assumption, one way I often think about it is, how often I ask myself

Given the input features x, given the features, given the same information available as well as learning algorithm.

If we were to go to human expert in this domain. Can a human experts actually or can human expert confidently predict the value of y. For this first example if we go to, you know an expert human English speaker. You go to someone that speaks English well, right, then a human expert in English just read most people like you and me will probably we would probably be able to predict what word should go in here, so a good English speaker can predict this well, and so this gives me confidence that x allows us to predict y accurately, but in contrast if we go to an expert in human prices. Like maybe an expert realtor, right, someone who sells houses for a living, if I just tell them the size of a house and I tell them what the price is well even an expert in pricing or selling houses wouldn't be able to tell me and so this is fine that for the housing price example knowing only the size doesn't give me enough information to predict the price of the house. So, let's say, this assumption holds.

Large data rationale

Use a learning algorithm with many parameters (e.g. logistic regression/linear regression with many features; neural network with many hidden units). low bias algorithms. ←

→ $J_{train}(\theta)$ will be small.

Use a very large training set (unlikely to overfit) low variance ←

→ $J_{train}(\theta) \approx J_{test}(\theta)$

$J_{test}(\theta)$ will be small

Let's see then, when having a lot of data could help. Suppose the features have enough information to predict the value of y. And let's suppose we use a learning algorithm with a large number of parameters so maybe logistic regression or linear regression with a large number of features. Or one thing that I sometimes do, one thing that I often do actually is using neural network with many hidden units. That would be another learning algorithm with a lot of parameters.

So these are all powerful learning algorithms with a lot of parameters that can fit very complex functions.

So, I'm going to call these, I'm

going to think of these as low-bias algorithms because you know we can fit very complex functions

and because we have a very powerful learning algorithm, they can fit very complex functions.

Chances are, if we run these algorithms on the data sets, it will be able to fit the training set well, and so hopefully the training error will be slow.

Now let's say, we use a massive, massive training set, in that case, if we have a huge training set, then hopefully even though we have a lot of parameters but if the training set is sort of even much larger than the number of parameters then hopefully these albums will be unlikely to overfit.

Right because we have such a massive training set and by unlikely to overfit what that means is that the training error will hopefully be close to the test error. Finally putting these two together that the train set error is small and the test set error is close to the training error what this two together imply is that hopefully the test set error

will also be small.

Another way to think about this is that in order to have a high performance learning algorithm we want it not to have high bias and not to have high variance.

So the bias problem we're going to address by making sure we have a learning algorithm with many parameters and so that gives us a low bias alorithm

and by using a very large training set, this ensures that we don't have a variance problem here. So hopefully our algorithm will have no variance and so is by pulling these two together, that we end up with a low bias and a low variance

learning algorithm and this allows us to do well on the test set. And fundamentally it's a key ingredients of assuming that the features have enough information and we have a rich class of functions that's why it guarantees low bias,

and then it having a massive training set that that's what guarantees more variance.

So this gives us a set of conditions rather hopefully some understanding of what's the sort of problem where if you have a lot of data and you train a learning algorithm with lot of parameters, that might be a good way to give a high performance learning algorithm and really, I think the key test that I often ask myself are first, can a human experts look at the features x and confidently predict the value of y. Because that's sort of a certification that y can be predicted accurately from the features x and second, can we actually get a large training set, and train the learning algorithm with a lot of parameters in the training set and if you can't do both then that's more often give you a very kind performance learning algorithm.

Having a large training set can help significantly improve a learning algorithm's performance. However, the large training set is **unlikely** to help when:

- [x] The features x do not contain enough information to predict y accurately (such as predicting a house's price from only its size), and we are using a simple learning algorithm such as logistic regression.

✓ **Correct**

- [ ] We are using a learning algorithm with a large number of features (i.e. one with "low bias").
- [x] The features x do not contain enough information to predict y accurately (such as predicting a house's price from only its size), even if we are using a neural network with a large number of hidden units.

✓ **Correct**

- [ ] We are not using regularization (e.g. the regularization parameter $\lambda = 0$).

Large margin classification

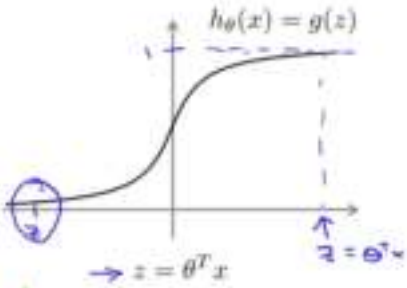
optimization objective

Support vector machine

By now, you've seen a range of difference learning algorithms. With supervised learning, the performance of many supervised learning algorithms will be pretty similar, and what matters less often will be whether you use learning algorithm a or learning algorithm b, but what matters more will often be things like the amount of data you create these algorithms on, as well as your skill in applying these algorithms. Things like your choice of the features you design to give to the learning algorithms, and how you choose the colorization parameter, and things like that. But, there's one more algorithm that is very powerful and is very widely used both within industry and academia, and that's called the support vector machine. And compared to both logistic regression and neural networks, the Support Vector Machine, or SVM sometimes gives a cleaner, and sometimes more powerful way of learning complex non-linear functions. And so let's take the next videos to talk about that. Later in this course, I will do a quick survey of a range of different supervisory algorithms just as a very briefly describe them. But the support vector machine, given its popularity and how powerful it is, this will be the last of the supervisory algorithms that I'll spend a significant amount of time on in this course as with our development other learning algorithms, we're gonna start by talking about the optimization objective. So, let's get started on this algorithm.

Alternative view of logistic regression

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$



If $y = 1$, we want $h_\theta(x) \approx 1$, $\theta^T x \gg 0$

If $y = 0$, we want $h_\theta(x) \approx 0$, $\theta^T x \ll 0$

In order to describe the support vector machine, I'm actually going to start with logistic regression, and show how we can modify it a bit, and get what is essentially the support vector machine. So in logistic regression, we have our familiar form of the hypothesis there and the sigmoid activation function shown on the right.

And in order to explain some of the math, I'm going to use z to denote theta transpose axiom.

Now let's think about what we would like logistic regression to do. If we have an example with y equals one and by this I mean an example in either the training set or the test set or the cross-validation set, but when y is equal to one then we're sort of hoping that h of x will be close to one. Right, we're hoping to correctly classify that example. And what having x subscript 1, what that means is that theta transpose x must be must larger than 0. So there's greater than, greater than sign that means much, much greater than 0. And that's because it is z, the theta of transpose x is when z is much bigger than 0 is far to the right of the sphere. That the outputs of logistic progression becomes close to one.

Conversely, if we have an example where y is equal to zero, then what we're hoping for is that the hypothesis will output a value close to zero. And that corresponds to theta transpose x of z being much less than zero because that corresponds to a hypothesis of putting a value close to zero.

①

If you look at the cost function of logistic regression, what you'll find is that each example (x,y) contributes a term like this to the overall cost function, right?

So for the overall cost function, we will also have a sum over all the chain examples and the 1 over m term, that this expression here, that's the term that a single training example contributes to the overall objective function so we can just rush them.

Now if I take the definition for the fall of my hypothesis and plug it in over here, then what I get is that each training example contributes this term, ignoring the one over M but it contributes that term to my overall cost function for logistic regression.

Now let's consider two cases of when y is equal to one and when y is equal to zero. In the first case, let's suppose that y is equal to 1. In that case, only this first term in the objective matters, because this one minus y term would be equal to zero if y is equal to one.

So when y is equal to one, when in our example x comma y, when y is equal to 1 what we get is this term. Minus log one over one, plus E to the negative Z where as similar to the last line I'm using Z to denote data transposed X and of course in a cost I should have this minus line that we just had if Y is equal to one so that's equal to one I just simplify in a way in the expression that I have written down here.

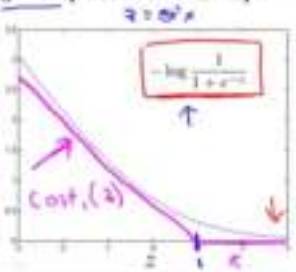
And if we plot this function as a function of z, what you find is that you get this curve shown on the lower left of the slide. And thus, we also see that when z is equal to large, that is, when theta transpose x is large, that corresponds to a value of z that gives us a fairly small value, a very, very small contribution to the consumption. And this kinda explains why, when logistic regression sees a positive example, with y=1, it tries to set theta transport x to be very large because that corresponds to this term, in the cross function, being small. Now, to fill the support vec machine, here's what we're going to do. We're gonna take this cross function, this minus log 1 over 1 plus e to negative z, and modify it a little bit. Let me take this point 1 over here, and let me draw the cross functions you're going to use. The new pass functions can be flat from here on out, and then we draw something that grows as a straight line, similar to logistic regression. But this is going to be a straight line at this portion. So the curve that I just drew in magenta, and the curve I just drew purple and magenta, so if it's pretty close approximation to the cross function used by logistic regression. Except it is now made up of two line segments, there's this flat portion on the right, and then there's this straight line portion on the left. And don't worry too much about the slope of the straight line portion. It doesn't matter that much. But that's the new cost function we're going to use for when y is equal to one, and you can imagine it should do something pretty similar to logistic regression. But turns out, that this will give the support vector machine computational advantages and give us, later on, an easier optimization problem

Alternative view of logistic regression

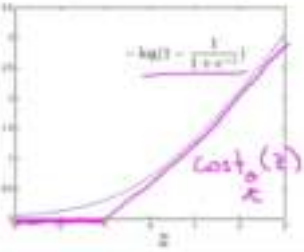
Cost of example: $-(y \log h_\theta(x) + (1-y)\log(1-h_\theta(x)))$

$$= -y \log \frac{1}{1+e^{-\theta^T x}} - (1-y)\log\left(1 - \frac{1}{1+e^{-\theta^T x}}\right)$$

If $y = 1$ (want $\theta^T x \gg 0$):



If $y = 0$ (want $\theta^T x \ll 0$):



②

that would be easier for software to solve. We just talked about the case of y equals one. The other case is if y is equal to zero. In that case, if you look at the cost, then only the second term will apply because the first term goes away, right? If y is equal to zero, then you have a zero here, so you're left only with the second term of the expression above. And so the cost of an example, or the contribution of the cost function, is going to be given by this term over here. And if you plot that as a function of z, to have pure z on the horizontal axis, you end up with this one. And for the support vector machine, once again, we're going to replace this blue line with something similar and at the same time we replace it with a new cost, this flat out here, this 0 out here. And that then grows as a straight line, like so. So let me give these two functions names. This function on the left I'm going to call cost subscript 1 of z, and this function of the right I'm gonna call cost subscript 0 of z. And the subscript just refers to the cost corresponding to when y is equal to 1, versus when y is equal to zero. Armed with these definitions, we're now ready to build a support vector machine. ~~Here's the cost function, j of theta, that we have for logistic regression. In case~~

Support vector machine

Logistic regression:

$$\min_\theta \frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)}\underbrace{\left(-\log h_\theta(x^{(i)})\right)}_{\text{cost}_1(\theta^T x^{(i)})} + (1-y^{(i)})\underbrace{\left(-\log(1-h_\theta(x^{(i)}))\right)}_{\text{cost}_0(\theta^T x^{(i)})}\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$

Support vector machine:

$$\min_\theta C\sum_{i=1}^{m}\left[y^{(i)}\text{cost}_1(\theta^T x^{(i)}) + (1-y^{(i)})\text{cost}_0(\theta^T x^{(i)})\right] + \frac{1}{2}\sum_{j=1}^{n}\theta_j^2$$

$$\Rightarrow \min_\theta C\sum_{i=1}^{m}\left[y^{(i)}\text{cost}_1(\theta^T x^{(i)}) + (1-y^{(i)})\text{cost}_0(\theta^T x^{(i)})\right] + \frac{1}{2}\sum_{i=1}^{n}\theta_j^2$$

①

~~vector machine~~. Here's the cost function, j of theta, that we have for logistic regression. In case this equation looks a bit unfamiliar, it's because previously we had a minus sign outside, but here what I did was I instead moved the minus signs inside these expressions, so it just makes it look a little different.

For the support vector machine what we're going to do is essentially take this and replace this with cost1 of z, that is cost1 of theta transpose x. And we're going to take this and replace it with cost0 of z, that is cost0 of theta transpose x. Where the cost one function is what we had on the previous slide that looks like this. And the cost zero function, again what we had on the previous slide, and it looks like this. So what we have for the support vector machine is a minimization problem of one over M, the sum of Y i times cost one, theta transpose X i, plus one minus Y i times cause zero of theta transpose X i, and then plus my usual regularization parameter.

Like so. Now, by convention, for the support of vector machine, we're actually write things slightly different. We re-parameterize this just very slightly differently.

First, we're going to get rid of the 1 over m terms, and this just this happens to be a slightly different convention that people use for support vector machines compared to or just a progression. But here's what I mean. You're one way to do this, we're just gonna get rid of these one over m terms and this should give you me the same optimal value of beta right? Because one over m is just as constant so whether I solved this minimization problem with one over n in front or not. I should end up with the same optimal value for theta. Here's what I mean, to give you an example, suppose I had a minimization problem. Minimize over a long number U of U minus five squared plus one. Well, the minimum of this happens to be U equals five.

Now if I were to take this objective function and multiply it by 10. So here my minimization problem is min over U, 10 U minus five squared plus 10. Well the value of U that minimizes this is still U equals five right? So multiply something that you're minimizing over, by some constant, 10 in this case, it does not change the value of U that gives us, that minimizes this function. So the same way, what I've done is by crossing out the M is all I'm doing is multiplying my objective function by some constant M and it doesn't change the value of theta. That achieves the minimum. The second bit of notational change, which is just, again, the more standard convention when using SVMs instead of logistic regression, is the following. So for logistic regression, we add two terms to the objective function. The first is this term, which is the cost that comes from the training set and the second is this row, which is the regularization term.

②

And what we had was we had a, we control the trade off between these by saying, what we want is A plus, and then my regularization parameter lambda. And then times some other term B, where I guess I'm using your A to denote this first term, and I've using B to denote the second term, maybe without the lambda.

And instead of prioritizing this as A plus lambda B, and so what we did was by setting different values for this regularization parameter lambda, we could trade off the relative weight between how much we wanted the training set well, that is, minimizing A, versus how much we care about keeping the values of the parameter small, so that will be, the parameter is B for the support vector machine, just by convention, we're going to use a different parameter. So instead of using lambda here to control the relative waiting between the first and second terms. We're instead going to use a different parameter which by convention is called C and is set to minimize C times a + B. So for logistic regression, if we set a very large value of lambda, that means you will give B a very high weight. Here is that if we set C to be a very small value, then that responds to giving B a much larger rate than C, than A. So this is just a different way of controlling the trade off, it's just a different way of prioritizing how much we care about optimizing the first term, versus how much we care about optimizing the second term. And if you want you can think of this as the parameter C playing a role similar to 1 over lambda. And it's not that it's two equations or these two expressions will be equal. This equals 1 over lambda, that's not the case. It's rather that if C is equal to 1 over lambda, then these two optimization objectives should give you the same value the same optimal value for theta so we just filling that in I'm guess cross out lambda here and write in the constant C there.

So that gives us our overall optimization objective function for the support vector machine. And if you minimize that function, then what you have is the parameters learned by the SVM.

SVM hypothesis

$$\min_\theta C\sum_{i=1}^{m}\left[y^{(i)}cost_1(\theta^T x^{(i)}) + (1-y^{(i)})cost_0(\theta^T x^{(i)})\right] + \frac{1}{2}\sum_{i=1}^{n}\theta_j^2$$

Hypothesis:

$$h_\theta(x) \begin{cases} 1 & \text{if } \theta^T x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Consider the following minimization problems:

1. $\min_\theta \frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}cost_1(\theta^T x^{(i)}) + (1-y^{(i)})cost_0(\theta^T x^{(i)})\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$

2. $\min_\theta C\sum_{i=1}^{m}\left[y^{(i)}cost_1(\theta^T x^{(i)}) + (1-y^{(i)})cost_0(\theta^T x^{(i)})\right] + \frac{1}{2}\sum_{j=1}^{n}\theta_j^2$

These two optimization problems will give the same value of θ (i.e., the same value of θ gives the optimal solution to both problems).

- [ ] $C = \lambda$
- [ ] $C = -\lambda$
- [x] $C = \frac{1}{\lambda}$
- [ ] $C = \frac{1}{2\lambda}$

✓ Correct

Finally unlike logistic regression, the support vector machine doesn't output the probability is that what we have is we have this cost function, that we minimize to get the parameter's data, and what a support vector machine does is it just makes a prediction of y being equal to one or zero, directly. So the hypothesis will predict one

If theta transpose x is greater or equal to zero, and it will predict zero otherwise and so having learned the parameters theta, this is the form of the hypothesis for the support vector machine. So that was a mathematical definition of what a support vector machine does. In the next few videos, let's try to get back to intuition about what this optimization objective leads to and whether the source of the hypotheses SVM will learn and we'll also talk about how to modify this just a little bit to the complex nonlinear functions.

Large margin classification

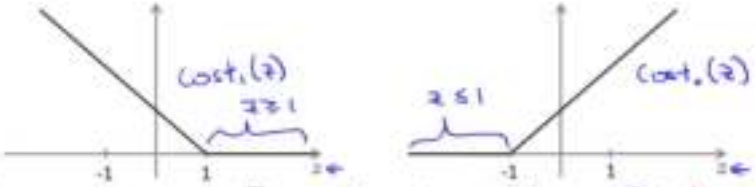
Large margin intuition

Support vector machine

Sometimes people talk about support vector machines, as large margin classifiers, in this video I'd like to tell you what that means, and this will also give us a useful picture of what an SVM hypothesis may look like. Here's my cost function for the support vector machine

Support Vector Machine

$$\min_\theta C\sum_{i=1}^{m}\left[y^{(i)}cost_1(\theta^T x^{(i)}) + (1-y^{(i)})cost_0(\theta^T x^{(i)})\right] + \frac{1}{2}\sum_{i=1}^{n}\theta_j^2$$



- If $y = 1$, we want $\theta^T x \geq 1$ (not just ≥ 0) $\quad \theta^T x \geq$ a lot 1
- If $y = 0$, we want $\theta^T x \leq -1$ (not just < 0) $\quad \theta^T x \leq$ a lot -1

$C = 100{,}000$

where here on the left I've plotted my cost 1 of z function that I used for positive examples and on the right I've plotted my

zero of 'Z' function, where I have 'Z' here on the horizontal axis. Now, let's think about what it takes to make these cost functions small.

If you have a positive example, so if y is equal to 1, then cost 1 of Z is zero only when Z is greater than or equal to 1. So in other words, if you have a positive example, we really want theta transpose x to be greater than or equal to 1 and conversely if y is equal to zero, look this cost zero of z function,

then it's only in this region where z is less than equal to 1 we have the cost is zero as z is equals to zero, and this is an interesting property of the support vector machine right, which is that, if you have a positive example so if y is equal to one, then all we really need is that theta transpose x is greater than equal to zero.

And that would mean that we classify correctly because if theta transpose x is greater than zero our hypothesis will predict zero. And similarly, if you have a negative example, then really all you want is that theta transpose x is less than zero and that will make sure we got the example right. But the support vector machine wants a bit more than that. It says, you know, don't just barely get the example right. So then don't just have it just a little bit bigger than zero. What I really want is for this to be quite a lot bigger than zero say maybe bit greater or equal to one and I want this to be much less than zero. Maybe I want it less than or equal to -1. And so this builds in an extra safety factor or safety margin factor into the support vector machine. Logistic regression does something similar too of course, but let's see what happens or let's see what the consequences of this are, in the context of the support vector machine.

Concretely, what I'd like to do next is consider a case case where we set this constant C to be a very large value, so let's imagine we set C to a very large value, may be a hundred thousand, some huge number.

Let's see what the support vector machine will do. ~~If C is very, very large, then when minimizing~~

Let's see what the support vector machine will do. If C is very, very large, then when minimizing this optimization objective, we're going to be highly motivated to choose a value, so that this first term is equal to zero.

So let's try to understand the optimization problem in the context of, what would it take to make this first term in the objective equal to zero, because you know, maybe we'll set C to some huge constant, and this will hope, this should give us additional intuition about what sort of hypotheses a support vector machine learns. So we saw already that whenever you have a training example with a label of y=1 if you want to make that first term zero, what you need is is to find a value of theta so that theta transpose x i is greater than or equal to 1. And similarly, whenever we have an example, with label zero, in order to make sure that the cost, cost zero of Z, in order to make sure that cost is zero we need that theta transpose x i is less than or equal to -1. So, if we think of our optimization problem as now, really choosing parameters and show that this first term is equal to zero, what we're left with is the following optimization problem. We're going to minimize that first term zero, so C times zero, because we're going to choose parameters so that's equal to zero, plus one half and then you know that second term and this first term is 'C' times zero, so let's just cross that out because I know that's going to be zero. And this will be subject to the constraint that theta transpose x(i) is greater than or equal to one, if y(i) is equal to one and theta transpose x(i) is less than or equal to minus one whenever you have a negative example and it turns out that when you solve this optimization problem, when you minimize this as a function of the parameters theta you get a very interesting decision boundary. ~~Concretely, if you look at a data set like this with positive and negative examples, this data~~

SVM Decision Boundary

$$\min_\theta C\sum_{i=1}^{m}\left[y^{(i)}cost_1(\theta^T x^{(i)}) + (1-y^{(i)})cost_0(\theta^T x^{(i)})\right] + \frac{1}{2}\sum_{i=1}^{n}\theta_j^2$$

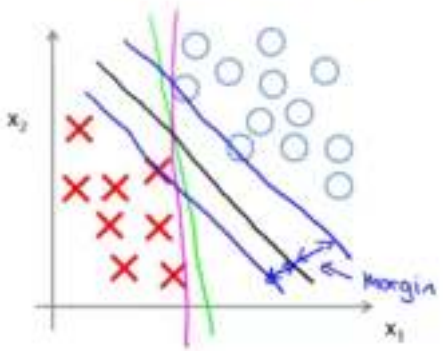
Whenever $y^{(i)} = 1$:

$\theta^T x^{(i)} \geq 1$

Whenever $y^{(i)} = 0$:

$\theta^T x^{(i)} \leq -1$

SVM Decision Boundary: Linearly separable case



Large margin classifier

you get a very interesting decision boundary. Concretely, if you look at a data set like this with positive and negative examples, this data is linearly separable and by that, I mean that there exists, you know, a straight line, altough there is many a different straight lines, they can separate the positive and negative examples perfectly. For example, here is one decision boundary that separates the positive and negative examples, but somehow that doesn't look like a very natural one, right? Or by drawing an even worse one, you know here's another decision boundary that separates the positive and negative examples but just barely. But neither of those seem like particularly good choices.

The Support Vector Machines will instead choose this decision boundary, which I'm drawing in black.

And that seems like a much better decision boundary than either of the ones that I drew in magenta or in green. The black line seems like a more robust separator, it does a better job of separating the positive and negative examples. And mathematically, what that does is, this black decision boundary has a larger distance.

That distance is called the margin, when I draw up this two extra blue lines, we see that the black decision boundary has some larger minimum distance from any of my training examples, whereas the magenta and the green lines they come awfully close to the training examples. and then that seems to do a less a good job separating the positive and negative classes than my black line. And so this distance is called the margin of the support vector machine and this gives the SVM a certain robustness, because it tries to separate the data with as a large a margin as possible.

So the support vector machine is sometimes also called a large margin classifier and this is actually a consequence of the optimization problem we wrote down on the previous slide. I know that you might be wondering how is it that the optimization problem I wrote down in the previous slide, how does that lead to this large margin classifier.

I know I haven't explained that yet. And in the next video I'm going to sketch a little bit of the intuition about why that optimization problem gives us this large margin classifier. But this is a useful feature to keep in mind if you are trying to understand what are the sorts of hypothesis that an SVM will choose. That is, trying to separate the positive and negative examples with as big a margin as possible.

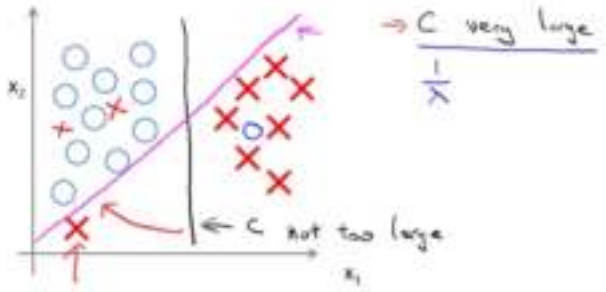
I want to say one last thing about large margin classifiers in this intuition, so we wrote out this large margin classification setting in the case of when C, that regularization concept, was very large, I think I set that to a hundred thousand or something. So given a dataset like this, maybe we'll choose that decision boundary that separate the positive and negative examples on large margin.

Now, the SVM is actually slightly more sophisticated than this large margin view might suggest. And in particular, if all you're doing is use a large margin classifier then your learning algorithms can be sensitive to outliers, so lets just add an extra positive example like that shown on the screen. If he had one example then it seems as if to separate data with a large margin,

maybe I'll end up learning a decision boundary like that, right? that is the magenta line and it's really not clear that based on the single outlier based on a single example and it's really not clear that it's actually a good idea to change my decision boundary from the black one over to the magenta one.

So, if C, if the regularization parameter C were very large, then this is actually what SVM will do, it will change the decision boundary from the black to the magenta one but if C were reasonably small if you were to use the C, not too large then you still end up with this black decision boundary. And of course if the data were not linearly separable so if you had some positive examples in here, or if you had some negative examples in here then the SVM will also do the right thing. And so this picture of a large margin classifier that's really, that's really the picture that gives better intuition only for the case of when the regulations parameter C is very large, and just to remind you this corresponds C plays a role similar to one over Lambda, where Lambda is the regularization parameter we had previously. And so it's only of one over Lambda is very large or equivalently if Lambda is very small that you end up with things like this Magenta decision boundary, but

Large margin classifier in presence of outliers



in practice when applying support vector machines, when C is not very very large like that,

it can do a better job ignoring the few outliers like here. And also do fine and do reasonable things even if your data is not linearly separable. But when we talk about bias and variance in the context of support vector machines which will do a little bit later, hopefully all of of this trade-offs involving the regularization parameter will become clearer at that time. So I hope that gives some intuition about how this support vector machine functions as a large margin classifier that tries to separate the data with a large margin, technically this picture of this view is true only when the parameter C is very large, which is

a useful way to think about support vector machines.

There was one missing step in this video which is, why is it that the optimization problem we wrote down on these slides, how does that actually lead to the large margin classifier, I didn't do that in this video, in the next video I will sketch a little bit more of the math behind that to explain that separate reasoning of how the optimization problem we wrote out results in a large margin classifier.

Large Margin Classification

Mathematics behind Large Margin Classification

In this video, I'd like to tell you a bit about the math behind large margin classification.

This video is optional, so please feel free to skip it. It may also give you better intuition about how the optimization problem of the support vex machine, how that leads to large margin classifiers.

In order to get started, let me first remind you of a couple of properties of what vector inner products look like.

Let's say I have two vectors U and V, that look like this. So both two dimensional vectors.

Then let's see what U transpose V looks like. And U transpose V is also called the inner products between the vectors U and V.

Use a two dimensional vector, so I can on plot it on this figure. So let's say that's the vector U. And what I mean by that is if on the horizontal axis that value takes whatever value U1 is and on the vertical axis the height of that is whatever U2 is the second component of the vector U. Now, one quantity that will be nice to have is the norm

of the vector U. So, these are, you know, double bars on the left and right that denotes the norm or length of U. So this just means, really the euclidean length of the vector U. And this is Pythagoras theorem is just equal to U1 squared plus U2 squared square root, right? And this is the length of the vector U. That's a real number. Just say you know, what is the length of this, what is the length of this vector down here. What is the length of this arrow that I just drew, is the normal view?

Now let's go back and look at the vector V because we want to compute the inner product. So V will be some other vector with, you know, some value V1, V2.

And so, the vector V will look like that, towards V like so.

Now let's go back and look at how to compute the inner product between U and V. Here's how you can do it. Let me take the vector V and project it down onto the vector U. So I'm going to take a orthogonal projection or a 90 degree projection, and project it down onto U like so.

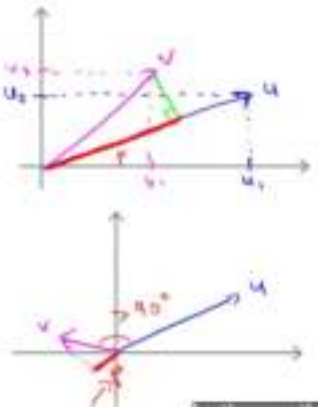
And what I'm going to do measure length of this red line that I just drew here. So, I'm going to call the length of that red line P. So, P is the length or is the magnitude of the projection

of the vector V onto the vector U. Let me just write that down. So, P is the length

of the projection of the vector V onto the vector U. And it is possible to show that and product U transpose V, that this is going to be equal to P times the norm or the length of the vector U. So, this is one way to compute the inner product. And if you actually do the geometry figure out what P is and figure out what the norm of U is. This should give you the same way, the same answer as the other way of computing unit product.

Right. Which is if you take U transpose V then U transposes this U1 U2, its a one by two matrix, I times V. And so this should actually give you U1, V1 plus U2, V2.

Vector Inner Product



$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$u^T v = ?$ $\quad [u_1 \; u_2] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$\|u\|$ = Length of vector u

$= \sqrt{u_1^2 + u_2^2} \in \mathbb{R}$

p = Length of projection of v onto u.

$u^T v = p \cdot \|u\|$ $\quad = v^T u$

$= u_1 v_1 + u_2 v_2$ $\quad p \in \mathbb{R}$

Signed

$u^T v = p \cdot \|u\|$

$p < 0$

And so the theorem of linear algebra that these two formulas give you the same answer.

And by the way, U transpose V is also equal to V transpose U. So if you were to do the same process in reverse, instead of projecting V onto U, you could project U onto V. Then, you know, do the same process, but with the rows of U and V reversed. And you would actually, you should actually get the same number whatever that number is. And just to clarify what's going on in this equation the norm of U is a real number and P is also a real number. And so U transpose V is the regular multiplication as two real numbers of the length of P times the normal view.

Just one last detail, which is if you look at the norm of P, P is actually signed so to the right.

And it can either be positive or negative.

So let me say what I mean by that, if U is a vector that looks like this and V is a vector that looks like this.

So if the angle between U and V is greater than ninety degrees. Then if I project V onto U, what I get is a projection it looks like this and so that length P. And in this case, I will still have that U transpose V is equal to P times the norm of U. Except in this example P will be negative.

So, you know, in inner products if the angle between U and V is less than ninety degrees, then P is the positive length for that red line whereas if the angle of this angle of here is greater than 90 degrees then P here will be negative of the length of the super line of that little line segment right over there. So the inner product between two vectors can also be negative if the angle between them is greater than 90 degrees. So that's how vector inner products work. We're going to use these properties of vector inner product to try to understand the support vector machine optimization objective over there. ~~Here is the optimization objective for the support vector~~

optimization objective over there. Here is the optimization objective for the support vector machine that we worked out earlier. Just for the purpose of this slide I am going to make one simplification or once just to make the objective easy to analyze and what I'm going to do is ignore the indexceptnums. So, we'll just ignore theta 0 and set that to be equal to 0. To

make things easier to plot, I'm also going to set N the number of features to be equal to 2. So, we have only 2 features,

X1 and X2.

Now, let's look at the objective function. The optimization objective of the SVM. What we have only two features. When N is equal to 2. This can be written, one half of theta one squared plus theta two squared. Because we only have two parameters, theta one and theta two.

What I'm going to do is rewrite this a bit. I'm going to write this as one half of theta one squared plus theta two squared and the square root squared. And the reason I can do that, is because for any number, you know, W, right, the

square roots of W and then squared, that's just equal to W. So square roots and squared should give you the same thing.

What you may notice is that this term inside is that's equal to the norm

or the length of the vector theta and what I mean by that is that if we write out the vector theta like this, as you know theta one, theta two. Then this term that I've just underlined in red, that's exactly the length, or the norm, of the vector theta. We are calling the definition of the norm of the vector that we have on the previous line.

And in fact this is actually equal to the length of the vector theta, whether you write it as theta zero, theta 1, theta 2. That is, if theta zero is equal to zero, as I assume here. Or just the length of theta 1, theta 2; but for this line I am going to ignore theta 0. So let me just, you know, treat theta as this, let me just write theta, the normal theta as this theta 1, theta 2 only, but the math works out either way, whether we include theta zero here or not. So it's not going to matter for the rest of our derivation.

And so finally this means that my optimization objective is equal to one half of the norm of theta squared.

So all the support vector machine is doing in the optimization objective is it's minimizing the squared norm of the square length of the parameter vector theta.

Now what I'd like to do is look at these terms, theta transpose X and understand better what they're doing. So given the parameter vector theta and given and example x, what is this is equal to? And on the previous slide, we figured out what U transpose V looks like, with different vectors U and V. And so we're going to take those definitions, you know, with theta and X(i) playing the roles of U and V.

And let's see what that picture looks like. So, let's say I plot. Let's say I look at just a single training example. Let's say I have a positive example the drawing was across there and let's say that is my example X(i), what that really means is

plotted on the horizontal axis some value X(i) 1 and on the vertical axis

X(i) 2. That's how I plot my training examples.

SVM Decision Boundary

$$\min_\theta \frac{1}{2}\sum_{j=1}^{n}\theta_j^2 = \frac{1}{2}(\theta_1^2+\theta_2^2) = \frac{1}{2}\left(\sqrt{\theta_1^2+\theta_2^2}\right)^2 = \frac{1}{2}\|\theta\|^2$$

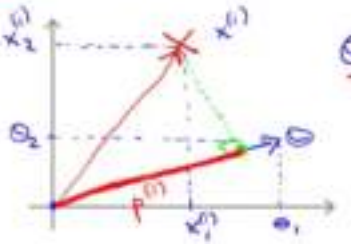
$$\text{s.t.}\quad \theta^T x^{(i)} \geq 1 \quad \text{if } y^{(i)}=1$$

$$\theta^T x^{(i)} \leq -1 \quad \text{if } y^{(i)}=0$$

Simplification: $\theta_0 = 0$. $n=2$

$\theta^T x^{(i)} = ?$ ($u^T v$)

$\theta^T x^{(i)} = p^{(i)} \cdot \|\theta\| = \theta_1 x_1^{(i)} + \theta_2 x_2^{(i)}$



And although we haven't been really thinking of this as a vector, what this really is, this is a vector from the origin from 0, 0 out to

the location of this training example.

And now let's say we have a parameter vector and I'm going to plot that as vector, as well. What I mean by that is if I plot theta 1 here and theta 2 there

so what is the inner product theta transpose X(i). While using our earlier method, the way we compute that is we take my example and project it onto my parameter vector theta.

And then I'm going to look at the length of this segment that I'm coloring in, in red. And I'm going to call that P superscript I to denote that this is a projection of the i-th training example

onto the parameter vector theta.

And so what we have is that theta transpose X(i) is equal to following what we have on the previous slide, this is going to be equal to

P times the length of the norm of the vector theta.

And this is of course also equal to theta 1 x1

plus theta 2 x2. So each of these is, you know, an equally valid way of computing the inner product between theta and X(i).

Okay. So where does this leave us? What this means is that, this constrains that theta transpose X(i) be greater than or equal to one or less than minus one. What this means is that it can replace the use of constraints that P(i) times X be greater than or equal to one. Because theta transpose X(i) is equal to P(i) times the norm of theta.

SVM Decision Boundary

$$\min_{\theta} \frac{1}{2}\sum_{j=1}^{n}\theta_j^2 = \frac{1}{2}\|\theta\|^2$$

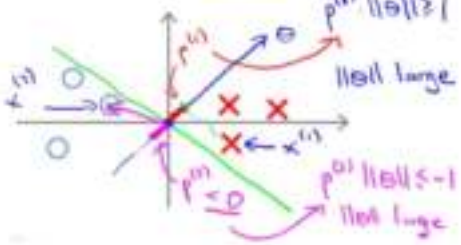
s.t. $p^{(i)} \cdot \|\theta\| \geq 1$ if $y^{(i)} = 1$

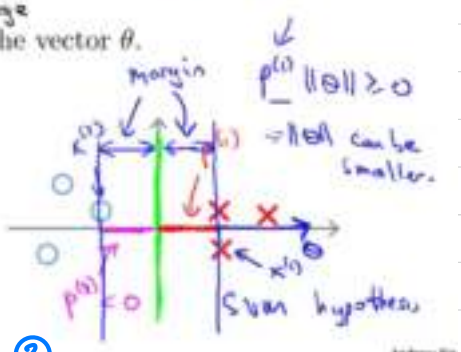
$p^{(i)} \cdot \|\theta\| \leq -1$ if $y^{(i)} = 1$

C very large

where $p^{(i)}$ is the projection of $x^{(i)}$ onto the vector θ.

Simplification: $\theta_0 = 0$





(1)

So writing that into our optimization objective. This is what we get where I have, instead of theta transpose X(i), I now have this P(i) times the norm of theta.

And just to remind you we worked out earlier too that this optimization objective can be written as one half times the norm of theta squared.

So, now let's consider the training example that we have at the bottom and for now, continuing to use the simplification that theta 0 is equal to 0. Let's see what decision boundary the support vector machine will choose.

Here's one option, let's say the support vector machine were to choose this decision boundary. This is not a very good choice because it has very small margins. This decision boundary comes very close to the training examples.

Let's see why the support vector machine will not do this.

For this choice of parameters it's possible to show that the parameter vector theta is actually at 90 degrees to the decision boundary. And so, that green decision boundary corresponds to a parameter vector theta that points in that direction.

And by the way, the simplification that theta 0 equals 0 that just means that the decision boundary must pass through the origin, (0,0) over there. So now, let's look at what this implies for the optimization objective.

(2)

Let's say that this example here.

Let's say that's my first example, you know,

X1.

If we look at the projection of this example onto my parameters theta.

That's the projection.

And so that little red line segment.

That is equal to P1. And that is going to be pretty small, right.

And similarly, if this

example here, if this happens to be X2, that's my second example.

Then, if I look at the projection of this this example onto theta.

You know. Then, let me draw this one in magenta.

This little magenta line segment, that's going to be P2. That's the projection of the second example onto my, onto the direction

of my parameter vector theta which goes like this.

And so, this little

projection line segment is getting pretty small.

P2 will actually be a negative number, right so P2 is in the opposite direction.

This vector has greater than 90 degree angle with my parameter vector theta, it's going to be less than 0.

(3)

And so what we're finding is that these terms P(i) are going to be pretty small numbers. So if we look at the optimization objective and see, well, for positive examples we need P(i) times the norm of theta to be bigger than either one.

But if P(i) over here, if P1 over here is pretty small, that means that we need the norm of theta to be pretty large, right? If

P1 of theta is small and we want P1 you know times in all of theta to be bigger than either one, well the only way for that to be true for the profit that these two numbers to be large if P1 is small, as we said we want the norm of theta to be large.

And similarly for our negative example, we need P2

times the norm of theta to be less than or equal to minus one. And we saw in this example already that P2 is going pretty small negative number, and so the only way for that to happen as well is for the norm of theta to be large, but what we are doing in the optimization objective is we are trying to find a setting of parameters where the norm of theta is small, and so you know, so this doesn't seem like such a good direction for the parameter vector and theta. In contrast, just look at a different decision boundary.

Here, let's say, this SVM chooses

that decision boundary.

Now the is going to be very different. If that is the decision boundary, here is the corresponding direction for theta. So, with the direction boundary you know, that vertical line that corresponds to it is possible to show using linear algebra that the way to get that green decision boundary is have the vector of theta be at 90 degrees to it, and now if you look at the projection of your data onto the vector

(4)

x, lets say its before this example is my example of x1. So when I project this on to x, or onto theta, what I find is that this is P1.

That length there is P1.

The other example, that example is and I do the same projection and what I find is that this length here is a P2 really that is going to be less than 0. And you notice that now P1 and P2, these lengths

of the projections are going to be much bigger, and so if we still need to enforce these constraints that P1 of the norm of theta is phase number one because P1 is so much bigger now. The normal can be smaller.

And so, what this means is that by choosing the decision boundary shown on the right instead of on the left, the SVM can make the norm of the parameters theta much smaller. So, if we can make the norm of theta smaller and therefore make the squared norm of theta smaller, which is why the SVM would choose this hypothesis on the right instead.

And this is how

the SVM gives rise to this large margin certification effect.

Mainly, if you look at this green line, if you look at this green hypothesis we want the projections of my positive and negative examples onto theta to be large, and the only way for that to hold true this is if surrounding the green line

There's this large margin, there's this large gap that separates

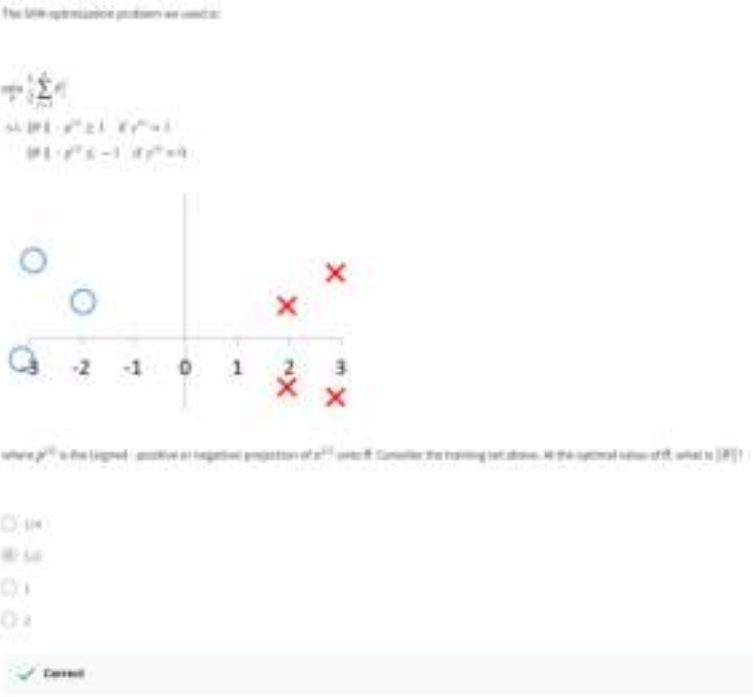
positive and negative examples is really the magnitude of this gap. The magnitude of this margin is exactly the values of P1, P2, P3 and so on. And so by making the margin large, by these lyrics P1, P2, P3 and so on that's the SVM can end up with a smaller value for the norm of theta which is what it is trying to do in the objective. And this is why this machine ends up with enlarge margin classifiers because itss trying to maximize the norm of these P1 which is the distance from the training examples to the decision boundary.

Finally, we did this whole derivation using this simplification that the parameter theta 0 must be equal to 0. The effect of that as I mentioned briefly, is that if theta 0 is equal to 0 what that means is that we are entertaining decision boundaries that pass through the origins of decision boundaries pass through the origin like that, if you allow theta zero to be non 0 then what that means is that you entertain the decision boundaries that did not cross through the origin, like that one I just drew. And I'm not going to do the full derivation that. It turns out that this same large margin proof works in pretty much in exactly the same way. And there's a generalization of this argument that we just went through them long ago through that shows that even when theta 0 is non 0, what the SVM is trying to do when you have this optimization objective.

Which again corresponds to the case of when C is very large.

But it is possible to show that, you know, when theta is not equal to 0 this support vector machine is still finding is really trying to find the large margin separator that between the positive and negative examples. So that explains how this support vector machine is a large margin classifier.

In the next video we will start to talk about how to take some of these SVM ideas and start to apply them to build a complex nonlinear classifiers.



Kernels

Kernels I

In this video, I'd like to start adapting support vector machines in order to develop complex nonlinear classifiers.

The main technique for doing that is something called kernels.

Let's see what this kernels are and how to use them.

If you have a training set that looks like this, and you want to find a nonlinear decision boundary to distinguish the positive and negative examples, maybe a decision boundary that looks like that.

One way to do so is to come up with a set of complex polynomial features, right? So, set of features that looks like this, so that you end up with a hypothesis X that predicts 1 if you know that theta 0 and plus theta 1 X1 plus dot dot dot all those polynomial features is greater than 0, and predict 0, otherwise.

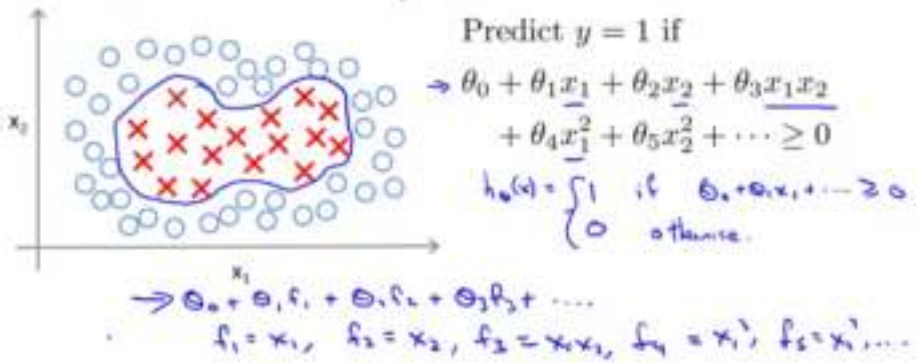
And another way of writing this, to introduce a level of new notation that I'll use later, is that we can think of a hypothesis as computing a decision boundary using this. So, theta 0 plus theta 1 f1 plus theta 2, f2 plus theta 3, f3 plus and so on. Where I'm going to use this new denotation f1, f2, f3 and so on to denote these new sort of features.

that I'm computing, so f1 is just X1, f2 is equal to X2, f3 is equal to this one here. So, X1X2. So, f4 is equal to

X1 squared where f5 is to be x2 squared and so on and we seen previously that coming up with these high order polynomials is one way to come up with lots more features.

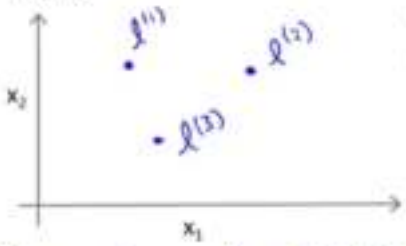
the question is, is there a different choice of features or is there better sort of features than this high order polynomials because you know it's not clear that this high order polynomial is what we want, and what we talked about computer vision talk about when the input is an image with lots of pixels. We also saw how using high order polynomials becomes very computationally expensive because there are a lot of these higher order polynomial terms.

So, is there a different or a better choice of the features that we can use to plug into this sort of hypothesis form. So, here is one idea for how to define new features f1, f2, f3.



Is there a different / better choice of the features $f_1, f_2, f_3, \ldots$?

Kernel



Given x, compute new feature depending on proximity to landmarks $l^{(1)}, l^{(2)}, l^{(3)}$

Given x:

$$f_1 = \text{similarity}(x, l^{(1)}) = \exp\left(-\frac{\|x - l^{(1)}\|^2}{2\sigma^2}\right)$$

$$f_2 = \text{similarity}(x, l^{(2)}) = \exp\left(-\frac{\|x - l^{(2)}\|^2}{2\sigma^2}\right)$$

$$f_3 = \text{similarity}(x, l^{(3)}) = \exp(\dots)$$

Kernel (Gaussian kernels) $k(x, l^{(1)})$

So, is there a different or a better choice of the features that we can use to plug into this sort of hypothesis form. So, here is one idea for how to define new features f1, f2, f3.

On this line I am going to define only three new features, but for real problems we can get to define a much larger number. But here's what I'm going to do in this phase of features X1, X2, and I'm going to leave X0 out of this, the intercept X0, but in this phase X1 X2, I'm going to just,

you know, manually pick a few points, and then call these points l1, we are going to pick a different point, let's call that l2 and let's pick the third one and call this one l3, and for now let's just say that I'm going to choose these three points manually. I'm going to call these three points line ups, so line up one, two, three. What I'm going to do is define my new features as follows, given an example X, let me define my first feature f1 to be some measure of the similarity between my training example X and my first landmark and this specific formula that I'm going to use to measure similarity is going to be this is E to the minus the length of X minus l1, squared, divided by two sigma squared.

So, depending on whether or not you watched the previous optional video, this notation, you know, this is the length of the vector W. And so, this thing here, this X minus l1, this is actually just the euclidean distance

squared, is the euclidean distance between the point x and the landmark l1. We will see more about this later.

But that's my first feature, and my second feature f2 is going to be, you know, similarity function that measures how similar X is to l2 and the game is going to be defined as the following function.

This is E to the minus of the square of the euclidean distance between X and the second landmark, that is what the enumerator is and then divided by 2 sigma squared and similarly f3 is, you know, similarity between X and l3, which is equal to, again, similar formula.

And what this similarity function is, the mathematical term for this, is that this is going to be a kernel function. And the specific kernel I'm using here, this is actually called a Gaussian kernel.

And so this formula, this particular choice of similarity function is called a Gaussian kernel. But the way the terminology goes is that, you know, in the abstract these different similarity functions are called kernels and we can have different similarity functions

and the specific example I'm giving here is called the Gaussian kernel. We'll see other examples of other kernels. But for now just think of these as similarity functions.

And so, instead of writing similarity between X and l, sometimes we also write this a kernel denoted you know, lower case k between x and one of my landmarks all right.

So let's see what a criminals actually do and why these sorts of similarity functions, why these expressions might make sense.

So let's take my first landmark. My landmark l1, which is one of those points I chose on my figure just now.

So the similarity of the kernel between x and l1 is given by this expression.

Just to make sure, you know, we are on the same page about what the numerator term is, the numerator can also be written as a sum from J equals 1 through N on sort of the distance. So this is the component wise distance between the vector X and the vector l. And again for the purpose of these slides I'm ignoring X0. So just ignoring the intercept term X0, which is always equal to 1.

So, you know, this is how you compute the kernel with similarity between X and a landmark.

So let's see what this function does. Suppose X is close to one of the landmarks.

Then this euclidean distance formula and the numerator will be close to 0, right. So, that is this term here, the distance was great, the distance using X and 0 will be close to zero, and so

f1, this is a simple feature, will be approximately E to the minus 0 and then the numerator squared over 2 is equal to squared

so that E to the 0, E to minus 0, E to 0 is going to be close to one.

And I'll put the approximation symbol here because the distance may not be exactly 0, but if X is closer to landmark this term will be close to 0 and so f1 would be close 1.

Conversely, if X is far from 01 then this first feature f1 will be E to the minus of some large number squared, divided divided by two sigma squared and E to the minus of a large number is going to be close to 0.

So what these features do is they measure how similar X is from one of your landmarks and the feature f is going to be close to one when X is close to your landmark and is going to be 0 or close to zero when X is far from your landmark. Each of these landmarks. On the previous line, I drew three landmarks, l1, l2,l3.

Each of these landmarks, defines a new feature f1, f2 and f3. That is, given the the training example X, we can now compute three new features: f1, f2, and f3, given, you know, the three landmarks that I wrote just now. But first, let's look at this exponentiation function, let's look at this similarity function and plot in some figures and just, you know, understand better what this really looks like.

Kernels and Similarity

$$f_1 = \text{similarity}(x, l^{(1)}) = \exp\left(-\frac{\|x - l^{(1)}\|^2}{2\sigma^2}\right) = \exp\left(-\frac{\sum_{j=1}^{n}(x_j - l_j^{(1)})^2}{2\sigma^2}\right)$$

If $x \approx l^{(1)}$:

$$f_1 \approx \exp\left(-\frac{0^2}{2\sigma^2}\right) \approx 1$$

If x if far from $l^{(1)}$:

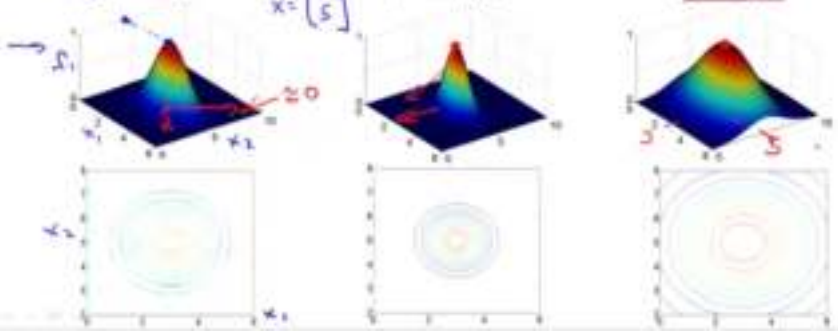
$$f_1 = \exp\left(-\frac{(\text{large number})^2}{2\sigma^2}\right) \approx 0.$$

$l^{(1)} \to f_1$, $l^{(2)} \to f_2$, $l^{(3)} \to f_3$

Example:

$l^{(1)} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $f_1 = \exp\left(-\frac{\|x - l^{(1)}\|^2}{2\sigma^2}\right)$

$\sigma^2 = 1$ $\quad x = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ $\quad \sigma^2 = 0.5$ $\quad \sigma^2 = 3$



For this example, let's say I have two features X1 and X2. And let's say my first landmark, l1 is at a location, 3 5. So

and let's say I set sigma squared equals one for now. If I plot what this feature looks like, what I get is this figure. So the vertical axis, the height of the surface is the value

of f1 and down here on the horizontal axis are, if I have some training example, and there

is x1 and there is x2. Given a certain training example, the training example here which shows the value of x1 and x2 at a height above the surface, shows the corresponding value of f1 and down below this is the same figure I had showed, using a quantifiable plot, with x1 on horizontal axis, x2 on horizontal axis and so, this figure on the bottom is just a contour plot of the 3D surface.

You notice that when X is equal to 3 5 exactly, then we the f1 takes on the value 1, because that's at the maximum and X moves away as X goes further away then this feature takes on values

that are close to 0.

And so, this is really a feature, f1 measures, you know, how close X is to the first landmark and if varies between 0 and one depending on how close X is to the first landmark l1.

Now the other was due on this slide is show the effects of varying this parameter sigma squared. So, sigma squared is the parameter of the Gaussian kernel and as you vary it, you get slightly different effects.

Let's set sigma squared to be equal to 0.5 and see what we get. We set sigma square to 0.5, what you find is that the kernel looks similar, except for the width of the bump becomes narrower. The contours shrink a bit too. So if sigma squared equals to 0.5 then as you start from X equals 3 5 and as you move away,

then the feature f1 falls to zero much more rapidly and conversely,

if you has increase since where three in that case and as I move away from, you know I. So this point here is really I, right, that's l1 is at location 3 5, right. So it's shown up here.

And if sigma squared is large, then as you move away from l1, the value of the feature falls away much more slowly.

So, given this definition of the features, let's see what source of hypothesis we can learn.

Given the training example X, we are going to compute these features

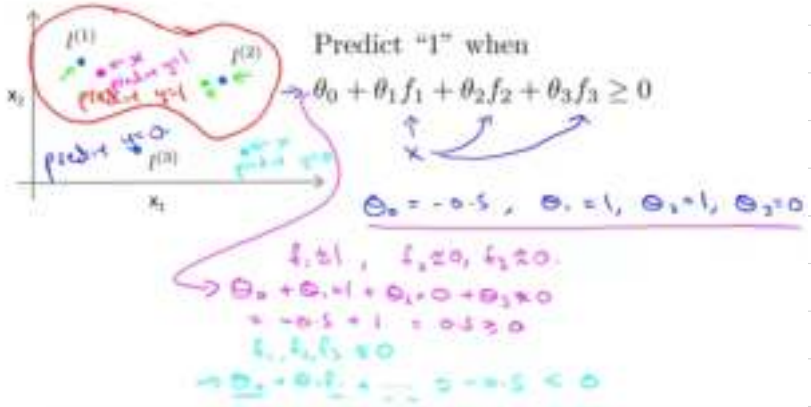
f1, f2, f3 and a

hypothesis is going to predict one when theta 0 plus theta 1 f1 plus theta 2 f2, and so on is greater than or equal to 0. For this particular example, let's say that I've already found a learning algorithm and let's say that, you know, somehow I ended up with these values of the parameter. So if theta 0 equals minus 0.5, theta 1 equals 1, theta 2 equals 1, and theta 3 equals 0 And what I want to do is consider what happens if we have a training example that takes

has location at this magenta dot, right where I just drew this dot over here. So let's say I have a training example X, what would my hypothesis predict? Well, if I look at this formula.

Because my training example X is close to l1, we have that f1 is going to be close to 1 the because my training example X is far from l2 and l3 I have that, you know, f2 would be close to 0 and f3 will be close to 0.

So, if I look at that formula, I have theta 0 plus theta 1 times 1 plus theta 2 times some value. Not exactly 0, but let's say close to 0. Then plus theta 3 times something close to 0.



And this is going to be equal to plugging in these values now.

So, that gives minus 0.5 plus 1 times 1 which is 1, and so on. Which is equal to 0.5 which is greater than or equal to 0. So, at this point, we're going to predict Y equals 1, because that's greater than or equal to zero.

Now let's take a different point. Now lets' say I take a different point, I'm going to draw this one in a different color, in cyan say, for a point out there, if that were my training example X, then if you make a similar computation, you find that f1, f2,

Ff3 are all going to be close to 0.

And so, we have theta 0 plus theta 1, f1, plus so on and this will be about equal to minus 0.5, because theta 0 is minus 0.5 and f1, f2, f3 are all zero. So this will be minus 0.5, this is less than zero. And so, at this point out there, we're going to predict Y equals zero.

And if you do this yourself for a range of different points, be sure to convince yourself that if you have a training example that's close to l2, say, then at this point we'll also predict Y equals one.

And in fact, what you end up doing is, you know, if you look around this boundary, this space, what we'll find is that for points near l1 and l2 we end up predicting positive. And for points far away from l1 and l2, that's for points far away from these two landmarks, we end up predicting that the class is equal to 0. As so, what we end up doing is that the decision boundary of this hypothesis would end up looking something like this where inside this red decision boundary would predict Y equals 1 and outside we predict

Y equals 0. And so this is how with this definition of the landmarks and of the kernel function. We can learn pretty complex non-linear decision boundary, like what I just drew where we predict positive when we're close to either one of the two landmarks. And we predict negative when we're very far away from any of the landmarks. And so this is part of the idea of kernels of and how we use them with the support vector machine, which is that we define these extra features using landmarks and similarity functions to learn more complex nonlinear classifiers.

So hopefully that gives you a sense of the idea of kernels and how we could use it to define new features for the Support Vector Machine.

But there are a couple of questions that we haven't answered yet. One is, how do we get these landmarks? How do we choose these landmarks? And another is, what other similarity functions, if any, can we use other than the one we talked about, which is called the Gaussian kernel. In the next video we give answers to these questions and put everything together to show how support vector machines with kernels can be a powerful way to learn complex nonlinear functions.

Kernels

Kernels II

In the last video, we started to talk about the kernels idea and how it can be used to define new features for the support vector machine. In this video, I'd like to throw in some of the missing details and, also, say a few words about how to use these ideas in practice. Such as, how they pertain to, for example, the bias variance trade-off in support vector machines.

In the last video, I talked about the process of picking a few landmarks. You know, l1, l2, l3 and that allowed us to define the similarity function also called the kernel or in this example if you have this similarity function this is a Gaussian kernel.

And that allowed us to build this form of a hypothesis function.

But where do we get these landmarks from? Where do we get l1, l2, l3 from? And it seems, also, that for complex learning problems, maybe we want a lot more landmarks than just three of them that we might choose by hand.

So in practice this is how the landmarks are chosen which is that given the machine learning problem. We have some data set of some some positive and negative examples. So, this is the idea here which is that we're gonna take the examples and for every training example that we have, we are just going to call it. We're just going to put landmarks as exactly the same locations as the training examples.

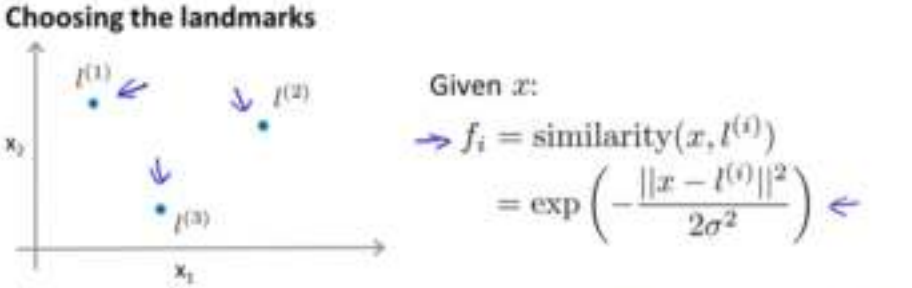
So if I have one training example if that is x1, well then I'm going to choose this is my first landmark to be at xactly the same location as my first training example.

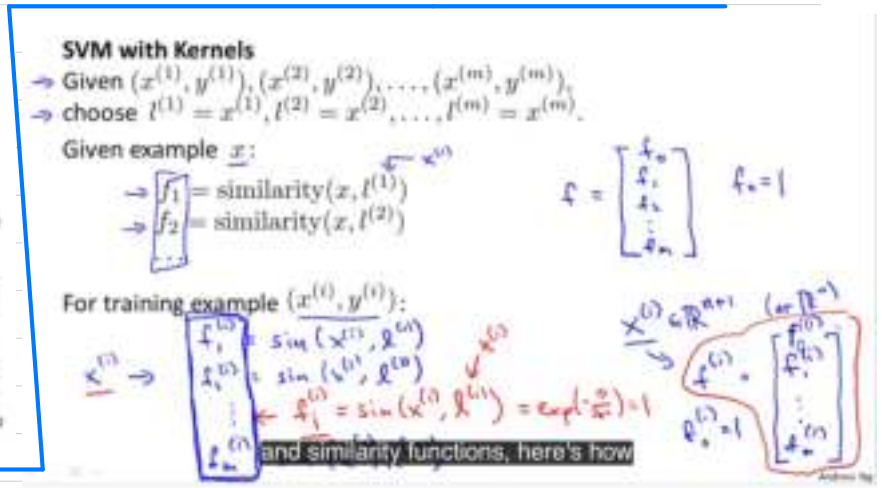
And if I have a different training example x2. Well we're going to set the second landmark to be the location of my second training example.

On the figure on the right, I used red and blue dots just as illustration, the color of this figure, the color of the dots on the figure on the right is not significant.

But what I'm going to end up with using this method is I'm going to end up with m landmarks of l1, l2

down to l(m) if I have m training examples with one landmark per location of my per location of each of my training examples. And this is nice because it is saying that my features are basically going to measure how close an example is to one of the things I saw in my training set. So, just to write this outline a little more concretely, given m training examples, I'm going to choose the the location of my landmarks to be exactly near the locations of my m training examples.





When you are given example x, and in this example x can be something in the training set, it can be something in the cross validation set, or it can be something in the test set. Given an example x we are going to compute, you know, these features as so f1, f2, and so on. Where l1 is actually equal to x1 and so on. And these then give me a feature vector. So let me write f as the feature vector. I'm going to take these f1, f2 and so on, and just group them into feature vector.

Take those down to fm.

And, you know, just by convention. If we want, we can add an extra feature f0, which is always equal to 1. So this plays a role similar to what we had previously. For x0, which was our interceptor.

So, for example, if we have a training example x(i), y(i),

the features we would compute for this training example will be as follows: given x(i), we will then map it to, you know, f1(i).

Which is the similarity. I'm going to abbreviate as SIM instead of writing out the whole word similarity, right?

And f2(i) equals the similarity between x(i) and l2, and so on, down to fm(i) equals

the similarity between x(i) and l(m).

And somewhere in the middle. Somewhere in this list, you know, at the i-th component, I will actually have one feature component which is f subscript i(i), which is going to be the similarity

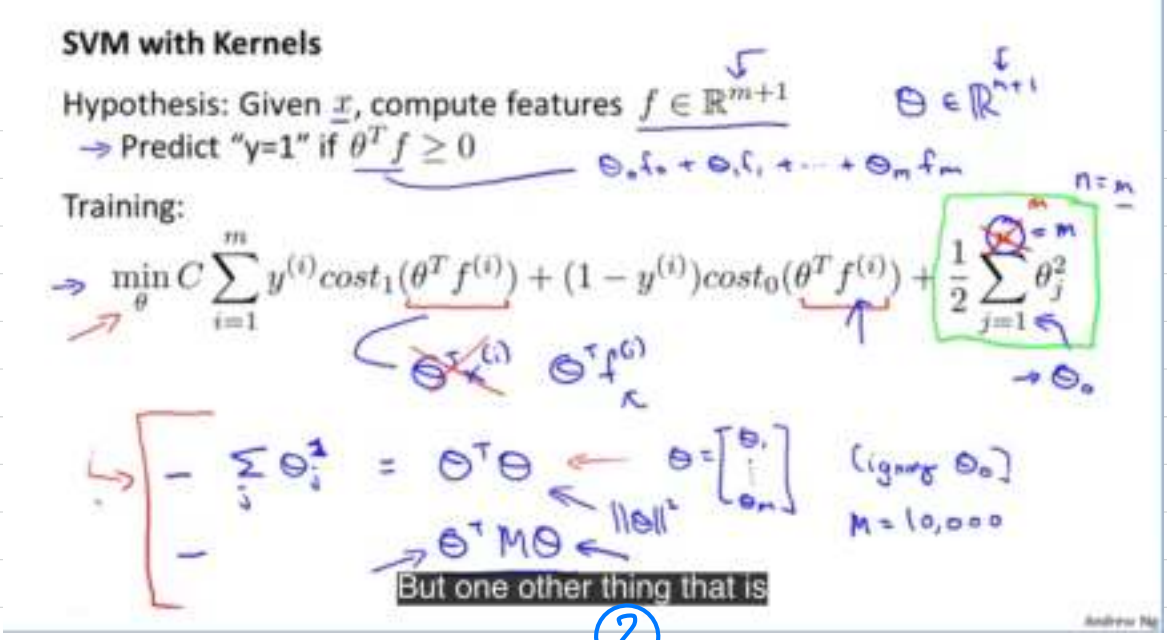
between x and l(i).

Where l(i) is equal to x(i), and so you know fi(i) is just going to be the similarity between x and itself.

And if you're using the Gaussian kernel this is actually e to the minus 0 over 2 sigma squared and so, this will be equal to 1 and that's okay. So one of my features for this training example is going to be equal to 1.

And then similar to what I have above. I can take all of these m features and group them into a feature vector. So instead of representing my example, using, you know, x(i) which is this what R(n) plus R(n) one dimensional vector.

Depending on whether you can set terms, is either R(n) or R(n) plus 1. We can now instead represent my training example using this feature vector f. I am going to write this f superscript i. Which is going to be taking all of these things and stacking them into a vector. So, f1(i) down to fm(i) and if you want and well, usually we'll also add this f0(i), where f0(i) is equal to 1. And so this vector here gives me my new feature vector with which to represent my training example. So given these kernels and similarity functions, here's how we use a simple vector machine. If you



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given these kernels and similarity functions, here's how we use a simple vector machine. If you already have a learning set of parameters theta, then if you given a value of x and you want to make a prediction.

What we do is we compute the features f, which is now an R(m) plus 1 dimensional feature vector.

And we have m here because we have m training examples and thus m landmarks and what we do is we predict 1 if theta transpose f is greater than or equal to 0. Right. So, if theta transpose f, of course, that's just equal to theta 0, f0 plus theta 1, f1 plus dot dot dot, plus theta m f(m). And so my parameter vector theta is also now going to be an m plus 1 dimensional vector. And we have m here because where the number of landmarks is equal to the training set size. So m was the training set size and now, the parameter vector theta is going to be m plus one dimensional.

So that's how you make a prediction if you already have a setting for the parameter's theta. How do you get the parameter's theta? Well you do that using the SVM learning algorithm, and specifically what you do is you would solve this minimization problem. You've minimized the parameter's theta of C times this cost function which we had before. Only now, instead of looking there instead of making predictions using theta transpose x(i) using our original features, x(i). Instead we've taken the features x(i) and replace them with a new features

so we are using theta transpose f(i) to make a prediction on the i'f training examples and we see that, you know, in both places here and it's by solving this minimization problem that you get the parameters for your Support Vector Machine.

And one last detail is because this optimization problem we really have n equals m features. That is here. The number of features we have.

Really, the effective number of features we have is dimension of f. So that n is actually going to be equal to m. So, if you want to, you can think of this as a sum, this really is a sum from j equals 1 through m. And then one way to think about this, is you can think of it as n being equal to m, because if f isn't a new feature, then we have m plus 1 features, with the plus 1 coming from the interceptor.

And here, we still do sum from j equal 1 through n, because similar to our earlier videos on regularization, we still do not regularize the parameter theta zero, which is why this is a sum for j equals 1 through m instead of j equals zero though m. So that's the support vector machine learning algorithm. That's one sort of, mathematical detail aside that I should mention, which is that in the way the support vector machine is implemented, this last term is actually done a little bit differently. So you don't really need to know about this last detail in order to use support vector machines, and in fact the equations that are written down here should give you all the intuitions that should need. But in the way the support vector machine is implemented, you know, that term, the sum of j of theta j squared right?

Another way to write this is this can be written as theta transpose theta if we ignore the parameter theta 0. So theta 1 down to theta m. Ignoring theta 0.

Then this sum of j of theta j squared that this can also be written theta transpose theta.

And what most support vector machine implementations do is actually replace this theta transpose theta, will instead, theta transpose times some matrix inside, that depends on the kernel you use, times theta. And so this gives us a slightly different distance metric. We'll use a slightly different measure instead of minimizing exactly

the norm of theta squared means that minimize something slightly similar to it. That's like a rescale version of the parameter vector theta that depends on the kernel. But this is kind of a mathematical detail. That allows the support vector machine software to run much more efficiently.

And the reason the support vector machine does this is with this modification. It allows it to scale to much bigger training sets. Because for example, if you have a training set with 10,000 training examples.

Then, you know, the way we define landmarks, we end up with 10,000 landmarks.

2

And so theta becomes 10,000 dimensional. And maybe that works, but when m becomes really, really big then solving for all of these parameters, you know, if m were 50,000 or a 100,000 then solving for all of these parameters can become expensive for the support vector machine optimization software, thus solving the minimization problem that I drew here. So kind of as mathematical detail, which again you really don't need to know about.

It actually modifies that last term a little bit to optimize something slightly different than just minimizing the norm squared of theta squared, of theta. But if you want, you can feel free to think of this as an kind of a n implementational detail that does change the objective a bit, but is done primarily for reasons of computational efficiency, so usually you don't really have to worry about this.

And by the way, in case your wondering why we don't apply the kernel's idea to other algorithms as well like logistic regression, it turns out that if you want, you can actually apply the kernel's idea and define the source of features using landmarks and so on for logistic regression. But the computational tricks that apply for support vector machines don't generalize well to other algorithms like logistic regression. And so, using kernels with logistic regression is going too very slow, whereas, because of computational tricks, like that embodied and how it modifies this and the details of how the support vector machine software is implemented, support vector machines and kernels tend go particularly well together. Whereas,

logistic regression and kernels, you know, you can do it, but this would run very slowly. And it won't be able to take advantage of advanced optimization techniques that people have figured out for the particular case of running a support vector machine with a kernel. But all this pertains only to how you actually implement software to minimize the cost function. I will say more about that in the next video, but you really don't need to know about how to write software to minimize this cost function because you can find very good off the shelf software for doing so.

And just as, you know, I wouldn't recommend writing code to invert a matrix or to compute a square root, I actually do not recommend writing software to minimize this cost function yourself, but instead to use off the shelf software packages that people have developed and so those software packages already embody these numerical optimization tricks,

so you don't really have to worry about them. But one other thing that is worth knowing about is when you're applying a support vector machine, how do you choose the parameters of the support vector machine?

SVM parameters:

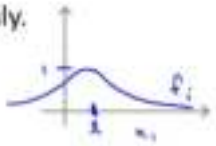
C ($= \frac{1}{\lambda}$). → Large C: Lower bias, high variance. (small λ)

→ Small C: Higher bias, low variance. (large λ)

σ^2 Large σ^2: Features f_i vary more smoothly.

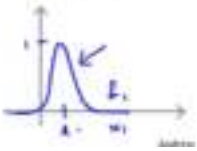
→ Higher bias, lower variance.

$\exp\left(-\frac{\|x-l^{(i)}\|^2}{2\sigma^2}\right)$



Small σ^2: Features f_i vary less smoothly.

Lower bias, higher variance.



① And the last thing I want to do in this video is say a little word about the bias and variance trade offs when using a support vector machine. When using an SVM, one of the things you need to choose is the parameter C which was in the optimization objective, and you recall that C played a role similar to 1 over lambda, where lambda was the regularization parameter we had for logistic regression.

So, if you have a large value of C, this corresponds to what we have back in logistic regression, of a small value of lambda meaning of not using much regularization. And if you do that, you tend to have a hypothesis with lower bias and higher variance.

Whereas if you use a smaller value of C then this corresponds to when we are using logistic regression with a large value of lambda and that corresponds to a hypothesis with higher bias and lower variance. And so, hypothesis with large C has a higher variance, and is more prone to overfitting, whereas hypothesis with small C has higher bias and is thus more prone to underfitting.

② So this parameter C is one of the parameters we need to choose. The other one is the parameter sigma squared, which appeared in the Gaussian kernel.

So if the Gaussian kernel sigma squared is large, then in the similarity function, which was this you know E to the minus x minus landmark

varies squared over 2 sigma squared.

In this one of the example; if I have only one feature, x1, if I have a landmark there at that location, if sigma squared is large, then, you know, the Gaussian kernel would tend to fall off relatively slowly

and so this would be my feature f(i), and so this would be smoother function that varies more smoothly, and so this will give you a hypothesis with higher bias and lower variance, because the Gaussian kernel that falls off smoothly, you tend to get a hypothesis that varies slowly, or

③ varies smoothly as you change the input x. Whereas in contrast, if sigma squared was small and if that's my landmark given my 1 feature x1, you know, my Gaussian kernel, my similarity function, will vary more abruptly. And in both cases I'd pick out 1, and so if sigma squared is small, then my features vary less smoothly. So if it's just higher slopes or higher derivatives here. And using this, you end up fitting hypotheses of lower bias and you can have higher variance.

And if you look at this week's points exercise, you actually get to play around with some of these ideas yourself and see these effects yourself.

So, that was the support vector machine with kernels algorithm. And hopefully this discussion of bias and variance will give you some sense of how you can expect this algorithm to behave as well.

Suppose you train an SVM and find it overfits your training data. Which of these would be a reasonable next step? Check all that apply.

- [ ] Increase C
- [x] Decrease C

✓ **Correct**

- [x] Increase σ^2

✓ **Correct**

- [ ] Decrease σ^2

SVMs in Practice

Using An SVM

① So far we've been talking about SVMs in a fairly abstract level. In this video I'd like to talk about what you actually need to do in order to run or to use an SVM.

The support vector machine algorithm poses a particular optimization problem. But as I briefly mentioned in an earlier video, I really do not recommend writing your own software to solve for the parameter's theta yourself.

So just as today, very few of us, or maybe almost essentially none of us would think of writing code ourselves to invert a matrix or take a square root of a number, and so on. We just, you know, call some library function to do that. In the same way, the software for solving the SVM optimization problem is very complex, and there have been researchers that have been doing essentially numerical optimization research for many years. So you come up with good software libraries and good software packages to do this. And I strongly recommend just using one of the highly optimized software libraries rather than trying to implement something yourself. And there are lots of good software libraries out there. The two that I happen to use the most often are the linear SVM but there are really lots of good software libraries for doing this that you know, you can link to many of the major programming languages that you may be using to code up learning algorithm. Even though you shouldn't be writing your own SVM optimization software, there are a few things you need to do, though. First is to come up with with some choice of the parameter's C. We talked a little bit of the bias/variance properties of this in the earlier video.

Second, you also need to choose the kernel or the similarity function that you want to use. So one choice might be if we decide not to use any kernel.

And the idea of no kernel is also called a linear kernel. So if someone says, I use an SVM with a linear kernel, what that means is you know, they use an SVM without using without using a kernel and it was a version of the SVM that just uses theta transpose X, right, that predicts 1 theta 0 plus theta 1 X1 plus so on plus theta N, X N is greater than equals 0.

This term linear kernel, you can think of this as you know this is the version of the SVM

that just gives you a standard linear classifier.

So that would be one reasonable choice for some problems, and you know, there would be many software libraries, like linear, was one example, out of many, one example of a software library that can train an SVM without using a kernel, also called a linear kernel. So, why would you want to do this? If you have a large number of features, if N is large, and M the number of training examples is small, then you know you have a huge number of features that if X, this is an X is an Rn, Rn +1. So if you have a huge number of features already, with a small training set, you know, maybe you want to just fit a linear decision boundary and not try to fit a very complicated nonlinear function, because might not have enough data. And you might risk overfitting, if you're trying to fit a very complicated function

Use SVM software package (e.g. liblinear, libsvm, ...) to solve for parameters θ.

Need to specify:

→ Choice of parameter C.

Choice of kernel (similarity function):

E.g. No kernel ("linear kernel")

Predict "y = 1" if $\theta^T x \geq 0$

$\theta_0 + \theta_1 x_1 + \cdots + \theta_n x_n \geq 0$

→ n large, m small $x \in \mathbb{R}^{n+1}$

Gaussian kernel:

$f_i = \exp\left(-\frac{||x - l^{(i)}||^2}{2\sigma^2}\right)$, where $l^{(i)} = x^{(i)}$.

$x \in \mathbb{R}^n$, n small and/or m large

Need to choose σ^2.

② decide to use a Gaussian

in a very high-dimensional feature space, but if your training set sample is small. So this would be one reasonable setting where you might decide to just not use a kernel, or equivalents to use what's called a linear kernel. A second choice for the kernel that you might make, is this Gaussian kernel, and this is what we had previously.

And if you do this, then the other choice you need to make is to choose this parameter sigma squared when we also talk a little bit about the bias variance tradeoffs

of how, if sigma squared is large, then you tend to have a higher bias, lower variance classifier, but if sigma squared is small, then you have a higher variance, lower bias classifier.

So when would you choose a Gaussian kernel? Well, if your omission of features X, I mean Rn, and if N is small, and, ideally, you know,

if n is large, right,

so that's if, you know, we have say, a two-dimensional training set, like the example I drew earlier. So n is equal to 2, but we have a pretty large training set. So, you know, I've drawn in a fairly large number of training examples, then maybe you want to use a kernel to fit a more complex nonlinear decision boundary, and the Gaussian kernel would be a fine way to do this. I'll say more towards the end of the video, a little bit more about when you might choose a linear kernel, a Gaussian kernel and so on.

But if concretely, if you decide to use a Gaussian kernel, then here's what you need to do.

Kernel (similarity) functions:

```
function f = kernel(x1,x2)
    f = exp( -|| x1 - x2 ||^2 / (2σ^2) )
return
```

$x \to \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix}$

→ Note: Do perform feature scaling before using the Gaussian kernel.

$x \in \mathbb{R}^n$

$||x - l||^2 \quad v = x - l$

$||v||^2 = v_1^2 + v_2^2 + \cdots + v_n^2$

$= (x_1 - l_1)^2 + (x_2 - l_2)^2 + \cdots + (x_n - l_n)^2$

1000 feet² 1-5 bedrooms

Depending on what support vector machine software package you use, it may ask you to implement a kernel function, or to implement the similarity function.

So if you're using an octave or MATLAB implementation of an SVM, it may ask you to provide a function to compute a particular feature of the kernel. So this is really computing f subscript i for one particular value of i, where

f here is just a single real number, so maybe I should move this better written f(i), but what you need to do is to write a kernel function that takes this input, you know,

a training example or a test example whatever it takes in some vector X and takes as input one of the landmarks and but only I've come down X1 and X2 here, because the landmarks are really training examples as well. But what you need to do is write software that takes this input, you know, X1, X2 and computes this sort of similarity function between them and return a real number.

And so what some support vector machine packages do is expect you to provide this kernel function that take this input you know, X1, X2 and returns a real number.

And then it will take it from there and it will automatically generate all the features, and so automatically take X and map it to f1, f2, down to fm using this function that you write, and generate all the features and train the support vector machine from there. But sometimes you do need to provide this function yourself. Other if you are using the Gaussian kernel, some SVM implementations will also include the Gaussian kernel and a few other kernels as well, since the Gaussian kernel is probably the most common kernel.

Gaussian and linear kernels are really the two most popular kernels by far. Just one implementational note. If you have features of very different scales, it is important

to perform feature scaling before using the Gaussian kernel. And here's why. If you imagine the computing this norm between X and L, right, so this term here, and the subtraction term over there.

What this is doing, the norm between X and L, that's really saying, you know, let's compute the vector V, which is equal to X minus L. And then let's compute the norm does vector V, which is the difference between X. So the norm of V is really

equal to X1 squared plus X2 squared plus dot dot dot, plus Xn squared. Because here X is in Rn, or Rn plus 1, but I'm going to ignore, you know, X0.

So, let's pretend X is an Rn, square on this left take is what makes this correct. So this is equal to that, right?

And so written differently, this is going to be X1 minus l1 squared, plus x2 minus l2 squared, plus dot dot dot plus Xn minus ln squared.

And now if your features

take on very different ranges of value. So take a housing prediction, for example, if your data is some data about houses, and if X is in the range of thousands of square feet, for the first feature, X1, that if your second feature, X2 is the number of bedrooms. So if this is in the range of one to five bedrooms, then

X1 minus l1 is going to be huge. This could be like a thousand squared, whereas X2 minus l2 is going to be much smaller and if that's the case, then in this term,

those distances will be almost essentially dominated by the sizes of the houses

and the number of bedrooms would be largely ignored.

So so as, to avoid this in order to make a machine work well, do perform feature scaling.

And that will sure that the SVM gives, you know, comparable amount of attention to all of your different features, and not just to in this example to size of houses were big movement here the features.

Other choices of kernel

Note: Not all similarity functions $\text{similarity}(x, l)$ make valid kernels.
(Need to satisfy technical condition called "Mercer's Theorem" to make sure SVM packages' optimizations run correctly, and do not diverge).

Many off-the-shelf kernels available:

- Polynomial kernel: $k(x, l) = (x^T l)^3$, $(x^T l)^2$, $(x^T l + 1)$, $(x^T l + 5)$, $(x^T l + \text{constant})^{\text{degree}}$
- More esoteric: String kernel, chi-square kernel, histogram intersection kernel, ... $\quad \text{sim}(x, l)$

1

When you try a support vector machines chances are by far the two most common kernels you use will be the linear kernel, meaning no kernel, or the Gaussian kernel that we talked about. And just one note of warning which is that not all similarity functions you might come up with are valid kernels. And the Gaussian kernel and the linear kernel and other kernels that you sometimes others will use, all of them need to satisfy a technical condition. It's called Mercer's Theorem and the reason you need to this is because support vector machine algorithms or implementations of the SVM have lots of clever numerical optimization tricks. In order to solve for the parameter's theta efficiently and in the original design envisaged, those are decision made to restrict our attention only to kernels that satisfy this technical condition called Mercer's Theorem. And what that does is, that makes sure that all of these SVM packages, all of these SVM software packages can use the large class of optimizations and get the parameter theta very quickly.

So, what most people end up doing is using either the linear or Gaussian kernel, but there are a few other kernels that also satisfy Mercer's theorem and that you may run across other people using, although I personally end up using other kernels you know, very, very rarely, if at all. Just to mention some of the other kernels that you may run across.

One is the polynomial kernel.

And for that the similarity between X and l is defined as, there are a lot of options, you can take X transpose l squared. So, here's one measure of how similar X and l are. If X and l are very close with each other, then the inner product will tend to be large.

And so, you know, this is a slightly

unusual kernel. That is not used that often, but

you may run across some people using it. This is one version of a polynomial kernel. Another is X transpose l cubed.

2

These are all examples of the polynomial kernel. X transpose l plus 1 cubed.

X transpose l plus maybe a number different then one 5 and, you know, to the power of 4 and

so the polynomial kernel actually has two parameters. One is, what number do you add over here? It could be 0. This is really plus 0 over there, as well as what's the degree of the polynomial over there. So the degree power and these numbers. And the more general form of the polynomial kernel is X transpose l, plus some constant and then to some degree in the X1 and so both of these are parameters for the polynomial kernel. So the polynomial kernel almost always or usually performs worse. And the Gaussian kernel does not use that much, but this is just something that you may run across. Usually it is used only for data where X and l are all strictly non negative, and so that ensures that these inner products are never negative.

And this captures the intuition that X and l are very similar to each other, then maybe the inner product between them will be large. They have some other properties as well but people tend not to use it much.

And then, depending on what you're doing, there are other, sort of more

esoteric kernels as well, that you may come across. You know, there's a string kernel, this is sometimes used if your input data is text strings or other types of strings. There are things like the chi-square kernel, the histogram intersection kernel, and so on. There are sort of more esoteric kernels that you can use to measure similarity between different objects. So for example, if you're trying to do some sort of text classification problem, where the input x is a string then maybe we want to find the similarity between two strings using the string kernel, but I personally you know end up very rarely, if at all, using these more esoteric kernels. I think I might have use the chi-square kernel, may be once in my life and the histogram kernel, may be once or twice in my life. I've actually never used the string kernel myself. But in case you've run across this in other applications. You know, if

you do a quick web search we do a quick Google search or quick Bing search you should have found definitions that these are the kernels as well. So

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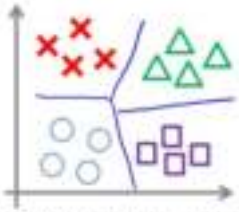
Suppose you are trying to decide among a few different choices of kernel and are also choosing parameters such as C, σ^2, etc. How should you make the choice?

- ○ Choose whatever performs best on the training data.
- ◉ Choose whatever performs best on the cross-validation data.
- ○ Choose whatever performs best on the test data.
- ○ Choose whatever gives the largest SVM margin.

✓ **Correct**

Multi-class classification



$y \in \{1, 2, 3, \ldots, K\}$

Many SVM packages already have built-in multi-class classification functionality.

→ Otherwise, use one-vs.-all method. (Train K SVMs, one to distinguish $y = i$ from the rest, for $i = 1, 2, \ldots, K$), get $\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(K)}$

Pick class i with largest $(\theta^{(i)})^T x$

Just two last details I want to talk about in this video. One in multiclass classification. So, you have four classes or more generally 3 classes output some appropriate decision boundary between your multiple classes. Most SVM, many SVM packages already have built-in multiclass classification functionality. So if your using a pattern like that, you just use the both that functionality and that should work fine. Otherwise, one way to do this is to use the one versus all method that we talked about when we are developing logistic regression. So what you do is you trade kSVM's if you have k classes, one to distinguish each of the classes from the rest. And this would give you k parameter vectors, so this will give you, upi Impw. theta 1, which is trying to distinguish class y equals one from all of the other classes, then you get the second parameter, vector theta 2, which is what you get when you, you know, have y equals 2 as the positive class and all the others as negative class and so on up to a parameter vector theta k, which is the parameter vector for distinguishing the final class key from anything else, and then lastly, this is exactly the same as the one versus all method we have for logistic regression. Where we you just predict the class i with the largest theta transpose X. So let's multiclass classification designate. For the more common cases that there is a good chance that whatever software package you use, you know, there will be a reasonable chance that are already have built in multiclass classification functionality, and so you don't need to worry about this result.

Logistic regression vs. SVMs

n = number of features ($x \in \mathbb{R}^{n+1}$), m = number of training examples

→ If n is large (relative to m): (e.g. $n \geq m$, $n = 10{,}000$, $m = 10 - 1000$)

→ Use logistic regression, or SVM without a kernel ("linear kernel")

→ If n is small, m is intermediate: ($n = 1 - 1000$, $m = 10 - 10{,}000$)

→ Use SVM with Gaussian kernel

If n is small, m is large: ($n = 1 - 1000$, $m = 50{,}000+$)

→ Create/add more features, then use logistic regression or SVM without a kernel

→ Neural network likely to work well for most of these settings, but may be slower to train.

Finally, we developed support vector machines starting off with logistic regression and then modifying the cost function a little bit. The last thing we want to do in this video is, just say a little bit about. when you will use one of these two algorithms, so let's say n is the number of features and m is the number of training examples.

So, when should we use one algorithm versus the other?

Well, if n is larger relative to your training set size, so for example,

if you take a business with a number of features this is much larger than m and this might be, for example, if you have a text classification problem, where you know, the dimension of the feature vector is I don't know, maybe, 10 thousand.

And if your training set size is maybe 10 you know, maybe, up to 1000. So, imagine a spam classification problem, where email spam, where you have 10,000 features corresponding to 10,000 words but you have, you know, maybe 10 training examples or maybe up to 1,000 examples.

So if n is large relative to m, then what I would usually do is use logistic regression or use it as the m without a kernel or use it with a linear kernel. Because, if you have so many features with smaller training sets, you know, a linear function will probably do fine, and you don't have really enough data to fit a very complicated nonlinear function. Now if is n is small and m is intermediate what I mean by this is n is maybe anywhere from 1 - 1000, 1 would be very small. But maybe up to 1000 features and if the number of training examples is maybe anywhere from 10, you know, 10 to maybe up to 10,000 examples. Maybe up to 50,000 examples. If m is pretty big like maybe 10,000 but not a million. Right? So if m is an intermediate size then often an SVM with a linear kernel will work well. We talked about this early as well, with the one concrete example, this would be if you have a two dimensional training set. So, if n is equal to 2 where you have, you know, drawing in a pretty large number of training examples.

So Gaussian kernel will do a pretty good job separating positive and negative classes.

One third setting that's of interest is if n is small but m is large. So if n is you know, again maybe 1 to 1000, could be larger. But if m was, maybe 50,000 and greater to millions.

So, 50,000, a 100,000, million, trillion.

You have very very large training set sizes, right.

So if this is the case, then a SVM of the Gaussian Kernel will be somewhat slow to run. Today's SVM packages, if you're using a Gaussian Kernel, tend to struggle a bit. If you have, you know, maybe 50 thousands okay, but if you have a million training examples, maybe or even a 100,000 with a massive value of m. Today's SVM packages are very good, but they can still struggle a little bit when you have a massive, massive trainings that size when using a Gaussian Kernel.

So in that case, what I would usually do is try to just manually create have more features and then use logistic regression or an SVM without the Kernel.

And in case you look at this slide and you see logistic regression or SVM without a kernel. In both of these places, I kind of paired them together. There's a reason for that, is that logistic regression and SVM without the kernel, those are really pretty similar algorithms and, you know, either logistic regression or SVM without a kernel will usually do pretty similar things and give pretty similar performance, but depending on your implementational details, one may be more efficient than the other. But, where one of these algorithms applies, logistic regression where SVM without a kernel, the other one is to likely to work pretty well as well. But along with the power of the SVM is when you use different kernels to learn complex nonlinear functions. And this regime, you know, when you have maybe up to 10,000 examples, maybe up to 50,000. And your number of features,

this is reasonably large. That's a very common regime and maybe that's a regime where a support vector machine with a kernel kernel will shine. You can do things that are much harder to do that will need logistic regression. And finally, where do neural networks fit in? Well for all of these problems, for all of these different regimes, a well designed neural network is likely to work well as well.

The one disadvantage, or the one reason that might not sometimes use the neural network is that, for some of these problems, the neural network might be slow to train. But if you have a very good SVM implementation package, that could run faster, quite a bit faster than your neural network.

And, although we didn't show this earlier, it turns out that the optimization problem that the SVM has is a convex

optimization problem and so the good SVM optimization software packages will always find the global minimum or something close to it. And so for the SVM you don't need to worry about local optima.

In practice local optima aren't a huge problem for neural networks but they all solve, so this is one less thing to worry about if you're using an SVM.

And depending on your problem, the neural network may be slower, especially in this sort of regime that the SVM. In case the guidelines they gave here, seem a little bit vague and if you're looking at some problems, you know,

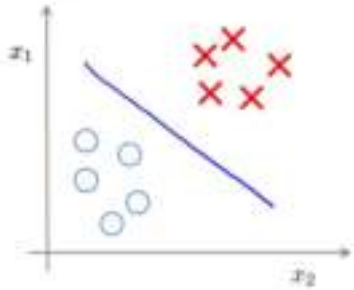
the guidelines are a bit vague, I'm still not entirely sure, should I use this algorithm or that algorithm, that's actually okay. When I face a machine learning problem, you know, sometimes its actually just not clear whether that's the best algorithm to use, but as you saw in the earlier videos, really, you know, the algorithm does matter, but what often matters even more is things like, how much data do you have. And how skilled are you, how good are you at doing error analysis and debugging learning algorithms, figuring out how to design new features and figuring out what other features to give you learning algorithms and so on. And often those things will matter more than what you are using logistic regression or an SVM. But having said that, the SVM is still widely perceived as one of the most powerful learning algorithms, and there is this regime of when there's a very effective way to learn complex non linear functions. And so I actually, together with logistic regressions, neural networks, SVM's, using those to speed learning algorithms you're I think very well positioned to build state of the art you know, machine learning systems for a wide region for applications and this is another very powerful tool to have in your arsenal. One that is used all over the place in Silicon Valley, so in industry and in the Academia, to build many high performance machine learning systems.

Clustering

Unsupervised Learning: introduction

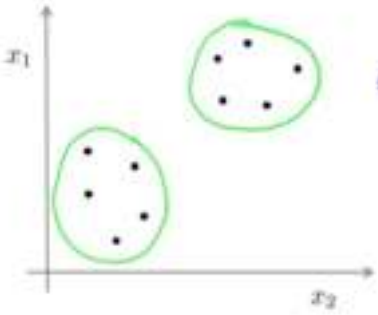
In this video, I'd like to start to talk about clustering. This will be exciting, because this is our first unsupervised learning algorithm, where we learn from unlabeled data instead from labelled data. So, what is unsupervised learning? I briefly talked about unsupervised learning at the beginning of the class but it's useful to contrast it with supervised learning. So, here's a typical supervised learning problem where we're given a labeled training set and the goal is to find the decision boundary that separates the positive label examples and the negative label examples. So, the supervised learning problem in this case is given a set of labels to fit a hypothesis to it. In contrast, in the unsupervised learning problem we're given data that does not have any labels associated with it. So, we're given data that looks like this. ~~Here's a set of points add in no labels,~~

Supervised learning



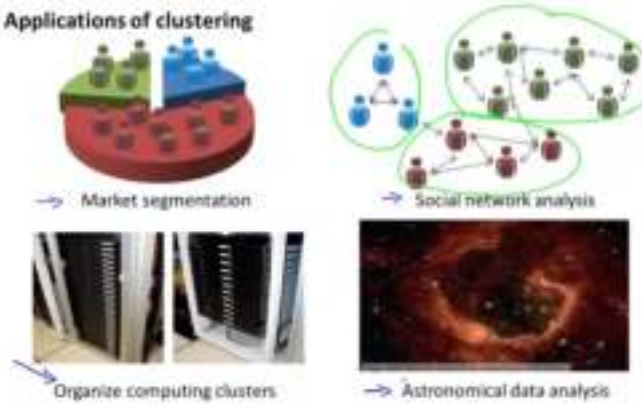
Training set: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), (x^{(3)}, y^{(3)}), \ldots, (x^{(m)}, y^{(m)})\}$

Unsupervised learning



Training set: $\{x^{(1)}, x^{(2)}, x^{(3)}, \ldots, x^{(m)}\}$

~~associated with it. So, we're given data that looks like this.~~ Here's a set of points add in no labels, and so, our training set is written just x1, x2, and so on up to x m and we don't get any labels y. And that's why the points plotted up on the figure don't have any labels with them. So, in unsupervised learning what we do is we give this sort of unlabeled training set to an algorithm and we just ask the algorithm find some structure in the data for us. Given this data set one type of structure we might have an algorithm find is that it looks like this data set has points grouped into two separate clusters and so an algorithm that finds clusters like the ones I've just circled is called a clustering algorithm. And this would be our first type of unsupervised learning, although there will be other types of unsupervised learning algorithms that we'll talk about later that finds other types of structure or other types of patterns in the data other than clusters. We'll talk about this after we've talked about clustering. So, what is clustering good for? ~~Early in this class I~~



this after we've talked about clustering. So, what is clustering good for? Early in this class I already mentioned a few applications. One is market segmentation where you may have a database of customers and want to group them into different market segments so you can sell to them separately or serve your different market segments better. Social network analysis. There are actually groups have done this things like looking at a group of people's social networks. So, things like Facebook, Google+, or maybe information about who other people that you email the most frequently and who are the people that they email the most frequently and to find coherence in groups of people. So, this would be another maybe clustering algorithm where you know want to find who are the coherent groups of friends in the social network? Here's something that one of my friends actually worked on which is, use clustering to organize computer clusters or to organize data centers better. Because if you know which computers in the data center in the cluster tend to work together, you can use that to reorganize your resources and how you layout the network and how you design your data center communications. And lastly, something that actually another friend worked on using clustering algorithms to understand galaxy formation and using that to understand astronomical data.

Which of the following statements are true? Check all that apply.

- In unsupervised learning, the training set is of the form $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$ without labels $y^{(i)}$.

  ✓ Correct

- Clustering is an example of unsupervised learning.

  ✓ Correct

- In unsupervised learning, you are given an unlabeled dataset and are asked to find "structure" in the data.

  ✓ Correct

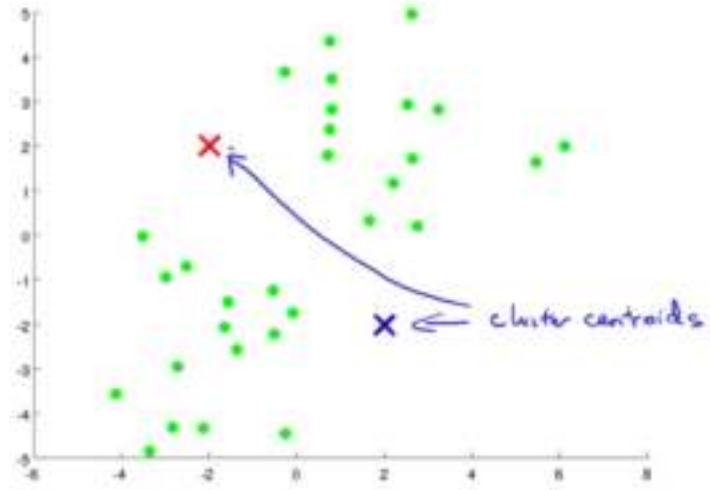
- Clustering is the only unsupervised learning algorithm.

clustering

K-means Algorithm

In the clustering problem we are given an unlabeled data set and we would like to have an algorithm automatically group the data into coherent subsets or into coherent clusters for us.

The K Means algorithm is by far the most popular, by far the most widely used clustering algorithm, and in this video I would like to tell you what the K Means Algorithm is and how it works.



The K means clustering algorithm is best illustrated in pictures. Let's say I want to take an unlabeled data set like the one shown here, and I want to group the data into two clusters.

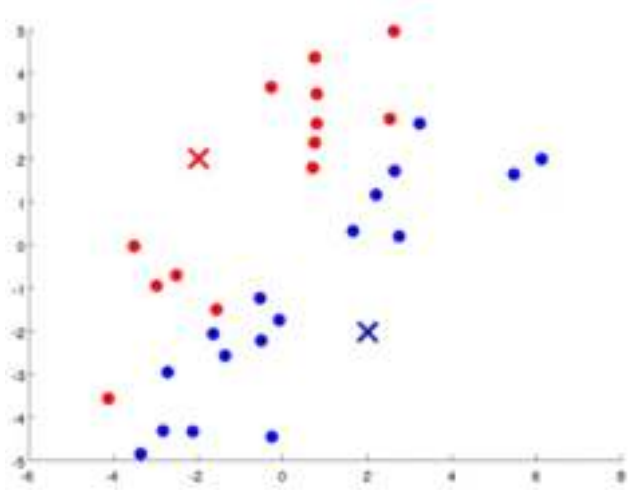
If I run the K Means clustering algorithm, here is what I'm going to do. The first step is to randomly initialize two points, called the cluster centroids. So, these two crosses here, these are called the Cluster Centroids

and I have two of them because I want to group my data into two clusters.

K Means is an iterative algorithm and it does two things.

First is a cluster assignment step, and second is a move centroid step. So, let me tell you what those things mean.

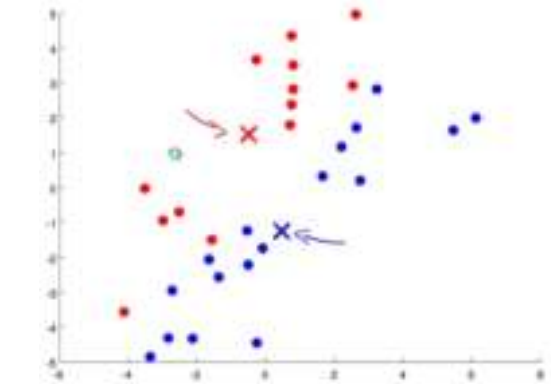
The first of the two steps in the loop of K means, is this cluster assignment step. What that means is that, it's going through each of the examples, each of these green dots shown here and depending on whether it's closer to the red cluster centroid or the blue cluster centroid, it is going to assign each of the data points to one of the two cluster centroids.



Specifically, what I mean by that, is to go through your data set and color each of the points either red or blue, depending on whether it is closer to the red cluster centroid or the blue cluster centroid, and I've done that in this diagram here.

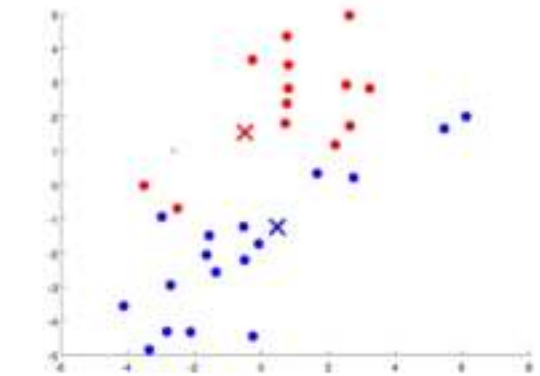
So, that was the cluster assignment step.

The other part of K means, in the loop of K means, is the move centroid step, and what we are going to do is, we are going to take the two cluster centroids, that is, the red cross and the blue cross, and we are going to move them to the average of the points colored the same colour. So what we are going to do is look at all the red points and compute the average, really the mean of the location of all the red points, and we are going to move the red cluster centroid there. And the same things for the blue cluster centroid, look at all the blue dots and compute their mean, and then move the blue cluster centroid there. So, let me do that now. We're going to move the cluster centroids as follows

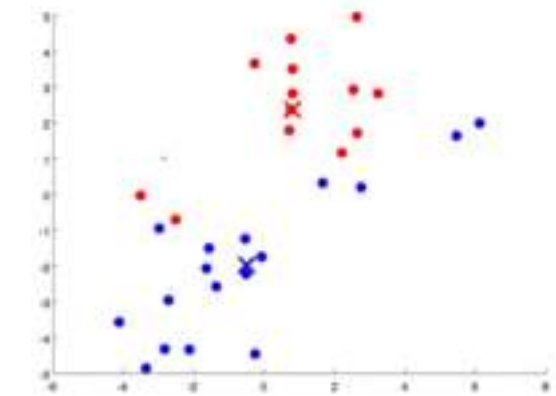


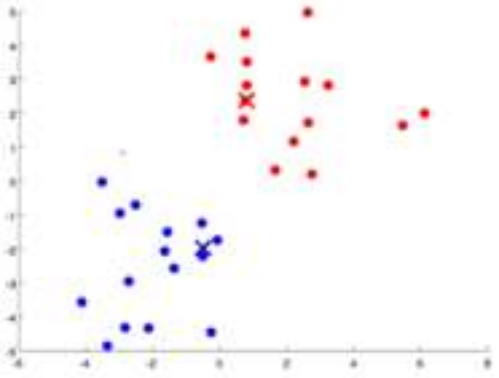
and I've now moved them to their new means. The red one moved like that and the blue one moved like that and the red one moved like that. And then we go back to another cluster assignment step, so we're again going to look at all of my unlabeled examples and depending on whether it's closer the red or the blue cluster centroid, I'm going to color them either red or blue. I'm going to assign each point to one of the two cluster centroids, so let me do that now.

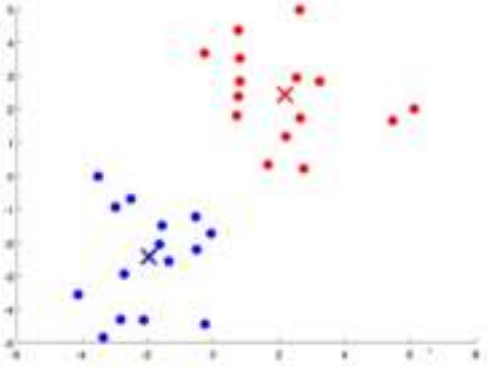
And so the colors of some of the points just changed.



And then I'm going to do another move centroid step. So I'm going to compute the average of all the blue points, compute the average of all the red points and move my cluster centroids like this, and so, let's do that again. Let me do one more cluster assignment step. So colour each point red or blue, based on what it's closer to and then do another move centroid step and we're done. And in fact if you keep running additional iterations of K means from here the cluster centroids will not change any further and the colours of the points will not change any further. And so, this is the, at this point, K means has converged and it's done a pretty good job finding the two clusters in this data. Let's write out the K means algorithm more formally.







K-means algorithm

Input:

- K (number of clusters)
- Training set $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$

$x^{(i)} \in \mathbb{R}^n$ (drop $x_0 = 1$ convention)

The K means algorithm takes two inputs. One is a parameter K, which is the number of clusters you want to find in the data. I'll later say how we might go about trying to choose k, but for now let's just say that we've decided we want a certain number of clusters and we're going to tell the algorithm how many clusters we think there are in the data set. And then K means also takes as input this sort of unlabeled training set of just the Xs and because this is unsupervised learning, we don't have the labels Y anymore. And for unsupervised learning of the K means I'm going to use the convention that XI is an RN dimensional vector. And that's why my training examples are now N dimensional rather N plus one dimensional vectors.

K-means algorithm

Randomly initialize K cluster centroids $\mu_1, \mu_2, \ldots, \mu_K \in \mathbb{R}^n$

Repeat {

for $i = 1$ to m

$c^{(i)}$:= index (from 1 to K) of cluster centroid closest to $x^{(i)}$

for $k = 1$ to K

μ_k := average (mean) of points assigned to cluster k

}

(1)

This is what the K means algorithm does.

The first step is that it randomly initializes k cluster centroids which we will call mu 1, mu 2, up to mu k. And so in the earlier diagram, the cluster centroids corresponded to the location of the red cross and the location of the blue cross. So there we had two cluster centroids, so maybe the red cross was mu 1 and the blue cross was mu 2, and more generally we would have k cluster centroids rather than just 2. Then the inner loop of k means does the following, we're going to repeatedly do the following.

First for each of my training examples, I'm going to set this variable CI to be the index 1 through K of the cluster centroid closest to Xi. So this was my cluster assignment step, where we took each of my examples and coloured it either red or blue, depending on which cluster centroid it was closest to. So CI is going to be a number from 1 to K that tells us, you know, is it closer to the red cross or is it closer to the blue cross,

and another way of writing this is I'm going to, to compute Ci, I'm going to take my ith example Xi and and I'm going to measure it's distance

(2)

to each of my cluster centroids, this is mu and then lower-case k, right, so capital K is the total number centroids and I'm going to use lower case k here to index into the different centroids.

But so, CI is going to, I'm going to minimize over my values of k and find the value of K that minimizes this distance between Xi and the cluster centroid, and then, you know, the value of k that minimizes this, that's what gets set in Ci. So, here's another way of writing out what Ci is.

If I write the norm between Xi minus Mu k,

then this is the distance between my ith training example Xi and the cluster centroid Mu subscript K, this is--this here, that's a lowercase K. So uppercase K is going to be used to denote the total number of cluster centroids, and this lowercase K's a number between one and capital K. I'm just using lower case K to index into my different cluster centroids.

Next is lower case k. So

that's the distance between the example and the cluster centroid and so what I'm going to do is find the value of K, of lower case k that minimizes this, and so the value of k that minimizes you know, that's what I'm going to set as Ci, and by convention here I've written the distance between Xi and the cluster centroid, by convention people actually tend to write this as the squared distance. So we think of Ci as picking the cluster centroid with the smallest squared distance to my training example Xi. But of course minimizing squared distance, and minimizing distance that should give you the same value of Ci, but we usually put in the square there, just as the convention that people use for K means. So that was the cluster assignment step.

The other in the loop of K means does the move centroid step.

And what that does is for each of my cluster centroids, so for lower case k equals 1 through K, it sets Mu k equals to the average of the points assigned to cluster. So as a concrete example, let's say that one of my cluster centroids, let's say cluster centroid two, has training examples, you know, 1, 5, 6, and 10 assigned to it. And what this means is, really this means that C1 equals

to C5 equals to

C6 equals to and similarly well c10 equals, too, right?

If we got that from the cluster assignment step, then that means examples 1,5,6 and 10 were assigned to the cluster centroid two.

Then in this move centroid step, what I'm going to do is just compute the average of these four things.

So X1 plus X5 plus X6 plus X10. And now I'm going to average them so here I have four points assigned to this cluster centroid, just take one quarter of that. And now Mu2 is going to be an n-dimensional vector. Because each of these example x1, x5, x6, x10

(3)

each of them were an n-dimensional vector, and I'm going to add up these things and, you know, divide by four because I have four points assigned to this cluster centroid, I end up with my move centroid step,

for my cluster centroid mu-2. This has the effect of moving mu-2 to the average of the four points listed here.

One thing that I've asked is, well here we said, let's let mu-k be the average of the points assigned to the cluster. But what if there is a cluster centroid no points with zero points assigned to it. In that case the more common thing to do is to just eliminate that cluster centroid. And if you do that, you end up with K minus one clusters

instead of k clusters. Sometimes if you really need k clusters, then the other thing you can do if you have a cluster centroid with no points assigned to it is you can just randomly reinitialize that cluster centroid, but it's more common to just eliminate a cluster if somewhere during K means it with no points assigned to that cluster centroid, and that can happen, altthough in practice it happens not that often. So that's the K means Algorithm.

Suppose you run k means and after the algorithm converges, you have: $c^{(1)} = 3, c^{(2)} = 3, c^{(3)} = 5, \ldots$

Which of the following statements are true? Check all that apply.

- [x] The third example $x^{(3)}$ has been assigned to cluster 5.

✓ Correct

- [x] The first and second training examples $x^{(1)}$ and $x^{(2)}$ have been assigned to the same cluster.

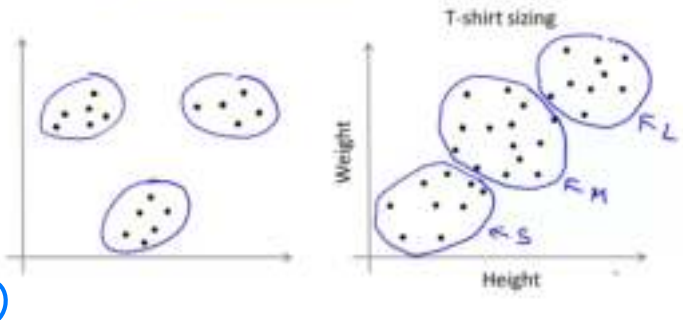
✓ Correct

- [ ] The second and third training examples have been assigned to the same cluster.
- [x] Out of all the possible values of $k \in \{1, 2, \ldots, K\}$ the value $k = 3$ minimizes $\|x^{(3)} - \mu_k\|^2$.

✓ Correct

K-means for non-separated clusters

S, M, L



(1)

Before wrapping up this video I just want to tell you about one other common application of K Means and that's to the problems with non well separated clusters.

Here's what I mean. So far we've been picturing K Means and applying it to data sets like that shown here where we have three pretty well separated clusters, and we'd like an algorithm to find maybe the 3 clusters for us. But it turns out that very often K Means is also applied to data sets that look like this where there may not be several very well separated clusters. Here is an example application, to t-shirt sizing.

Let's say you are a t-shirt manufacturer you've done is you've gone to the population that you want to sell t-shirts to, and you've collected a number of examples of the height and weight of these people in your population and so, well I guess height and weight tend to be positively highlighted so maybe you end up with a data set like this, you know, with a sample or set of examples of different peoples heights and weight. Let's say you want to size your t shirts. Let's say I want to design and sell t shirts of three sizes, small, medium and large. So how big should I make my small one? How big should I my medium? And how big should I make my large t-shirts.

(2)

One way to do that would to be to run my k means clustering logarithm on this data set that I have shown on the right and maybe what K Means will do is group all of these points into one cluster and group all of these points into a second cluster and group all of those points into a third cluster. So, even though the data, you know, before hand it didn't seem like we had 3 well separated clusters, K Means will kind of separate out the data into multiple pluses for you. And what you can do is then look at this first population of people and look at them and, you know, look at the height and weight, and try to design a small t-shirt so that it kind of fits this first population of people well and then design a medium t-shirt and design a large t-shirt. And this is in fact kind of an example of market segmentation

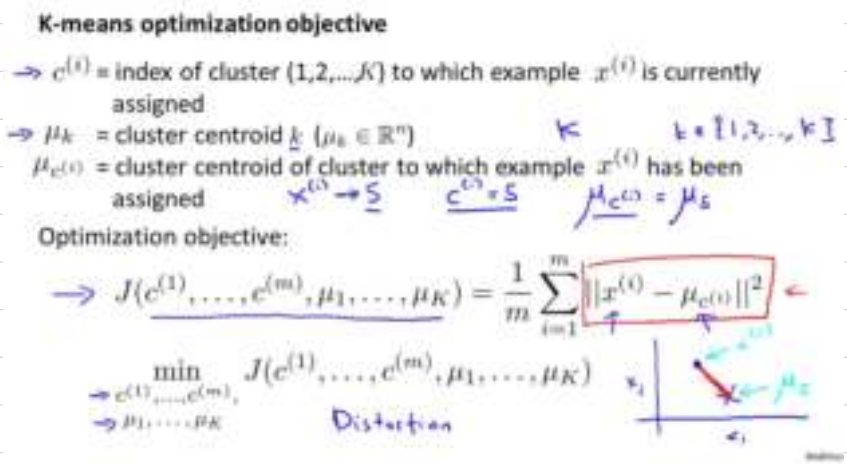
where you're using K Means to separate your market into 3 different segments. So you can design a product separately that is a small, medium, and large t-shirts,

that tries to suit the needs of each of your 3 separate sub-populations well. So that's the K Means algorithm. And by now you should know how to implement the K Means Algorithm and kind of get it to work for some problems. But in the next few videos what I want to do is really get more deeply into the nuts and bolts of K means and to talk a bit about how to actually get this to work really well.

Clustering

optimization objective

Most of the supervised learning algorithms we've seen, things like linear regression, logistic regression, and so on, all of those algorithms have an optimization objective or some cost function that the algorithm was trying to minimize. It turns out that k-means also has an optimization objective or a cost function that it's trying to minimize. And in this video I'd like to tell you what that optimization objective is. And the reason I want to do so is because this will be useful to us for two purposes. First, knowing what is the optimization objective of k-means will help us to debug the learning algorithm and just make sure that k-means is running correctly. And second, and perhaps more importantly, in a later video we'll talk about how we can use this to help k-means find better costs for this and avoid the local ultima. But we do that in a later video that follows this one. Just as a quick reminder while k-means is running we're going to be



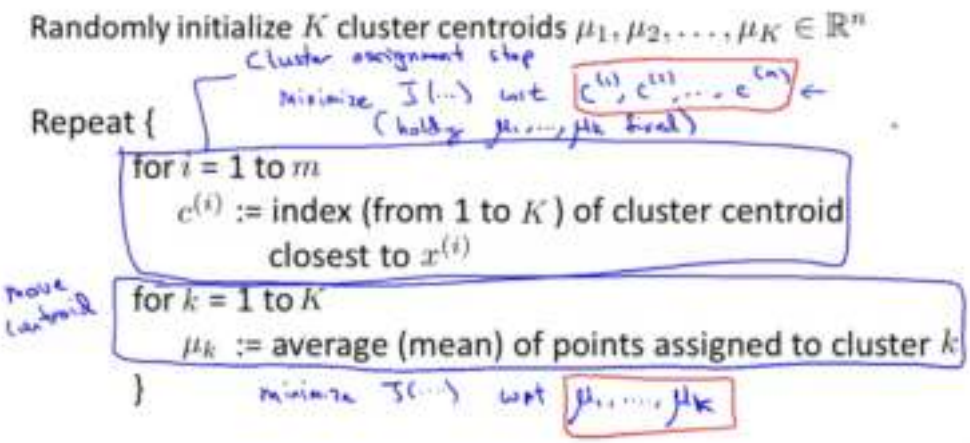
keeping track of two sets of variables. First is the ci's and that keeps track of the index or the number of the cluster, to which an example xi is currently assigned. And then the other set of variables we use is mu subscript k, which is the location of cluster centroid k. Again, for k-means we use capital K to denote the total number of clusters. And here lower case k is going to be an index into the cluster centroids and so, lower case k is going to be a number between one and capital K.

Now here's one more bit of notation, which is gonna use mu subscript ci to denote the cluster centroid of the cluster to which example xi has been assigned, right? And to explain that notation a little bit more, lets say that xi has been assigned to cluster number five. What that means is that ci, that is the index of xi, that that is equal to five. Right? Because having ci equals five, if that's what it means for the example xi to be assigned to cluster number five. And so mu subscript ci is going to be equal to mu subscript 5. Because ci is equal to five. And so this mu subscript ci is the cluster centroid of cluster number five, which is the cluster to which my example xi has been assigned. Out with this notation, we're now ready to write out what is the optimization objective of the k-means clustering algorithm and here it is. The cost function that k-means is minimizing is a function J of all of these parameters, c1 through cm and mu 1 through mu K. That k-means is varying as the algorithm runs. And the optimization objective is shown to the right, is the average of 1 over m of sum from i equals 1 through m of this term here.

That I've just drawn the red box around, right? The square distance between each example xi and the location of the cluster centroid to which xi has been assigned. So let's draw this and just let me explain this. Right, so here's the location of training example xi and here's the location of the cluster centroid to which example xi has been assigned. So to explain this in pictures, if here's x1, x2, and if a point here is my example xi, so if that is equal to my example xi, and if xi has been assigned to some cluster centroid, I'm gonna denote my cluster centroid with a cross, so if that's the location of mu 5, let's say. If x i has been assigned cluster centroid five as in my example up there, then this square distance, that's the square of the distance between the point xi and this cluster centroid to which xi has been assigned. And what k-means can be shown to be doing is that it is trying to define parameters ci and mu i. Trying to find c and mu to try to minimize this cost function J. This cost function is sometimes also called the distortion

cost function, or the distortion of the k-means algorithm. And just to provide a little bit more

K-means algorithm



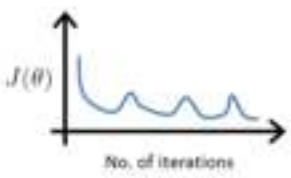
~~cost function, or the distortion of the k-means algorithm~~. And just to provide a little bit more detail, here's the k-means algorithm. Here's exactly the algorithm as we have written it out on the earlier slide. And what this first step of this algorithm is, this was the cluster assignment step

where we assigned each point to the closest centroid. And it's possible to show mathematically that what the cluster assignment step is doing is exactly Minimizing J, with respect to the variables c1, c2 and so on, up to cm, while holding the cluster centroids mu 1 up to mu K, fixed.

So what the cluster assignment step does is it doesn't change the cluster centroids, but what it's doing is this is exactly picking the values of c1, c2, up to cm. That minimizes the cost function, or the distortion function J. And it's possible to prove that mathematically, but I won't do so here. But it has a pretty intuitive meaning of just well, let's assign each point to a cluster centroid that is closest to it, because that's what minimizes the square of distance between the points in the cluster centroid. And then the second step of k-means, this second step over here. The second step was the move centroid step. And once again I won't prove it, but it can be shown mathematically that what the move centroid step does is it chooses the values of mu that minimizes J, so it minimizes the cost function J with respect to, wrt is my abbreviation for, with respect to, when it minimizes J with respect to the locations of the cluster centroids mu 1 through mu K. So if is really is doing is this taking the two sets of variables and partitioning them into two halves right here. First the c sets of variables and then you have the mu sets of variables. And what it does is it first minimizes J with respect to the variable c and then it minimizes J with respect to the variables mu and then it keeps on. And, so all that's all that k-means does. And now that we understand k-means as trying to minimize this cost function J, we can also use this to try to debug other any algorithm and just kind of make sure that our implementation of k-means is running correctly. So, we now understand the k-means algorithm as trying to optimize this cost function J, which is also called the distortion function.

We can use that to debug k-means and help make sure that k-means is converging and is running properly. And in the next video we'll also see how we can use this to help k-means find better clusters and to help k-means to avoid

Suppose you have implemented k-means and you think that it is running correctly, you want the cost function $J(c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K)$ to a function of the number of iterations. Your plot looks like this:



What does this mean?

- [ ] The learning rate is too large.
- [ ] The algorithm is working correctly.
- [ ] The algorithm is working, but it is too slow.
- [x] It is not possible for the cost function to sometimes increase. There must be a bug in the code.

Correct

Clustering

Random initialization

K-means algorithm

Randomly initialize K cluster centroids $\mu_1, \mu_2, \ldots, \mu_K \in \mathbb{R}^n$

Repeat {
  for i = 1 to m
    $c^{(i)}$:= index (from 1 to K) of cluster centroid closest to $x^{(i)}$
  for k = 1 to K
    μ_k := average (mean) of points assigned to cluster k
}

In this video, I'd like to talk about how to initialize

K-means and more importantly, this will lead into a discussion of how to make K-means avoid local optima as well. Here's the K-means clustering algorithm that we talked about earlier.

One step that we never really talked much about was this step of how you randomly initialize the cluster centroids. There are few different ways that one can imagine using to randomly initialize the cluster centroids. But, it turns out that there is one method that is much more recommended than most of the other options one might think about. So, let me tell you about that option since it's what often seems to work best.

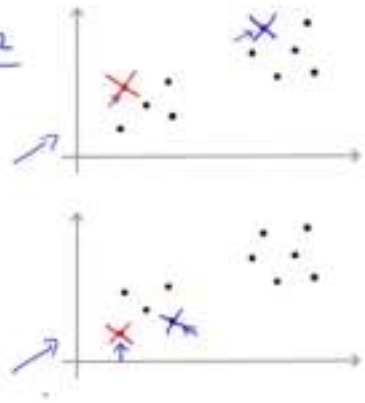
Here's how I usually initialize my cluster centroids.

Random initialization

Should have $K < m$

Randomly pick K training examples.

Set $\mu_1, \ldots, \mu_K$ equal to these K examples.



When running K-means, you should have the number of cluster centroids, K, set to be less than the number of training examples M. It would be really weird to run K-means with a number of cluster centroids that's, you know, equal or greater than the number of examples you have, right?

So the way I usually initialize K-means is, I would randomly pick k training examples. So, and, what I do is then set Mu1 of MuK equal to these k examples.

Let me show you a concrete example.

Lets say that k is equal to 2 and so on this example on the right let's say I want to find two clusters.

So, what I'm going to do in order to initialize my cluster centroids is, I'm going to randomly pick a couple examples. And let's say, I pick this one and I pick that one. And the way I'm going to initialize my cluster centroids is, I'm just going to initialize

my cluster centroids to be right on top of those examples. So that's my first cluster centroid and that's my second cluster centroid, and that's one random initialization of K-means.

The one I drew looks like a particularly good one. And sometimes I might get less lucky and maybe I'll end up picking that as my first random initial example, and that as my second one. And here I'm picking two examples because k equals 2. Some we have randomly picked two training examples and if I chose those two then I'll end up with, may be this as my first cluster centroid and that as my second initial location of the cluster centroid. So, that's how you can randomly initialize the cluster centroids. And so at initialization, your first cluster centroid Mu1 will be equal to x(i) for some randomly value of i and

Mu2 will be equal to x(j)

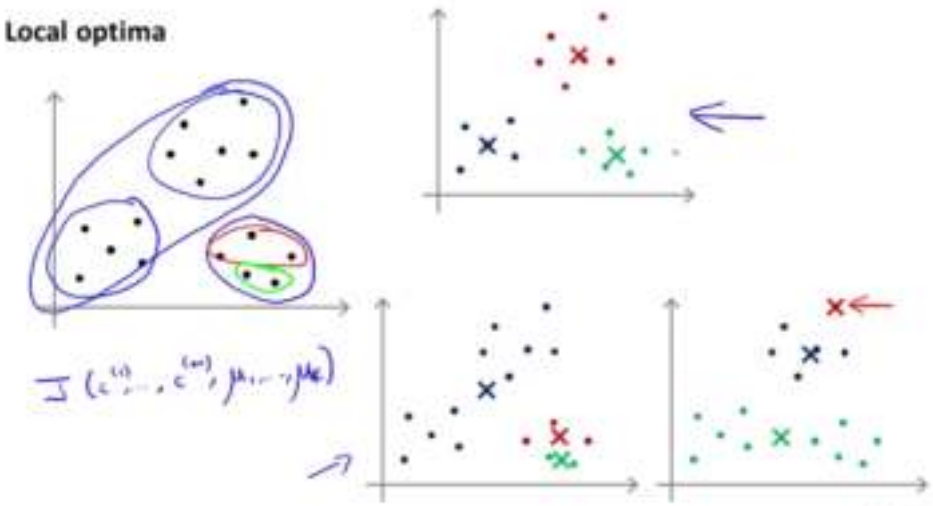
for some different randomly chosen value of j and so on, if you have more clusters and more cluster centroid.

And sort of the side common. I should say that in the earlier video where I first illustrated K-means with the animation.

In that set of slides. Only for the purpose of illustration. I actually used a different method of initialization for my cluster centroids. But the method described on this slide, this is really the recommended way. And the way that you should probably use, when you implement K-means.

So, as they suggested perhaps by these two illustrations on the right. You might really guess that K-means can end up converging to different solutions depending on exactly how the clusters were initialized, and so, depending on the random initialization.

K-means can end up at different solutions. And, in particular, K-means can actually end up at local optima.



If you're given the data sale like this. Well, it looks like, you know, there are three clusters, and so, if you run K-means and if it ends up at a good local optima this might be really the global optima, you might end up with that cluster ring. But if you had a particularly unlucky, random initialization, K-means can also get stuck at different local optima. So, in this example on the left it looks like this blue cluster has captured a lot of points of the left and then the they were on the green clusters each is captioned on the relatively small number of points. And so, this corresponds to a bad local optima because it has basically taken these two clusters and used them into 1 and furthermore, has split the second cluster into two separate sub-clusters like so, and it has also taken the second cluster and split it into two separate sub-clusters like so, and so, both of these examples on the lower right correspond to different local optima of K-means and in fact, in this example here, the cluster, the red cluster has captured only a single optima example. And the term local optima, by the way, refers to local optima of this distortion function J, and what these solutions on the lower left, what these local optima correspond to is really solutions where K-means has gotten stuck to the local optima and it's not doing a very good job minimizing this distortion function J. So, if you're worried about K-means getting stuck in local optima, if you want to increase the odds of K-means finding the best possible clustering, like that shown on top here, what we can do, is try multiple, random initializations. So, instead of just initializing K-means once and hopping that that works, what we can do is, initialize K-means lots of times and run K-means lots of times, and use that to try to make sure we get as good a solution, as good a local or global optima as possible.

Random initialization

For i = 1 to 100 {

50 – 1000

→ Randomly initialize K-means.
Run K-means. Get $c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K$.
Compute cost function (distortion)
→ $J(c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K)$
}

Pick clustering that gave lowest cost $J(c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K)$

K = 2 - 10

Concretely, here's how you could go about doing that. Let's say, I decide to run K-meanss a hundred times so I'll execute this loop a hundred times and it's fairly typical a number of times when came to will be something from 50 up to may be 1000.

So, let's say you decide to say K-means one hundred times.

So what that means is that we would randomly initialize K-means. And for each of these one hundred random intializations we would run K-means and that would give us a set of clusteringings, and a set of cluster centroids, and then we would then compute the distortion J, that is compute this cause function on

the set of cluster assignments and cluster centroids that we got.

Finally, having done this whole procedure a hundred times. You will have a hundred different ways of clustering the data and then finally what you do is all of these hundred ways you have found of clustering the data, just pick one, that gives us the lowest cost. That gives us the lowest distortion. And it turns out that if you are running K-means with a fairly small number of clusters , so you know if the number of clusters is anywhere from two up to maybe 10 - then doing multiple random initializations can often, can sometimes make sure that you find a better local optima. Make sure you find the better clustering data. But if K is very large, so, if K is much greater than 10, certainly if K were, you know, if you were trying to find hundreds of clusters, then,

having multiple random initializations is less likely to make a huge difference and there is a much higher chance that your first random initialization will give you a pretty decent solution already

and doing, doing multiple random initializations will probably give you a slightly better solution but, but maybe not that much. But it's really in the regime of where you have a relatively small number of clusters, especially if you have, maybe 2 or 3 or 4 clusters that random initialization could make a huge difference in terms of making sure you do a good job minimizing the distortion function and giving you a good clustering.

So, that's K-means with random initialization.

If you're trying to learn a clustering with a relatively small number of clusters, 2, 3, 4, 5, maybe, 6, 7, using

multiple random initializations can sometimes, help you find much better clustering of the data. But, even if you are learning a large number of clusters, the initialization, the random initialization method that I describe here. That should give K-means a reasonable starting point to start from for finding a good set of clusters.

Which of the following is the recommended way to initialize k-means?

- ○ Pick a random integer i from $\{1, \ldots, k\}$. Set $\mu_1 = \mu_2 = \cdots = \mu_k = x^{(i)}$.
- ○ Pick k distinct random integers $i_1, \ldots, i_k$ from $\{1, \ldots, k\}$. Set $\mu_1 = x^{(i_1)}, \mu_2 = x^{(i_2)}, \ldots, \mu_k = x^{(i_k)}$.
- ◉ Pick k distinct random integers $i_1, \ldots, i_k$ from $\{1, \ldots, m\}$. Set $\mu_1 = x^{(i_1)}, \mu_2 = x^{(i_2)}, \ldots, \mu_k = x^{(i_k)}$.
- ○ Set every element of $\mu_i \in \mathbb{R}^n$ to a random value between $-\epsilon$ and ϵ, for some small ϵ.

✓ Correct

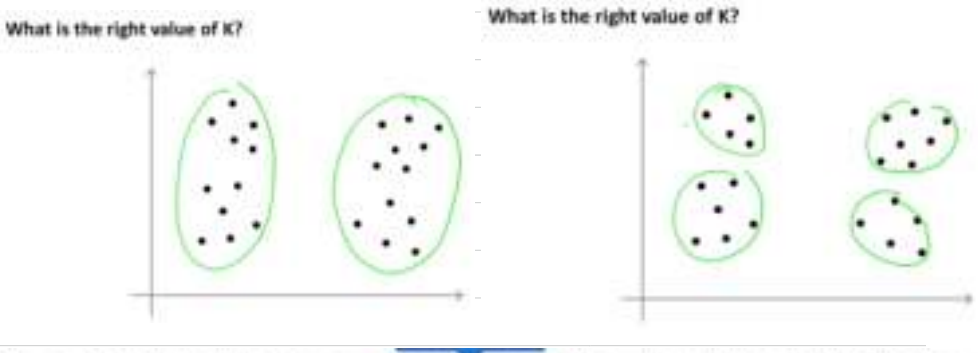
clustering

Choosing the number of clusters

In this video I'd like to talk about one last detail of K-means clustering which is how to choose the number of clusters, or how to choose the value of the parameter

capsule K. To be honest, there actually isn't a great way of answering this or doing this automatically and by far the most common way of choosing the number of clusters, is still choosing it manually by looking at visualizations or by looking at the output of the clustering algorithm or something else.

But I do get asked this question quite a lot of how do you choose the number of clusters, and so I just want to tell you know what are peoples' current thinking on it although, the most common thing is actually to choose the number of clusters by hand.



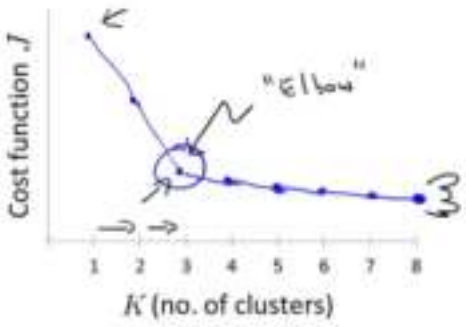
A large part of why it might not always be easy to choose the number of clusters is that it is often generally ambiguous how many clusters there are in the data.

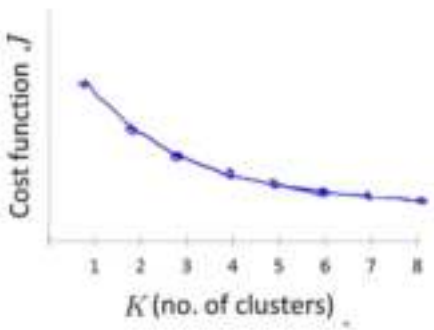
Looking at this data set some of you may see four clusters and that would suggest using K equals 4. Or some of you may see two clusters and that will suggest K equals 2 and now this may see three clusters.

And so, looking at the data set like this, the true number of clusters, it actually seems genuinely ambiguous to me, and I don't think there is one right answer. And this is part of our supervised learning. We are aren't given labels, and so there isn't always a clear cut answer. And this is one of the things that makes it more difficult to say, have an automatic algorithm for choosing how many clusters to have.

Choosing the value of K

Elbow method:





When people talk about ways of choosing the number of clusters, one method that people sometimes talk about is something called the Elbow Method. Let me just tell you a little bit about that, and then mention some of its advantages but also shortcomings. So the Elbow Method, what we're going to do is vary K, which is the total number of clusters. So, we're going to run K-means with one cluster, that means really, everything gets grouped into a single cluster and compute the cost function or compute the distortion J and plot that here. And then we're going to run K means with two clusters, maybe with multiple random initial agents, maybe not. But then, you know, with two clusters we should get, hopefully, a smaller distortion,

and so plot that there. And then run K-means with three clusters, hopefully, you get even smaller distortion and plot that there. I'm gonna run K-means with four, five and so on. And so we end up with a curve showing how the distortion, you know, goes down as we increase the number of clusters. And so we get a curve that maybe looks like this.

And if you look at this curve, what the Elbow Method does it says "Well, let's look at this plot. Looks like there's a clear elbow there". Right, this is, would be by analogy to the human arm where, you know, if you imagine that you reach out your arm, then, this is your shoulder joint, this is your elbow joint and I guess, your hand is at the end over here. And so this is the Elbow Method. Then you find this sort of pattern where the distortion goes down rapidly from 1 to 2, and 2 to 3, and then you reach an elbow at 3, and then the distortion goes down very slowly after that. And then it looks like, you know what, maybe using three clusters is the right number of clusters, because that's the elbow of this curve, right? That it goes down, distortion goes down rapidly until K equals 3, really goes down very slowly after that. So let's pick K equals 3.

If you apply the Elbow Method, and if you get a plot that actually looks like this, then, that's pretty good, and this would be a reasonable way of choosing the number of clusters.

It turns out the Elbow Method isn't used that often, and one reason is that, if you actually use this on a clustering problem, it turns out that fairly often, you know, you end up with a curve that looks much more ambiguous, maybe something like this. And if you look at this, I don't know, maybe there's no clear elbow, but it looks like distortion continuously goes down, maybe 3 is a good number, maybe 4 is a good number, maybe 5 is also not bad. And so, if you actually do this in a practice, you know, if your plot looks like the one on the left and that's great. It gives you a clear answer, but just as often, you end up with a plot that looks like the one on the right and is not clear where the ready location of the elbow is. It makes it harder to choose a number of clusters using this method. So maybe the quick summary of the Elbow Method is that is worth the shot but I wouldn't necessarily,

you know, have a very high expectation of it working for any particular problem.

Suppose you run k-means using k = 3 and k = 5. You find that the cost function J is much higher for k = 5 than for k = 3. What can you conclude?

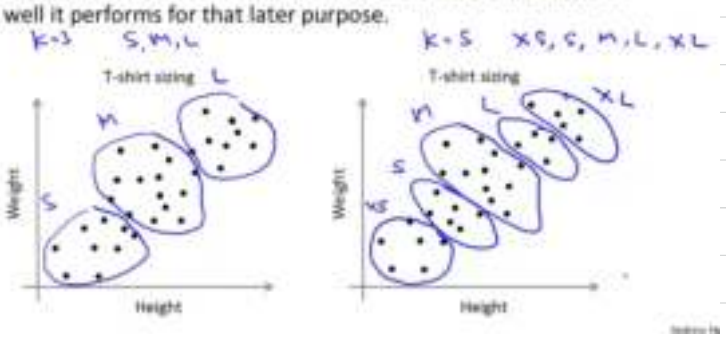
- [ ] This is mathematically impossible. There must be a bug in the code.
- [ ] The correct number of clusters is k = 3.
- [x] In the run with k = 5, k-means got stuck in a bad local minimum. You should try re-running k-means with multiple random initializations.
- [ ] In the run with k = 3, k-means got lucky. You should try re-running k-means with k = 3 and different random initializations until it performs no better than with k = 5.

✓ **Correct**

Choosing the value of K

Sometimes, you're running K-means to get clusters to use for some later/downstream purpose. Evaluate K-means based on a metric for how well it performs for that later purpose.

E.g.



Finally, here's one other way of how, thinking about how you choose the value of K, very often people are running K-means in order you get clusters for some later purpose, or for some sort of downstream purpose. Maybe you want to use K-means in order to do market segmentation, like in the T-shirt sizing example that we talked about. Maybe you want K-means to organize a computer cluster better, or maybe a learning cluster for some different purpose, and so, if that later, downstream purpose, such as market segmentation. If that gives you an evaluation metric, then often, a better way to determine the number of clusters, is to see how well different numbers of clusters serve that later downstream purpose.

Let me step through a specific example.

Let me go through the T-shirt size example again, and I'm trying to decide, do I want three T-shirt sizes? So, I choose K equals 3, then I might have small, medium and large T-shirts. Or maybe, I want to choose K equals 5, and then I might have, you know, extra small, small, medium, large and extra large T-shirt sizes. So, you can have like 3 T-shirt sizes or four or five T-shirt sizes. We could also have four T-shirt sizes, but I'm just showing three and five here, just to simplify this slide for now.

So, if I run K-means with K equals 3, maybe I end up with, that's my small

and that's my medium and that's my large.

Whereas, if I run K-means with 5 clusters, maybe I end up with, those are my extra small T-shirts, these are my small, these are my medium, these are my large and these are my extra large.

And the nice thing about this example is that, this then maybe gives us another way to choose whether we want 3 or 4 or 5 clusters,

and in particular, what you can do is, you know, think about this from the perspective of the T-shirt business and ask: "Well if I have five segments, then how well will my T-shirts fit my customers and so, how many T-shirts can I sell? How happy will my customers be?" What really makes sense, from the perspective of the T-shirt business, in terms of whether, I want to have Goer T-shirt sizes so that my T-shirts fit my customers better. Or do I want to have fewer T-shirt sizes so that I make fewer sizes of T-shirts. And I can sell them to the customers more cheaply. And so, the t-shirt selling business, that might give you a way to decide, between three clusters versus five clusters.

So, that gives you an example of how a later downstream purpose like the problem of deciding what T-shirts to manufacture, how that can give you an evaluation metric for choosing the number of clusters. For those of you that are doing the program exercises, if you look at this week's program exercise associative K-means, that's an example there of using K-means for image compression. And so if you were trying to choose how many clusters to use for that problem, you could also, again use the evaluation metric of image compression to choose the number of clusters, K? So, how good do you want the image to look versus, how much do you want to compress the file size of the image, and, you know, if you do the programming exercise, what I've just said will make more sense at that time.

So, just summarize, for the most part, the number of customers K is still chosen by hand by human input or human insight. One way to try to do so is to use the Elbow Method, but I wouldn't always expect that to work well, but I think the better way to think about how to choose the number of clusters is to ask, for what purpose are you running K-means?

And then to think, what is the number of clusters K that serves that, you know, whatever later purpose that you actually run the K-means for.

Motivation

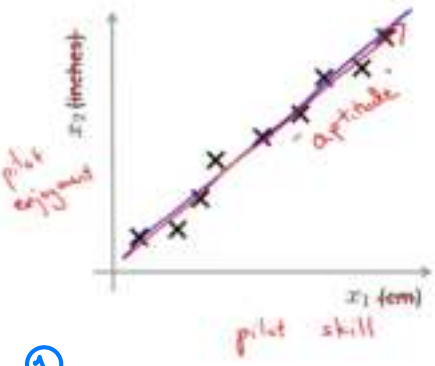
Motivation I : Data Compression

In this video, I'd like to start talking about a second type of unsupervised learning problem called dimensionality reduction.

There are a couple of different reasons why one might want to do dimensionality reduction. One is data compression, and as we'll see later, a few videos later, data compression not only allows us to compress the data and have it therefore use up less computer memory or disk space, but it will also allow us to speed up our learning algorithms.

But first, let's start by talking about what is dimensionality reduction.

Data Compression



Reduce data from 2D to 1D

1

As a motivating example, let's say that we've collected a data set with many, many, many features, and I've plotted just two of them here.

And let's say that unknown to us two of the features were actually the length of something in centimeters, and a different feature, x2, is the length of the same thing in inches.

So, this gives us a highly redundant representation and maybe instead of having two separate features x1 then x2, both of which basically measure the length, maybe what we want to do is reduce the data to one-dimensional and just have one number measuring this length. In case this example seems a bit contrived, this centimeter and inches example is actually not that unrealistic, and not that different from things that I see happening in industry.

If you have hundreds or thousands of features, it is often this easy to lose track of exactly what features you have. And sometimes may have a few different engineering teams, maybe one engineering team gives you two hundred features, a second engineering team gives you another three hundred features, and a third engineering team gives you five hundred features so you have a thousand features all together, and it actually becomes hard to keep track of you know, exactly which features you got from which team, and it's actually not that want to have highly redundant features like these. And so if the length in centimeters were rounded off to the nearest centimeter

2

And so on through my M examples.

So, just to summarize, if we allow ourselves to approximate

the original data set by projecting all of my original examples onto this green line over here, then I need only one number, I need only real number to specify the position of a point on the line, and so what I can do is therefore use just one number to represent the location of each of my training examples after they've been projected onto that green line.

So this is an approximation to the original training self because I have projected all of my training examples onto a line. But now, I need to keep around only one number for each of my examples.

And so this halves the memory requirement, or a space requirement, or what have you, for how to store my data.

And perhaps more interestingly, more importantly, what we'll see later, in the later video as well is that this will allow us to make our learning algorithms run more quickly as well. And that is actually, perhaps, even the more interesting application of this data compression rather than reducing the memory or disk space requirement for storing the data.

2

and lengthened inches was rounded off to the nearest inch. Then, that's why these examples don't lie perfectly on a straight line, because of, you know, round-off error to the nearest centimeter or the nearest inch. And if we can reduce the data to one dimension instead of two dimensions, that reduces the redundancy.

For a different example, again maybe when there seems fairly less contrives. For may years I've been working with autonomous helicopter pilots.

Or I've been working with pilots that fly helicopters.

And so.

If you were to measure--if you were to, you know, do a survey or do a test of these different pilots--you might have one feature, x1, which is maybe the skill of these helicopter pilots, and maybe "x2" could be the pilot enjoyment. That is, you know, how much they enjoy flying, and maybe these two features will be highly correlated. And

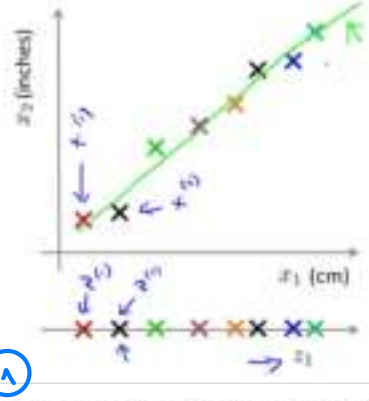
what you really care about might be this sort of

this sort of, this direction, a different feature that really measures pilot aptitude.

And I'm making up the name aptitude of course, but again, if you highly correlated features, maybe you really want to reduce the dimension.

So, let me say a little bit more about what it really means to reduce the dimension of the data from 2 dimensions down from 2D to 1 dimensional or to 1D. Let me color in these examples by using different

Data Compression



Reduce data from 2D to 1D

$x^{(1)} \in \mathbb{R}^2 \rightarrow z^{(1)} \in \mathbb{R}$

$x^{(2)} \in \mathbb{R}^2 \rightarrow z^{(2)} \in \mathbb{R}$

$x^{(m)} \rightarrow z^{(m)}$

1

colors. And in this case by reducing the dimension what I mean is that I would like to find maybe this line, this, you know, direction on which most of the data seems to lie and project all the data onto that line which is true, and by doing so, what I can do is just measure the position of each of the examples on that line. And what I can do is come up with a new feature, z1,

and to specify the position on the line I need only one number, so it says z1 is a new feature that specifies the location of each of those points on this green line. And what this means, is that where as previously if I had an example x1, maybe this was my first example, x1. So in order to represent x1 originally x1.

I needed a two dimensional number, or a two dimensional feature vector. Instead now I can represent

z1. I could use just z1 to represent my first

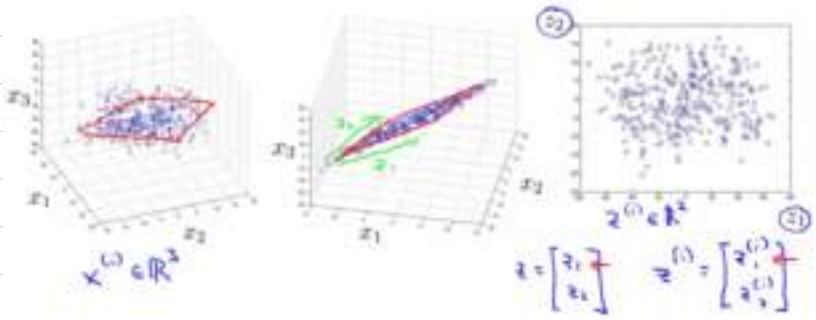
example, and that's going to be a real number. And similarly x2 you know, if x2 is my second example there,

then previously, whereas this required two numbers to represent if I instead compute the projection

of that black cross onto the line. And now I only need one real number which is z2 to represent the location of this point z2 on the line.

Data Compression

Reduce data from 3D to 2D



On the previous slide we showed an example of reducing data from 2D to 1D. On this slide, I'm going to show another example of reducing data from three dimensional 3D to two dimensional 2D.

By the way, in the more typical example of dimensionality reduction we might have a thousand dimensional data or 1000D data that we might want to reduce to let's say a hundred dimensional or 100D, but because of the limitations of what I can plot on the slide. I'm going to use examples of 3D to 2D, or 2D to 1D.

So, let's have a data set like that shown here. And so, I would have a set of examples x(i) which are points in r3. So, I have three dimension examples. I know it might be a little bit hard to see this on the slide, but I'll show a 3D point cloud in a little bit. And it might be hard to see here, but all of this data maybe lies roughly on the plane, like so.

And so what we can do with dimensionality reduction, is take all of this data and project the data down onto a two dimensional plane. So, here what I've done is, I've taken all the data and I've projected all of the data, so that it all lies on the plane.

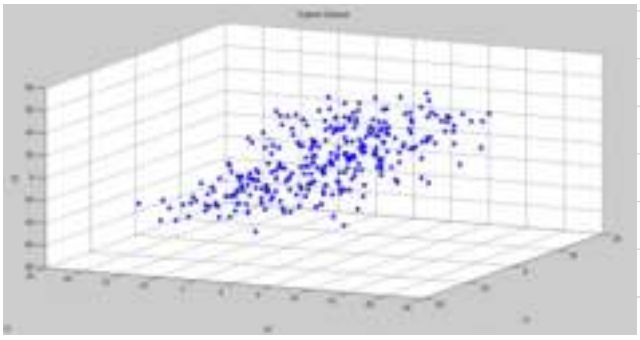
Now, finally, in order to specify the location of a point within a plane, we need two numbers, right? We need to, maybe, specify the location of a point along this axis. So, we need two numbers, maybe called z1 and z2 to specify the location of a point within a plane. And so, what that means, is that we can now represent each example, each training example, using two numbers that I've drawn here, z1, and z2.

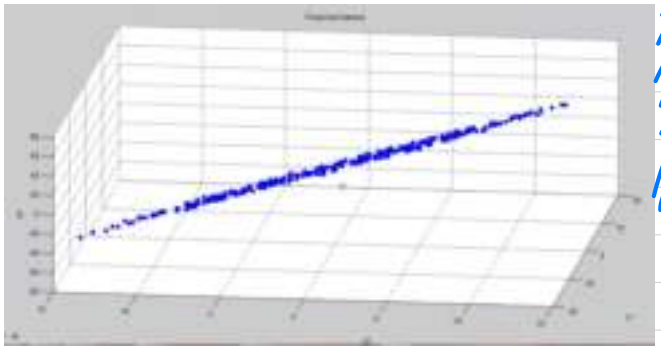
So, our data can be represented using vector z which are in r2.

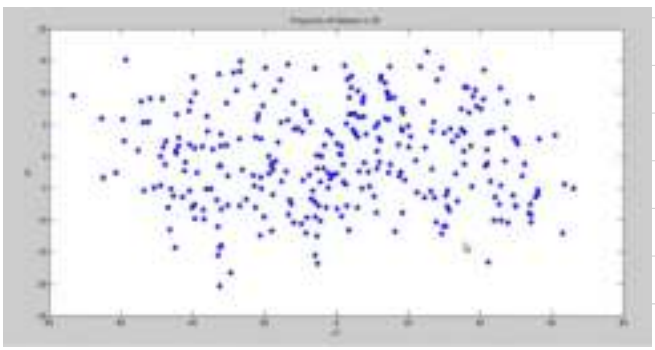
And these subscript, z subscript 1, z subscript 2, what I just mean by that is that my vectors here, z, you know, are two dimensional vectors. z1, z2. And so if I have some particular examples, z(i), or that's the two dimensional vector, z(i)1, z(i)2.

And on the previous slide when I was reducing data to one dimensional data then I had only z1, right? And that is what a z1 subscript 1 on the previous slide was, but here I have two dimensional data, so I have z1 and z2 as the two components of the data.

Now, let me just make sure that these figures make sense. So let me just reshow these exact three figures again but with 3D plots. So the process we went through was that shown in the lab is the







optimal data set, in the middle the data set projects on the 2D, and on the right the 2D data sets with z1 and z2 as the axis. Let's look at them a little bit further. Here's my original data set, shown on the left, and so I had started off with a 3D point cloud like so, where the axis are labeled x1, x2, x3, and so there's a 3D point but most of the data, maybe roughly lies on some, you know, not too far from some 2D plain.

So, what we can do is take this data and here's my middle figure. I'm going to project it onto 2D. So, I've projected this data so that all of it now lies on this 2D surface. As you can see all the data lies on a plane, 'cause we've projected everything onto a plane, and so what this means is that now I need only two numbers, z1 and z2, to represent the location of point on the plane.

And so that's the process that we can go through to reduce our data from three dimensional to two dimensional. So that's dimensionality reduction and how we can use it to compress our data.

And as we'll see later this will allow us to make some of our learning algorithms run much later as well, but we'll get to that only in a later video.

Motivation

Motivation II : Visualization

Data Visualization

In the last video, we talked about dimensionality reduction for the purpose of compressing the data. In this video, I'd like to tell you about a second application of dimensionality reduction and that is to visualize the data. For a lot of machine learning applications, it really helps us to develop effective learning algorithms, if we can understand our data better. If there is some way of visualizing the data better, and so, dimensionality reduction offers us, often, another useful tool to do so. Let's start with an example.

Data Visualization

Country	x_1 GDP (trillions of US$)	x_2 Per capita GDP (thousands of intl. $)	x_3 Human Develop-ment Index	x_4 Life expectancy	x_5 Poverty Index (Gini as percentage)	x_6 Mean household income (thousands of US$)	...
Canada	1.577	39.17	0.908	80.7	32.6	67.293	...
China	5.878	7.54	0.687	73	46.9	10.22	...
India	1.632	3.41	0.547	64.7	36.8	0.735	...
Russia	1.48	19.84	0.755	65.5	39.9	0.72	...
Singapore	0.223	56.69	0.866	80	42.5	67.1	...
USA	14.527	46.86	0.91	78.3	40.8	84.3	...
...	...	...	...	...	...	...	

[resources from en.wikipedia.org]

of Canada: let's say we

Let's say we've collected a large data set of many statistics and facts about different countries around the world. So, maybe the first feature, X1 is the country's GDP, or the Gross Domestic Product, and X2 is a per capita, meaning the per person GDP, X3 human development index, life expectancy, X5, X6 and so on. And we may have a huge data set like this, where, you know, maybe 50 features for every country, and we have a huge set of countries.

So is there something we can do to try to understand our data better? I've given this huge table of numbers. How do you visualize this data? If you have 50 features, it's very difficult to plot 50-dimensional

data. What is a good way to examine this data?

Data Visualization

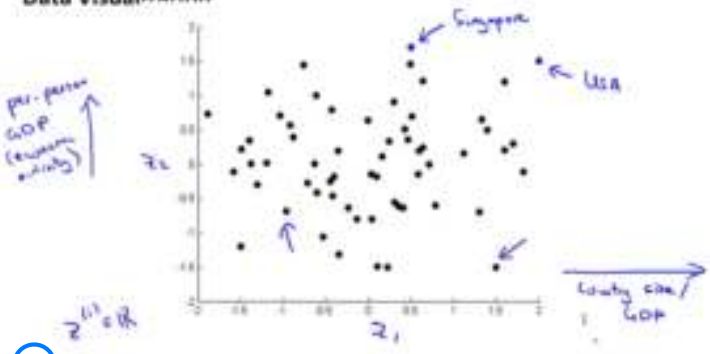
Country	z_1	z_2
Canada	1.6	1.2
China	1.7	0.3
India	1.6	0.2
Russia	1.4	0.5
Singapore	0.5	1.7
USA	2	1.5
...	...	...

Reduce data from 50D to 2D

Using dimensionality reduction, what we can do is, instead of having each country represented by this featured vector, xi, which is 50-dimensional, so instead of, say, having a country like Canada, instead of having 50 numbers to represent the features of Canada, let's say we can come up with a different feature representation that is these z vectors, that is in R2.

If that's the case, if we can have just a pair of numbers, z1 and z2 that somehow, summarizes my 50 numbers, maybe what we can do [xx] is to plot these countries in R2 and use that to try to understand the space in [xx] of features of different countries [xx] the better and so, here, what you can do is reduce the data from 50 D, from 50 dimensions to 2D, so you can plot this as a 2 dimensional plot, and, when you do that, it turns out that, if you look at the output of the Dimensionality Reduction algorithms, it usually doesn't astride a physical meaning to these new features you want [xx] to. It's often up to us to figure out you know, roughly what these features means.

Data Visualization



(1) But, And if you plot those features, here is what you might find. So, here, every country is represented by a point Zi, which is an R2 and so each of these. Dots, and this figure represents a country, and so, here's Z1 and here's Z2, and [xx] [xx] of these. So, you might find, for example, That the horizontal axis the Z1 axis

corresponds roughly to the overall country size, or the overall economic activity of a country. So the overall GDP, overall economic size of a country. Whereas the vertical axis in our data might correspond to the per person GDP. Or the per person well being, or the per person economic activity, and, you might find that, given these 50 features, you know, these are really the 2 main dimensions of the deviation, and so, out here you may have a country like the U.S.A., which is a relatively large GDP, you know, is a very large GDP and a relatively high per-person GDP as well. Whereas here you might have a country like Singapore, which actually has a very high per person GDP as well, but because Singapore is a much smaller country the overall

economy size of Singapore

is much smaller than the US.

And, over here, you would have countries where individuals

are unfortunately some are less well off, maybe shorter life expectancy,

less health care, less economic

(2) maturity that's why smaller countries, whereas a point like this will correspond to a country that has a fair, has a substantial amount of economic activity, but where individuals tend to be somewhat less well off. So you might find that the axes Z1 and Z2 can help you to most succinctly capture really what are the two main dimensions of the variations

amongst different countries.

Such as the overall economic activity of the country projected by the size of the country's overall economy as well as the per-person individual well-being, measured by per-person

GDP, per-person healthcare, and things like that.

So that's how you can use dimensionality reduction, in order to reduce data from 50 dimensions or whatever, down to two dimensions, or maybe down to three dimensions, so that you can plot it and understand your data better.

In the next video, we'll start to develop a specific algorithm, called PCA, or Principal Component Analysis, which will allow us to do this and also

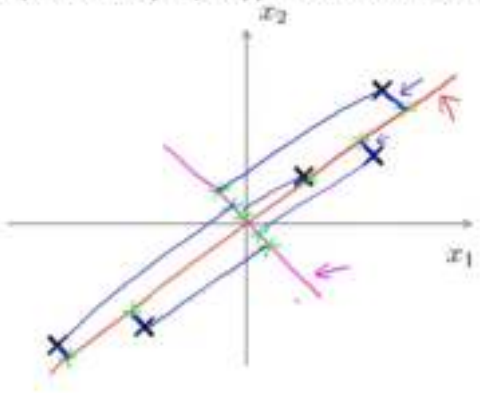
do the earlier application I talked about of compressing the data.

Principal Component Analysis

Principal Component Analysis Problem Formulation

For the problem of dimensionality reduction, by far the most popular, by far the most commonly used algorithm is something called principle components analysis, or PCA. In this video, I'd like to start talking about the problem formulation for PCA. In other words, let's try to formulate, precisely, exactly what we would like PCA to do. Let's say we have a data set like this. So, this is a

Principal Component Analysis (PCA) problem formulation



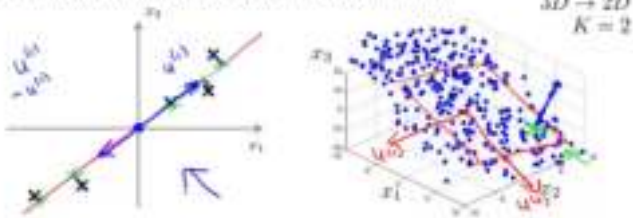
$x \in \mathbb{R}^2$

data set of examples x and R2 and let's say I want to reduce the dimension of the data from two-dimensional to one-dimensional. In other words, I would like to find a line onto which to project the data. So what seems like a good line onto which to project the data, it's a line like this, might be a pretty good choice. And the reason we think this might be a good choice is that if you look at where the projected versions of the point scales, so I take this point and project it down here. Get that, this point gets projected here, to here, to here, to here. What we find is that the distance between each point and the projected version is pretty small. That is, these blue line segments are pretty short. So what PCA does formally is it tries to find a lower dimensional surface, really a line in this case, onto which to project the data so that the sum of squares of these little blue line segments is minimized. The length of those blue line segments, that's sometimes also called the projection error. And so what PCA does is it tries to find a surface onto which to project the data so as to minimize that. As an aside, before applying PCA, it's standard practice to first perform mean normalization at feature scaling so that the features x1 and x2 should have zero mean, and should have comparable ranges of values. I've already done this for this example, but I'll come back to this later and talk more about feature scaling and the normalization in the context of PCA later.

But coming back to this example, in contrast to the red line that I just drew, here's a different line onto which I could project my data, which is this magenta line. And, as we'll see, this magenta line is a much worse direction onto which to project my data, right? So if I were to project my data onto the magenta line, we'd get a set of points like that. And the projection errors, that is these blue line segments, will be huge. So these points have to move a huge distance in order to get projected onto the magenta line. And so that's why PCA, principal components analysis, will choose something like the red line rather than the magenta line down here.

Principal Component Analysis (PCA) problem formulation

$3D \to 2D$
$K = 2$



Reduce from 2-dimension to 1-dimension: Find a direction (a vector $u^{(1)} \in \mathbb{R}^n$) onto which to project the data so as to minimize the projection error.

Reduce from n-dimension to k-dimension: Find k vectors $u^{(1)}, u^{(2)}, \ldots, u^{(k)}$ onto which to project the data, so as to minimize the projection error.

(1)

Let's write out the PCA problem a little more formally. The goal of PCA, if we want to reduce data from two-dimensional to one-dimensional is, we're going to try find a vector that is a vector u1, which is going to be an Rn, so that would be an R2 in this case. I'm gonna find the direction onto which to project the data, so it's to minimize the projection error. So, in this example I'm hoping that PCA will find this vector, which I wanna call u[1], so that when I project the data onto the line that I define by extending out this vector, I end up with pretty small reconstruction errors. And that reference of data that looks like this. And by the way, I should mention that where the PCA gives me u[1] or -u[1], doesn't matter. So if it gives me a positive vector in this direction, that's fine. If it gives me the opposite vector facing in the opposite direction, so that would be like minus u[1]. Let's draw that in blue instead, right? But it gives a positive u[1] or negative u[1], it doesn't matter because each of these vectors defines the same red line onto which I'm projecting my data.

So this is a case of reducing data from two-dimensional to one-dimensional. In the more general case we have n-dimensional data and we'll want to reduce it to k-dimensions. In that case we

(2)

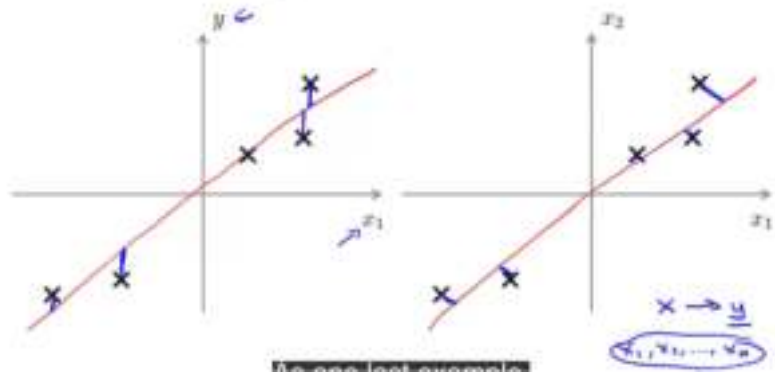
want to find not just a single vector onto which to project the data but we want to find k-dimensions onto which to project the data. So as to minimize this projection error. So here's the example. If I have a 3D point cloud like this, then maybe what I want to do is find vectors. So find a pair of vectors. And I'm gonna call these vectors. Let's draw these in red. I'm going to find a pair of vectors, sustained from the origin. Here's u(1), and here's my second vector, u(2). And together, these two vectors define a plane, or they define a 2D surface, right? Like this with a 2D surface onto which I am going to project my data. For those of you that are familiar with linear algebra, for this year they're really experts in linear algebra, the formal definition of this is that we are going to find the set of vectors u(1), u(2), maybe up to u(k). And what we're going to do is project the data onto the linear subspace spanned by this set of k vectors. But if you're not familiar with linear algebra, just think of it as finding k directions instead of just one direction onto which to project the data. So finding a k-dimensional surface is really finding a 2D plane in this case, shown in this figure, where we can define the position of the points in a plane using k

(3)

directions. And that's why for PCA we want to find k vectors onto which to project the data. And so more formally in PCA, what we want to do is find this way to project the data so as to minimize the sort of projection distance, which is the distance between the points and the projections. And so in this 3D example too. Given a point we would take the point and project it onto this 2D surface.

We are done with that. And so the projection error would be, the distance between the point and where it gets projected down to my 2D surface. And so what PCA does is I try to find the line, or a plane, or whatever, onto which to project the data, to try to minimize that square projection, that 90 degree or that orthogonal projection error. Finally, one question I sometimes get asked is how does PCA relate to linear regression? Because when explaining PCA, I sometimes end up drawing diagrams like these and that looks a little bit like linear regression.

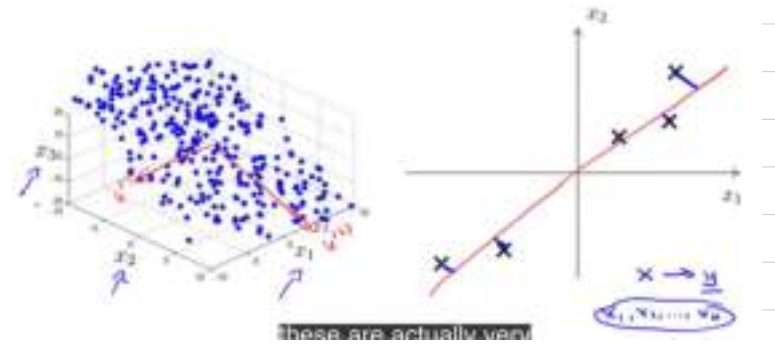
PCA is not linear regression



It turns out PCA is not linear regression, and despite some cosmetic similarity, these are actually totally different algorithms. If we were doing linear regression, what we would do would be, on the left we would be trying to predict the value of some variable y given some info features x. And so linear regression, what we're doing is we're fitting a straight line so as to minimize the square error between point and this straight line. And so what we're minimizing would be the squared magnitude of these blue lines. And notice that I'm drawing these blue lines vertically. That these blue lines are the vertical distance between the point and the value predicted by the hypothesis. Whereas in contrast, in PCA, what it does is it tries to minimize the magnitude of these blue lines, which are drawn at an angle. These are really the shortest orthogonal distances. The shortest distance between the point x and this red line.

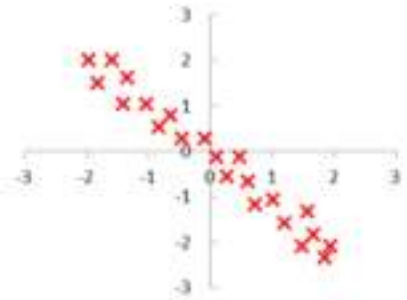
And this gives very different effects depending on the dataset. And more generally, when you're doing linear regression, there is this distinguished variable y they we're trying to predict. All that linear regression as well as taking all the values of x and try to use that to predict y. Whereas in PCA, there is no distinguish, or there is no special variable y that we're trying to predict. And instead, we have a list of features, x1, x2, and so on, up to xn, and all of these features are treated equally, so no one of them is special. As one last example, if I have three-dimensional data and I

PCA is not linear regression



want to reduce data from 3D to 2D, so maybe I wanna find two directions, u(1) and u(2), onto which to project my data. Then what I have is I have three features, x1, x2, x3, and all of these are treated alike. All of these are treated symmetrically and there's no special variable y that I'm trying to predict. And so PCA is not a linear regression, and even though at some cosmetic level they might look related, these are actually very different algorithms. So hopefully you now understand what PCA is doing, it's trying to find a lower dimensional surface onto which to project the data, so as to minimize this squared projection error. To minimize the square distance between each point and the location of where it gets projected. In the next video, we'll start to talk about how to actually find this lower dimensional surface onto which to project the data.

Suppose you run PCA on the dataset below. Which of the following would be a reasonable vector $u^{(1)}$ onto which to project the data? (By convention, we choose $u^{(1)}$ so that $\|u^{(1)}\| = \sqrt{(u_1^{(1)})^2 + (u_2^{(1)})^2}$, the length of the vector $u^{(1)}$, equals 1.)



- ☐ $u^{(1)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- ○ $u^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- ○ $u^{(1)} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$
- ◉ $u^{(1)} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

✓ Correct

Principal Component Analysis

Principal Component Analysis Algorithm

In this video I'd like to tell you about the principle components analysis algorithm.

And by the end of this video you know to implement PCA for yourself. And use it reduce the dimension of your data. Before applying PCA, there is a data pre-processing step which you should always do. Given the trading sets of the examples is important to always perform mean normalization,

and then depending on your data, maybe perform feature scaling as well.

this is very similar to the mean normalization and feature scaling process that we have for supervised learning. In fact it's exactly the same procedure except that we're doing it now to our unlabeled

data, X1 through Xm. So for mean normalization we first compute the mean of each feature and then we replace each feature, X, with X minus its mean, and so this makes each feature now have exactly zero mean

The different features have very different scales. So for example, if x1 is the size of a house, and x2 is the number of bedrooms, to use our earlier example, we then also scale each feature to have a comparable range of values. And so, similar to what we had with supervised learning, we would take x, i substitute j, that's the j feature

and so we would subtract of the mean, now that's what we have on top, and then divide by sj. Here, sj is some measure of the beta values of feature j. So, it could be the max minus min value, or more commonly, it is the standard deviation of feature j. Having done this sort of data pre-processing, here's what the PCA algorithm does.

Data preprocessing

Training set: $x^{(1)}, x^{(2)}, \ldots, x^{(m)}$ ←

Preprocessing (feature scaling/mean normalization):

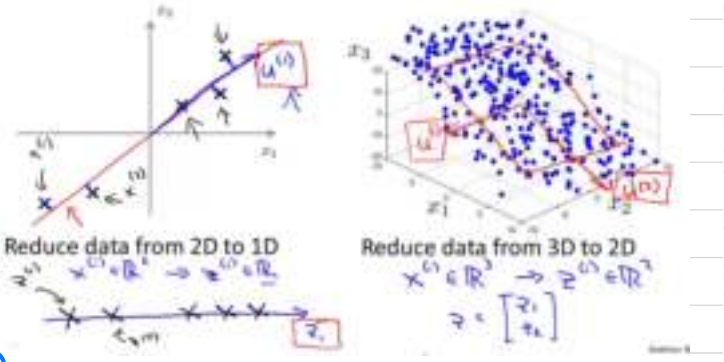
$\mu_j = \frac{1}{m} \sum_{i=1}^{m} x_j^{(i)}$

Replace each $x_j^{(i)}$ with $x_j - \mu_j$.

If different features on different scales (e.g., x_1 = size of house, x_2 = number of bedrooms), scale features to have comparable range of values.

$x_j^{(i)} \leftarrow \frac{x_j^{(i)} - \mu_j}{s_j}$

Principal Component Analysis (PCA) algorithm



(1)

We saw from the previous video that what PCA does is, it tries to find a lower dimensional sub-space onto which to project the data, so as to minimize the squared projection errors, sum of the squared projection errors, as the square of the length of those blue lines that and so what we wanted to do specifically is find a vector, u1, which specifies that direction or in the 2D case we want to find two vectors, u1 and

u2, to define this surface onto which to project the data.

So, just as a quick reminder of what reducing the dimension of the data means, for this example on the left we were given the examples xI, which are in r2. And what we like to do is find a set of numbers zI in r push to represent our data.

So that's what from reduction from 2D to 1D means.

So specifically by projecting

data onto this red line there. We need only one number to specify the position of the points on the line. So i'm going to call that number

z or z1. Z here [xx] real number, so that's like a one dimensional vector. So z1 just refers to the first component of this, you know, one by one matrix, or this one dimensional vector.

And so we need only one number to specify the position of a point. So if this example here was my example

(2)

X1, then maybe that gets mapped here. And if this example was X2 maybe that example gets mapped And so this point here will be Z1 and this point here will be Z2, and similarly we would have those other points for These, maybe X3, X4, X5 get mapped to Z1, Z2, Z3.

So What PCA has to do is we need to come up with a way to compute two things. One is to compute these vectors,

u1, and in this case u1 and u2. And the other is how do we compute these numbers,

Z. So on the example on the left we're reducing the data from 2D to 1D.

In the example on the right, we would be reducing data from 3 dimensional as in r3, to zi, which is now two dimensional. So these z vectors would now be two dimensional. So it would be z1 z2 like so, and so we need to give away to compute these new representations, the z1 and z2 of the data as well. So how do you compute all of these quantities? It turns out that a mathematical derivation, also the mathematical proof, for what is the right value U1, U2, Z1, Z2, and so on. That mathematical proof is very complicated and beyond the scope of the course. But once you've done [xx] it turns out that the procedure to actually find the value of u1 that you want is not that hard, even though so that the mathematical proof that this value is the correct value is someone more involved and more than i want to get into. But let me just describe the specific procedure that you have to implement in order to compute all of these things, the vectors, u1, u2,

the vector z. Here's the procedure.

①

the vector z. Here's the procedure.

Save Note

Let's say we want to reduce the data to n dimensions to k dimension What we're

going to do is first compute something called the covariance matrix, and the covariance matrix is commonly denoted by this Greek alphabet which is the capital Greek alphabet sigma.

It's a bit unfortunate that the Greek alphabet sigma looks exactly like the summation symbols. So this is the Greek alphabet Sigma is used to denote a matrix and this here is a summation symbol. So hopefully in these slides there won't be ambiguity about which is Sigma Matrix, the matrix, which is a summation symbol, and hopefully it will be clear from context when I'm using each one. How do you compute this matrix let's say we want to store it in an octave variable called sigma. What we need to do is compute something called the eigenvectors of the matrix sigma.

And an octave, the way you do that is you use this command, u s v equals s v d of sigma.

SVD, by the way, stands for singular value decomposition.

This is a Much more advanced single value composition.

It is much more advanced linear algebra than you actually need to know but now it turns out that when sigma is equal to matrix there is a few ways to compute these are high in vectors and if you are an expert in linear algebra and if you've heard of high in vectors before you may know that there is another octet function called I, which can also be used to compute the same thing, and it turns out that the SVD function and the I function it will give you the same vectors, although SVD is a little more numerically stable. So I tend to use SVD, although I have a few friends that use the I function to do this as wellbut when you apply this to a covariance matrix sigma it gives you the same thing. This is because the covariance matrix always satisfies a mathematical Property called symmetric positive definite You really don't need to know what that means, but the SVD

Principal Component Analysis (PCA) algorithm

Reduce data from n-dimensions to k-dimensions

Compute "covariance matrix":

$$\Sigma = \frac{1}{m}\sum_{i=1}^{n}(x^{(i)})(x^{(i)})^T$$

Compute "eigenvectors" of matrix Σ:

```
[U,S,V] = svd(Sigma);
```

$$U = \begin{bmatrix} | & | & & | \\ u^{(1)} & u^{(2)} & \cdots & u^{(n)} \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{n \times n}$$

②

and I-functions are different functions but when they are applied to a covariance matrix which can be proved to always satisfy this

mathematical property, they'll always give you the same thing.

Okay, that was probably much more linear algebra than you needed to know. In case none of that made sense, don't worry about it. All you need to know is that this system command you should implement in Octave. And if you're implementing this in a different language than Octave or MATLAB, what you should do is find the numerical linear algebra library that can compute the SVD or singular value decomposition, and there are many such libraries for probably all of the major programming languages. People can use that to compute the matrices u, s, and d of the covariance matrix sigma. So just to fill in some more details, this covariance matrix sigma will be an n by n matrix. And one way to see that is if you look at the definition

this is an n by 1 vector and this here I transpose is 1 by N so the product of these two things is going to be an N by N matrix.

1xN transfers, 1xN, so there's an NxN matrix and when we add up all of these you still have an NxN matrix.

And what the SVD outputs three matrices, u, s, and v. The thing you really need out of the SVD is the u matrix.

The u matrix will also be a NxN matrix.

And if we look at the columns of the U matrix it turns out that the columns

of the U matrix will be exactly those vectors, u1, u2 and so on.

So u, will be matrix.

And if we want to reduce the data from n dimensions down to k dimensions, then what we need to do is take the first k vectors.

that gives us u1 up to uK which gives us the K direction onto which we want to project the data.

the rest of the procedure from this SVD numerical linear algebra routine we get this matrix u. We'll call these columns u1-uN.

So, just to wrap up the description of the rest of the procedure, from the SVD numerical linear algebra routine we get these matrices u, s, and d. we're going to use the first K columns of this matrix to get u1-uK.

Now the other thing we need to is take my original data set, X which is an RN And find a lower dimensional representation Z, which is a R K for this data. So the way we're going to do that is take the first K Columns of the U matrix.

Construct this matrix.

Stack up U1, U2 and

so on up to U K in columns. It's really basically taking, you know, this part of the matrix, the first K columns of this matrix.

And so this is going to be an N by K matrix. I'm going to give this matrix a name. I'm going to call this matrix U, subscript "reduce," sort of a reduced version of the U matrix maybe. I'm going to use it to reduce the dimension of my data.

And the way I'm going to compute Z is going to let Z be equal to this U reduce matrix transpose times X. Or alternatively, you know, to write down what this transpose means. When I take this transpose of this U matrix, what I'm going to end up with is these vectors now in rows. I have U1 transpose down to UK transpose.

Then take that times X, and that's how I get my vector Z. Just to make sure that these dimensions make sense,

this matrix here is going to be k by n and x here is going to be n by 1 and so the product here will be k by 1. And so z is k dimensional, is a k dimensional vector, which is exactly what we wanted. And of course these x's here right, can be Examples in our training set can be examples in our cross validation set, can be examples in our test set, and for example if you know, I wanted to take training example i, I can write this as xi

Xi and that's what will give me Zi over there. So, to summarize, here's the PCA algorithm so one

Principal Component Analysis (PCA) algorithm

From `[U,S,V] = svd(Sigma)`, we get:

$$U = \begin{bmatrix} | & | & & | \\ u^{(1)} & u^{(2)} & \cdots & u^{(n)} \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{n \times n}$$

$$z = U_{reduce}^T x$$

Principal Component Analysis (PCA) algorithm summary

→ After mean normalization (ensure every feature has zero mean) and optionally feature scaling:

$$\text{Sigma} = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)})(x^{(i)})^T$$

```
[U,S,V] = svd(Sigma);
Ureduce = U(:,1:k);
z = Ureduce'*x;
```

$X = \begin{bmatrix} — (x^{(1)})^T — \\ \vdots \\ — (x^{(m)})^T — \end{bmatrix}$

```
Sigma = (1/m)* X' * X;
```

$x \in \mathbb{R}^n$ $x_0 = 1$

~~Xi and that's what will give me Zi over there.~~ So, to summarize, here's the PCA algorithm on one slide.

After mean normalization, to ensure that every feature is zero mean and optional feature scaling whichYou really should do feature scaling if your features take on very different ranges of values. After this pre-processing we compute the carrier matrix Sigma like so by the way if your data is given as a matrix like hits if you have your data Given in rows like this. If you have a matrix X which is your time trading sets written in rows where x1 transpose down to x1 transpose,

this covariance matrix sigma actually has a nice vectorizing implementation.

You can implement in octave, you can even run sigma equals 1 over m, times x, which is this matrix up here, transpose times x and this simple expression, that's the vectorize implementation of how to compute the matrix sigma.

I'm not going to prove that today. This is the correct vectorization whether you want, you can either numerically test this on yourself by trying out an octave and making sure that both this and this implementations give the same answers or you Can try to prove it yourself mathematically.

Either way but this is the correct vectorizing implementation, without compusingnext

we can apply the SVD routine to get u, s, and d. And then we grab the first k columns of the u matrix you reduce and finally this defines how we go from a feature vector x to this reduce dimension representation z. And similar to k Means if you're apply PCA, they way you'd apply this is with vectors X and RN. So, this is not done with X-0 1. So that was the PCA algorithm.

One thing I didn't do is give a mathematical proof that this There it actually give the projection of the data onto the K dimensional subspace onto the K dimensional surface that actually

minimizes the square projection error Proof of that is beyond the scope of this course. Fortunately the PCA algorithm can be implemented in not too many lines of code. and if you implement this in octave or algorithm, you actually get a very effective dimensionality reduction algorithm.

So, that was the PCA algorithm.

One thing I didn't do was give a mathematical proof that the U1 and U2 and so on and the Z and so on you get out of this procedure is really the choices that would minimize these squared projection error. Right, remember we said What PCA tries to do is try to find a surface or line onto which to project the data so as to minimize to square projection error. So I didn't prove that this that, and the mathematical proof of that is beyond the scope of this course. But fortunately the PCA algorithm can be implemented in not too many lines of octave code. And if you implement this, this is actually what will work, or this will work well, and if you implement this algorithm, you get a very effective dimensionality reduction algorithm. That does do the right thing of minimizing this square projection error.

In PCA, we obtain $z \in \mathbb{R}^k$ from $x \in \mathbb{R}^n$ as follows:

$$z = \begin{bmatrix} | & | & & | \\ u^{(1)} & u^{(2)} & \cdots & u^{(k)} \\ | & | & & | \end{bmatrix}^T x = \begin{bmatrix} — & (u^{(1)})^T & — \\ — & (u^{(2)})^T & — \\ & \vdots & \\ — & (u^{(k)})^T & — \end{bmatrix} x$$

Which of the following is a correct expression for z_j?

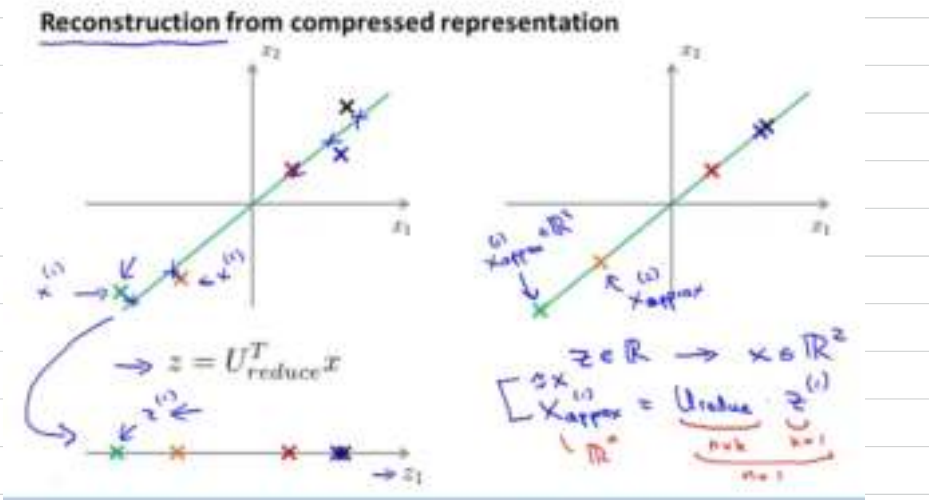
- $z_j = (u^{(k)})^T x$
- $z_j = (u^{(j)})^T x_j$
- $z_j = (u^{(j)})^T x_k$
- $z_j = (u^{(j)})^T x$

✓ **Correct**

Applying PCA

Reconstruction from Compressed Representation

In some of the earlier videos, I was talking about PCA as a compression algorithm where you may have say, 1,000-dimensional data and compress it to 100-dimensional feature vector. Or have three-dimensional data and compress it to a two-dimensional representation. So, if this is a compression algorithm, there should be a way to go back from this compressed representation back to an approximation of your original high-dimensional data. So given z_i, which may be 100-dimensional, how do you go back to your original representation, x_i which was maybe a 1000-dimensional. In this video, I'd like to describe how to do that.



In the PCA algorithm, we may have an example like this, so maybe that's my example x1, and maybe that's my example x2. And what we do is we take these examples, and we project them onto this one dimensional surface. And then now we need to use a real number, say z1, to specify the location of these points after they've been projected onto this one dimensional surface. So, given the point z1, how can we go back to this original two dimensional space? In particular, given the point z, which is R, can we map this back to some approximate representation x and R2 of whatever the original value of the data was? So whereas z equals U reduce transpose x, if you want to go in the opposite direction, the equation for that is, we're going to write x approx equals U reduce, times z. And again, just to check the dimensions, here U reduce is going to be an n by k dimensional vector, z is going to be k by one dimensional vector. So you multiply these out that's going to be n by one, so x approx is going to be an n dimensional vector. And so the intent of PCA, that is if the square projection error is not too big, is that this x approx will be close to whatever was the original value of x that you have used to derive z in the first place. To show a picture of what this looks like, this is what it looks like. What you get back of this procedure are points that lie on the projection of that, onto the green line. So to take our early example, if we started off with this value of x1, and we got this value of z1, if you plug z1 through this formula to get x1 approx, then this point here, that would be x1 approx, which is going to be in R2. And similarly, if you do the same procedure, this would be x2 approx. And that's a pretty decent approximation to the original data. So that's how you go back from your low dimensional representation z, back to an uncompressed representation of the data. We get back an approximation to your original data x. And we also call this process reconstruction of the original data where we think of trying to reconstruct the original value of x from the compressed representation.

So, given an unlabeled data set, you now know how to apply PCA and take your high dimensional features x and map that to this lower-dimensional representation z. And from this video hopefully you now also know how to take these low-representation z and map it back up to an approximation of your original high-dimensional data.

Now that you know how to implement and apply PCA, what I'd like to do next is talk about some of the mechanics of how to actually use PCA well. And in particular in the next video, I'd like to talk about how to choose k, which is how to choose the dimension of the reduced representation vector z.

Applying PCA

Choosing the number of Principal Components

In the PCA algorithm we take N dimensional features and reduce them to some K dimensional feature representation.

This number K is a parameter of the PCA algorithm.

This number K is also called the number of principle components or the number of principle components that we've retained.

And in this video I'd like to give you some guidelines, tell you about how people tend to think about how to choose this parameter K for PCA.

In order to choose k, that is to choose the number of principal components, here are a couple of useful concepts.

Choosing k (number of principal components)

Average squared projection error: $\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)} - x_{approx}^{(i)}\|^2$

Total variation in the data: $\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)}\|^2$

Typically, choose k to be smallest value so that

$$\frac{\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)} - x_{approx}^{(i)}\|^2}{\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)}\|^2} \leq 0.01$$

"99% of variance is retained"

(1)

What PCA tries to do is it tries to minimize

the average squared projection error. So it tries to minimize this quantity, which I'm writing down, which is the difference between the original data X and the projected version, X approx i, which was defined last video, so it tries to minimize the squared distance between x and it's projection onto that lower dimensional surface.

So that's the average square projection error.

Also let me define the total variation in the data to be the average length squared of these examples Xi so the total variation in the data is the average of my training sets of the length of each of my training examples. And this one says, "On average, how far are my training examples from the vector, from just being all zeros?" How far is, how far on average are my training examples from the origin? When we're trying to choose k, a

pretty common rule of thumb for choosing k is to choose the smaller values so that the ratio between these is less than 0.01. So in other words, a pretty common way to think about how we choose k is we want the average squared projection error. That is the average distance between x and it's projections

(2)

divided by the total variation of the data. That is how much the data varies.

We want this ratio to be less than, let's say, 0.01. Or to be less than 1%, which is another way of thinking about it.

And the way most people think about choosing K is rather than choosing K directly the way most people talk about it is as what this number is, whether it is 0.01 or some other number. And if it is 0.01, another way to say this to use the language of PCA is that 99% of the variance is retained.

I don't really want to, don't worry about what this phrase really means technically but this phrase "99% of variance is retained" just means that this quantity on the left is less than 0.01. And so, if you

are using PCA and if you want to tell someone, you know, how many principle components you've retained it would be more common to say well, I chose k so that 99% of the variance was retained. And that's kind of a useful thing to know, it means that you know, the average squared projection error divided by the total variation that was at most 1%. That's kind of an insightful thing to think about, whereas if you tell someone that, "Well I had to 100 principle components" or "k was equal to 100 in a thousand dimensional data" it's a little hard for people to interpret

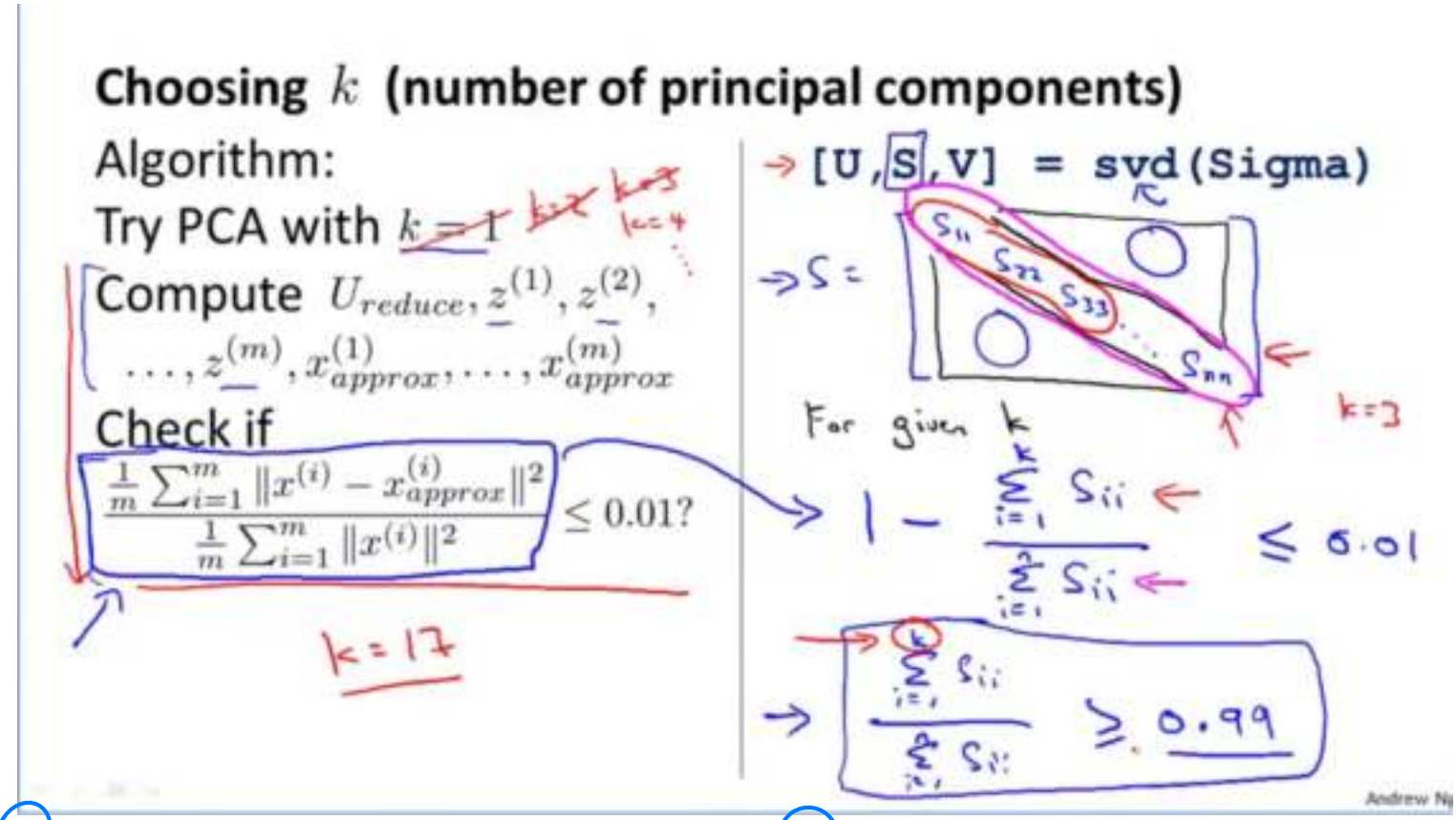
(3)

that. So this number 0.01 is what people often use. Other common values is 0.05,

and so this would be 5%, and if you do that then you go and say well 95% of the variance is retained and, you know other numbers maybe 90% of the variance is

retained, maybe as low as 85%. So 90% would correspond to say

0.10, kinda 10%. And so range of values from, you know, 90, 95, 99, maybe as low as 85% of the variables contained would be a fairly typical range in values. Maybe 95 to 99 is really the most common range of values that people use. For many data sets you'd be surprised, in order to retain 99% of the variance, you can often reduce the dimension of the data significantly and still retain most of the variance. Because for most real life data says many features are just highly correlated, and so it turns out to be possible to compress the data a lot and still retain you know 99% of the variance or 95% of the variance. So how do you implement this? Well, here's one



(1) 99% of the variance or 95% of the variance. So how do you implement this? Well, here's one algorithm that you might use. You may start off, if you want to choose the value of k, we might start off with k equals 1. And then we run through PCA. You know, so we compute, you reduce, compute z1, z2, up to zm. Compute all of those x1 approx and so on up to xm approx and then we check if 99% of the variance is retained.

Then we're good and we use k equals 1. But if it isn't then what we'll do we'll next try K equals 2. And then we'll again run through this entire procedure and check, you know is this expression satisfied. Is this less than 0.01. And if not then we do this again. Let's try k equals 3, then try k equals 4, and so on until maybe we get up to k equals 17 and we find 99% of the data have is retained and then

we use k equals 17, right? That is one way to choose the smallest value of k, so that and 99% of the variance is retained.

But as you can imagine, this procedure seems horribly inefficient

we're trying k equals one, k equals two, we're doing all these calculations.

(2) Fortunately when you implement PCA it actually, in this step, it actually gives us a quantity that makes it much easier to compute these things as well. Specifically when you're calling SVD to get these matrices u, s, and d, when you're calling usvd on the covariance matrix sigma, it also gives us back this matrix S and what S is, is going to be a square matrix an N by N matrix in fact, that is diagonal. So is diagonal entries s one one, s two two, s three three down to s n n are going to be the only non-zero elements of this matrix, and everything off the diagonals is going to be zero. Okay? So those big O's that I'm drawing, by that what I mean is that everything off the diagonal of this matrix all of those entries there are going to be zeros.

And so, what is possible to show, and I won't prove this here, and it turns out that for a given value of k, this quantity over here can be computed much more simply. And that quantity can be computed as one minus sum from i equals 1 through K of Sii divided by sum from I equals 1 through N of Sii.

So just to say that it words, or just to take another view of how to explain that, if K equals 3 let's say.

What we're going to do to compute the numerator is sum from one– I equals 1 through 3 of of Sii, so just compute the sum of these first three elements.

(3) So that's the numerator.

And then for the denominator, well that's the sum of all of these diagonal entries.

And one minus the ratio of that, that gives me this quantity over here, that I've circled in blue. And so, what we can do is just test if this is less than or equal to 0.01. Or equivalently, we can test if the sum from i equals 1 through k, s-i-i divided by sum from i equals 1 through n, s-i-i if this is greater than or equal to 4.99, if you want to be sure that 99% of the variance is retained.

And so what you can do is just slowly increase k, set k equals one, set k equals two, set k equals three and so on, and just test this quantity

to see what is the smallest value of k that ensures that 99% of the variance is retained.

And if you do this, then you need to call the SVD function only once. Because that gives you the S matrix and once you have the S matrix, you can then just keep on doing this calculation by increasing the value of K in the numerator and so you don't need keep to calling SVD over and over again to test out the different values of K. So this procedure is much more efficient, and this can allow you to select the value of K without needing to run PCA from scratch over and over. You just run SVD once, this gives you all of these diagonal numbers, all of these numbers S11, S22 down to SNN, and then you can just you know, vary K in this expression to find the smallest value of K, so that 99% of the variance is retained. So to summarize, the way that I often use, the way

Choosing k (number of principal components)

→ `[U,S,V] = svd(Sigma)`

Pick smallest value of k for which

$$\frac{\sum_{i=1}^{k} S_{ii}}{\sum_{i=1}^{m} S_{ii}} \geq 0.99$$

k=100

(99% of variance retained)

of K, so that 99% of the variance is retained. So to summarize, the way that I often use, the way that I often choose K when I am using PCA for compression is I would call SVD once in the covariance matrix, and then I would use this formula and pick the smallest value of K for which this expression is satisfied.

And by the way, even if you were to pick some different value of K, even if you were to pick the value of K manually, you know maybe you have a thousand dimensional data and I just want to choose K equals one hundred. Then, if you want to explain to others what you just did, a good way to explain the performance of your implementation of PCA to them, is actually to take this quantity and compute what this is, and that will tell you what was the percentage of variance retained. And if you report that number, then, you know, people that are familiar with PCA, and people can use this to get a good understanding of how well your hundred dimensional representation is approximating your original data set, because there's 99% of variance retained. That's really a measure of your square of construction error, that ratio being 0.01, just gives people a good intuitive sense of whether your implementation of PCA is finding a good approximation of your original data set.

So hopefully, that gives you an efficient procedure for choosing the number K. For choosing what dimension to reduce your data to, and if you apply PCA to very high dimensional data sets, you know, to like a thousand dimensional data, very often, just because data sets tend to have highly correlated features, this is just a property of most of the data sets you see,

you often find that PCA will be able to retain ninety nine per cent of the variance or say, ninety five ninety nine, some high fraction of the variance, even while compressing the data by a very large factor.

Previously, we said that PCA chooses a direction $u^{(1)}$ (or k directions $u^{(1)}, \ldots, u^{(k)}$) onto which to project the data so as to minimize the (squared) projection error. Another way to say the same is that PCA tries to minimize:

- $\frac{1}{m}\sum_{i=1}^{m} \|x^{(i)}\|^2$
- $\frac{1}{m}\sum_{i=1}^{m} \|x_{\text{approx}}^{(i)}\|^2$
- ◉ $\frac{1}{m}\sum_{i=1}^{m} \|x^{(i)} - x_{\text{approx}}^{(i)}\|^2$
- $\frac{1}{m}\sum_{i=1}^{m} \|x^{(i)} + x_{\text{approx}}^{(i)}\|^2$

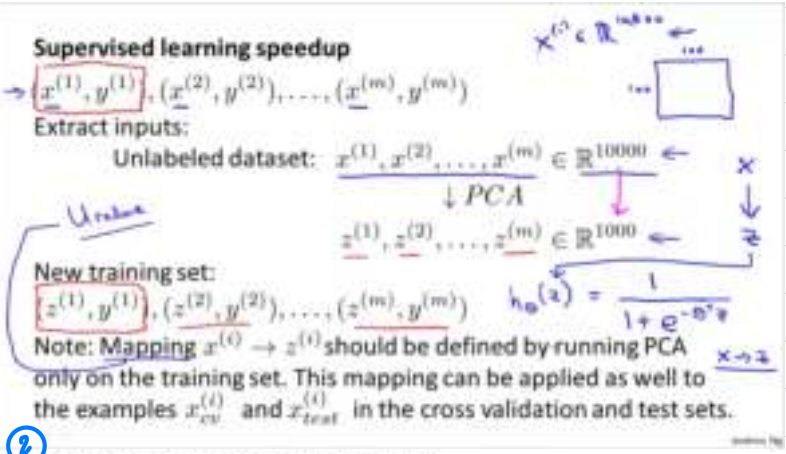
✓ Correct

Applying PCA

Advice for Applying PCA

In an earlier video, I had said that PCA can be sometimes used to speed up the running time of a learning algorithm.

In this video, I'd like to explain how to actually do that, and also say some, just try to give some advice about how to apply PCA.



(1) Here's how you can use PCA to speed up a learning algorithm, and this supervised learning algorithm speed up is actually the most common use that I personally make of PCA. Let's say you have a supervised learning problem, note this is a supervised learning problem with inputs X and labels Y, and let's say that your examples xi are very high dimensional. So, lets say that your examples, xi are 10,000 dimensional feature vectors.

One example of that, would be, if you were doing some computer vision problem, where you have a 100x100 images, and so if you have 100x100, that's 10000 pixels, and so if xi are, you know, feature vectors that contain your 10000 pixel intensity values, then you have 10000 dimensional feature vectors.

So with very high-dimensional feature vectors like this, running a learning algorithm can be slow, right? Just, if you feed 10,000 dimensional feature vectors into logistic regression, or a new network, or support vector machine or what have you, just because that's a lot of data, that's 10,000 numbers.

(2) it can make your learning algorithm run more slowly.

Fortunately with PCA we'll be able to reduce the dimension of this data and so make our algorithms run more efficiently. Here's how you do that. We are going first check our labeled training set and extract just the inputs, we're just going to extract the X's and temporarily put aside the Y's. So this will now give us an unlabelled training set x1 through xm which are maybe there's a ten thousand dimensional data, ten thousand dimensional examples we have. So just extract the input vectors

x1 through xm.

Then we're going to apply PCA and this will give me a reduced dimension representation of the data, so instead of 10,000 dimensional feature vectors I now have maybe one thousand dimensional feature vectors. So that's like a 10x savings.

So this gives me, if you will, a new training set. So whereas previously I might have had an example x1, y1, my first training input, is now represented by z1. And so we'll have a new sort of training example,

which is Z1 paired with y1.

(3) And similarly Z2, Y2, and so on, up to ZM, YM. Because my training examples are now represented with this much lower dimensional representation Z1, Z2, up to ZM. Finally, I can take this

reduced dimension training set and feed it to a learning algorithm maybe a neural network, maybe logistic regression, and I can learn the hypothesis H, that takes this input, these low-dimensional representations Z and tries to make predictions.

So if I were using logistic regression for example, I would train a hypothesis that outputs, you know, one over one plus E to the negative-theta transpose

Z, that takes this input to one these z vectors, and tries to make a prediction.

And finally, if you have a new example, maybe a new test example X. What you do is you would take your test example x,

map it through the same mapping that was found by PCA to get you your corresponding z. And that z then gets fed to this hypothesis, and this hypothesis then makes a prediction on your input x.

(4) One final note, what PCA does is it defines a mapping from x to z and this mapping from x to z should be defined by running PCA only on the training sets. And in particular, this mapping that PCA is learning, right, this mapping, what that does is it computes the set of parameters. That's the feature scaling and mean normalization. And there's also computing this matrix U reduced. But all of these things that U reduce, that's like a parameter that is learned by PCA and we should be fitting our parameters only to our training sets and not to our cross validation or test sets and so these things the U reduced so on, that should be obtained by running PCA only on your training set. And then having found U reduced, or having found the parameters for feature scaling where the mean normalization and scaling the scale that you divide the features by to get them on to comparable scales. Having found all those parameters on the training set, you can then apply the same mapping to other examples that may be In your cross-validation sets or in your test sets, OK? Just to summarize, when you're running PCA, run your PCA only on the training set portion of the data not the cross-validation set or the test set portion of your data. And that defines the mapping from x to z and you can then apply that mapping to your cross-validation set and your test set and by the way in this example I talked about reducing the data from ten thousand dimensional to one thousand dimensional, this is actually not that unrealistic. For many problems we actually reduce the dimensional data. You

know by 5x maybe by 10x and still retain most of the variance and we can do this barely effecting the performance,

in terms of classification accuracy, let's say, barely affecting the classification accuracy of the learning algorithm. And by working with lower dimensional data our learning algorithm can often run much much faster. To summarize, we've so far talked about the following applications of

Application of PCA

- Compression
  - Reduce memory/disk needed to store data
  - Speed up learning algorithm ←

    Choose k by % of variance retain
- Visualization

  $k=2$ or $k=3$

~~run much much faster~~ To summarize, we've so far talked about the following applications of PCA.

First is the compression application where we might do so to reduce the memory or the disk space needed to store data and we just talked about how to use this to speed up a learning algorithm. In these applications, in order to choose K, often we'll do so according to, figuring out what is the percentage of variance retained, and so for this learning algorithm, speed up application often will retain 99% of the variance. That would be a very typical choice for how to choose k. So that's how you choose k for these compression applications.

Whereas for visualization applications

while usually we know how to plot only two dimensional data or three dimensional data,

and so for visualization applications, we'll usually choose k equals 2 or k equals 3, because we can plot only 2D and 3D data sets.

So that summarizes the main applications of PCA, as well as how to choose the value of k for these different applications.

I should mention that there is often one frequent misuse of PCA and you sometimes hear about others doing this hopefully not too often. I just want to mention this so that you know not to do it. ~~And there is one bad use of PCA, which is to try to use it to prevent over-fitting.~~

Bad use of PCA: To prevent overfitting

→ Use $z^{(i)}$ instead of $x^{(i)}$ to reduce the number of features to $k < n$. — 1000 10000

Thus, fewer features, less likely to overfit.

Bad!

This might work OK, but isn't a good way to address overfitting. Use regularization instead.

→ $$\min_\theta \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2$$ ←

② ① → And there is one bad use of PCA, which iss to try to use it to prevent over-fitting.

Here's the reasoning.

This is not a great way to use PCA, but here's the reasoning behind this method, which is,you know if we have Xi, then maybe we'll have n features, but if we compress the data, and use Zi instead and that reduces the number of features to k, which could be much lower dimensional. And so if we have a much smaller number of features, if k is 1,000 and n is 10,000, then if we have only 1,000 dimensional data, maybe we're less likely to over-fit than if we were using 10,000-dimensional

data with like a thousand features. So some people think of PCA as a way to prevent over-fitting. But just to emphasize this is a bad application of PCA and I do not recommend doing this. And it's not that this method works badly. If you want to use this method to reduce the dimensional data, to try to prevent over-fitting, it might actually work OK. But this just is not a good way to address over-fitting and instead, if you're worried about over-fitting, there is a much better way to address it, to use regularization instead of using PCA to reduce the dimension of the data. And the reason is, if

you think about how PCA works, it does not use the labels y. You are just looking at your inputs xi, and you're using that to find a lower-dimensional approximation to your data. So what PCA does, is it throws away some information.

It throws away or reduces the dimension of your data without knowing what the values of y is, so this is probably okay using PCA this way is probably okay if, say 99 percent of the variance is retained, if you're keeping most of the variance, but it might also throw away some valuable information. And it turns out that if you're retaining 99% of the variance or 95% of the variance or whatever, it turns out that just using regularization will often give you at least as good a method for preventing over-fitting

and regularization will often just work better, because when you are applying linear regression or logistic regression or some other method with regularization, well, this minimization problem actually knows what the values of y are, and so is less likely to throw away some valuable information, whereas PCA doesn't make use of the labels and is more likely to throw away valuable information. So, to summarize, it is a good use of PCA, if your main motivation to speed up your learning algorithm, but using PCA to prevent over-fitting, that is not a good use of PCA, and using regularization instead is really what many people would recommend doing instead. Finally, one last misuse of PCA. And so I should say PCA is a very useful algorithm, I often use it for

PCA is sometimes used where it shouldn't be

Design of ML system:

- Get training set $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$
- ~~Run PCA to reduce $x^{(i)}$ in dimension to get $z^{(i)}$~~
- Train logistic regression on $\{(z^{(1)}, y^{(1)}), \ldots, (z^{(m)}, y^{(m)})\}$
- Test on test set: Map $x_{test}^{(i)}$ to $z_{test}^{(i)}$. Run $h_\theta(z)$ on $\{(z_{test}^{(1)}, y_{test}^{(1)}), \ldots, (z_{test}^{(m)}, y_{test}^{(m)})\}$

→ How about doing the whole thing without using PCA?

→ Before implementing PCA, first try running whatever you want to do with the original/raw data $x^{(i)}$. Only if that doesn't do what you want, then implement PCA and consider using $z^{(i)}$.

Finally, one last misuse of PCA. And so I should say PCA is a very useful algorithm, I often use it for the compression on the visualization purposes.

But, what I sometimes see, is also people sometimes use PCA where it shouldn't be. So, here's a pretty common thing that I see, which is if someone is designing a machine-learning system, they may write down the plan like this: let's design a learning system. Get a training set and then, you know, what I'm going to do is run PCA, then train logistic regression and then test on my test data. So often at the very start of a project, someone will just write out a project plan than says lets do these four steps with PCA inside.

Before writing down a project plan the incorporates PCA like this, one very good question to ask is, well, what if we were to just do the whole without using PCA. And often people do not consider this step before coming up with a complicated project plan and implementing PCA and so on. And sometime, and so specifically, what I often advise people is, before you implement PCA, I would first suggest that, you know, do whatever it is, take whatever it is you want to do and first consider doing it with your original raw data xi, and only if that doesn't do what you want, then implement PCA before using Zi.

So, before using PCA you know, instead of reducing the dimension of the data, I would consider well, let's ditch this PCA step, and I would consider, let's just train my learning algorithm on my original data. Let's just use my original raw inputs xi, and I would recommend, instead of putting PCA into the algorithm, just try doing whatever it is you're doing with the xi first. And only if you have a reason to believe that doesn't work, so that only if your learning algorithm ends up running too slowly, or only if the memory requirement or the disk space requirement is too large, so you want to compress your representation, but if only using the xi doesn't work, only if you have evidence or strong reason to believe that using the xi won't work, then implement PCA and consider using the compressed representation.

Because what I do see, is sometimes people start off with a project plan that incorporates PCA inside, and sometimes they, whatever they're doing will work just fine, even with out using PCA instead. So, just consider that as an alternative as well, before you go to spend a lot of time to get PCA in, figure out what k is and so on. So, that's it for PCA. Despite these last sets of comments, PCA is an incredibly useful algorithm, when you use it for the appropriate applications and I've actually used PCA pretty often and for me, I use it mostly to speed up the running time of my learning algorithms. But I think, just as common an application of PCA, is to use it to compress data, to reduce the memory or disk space requirements, or to use it to visualize data.

And PCA is one of the most commonly used and one of the most powerful unsupervised learning algorithms. And with what you've learned in these videos, I think hopefully you'll be able to implement PCA and use them through all of these purposes as well.

Which of the following are good / recommended applications of PCA? Select all that apply.

- [x] To compress the data so it takes up less computer memory / disk space

✓ Correct

- [x] To reduce the dimension of the input data so as to speed up a learning algorithm

✓ Correct

- [ ] Instead of using regularization, use PCA to reduce the number of features to reduce overfitting
- [x] To visualize high-dimensional data (by choosing k = 2 or k = 3)

✓ Correct

Density Estimation

Problem Motivation

In this next set of videos, I'd like to tell you about a problem called Anomaly Detection.

This is a reasonably commonly use you type machine learning. And one of the interesting aspects is that it's mainly for unsupervised problem, that there's some aspects of it that are also very similar to sort of the supervised learning problem.

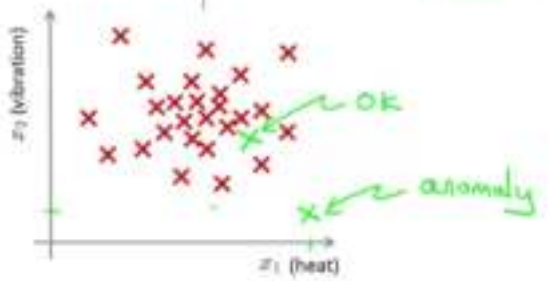
Anomaly detection example

Aircraft engine features:

- x_1 = heat generated
- x_2 = vibration intensity
- ...

Dataset: $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$

New engine: x_{test}



(1)

So, what is anomaly detection? To explain it. Let me use the motivating example of: Imagine that you're a manufacturer of aircraft engines, and let's say that as your aircraft engines roll off the assembly line, you're doing, you know, QA or quality assurance testing, and as part of that testing you measure features of your aircraft engine, like maybe, you measure the heat generated, things like the vibrations and so on. I share some friends that worked on this problem a long time ago, and these were actually the sorts of features that they were collecting off actual aircraft engines so you now have a data set of X1 through Xm, if you have manufactured m aircraft engines, and if you plot your data, maybe it looks like this. So, each point here, each cross here as one of your unlabeled examples.

So, the anomaly detection problem is the following.

Let's say that on, you know, the next day, you have a new aircraft engine that rolls off the assembly line and your new aircraft engine has some set of features x-test. What the anomaly detection problem is, we want to know if this aircraft engine is anomalous in any way, in other words, we want to know if, maybe, this engine should undergo further testing

(2)

because, or if it looks like an okay engine, and so it's okay to just ship it to a customer without further testing.

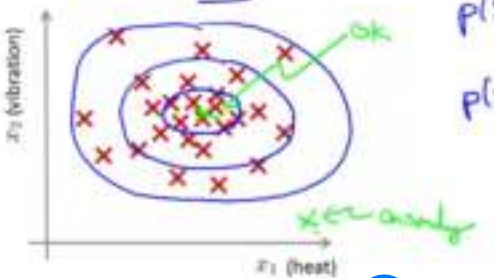
So, if your new aircraft engine looks like a point over there, well, you know, that looks a lot like the aircraft engines we've seen before, and so maybe we'll say that it looks okay. Whereas, if your new aircraft engine, if x-test, you know, were a point that were out here, so that if X1 and X2 are the features of this new example. If x-tests were all the way out there, then we would call that an anomaly.

and maybe send that aircraft engine for further testing before we ship it to a customer, since it looks very different than the rest of the aircraft engines we've seen before. More formally in the

Density estimation

- Dataset: $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$
- Is x_{test} anomalous?

Model $p(x)$



$p(x_{test}) < \varepsilon \rightarrow$ flag anomaly

$p(x_{test}) \geq \varepsilon \rightarrow$ OK

(1)

looks very different than the rest of the aircraft engines we've seen before. More formally in the anomaly detection problem, we're give some data sets, x1 through Xm of examples, and we usually assume that these end examples are normal or non-anomalous examples, and we want an algorithm to tell us if some new example x-test is anomalous. The approach that we're going to take is that given this training set, given the unlabeled training set, we're going to build a model for p of x. In other words, we're going to build a model for the probability of x, where x are these features of, say, aircraft engines.

And so, having built a model of the probability of x we're then going to say that for the new aircraft engine, if p of x-test is less than some epsilon then we flag this as an anomaly.

So we see a new engine that, you know, has very low probability under a model p of x that we estimate from the data, then we flag this anomaly, whereas if p of x-test is, say, greater than or equal to some small threshold.

(2)

Then we say that, you know, okay, it looks okay.

And so, given the training set, like that plotted here, if you build a model, hopefully you will find that aircraft engines, or hopefully the model p of x will say that points that lie, you know, somewhere in the middle, that's pretty high probability,

whereas points a little bit further out have lower probability.

Points that are even further out have somewhat lower probability, and the point that's way out here, the point that's way out there, would be an anomaly.

Whereas the point that's way in there, right in the middle, this would be okay because p of x right in the middle of that would be very high cause we've seen a lot of points in that region.

Anomaly detection example

- Fraud detection:
  - $x^{(i)}$ = features of user i's activities
  - Model $p(x)$ from data.
  - Identify unusual users by checking which have $p(x) < \epsilon$
- Manufacturing
- Monitoring computers in a data center.
  - $x^{(i)}$ = features of machine i

  x_1 = memory use, x_2 = number of disk accesses/sec,

  x_3 = CPU load, x_4 = CPU load/network traffic.

  ...

Here are some examples of applications of anomaly detection. Perhaps the most common application of anomaly detection is actually for detection if you have many users, and if each of your users take different activities, you know maybe on your website or in the physical plant or something, you can compute features of the different users activities.

And what you can do is build a model to say, you know, what is the probability of different users behaving different ways. What is the probability of a particular vector of features of a users behavior so you know examples of features of a users activity may be on the website it'd be things like,

maybe x1 is how often does this user log in, x2, you know, maybe the number of what pages visited, or the number of transactions, maybe x3 is, you know, the number of posts of the users on the forum, feature x4 could be what is the typing speed of the user and some websites can actually track that was the typing speed of this user in characters per second. And so you can model p of x based on this sort of data.

And finally having your model p of x, you can try to identify users that are behaving very strangely on your website by checking which ones have probably effects less than epsilon and maybe send the profiles of those users for further review.

Or demand additional identification from those users, or some such to guard against you know, strange behavior or fraudulent behavior on your website.

This sort of technique will tend of flag the users that are behaving unusually, not just

users that maybe behaving fraudulently. So not just constantly having stolen or users that are trying to do funny things, or just find unusual users. But this is actually the technique that is used by many online

websites that sell things to try identify users behaving strangely that might be indicative of either fraudulent behavior or of computer accounts that have been stolen.

Another example of anomaly detection is manufacturing. So, already talked about the aircraft engine thing where you can find unusual, say, aircraft engines and send those for further review.

A third application would be monitoring computers in a data center. I actually have some friends who work on this too. So if you have a lot of machines in a computer cluster or in a data center, we can do things like compute features at each machine. So maybe some features capturing you know, how much memory used, number of disc accesses, CPU load. As well as more complex features like what is the CPU load on this machine divided by the amount of network traffic on this machine? Then given the dataset of how your computers in your data center usually behave, you can model the probability of x, so you can model the probability of these machines having

different amounts of memory use or probability of these machines having different numbers of disc accesses or different CPU loads and so on. And if you ever have a machine whose probability of x, p of x, is very small then you know that machine is behaving unusually

and maybe that machine is about to go down, and you can flag that for review by a system administrator.

And this is actually being used today by various data centers to watch out for unusual things happening on their machines.

So, that's anomaly detection.

In the next video, I'll talk a bit about the Gaussian distribution and review properties of the Gaussian probability distribution, and in videos after that, we will apply it to develop an anomaly detection algorithm.

Your anomaly detection system flags x as anomalous whenever $p(x) \leq \epsilon$. Suppose your system is flagging too many things as anomalous that are not actually so (similar to supervised learning, these mistakes are called false positives). What should you do?

- Try increasing ϵ.
- Try decreasing ϵ.

✓ Correct

Density Estimation

Gaussian Distribution

In this video, I'd like to talk about the Gaussian distribution which is also called the normal distribution. In case you're already intimately familiar with the Gaussian distribution, it's probably okay to skip this video, but if you're not sure or if it has been a while since you've worked with the Gaussian distribution or normal distribution then please do watch this video all the way to the end. And in the video after this we'll start applying the Gaussian distribution to developing an anomaly detection algorithm.

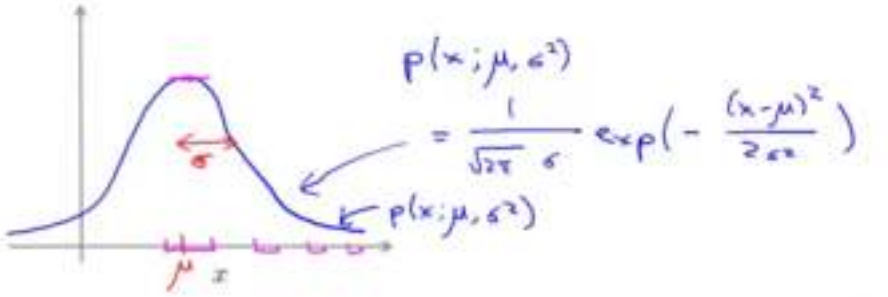
Gaussian (Normal) distribution

Say $x \in \mathbb{R}$. If x is a distributed Gaussian with mean μ, variance σ^2.

$x \sim \mathcal{N}(\mu, \sigma^2)$

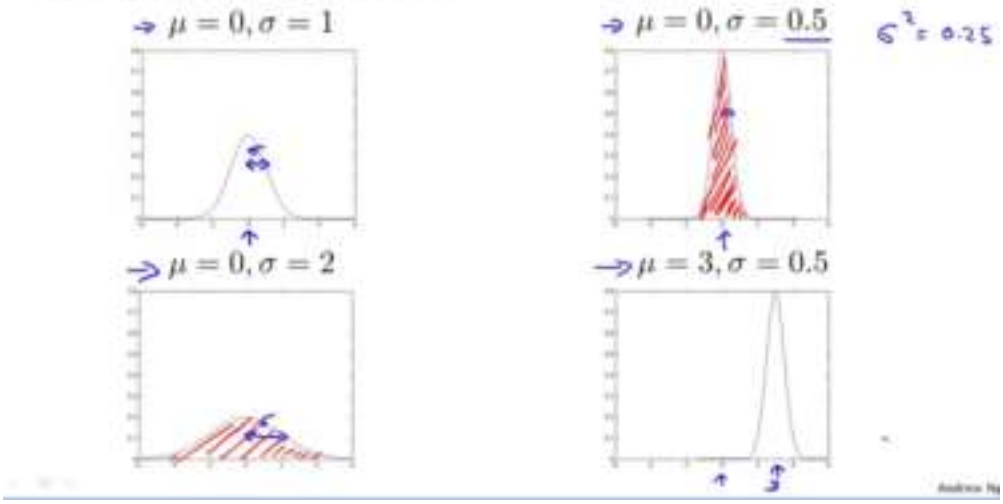
~ "distributed as"

σ standard deviation



Let's say x is a row value's random variable, so x is a row number. If the probability distribution of x is Gaussian with mean mu and variance sigma squared. Then, we'll write this as x, the random variable. Tilde, this little tilde, this is distributed as.

And then to denote a Gaussian distribution, sometimes I'm going to write script N parentheses mu comma sigma script. So this script N stands for normal since Gaussian and normal they mean the thing are synonyms. And the Gaussian distribution is parametarized by two parameters, by a mean parameter which we denote mu and a variance parameter which we denote via sigma squared. If we plot the Gaussian distribution or Gaussian probability density. It'll look like the bell shaped curve which you may have seen before. And so this bell shaped curve is paramafied by those two parameters, mu and sequel. And the location of the center of this bell shaped curve is the mean mu. And the width of this bell shaped curve, roughly that, is this parameter, sigma, is also called one standard deviation, and so this specifies the probability of x taking on different values. So, x taking on values here in the middle here it's pretty high, since the Gaussian density here is pretty high, whereas x taking on values further, and further away will be diminishing in probability. Finally just for completeness let me write out the formula for the Gaussian distribution. So the probability of x, and I'll sometimes write this as the p (x) when we write this as P (x ; mu, sigma squared), and so this denotes that the probability of X is parameterized by the two parameters mu and sigma squared. And the formula for the Gaussian density is this 1/ root 2 pi, sigma e (-(x-mu/g) squared/2 sigma squared. So there's no need to memorize this formula. This is just the formula for the bell-shaped curve over here on the left. There's no need to memorize it, and if you ever need to use this, you can always look this up. And so that figure on the left, that is what you get if you take a fixed value of mu and take a fixed value of sigma, and you plot P(x) so this curve here. This is really p(x) plotted as a function of X for a fixed value of Mu and of sigma squared. And by the way sometimes it's easier to think in terms of sigma squared that's called the variance. And sometimes is easier to think in terms of sigma. So sigma is called the standard deviation, and so it specifies the width of this Gaussian probability density, where as the square sigma, or sigma squared, is called the variance.



Let's look at some examples of what the Gaussian distribution looks like. If mu equals zero, sigma equals one. Then we have a Gaussian distribution that's centered around zero, because that's mu and the width of this Gaussian, so that's one standard deviation is sigma over there.

Let's look at some examples of Gaussians. If mu is equal to zero and sigma equals one, then that corresponds to a Gaussian distribution that is centered at zero, since mu is zero, and the width of this Gaussian

is is controlled by sigma by that variance parameter sigma.

Here's another example.

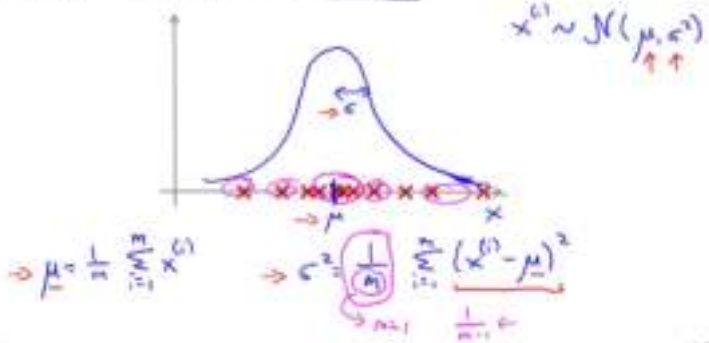
That same mu is equal to 0 and sigma is equal to .5 so the standard deviation is .5 and the variance sigma squared would therefore be the square of 0.5 would be 0.25 and in that case the Gaussian distribution, the Gaussian probability density goes like this.

Is also sent as zero. But now the width of this is much smaller because the smaller the area is, the width of this Gaussian density is roughly half as wide. But because this is a probability distribution, the area under the curve, that's the shaded area there. That area must integrate to one this is a property of probability distributing. So this is a much taller Gaussian density because this half is Y but half the standard deviation but it twice as tall. Another example is sigma is equal to 2 then you get a much fatter a much wider Gaussian density and so here the sigma parameter controls that Gaussian distribution has a wider width. And once again, the area under the curve, that is the shaded area, will always integrate to one, that's the property of probability distributions and because it's wider it's also half as tall in order to still integrate to the same thing.

And finally one last example would be if we now change the mu parameters as well. Then instead of being centered at 0 we now have a Gaussian distribution that's centered at 3 because this shifts over the entire Gaussian distribution.

Parameter estimation

⇒ Dataset: $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$ $x^{(i)} \in \mathbb{R}$



(7)

Next, let's talk about the Parameter estimation problem. So what's the parameter estimation problem? Let's say we have a dataset of m examples so exponents x m and lets say each of this example is a row number. Here in the figure I've plotted an example of the dataset so the horizontal axis is the x axis and either will have a range of examples of x, and I've just plotted them on this figure here. And the parameter estimation problem is, let's say I suspect that these examples came from a Gaussian distribution. So let's say I suspect that each of my examples, x i, was distributed. That's what this tilde thing means. Let's not suspect that each of these examples were distributed according to a normal distribution, or Gaussian distribution, with some parameter mu and some parameter sigma square. But I don't know what the values of these parameters are. The problem of parameter estimation is, given my data set, I want to try to figure out, well I want to estimate what are the values of mu and sigma squared. So if you're given a data set like this, it looks like maybe if I estimate what Gaussian distribution the data came from, maybe that might be roughly the Gaussian distribution it came from. With mu being the center of the distribution, sigma standing for the deviation controlling the width of this Gaussian distribution. Seems like a reasonable fit to the data. Because, you know, looks like the data has a very high probability of being in the central region, and a low probability of being further out, even though probability of being further out, and so on. So maybe this is a reasonable estimate of mu and sigma squared. That is, if it corresponds to a Gaussian distribution function that looks like this.

So what I'm going to do is just write out the formula the standard formulas for estimating the parameters Mu and sigma squared. Our estimate or the way we're going to estimate mu is going to be just the average of my example. So mu is the mean parameter. Just take my training set, take my m examples and average them. And that just means the center of this distribution.

How about sigma squared? Well, the variance, I'll just write out the standard formula again, I'm going to estimate as sum over one through m of x i minus mu squared. And so this mu here is actually the mu that I compute over here using this formula. And what the variance is, or one interpretation of the variance is that if you look at this term, that's the square difference between the value I got in my example minus the mean. Minus the center, minus the mean of the distribution. And so in the variance I'm gonna estimate as just the average of the square differences between my examples, minus the mean.

(2)

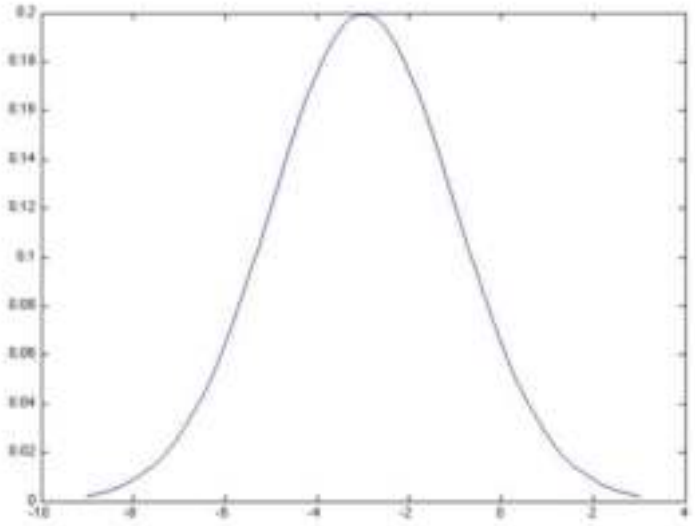
And as a side comment, only for those of you that are experts in statistics. If you're an expert in statistics, and if you've heard of maximum likelihood estimation, then these parameters, these estimates, are actually the maximum likelihood estimates of the primes of mu and sigma squared but if you haven't heard of that before don't worry about it, all you need to know is that these are the two standard formulas for how to figure out what are mu and Sigma squared given the data set. Finally one last side comment again only for those of you that have maybe taken the statistics class before but if you've taken statistics This class before. Some of you may have seen the formula here where this is M-1 instead of M so this first term becomes 1/M-1 instead of 1/M. In machine learning people tend to learn 1/M formula but in practice whether it is 1/M or 1/M-1 it makes essentially no difference assuming M is reasonably large. a reasonably large training set size. So just in case you've seen this other version before. In either version it works just about equally well but in machine learning most people tend to use 1/M in this formula.And the two versions have slightly different theoretical properties like these are different math properties. Bit of practice it really makes makes very little difference, if any.

So, hopefully you now have a good sense of what the Gaussian distribution looks like, as well as how to estimate the parameters mu and sigma squared of Gaussian distribution if you're given a training set, that is if you're given a set of data that you suspect comes from a Gaussian distribution with unknown parameters, mu and sigma squared. In the next video, we'll start to take this and apply it to develop an anomaly detection algorithm.

The formula for the Gaussian density is:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Which of the following is the formula for the density to the right?

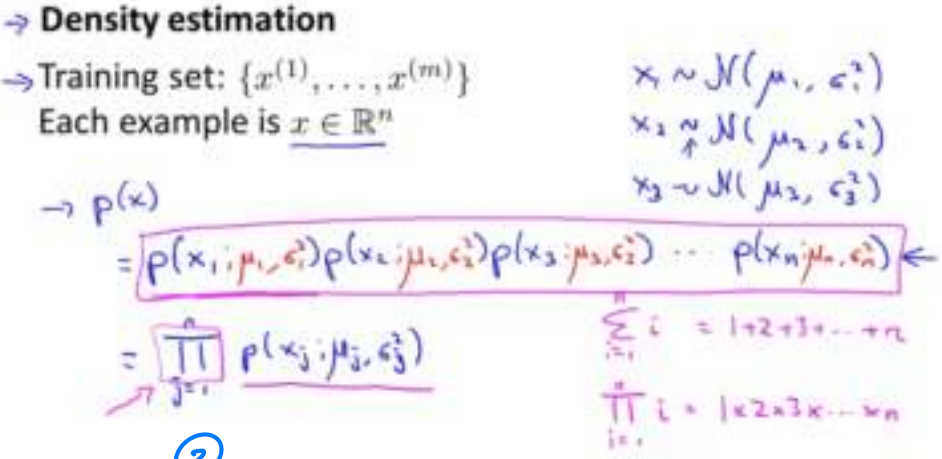


- ○ $p(x) = \frac{1}{\sqrt{2\pi \times 2}} \exp\left(-\frac{(x-3)^2}{2 \times 4}\right)$
- ○ $p(x) = \frac{1}{\sqrt{2\pi \times 4}} \exp\left(-\frac{(x-3)^2}{2 \times 2}\right)$
- ◉ $p(x) = \frac{1}{\sqrt{2\pi \times 2}} \exp\left(-\frac{(x+3)^2}{2 \times 4}\right)$
- ○ $p(x) = \frac{1}{\sqrt{2\pi \times 4}} \exp\left(-\frac{(x+3)^2}{2 \times 2}\right)$

✓ **Correct**

Density Estimation

Algorithm



(1)

In the last video, we talked about the Gaussian distribution. In this video lets apply that to develop an anomaly detection algorithm.

Let's say that we have an unlabeled training set of M examples, and each of these examples is going to be a feature in Rn so your training set could be,

feature vectors from the last M aircraft engines being manufactured. Or it could be features from m users or something else.

The way we are going to address anomaly detection, is we are going to model p of x from the data sets. We're going to try to figure out what are high probability features, what are lower probability types of features. So, x is a vector and what we are going to do is model p of x, as probability of x1, that is of the first component of x, times the probability of x2, that is the probability of the second feature, times the probability of the third feature, and so on up to the probability of the final feature of Xn. Now I'm leaving space here cause I'll fill in something in a minute.

So, how do we model each of these terms, p of X1, p of X2, and so on.

What we're going to do, is assume that the feature, X1, is distributed according to a Gaussian distribution, with some mean, which you want to write as mu1 and some variance, which I'm going to write as sigma squared 1,

(2)

and so p of X1 is going to be a Gaussian probability distribution, with mean mu1 and variance sigma squared 1. And similarly I'm going to assume that X2 is distributed, Gaussian, that's what this little tilda stands for, that means distributed Gaussian with mean mu2 and Sigma squared 2, so it's distributed according to a different Gaussian, which has a different set of parameters, mu2 sigma square 2. And similarly, you know, X3 is yet another Gaussian, so this can have a different mean and a different standard deviation than the other features, and so on, up to XN.

And so that's my model.

Just as a side comment for those of you that are experts in statistics, it turns out that this equation that I just wrote out actually corresponds to an independence assumption on the values of the features x1 through xn. But in practice it turns out that the algorithm of this fragment, it works just fine, whether or not these features are anywhere close to independent and even if independence assumption doesn't hold true this algorithm works just fine. But in case you don't know those terms I just used independence assumptions and so on, don't worry about it. You'll be able to understand it and implement this algorithm just fine and that comment was really meant only for the experts in statistics.

Finally, in order to wrap this up, let me take this expression and write it a little bit more compactly. So, we're going to write this is a product from J equals one through N, of P of XJ parameterized by mu j comma sigma squared

(3)

j. So this funny symbol here, there is capital Greek alphabet pi, that funny symbol there corresponds to taking the product of a set of values. And so, you're familiar with the summation notation, so the sum from i equals one through n, of i. This means 1 + 2 + 3 plus dot dot dot, up to n. Where as this funny symbol here, this product symbol, right product from i equals 1 through n of i. Then this means that, it's just like summation except that we're now multiplying. This becomes 1 times 2 times 3 times up

to N. And so using this product notation, this product from j equals 1 through n of this expression. It's just more compact, it's just shorter way for writing out this product of of all of these terms up there. Since we're are taking these p of x j given mu j comma sigma squared j terms and multiplying them together.

And, by the way the problem of estimating this distribution p of x, they're sometimes called

the problem of density estimation. Hence the title of the slide.

Anomaly detection algorithm

1. Choose features x_j that you think might be indicative of anomalous examples.
2. Fit parameters $\mu_1, \ldots, \mu_n, \sigma_1^2, \ldots, \sigma_n^2$

$$\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}$$

$$\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m} (x_j^{(i)} - \mu_j)^2$$

3. Given new example x, compute $p(x)$:

$$p(x) = \prod_{j=1}^{n} p(x_j; \mu_j, \sigma_j^2) = \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left(-\frac{(x_j - \mu_j)^2}{2\sigma_j^2}\right)$$

Anomaly if $p(x) < \varepsilon$

So putting everything together, here is our anomaly detection algorithm.

The first step is to choose features, or come up with features xi that we think might be indicative of anomalous examples. So what I mean by that, is, try to come up with features, so that when there's an unusual user in your system that may be doing fraudulent things, or when the aircraft engine examples, you know there's something funny, something strange about one of the aircraft engines. Choose features X I, that you think might take on unusually

large values, or unusually small values, for what an anomalous example might look like. But more generally, just try to choose features that describe general

properties of the things that you're collecting data on. Next, given a training set, of M, unlabled examples,

X1 through X M, we then fit the parameters, mu 1 through mu n, and sigma squared 1 through sigma squared n, and so these were the formulas similar to the formulas we have in the previous video, that we're going to use the estimate each of these parameters, and just to give some interpretation, mu J, that's my average value of the j feature. Mu j goes in this term p of xj. which is parametrized by mu J and sigma squared J. And so this says for the mu J just take the mean

over my training set of the values of the j feature. And, just to mention, that you do this, you compute these formulas for j equals one through n. So use these formulas to estimate mu 1, to estimate mu 2, and so on up to mu n, and similarly for sigma squared, and it's also possible to come up with vectorized versions of these. So if you think of mu as a vector, so mu if is a vector there's mu 1, mu 2, down to mu n, then a vectorized version of that set of parameters can be written like so sum from 1 equals one through n xi. So, this formula that I just wrote out estimates this xi as the feature vectors that estimates mu for all the values of n simultaneously.

And it's also possible to come up with a vectorized formula for estimating sigma squared j. Finally,

when you're given a new example, so when you have a new aircraft engine and you want to know is this aircraft engine anomalous.

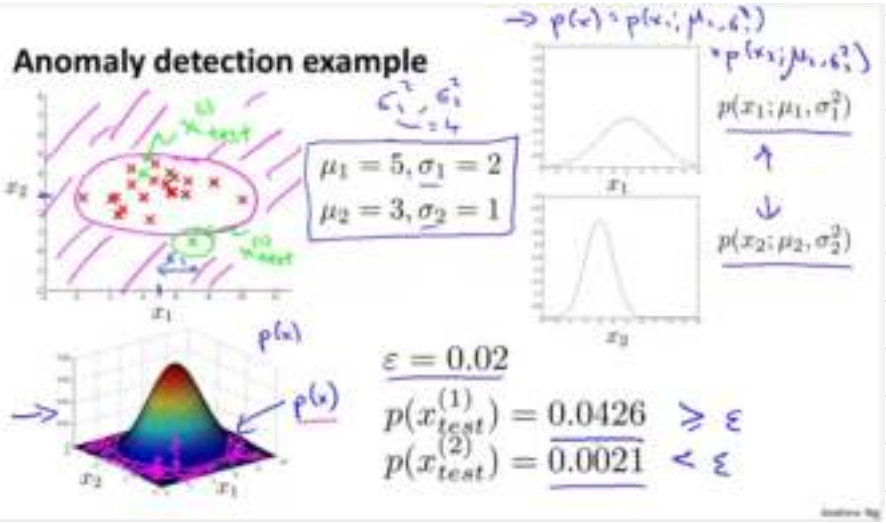
What we need to do is then compute p of x, what's the probability of this new example?

So, p of x is equal to this product, and what you implement, what you compute, is this formula and where over here, this thing here this is just the formula for the Gaussian probability, so you compute this thing, and finally if this probability is very small, then you flag this thing as an anomaly.

Given a training set $\{x^{(1)}, \ldots, x^{(m)}\}$, how would you estimate each μ_j and σ_j^2 (Note $\mu_j \in \mathbb{R}, \sigma_j^2 \in \mathbb{R}$.)

- $\mu_j = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$, $\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m} (x^{(i)} - \mu_j)^2$
- $\mu_j = \frac{1}{m}\sum_{i=1}^{m} (x_j^{(i)})^2$, $\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m} (x_j^{(i)} - \mu_j)^2$
- $\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}$, $\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m} (x^{(i)} - \mu_j)^2$
- (selected) $\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}$, $\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m} (x_j^{(i)} - \mu_j)^2$

✓ Correct



1

Here's an example of an application of this method.

Let's say we have this data set plotted on the upper left of this slide.

If you look at this, well, lets look the feature of x1. If you look at this data set, it looks like on average, the features x1 has a mean of about 5

and the standard deviation, if you only look at just the x1 values of this data set has the standard deviation of maybe 2. So that sigma 1 and looks like x2 the values of the features as measured on the vertical axis, looks like it has an average value of about 3, and a standard deviation of about 1. So if you take this data set and if you estimate mu1, mu2, sigma1, sigma2, this is what you get. And again, I'm writing sigma here, I'm think about standard deviations, but the formula on the previous 5 actually gave the estimates of the squares of theses things, so sigma squared 1 and sigma squared 2. So, just be careful whether you are using sigma 1, sigma 2, or sigma squared 1 or sigma squared 2. So, sigma squared 1 of course would be equal to 4, for

example, as the square of 2. And in pictures what p of x1 parametrized by mu1 and sigma squared 1 and p of x2, parametrized by mu 2 and sigma squared 2, that would look like these two distributions over here.

And, turns out that if were to plot of p of x, right, which is the product of these two things, you can actually get a surface plot that looks like this. This is a plot of p of x, where the height above of this, where the height of this surface at a particular point, so given a particular x1 x2 values of x2 if x1 equals 2, x equal 2, that's this point. And the height of this 3-D surface here, that's p

2

of x. So p of x, that is the height of this plot, is literally just p of x1

parametrized by mu 1 sigma squared 1, times p of x2 parametrized by mu 2 sigma squared 2. Now, so this is how we fit the parameters to this data. Let's see if we have a couple of new examples. Maybe I have a new example there.

Is this an anomaly or not? Or, maybe I have a different example, maybe I have a different second example over there. So, is that an anomaly or not? They way we do that is, we would set some value for Epsilon, let's say I've chosen Epsilon equals 0.02. I'll say later how we choose Epsilon.

But let's take this first example, let me call this example X1 test. And let me call the second example

X2 test. What we do is, we then compute p of X1 test, so we use this formula to compute it and this looks like a pretty large value. In particular, this is greater than, or greater than or equal to epsilon. And so this is a pretty high probability at least bigger than epsilon, so we'll say that X1 test is not an anomaly.

Whereas, if you compute p of X2 test, well that is just a much smaller value. So this is less than epsilon and so we'll say that that is indeed an anomaly, because it is much smaller than that epsilon that we then chose.

And in fact, I'd improve it here. What this is really saying is that, you look through the 3d surface plot

3

It's saying that all the values of x1 and x2 that have a high height above the surface, corresponds to an a non-anomalous example of an OK or normal example. Whereas all the points far out here, all the points out here, all of those points have very low probability, so we are going to flag those points as anomalous, and so it's gonna define some region, that maybe looks like this, so that everything outside this, it flags as anomalous,

whereas the things inside this ellipse I just drew, if it considers okay, or non-anomalous, not anomalous examples. And so this example x2 test lies outside that region, and so it has very small probability, and so we consider it an anomalous example.

In this video we talked about how to estimate p of x, the probability of x, for the purpose of developing an anomaly detection algorithm.

And in this video, we also stepped through an entire process of giving data set, we have, fitting the parameters, doing parameter estimations. We get mu and sigma parameters, and then taking new examples and deciding if the new examples are anomalous or not.

In the next few videos we will delve deeper into this algorithm, and talk a bit more about how to actually get this to work well.

Building an Anomaly Detection System

Developing and evaluating an Anomaly Detection System

The importance of real-number evaluation

When developing a learning algorithm (choosing features, etc.), making decisions is much easier if we have a way of evaluating our learning algorithm.

→ Assume we have some labeled data, of anomalous and non-anomalous examples. ($y = 0$ if normal, $y = 1$ if anomalous).

→ Training set: $x^{(1)}, x^{(2)}, \ldots, x^{(m)}$ (assume normal examples/not anomalous)

→ Cross validation set: $(x_{cv}^{(1)}, y_{cv}^{(1)}), \ldots, (x_{cv}^{(m_{cv})}, y_{cv}^{(m_{cv})})$

→ Test set: $(x_{test}^{(1)}, y_{test}^{(1)}), \ldots, (x_{test}^{(m_{test})}, y_{test}^{(m_{test})})$

$y = 1$

(1)

In the last video, we developed an anomaly detection algorithm. In this video, I like to talk about the process of how to go about developing a specific application of anomaly detection to a problem and in particular this will focus on the problem of how to evaluate an anomaly detection algorithm. In previous videos, we've already talked about the importance of real number evaluation and this captures the idea that when you're trying to develop a learning algorithm for a specific application, you need to often make a lot of choices like, you know, choosing what features to use and then so on. And making decisions about all of these choices is often much easier, and if you have a way to evaluate your learning algorithm that just gives you back a number.

So if you're trying to decide,

you know, I have an idea for one extra feature, do I include this feature or not. If you can run the algorithm with the feature, and run the algorithm without the feature, and just get back a number that tells you, you know, did it improve or worsen performance to add this feature? Then it gives you a much better way, a much simpler way, with which to decide whether or not to include that feature.

So in order to be able to develop an anomaly detection system quickly, it would be a really helpful to have a way of evaluating an anomaly detection system.

(2)

In order to do this, in order to evaluate an anomaly detection system, we're actually going to assume have some labeled data. So, so far, we'll be treating anomaly detection as an unsupervised learning problem, using unlabeled data. But if you have some labeled data that specifies what are some anomalous examples, and what are some non-anomalous examples, then this is how we actually think of as the standard way of evaluating an anomaly detection algorithm. So taking the aircraft engine example again. Let's say that, you know, we have some label data of just a few anomalous examples of some aircraft engines that were manufactured in the past that turns out to be anomalous. Turned out to be flawed or strange in some way. Let's say we use we also have some non-anomalous examples, so some perfectly okay examples. I'm going to use y equals 0 to denote the normal or the non-anomalous example and y equals 1 to denote the anomalous examples.

The process of developing and evaluating an anomaly detection algorithm is as follows.

We're going to think of it as a training set and talk about the cross validation in test sets later, but the training set we usually think of this as still the unlabeled

training set. And so this is our large collection of normal, non-anomalous or not anomalous examples.

And usually we think of this as being as non-anomalous, but it's actually okay even if a few anomalies slip into your unlabeled training set. And next we are going to define a cross validation set and a test set, with which to evaluate a particular anomaly detection algorithm. So, specifically, for both the cross validation test sets we're going to assume that, you know, we can include a few examples in the cross validation set and the test set that contain examples that are known to be anomalous. So the test sets say we have a few examples with y equals 1 that correspond to anomalous aircraft engines.

So here's a specific example.

Let's say that, altogether, this is the data that we have. We have manufactured 10,000 examples of engines that, as far as we know we're perfectly normal, perfectly good aircraft engines. And again, it turns out to be okay even if a few flawed engine slips into the set of 10,000 is actually okay, but we kind of assumed that the vast majority of these 10,000 examples are, you know, good and normal non-anomalous engines. And let's say that, you know, historically, however long we've been running on manufacturing plant, let's say that we end up getting features, getting 20 to 50 anomalous engines as well. And for a pretty typical application of anomaly detection, you know, the number non-anomalous

examples, that is with y equals 1, we may have anywhere from, you know, 20 to 50. It would be a pretty typical range of examples, number of examples that we have with y equals 1. And usually we will have a much larger number of good examples.

So, given this data set,

a fairly typical way to split it into the training set, cross validation set and test set would be as follows.

Let's take 10,000 good aircraft engines and put 6,000 of that into the unlabeled training set. So, I'm calling this an unlabeled training set but all of these examples are really ones that correspond to y equals 0, as far as we know. And so, we will use this to fit p of x, right. So, we will use these 6000 engines to fit p of x, which is that p of x one parametrized by Mu 1, sigma squared 1, up to p of Xn parametrized by Mu N sigma squared

n. And so it would be these 6,000 examples that we would use to estimate the parameters Mu 1, sigma squared 1, up to Mu N, sigma squared N. And so that's our training set of all, you know, good, or the vast majority of good examples.

Next we will take our good aircraft engines and put some number of them in a cross validation set plus some number of them in the test sets. So 6,000 plus 2,000 plus 2,000, that's how we split up our 10,000 good aircraft engines. And then we also have 20 flawed aircraft engines, and we'll take that and maybe split it up, you know, put ten of them in the cross validation set and put ten of them in the test sets. And in the next slide we will talk about how to actually use this to evaluate the anomaly detection algorithm.

So what I have just described here is a you know probably the recommend a good way of splitting the labeled and unlabeled example. The good and the flawed aircraft engines. Where we use like a 60, 20, 20% split for the good engines and we take the flawed engines, and we put them just in the cross validation set, and just in the test set, then we'll see in the next slide why that's the case.

Aircraft engines motivating example

10000 good (normal) engines

20 flawed engines (anomalous) 2-50 $y=1$

Training set: 6000 good engines ($y=0$) $p(x) = p(x_1; \mu_1, \sigma_1^2) \cdots p(x_n; \mu_n, \sigma_n^2)$

CV: 2000 good engines ($y=0$), 10 anomalous ($y=1$)

Test: 2000 good engines ($y=0$), 10 anomalous ($y=1$)

Alternative:

Training set: 6000 good engines

CV: 4000 good engines ($y=0$), 10 anomalous ($y=1$)

Test: 4000 good engines ($y=0$), 10 anomalous ($y=1$)

Just as an aside, if you look at how people apply anomaly detection algorithms, sometimes you see other peoples' split the data differently as well. So, another alternative, this is really not a recommended alternative, but some people want to take off your 10,000 good engines, maybe put 6000 of them in your training set and then put the same 4000 in the cross validation

set and the test set. And so, you know, we like to think of the cross validation set and the test set as being completely different data sets to each other.

But you know, in anomaly detection, you know, for sometimes you see people, sort of, use the same set of good engines in the cross validation sets, and the test sets, and sometimes you see people use exactly the same sets of anomalous

engines in the cross validation set and the test set. And so, all of these are considered, you know, less good practices and definitely less recommended.

Certainly using the same data in the cross validation set and the test set, that is not considered a good machine learning practice. But, sometimes you see people do this too.

Algorithm evaluation

Fit model $p(x)$ on training set $\{x^{(1)}, \ldots, x^{(m)}\}$

On a cross validation/test example x, predict

$$y = \begin{cases} 1 & \text{if } p(x) < \varepsilon \text{ (anomaly)} \\ 0 & \text{if } p(x) \geq \varepsilon \text{ (normal)} \end{cases}$$

Possible evaluation metrics:

- True positive, false positive, false negative, true negative
- Precision/Recall
- F_1-score

Can also use cross validation set to choose parameter ε

So, given the training cross validation and test sets, here's how you evaluate or here is how you develop and evaluate an algorithm.

First, we take the training sets and we fit the model p of x. So, we fit, you know, all these Gaussians to my m unlabeled examples of aircraft engines, and these, I am calling them unlabeled examples, but these are really examples that we're assuming our goods are the normal aircraft engines.

Then imagine that your anomaly detection algorithm is actually making prediction. So, on the cross validation of the test set, given that, say, test example X, think of the algorithm as predicting that y is equal to 1, p of x is less than epsilon, we must be taking zero, if p of x is greater than or equal to epsilon.

So, given x, it's trying to predict, what is the label, given y equals 1 corresponding to an anomaly or is it y equals 0 corresponding to a normal example? So given the training, cross validation, and

Algorithm evaluation

→ Fit model $p(x)$ on training set $\{x^{(1)}, \ldots, x^{(m)}\}$

→ On a cross validation/test example x, predict

$$y = \begin{cases} 1 & \text{if } p(x) < \varepsilon \text{ (anomaly)} \\ 0 & \text{if } p(x) \geq \varepsilon \text{ (normal)} \end{cases}$$

$(x^{(i)}_{test}, y^{(i)}_{test})$

$y = 0$

Possible evaluation metrics:

→ - True positive, false positive, false negative, true negative

→ - Precision/Recall

→ - F_1-score ←

CV

Test set

Can also use cross validation set to choose parameter ε ←

(1)

or is it y equals 0 corresponding to a normal example? So given the training, cross validation, and test sets. How do you develop an algorithm? And more specifically, how do you evaluate an anomaly detection algorithm? Well, to this whole, the first step is to take the unlabeled training set, and to fit the model p of x lead training data. So you take this, you know on I'm coming, unlabeled training set, but really, these are examples that we are assuming, vast majority of which are normal aircraft engines, not because they're not anomalies and it will fit the model p of x. It will fit all those parameters for all the Gaussians on this data.

Next on the cross validation of the test set, we're going to think of the anomaly detection algorithm as trying to predict the value of y. So in each of like say test examples. We have these X-I tests,

Y-I test, where y is going to be equal to 1 or 0 depending on whether this was an anomalous example.

So given input x in my test set, my anomaly detection algorithm think of it as predicting the y as 1 if p of x is less than epsilon. So predicting that it is an anomaly, it is probably is very low. And we think of the algorithm is predicting that y is equal to 0. If p of x is greater then or equals epsilon. So predicting those normal example if the p of x is reasonably large.

And so we can now think of the anomaly detection algorithm as making predictions for what are the values of these y labels in the test sets or on the cross validation set. And this puts us somewhat more similar to the supervised learning setting, right? Where we have label test set and our algorithm is making predictions on these labels and so we can evaluate if you know by seeing how often it gets these labels right.

Of course these labels are will be very skewed because y equals zero, that is normal examples, usually be much more common than y equals 1 than anomalous examples.

But, you know, this is much closer to the source of evaluation metrics we can use in supervised learning.

So what's a good evaluation metric to use. Well, because the data is very skewed, because y equals 0 is much more common, classification accuracy would not be a good the evaluation metrics. So, we talked about this in the earlier video.

So, if you have a very skewed data set, then predicting y equals 0 all the time, will have very high classification accuracy.

(2)

Instead, we should use evaluation metrics, like computing the fraction of true positives, false positives, false negatives, true negatives or compute the position of the v curve of this algorithm or do things like compute the f1 score, right, which is a single real number way of summarizing the position and the recall numbers. And so these would be ways to evaluate an anomaly detection algorithm on your cross validation set or on your test set.

Finally, earlier in the anomaly detection algorithm, we also had this parameter epsilon, right? So, epsilon is this threshold that we would use to decide when to flag something as an anomaly.

And so, if you have a cross validation set, another way to and to choose this parameter epsilon, would be to try a different, try many different values of epsilon, and then pick the value of epsilon that, let's say, maximizes f1 score, or that otherwise does well on your cross validation set.

And more generally, the way to reduce the training, testing, and cross validation sets, is that

when we are trying to make decisions, like what features to include, or trying to, you know, tune the parameter epsilon, we would then continually evaluate the algorithm on the cross validation sets and make all those decisions like what features did you use, you know, how to set epsilon, use that, evaluate the algorithm on the cross validation set, and then when we've picked the set of features, when we've found the value of epsilon that we're happy with, we can then take the final model and evaluate it, you know, do the final evaluation of the algorithm on the test sets.

So, in this video, we talked about the process of how to evaluate an anomaly detection algorithm, and again, having being able to evaluate an algorithm, you know, with a single real number evaluation, with a number like an F1 score that often allows you to much more efficient use of your time when you are trying to develop an anomaly detection system. And we try to make these sorts of decisions. I have to chose epsilon, what features to include, and so on. In this video, we started to use a bit of labeled data in order to evaluate the anomaly detection algorithm and this takes us a little bit closer to a supervised learning setting.

In the next video, I'm going to say a bit more about that. And in particular we'll talk about when should you be using an anomaly detection algorithm and when should we be thinking about using supervised learning instead, and what are the differences between these two formalisms.

Suppose you have fit a model p(x). When evaluating on the cross validation set or test set, your algorithm predicts:

$$y = \begin{cases} 1 & \text{if } p(x) \leq \varepsilon \\ 0 & \text{if } p(x) > \varepsilon \end{cases}$$

Is classification accuracy a good way to measure the algorithm's performance?

- [ ] Yes, because we have labels in the cross validation / test sets.
- [ ] No, because we do not have labels in the cross validation / test sets.
- [x] No, because of skewed classes (so an algorithm that always predicts y = 0 will have high accuracy).
- [ ] No for the cross validation set; yes for the test set.

✓ **Correct**

Building an Anomaly Detection System

Anomaly Detection Vs Supervised Learning

In the last video we talked about the process of evaluating an anomaly detection algorithm. And there we started to use some label data with examples that we knew were either anomalous or not anomalous with Y equals one, or Y equals 0. And so, the question then arises of, and if we have the label data, that we have some examples and know the anomalies, and some of them will not be anomalies. Why don't we just use a supervisor on half of them? So why don't we just use logistic regression, or a neuro network to try to learn directly from our labeled data to predict whether Y equals one or Y equals 0. In this video, I'll try to share with you some of the thinking and some guidelines for when you should probably use an anomaly detection algorithm, and whether it might be more fruitful instead of using a supervisor in the algorithm.

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This slide shows what are the settings under which you should maybe use anomaly detection versus when supervised learning might be more fruitful. If you have a problem with a very small number of positive examples, and remember the examples of y equals one are the anomaly examples. Then you might consider using an anomaly detection algorithm instead. So, having 0 to 20, it may be up to 50 positive examples, might be pretty typical. And usually we have such a small positive, set of positive examples, we're going to save the positive examples just for the cross validation set in the test set. And in contrast, in a typical normal anomaly detection setting, we will often have a relatively large number of negative examples of the normal examples of normal aircraft engines. And we can then use this very large number of negative examples With which to fit the model p(x). And so there's this idea that in many anomaly detection applications, you have very few positive examples and lots of negative examples. And when we're doing the process of estimating p(x), affecting all those Gaussian parameters, we need only negative examples to do that. So if you have a lot negative data, we can still fit p(x) pretty well. In contrast, for supervised learning, more typically we would have a reasonably large number of both positive and negative examples. And so this is one way to look at your problem and decide if you should use an anomaly detection algorithm or a supervised. Here's another way that people often think about anomaly detection. So for anomaly detection applications, often there are very different types of anomalies. So think about so many different ways for go wrong. There are so many things that could go wrong that could the aircraft engine. And so if that's the case, and if you have a pretty small set of positive examples, then it can be hard for an algorithm, difficult for an algorithm to learn from your small set of positive examples what the anomalies look like. And in particular, you know future anomalies may look nothing like the ones you've seen so far. So maybe in your set of positive examples, maybe you've seen 5 or 10 or 20 different ways that an aircraft engine could go wrong. But maybe tomorrow, you need to detect a totally new set, a totally new type of anomaly. A totally new way for an aircraft engine to be broken, that you've just never seen before. And if that's the case, it might be more promising to just model the negative examples with this sort of calcium model p of x instead of try to hard to model the positive examples. Because tomorrow's anomaly may be nothing like the ones you've seen so far.

In contrast, in some other problems, you have enough positive examples for an algorithm to get a sense of what the positive examples are like. In particular, if you think that future positive examples are likely to be similar to ones in the training set; then in that setting, it might be more reasonable to have a supervisor in the algorithm that looks at all of the positive examples, looks at all of the negative examples, and uses that to try to distinguish between positives and negatives.

Hopefully, this gives you a sense of if you have a specific problem, should you think about using an anomaly detection algorithm, or a supervised learning algorithm.

Anomaly detection	vs.	**Supervised learning**
Very small number of positive examples ($y = 1$). (0-20 is common).		Large number of positive and negative examples.
Large number of negative ($y = 0$) examples. $p(x)$		
Many different "types" of anomalies. Hard for any algorithm to learn from positive examples what the anomalies look like; future anomalies may look nothing like any of the anomalous examples we've seen so far.		Enough positive examples for algorithm to get a sense of what positive examples are like, future positive examples likely to be similar to ones in training set.

Spam

(2)

And a key difference really is that in anomaly detection, often we have such a small number of positive examples that it is not possible for a learning algorithm to learn that much from the positive examples. And so what we do instead is take a large set of negative examples and have it just learn a lot, learn p(x) from just the negative examples. Of the normal [INAUDIBLE] and we've reserved the small number of positive examples for evaluating our algorithms to use in the either the transvalidation set or the test set.

And just as a side comment about this many different types of easier. In some earlier videos we talked about the email spam examples. In those examples, there are actually many different types of spam email, right? There's spam email that's trying to sell you things. Spam email trying to steal your passwords, this is called phishing emails and many different types of spam emails. But for the spam problem we usually have enough examples of spam email to see most of these different types of spam email because we have a large set of examples of spam. And that's why we usually think of spam as a supervised learning setting even though there are many different types of

Anomaly detection	vs.	Supervised learning
• Fraud detection $y=1$		• Email spam classification
• Manufacturing (e.g. aircraft engines)		• Weather prediction (sunny/rainy/etc).
• Monitoring machines in a data center		• Cancer classification
⋮		⋮

If we look at some applications of anomaly detection versus supervised learning we'll find fraud detection. If you have many different types of ways for people to try to commit fraud and a relatively small number of fraudulent users on your website, then I use an anomaly detection algorithm. I should say, if you have, if you're a very major online retailer and if you actually have had a lot of people commit fraud on your website, so you actually have a lot of examples of y=1, then sometimes fraud detection could actually shift over to the supervised learning column. But, if you haven't seen that many examples of users doing strange things on your website, then more frequently fraud detection is actually treated as an anomaly detection algorithm rather than a supervised learning algorithm.

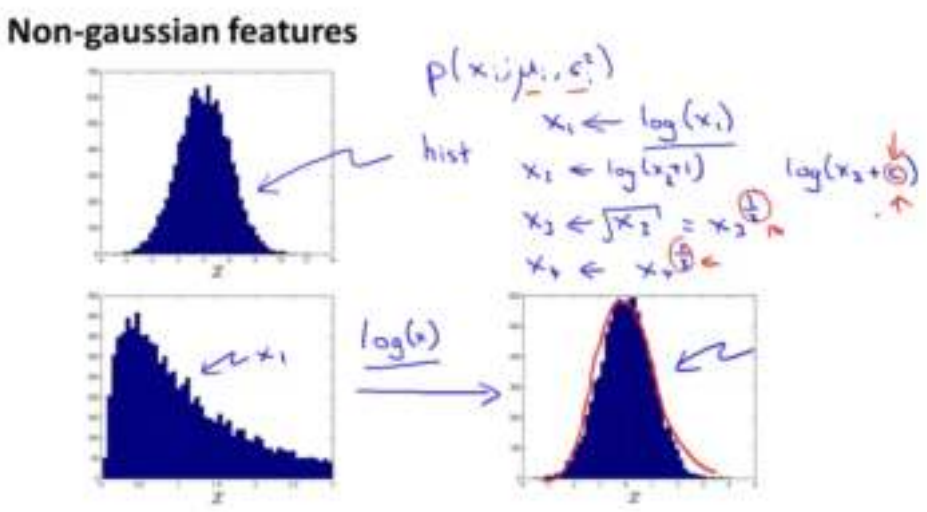
Other examples, we've talked about manufacturing already. Hopefully, you see more and more examples are not that many anomalies but if again for some manufacturing processes, if you manufacture in very large volumes and you see a lot of bad examples, maybe manufacturing can shift to the supervised learning column as well. But if you haven't seen that many bad examples of so to do the anomaly detection monitoring machines in a data center [INAUDIBLE] similar source of apply. Whereas, you must have classification, weather prediction, and classifying cancers. If you have equal numbers of positive and negative examples.

Your positive and your negative examples, then we would tend to treat all of these as supervisor problems. So hopefully, that gives you a sense of one of the properties of a learning problem that would cause you to treat it as an anomaly detection problem versus a supervisory problem. And for many other problems that are faced by various technology companies and so on, we actually are in the settings where we have very few or sometimes zero positive training examples. There's just so many different types of anomalies that we've never seen them before. And for those sorts of problems, very often the algorithm that is used is an anomaly detection algorithm.

Building an Anomaly Detection System

Choosing What Features to Use

By now you've seen the anomaly detection algorithm and we've also talked about how to evaluate an anomaly detection algorithm. It turns out, that when you're applying anomaly detection, one of the things that has a huge effect on how well it does, is what features you use, and what features you choose, to give the anomaly detection algorithm. So in this video, what I'd like to do is say a few words, give some suggestions and guidelines for how to go about designing or selecting features give to an anomaly detection algorithm.



In our anomaly detection algorithm, one of the things we did was model the features using this sort of Gaussian distribution. With xi to mu i, sigma squared i, lets say. And so one thing that I often do would be to plot the

data or the histogram of the data, to make sure that the data looks vaguely Gaussian before feeding it to my anomaly detection algorithm. And, it'll usually work okay, even if your data isn't Gaussian, but this is sort of a nice sanitary check to run. And by the way, in case your data looks non-Gaussian, the algorithms will often work just find. But, concretely if I plot the data like this, and if it looks like a histogram like this, and the way to plot a histogram is to use the HIST, or the HIST command in Octave, but it looks like this, this looks vaguely Gaussian, so if my features look like this, I would be pretty happy feeding into my algorithm. But if I were to plot a histogram of my data, and it were to look like this well, this doesn't look at all like a bell shaped curve, this is a very asymmetric distribution, it has a peak way off to one side.

If this is what my data looks like, what I'll often do is play with different transformations of the data in order to make it look more Gaussian. And again the algorithm will usually work okay, even if you don't. But if you use these transformations to make your data more gaussian, it might work a bit better.

So given the data set that looks like this, what I might do is take a log transformation of the data and if I do that and re-plot the histogram, what I end up with in this particular example, is a histogram that looks like this. And this looks much more Gaussian, right? This looks much more like the classic bell shaped curve, that we can fit with some mean and variance parameter sigma.

So what I mean by taking a log transform, is really that if I have some feature x1 and then the histogram of x1 looks like this then I might take my feature x1 and replace it with log of x1 and this is my new x1 that I'll plot to the histogram over on the right, and this looks much more Guassian.

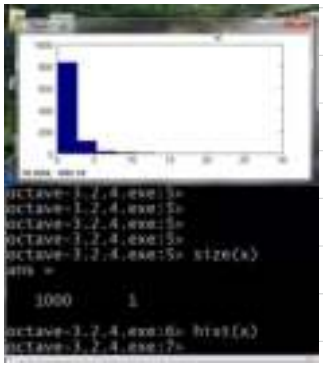
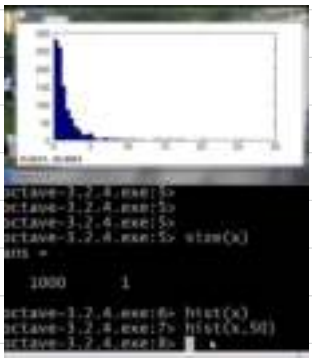
Rather than just a log transform some other things you can do, might be, let's say I have a different feature x2, maybe I'll replace that will log x plus 1, or more generally with log

x with x2 and some constant c and this constant could be something that I play with, to try to make it look as Gaussian as possible.

Or for a different feature x3, maybe I'll replace it with x3,

I might take the square root. The square root is just x3 to the power of one half, right?

And this one half is another example of a parameter I can play with. So, I might have x4 and maybe I might instead replace that with x4 to the power of something else, maybe to the power of 1/3. And these, all of these, this one, this exponent parameter, or the C parameter, all of these are examples of parameters that you can play with in order to make your data look a little bit more Gaussian.

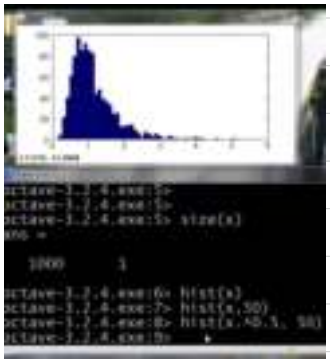
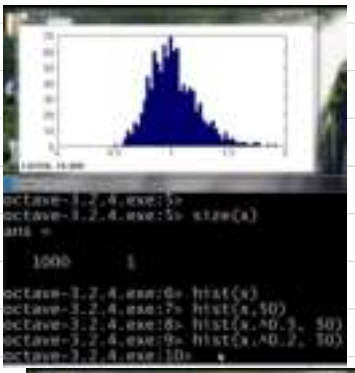



So, let me show you a live demo of how I actually go about playing with my data to make it look more Gaussian. So, I have already loaded in to octave here a set of features x I have a thousand examples loaded over there. So let's pull up the histogram of my data.

Use the hist x command. So there's my histogram.

By default, I think this uses 10 bins of histograms, but I want to see a more fine grid histogram. So we do hist to the x, 50, so, this plots it in 50 different bins. Okay, that looks better. Now, this doesn't look very Gaussian, does it? So, lets start playing around with the data. Lets try a hist of x to the 0.5. So we take the square root of the data, and plot that histogram.

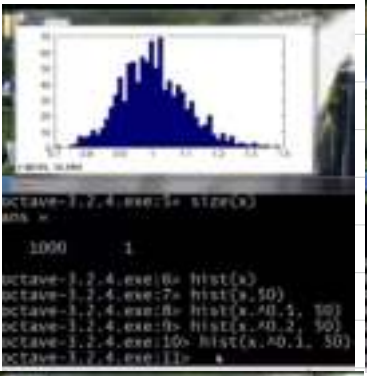
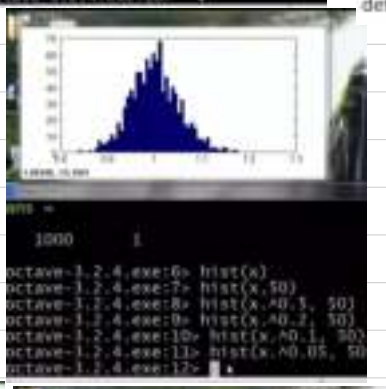
And, okay, it looks a little bit more Gaussian, but not quite there, so let's play at the 0.5 parameter. Let's see.

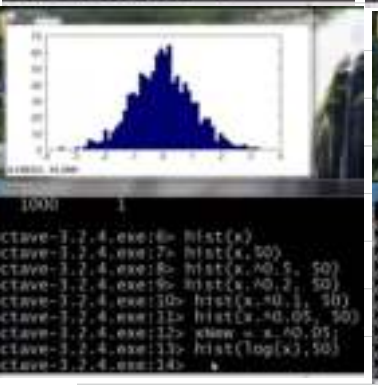
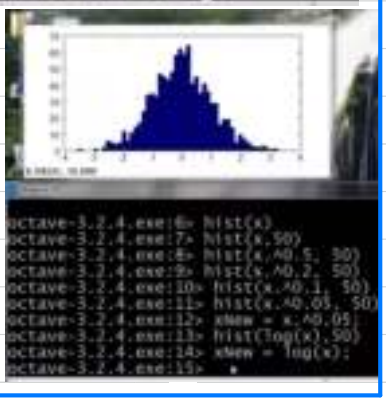



Set this to 0.2. Looks a little bit more Gaussian.

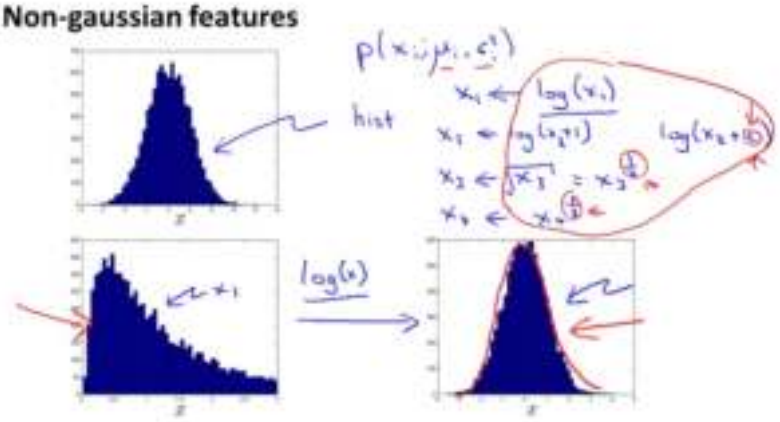
Let's reduce a little bit more 0.1.

Yeah, that looks pretty good. I could actually just use 0.1. Well, let's reduce it to 0.05. And, you know? Okay, this looks pretty Gaussian, so I can define a new feature which is x mu equals x to the 0.05, and now my new feature x Mu looks more Gaussian than my previous one and then I might instead use this new feature to feed into my anomaly detection algorithm. And of course, there is more than one way to do this. You could also have hist of log of x, that's another example of a transformation you can use. And, you know, that also look pretty Gaussian. So, I can also define x mu equals log of x. and that would be another pretty good choice of a feature to use.

Non-gaussian features



So to summarize, if you plot a histogram with the data, and find that it looks pretty non-Gaussian, it's worth playing around a little bit with different transformations like these, to see if you can make your data look a little bit more Gaussian, before you feed it to your learning algorithm, although even if you don't, it might work okay. But I usually do take this step. ~~Now, the second~~

(1)

although even if you don't, it might work okay, but I usually do take this step. Now, the second thing I want to talk about is, how do you come up with features for an anomaly detection algorithm.

And the way I often do so, is via an error analysis procedure.

So what I mean by that, is that this is really similar to the error analysis procedure that we have for supervised learning, where we would train a complete algorithm, and run the algorithm on a cross validation set, and look at the examples it gets wrong, and see if we can come up with extra features to help the algorithm do better on the examples that it got wrong in the cross-validation set.

So lets try to reason through an example of this process. In anomaly detection, we are hoping that p of x will be large for the normal examples and it will be small for the anomalous examples.

And so a pretty common problem would be if p of x is comparable, maybe both are large for both the normal and the anomalous examples.

Lets look at a specific example of that. Let's say that this is my unlabeled data. So, here I have just one feature, x1 and so I'm gonna fit a Gaussian to this.

And maybe my Gaussian that I fit to my data looks like that.

And now let's say I have an anomalous example, and let's say that my anomalous example takes on an x value of 2.5. So I plot my anomalous example there. And you know, it's kind of buried in the middle of a bunch of normal examples, and so,

just this anomalous example that I've drawn in green, it gets a pretty high probability, where it's the height of the blue curve, and the algorithm fails to flag this as an anomalous example.

Now, if this were maybe aircraft engine manufacturing or something, what I would do is, I would actually look at my training examples and look at what went wrong with that particular aircraft engine, and see, if looking at that example can inspire me to come up with a new feature x2, that helps to distinguish between this bad example, compared to the rest of my red examples, compared to all

of my normal aircraft engines.

And if I managed to do so, the hope would be then, that, if I can create a new feature, X2, so that when I re-plot my data, if I take all my normal examples of my training set, hopefully I find that all my training examples are these red crosses here. And hopefully, if I find that for my anomalous example, the feature x2 takes on the the unusual value. So for my green example here, this anomaly, right, my X1 value, is still 2.5. Then maybe my X2 value, hopefully it takes on a very large value like 3.5 over there,

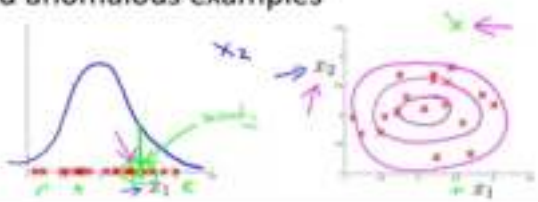
or a very small value.

Error analysis for anomaly detection

Want $p(x)$ large for normal examples x.
$p(x)$ small for anomalous examples x.

Most common problem:

$p(x)$ is comparable (say, both large) for normal and anomalous examples



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But now, if I model my data, I'll find that my anomaly detection algorithm gives high probability to data in the central regions, slightly lower probability to that, sightly lower probability to that. An example that's all the way out there, my algorithm will now give very low probability

to. And so, the process of this is, really look at the

mistakes that it is making. Look at the anomaly that the algorithm is failing to flag, and see if that inspires you to create some new feature. So find something unusual about that aircraft engine and use that to create a new feature, so that with this new feature it becomes easier to distinguish the anomalies from your good examples. And so that's the process of error analysis

and using that to create new features for anomaly detection. Finally, let me share with you my

Monitoring computers in a data center

Choose features that might take on unusually large or small values in the event of an anomaly.

- x_1 = memory use of computer
- x_2 = number of disk accesses/sec
- x_3 = CPU load
- x_4 = network traffic

$x_5 = \frac{\text{CPU load}}{\text{network traffic}}$ $\qquad$ $x_6 = \frac{(\text{CPU load})^2}{\text{network traffic}}$

(1)

and using that to create new features for anomaly detection. Finally, let me share with you my thinking on how I usually go about choosing features for anomaly detection.

So, usually, the way I think about choosing features is I want to choose features that will take on either very, very large values, or very, very small values, for examples that I think might turn out to be anomalies.

So let's use our example again of monitoring the computers in a data center. And so you have lots of machines, maybe thousands, or tens of thousands of machines in a data center. And we want to know if one of the machines, one of our computers is acting up, so doing something strange. So here are examples of features you may choose, maybe memory used, number of disc accesses, CPU load, network traffic.

But now, lets say that I suspect one of the failure cases, let's say that in my data set I think that CPU load the network traffic tend to grow linearly with each other. Maybe I'm running a bunch of web servers, and so, here if one of my servers is serving a lot of users, I have a very high CPU load, and have a very high network traffic.

But let's say, I think, let's say I have a suspicion, that one of the failure cases is if one of my computers has a job that gets stuck in some infinite loop. So if I think one of the failure cases, is one of my machines, one of my web servers--server code-- gets stuck in some infinite loop, and so the CPU load grows, but the network traffic doesn't because it's just spinning it's wheels and doing a lot of CPU work, you know, stuck in some infinite loop. In that case, to detect that type of anomaly, I might create a new feature, X5, which might be CPU load

(2)

divided by network traffic.

And so here X5 will take on a unusually large value if one of the machines has a very large CPU load but not that much network traffic and so this will be a feature that will help your anomaly detection capture, a certain type of anomaly. And you can also get creative and come up with other features as well. Like maybe I have a feature x6 thats CPU load squared divided by network traffic.

And this would be another variant of a feature like x5 to try to capture anomalies where one of your machines has a very high CPU load, that maybe doesn't have a commensurately large network traffic.

And by creating features like these, you can start to capture

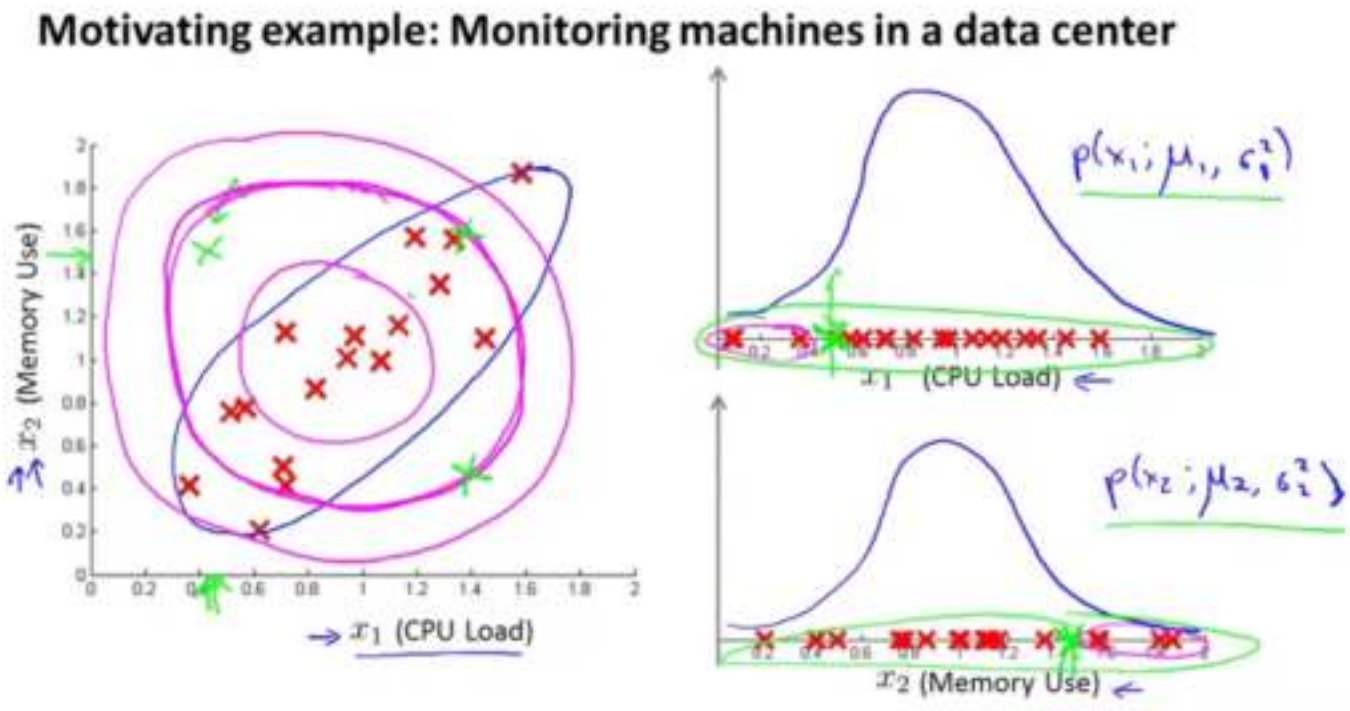
anomalies that correspond to

unusual combinations of values of the features.

So in this video we talked about how to and take a feature, and maybe transform it a little bit, so that it becomes a bit more Gaussian, before feeding into an anomaly detection algorithm. And also the error analysis in this process of creating features to try to capture different types of anomalies. And with these sorts of guidelines hopefully that will help you to choose good features, to give to your anomaly detection algorithm, to help it capture all sorts of anomalies.

Multivariate Gaussian Distribution (optional)

Multivariate Gaussian Distribution



(1)

In this and the next video, I'd like to tell you about one possible extension to the anomaly detection algorithm that we've developed so far. This extension uses something called the multivariate Gaussian distribution, and it has some advantages, and some disadvantages, and it can sometimes catch some anomalies that the earlier algorithm didn't.

To motivate this, let's start with an example.

Let's say that so our unlabeled data looks like what I have plotted here. And I'm going to use the example of monitoring machines in the data center, monitoring computers in the data center. So my two features are x1 which is the CPU load and x2 which is maybe the memory use.

So if I take my two features, x1 and x2, and I model them as Gaussians then here's a plot of my X1 features, here's a plot of my X2 features, and so if I fit a Gaussian to that, maybe I'll get a Gaussian like this, so here's P of X 1, which depends on the parameters mu 1, and sigma squared 1, and here's my memory used, and, you know, maybe I'll get a Gaussian that looks like this, and this is my P of X 2, which depends on mu 2 and sigma squared 2. And so this is how the anomaly detection algorithm models X1 and X2.

Now let's say that in the test sets I have an example that looks like this.

(2)

The location of that green cross, so the value of X 1 is about 0.4, and the value of X 2 is about 1.5. Now, if you look at the data, it looks like, yeah, most of the data data lies in this region, and so that green cross is pretty far away from any of the data I've seen. It looks like that should be raised as an anomaly. So, in my data, in my, in the data of my good examples, it looks like, you know, the CPU load, and the memory use, they sort of grow linearly with each other. So if I have a machine using lots of CPU, you know memory use will also be high, whereas this example, this green example it looks like here, the CPU load is very low, but the memory use is very high, and I just have not seen that before in my training set. It looks like that should be an anomaly.

But let's see what the anomaly detection algorithm will do. Well, for the CPU load, it puts it at around there 0.5 and this reasonably high probability is not that far from other examples we've seen, maybe, whereas, for the memory use, this appointment, 0.5, whereas for the memory use, it's about 1.5, which is there. Again, you know, it's all to us, it's not terribly Gaussian, but the value here and the value here is not that different from many other examples we've seen, and so P of X 1, will be pretty high,

(3)

reasonably high. P of X 2 reasonably high. I mean, if you look at this plot right, this point here, it doesn't look that bad, and if you look at this plot, you know across here, doesn't look that bad. I mean, I have had examples with even greater memory used, or with even less CPU use, and so this example doesn't look that anomalous.

And so, an anomaly detection algorithm will fail to flag this point as an anomaly. And it turns out what our anomaly detection algorithm is doing is that it is not realizing that this blue ellipse shows the high probability region, is that, one of the thing is that, examples here, a high probability, and the examples, the next circle

of from a lower probably, and examples here are even lower probability, and somehow, here are things that are, green cross there, it's pretty high probability,

and in particular, it tends to think that, you know, everything in this region, everything on the line that I'm circling over, has, you know, about equal probability, and it doesn't realize that something out here actually has much lower probability than something over there.

Multivariate Gaussian (Normal) distribution

→ $x \in \mathbb{R}^n$. Don't model $p(x_1), p(x_2), \ldots$, etc. separately. Model $p(x)$ all in one go.

Parameters: $\mu \in \mathbb{R}^n$, $\Sigma \in \mathbb{R}^{n \times n}$ (covariance matrix)

$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

$|\Sigma|$ = determinant of Σ | det (Sigma)

Parameters μ, Σ

$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

(1)

So, in order to fix this, we can, we're going to develop a modified version of the anomaly detection algorithm, using something called the multivariate Gaussian distribution also called the multivariate normal distribution.

So here's what we're going to do. We have features x which are in Rn and instead of P of X 1, P of X 2, separately, we're going to model P of X, all in one go, so model P of X, you know, all at the same time.

So the parameters of the multivariate Gaussian distribution are mu, which is a vector, and sigma, which is an n by n matrix, called a covariance matrix,

and this is similar to the covariance matrix that we saw when we were working with the PCA, with the principal components analysis algorithm.

For the second complete is, let me just write out the formula

(2)

for the multivariate Gaussian distribution. So we say that probability of X, and this is parameterized by my parameters mu and sigma that the probability of x is equal to once again there's absolutely no need to memorize this formula.

You know, you can look it up whenever you need to use it, but this is what the probability of X looks like.

Transverse, 2nd inverse, X minus mu.

And this thing here,

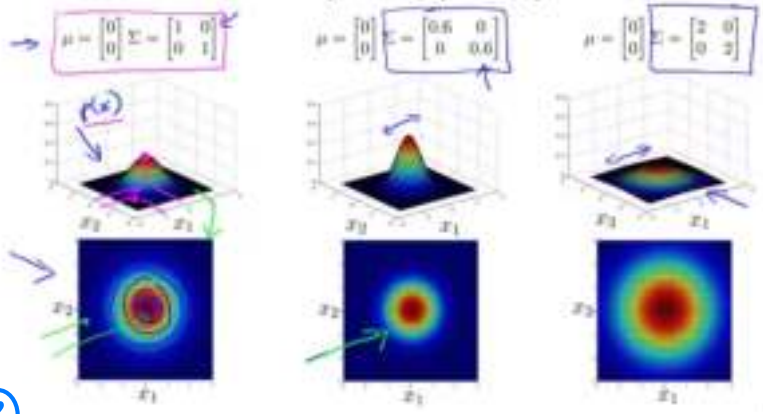
the absolute value of sigma, this thing here when you write this symbol, this is called the determinant of sigma and this is a mathematical function of a matrix and you really don't need to know what the determinant of a matrix is, but really all you need to know is that you can compute it in octave by using the octave command DET of

sigma. Okay, and again, just be clear, alright? In this expression, these sigmas here, these are just n by n matrix. This is not a summation and you know, the sigma there is an n by n matrix.

So that's the formula for P of X, but it's more interestingly, or more importantly,

So that's the formula for P of X, but it's more interestingly, or more importantly,what does P of X actually looks like?

Multivariate Gaussian (Normal) examples



So that's the formula for P of X, but it's more interestingly, or more importantly,

what does P of X actually looks like? Lets look at some examples of multivariate Gaussian distributions.

So let's take a two dimensional example, say if I have N equals 2, I have two features, X 1 and X 2.

Lets say I set MU to be equal to 0 and sigma to be equal to this matrix here. With 1s on the diagonals and 0s on the off-diagonals, this matrix is sometimes also called the identity matrix.

In that case, p of x will look like this, and what I'm showing in this figure is, you know, for a specific value of X1 and for a specific value of X2, the height of this surface the value of p of x. And so with this setting the parameters

p of x is highest when X1 and X2 equal zero 0, so that's the peak of this Gaussian distribution,

and the probability falls off with this sort of two dimensional Gaussian or this bell shaped two dimensional bell-shaped surface.

Down below is the same thing but plotted using a contour plot instead, or using different colors, and so this heavy intense red in the middle, corresponds to the highest values, and then the values decrease with the yellow being slightly lower values the cyan being lower values and this deep blue being the lowest values so this is really the same figure but plotted viewed from the top instead, using colors instead.

And so, with this distribution,

you see that it faces most of the probability near 0,0 and then as you go out from 0,0 the probability of X1 and X2 goes down.

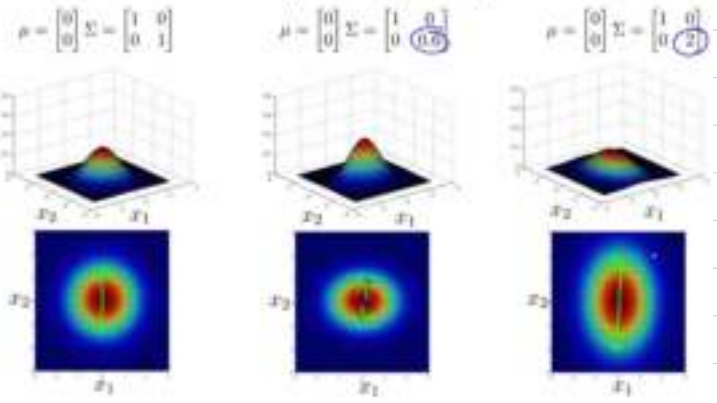
Now lets try varying some of the parameters and see what happens. So let's take sigma and change it so let's say sigma shrinks a little bit. Sigma is a covariance matrix and so it measures the variance or the variability of the features X1 X2. So if the shrink sigma then what you get is what you get is that the width of this bump diminishes

and the height also increases a bit, because the area under the surface is equal to 1. So the integral of the volume under the surface is equal to 1, because probability distribution must integrate to one. But, if you shrink the variance,

it's kinda like shrinking sigma squared, you end up with a narrower distribution, and one that's a little bit taller. And so you see here also the concentric ellipsis has shrunk a little bit. Whereas in contrast if you were to increase sigma

to 2 2 on the diagonals, so it is now two times the identity then you end up with a much wider and much flatter Gaussian. And so the width of this is much wider. This is hard to see but this is still a bell shaped bump, it's just flattened down a lot, it has become much wider and so the variance or the variability of X1 and X2 just becomes wider.

Multivariate Gaussian (Normal) examples



Here are a few more examples. Now lets try varying one of the elements of sigma at the time. Let's say I send sigma to 0.6 there, and 1 over there.

What this does, is this reduces the variance of

the first feature, X 1, while keeping the variance of the second feature X 2, the same. And so with this setting of parameters, you can model things like that. X 1 has smaller variance, and X 2 has larger variance. Whereas if I do this, if I set this matrix to 2, 1 then you can also model examples where you know here

we'll say X1 can have take on a large range of values whereas X2 takes on a relatively narrower range of values. And that's reflected in this figure as well, you know where, the distribution falls off more slowly as X 1 moves away from 0, and falls off very rapidly as X 2 moves away from 0.

Multivariate Gaussian (Normal) examples

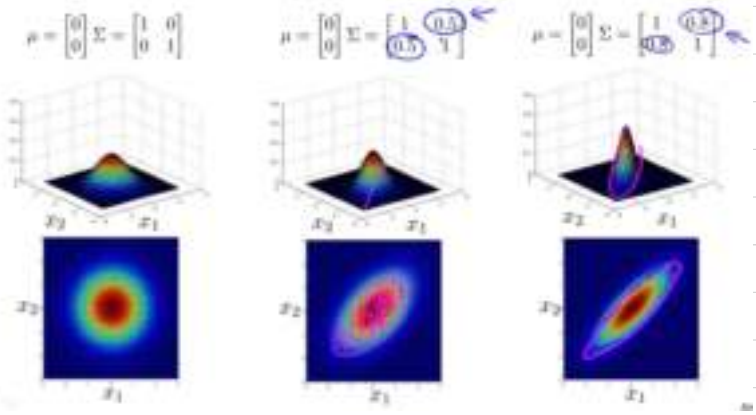
And similarly if we were to modify this element of the matrix instead, then similar to the previous

slide, except that here where you know playing around here saying that X2 can take on a very small range of values and so here if this is 0.6, we notice now X2

tends to take on a much smaller range of values than the original example,

whereas if we were to set sigma to be equal to 2 then that's like saying X2 you know, has a much larger range of values.

Multivariate Gaussian (Normal) examples



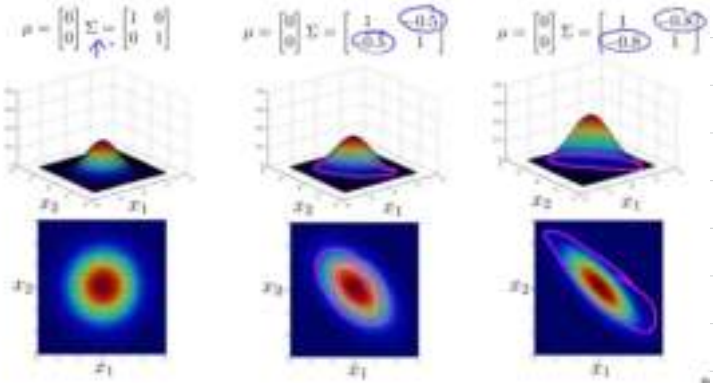
Now, one of the cool things about the multivariate Gaussian distribution is that you can also use it to model correlations between the data. That is we can use it to model the fact that X1 and X2 tend to be highly correlated with each other for example. So specifically if you start to change the off diagonal entries of this covariance matrix you can get a different type of Gaussian distribution.

And so as I increase the off-diagonal entries from .5 to .8, what I get is this distribution that is more and more thinly peaked along this sort of x equals y line. And so here the contour says that x and y tend to grow together and the things that are with large probability are if either X1 is large and Y2 is large or X1 is small and Y2 is small. Or somewhere in between. And as this entry, 0.8 gets large, you get a Gaussian distribution, that's sort of where all the probability lies on this sort of narrow region,

where x is approximately equal to y. This is a very tall, thin distribution you know line mostly along this line

central region where x is close to y. So this is if we set these entries to be positive entries.

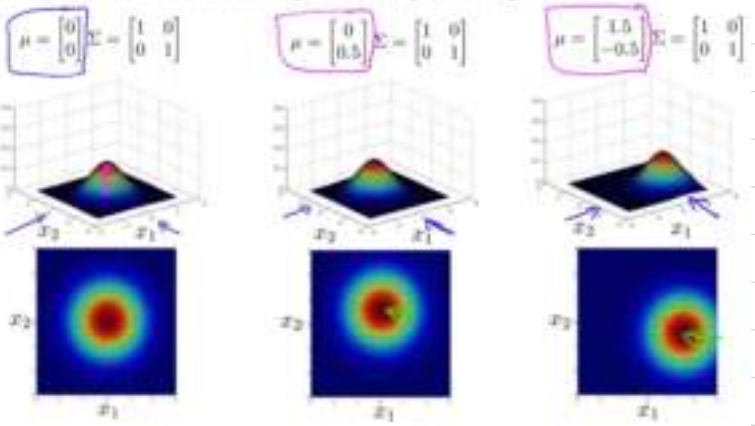
Multivariate Gaussian (Normal) examples



central region where x is close to y. So this is if we set these entries to be positive entries. In contrast if we set these to negative values, as I decreases it to -.5 down to -.8, then what we get is a model where we put most of the probability in this sort of negative X one in the next 2 correlation region, and so, most of the probability now lies in this region, where X 1 is about equal to -X 2, rather than X 1 equals X 2. And so this captures a sort of negative correlation between x1

and x2. And so this is a hopefully this gives you a sense of the different distributions that the multivariate Gaussian distribution can capture.

Multivariate Gaussian (Normal) examples

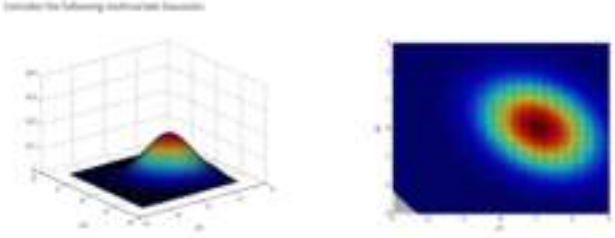


So follow up in varying, the covariance matrix sigma, the other thing you can do is also, vary the mean parameter mu, and so operationally, we have mu equal 0 0, and so the distribution was centered around X 1 equals 0, X2 equals 0, so the peak of the distribution is here, whereas, if we vary the values of mu, then that varies the peak of the distribution and so, if mu equals 0, 0.5, the peak is at, you know, X1 equals zero, and X2 equals 0.5, and so the peak or the center of this distribution has shifted,

and if mu was 1.5 minus 0.5 then OK,

and similarly the peak of the distribution has now shifted to a different location, corresponding to where, you know, X1 is 1.5 and X2 is -0.5, and so varying the mu parameter, just shifts around the center of this whole distribution. So, hopefully, looking at all these different pictures gives you a sense of the sort of probability distributions that the Multivariate Gaussian Distribution allows you to capture. And the key advantage of it is it allows you to capture, when you'd expect two different features to be positively correlated, or maybe negatively correlated.

In the next video, we'll take this multivariate Gaussian distribution and apply it to anomaly detection.



Multivariate Gaussian Distribution (Optional)

Anomaly Detection using the multivariate Gaussian Distribution

In the last video we talked about the Multivariate Gaussian Distribution

and saw some examples of the sorts of distributions you can model, as you vary the parameters, mu and sigma. In this video, let's take those ideas, and apply them to develop a different anomaly detection algorithm.

To recap the multivariate Gaussian distribution and the multivariate normal distribution has two parameters, mu and sigma. Where mu this an n dimensional vector and sigma,

the covariance matrix, is an n by n matrix.

And here's the formula for the probability of X, as parameterized by mu and sigma, and as you vary mu and sigma, you can get a range of different distributions, like, you know, these are three examples of the ones that we saw in the previous video.

So let's talk about the parameter fitting or the parameter estimation problem. The question, as usual, is if I have a set of examples X1 through XM and here each of these examples is an n dimensional vector and I think my examples come from a multivariate Gaussian distribution.

How do I try to estimate my parameters mu and sigma? Well the standard formulas for estimating them is you set mu to be just the average of your training examples.

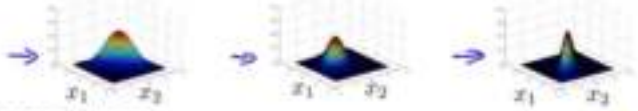
And you set sigma to be equal to this. And this is actually just like the sigma that we had written out, when we were using the PCA or the Principal Components Analysis algorithm.

So you just plug in these two formulas and this would give you your estimated parameter mu and your estimated parameter sigma.

Multivariate Gaussian (Normal) distribution

Parameters μ, Σ $\quad \mu \in \mathbb{R}^n \quad \Sigma \in \mathbb{R}^{n \times n}$

$$p(x; \mu, \Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$



Parameter fitting:

Given training set $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$ $\quad x \in \mathbb{R}^n$

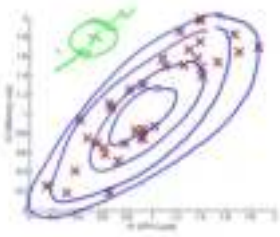
$$\mu = \frac{1}{m} \sum_{i=1}^{m} x^{(i)} \qquad \Sigma = \frac{1}{m} \sum_{i=1}^{m} (x^{(i)} - \mu)(x^{(i)} - \mu)^T$$

Anomaly detection with the multivariate Gaussian

1. Fit model $p(x)$ by setting

$$\mu = \frac{1}{m} \sum_{i=1}^{m} x^{(i)}$$

$$\Sigma = \frac{1}{m} \sum_{i=1}^{m} (x^{(i)} - \mu)(x^{(i)} - \mu)^T$$



2. Given a new example x, compute

$$p(x) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

Flag an anomaly if $p(x) < \varepsilon$

So given the data set here is how you estimate mu and sigma. Let's take this method and just plug it into an anomaly detection algorithm. So how do we put all of this together to develop an anomaly detection algorithm? Here 's what we do. First we take our training set, and we fit the model, we fit P of X, by, you know, setting mu and sigma as described

on the previous slide.

Next when you are given a new example X. So if you are given a test example,

lets take an earlier example to have a new example out here. And that is my test example.

Given the new example X, what we are going to do is compute P of X, using this formula for the multivariate Gaussian distribution.

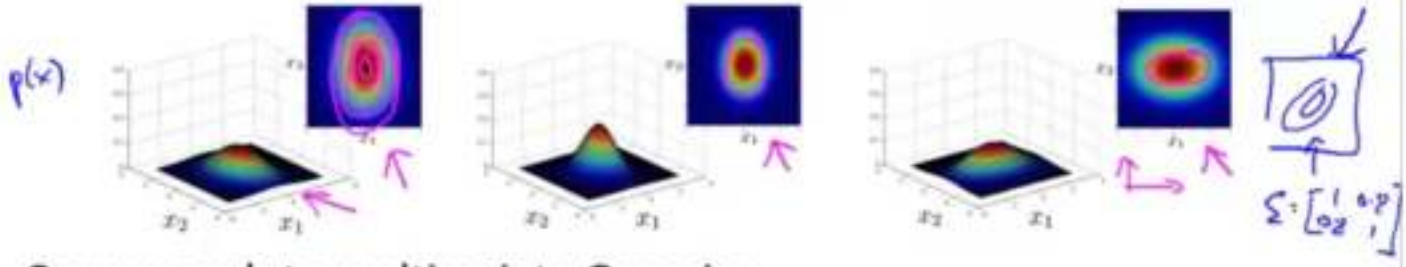
And then, if P of X is very small, then we flagged it as an anomaly, whereas, if P of X is greater than that parameter epsilon, then we don't flag it as an anomaly. So it turns out, if we were to fit a multivariate Gaussian distribution to this data set, so just the red crosses, not the green example, you end up with a Gaussian distribution that places lots of probability in the central region, slightly less probability here, slightly less probability here, slightly less probability here,

and very low probability at the point that is way out here.

And so, if you apply the multivariate Gaussian distribution to this example, it will actually correctly flag that example. as an anomaly.

Relationship to original model

Original model: $p(x) = p(x_1; \mu_1, \sigma_1^2) \times p(x_2; \mu_2, \sigma_2^2) \times \cdots \times p(x_n; \mu_n, \sigma_n^2)$



Corresponds to multivariate Gaussian

$$p(x; \mu, \Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

where $\Sigma = \begin{bmatrix} \sigma_1^2 & & 0 \\ & \sigma_2^2 & \\ & & \ddots \\ 0 & & & \sigma_n^2 \end{bmatrix}$

Andrew Ng

(1)

Finally it's worth saying a few words about what is the relationship between the multivariate Gaussian distribution model, and the original model, where we were modeling P of X as a product of this P of X1, P of X2, up to P of Xn.

It turns out that you can prove mathematically, I'm not going to do the proof here, but you can prove mathematically that this relationship, between the multivariate Gaussian model and this original one. And in particular, it turns out that the original model corresponds to multivariate Gaussians, where the contours of the Gaussian are always axis aligned.

So all three of these are examples of Gaussian distributions that you can fit using the original model. It turns out that that corresponds to multivariate Gaussian, where, you know, the ellipsis here, the contours of this distribution--it turns out that this model actually corresponds to a special case of a multivariate Gaussian distribution. And in particular, this special case is defined by constraining

the distribution of p of x, the multivariate a Gaussian distribution of p of x, so that the contours of the probability density function, of the probability distribution function, are axis aligned. And so you can get a p of x with a multivariate Gaussian that looks like this, or like this, or like this. And you notice, that in all 3 of these examples, these ellipses, or these ovals that I'm drawing, have their axes aligned with the X1 X2 axes.

(2)

And what we do not have, is a set of contours that are at an angle, right? And this corresponded to examples where sigma is equal to 1 1, 0.8, 0.8. Let's say, with non-0 elements on the off diagonals. So, it turns out that it's possible to show mathematically that this model actually is the same as a multivariate Gaussian distribution but with a constraint. And the constraint is that the covariance matrix sigma must have 0's on the off diagonal elements. In particular, the covariance matrix sigma, this thing here, it would be sigma squared 1, sigma squared 2, down to sigma squared n, and then everything on the off diagonal entries, all of these elements

above and below the diagonal of the matrix, all of those are going to be zero.

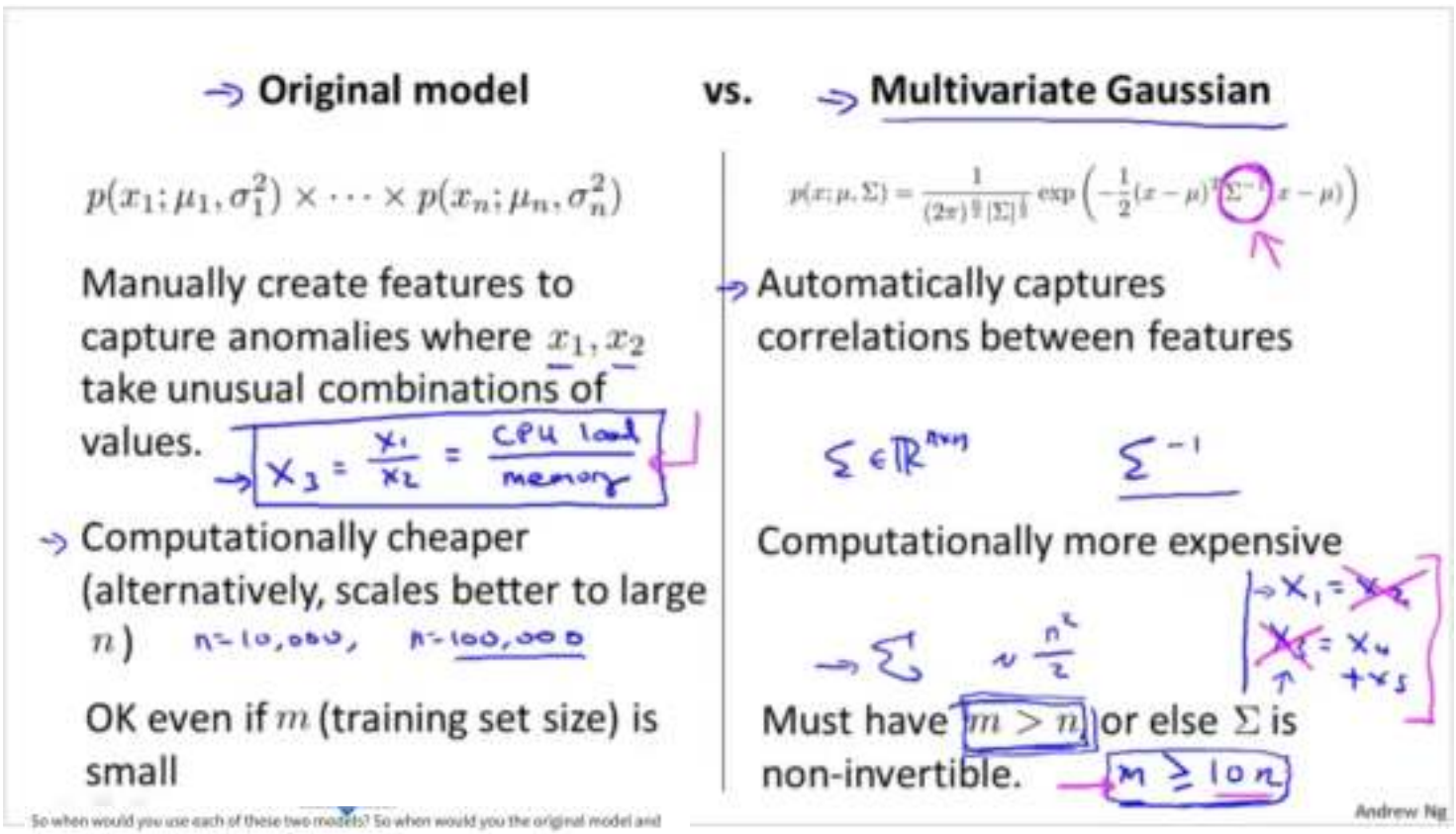
And in fact if you take these values of sigma, sigma squared 1, sigma squared 2, down to sigma squared n, and plug them into here, and you know, plug them into this covariance matrix, then the two models are actually identical. That is, this new model,

using a multivariate Gaussian distribution,

corresponds exactly to the old model, if the covariance matrix sigma, has only 0 elements off the diagonals, and in pictures that corresponds to having Gaussian distributions,

where the contours of this distribution function are axis aligned. So you aren't allowed to model the correlations between the diffrent features.

So in that sense the original model is actually a special case of this multivariate Gaussian model.



So when would you use each of these two models? So when would you the original model and when would you use the multivariate Gaussian model?

The original model is probably used somewhat more often,

and whereas the multivariate Gaussian distribution is used somewhat less but it has the advantage of being able to capture correlations between features. So

suppose you want to capture anomalies where you have different features say where features x1, x2 take on unusual combinations of values so in the earlier example, we had that example where the anomaly was with the CPU load and the memory use taking on unusual combinations of values, if

you want to use the original model to capture that, then what you need to do is create an extra feature, such as X3 equals X1/X2, you know equals maybe the CPU load divided by the memory used, or something, and you

need to create extra features if there's unusual combinations of values where X1 and X2 take on an unusual combination of values even though X1 by itself and X2 by itself

looks like it's taking a perfectly normal value. But if you're willing to spend the time to manually create an extra feature like this, then the original model will work fine. Whereas in contrast, the multivariate Gaussian model can automatically capture correlations between different features. But the original model has some other more significant advantages, too, and one huge advantage of the original model

is that it is computationally cheaper, and another view on this is that is scales better to very large values of n and very large numbers of features, and so even if n were ten thousand,

or even if n were equal to a hundred thousand, the original model will usually work just fine. Whereas in contrast for the multivariate Gaussian model notice here, for example, that we need to compute the inverse of the matrix sigma where sigma is an n by n matrix

and so computing sigma if sigma is a hundred thousand by a hundred thousand matrix that is going to be very computationally expensive. And so the multivariate Gaussian model scales less well to large values of N. And finally for the original model, it turns out to work out ok even if you have a relatively small training set this is the small unlabeled examples that we use to model p of x

of course, and this works fine, even if M is, you

know, maybe 50, 100, works fine. Whereas for the multivariate Gaussian, it is sort of a mathematical property of the algorithm that you must have m greater than n, so that the number of examples is greater than the number of features you have. And there's a mathematical property of the way we estimate the parameters

that if this is not true, so if m is less than or equal to n, then this matrix isn't even invertible, that is this matrix is singular, and so you can't even use the multivariate Gaussian model unless you make some changes to it. But a

typical rule of thumb that I use is, I will use the multivariate Gaussian model only if m is much greater than n, so this is sort of the

narrow mathematical requirement, but in practice, I would use the multivariate Gaussian model, only if m were quite a bit bigger than n. So if m were greater than or equal to 10 times n, let's say, might be a reasonable rule of thumb, and if

it doesn't satisfy this, then the multivariate Gaussian model has a lot of parameters, right, so this covariance matrix sigma is an n by n matrix, so it has, you know, roughly n squared parameters, because it's a symmetric matrix, it's actually closer to n squared over 2 parameters, but this is a lot of parameters, so you need make sure you have a fairly large value for m, make sure you have enough data to fit all these parameters. And m greater than or equal to 10 n would be a reasonable rule of thumb to make sure that you can estimate this covariance matrix sigma reasonably well.

So in practice the original model shown on the left that is used more often. And if you suspect that you need to capture correlations between features what people will often do is just manually design extra features like these to capture specific unusual combinations of values. But in problems where you have a very large training set or m is very large and n is

not too large, then the multivariate Gaussian model is well worth considering and may work better as well, and can

save you from having to spend your time to manually create extra features in case

the anomalies turn out to be captured by unusual combinations of values of the features.

Finally I just want to briefly mention one somewhat technical property, but if you're fitting multivariate Gaussian model, and if you find that the covariance matrix sigma is singular, or you find it's non-invertible, they're usually 2 cases for this. One is if it's failing to satisfy this m greater than n condition, and the second case is if you have redundant features. So by redundant features, I mean, if you have 2 features that are the same. Somehow you accidentally made two copies of the feature, so your x1 is just equal to x2. Or if you have redundant features like maybe

your features X3 is equal to feature X4, plus feature X5. Okay, so if you have highly redundant features like these, you know, where if X3 is equal to X4 plus X5, well X3 doesn't contain any extra information, right? You just take these 2 other features, and add them togethes

And if you have this sort of redundant features, duplicated features, or this sort of features, then sigma may be non-invertible.

And so there's a debugging set-- this should very rarely happen, so you probably won't run into this, it is very unlikely that you have to worry about this-- but in case you implement a multivariate Gaussian model you find that sigma is non-invertible.

What I would do is first make sure that M is quite a bit bigger than N, and if it is then, the second thing I do, is just check for redundant features. And so if there are 2 features that are equal, just get rid of one of them, or if you have redundant if these , X3 equals X4 plus X5, just get rid of the redundant feature, and then it should work fine again. As an aside for those of you who are experts in linear algebra, by redundant features, what I mean is the formal term is features that are linearly dependent. But in practice what that really means is one of these problems tripping up the algorithm if you just make you features non-redundant , that should solve the problem of sigma being non-invertible. But once again the odds of your running into this at all are pretty low so chances are, you can just apply the multivariate Gaussian model, without having to worry about sigma being non-invertible, so long as m is greater than or equal to n. So that's it for anomaly detection, with the multivariate Gaussian distribution. And if you apply this method you would be able to have an anomaly detection algorithm that automatically captures positive and negative correlations between your different features and flags an anomaly if it sees is unusual combination of the values of the features.

Consider applying anomaly detection using a training set $\{x^{(1)}, \ldots, x^{(m)}\}$ where $x^{(i)} \in \mathbb{R}^n$. Which of the following statements are true? Check all that apply.

- [x] The original model $p(x_1; \mu_1, \sigma_1^2) \times \cdots \times p(x_n; \mu_n, \sigma_n^2)$ corresponds to a multivariate Gaussian where the contours of $p(x; \mu, \Sigma)$ are axis-aligned.

✓ **Correct**

- [ ] Using the multivariate Gaussian model is advantageous when m (the training set size) is very small ($m < n$).
- [x] The multivariate Gaussian model can automatically capture correlations between different features in x.

✓ **Correct**

- [x] The original model can be more computationally efficient than the multivariate Gaussian model, and thus might scale better to very large values of n (number of features).

✓ **Correct**

Predicting Movie Ratings

Recommender Systems

Problem Formulation

In this next set of videos, I would like to tell you about recommender systems.

There are two reasons, I had two motivations for why I wanted to talk about recommender systems.

The first is just that it is an important application of machine learning. Over the last few years, occasionally I visit different, you know, technology companies here in Silicon Valley and I often talk to people working on machine learning applications there and so I've asked people what are the most important applications of machine learning or what are the machine learning applications that you would most like to get an improvement in the performance of. And one of the most frequent answers I heard was that there are many groups out in Silicon Valley now, trying to build better recommender systems.

So, if you think about what the websites are like Amazon, or what Netflix or what eBay, or what iTunes Genius, made by Apple does, there are many websites or systems that try to recommend new products to use. So, Amazon recommends new books to you, Netflix try to recommend new movies to you, and so on. And these sorts of recommender systems, that look at what books you may have purchased in the past, or what movies you have rated in the past, but these are the systems that are responsible

for today, a substantial fraction of Amazon's revenue and for a company like Netflix, the recommendations

that they make to the users is also responsible for a substantial fraction of the movies watched by their users. And so an improvement in performance of a recommender system can have a substantial and immediate impact on the bottom line of many of these companies. Recommender systems is kind of a funny problem, within academic machine learning so that we could go to an academic machine learning conference,

the problem of recommender systems, actually receives relatively little attention, or at least it's sort of a smaller fraction of what goes on within Academia. But if you look at what's happening, many technology companies, the ability to build these systems seems to be a high priority for many companies. And that's one of the reasons why I want to talk about them in this class.

The second reason that I want to talk about recommender systems is that as we approach the last few sets of videos

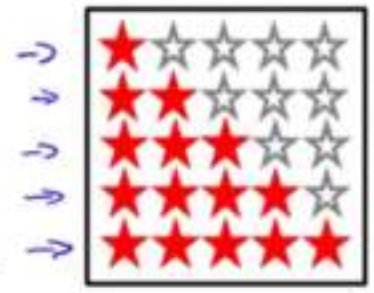
of this class I wanted to talk about a few of the big ideas in machine learning and share with you, you know, some of the big ideas in machine learning. And we've already seen in this class that features are important for machine learning, the features you choose will have a big effect on the performance of your learning algorithm. So there's this big idea in machine learning, which is

that for some problems, maybe not all problems, but some problems, there are algorithms that can try to automatically learn a good set of features for you. So rather than trying to hand design, or hand code the features, which is mostly what we've been doing so far, there are a few settings where you might be able to have an algorithm, just to learn what feature to use, and the recommender systems is just one example of that sort of setting. There are many others, but engraved through recommender systems, will be able to go a little bit into this idea of learning the features and you'll be able to see at least one example of this, I think, big idea in machine learning as well.

So, without further ado, let's get started, and talk about the recommender system problem formulation.

Example: Predicting movie ratings

→ User rates movies using ~~one~~ zero to five stars



Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)
Love at last	5	5	0	0
Romance forever	5	? → 4.5	? → 0	0
Cute puppies of love	? → 5	4	0	? → 0
Nonstop car chases	0	0	5	4
Swords vs. karate	0	0	5	? → 4

$n_u = 4$ $n_m = 5$

→ n_u = no. users

→ n_m = no. movies

→ $r(i,j)$ = 1 if user j has rated movie i

$y^{(i,j)}$ = rating given by user j to movie i (defined only if $r(i,j)=1$)

$0, \ldots, 5$

(1)

As my running example, I'm going to use the modern problem of predicting movie ratings. So, here's a problem. Imagine that you're a website or a company that sells or rents out movies, or what have you. And so, you know, Amazon, and Netflix, and I think iTunes are all examples

of companies that do this, and let's say you let your users rate different movies, using a 1 to 5 star rating. So, users may, you know, something one, two, three, four or five stars.

In order to make this example just a little bit nicer, I'm going to allow 0 to 5 stars as well, because that just makes some of the math come out just nicer. Although most of these websites use the 1 to 5 star scale.

So here, I have 5 movies. You know, Love That Lasts, Romance Forever, Cute Puppies of Love, Nonstop Car Chases, and Swords vs. Karate. And we have 4 users, which, calling, you know, Alice, Bob, Carol, and Dave, with initials A, B, C, and D, we'll call them users 1, 2, 3, and 4. So, let's say Alice really likes Love That Lasts and rates that 5 stars, likes Romance Forever, rates it 5 stars. She did not watch Cute Puppies of Love, and did rate it, so we don't have a rating for that, and Alice really did not like Nonstop Car Chases or Swords vs. Karate. And a different user Bob, user two, maybe rated a different set of movies, maybe she likes to Love at Last, did not to watch Romance Forever, just have a rating of 4, a 0, a 0, and maybe our 3rd user, rates this 0, did not watch that one, 0, 5, 5, and, you know, let's just fill in some of the numbers.

(2)

And so just to introduce a bit of notation, this notation that we'll be using throughout, I'm going to use NU to denote the number of users. So in this example, NU will be equal to 4. So the u subscript stands for users and Nm, going to use to denote the number of movies, so here I have five movies so Nm equals equals 5. And you know for this example, I have for this example, I have loosely

3 maybe romantic or romantic comedy movies and 2 action movies and you know, if you look at this small example, it looks like Alice and Bob are giving high ratings to these romantic comedies or movies about love, and giving very low ratings about the action movies, and for Carol and Dave, it's the opposite, right? Carol and Dave, users three and four, really like the action movies and give them high ratings, but don't like the romance and love- type movies as much.

Specifically, in the recommender system problem, we are given the following data. Our data comprises the following: we have these values r(i, j), and r(i, j) is 1 if user J has rated movie I. So our users rate only some of the movies, and so, you know, we don't have ratings for those movies. And whenever r(i, j) is equal to 1, whenever user j has rated movie i, we also get this number y(i, j), which is the rating given by user j to movie i. And so, y(i, j) would be a number from zero to five, depending on the star rating, zero to five stars that user gave that particular movie.

(3)

So, the recommender system problem is given this data that has give these r(i, j)'s and the y(i, j)'s to look through the data and look at all the movie ratings that are missing and to try to predict what these values of the question marks should be. In the particular example, I have a very small number of movies and a very small number of users and so most users have rated most movies but in the realistic settings your users each of your users may have rated only a minuscule fraction of your movies but looking at this data, you know, if Alice and Bob both like the romantic movies maybe we think that Alice would have given this a five. Maybe we think Bob would have given this a 4.5 or some high value, as we think maybe Carol and Dave were doing these very low ratings. And Dave, well, if Dave really likes action movies, maybe he would have given Swords and Karate a 4 rating or maybe a 5 rating, okay? And so, our job in developing a recommender system is to come up with a learning algorithm that can automatically go fill in these missing values for us so that we can look at, say, the movies that the user has not yet watched, and recommend new movies to that user to watch. You try to predict what else might be interesting to a user.

So that's the formalism of the recommender system problem.

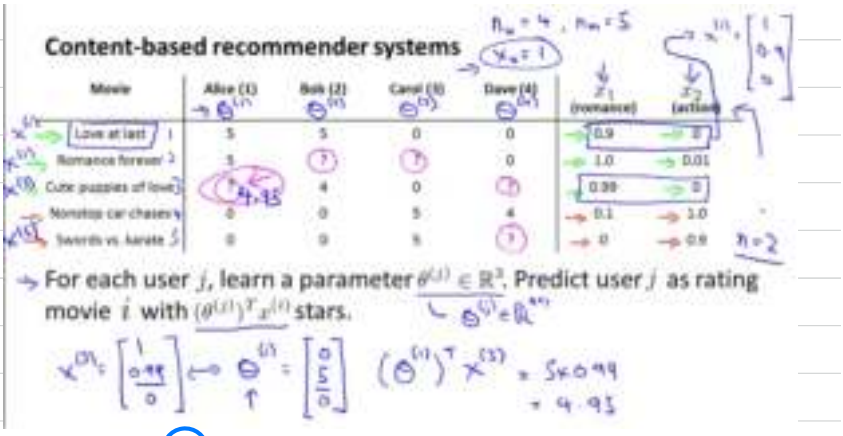
In the next video we'll start to develop a learning algorithm to address this problem.

Predicting Movie Ratings

Recommender Systems

Content Based Recommendations

In the last video, we talked about the recommender systems problem where for example you might have a set of movies and you may have a set of users, each who have rated some subset of the movies. They've rated the movies one to five stars or zero to five stars. And what we would like to do is look at these users and predict how they would have rated other movies that they have not yet rated. In this video I'd like to talk about our first approach to building a recommender system. This approach is called content based recommendations. Here's our data



① recommender system. This approach is called content based recommendations. Here's our data set from before and just to remind you of a bit of notation, I was using nu to denote the number of users and so that's equal to 4, and nm to denote the number of movies, I have 5 movies.

So, how do I predict what these missing values would be?

Let's suppose that for each of these movies I have a set of features for them. In particular, let's say that for each of the movies have two features which I'm going to denote x1 and x2. Where x1 measures the degree to which a movie is a romantic movie and x2 measures the degree to which a movie is an action movie. So, if you take a movie, Love at last, you know it's 0.9 rating on the romance scale. This is a highly romantic movie, but zero on the action scale. So, almost no action in that movie. Romance forever is a 1.0, lot of romance and 0.01 action. I don't know, maybe there's a minor car crash in that movie or something. So there's a little bit of action. Skipping one, let's do Swords vs karate, maybe that has a 0 romance rating and no romance at all in that but plenty of action. And Nonstop car chases, maybe again there's a tiny bit of romance in that movie but mainly action. And Cute puppies of love mainly a romance movie with no action at all.

③ So let's say you want to make a prediction for what Alice will think of the movie Cute puppies of love. Well that movie is going to have some parameter vector x3 where we have that x3 is going to be equal to 1, which is my intercept term and then 0.99 and then 0. And let's say, for this example, let's say that we've somehow already gotten a parameter vector theta 1 for Alice. We'll say it later exactly how we come up with this parameter vector.

② So if we have features like these, then each movie can be represented with a feature vector. Let's take movie one. So let's call these movies 1, 2, 3, 4, and 5. But my first movie, Love at last, I have my two features, 0.9 and 0. And so these are features x1 and x2. And let's add an extra feature as usual, which is my interceptor feature x0 = 1. And so putting these together I would then have a feature x1. The superscript 1 denotes it's the feature vector for my first movie, and this feature vector is equal to 1. The first 1 there is this interceptor. And then my two feature is 0.90 like so. So for Love at last I would have a feature vector x1, for the movie Romance forever I may have a software feature of vector x2, and so on, and for Swords vs karate I would have a different feature vector x superscript 5. Also, consistence with our earlier node notation that we were using, we're going to set n to be the number of features not counting this x0 interceptor. So n is equal to 2 because it's we have two features x1 and x2 capturing the degree of romance and the degree of action in each movie. Now in order to make predictions here's one thing that we do which is that we could treat predicting the ratings of each user as a separate linear regression problem. So specifically, let's say that for each user j, we're going to learn the parameter vector theta j, which would be an R3 in this case. More generally, theta [j] would be an R (n+1), where n is the number of features not counting the set term. And we're going to predict user j as rating movie i with just the inner product between parameters vectors theta and the features xi. So let's take a specific example. Let's take user 1, so that would be Alice. And associated with Alice would be some parameter vector theta 1. And our second user, Bob, will be associated a different parameter vector theta 2. Carol will be associated with a different parameter vector theta 3 and Dave a different parameter vector theta 4.

④ But let's just say for now that some unspecified learning algorithm has learned the parameter vector theta 1 and is equal to this 0,5,0. So our prediction for this entry is going to be equal to theta 1, that is Alice's parameter vector, transpose x3, that is the feature vector for the Cute puppies of love movie, number 3. And so the inner product between these two vectors is gonna be 5 times 0.99, which is equal to 4.95. And so my prediction for this value over here is going to be 4.95. And maybe that seems like a reasonable value if indeed this is my parameter vector theta 1. So, all we're doing here is we're applying a different copy of this linear regression for each user, and we're saying that what Alice does is Alice has some parameter vector theta 1 that she uses, that we use to predict her ratings as a function of how romantic and how action packed a movie is. And Bob and Carol and Dave, each of them have a different linear function of the romanticness and actionness, or degree of romance and degree of action in a movie and that that's how we're gonna predict that their star ratings.

Problem formulation

- $r(i,j) = 1$ if user j has rated movie i (0 otherwise)
- $y^{(i,j)}$ = rating by user j on movie i (if defined)
- $\theta^{(j)}$ = parameter vector for user j
- $x^{(i)}$ = feature vector for movie i
- For user j, movie i, predicted rating: $(\theta^{(j)})^T(x^{(i)})$ $\quad \theta^{(j)} \in \mathbb{R}^{n+1}$
- $m^{(j)}$ = no. of movies rated by user j

To learn $\theta^{(j)}$:

$$\min_{\theta^{(j)}} \frac{1}{2} \sum_{i:r(i,j)=1} \left((\theta^{(j)})^T(x^{(i)}) - y^{(i,j)}\right)^2 + \frac{\lambda}{2} \sum_{k=1}^{n} (\theta_k^{(j)})^2$$

Andrew Ng

(1)

More formally, here's how we can write down the problem. Our notation is that r(i,j) is equal to 1 if user j has rated movie i and y(i,j) is the rating of that movie, if that rating exists.

That is, if that user has actually rated that movie.

And, on the previous slide we also defined these, theta j, which is a parameter for the user xi, which is a feature vector for a specific movie. And for each user and each movie, we predict that rating as follows. So let me introduce just temporarily introduce one extra bit of notation mj. We're gonna use mj to denote the number of users rated by movie j. We don't need this notation only for this line. Now in order to learn the parameter vector for theta j, well how do we do so. This is basically a linear regression problem. So what we can do is just choose a parameter vector theta j so that the predicted values here are as close as possible to the values that we observed in our training sets and the values we observed in our data. So let's write that down. In order to learn the parameter vector theta j, let's minimize over the parameter vector theta j of sum,

and I want to sum over all movies that user j has rated. So we write it as sum over all values of i. That's a r(i,j) equals 1. So the way to read this summation syntax is this is summation over all the values of i, so the r(i,j) is equal to 1. So you'll be summing over all the movies that user j has rated.

(2)

And then I'm going to compute theta j, transpose x i. So that's the prediction of using j's rating on movie i, - y (i,j). So that's the actual observed rating squared. And then, let me just divide by the number of movies that user j has actually rated. So let's just divide by 1 over 2m j. And so this is just like the least squares regressions. It's just like linear regression, where we want to choose the parameter vector theta j to minimize this type of squared error term.

And if you want, you can also add in irregularization terms so plus lambda over 2m and this is really 2mj because we have mj examples. User j has rated that many movies, it's not like we have that many data points with which to fit the parameters of theta j. And then let me add in my usual regularization term here of theta j k squared. As usual, this sum is from k equals 1 through n, so here, theta j is going to be an n plus 1 dimensional vector, where in our early example n was equal to 2. But more broadly, more generally n is the number of features we have per movie. And so as usual we don't regularize over theta 0. We don't regularize over the bias terms. The sum is from k equals 1 through n.

So if you minimize this as a function of theta j you get a good solution, you get a pretty good estimate of a parameter vector theta j with which to make predictions for user j's movie ratings. For recommender systems, I'm gonna change this notation a little bit. So to simplify the subsequent math, I with to get rid of this term mj. So that's just a constant, right? So I can delete it without changing the value of theta j that I get out of this optimization. So if you imagine taking this whole equation, taking this whole expression and multiplying it by mj, get rid of that constant. And when I minimize this, I should still get the same value of theta j as before. So just to

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Optimization objective:

To learn $\theta^{(j)}$ (parameter for user j):

$$\min_{\theta^{(j)}} \frac{1}{2} \sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)}\right)^2 + \frac{\lambda}{2} \sum_{k=1}^{n} (\theta_k^{(j)})^2$$

To learn $\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(n_u)}$:

$$\min_{\theta^{(1)},\ldots,\theta^{(n_u)}} \frac{1}{2} \sum_{j=1}^{n_u} \sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)}\right)^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^{n} (\theta_k^{(j)})^2$$

$\theta^{(1)}, \ldots, \theta^{(n_u)}$

constant. And when I minimize this, I should still get the same value of theta j as before. So just to repeat what we wrote on the previous slide, here's our optimization objective. In order to learn theta j which is the parameter for user j, we're going to minimize over theta j of this optimization objectives. So this is our usual squared error term and then this is our regularizations term. Now of course in building a recommender system, we don't just want to learn parameters for a single user. We want to learn parameters for all of our users. I have n subscript u users, so I want to learn all of these parameters. And so, what I'm going to do is take this optimization objective and just add the mixture summation there. So this expression here with the one half on top of this is exactly the same as what we had on top. Except that now instead of just doing this for a specific user theta j, I'm going to sum my objective over all of my users and then minimize this overall optimization objective, minimize this overall cost on. And when I minimize this as a function of theta 1, theta 2, up to theta nu, I will get a separate parameter vector for each user. And I can then use that to make predictions for all of my users, for all of my n subscript users.

Optimization algorithm:

$$\min_{\theta^{(1)},\ldots,\theta^{(n_u)}} \frac{1}{2}\sum_{j=1}^{n_u}\sum_{i:r(i,j)=1}\left((\theta^{(j)})^T x^{(i)} - y^{(i,j)}\right)^2 + \frac{\lambda}{2}\sum_{j=1}^{n_u}\sum_{k=1}^{n}(\theta_k^{(j)})^2$$

$J(\theta^{(1)},\ldots,\theta^{(n_u)})$

Gradient descent update:

$$\theta_k^{(j)} := \theta_k^{(j)} - \alpha \sum_{i:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)}) x_k^{(i)} \quad (\text{for } k = 0)$$

$$\theta_k^{(j)} := \theta_k^{(j)} - \alpha \left(\sum_{i:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)}) x_k^{(i)} + \lambda \theta_k^{(j)} \right) \quad (\text{for } k \neq 0)$$

$\frac{\partial}{\partial \theta_k^{(j)}} J(\theta^{(1)},\ldots,\theta^{(n_u)})$

(1)

So putting everything together, this was our optimization objective on top. And to give this thing a name, I'll just call this J(theta1, …, theta nu). So j as usual is my optimization objective, which I'm trying to minimize.

Next, in order to actually do the minimization, if you were to derive the gradient descent update, these are the equations that you would get. So you take theta j, k, and subtract from an alpha, which is the learning rate, times these terms over here on the right. So there's slightly different cases when k equals 0 and when k does not equal 0. Because our regularization term here regularizes only the values of theta jk for k not equal to 0, so we don't regularize theta 0, so with slightly different updates when k equals 0 and k is not equal to 0. And this term over here, for example, is just the partial derivative with respect to your parameter,

that of your optimization objective. Right and so this is just gradient descent and I've already computed the derivatives and plugged them into here.

(2)

And if this gradient descent update look a lot like what we have here for linear regression. That's because these are essentially the same as linear regression. The only minor difference is that for linear regression we have these 1 over m terms, this really would've been 1 over mj. But because earlier when we are deriving the optimization objective, we got rid of this, that's why we don't have this 1 over m term. But otherwise, it's really some of my training examples of the ever times xk plus that regularization term, plus that term of regularization contributes to the derivative. And so if you're using gradient descent here's how you can minimize the cost function j to learn all the parameters. And using these formulas for the derivative if you want, you can also plug them into a more advanced optimization algorithm, like conjugate gradient or LBFGS or what have you. And use that to try to minimize the cost function j as well.

So hopefully you now know how you can apply essentially a deviation on linear regression in order to predict different movie ratings by different users. This particular algorithm is called a content based recommendations, or a content based approach, because we assume that we have available to us features for the different movies. And so where features that capture what is the content of these movies, of how romantic is this movie, how much action is in this movie. And we're really using features of a content of the movies to make our predictions.

But for many movies, we don't actually have such features. Or maybe very difficult to get such features for all of our movies, for all of whatever items we're trying to sell. And so, in the next video, we'll start to talk about an approach to recommender systems that isn't content based and does not assume that we have someone else giving us all of these features for all of the movies in our data set.

Collaborative Filtering

Recommender Systems

Collaborative Filtering

In this video we'll talk about an approach to building a recommender system that's called collaborative filtering.

The algorithm that we're talking about has a very interesting property that it does what is called feature learning and by that I mean that this will be an algorithm that can start to learn for itself what features to use.

Problem motivation

Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)	x_1 (romance)	x_2 (action)
Love at last	5	5	0	0	?	?
Romance forever	5	?	?	0	?	?
Cute puppies of love	?	4	0	?	?	?
Nonstop car chases	0	0	5	4	?	?
Swords vs. karate	0	0	5	?	?	?

$\theta^{(1)}$ $\theta^{(2)}$ $\theta^{(3)}$ $\theta^{(4)}$

(1)

Here was the data set that we had and we had assumed that for each movie, someone had come and told us how romantic that movie was and how much action there was in that movie.

But as you can imagine it can be very difficult and time consuming and expensive to actually try to get someone to, you know, watch each movie and tell you how romantic each movie and how action packed is each movie, and often you'll want even more features than just these two. So where do you get these features from?

So let's change the problem a bit and suppose that we have a data set where we do not know the values of these features. So we're given the data set of movies and of how the users rated them, but we have no idea how romantic each movie is and we have no idea how action packed each movie is so I've replaced all of these things with question marks.

But now let's make a slightly different assumption.

Let's say we've gone to each of our users, and each of our users has told has told us how much they like the romantic movies and how much they like action packed movies. So Alice has associated a current of theta 1. Bob theta 2. Carol theta 3. Dave theta 4. And let's say we also use this and that Alice tells us that she really likes romantic movies and so there's a five there which is the multiplier associated with X1 and lets say that Alice tells us she really doesn't like action movies and so there's a 0 there.

(2)

And Bob tells us something similar

so we have theta 2 over here. Whereas Carol tells us that

she really likes action movies which is why there's a 5 there, that's the multiplier associated with X2, and remember there's also

X0 equals 1 and let's say that Carol tells us she doesn't like romantic movies and so on, similarly for Dave. So let's assume that somehow we can go to users and each user J just tells us what is the value of theta J for them. And so basically specifies to us of how much they like different types of movies.

If we can get these parameters theta from our users then it turns out that it becomes possible to try to infer what are the values of x1 and x2 for each movie.

Let's look at an example. Let's look at movie 1.

So that movie 1 has associated with it a feature vector x1. And you know this movie is called Love at last but let's ignore that. Let's pretend we don't know what this movie is about, so let's ignore the title of this movie.

(3)

All we know is that Alice loved this movie. Bob loved this movie. Carol and Dave hated this movie.

So what can we infer? Well, we know from the feature vectors that Alice and Bob love romantic movies because they told us that there's a 5 here. Whereas Carol and Dave, we know that they hate romantic movies and that they love action movies. So because those are the parameter vectors that you know, users 3 and 4, Carol and Dave, gave us.

And so based on the fact that movie 1 is loved by Alice and Bob and hated by Carol and Dave, we might reasonably conclude that this is probably a romantic movie, it is probably not much of an action movie.

this example is a little bit mathematically simplified but what we're really asking is what feature vector should X1 be so that theta 1 transpose x1 is approximately equal to 5, that's Alice's rating, and theta 2 transpose x1 is also approximately equal to 5,

and theta 3 transpose x1 is approximately equal to 0, so this would be Carol's rating, and

theta 4 transpose X1 is approximately equal to 0. And from this it looks like, you know, X1 equals one that's the intercept term, and then 1.0, 0.0, that makes sense given what we know of Alice, Bob, Carol, and Dave's preferences for movies and the way they rated this movie.

And so more generally, we can go down this list and try to figure out what might be reasonable features for these other movies as well.

Consider the following movie ratings:

-	User 1	User 2	User 3	(romance)
Movie 1	0	1.5	2.5	?

Note that there is only one feature x_1. Suppose that:

$$\theta^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \theta^{(2)} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \theta^{(3)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

What would be a reasonable value for $x_1^{(1)}$ (the value denoted "?" in the table above)?

- (●) 0.5
- ( ) 1
- ( ) 2
- ( ) Any of these values would be equally reasonable.

✓ **Correct**

Optimization algorithm

Given $\theta^{(1)}, \ldots, \theta^{(n_u)}$, to learn $x^{(i)}$:

$$\min_{x^{(i)}} \frac{1}{2} \sum_{j:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{k=1}^{n} (x_k^{(i)})^2$$

Given $\theta^{(1)}, \ldots, \theta^{(n_u)}$, to learn $x^{(1)}, \ldots, x^{(n_m)}$:

$$\min_{x^{(1)}, \ldots, x^{(n_m)}} \frac{1}{2} \sum_{i=1}^{n_m} \sum_{j:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^{n} (x_k^{(i)})^2$$

Let's formalize this problem of learning the features $X^{(i)}$. Let's say that our users have given us their preferences. So let's say that our users have come and, you know, told us these values for theta 1 through theta of N_U and we want to learn the feature vector $X^{(i)}$ for movie number I. What we can do is therefore pose the following optimization problem. So we want to sum over all the indices J for which we have a rating for movie I because we're trying to learn the features for movie I that is this feature vector $X^{(i)}$.

So and then what we want to do is minimize this squared error, so we want to choose features $X^{(i)}$, so that, you know, the predictive value of how user J rates movie I will be similar, will be not too far in the squared error sense of the actual value $Y^{(i,j)}$ that we actually observe in the rating of user J

on movie I. So, just to summarize what this term does is it tries to choose features $X^{(i)}$ so that for all the users J that have rated that movie, the algorithm also predicts a value for how that user would have rated that movie that is not too far, in the squared error sense, from the actual value that the user had rated that movie.

So that's the squared error term. As usual, we can also add this sort of regularization term to prevent the features from becoming too big.

So this is how we would learn the features for one specific movie but what we want to do is learn all the features for all the movies and so what I'm going to do is add this extra summation here so I'm going to sum over all ${n_m}$ movies, N subscript m movies, and minimize this objective on top that sums of all movies. And if you do that, you end up with the following optimization problem.

And if you minimize this, you have hopefully a reasonable set of features for all of your movies.

Suppose you use gradient descent to minimize:

$$\min_{x^{(1)}, \ldots, x^{(n_m)}} \frac{1}{2} \sum_{i=1}^{n_m} \sum_{j:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^{n} (x_k^{(i)})^2$$

Which of the following is a correct gradient descent update rule for $i \neq 0$?

- [x] $x_k^{(i)} := x_k^{(i)} + \alpha \left(\sum_{j:r(i,j)=1} \left((\theta^{(j)})^T (x^{(i)}) - y^{(i,j)} \right) \theta_k^{(j)} \right)$
- [ ] $x_k^{(i)} := x_k^{(i)} - \alpha \left(\sum_{j:r(i,j)=1} \left((\theta^{(j)})^T (x^{(i)}) - y^{(i,j)} \right) \theta_k^{(j)} \right)$
- [ ] $x_k^{(i)} := x_k^{(i)} + \alpha \left(\sum_{j:r(i,j)=1} \left((\theta^{(j)})^T (x^{(i)}) - y^{(i,j)} \right) \theta_k^{(j)} + \lambda x_k^{(i)} \right)$
- [●] $x_k^{(i)} := x_k^{(i)} - \alpha \left(\sum_{j:r(i,j)=1} \left((\theta^{(j)})^T (x^{(i)}) - y^{(i,j)} \right) \theta_k^{(j)} + \lambda x_k^{(i)} \right)$

✓ **Correct**

Collaborative filtering

Given $x^{(1)}, \ldots, x^{(n_m)}$ (and movie ratings),
can estimate $\theta^{(1)}, \ldots, \theta^{(n_u)}$

Given $\theta^{(1)}, \ldots, \theta^{(n_u)}$,
can estimate $x^{(1)}, \ldots, x^{(n_m)}$

Guess $\theta \to x \to \theta \to x \to \theta \to x \to \cdots$

(1)

So putting everything together, what we, the algorithm we talked about in the previous video and the algorithm that we just talked about in this video. In the previous video, what we showed was that you know, if you have a set of movie ratings, so if you have the data the rij's and then you have the yij's that will be the movie ratings.

Then given features for your different movies we can learn these parameters theta. So if you knew the features, you can learn the parameters theta for your different users.

And what we showed earlier in this video is that if your users are willing to give you parameters, then you can estimate features for the different movies.

So this is kind of a chicken and egg problem. Which comes first? You know, do we want if we can get the thetas, we can know the Xs. If we have the Xs, we can learn the thetas.

And what you can do is, and then this actually works, what you can do is in fact randomly guess some value of the thetas.

(2)

Now based on your initial random guess for the thetas, you can then go ahead and use the procedure that we just talked about in order to learn features for your different movies.

Now given some initial set of features for your movies you can then use this first method that we talked about in the previous video to try to get an even better estimate for your parameters theta. Now that you have a better setting of the parameters theta for your users, we can use that to maybe even get a better set of features and so on. We can sort of keep iterating, going back and forth and optimizing theta, x theta, x theta, nd this actually works and if you do this, this will actually cause your album to converge to a reasonable set of features for you movies and a reasonable set of parameters for your different users.

So this is a basic collaborative filtering algorithm. This isn't actually the final algorithm that we're going to use. In the next video we are going to be able to improve on this algorithm and make it quite a bit more computationally efficient. But, hopefully this gives you a sense of how you can formulate a problem where you can simultaneously learn the parameters and simultaneously learn the features from the different movies.

(3)

And for this problem, for the recommender system problem, this is possible only because each user rates multiple movies and hopefully each movie is rated by multiple users. And so you can do this back and forth process to estimate theta and x. So to summarize, in this video we've seen an initial collaborative filtering algorithm. The term collaborative filtering refers to the observation that when you run this algorithm with a large set of users, what all of these users are effectively doing are sort of collaboratively--or collaborating to get better movie ratings for everyone because with every user rating some subset with the movies, every user is helping the algorithm a little bit to learn better features,

and then by helping -- by rating a few movies myself, I will be helping

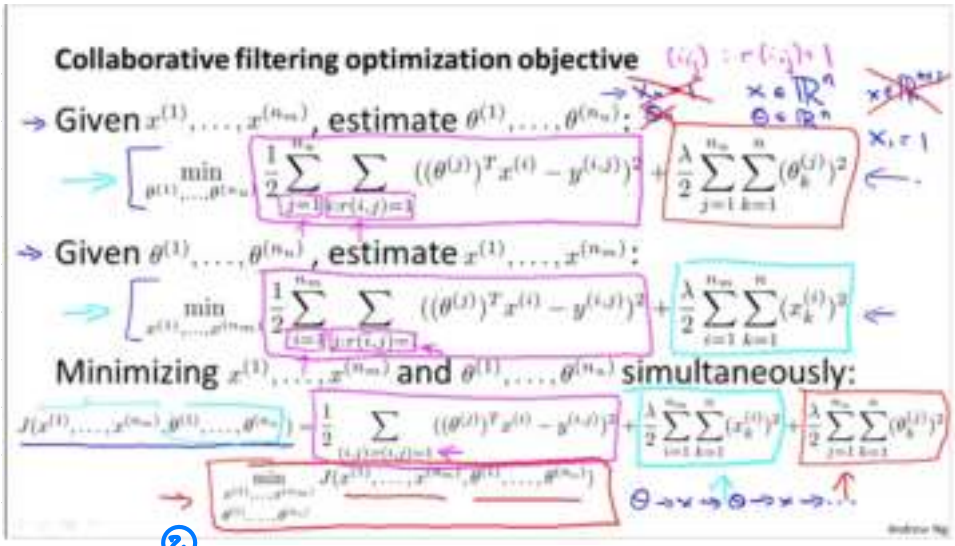
the system learn better features and then these features can be used by the system to make better movie predictions for everyone else. And so there is a sense of collaboration where every user is helping the system learn better features for the common good. This is this collaborative filtering. And, in the next video what we going to do is take the ideas that have worked out, and try to develop a better an even better algorithm, a slightly better technique for collaborative filtering.

Collaborative Filtering

Recommender Systems

Collaborative Filtering Algorithm

In the last couple videos, we talked about the ideas of how, first, if you're given features for movies, you can use that to learn parameters data for users. And second, if you're given parameters for the users, you can use that to learn features for the movies. In this video we're going to take those ideas and put them together to come up with a collaborative filtering algorithm.



① So one of the things we worked out earlier is that if you have features for the movies then you can solve this minimization problem to find the parameters theta for your users. And then we also worked that out, if you are given the parameters theta, you can also use that to estimate the features x, and you can do that by solving this minimization problem.

So one thing you could do is actually go back and forth. Maybe randomly initialize the parameters and then solve for theta, solve for x, solve for theta, solve for x. But, it turns out that there is a more efficient algorithm that doesn't need to go back and forth between the x's and the thetas, but that can solve for theta and x simultaneously. And here it is. What we are going to do, is basically take both of these optimization objectives, and put them into the same objective. So I'm going to define the new optimization objective j, which is a cost function, that is a function of my features x and a function of my parameters theta. And, it's basically the two optimization objectives I had on top, but I put together.

So, in order to explain this, first, I want to point out that this term over here, this squared error term, is the same as this squared error term and the summations look a little bit different, but let's see what the summations are really doing. The first summation is sum over all users J and then sum over all movies rated by that user.

② So, this is really summing over all pairs IJ, that correspond to a movie that was rated by a user. Sum over J says, for every user, the sum of all the movies rated by that user.

This summation down here, just does things in the opposite order. This says for every movie I, sum over all the users J that have rated that movie and so, you know these summations, both of these are just summations over all pairs ij for which r of i J is equal to 1. It's just something over all the user movie pairs for which you have a rating.

and so those two terms up there is just exactly this first term, and I've just written the summation here explicitly,

where I'm just saying the sum of all pairs IJ, such that RIJ is equal to 1. So what we're going to do is define a combined optimization objective that we want to minimize in order to solve simultaneously for x and theta.

③ And then the other terms in the optimization objective are this, which is a regularization in terms of theta. So that came down here and the final piece is this term which is my optimization objective for the x's and that became this. And this optimization objective j actually has an interesting property that if you were to hold the x's constant and just minimize with respect to the thetas then you'd be solving exactly this problem, whereas if you were to do the opposite, if you were to hold the thetas constant, and minimize j only with respect to the x's, then it becomes equivalent to this. Because either this term or this term is constant if you're minimizing only the respective x's or only respective thetas. So here's an optimization objective that puts together my cost functions in terms of x and in terms of theta.

And in order to come up with just one optimization problem, what we're going to do, is treat this cost function, as a function of my features x and of my user pro user parameters data and just minimize this whole thing, as a function of both the Xs and a function of the thetas.

④ And really the only difference between this and the older algorithm is that, instead of going back and forth, previously we talked about minimizing with respect to theta then minimizing with respect to x, whereas minimizing with respect to theta, minimizing with respect to x and so on. In this new version instead of sequentially going between the 2 sets of parameters x and theta, what we are going to do is just minimize with respect to both sets of parameters simultaneously.

Finally one last detail is that when we're learning the features this way. Previously we have been using this convention that we have a feature x0 equals one that corresponds to an interceptor.

When we are using this sort of formalism where we're are actually learning the features, we are actually going to do away with this convention.

And so the features we are going to learn x, will be in Rn.

⑤ Whereas previously we had features x and Rn + 1 including the intercept term. By getting rid of x0 we now just have x in Rn.

And so similarly, because the parameters theta is in the same dimension, we now also have theta in RN because if there's no x0, then there's no need parameter theta 0 as well.

And the reason we do away with this convention is because we're now learning all the features, right? So there is no need to hard code the feature that is always equal to one. Because if the algorithm really wants a feature that is always equal to 1, it can choose to learn one for itself. So if the algorithm chooses, it can set the feature X1 equals 1. So there's no need to hard code the feature of 001, the algorithm now has the flexibility to just learn it by itself. So, putting everything together, here is our collaborative filtering algorithm.

Collaborative filtering algorithm

1. Initialize $x^{(1)}, \ldots, x^{(n_m)}, \theta^{(1)}, \ldots, \theta^{(n_u)}$ to small random values.
2. Minimize $J(x^{(1)}, \ldots, x^{(n_m)}, \theta^{(1)}, \ldots, \theta^{(n_u)})$ using gradient descent (or an advanced optimization algorithm). E.g. for every $j = 1, \ldots, n_u, i = 1, \ldots, n_m$:

$$x_k^{(i)} := x_k^{(i)} - \alpha \left(\sum_{j:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)}) \theta_k^{(j)} + \lambda x_k^{(i)} \right)$$

$$\theta_k^{(j)} := \theta_k^{(j)} - \alpha \left(\sum_{i:r(i,j)=1} ((\theta^{(j)})^T x^{(i)} - y^{(i,j)}) x_k^{(i)} + \lambda \theta_k^{(j)} \right)$$

3. For a user with parameters θ and a movie with (learned) features x, predict a star rating of $\theta^T x$.

Andrew Ng

~~together~~, here is our collaborative filtering algorithm.

first we are going to initialize x and theta to small random values.

And this is a little bit like neural network training, where there we were also initializing all the parameters of a neural network to small random values.

Next we're then going to minimize the cost function using great intercepts or one of the advance optimization algorithms.

So, if you take derivatives you find that the great intercept like these and so this term here is the partial derivative of the cost function,

I'm not going to write that out, with respect to the feature value Xik and similarly

this term here is also a partial derivative value of the cost function with respect to the parameter theta that we're minimizing.
And just as a reminder, in this formula that we no longer have this X0 equals 1 and so we have that x is in Rn and theta is a Rn.

In this new formalism, we're regularizing every one of our perimeters theta, you know, every one of our parameters Xn.

There's no longer the special case theta zero, which was regularized differently, or which was not regularized compared to the parameters theta 1 down to theta. So there is now no longer a theta 0, which is why in these updates, I did not break out a special case for k equals 0.

So we then use gradient descent to minimize the cost function j with respect to the features x and with respect to the parameters theta.

And finally, given a user, if a user has some parameters, theta, and if there's a movie with some sort of learned features x, we would then predict that that movie would be given a star rating by that user of theta transpose j. Or just to fill those in, then we're saying that if user J has not yet rated movie I, then what we do is predict that user J is going to rate movie I according to theta J transpose Xi.

So that's the collaborative filtering algorithm and if you implement this algorithm you actually get a pretty decent algorithm that will simultaneously learn good features for hopefully all the movies as well as learn parameters for all the users and hopefully give pretty good predictions for how different users will rate different movies that they have not yet rated

In the algorithm we described, we initialized $x^{(1)}, \ldots, x^{(n_m)}$ and $\theta^{(1)}, \ldots, \theta^{(n_u)}$ to small random values. Why is this?

- [ ] This step is optional. Initializing to all 0's would work just as well.
- [ ] Random initialization is always necessary when using gradient descent on any problem.
- [ ] This ensures that $x^{(i)} \neq \theta^{(j)}$ for any i, j.
- [x] This serves as symmetry breaking (similar to the random initialization of a neural network's parameters) and ensures the algorithm learns features $x^{(1)}, \ldots, x^{(n_m)}$ that are different from each other.

✓ Correct

Low Rank Matrix Factorization

Recommender System

Vectorization: Low rank matrix factorization

In the last few videos, we talked about a collaborative filtering algorithm. In this video I'm going to say a little bit about the vectorization implementation of this algorithm. And also talk a little bit about other things you can do with this algorithm. For example, one of the things you can do is, given one product can you find other products that are related to this so that for example, a user has recently been looking at one product. Are there other related products that you could recommend to this user? So let's see what we could do about that.

What I'd like to do is work out an alternative way of writing out the predictions of the collaborative filtering algorithm.

Collaborative filtering

Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)
Love at last	5	5	0	0
Romance forever	5	?	?	0
Cute puppies of love	?	4	0	?
Nonstop car chases	0	0	5	4
Swords vs. karate	0	0	5	?

$n_m = 5$, $n_u = 4$

$$Y = \begin{bmatrix} 5 & 5 & 0 & 0 \\ 5 & ? & ? & 0 \\ ? & 4 & 0 & ? \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 5 & 0 \end{bmatrix}$$

$y^{(i,j)}$

To start, here is our data set with our five movies and what I'm going to do is take all the ratings by all the users and group them into a matrix. So, here we have five movies and four users, and so this matrix y is going to be a 5 by 4 matrix. It's just you know, taking all of the elements, all of this data.

Including question marks, and grouping them into this matrix. And of course the elements of this matrix of the (i, j) element of this matrix is really what we were previously writing as y superscript i, j. It's the rating given to movie i by user j. Given this matrix y of all the ratings that we have,

there's an alternative way of writing out all the predictive ratings of the algorithm. And, in particular if you look at what a certain user predicts on a certain movie, what user j predicts on movie i is given by this formula.

And so, if you have a matrix of the predicted ratings, what you would have is the following

matrix where the i, j entry

So this corresponds to the rating that we predict using j will give to movie i

is exactly equal to that theta j transpose xi, and so, you know, this is a matrix where this first element the one-one element is a predictive rating of user one or movie one and this element, this is the one-two element is the predicted rating of user two on movie one, and so on, and this is the predicted rating of user one on the last movie and if you want, you know, this rating is what we would have predicted for this value

Collaborative filtering

Predicted ratings:

$$Y = \begin{bmatrix} 5 & 5 & 0 & 0 \\ 5 & ? & ? & 0 \\ ? & 4 & 0 & ? \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 5 & 0 \end{bmatrix}$$

$X\,(\Theta)^T$

$(\Theta^{(j)})^T(x^{(i)})$

(i,j)

$$X = \begin{bmatrix} -(x^{(1)})^T- \\ -(x^{(2)})^T- \\ \vdots \\ -(x^{(n_m)})^T- \end{bmatrix} \qquad \Theta = \begin{bmatrix} -(\theta^{(1)})^T- \\ -(\theta^{(2)})^T- \\ \vdots \\ -(\theta^{(n_u)})^T- \end{bmatrix}$$

→ Low rank matrix factorization

and this rating is what we would have predicted for that value, and so on.

Now, given this matrix of predictive ratings there is then a simpler or vectorized way of writing these out. In particular if I define the matrix x, and this is going to be just like the matrix we had earlier for linear regression to be

sort of x1 transpose x2

transpose down to

x of nm transpose. So I'm take all the features for my movies and stack them in rows. So if you think of each movie as one example and stack all of the features of the different movies and rows. And if we also to find a matrix capital theta,

and what I'm going to do is take each of the per user parameter vectors, and stack them in rows, like so. So that's theta 1, which is the parameter vector for the first user.

And, you know, theta 2, and so, you must stack them in rows like this to define a matrix capital theta and so I have

nu parameter vectors all stacked in rows like this.

Now given this definition for the matrix x and this definition for the matrix theta in order to have a vectorized way of computing the matrix of all the predictions you can just compute x times

the matrix theta transpose, and that gives you a vectorized way of computing this matrix over here.

To give the collaborative filtering algorithm that you've been using another name. The algorithm that we're using is also called low rank

matrix factorization.

And so if you hear people talk about low rank matrix factorization that's essentially exactly the algorithm that we have been talking about. And this term comes from the property that this matrix x times theta transpose has a mathematical property in linear algebra called that this is a low rank matrix and so that's what gives rise to this name low rank matrix factorization for these algorithms, because of this low rank property of this matrix x theta transpose.

In case you don't know what low rank means or in case you don't know what a low rank matrix is, don't worry about it. You really don't need to know that in order to use this algorithm. But if you're an expert in linear algebra, that's what gives this algorithm, this other name of low rank matrix factorization. Finally, having run the collaborative filtering algorithm here's something else that you can do which is use the learned features in order to find related movies.

Let $X = \begin{bmatrix} - & (x^{(1)})^T & - \\ & \vdots & \\ - & (x^{(n_m)})^T & - \end{bmatrix}$, $\Theta = \begin{bmatrix} - & (\theta^{(1)})^T & - \\ & \vdots & \\ - & (\theta^{(n_u)})^T & - \end{bmatrix}$.

What is another way of writing the following:

$$\begin{bmatrix} (x^{(1)})^T(\theta^{(1)}) & \cdots & (x^{(1)})^T(\theta^{(n_u)}) \\ \vdots & \ddots & \vdots \\ (x^{(n_m)})^T(\theta^{(1)}) & \cdots & (x^{(n_m)})^T(\theta^{(n_u)}) \end{bmatrix}$$

- $X\Theta$
- $X^T\Theta$
- $X\Theta^T$
- $\Theta^T X^T$

✓ Correct

Finding related movies

For each product i, we learn a feature vector $x^{(i)} \in \mathbb{R}^n$.

→ x_1 = romance, x_2 = action, x_3 = comedy, x_4 = ...

How to find movies j related to movie i?

small $\|x^{(j)} - x^{(i)}\|$ → movie j and i are "similar"

5 most similar movies to movie i:

→ Find the 5 movies j with the smallest $\|x^{(j)} - x^{(i)}\|$.

(1)

Specifically for each product i really for each movie i, we've

learned a feature vector xi. So, you know, when you learn a certain features without really know that can the advance what the different features are going to be, but if you run the algorithm and perfectly the features will tend to capture what are the important aspects of these different movies or different products or what have you. What are the important aspects that cause some users to like certain movies and cause some users to like different sets of movies. So maybe you end up learning a feature, you know, where x1 equals romance, x2 equals action similar to an earlier video and maybe you learned a different feature x3 which is a degree to which this is a comedy. Then some feature x4 which is, you know, some other thing. And you have N features all together and after

you have learned features it's actually often pretty difficult to go in to the learned features and come up with a human understandable interpretation of what these features really are. But in practice, you know, the features even though these features can be hard to visualize. It can be hard to figure out just what these features are.

(2)

Usually, it will learn features that are very meaningful for capturing whatever are the most important or the most salient properties of a movie that causes you to like or dislike it. And so now let's say we want to address the following problem.

Say you have some specific movie i and you want to find other movies j that are related to that movie. And so well, why would you want to do this? Right, maybe you have a user that's browsing movies, and they're currently watching movie j, than what's a reasonable movie to recommend to them to watch after they're done with movie j? Or if someone's recently purchased movie j, well, what's a different movie that would be reasonable to recommend to them for them to consider purchasing.

So, now that you have learned these feature vectors, this gives us a very convenient way to measure how similar two movies are. In particular, movie i has a feature vector xi. and so if you can find a different movie, j, so that the distance between xi and xj is small,

(3)

then this is a pretty strong indication that, you know, movies j and i are somehow similar. At least in the sense that some of them likes movie i, maybe more likely to like movie j as well. So, just to recap, if your user is looking at some movie i and if you want to find the 5 most similar movies to that movie in order to recommend 5 new movies to them, what you do is find the five movies j, with the smallest distance between the features between these different movies. And this could give you a few different movies to recommend to your user. So with that, hopefully, you now know how to use a vectorized implementation to compute all the predicted ratings of all the users and all the movies, and also how to do things like use learned features to find what might be movies and what might be products that aren't related to each other.

Low Rank Matrix Factorization

Recommender System

Implementational Detail: Mean Normalization

By now you've seen all of the main pieces of the recommender system algorithm or the collaborative filtering algorithm.

In this video I want to just share one last implementational detail,

namely mean normalization, which can sometimes just make the algorithm work a little bit better.

Users who have not rated any movies

Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)	Eve (5)
Love at last	5	5	0	0	?
Romance forever	5	?	?	0	?
Cute puppies of love	?	4	0	?	?
Nonstop car chases	0	0	5	4	?
Swords vs. karate	0	0	5	?	?

$$Y = \begin{bmatrix} 5 & 5 & 0 & 0 & ? \\ 5 & ? & ? & 0 & ? \\ ? & 4 & 0 & ? & ? \\ 0 & 0 & 5 & 4 & ? \\ 0 & 0 & 5 & 0 & ? \end{bmatrix}$$

$$\min_{\theta^{(1)},\dots,\theta^{(n_u)}} \frac{1}{2} \sum_{(i,j):r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)}\right)^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^{n} (\theta_k^{(j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^{n} (x_k^{(i)})^2$$

$n = 2$ $\quad \theta^{(1)}, \theta^{(2)}, \theta^{(3)}, \theta^{(4)} \in \mathbb{R}^2$ $\quad \theta^{(5)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\frac{\lambda}{2}\left[(\theta_1^{(5)})^2 + (\theta_2^{(5)})^2\right]$$

$$(\theta^{(5)})^T x^{(i)} = 0$$

(1)

To motivate the idea of mean normalization, let's

consider an example of where there's a user that has not rated any movies.

So, in addition to our four users, Alice, Bob, Carol, and Dave, I've added a fifth user, Eve, who hasn't rated any movies.

Let's see what our collaborative filtering algorithm will do on this user.

Let's say that n is equal to 2 and so we're going to learn two features and we are going to have to learn a parameter vector theta 5, which is going to be in R2, remember this is now vectors in Rn not Rn+1,

we'll learn the parameter vector theta 5 for our user number 5, Eve.

So if we look in the first term in this optimization objective, well the user Eve hasn't rated any movies, so there are no movies for which Rij is equal to one for the user Eve and so this first term plays no role at all in determining theta 5

(2)

because there are no movies that Eve has rated.

And so the only term that effects theta 5 is this term. And so we're saying that we want to choose vector theta 5 so that the last regularization term is as small as possible. In other words we want to minimize this lambda over 2 theta 5 subscript 1 squared

plus theta 5 subscript 2 squared so that's the component of the regularization term that corresponds to user 5, and of course if your goal is to minimize this term, then what you're going to end up with is just theta 5 equals 0 0.

Because a regularization term is encouraging us to set parameters close to 0 and if there is no data to try to pull the parameters away from 0, because this first term

doesn't effect theta 5, we just end up with theta 5 equals the vector of all zeros. And so when we go to predict how user 5 would rate any movie, we have that theta 5 transpose xi,

(3)

for any i, that's just going

to be equal to zero. Because theta 5 is 0 for any value of x, this inner product is going to be equal to 0. And what we're going to have therefore, is that we're going to predict that Eve is going to rate every single movie with zero stars.

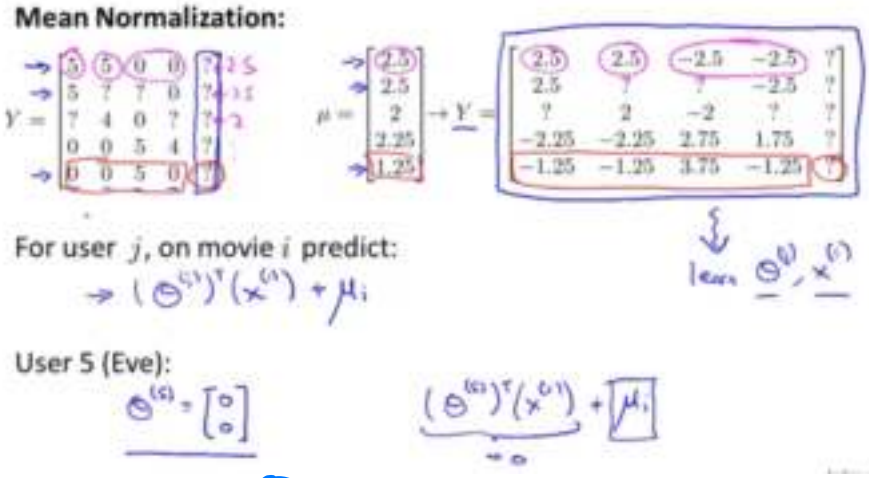
But this doesn't seem very useful does it? I mean if you look at the different movies, Love at Last, this first movie, a couple people rated it 5 stars.

And for even the Swords vs. Karate, someone rated it 5 stars. So some people do like some movies. It seems not useful to just predict that Eve is going to rate everything 0 stars. And in fact if we're predicting that eve is going to rate everything 0 stars, we also don't have any good way of recommending any movies to her, because you know all of these movies are getting exactly the same predicted rating for Eve so there's no one movie with a higher predicted rating that we could recommend to her, so, that's not very good.

The idea of mean normalization will let us fix this problem. So here's how it works.

As before let me group all of my movie ratings into this matrix Y, so just take all of these ratings and group them into matrix Y. And this column over here of all question marks corresponds to Eve's not having rated any movies.

Now to perform mean normalization what I'm going to do is compute the average rating that each movie obtained. And I'm going to store that in a vector that we'll call mu. So the first movie got two 5-star and two 0-star ratings, so the average of that is a 2.5-star rating. The second movie had an average of 2.5-stars and so on. And the final movie that has 0, 0, 5, 0. And the average of 0, 0, 5, 0, that averages out to an average of 1.25 rating. And what I'm going to do is look at all the movie ratings and I'm going to subtract off the mean rating. So this first element 5 I'm going to subtract off 2.5 and that gives me 2.5.



And the second element 5 subtract off of 2.5, get a 2.5. And then the 0, 0, subtract off 2.5 and you get -2.5, -2.5. In other words, what I'm going to do is take my matrix of movie ratings, take this wide matrix, and subtract form each row the average rating for that movie.

So, what I'm doing is just normalizing each movie to have an average rating of zero.

And so just one last example. If you look at this last row, 0 0 5 0. We're going to subtract 1.25, and so I end up with

these values over here. So now and of course the question marks stay a question

mark. So each movie in this new matrix Y has an average rating of 0.

What I'm going to do then, is take this set of ratings and use it with my collaborative filtering algorithm. So I'm going to pretend that this was the data that I had gotten from my users, or pretend that these are the actual ratings I had gotten from the users, and I'm going to use this as my data set with which to learn my parameters theta J and my features XI - from these mean normalized movie ratings.

When I want to make predictions of movie ratings, what I'm going to do is the following: for user J on movie I, I'm gonna predict theta J transpose XI, where X and theta are the parameters that I've learned from this mean normalized data set. But, because on the data set, I had subtracted off the means in order to make a prediction on movie i, I'm going to need to add back in the mean, and so I'm going to add back in mu i. And so that's going to be my prediction where in my training data subtracted off all the means and so when we make predictions and we need

to add back in these means mu i for movie i. And so specifically if you user 5 which is Eve, the same argument as the previous slide still applies in the sense that Eve had not rated any movies and so the learned parameter for user 5 is still going to be equal to 0, 0. And so what we're going to get then is that on a particular movie i we're going to predict for Eve theta 5, transpose xi plus

add back in mu i and so this first component is going to be equal to zero, if theta five is equal to zero. And so on movie i, we are going to end a predicting mu i. And, this actually makes sense. It means that on movie 1 we're going to predict Eve rates it 2.5. On movie 2 we're gonna predict Eve rates it 2.5. On movie 3 we're gonna predict Eve rates it at 2 and so on. This actually makes sense, because it says that if Eve hasn't rated any movies and we just don't know anything about this new user Eve, what we're going to do is just predict for each of the movies, what are the average rating that those movies got.

Finally, as an aside, in this video we talked about mean normalization, where we normalized each row of the matrix y,

to have mean 0. In case you have some movies with no ratings, so it is analogous to a user who hasn't rated anything, but in case you have some movies with no ratings, you can also play with versions of the algorithm, where you normalize the different columns to have means zero, instead of normalizing the rows to have mean zero, although that's maybe less important, because if you really have a movie with no rating, maybe you just shouldn't recommend that movie to anyone, anyway. And so, taking care of the case of a user who hasn't rated anything might be more important than taking care of the case of a movie that hasn't gotten a single rating.

So to summarize, that's how you can do mean normalization as a sort of pre-processing step for collaborative filtering. Depending on your data set, this might some times make your implementation work just a little bit better.

We talked about mean normalization. However, unlike some other applications of feature scaling, we did not scale the movie ratings by dividing by the range (max - min value). This is because:

- This sort of scaling is not useful when the value being predicted is real-valued.
- All the movie ratings are already comparable (e.g., 0 to 5 stars), so they are already on similar scales.
- Subtracting the mean is mathematically equivalent to dividing by the range.
- This makes the overall algorithm significantly more computationally efficient.

✓ Correct

Gradient Descent with Large Datasets

Large scale Machine Learning

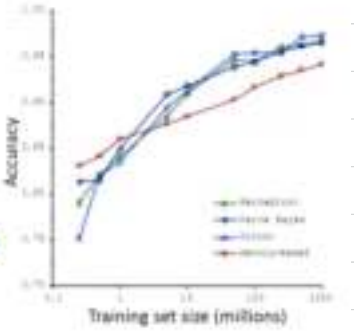
Learning with Large Datasets

In the next few videos, we'll talk about large scale machine learning. That is, algorithms but viewing with big data sets. If you look back at a recent 5 or 10-year history of machine learning. One of the reasons that learning algorithms work so much better now than even say, 5-years ago, is just the sheer amount of data that we have now and that we can train our algorithms on. In these next few videos, we'll talk about algorithms for dealing when we have such massive data sets.

Machine learning and data

Classify between confusable words.
E.g., {to, two, too}, {then, than}.

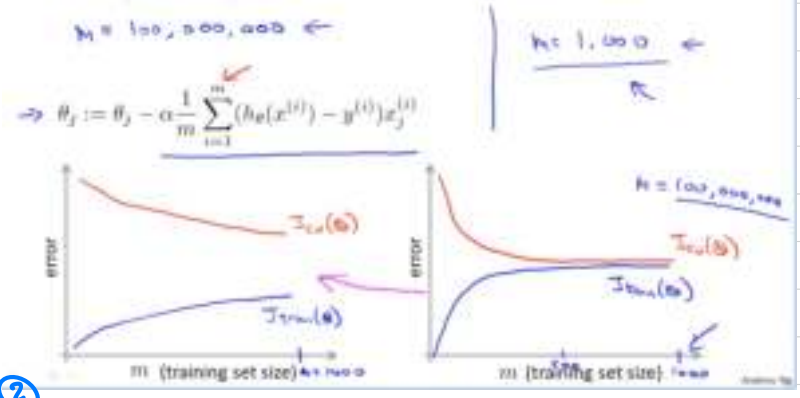
For breakfast I ate _____ eggs.



→ "It's not who has the best algorithm that wins.
It's who has the most data."

So why do we want to use such large data sets? We've already seen that one of the best ways to get a high performance machine learning system, is if you take a low-bias learning algorithm, and train that on a lot of data. And so, one early example we have already seen was this example of classifying between confusable words. So, for breakfast, I ate two (TWO) eggs and we saw in this example, these sorts of results, where, you know, so long as you feed the algorithm a lot of data, it seems to do very well. And so it's results like these that has led to the saying in machine learning that often it's not who has the best algorithm that wins. It's who has the most data. So

Learning with large datasets



① learning that often it's not who has the best algorithm that wins. It's who has the most data. So you want to learn from large data sets, at least when we can get such large data sets. But learning with large data sets comes with its own unique problems, specifically, computational problems. Let's say your training set size is M equals 100,000,000. And this is actually pretty realistic for many modern data sets. If you look at the US Census data set, if there are, you know, 300 million people in the US, you can usually get hundreds of millions of records. If you look at the amount of traffic that popular websites get, you easily get training sets that are much larger than hundreds of millions of examples. And let's say you want to train a linear regression model, or maybe a logistic regression model, in which case this is the gradient descent rule. And if you look at what you need to do to compute the gradient, which is this term over here, then when M is a hundred million, you need to carry out a summation over a hundred million terms, in order to compute these derivatives terms and to perform a single step of decent. Because of the computational expense of summing over a hundred million entries in order to compute just one step of gradient descent, in the next few videos we've spoken about techniques for either replacing this with something else or to find more efficient ways to compute this derivative. By the end of this

② sequence of videos on large scale machine learning, you know how to fit models, linear regression, logistic regression, neural networks and so on even today's data sets with, say, a hundred million examples. Of course, before we put in the effort into training a model with a hundred million examples, We should also ask ourselves, well, why not use just a thousand examples. Maybe we can randomly pick the subsets of a thousand examples out of a hundred million examples and train our algorithm on just a thousand examples. So before investing the effort into actually developing and the software needed to train these massive models is often a good sanity check, if training on just a thousand examples might do just as well. The way to sanity check of using a much smaller training set might do just as well, that is if using a much smaller n equals 1000 size training set, that might do just as well, it is the usual method of plotting the learning curves, so if you were to plot the learning curves and if your training objective were to look like this, that's J train theta. And if your cross-validation set objective, Jcv of theta would look like this, then this looks like a high-variance learning algorithm, and we will be more confident that adding extra training examples would improve performance. Whereas in

③ contrast if you were to plot the learning curves, if your training objective were to look like this, and if your cross-validation objective were to look like that, then this looks like the classical high-bias learning algorithm. And in the latter case, you know, if you were to plot this up to, say, m equals 1000 and so that is m equals 500 up to m equals 1000, then it seems unlikely that increasing m to a hundred million will do much better and then you'd be just fine sticking to n equals 1000, rather than investing a lot of effort to figure out how the scale of the algorithm. Of course, if you were in the situation shown by the figure on the right, then one natural thing to do would be to add extra features, or add extra hidden units to your neural network and so on, so that you end up with a situation closer to that on the left, where maybe this is up to n equals 1000, and this then gives you more confidence that trying to add infrastructure to change the algorithm to use much more than a thousand examples that might actually be a good use of your time. So in large-scale machine learning, we like to come up with computationally reasonable ways, or computationally efficient ways, to deal with very big data sets. In the next few videos, we'll see two main ideas. The first is called stochastic gradient descent and the second is called Map Reduce, for viewing with very big data sets. And after you've learned about these methods, hopefully that will allow you to scale up your learning algorithms to big data and allow you to get much better performance on many different applications.

Gradient Descent with Large Datasets

Large scale Machine learning

Stochastic Gradient Descent

For many learning algorithms, among them linear regression, logistic regression and neural networks, the way we derive the algorithm was by coming up with a cost function or coming up with an optimization objective. And then using an algorithm like gradient descent to minimize that cost function. We have a very large training set gradient descent becomes a computationally very expensive procedure. In this video, we'll talk about a modification to the basic gradient descent algorithm called Stochastic gradient descent, which will allow us to scale these algorithms to much bigger training sets. ~~Suppose you are training a linear regression model using~~

Linear regression with gradient descent

$$h_\theta(x) = \sum_{j=0}^{n} \theta_j x_j$$

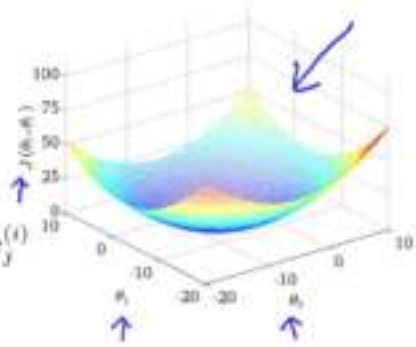
$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

Repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(for every $j = 0, \ldots, n$)

}



~~algorithms to much bigger training sets~~. Suppose you are training a linear regression model using gradient descent. As a quick recap, the hypothesis will look like this, and the cost function will look like this, which is the sum of one half of the average square error of your hypothesis on your m training examples, and the cost function we've already seen looks like this sort of bow-shaped function. So, plotted as function of the parameters theta 0 and theta 1, the cost function J is a sort of a bow-shaped function. And gradient descent looks like this, where in the inner loop of gradient descent you repeatedly update the parameters theta using that expression. Now in the rest of this video, I'm going to keep using linear regression as the running example. But the ideas here, the ideas of Stochastic gradient descent is fully general and also applies to other learning algorithms like logistic regression, neural networks and other algorithms that are based on training gradient descent on a specific training set. So here's a picture of what gradient descent

Linear regression with gradient descent

$$h_\theta(x) = \sum_{j=0}^{n} \theta_j x_j$$

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

Repeat {

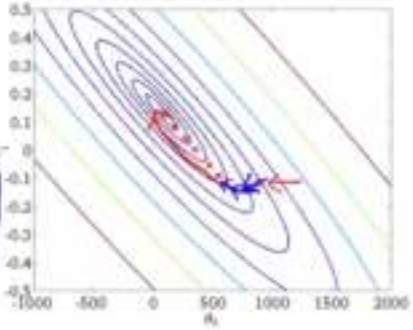
$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(for every $j = 0, \ldots, n$)

}

$m = 300,000,000$

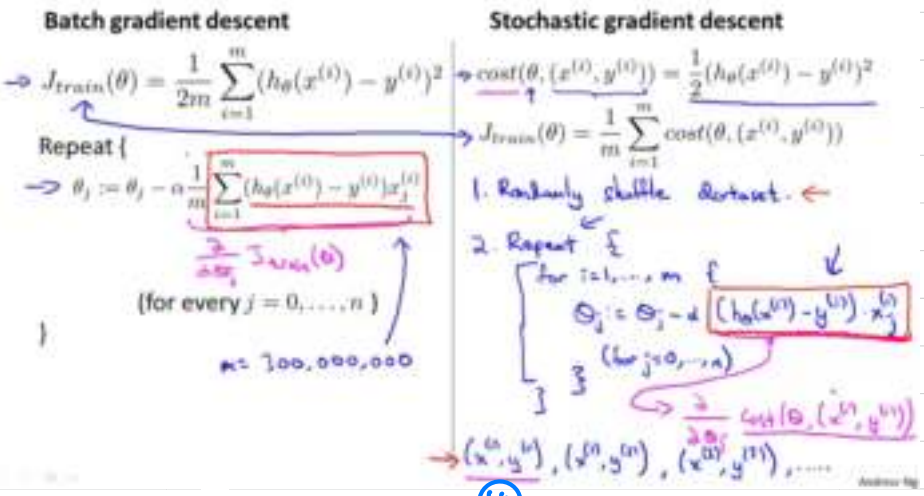
Batch gradient descent



① does, if the parameters are initialized to the point there then as you run gradient descent different iterations of gradient descent will take the parameters to the global minimum. So take a trajectory that looks like that and heads pretty directly to the global minimum. Now, the problem with gradient descent is that if m is large. Then computing this derivative term can be very expensive, because the surprise, summing over all m examples. So if m is 300 million, alright. So in the United States, there are about 300 million people. And so the US or United States census data may have on the order of that many records. So you want to fit the linear regression model to that then you need to sum over 300 million records. And that's very expensive. To give the algorithm a name, this particular version of gradient descent is also called Batch gradient descent. And the term Batch refers to the fact that we're looking at all of the training examples at a time. We call it sort of a batch of all of the training examples. And it really isn't the, maybe the best name but this is what machine learning people call this particular version of gradient descent. And if you imagine really that you have 300 million census records stored away on disc

② The way this algorithm works is you need to read into your computer memory all 300 million records in order to compute this derivative term. You need to stream all of these records through computer because you can't store all your records in computer memory. So you need to read through them and slowly, you know, accumulate the sum in order to compute the derivative. And then having done all that work, that allows you to take one step of gradient descent. And now you need to do the whole thing again. You know, scan through all 300 million records, accumulate these sums. And having done all that work, you can take another little step using gradient descent. And then do that again. And then you take yet a third step. And so on. And so it's gonna take a long time in order to get the algorithm to converge. In contrast to Batch gradient descent, what we are going to do is come up with a different algorithm that doesn't need to look at all the training examples in every single iteration, but that needs to look at only a single training example in one iteration. ~~Before moving on to the new algorithm, here's just a Batch~~

training example in one iteration. Before moving on to the new algorithm, here's just a Batch gradient descent algorithm written out again with that being the cost function and that being the update and of course this term here, that's used in the gradient descent rule, that is the partial derivative with respect to the parameters theta J of our optimization objective, J train of theta. Now, let's look at the more efficient algorithm that scales better to large data sets. In order to work off the algorithms called Stochastic gradient descent, this vectors the cost function in a slightly different way then they define the cost of the parameter theta with respect to a training example x(i), y(i) to be equal to one half times the squared error that my hypothesis incurs on that example, x(i), y(i). So this cost function term really measures how well is my hypothesis doing on a single example x(i), y(i). Now you notice that the overall cost function j train can now be written in this equivalent form. So j train is just the average over my m training examples of the cost of my hypothesis on that example x(i), y(i). Armed with this view of the cost function for linear regression, let me now write out what Stochastic gradient descent does. The first step of



looking at only this first example, it's gonna take like a basically a little gradient descent step with respect to the cost of just this first training example. So in other words, we're going to look at the first example and modify the parameters a little bit to fit just the first training example a little bit better. Having done this inside this inner for-loop is then going to go on to the second training example. And what it's going to do there is take another little step in parameter space, so modify the parameters just a little bit to try to fit just a second training example a little bit better. Having done that, is then going to go onto my third training example. And modify the parameters to try to fit just the third training example a little bit better, and so on until you know, you get through the entire training set. And then this ultra repeat loop may cause it to take multiple passes over the entire training set. This view of Stochastic gradient descent also motivates why we wanted to start by randomly shuffling the data set. This doesn't show us that when we scan through the training site here, that we end up visiting the training examples in some sort of randomly sorted order. Depending on whether your data already came randomly sorted or whether it came

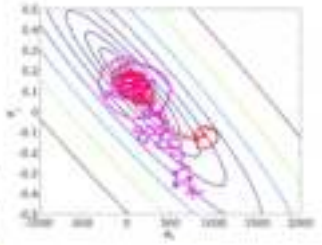
Stochastic gradient descent is to randomly shuffle the data set. So by that I just mean randomly shuffle, or randomly reorder your m training examples. It's sort of a standard pre-processing step, come back to this in a minute. But the main work of Stochastic gradient descent is then done in the following. We're going to repeat for i equals 1 through m. So we'll repeatedly scan through my training examples and perform the following update. Gonna update the parameter theta j as theta j minus alpha times h of x(i) minus y(i) times x(i)j. And we're going to do this update as usual for all values of j. Now, you notice that this term over here is exactly what we had inside the summation for Batch gradient descent. In fact, for those of you that are calculus is possible to show that that term here, that's this term here, is equal to the partial derivative with respect to my parameter theta j of the cost of the parameters theta on x(i), y(i). Where cost is of course this thing that was defined previously. And just the wrap of the algorithm, let me close my curly braces over there. So what Stochastic gradient descent is doing is it is actually scanning through the training examples. And first it's gonna look at my first training example x(1), y(1). And then

originally sorted in some strange order, in practice this would just speed up the conversions to Stochastic gradient descent just a little bit. So in the interest of safety, it's usually better to randomly shuffle the data set if you aren't sure if it came to you in randomly sorted order. But more importantly another view of Stochastic gradient descent is that it's a lot like descent but rather than wait to sum up these gradient terms over all m training examples, what we're doing is we're taking this gradient term using just one single training example and we're starting to make progress in improving the parameters already. So rather than, you know, waiting 'till taking a path through all 300,000 United States Census records, say, rather than needing to scan through all of the training examples before we can modify the parameters a little bit and make progress towards a global minimum. For Stochastic gradient descent instead we just need to look at a single training example and we're already starting to make progress in this case of parameters towards, moving the parameters towards the global minimum. So, here's the algorithm written

towards, moving the parameters towards the global minimum. So, here's the algorithm written out again where the first step is to randomly shuffle the data and the second step is where the real work is done, where that's the update with respect to a single training example x(i), y(i). So, let's see what this algorithm does to the parameters. Previously, we saw that when we are using Batch gradient descent, that is the algorithm that looks at all the training examples in time, Batch gradient descent will tend to, you know, take a reasonably straight line trajectory to get to the global minimum like that. In contrast with Stochastic gradient descent every iteration is going to be much faster because we don't need to sum up over all the training examples. But every iteration is just trying to fit single training example better. So, if we were to start stochastic gradient descent, oh, let's start stochastic gradient descent at a point like that. The first iteration, you know, may take the parameters in that direction and maybe the second iteration looking at just the second example maybe just by chance, we get more unlucky and actually head in a bad direction with the parameters like that. In the third iteration where we tried to modify the parameters to fit just the third training examples better, maybe we'll end up heading in that direction. And then we'll look at the fourth training example and we will do that. The fifth example, sixth example, 7th and so on. And as you run Stochastic gradient descent, what you find is that it will generally move the parameters in the direction of the global minimum, but not always. And so take some more random-looking, circuitous path to watch the global minimum. And in fact as you run Stochastic gradient descent it doesn't actually converge in the same same sense as Batch gradient descent does and what it ends up doing is wandering around continuously in some region that's in some region close to the global minimum, but it doesn't just get to the global minimum and stay there. But in practice this isn't a problem because, you know, so long as the parameters end up in some region there maybe it is pretty close to the global minimum. So, as parameters end up pretty close to the global minimum, that will be a pretty good hypothesis and so usually running Stochastic gradient descent we get a parameter near the global minimum and that's good enough for, you know, essentially any, most practical purposes. Just one final detail. In Stochastic gradient descent, we had this outer loop repeat which says to do this inner loop multiple times. So, how many times do we repeat this outer loop? Depending on the size of the training set, doing this loop just a single time may be enough.

Stochastic gradient descent

1. Randomly shuffle (reorder) training examples
2. Repeat { 1-10×
   for $i := 1, \ldots, m$ {
   $\theta_j := \theta_j - \alpha(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$
   (for every $j = 0, \ldots, n$)
   }
   } $m = 300,000,000$



And up to, you know, maybe 10 times may be typical so we may end up repeating this inner loop anywhere from once to ten times. So if we have a you know, truly massive data set like the this US census gave us that example that I've been talking about with 300 million examples, it is possible that by the time you've taken just a single pass through your training set. So, this is for i equals 1 through 300 million. It's possible that by the time you've taken a single pass through your data set you might already have a perfectly good hypothesis. In which case, you know, this inner loop you might need to do only once if m is very, very large. But in general taking anywhere from 1 through 10 passes through your data set, you know, maybe fairly common. But really it depends on the size of your training set. And if you contrast this to Batch gradient descent. With Batch gradient descent, after taking a pass through your entire training set, you would have taken just one single gradient descent steps. So one of these little baby steps of gradient descent where you just take one small gradient descent step and this is why Stochastic gradient descent can be much faster. So, that was the Stochastic gradient descent algorithm. And if you implement it, hopefully that will allow you to scale up many of your learning algorithms to much bigger data sets and get much more performance that way.

Which of the following statements about stochastic gradient descent are true? Check all that apply.

- [x] When the training set size m is very large, stochastic gradient descent can be much faster than gradient descent.

✓ Correct

- [x] The cost function $J_{train}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$ should go down with every iteration of batch gradient descent (assuming a well tuned learning rate α) but not necessarily with stochastic gradient descent.

✓ Correct

- [ ] Stochastic gradient descent is applicable only to linear regression but not to other models (such as logistic regression or neural networks).
- [x] Before beginning the main loop of stochastic gradient descent, it is a good idea to "shuffle" your training data into a random order.

✓ Correct

Gradient Descent with Large Datasets

Large scale Machine Learning

Mini-batch gradient descent

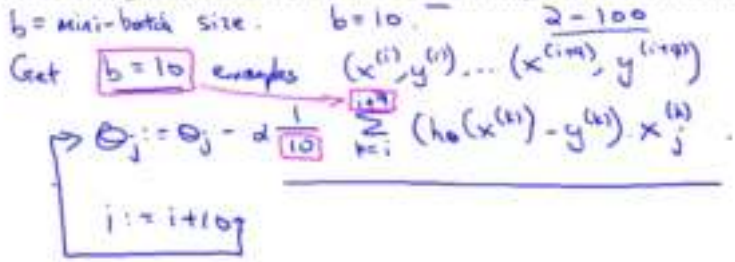
In the previous video, we talked about Stochastic gradient descent, and how that can be much faster than Batch gradient descent. In this video, let's talk about another variation on these ideas is called Mini-batch gradient descent they can work sometimes even a bit faster than stochastic gradient descent. To summarize the algorithms we talked about so far. In Batch gradient descent ~~gradient descent. To summarize the algorithms we talked about so far. In Batch gradient descent~~ we will use all m examples in each generation. Whereas in Stochastic gradient descent we will use a single example in each generation. What Mini-batch gradient descent does is somewhere in between. Specifically, with this algorithm we're going to use b examples in each iteration where b is a parameter called the "mini batch size" so the idea is that this is somewhat in-between Batch gradient descent and Stochastic gradient descent. This is just like batch gradient descent, except that I'm going to use a much smaller batch size. A typical choice for the value of b might be b equals 10, lets say, and a typical range really might be anywhere from b equals 2 up to b equals 100. So that will be a pretty typical range of values for the Mini-batch size. And the idea is that rather than using one example at a time or m examples at a time we will use b examples at a time. So let me just write this out informally, we're going to get, let's say, b. For this example, let's

Mini-batch gradient descent

→ Batch gradient descent: Use all m examples in each iteration

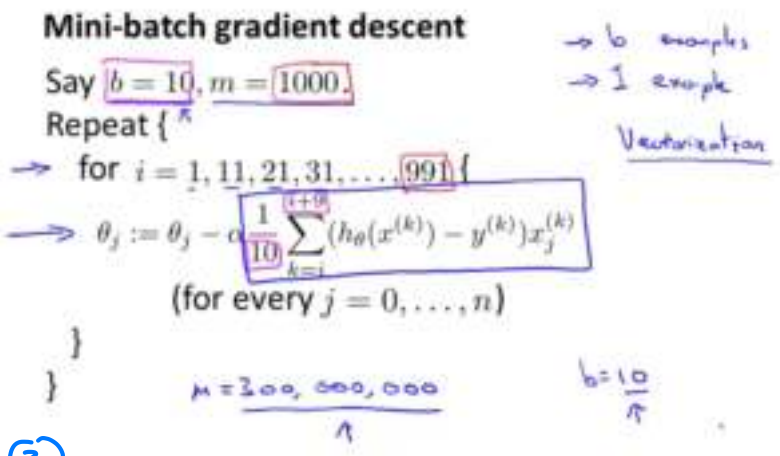
→ Stochastic gradient descent: Use 1 example in each iteration

Mini-batch gradient descent: Use b examples in each iteration



say b equals 10. So we're going to get, the next 10 examples from my training set so that may be some set of examples xi, yi. If it's 10 examples then the indexing will be up to x (i+9), y (i+9) so that's 10 examples altogether and then we'll perform essentially a gradient descent update using these 10 examples. So, that's any rate times one tenth times sum over k equals i through i+9 of h subscript theta of x(k) minus y(k) times x(k)j. And so in this expression, where summing the gradient terms over my ten examples. So, that's number ten, that's, you know, my mini batch size and just i+9 again, the 9 comes from the choice of the parameter b, and then after this we will then increase, you know, i by tenth, we will go on to the next ten examples and then keep moving like this. So just to write out the entire algorithm in full. ~~In order to simplify the indexing for this~~

like this. So just to write out the entire algorithm in full. In order to simplify the indexing for this one at the right top, I'm going to assume we have a mini-batch size of ten and a training set size of a thousand, what we're going to do is have this sort of form, for i equals 1 and that in 21's the stepping, in steps of 10 because we look at 10 examples at a time. And then we perform this sort of gradient descent update using ten examples at a time so this 10 and this i+9 those are consequence of having chosen my mini-batch to be ten. And you know, this ultimate four-loop, this ends at 991 here because if I have 1000 training samples then I need 100 steps of size 10 in order to get through my training set. So this is mini-batch gradient descent. Compared to batch gradient descent, this also allows us to make progress much faster. So we have again our running example of, you know, U.S. Census data with 300 million training examples, then what we're saying is after looking at just the first 10 examples we can start to make progress in improving the parameters theta so we don't need to scan through the entire training set. We just need to look at



the first 10 examples and this will start letting us make progress and then we can look at the second ten examples and modify the parameters a little bit again and so on. So, that is why Mini-batch gradient descent can be faster than batch gradient descent. Namely, you can start making progress in modifying the parameters after looking at just ten examples rather than needing to wait 'till you've scan through every single training example of 300 million of them. So, how about Mini-batch gradient descent versus Stochastic gradient descent. So, why do we want to look at b examples at a time rather than look at just a single example at a time as the Stochastic gradient descent? The answer is in vectorization. In particular, Mini-batch gradient descent is likely to outperform Stochastic gradient descent only if you have a good vectorized implementation. In that case, the sum over 10 examples can be performed in a more vectorized way which will allow you to partially parallelize your computation over the ten examples. So, in other words, by using appropriate vectorization to compute the rest of the terms, you can sometimes partially use the good numerical algebra libraries and parallelize your gradient computations over the b examples, whereas if you were looking at just a single example of time with Stochastic gradient descent then, you know, just looking at one example at a time there isn't much to parallelize over. At least there is less to parallelize over. One disadvantage of Mini-batch gradient descent is that there is now this extra parameter b, the Mini-batch size which you may have to fiddle with, and which may therefore take time. But if you have a good vectorized implementation this can sometimes run even faster that Stochastic gradient descent. So that was Mini-batch gradient descent which is an algorithm that in some sense does something that's somewhat in between what Stochastic gradient descent does and what Batch gradient descent does. And if you choose their reasonable value of b. I usually use b equals 10, but, you know, other values, anywhere from say 2 to 100, would be reasonably common. So we choose value of b and if you use a good vectorized implementation, sometimes it can be faster than both Stochastic gradient descent and faster than Batch gradient descent.

Suppose you use mini-batch gradient descent on a training set of size m, and you use a mini-batch size of b. The algorithm becomes the same as batch gradient descent if:

- ○ b = 1
- ○ b = m / 2
- ◉ b = m
- ○ None of the above

✓ **Correct**

Gradient Descent with Large Datasets

Large scale Machine Learning

Stochastic gradient descent Convergence

You now know about the stochastic gradient descent algorithm. But when you're running the algorithm, how do you make sure that it's completely debugged and is converging okay? Equally important, how do you tune the learning rate alpha with Stochastic Gradient Descent. In this video we'll talk about some techniques for doing these things, for making sure it's converging and for picking the learning rate alpha. Back when we were using batch gradient descent, our

Checking for convergence

- Batch gradient descent:
  - Plot $J_{train}(\theta)$ as a function of the number of iterations of gradient descent
  - $J_{train}(\theta) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$ — $m = 300,000,000$
- Stochastic gradient descent:
  - $cost(\theta, (x^{(i)}, y^{(i)})) = \frac{1}{2}(h_\theta(x^{(i)}) - y^{(i)})^2$ — $(x^{(i)}, y^{(i)})$, $(x^{(i+1)}, y^{(i+1)})$, ...
  - During learning, compute $cost(\theta, (x^{(i)}, y^{(i)}))$ before updating θ using $(x^{(i)}, y^{(i)})$.
  - Every 1000 iterations (say), plot $cost(\theta, (x^{(i)}, y^{(i)}))$ averaged over the last 1000 examples processed by algorithm.

(1) and for picking the learning rate alpha. Back when we were using batch gradient descent, our standard way for making sure that gradient descent was converging was we would plot the optimization cost function as a function of the number of iterations. So that was the cost function and we would make sure that this cost function is decreasing on every iteration. When the training set sizes were small, we could do that because we could compute the sum pretty efficiently. But when you have a massive training set size then you don't want to have to pause your algorithm periodically. You don't want to have to pause stochastic gradient descent periodically in order to compute this cost function since it requires a sum of your entire training set size. And the whole point of stochastic gradient was that you wanted to start to make progress after looking at just a single example without needing to occasionally scan through your entire training set right in the middle of the algorithm, just to compute things like the cost function of the entire training set. So for stochastic gradient descent, in order to check the algorithm is converging, here's what we can do instead. Let's take the definition of the cost that

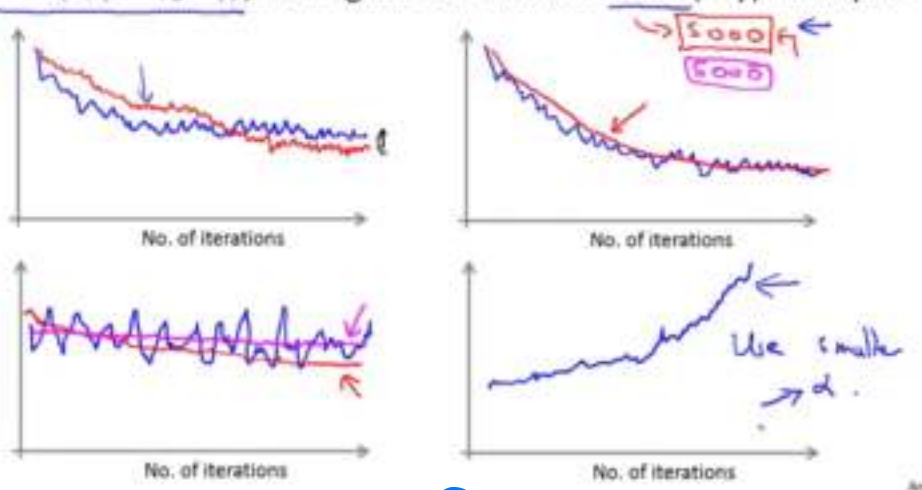
(2) we had previously. So the cost of the parameters theta with respect to a single training example is just one half of the square error on that training example. Then, while stochastic gradient descent is learning, right before we train on a specific example. So, in stochastic gradient descent we're going to look at the examples xi, yi, in order, and then sort of take a little update with respect to this example. And we go on to the next example, xi plus 1, yi plus 1, and so on, right? That's what stochastic gradient descent does. So, while the algorithm is looking at the example xi, yi, but before it has updated the parameters theta using that an example, let's compute the cost of that example. Just to say the same thing again, but using slightly different words. A stochastic gradient descent is scanning through our training set right before we have updated theta using a specific training example x(i) comma y(i) let's compute how well our hypothesis is doing on that training example. And we want to do this before updating theta because if we've just updated theta using example, you know, that it might be doing better on that example than what would

(3) be representative. Finally, in order to check for the convergence of stochastic gradient descent, what we can do is every, say, every thousand iterations, we can plot these costs that we've been computing in the previous step. We can plot those costs average over, say, the last thousand examples processed by the algorithm. And if you do this, it kind of gives you a running estimate of how well the algorithm is doing. on, you know, the last 1000 training examples that your algorithm has seen. So, in contrast to computing J<u>train periodically which needed to scan through the entire training set.</u> With this other procedure, well, as part of stochastic gradient descent, it doesn't cost much to compute these costs as well right before updating to parameter theta. And all we're doing is every thousand integrations or so, we just average the last 1,000 costs that we computed and plot that. And by looking at those plots, this will allow us to check if stochastic gradient descent is converging. So here are a few examples of what these plots might

stochastic gradient descent is converging. So here are a few examples of what these plots might look like. Suppose you have plotted the cost average over the last thousand examples, because these are averaged over just a thousand examples, they are going to be a little bit noisy and so, it may not decrease on every single iteration. Then if you get a figure that looks like this, So the plot is noisy because it's average over, you know, just a small subset, say a thousand training examples. If you get a figure that looks like this, you know that would be a pretty decent run with the algorithm, maybe, where it looks like the cost has gone down and then this plateau that looks kind of flattened out, you know, starting from around that point. look like, this is what your cost looks like then maybe your learning algorithm has converged. If you want to try using a smaller learning rate, something you might see is that the algorithm may initially learn more slowly so the cost goes down more slowly. But then eventually you have a smaller learning rate is actually possible for the algorithm to end up at a, maybe very slightly better solution. So the red line may represent the behavior of stochastic gradient descent using a slower, using a smaller learning rate.

Checking for convergence

Plot $cost(\theta, (x^{(i)}, y^{(i)}))$, averaged over the last 1000 (say) examples



well your learning learning algorithm is doing is, sort of, maybe it's more delayed because you get one data point on your plot only every 5,000 examples rather than every 1,000 examples. Along a similar vein some times you may run a gradient descent and end up with a plot that looks like this. And with a plot that looks like this, you know, it looks like the cost just is not decreasing at all. It looks like the algorithm is just not learning. It's just, looks like this here a flat curve and the cost is just not decreasing. But again if you were to increase this to averaging over a larger number of examples it is possible that you see something like this red line it looks like the cost actually is decreasing, it's just that the blue line averaging over 2, 3 examples, the blue line was too noisy so you couldn't see the actual trend in the cost actually decreasing and possibly averaging over 5,000 examples instead of 1,000 may help. Of course we averaged over a larger number examples that we've averaged here over 5,000 examples, I'm just using a different color, it is also possible that you that see a learning curve ends up looking like this. That it's still flat even when you average over a larger number of examples. And as you get that, then that's maybe

And the reason this is the case is because, you remember, stochastic gradient descent doesn't just converge to the global minimum, is that what it does is the parameters will oscillate a bit around the global minimum. And so by using a smaller learning rate, you'll end up with smaller oscillations. And sometimes this little difference will be negligible and sometimes with a smaller than you can get a slightly better value for the parameters. Here are some other things that might happen. Let's say you run stochastic gradient descent and you average over a thousand examples when plotting these costs. So, you know, here might be the result of another one of these plots. Then again, it kind of looks like it's converged. If you were to take this number, a thousand, and increase to averaging over 5 thousand examples. Then it's possible that you might get a smoother curve that looks more like this. And by averaging over, say 5,000 examples instead of 1,000, you might be able to get a smoother curve like this. And so that's the effect of increasing the number of examples you average over. The disadvantage of making this too big of course is that now you get one date point only every 5,000 examples. And so the feedback you get on how

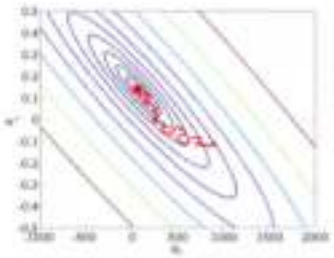
just a more firm verification that unfortunately the algorithm just isn't learning much for whatever reason. And you need to either change the learning rate or change the features or change something else about the algorithm. Finally, one last thing that you might see would be if you were to plot these curves and you see a curve that looks like this, where it actually looks like it's increasing. And if that's the case then this is a sign that the algorithm is diverging. And what you really should do is use a smaller value of the learning rate alpha. So hopefully this gives you a sense of the range of phenomena you might see when you plot these cost average over some range of examples as well as suggests the sorts of things you might try to do in response to seeing different plots. So if the plots looks too noisy, or if it wiggles up and down too much, then try increasing the number of examples you're averaging over so you can see the overall trend in the plot better. And if you see that the errors are actually increasing, the costs are actually increasing, try using a smaller value of alpha. Finally, it's worth examining the issue of the learning rate just a little bit more. We saw that when we run stochastic gradient descent, the algorithm will start here

Stochastic gradient descent

$$cost(\theta, (x^{(i)}, y^{(i)})) = \frac{1}{2}(h_\theta(x^{(i)}) - y^{(i)})^2$$

$$J_{train}(\theta) = \frac{1}{2m}\sum_{i=1}^{m} cost(\theta, (x^{(i)}, y^{(i)}))$$

1. Randomly shuffle dataset.
2. Repeat {
   for $i := 1, \ldots, m$ {
   $\theta_j := \theta_j - \alpha(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$
   (for $j = 0, \ldots, n$)
   }
   }



Learning rate α is typically held constant. Can slowly decrease α over time if we want θ to converge. (E.g. $\alpha = \frac{const1}{iterationNumber + const2}$)

① ~~Try using a smaller value of alpha~~. Finally, it's worth examining the issue of the learning rate just a little bit more. We saw that when we run stochastic gradient descent, the algorithm will start here and sort of meander towards the minimum And then it won't really converge, and instead it'll wander around the minimum forever. And so you end up with a parameter value that is hopefully close to the global minimum that won't be exact at the global minimum. In most typical implementations of stochastic gradient descent, the learning rate alpha is typically held constant. And so what you we end up is exactly a picture like this. If you want stochastic gradient descent to actually converge to the global minimum, there's one thing which you can do which is you can slowly decrease the learning rate alpha over time. So, a pretty typical way of doing that would be to set alpha equals some constant 1 divided by iteration number plus constant 2. So, iteration number is the number of iterations you've run of stochastic gradient descent, so it's really the number of training examples you've seen And const 1 and const 2 are additional parameters of the algorithm that you might have to play with a bit in order to get good

② performance. One of the reasons people tend not to do this is because you end up needing to spend time playing with these 2 extra parameters, constant 1 and constant 2, and so this makes the algorithm more finicky. You know, it's just more parameters able to fiddle with in order to make the algorithm work well. But if you manage to tune the parameters well, then the picture you can get is that the algorithm will actually around towards the minimum, but as it gets closer because you're decreasing the learning rate the meanderings will get smaller and smaller until it pretty much just to the global minimum. I hope this makes sense, right? And the reason this formula makes sense is because as the algorithm runs, the iteration number becomes large So alpha will slowly become small, and so you take smaller and smaller steps until it hopefully converges to the global minimum. So if you do slowly decrease alpha to zero you can end up with

③ a slightly better hypothesis. But because of the extra work needed to fiddle with the constants and because frankly usually we're pretty happy with any parameter value that is, you know, pretty close to the global minimum. Typically this process of decreasing alpha slowly is usually not done and keeping the learning rate alpha constant is the more common application of stochastic gradient descent although you will see people use either version. To summarize in this video we talk about a way for approximately monitoring how the stochastic gradient descent is doing in terms for optimizing the cost function. And this is a method that does not require scanning over the entire training set periodically to compute the cost function on the entire training set. But instead it looks at say only the last thousand examples or so. And you can use this method both to make sure the stochastic gradient descent is okay and is converging or to use it to tune the learning rate alpha.

Which of the following statements about stochastic gradient descent are true? Check all that apply.

- [ ] Picking a learning rate α that is very small has no disadvantage and can only speed up learning.
- [x] If we reduce the learning rate α (and run stochastic gradient descent long enough), it's possible that we may find a set of better parameters than with larger α.

✓ Correct

- [ ] If we want stochastic gradient descent to converge to a (local) minimum rather than wander all "oscillate" around it, we should slowly increase α over time.
- [x] If we plot $cost(\theta, (x^{(i)}, y^{(i)}))$ (averaged over the last 1000 examples) and stochastic gradient descent does not seem to be reducing the cost, one possible problem may be that the learning rate α is poorly tuned.

✓ Correct

Advanced Topics

Large scale machine learning

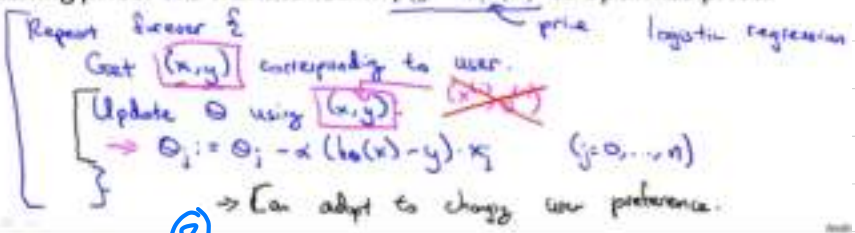
Online learning

In this video, I'd like to talk about a new large-scale machine learning setting called the online learning setting. The online learning setting allows us to model problems where we have a continuous flood or a continuous stream of data coming in and we would like an algorithm to learn from that. Today, many of the largest websites, or many of the largest website companies use different versions of online learning algorithms to learn from the flood of users that keep on coming to, back to the website. Specifically, if you have a continuous stream of data generated by a continuous stream of users coming to your website, what you can do is sometimes use an online learning algorithm to learn user preferences from the stream of data and use that to optimize some of the decisions on your website.

Online learning

Shipping service website where user comes, specifies origin and destination, you offer to ship their package for some asking price, and users sometimes choose to use your shipping service ($y = 1$), sometimes not ($y = 0$).

Features x capture properties of user, of origin/destination and asking price. We want to learn $p(y = 1|x; \theta)$ to optimize price.



(1) Suppose you run a shipping service, so, you know, users come and ask you to help ship their package from location A to location B and suppose you run a website, where users repeatedly come and they tell you where they want to send the package from, and where they want to send it to (so the origin and destination) and your website offers to ship the package for some asking price, so I'll ship your package for $50, I'll ship it for $20. And based on the price that you offer to the users, the users sometimes chose to use a shipping service, that's a positive example and sometimes they go away and they do not choose to purchase your shipping service. So let's say that we want a learning algorithm to help us to optimize what is the asking price that we want to offer to our users. And specifically, let's say we come up with some sort of features that capture properties of the users. If we know anything about the demographics, they capture, you know, the origin and destination of the package, where they want to ship the package. And what is the price that we offer to them for shipping the package, and what we want to do is learn what is the probability that they will elect to ship the package, using our shipping service given these features, and again just as a reminder these features X also captures the price that we're asking for. And so if we could estimate the chance that they'll agree to use our service for any given price, then we can try to pick a price so that they have a pretty high probability of choosing our website while simultaneously hopefully offering us a fair return, offering us a fair profit for shipping their package. So if we can learn this property of y equals 1 given any price and given the other features we could really use this to choose appropriate prices as new users come to us. So in order to model the probability of y equals 1, what we can do is use logistic regression or neural network or some other algorithm like that. But let's start with logistic regression.

(2) Now if you have a website that just runs continuously, here's what an online learning algorithm would do. I'm gonna write repeat forever. This just means that our website is going to, you know, keep on staying up. What happens on the website is occasionally a user will come and for the user that comes we'll get some x,y pair corresponding to a customer or to a user on the website. So the features x are, you know, the origin and destination specified by this user and the price that we happened to offer to them this time around, and y is either one or zero depending one whether or not they chose to use our shipping service. Now once we get this [x,y] pair, what an online learning algorithm does is then update the parameters theta using just this example x,y, and in particular we would update my parameters theta as Theta j get updated as Theta j minus the learning rate alpha times my usual gradient descent rule for logistic regression. So we do this for j equals zero up to n, and that's my close curly brace. So, for other learning algorithms instead of writing X-Y, right, I was writing things like Xi, Yi but in this online learning setting where actually discarding the notion of there being a fixed training set instead we have an algorithm. Now what happens as we get an example and then we learn using that example like so and then we throw that example away. We discard that example and we never use it again and so that's why we just look at one example at a time. We learn from that example. We discard it. Which is why, you know, we're also doing away with this notion of there being this sort of fixed training set indexed by i. And, if you really run a major website where you really have a continuous stream of users coming, then this sort of online learning algorithm is actually a pretty reasonable algorithm. Because of data is essentially free if you have so much data, that data is essentially unlimited then there is really may be no need to look at a training example more than once. Of course if we had only a small number of users then rather than using an online learning algorithm like this, you might be better off saving away all your data in a fixed training set and then running some algorithm over that training set. But if you really have a continuous stream of data, then an online learning algorithm can be very effective. I should mention also that one interesting effect of this sort of online learning algorithm is that it can adapt to changing user preferences.

(3) And in particular, if over time because of changes in the economy maybe users start to become more price sensitive and willing to pay, you know, less willing to pay high prices. Or if they become less price sensitive and they're willing to pay higher prices. Or if different things become more important to users, if you start to have new types of users coming to your website. This sort of online learning algorithm can also adapt to changing user preferences and kind of keep track of what your changing population of users may be willing to pay for. And it does that because if your pool of users changes, then these updates to your parameters theta will just slowly adapt your parameters to whatever your latest pool of users looks like. Here's another example of a cool

Other online learning example:

Product search (learning to search)

User searches for "Android phone 1080p camera"

Have 100 phones in store. Will return 10 results.

→ x = features of phone, how many words in user query match name of phone, how many words in query match description of phone, etc.

→ $y = 1$ if user clicks on link. $y = 0$ otherwise.

→ Learn $p(y = 1|x; \theta)$. predicted CTR

→ Use to show user the 10 phones they're most likely to click on.

Other examples: Choosing special offers to show user; customized selection of news articles; product recommendation; ...

your parameters to whatever your latest pool of users looks like. Here's another example of a sort of application to which you might apply online learning. this is an application in product search in which we want to apply learning algorithm to learn to give good search listings to a user. Let's say you run an online store that sells phones - that sells mobile phones or sells cell phones. And you have a user interface where a user can come to your website and type in the query like "Android phone 1080p camera". So 1080p is a type of a specification for a video camera that you might have on a phone, a cell phone, a mobile phone. Suppose, suppose we have a hundred phones in our store. And because of the way our website is laid out, when a user types in a query, if it was a search query, we would like to find a choice of ten different phones to show what to offer to the user. What we'd like to do is have a learning algorithm help us figure out what are the ten phones out of the 100 we should return the user in response to a user-search query like the one here. Here's how we can go about the problem. For each phone and given a specific user query, we can construct a feature vector X. So the feature vector X might capture different properties of the phone. It might capture things like, how similar the user search query is in the phones. We capture things like how many words in the user search query match the name of the phone, how many words in the user search query match the description of the phone and so on. So the features x capture properties of the phone and it captures things about how similar or how well the phone matches the user query along different dimensions. What we like to do is estimate the probability that a user will click on the link for a specific phone, because we want to show the user phones that they are likely to want to buy, want to show the user phones that they have high probability of clicking on in the web browser. So I'm going to define y equals one if the user clicks on the link for a phone and y equals zero otherwise and what I would like to do is learn the probability the user will click on a specific phone given, you know, the features x, which capture properties of the phone and how well the query matches the phone. To give this problem a name in the language of people that run websites like this, the problem of learning this is actually called the problem of learning the predicted click through rate, the predicted CTR. It just means learning the probability that the user will click on the specific link that you offer them, so CTR is an abbreviation for click through rate. And if you can estimate the predicted click through rate for any particular phone, what we can do is use this to show the user the ten phones that are most likely to click on, because out of the hundred phones, we can compute this for each of the 100 phones and just select the 10 phones that the user is most likely to click on, and this will be a pretty reasonable way to decide what ten results to show to the user. Just to be clear, suppose that every time a user does a search, we return ten results what that will do is it will actually give us ten x,y pairs, this actually gives us ten training examples every time a user comes to our website because, because for the ten phone that we chose to show the user, for each of those 10 phones we get a feature vector X, and for each of those 10 phones we show the user we will also get a value for y, we will also observe the value of y, depending on whether or not we clicked on that url or not and so, one way to run a website like this would be to continuously show the user, you know, your ten best guesses for what other phones they might like and so, each time a user comes you would get ten examples, ten x,y pairs, and then use an online learning algorithm to update the parameters using essentially 10 steps of gradient descent on these 10 examples, and then you can throw the data away, and if you really have a continuous stream of users coming to your website, this would be a pretty reasonable way to learn parameters for your algorithm so as to show the ten phones to your users that may be most promising and the most likely to click on. So, this is a product search problem or learning to rank phones, learning to search for phones example. So, I'll quickly mention a few others. One is, if you have a website and you're trying to decide, you know, what special offer to show the user, this is very similar to phones, or if you have a website and you show different users different news articles. So, if you're a news aggregator website, then you can again use a similar system to select, to show to the user, you know, what are the news articles that they are most likely to be interested in and what are the news articles that they are most likely to click on. Closely related to special offers, will we profit from recommendations. And in fact, if you have a collaborative filtering system, you can even imagine a collaborative filtering system giving you additional features to feed into a logistic regression classifier to try to predict the click through rate for different products that you might recommend to a user. Of course, I should say that any of these problems could also have been formulated as a standard machine learning problem, where you have a fixed training set. Maybe, you can run your website for a few days and then save away a training set, a fixed training set, and run a learning algorithm on that. But these are the actual sorts of problems, where you do see large companies get so much data, that there's really maybe no need to save away a fixed training set, but instead you can use an online learning algorithm to just learn continuously. from the data that users are generating on your website.

So, that was the online learning setting and as we saw, the algorithm that we apply to it is really very similar to this schotastic gradient descent algorithm, only instead of scanning through a fixed training set, we're instead getting one example from a user, learning from that example, then discarding it and moving on. And if you have a continuous stream of data for some application, this sort of algorithm may be well worth considering for your application. And of course, one advantage of online learning is also that if you have a changing pool of users, or if the things you're trying to predict are slowly changing like your user taste is slowly changing, the online learning algorithm can slowly adapt your learned hypothesis to whatever the latest sets of user behaviors are like as well.

Advanced Topics

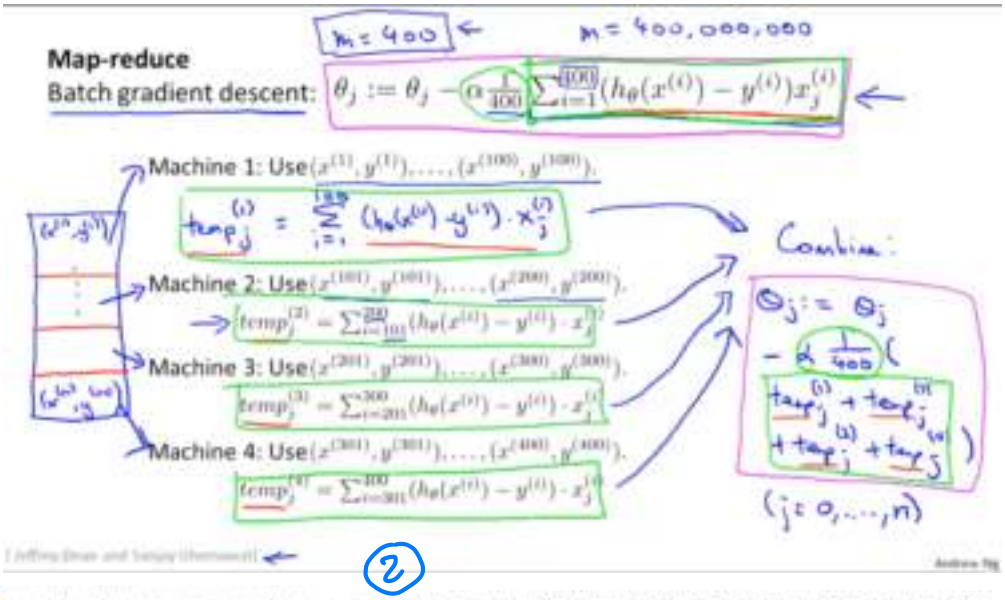
Large scale machine learning

Map-reduce and data parallelism

In the last few videos, we talked about stochastic gradient descent, and, you know, other variations of the stochastic gradient descent algorithm, including those adaptations to online learning, but all of those algorithms could be run on one machine, or could be run on one computer.

And some machine learning problems are just too big to run on one machine, sometimes maybe you just so much data you just don't ever want to run all that data through a single computer, no matter what algorithm you would use on that computer.

So in this video I'd like to talk about different approach to large scale machine learning, called the map reduce approach. And even though we have quite a few videos on stochastic gradient descent and we're going to spend relative less time on map reduce--don't judge the relative importance of map reduce versus the gradient descent based on the amount amount of time I spend on these ideas in particular. Many people will say that map reduce is at least an equally important, and some would say an even more important idea compared to gradient descent, only it's relatively simpler to explain, which is why I'm going to spend less time on it, but using these ideas you might be able to scale learning algorithms to even far larger problems than is possible using stochastic gradient descent.



[Jeffrey Dean and Sanjay Ghemawat]

Andrew Ng

(1)

Here's the idea. Let's say we want to fit a linear regression model or a logistic regression model or some such, and let's start again with batch gradient descent, so that's our batch gradient descent learning rule. And to keep the writing on this slide tractable, I'm going to assume throughout that we have m equals 400 examples. Of course, by our standards, in terms of large scale machine learning, you know m might be pretty small and so, this might be more commonly applied to problems, where you have maybe closer to 400 million examples, or some such, but just to make the writing on the slide simpler, I'm going to pretend we have 400 examples. So in that case, the batch gradient descent learning rule has this 400 and the sum from i equals 1 through 400 through my 400 examples here, and if m is large, then this is a computationally expensive step.

So, what the MapReduce idea does is the following, and I should say the map reduce idea is due to two researchers, Jeff Dean and Sanjay Gimawat. Jeff Dean, by the way, is one of the most legendary engineers in all of Silicon Valley and he kind of built a large fraction of the architectural infrastructure that all of Google runs on today.

But here's the map reduce idea. So, let's say I have some training set, if we want to denote by this box here of X Y pairs,

(2)

where it's X1, Y1, down to my 400 examples, Xm, Ym. So, that's my training set with 400 training examples.

In the MapReduce idea, one way to do, is split this training set in to different subsets.

I'm going to assume for this example that I have 4 computers, or 4 machines to run in parallel on my training set, which is why I'm splitting this into 4 machines. If you have 10 machines or 100 machines, then you would split your training set into 10 pieces or 100 pieces or what have you.

And what the first of my 4 machines is to do, say, is use just the first one quarter of my training set--so use just the first 100 training examples.

And in particular, what it's going to do is look at this summation, and compute that summation for just the first 100 training examples.

So let me write that up I'm going to compute a variable

temp 1 to superscript 1 the first machine J equals

sum from equals 1 through 100, and then I'm going to plug in exactly that term there--so I have h-theta, Xi, minus Yi

(3)

times Xj, right? So that's just that gradient descent term up there. And then similarly, I'm going to take the second quarter of my data and send it to my second machine, and my second machine will use training examples 101 through 200 and you will compute similar variables of a temp to j which is the same sum for index from examples 101 through 200. And similarly machines 3 and 4 will use the third quarter and the fourth quarter of my training set. So now each machine has to sum over 100 instead of over 400 examples and so has to do only a quarter of the work and thus presumably it could do it about four times as fast.

Finally, after all these machines have done this work, I am going to take these temp variables

and put them back together. So I take these variables and send them all to a You know centralized master server and what the master will do is combine these results together, and in particular, it will update my parameters theta j according to theta j gets updated as theta j

minus Of the learning rate alpha times one over 400 times temp, 1, J, plus temp 2j plus temp 3j

plus temp 4j and of course we have to do this separately for j equals 0. You know, up to and within this number of features.

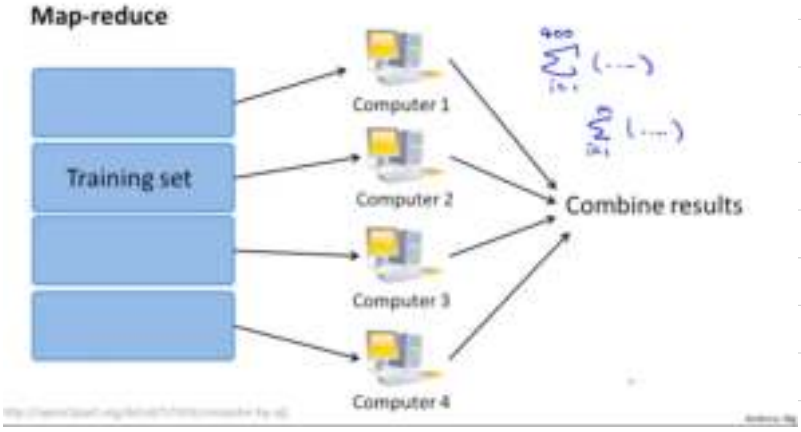
(4)

So operating this equation into I hope it's clear. So what this equation

is doing is exactly the same is that when you have a centralized master server that takes the results, the ten one j the ten two j ten three j and ten four j and adds them up and so of course the sum of these four things.

Right, that's just the sum of this, plus the sum of this, plus the sum of this, plus the sum of that, and those four things just add up to be equal to this sum that we've originally computing a batch stream descent.

And then we have the alpha times 1 of 400, alpha times 1 of 100, and this is exactly equivalent to the batch gradient descent algorithm, only, instead of needing to sum over all four hundred training examples on just one machine, we can instead divide up the work load on four machines.



So, here's what the general picture of the MapReduce technique looks like.

We have some training sets, and if we want to paralyze across four machines, we are going to take the training set and split it, you know, equally. Split it as evenly as we can into four subsets.

Then we are going to take the 4 subsets of the training data and send them to 4 different computers. And each of the 4 computers can compute a summation over just one quarter of the training set, and then finally take each of the computers takes the results, sends them to a centralized server, which then combines the results together. So, on the previous line in that example, the bulk of the work in gradient descent, was computing the sum from i equals 1 to 400 of something. So more generally, sum from i equals 1 to m of that formula for gradient descent. And now, because each of the four computers can do just a quarter of the work, potentially you can get up to a 4x speed up.

In particular, if there were no network latencies and no costs of the network communications to send the data back and forth, you can potentially get up to a 4x speed up. Of course, in practice, because of network latencies, the overhead of combining the results afterwards and other factors, in practice you get slightly less than a 4x speedup. But, none the less, this sort of macro juice approach does offer us a way to process much larger data sets than is possible using a single computer.

Map-reduce and summation over the training set

Many learning algorithms can be expressed as computing sums of functions over the training set.

E.g. for advanced optimization, with logistic regression, need:

$$J_{train}(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) - (1-y^{(i)}) \log(1-h_\theta(x^{(i)}))$$

$$\frac{\partial}{\partial \theta_j} J_{train}(\theta) = \frac{1}{m}\sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$$

(1)

If you are thinking of applying Map Reduce to some learning algorithm, in order to speed this up. By paralleling the computation over different computers, the key question to ask yourself is, can your learning algorithm be expressed as a summation over the training set? And it turns out that many learning algorithms can actually be expressed as computing sums of functions over the training set and the computational expense of running them on large data sets is because they need to sum over a very large training set. So, whenever your learning algorithm can be expressed as a sum of the training set and whenever the bulk of the work of the learning algorithm can be expressed as the sum of the training set, then map reviews might a good candidate

for scaling your learning algorithms through very, very good data sets.

Lets just look at one more example.

Let's say that we want to use one of the advanced optimization algorithm. So, things like, you know, l, b, f, g, s constant gradient and so on, and let's say we want to train a logistic regression of the algorithm. For that, we need to compute two main quantities. One is for the advanced optimization algorithms like, you know, LPF and constant gradient. We need to provide it a routine to compute the cost function of the optimization objective.

(2)

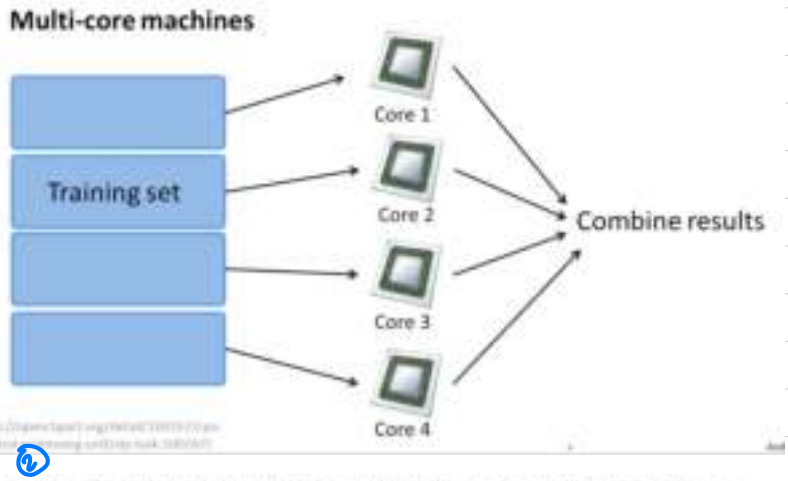
And so for logistic regression, you remember that a cost function has this sort of sum over the training set, and so if youre paralizing over ten machines, you would split up the training set onto ten machines and have each of the ten machines compute the sum of this quantity over just one tenth of the training

data. Then, the other thing that the advanced optimization algorithms need, is a routine to compute these partial derivative terms. Once again, these derivative terms, for which it's a logistic regression, can be expressed as a sum over the training set, and so once again, similar to our earlier example, you would have each machine compute that summation

over just some small fraction of your training data.

And finally, having computed all of these things, they could then send their results to a centralized server, which can then add up the partial sums. This corresponds to adding up those tenth i or tenth ij variables, which were computed locally on machine number i, and so the centralized server can sum these things up and get the overall cost function and get the overall partial derivative, which you can then pass through the advanced optimization algorithm.

So, more broadly, by taking other learning algorithms and expressing them in sort of summation form or by expressing them in terms of computing sums of functions over the training set, you can use the MapReduce technique to parallelize other learning algorithms as well, and scale them to very large training sets.



back and forth, all that happens within a single machine. And so network latency becomes much less of an issue compared to if you were using this to over different computers within the data sensor. Finally, one last caveat on parallelizing within a multi-core machine. Depending on the details of your implementation, if you have a multi-core machine and if you have certain numerical linear algebra libraries.

It turns out that the sum numerical linear algebra libraries

that can automatically parallelize their linear algebra operations across multiple cores within the machine.

So if you're fortunate enough to be using one of those numerical linear algebra libraries and certainly this does not apply to every single library. If you're using one of those libraries and. If you have a very good vectorizing implementation of the learning algorithm.

Sometimes you can just implement you standard learning algorithm in a vectorized fashion and not worry about parallelization and numerical linear algebra libararies

Finally, as one last comment, so far we have been discussing MapReduce algorithms as allowing you to parallelize over multiple computers, maybe multiple computers in a computer cluster or over multiple computers in the data center.

It turns out that sometimes even if you have just a single computer,

MapReduce can also be applicable.

In particular, on many single computers now, you can have multiple processing cores. You can have multiple CPUs, and within each CPU you can have multiple proc cores. If you have a large training set, what you can do if, say, you have a computer with 4 computing cores, what you can do is, even on a single computer you can split the training sets into pieces and send the training set to different cores within a single box, like within a single desktop computer or a single server and use MapReduce this way to divvy up work load. Each of the cores can then carry out the sum over, say, one quarter of your training set, and then they can take the partial sums and combine them, in order to get the summation over the entire training set. The advantage of thinking about MapReduce this way, as paralyzing over cause within a single machine, rather than parallelizing over multiple machines is that, this way you don't have to worry about network latency, because all the communication, all the sending of the [xx]

could take care of some of it for you. So you don't need to implement [xx] but, for other any problems, taking advantage of this sort of map reducing commentation, finding and using this MapReduce formulation and to paralelize a cross coarse except yourself might be a good idea as well and could let you speed up your learning algorithm.

In this video, we talked about the MapReduce approach to parallelizing machine learning by taking a data and spreading them across many computers in the data center. Although these ideas are critical to paralysing across multiple cores within a single computer

as well. Today there are some good open source implementations of MapReduce, so there are many users in open source system called Hadoop and using either your own implementation or using someone else's open source implementation, you can use these ideas to parallelize learning algorithms and get them to run on much larger data sets than is possible using just a single machine.

Suppose you apply the map-reduce method to train a neural network on ten machines. In each iteration, what will each of the machines do?

- [ ] Compute either forward propagation or back propagation on 1/5 of the data.
- [x] Compute forward propagation and back propagation on 1/10 of the data to compute the derivative with respect to that 1/10 of the data.
- [ ] Compute only forward propagation on 1/10 of the data. (The centralized machine then performs back propagation on all the data).
- [ ] Compute back propagation on 1/10 of the data (after the centralized machine has computed forward propagation on all of the data).

✓ **Correct**

Application example: Photo OCR

Problem Description and Pipeline

In this and the next few videos, I want to tell you about a machine learning application example, or a machine learning application history centered around an application called Photo OCR. There are three reasons why I want to do this, first I wanted to show you an example of how a complex machine learning system can be put together.

Second, once told the concepts of a machine learning a type line and how to allocate resources when you're trying to decide what to do next. And this can either be in the context of you working by yourself on the big application Or it can be the context of a team of developers trying to build a complex application together.

And then finally, the Photo OCR problem also gives me an excuse to tell you about just a couple more interesting ideas for machine learning. One is some ideas of how to apply machine learning to computer vision problems, and second is the idea of artificial data synthesis, which we'll see in a couple of videos. So, let's start by talking about what is the Photo OCR problem.

The Photo OCR problem

Photo OCR stands for Photo Optical Character Recognition.

With the growth of digital photography and more recently the growth of camera in our cell phones we now have tons of visual pictures that we take all over the place. And one of the things that has interested many developers is how to get our computers to understand the content of these pictures a little bit better. The photo OCR problem focuses on how to get computers to read the text to the purest in images that we take.

Given an image like this it might be nice if a computer can read the text in this image so that if you're trying to look for this picture again you type in the words, lulu bees and and have it automatically pull up this picture, so that you're not spending lots of time digging through your photo collection Maybe hundreds of thousands of pictures in. The Photo OCR problem does exactly this, and it does so in several steps. First, given the picture it has to look through the image and detect where there is text in the picture.

And after it has done that or if it successfully does that it then has to look at these text regions and actually read the text in those regions, and hopefully if it reads it correctly, it'll come

up with these transcriptions of what is the text that appears in the image. Whereas OCR, or optical character recognition of scanned documents is relatively easier problem, doing OCR from photographs today is still a very difficult machine learning problem, and you can do this. Not only can this help our computers to understand the content of our though images better, there are also applications like helping blind people, for example, if you could provide to a blind person a camera that can look at what's in front of them, and just tell them the words that my be on the street sign in front of them. With car navigation systems. For example, imagine if your car could read the street signs and help you navigate to your destination.

Photo OCR pipeline

In order to perform photo OCR, here's what we can do. First we can go through the image and find the regions where there's text and image. So, shown here is one example of text and image that the photo OCR system may find.

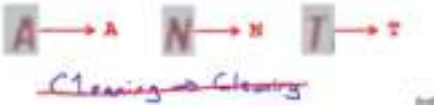
→ 1. Text detection

Second, given the rectangle around that text region, we can then do character segmentation, where we might take this text box that says "Antique Mall" and try to segment it out into the locations of the individual characters.

→ 2. Character segmentation

And finally, having segmented out into individual characters, we can then run a crossfire, which looks at the images of the visual characters, and tries to figure out the first character's an A, the second character's an N, the third character is a T, and so on, so that up by doing all this how that

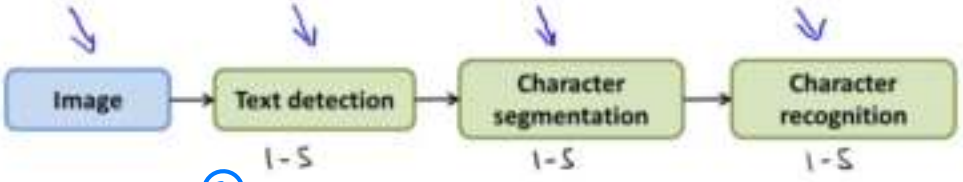
→ 3. Character classification



hopefully you can then figure out that this phrase is Rulegee's antique mall and similarly for some of the other words that appear in that image. I should say that there are some photo OCR systems that do even more complex things, like a bit of spelling correction at the end. So if, for example, your character segmentation and character classification system tells you that it sees the word c l e a n i n g. Then, you know, a sort of spelling correction system might tell you that this is probably the word 'cleaning', and your character classification algorithm had just mistaken the l for a 1. But for the purpose of what we want to do in this video, let's ignore this last step and just focus on the system that does these three steps of text detection, character segmentation, and character classification.

A system like this is what we call a machine learning pipeline.

Photo OCR pipeline



1

In particular, here's a picture showing the photo OCR pipeline. We have an image, which then fed to the text detection system text regions, we then segment out the characters--the individual characters in the text--and then finally we recognize the individual characters.

In many complex machine learning systems, these sorts of pipelines are common, where you can have multiple modules--in this example, the text detection, character segmentation, character recognition modules--each of which may be machine learning component, or sometimes it may not be a machine learning component but to have a set of modules that act one after another on some piece of data in order to produce the output you want, which in the photo OCR example is to find the transcription of the text that appeared in the image. If you're designing a machine learning system one of the most important decisions will often be what exactly is the pipeline that you want to put together. In other words, given the photo OCR problem, how do you break this problem down into a sequence of different modules. And you design the pipeline and each the performance of each of the modules in your pipeline. will often have a big impact on the final performance of your algorithm.

2

If you have a team of engineers working on a problem like this is also very common to have different individuals work on different modules. So I could easily imagine tech easily being the of anywhere from 1 to 5 engineers, character segmentation maybe another 1-5 engineers, and character recognition being another 1-5

engineers, and so having a pipeline like often offers a natural way to divide up the workload amongst different members of an engineering team, as well. Although, or course, all of this work could also be done by just one person if that's how you want to do it.

In complex machine learning systems the idea of a pipeline, of a machine of a pipeline, is pretty pervasive.

And what you just saw is a specific example of how a Photo OCR pipeline might work. In the next few videos I'll tell you a little bit more about this pipeline, and we'll continue to use this as an example to illustrate--I think--a few more key concepts of machine learning.

When someone refers to a "machine learning pipeline," he or she is referring to:

- ○ A PhotoOCR system.
- ○ A character recognition system.
- ◉ A system with many stages / components, several of which may use machine learning.
- ○ An application in plumbing. (Haha.)

✓ **Correct**

Application example: Photo OCR

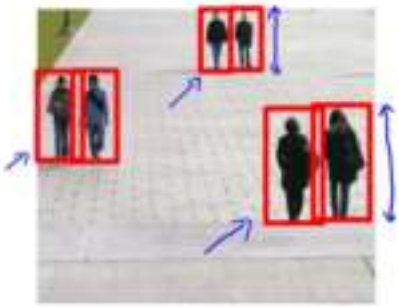
Sliding windows

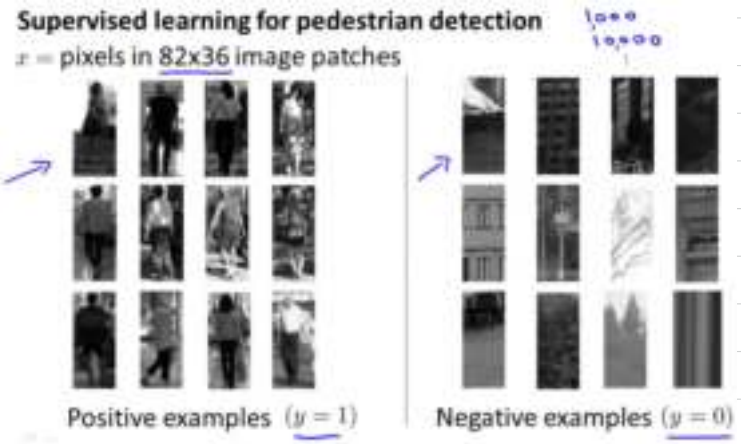
In the previous video, we talked about the photo OCR pipeline and how that worked. In which we would take an image and pass the Through a sequence of machine learning components in order to try to read the text that appears in an image. In this video I like to. A little bit more about how the individual components of the pipeline works. In particular most of this video will center around the discussion. of whats called a sliding windows. The first stage of the filter was the Text detection where we look at an image like this and try to find the regions of text that appear in this image. Text detection is an unusual problem in computer vision. Because depending on the length of the text you're trying to find, these rectangles that you're trying to find can have different aspect.

So in order to talk about detecting things in images let's start with a simpler example of pedestrian detection and we'll then later go back to. Ideas that were developed in pedestrian detection and apply them to text detection.

So in pedestrian detection you want to take an image that looks like this and the whole idea is the individual pedestrians that appear in the image. So there's one pedestrian that we found, there's a second one, a third one a fourth one, a fifth one. And a one. This problem is maybe slightly simpler than text detection just for the reason that the aspect ratio of most pedestrians are pretty similar. Just using a fixed aspect ratio for these rectangles that we're trying to find. So by aspect ratio I mean the ratio between the height and the width of these rectangles.

They're all the same. for different pedestrians but for text detection the height and width ratio is different for different lines of text Although for pedestrian detection, the pedestrians can be different distances away from the camera and so the height of these rectangles can be different depending on how far away they are. but the aspect ratio is the same. In order to build a





depending on how far away they are. but the aspect ratio is the same. In order to build a pedestrian detection system here's how you can go about it. Let's say that we decide to standardize on this aspect ratio of 82 by 36 and we could have chosen some rounded number like 80 by 40 or something, but 82 by 36 seems alright.

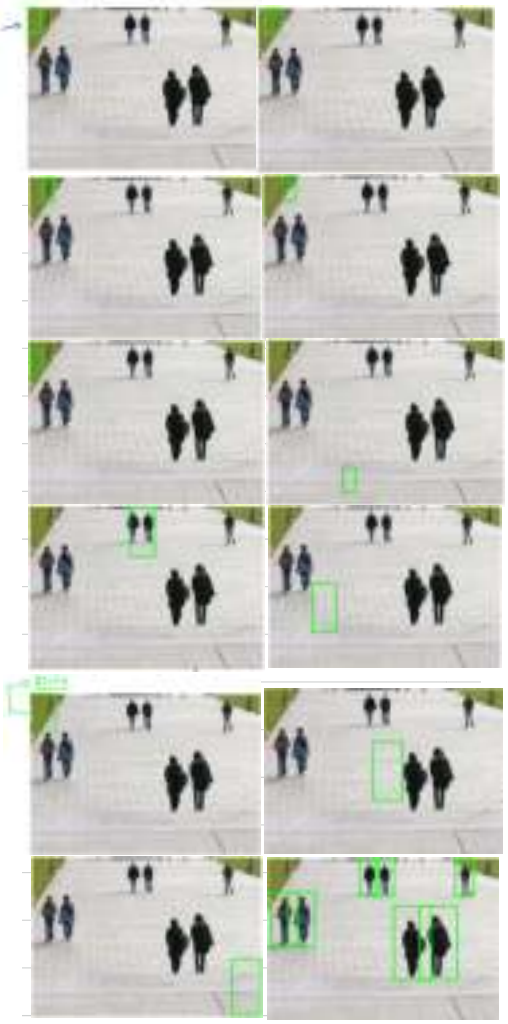
What we would do is then go out and collect large training sets of positive and negative examples. Here are examples of 82 X 36 image patches that do contain pedestrians and here are examples of images that do not.

On this slide I show 12 positive examples of y1 and 12 examples of y0.

In a more typical pedestrian detection application, we may have anywhere from a 1,000 training examples up to maybe 10,000 training examples, or even more if you can get even larger training sets. And what you can do, is then train in your network or some other learning algorithm to take this input, an MS patch of dimension 82 by 36, and to classify 'y' and to classify that image patch as either containing a pedestrian or not.

So this gives you a way of applying supervised learning in order to take an image patch can determine whether or not a pedestrian appears in that image capture.

Sliding window detection



Now, lets say we get a new image, a test set image like this and we want to try to find a pedestrian's picture image.

What we would do is start by taking a rectangular patch of this image. Like that shown up here, so that's maybe a 82 X 36 patch of this image, and run that image patch through our classifier to determine whether or not there is a pedestrian in that image patch, and hopefully our classifier will return y equals 0 for that patch, since there is no pedestrian.

Next, we then take that green rectangle and we slide it over a bit and then run that new image patch through our classifier to decide if there's a pedestrian there.

And having done that, we then slide the window further to the right and run that patch through the classifier again. The amount by which you shift the rectangle over each time is a parameter, that's sometimes called the step size of the parameter, sometimes also called the slide parameter, and if you step this one pixel at a time. So you can use the step size or stride of 1, that usually performs best, that is more cost effective, and so using a step size of maybe 4 pixels at a time, or eight pixels at a time or some large number of pixels might be more common, since you're then moving the rectangle a little bit more each time. So, using this process, you continue stepping the rectangle over to the right a bit at a time and running each of these patches through a classifier, until eventually, as you slide this window over the different locations in the image, first starting with the first row and then we go further rows in the image, you would then run all of these different image patches at some step size or some stride through your classifier.

Now, that was a pretty small rectangle, that would only detect pedestrians of one specific size. What we do next is start to look at larger image patches. So now let's take larger images patches, like those shown here and run those through the crossfire as well.

And by the way when I say take a larger image patch, what I really mean is when you take an image patch like this, what you're really doing is taking that image patch, and resizing it down to 82 X 36, say. So you take this larger patch and re-size it to be smaller image and then it would be the smaller size image that is what you would pass through your classifier to try and decide if there is a pedestrian in that patch.

And finally you can do this at an even larger scales and run that side of Windows to the end And after this whole process hopefully your algorithm will detect whether theres pedestrian appears in the image, so thats how you train a the classifier, and then use a sliding windows classifier, or use a sliding windows detector in order to find pedestrians in the image.

Text detection

Let's have a turn to the text detection example and talk about that stage in our photo OCR pipeline, where our goal is to find the text regions in unit.

1

similar to pedestrian detection you can come up with a label training set with positive examples and negative examples with examples corresponding to regions where text appears. So instead of trying to detect pedestrians, we're now trying to detect texts. And so positive examples are going to be patches of images where there is text. And negative examples is going to be patches of images where there isn't text. Having trained this we can now apply it to a new image, into a test

set image. So here's the image that we've been using as example.

Now, last time we run, for this example we are going to run a sliding windows at just one fixed scale just for purpose of illustration, meaning that I'm going to use just one rectangle size. But lets say I run my little sliding windows classifier on lots of little image patches like this if i do that, what Ill end up with is a result like this where the white region show where my text detection system has found text and so the axis' of these two figures are the same. So there is a region up here, of course also a region up here, so the fact that this black up here represents that the classifier does not think it's found any texts up there, whereas the fact that there's a lot of white stuff here, that reflects that classifier thinks that it's found a bunch of texts. over there on the image. What i have done on this image on the lower left is actually use white to show where the classifier thinks it has found text. And different shades of grey correspond to the probability that was output by the classifier, so like the shades of grey corresponds to where it thinks it might have found text but has lower confidence the bright white response to whether the classifier,

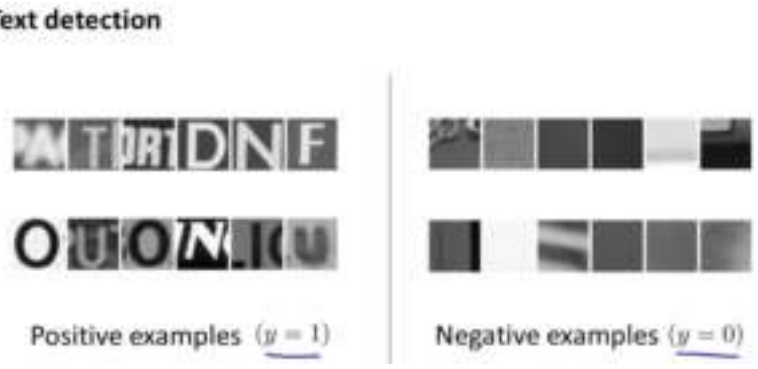
up with a very high probability, estimated probability of there being pedestrians in that location.

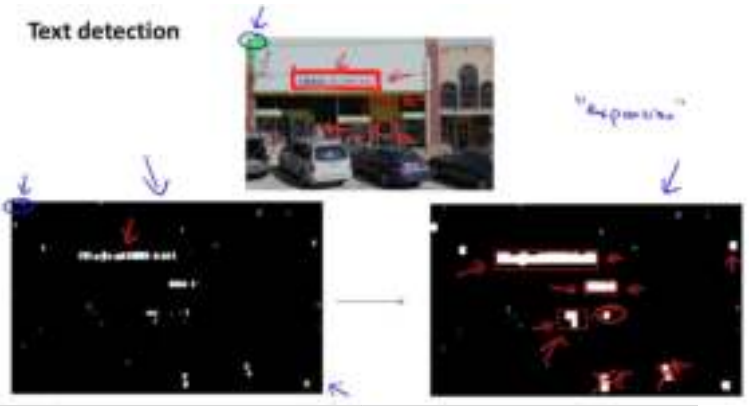
We aren't quite done yet because what we actually want to do is draw rectangles around all the region where this text in the image, so were going to take one more step which is we take the output of the classifier and apply to it what is called an expansion operator.

So what that does is, it take the image here,

and it takes each of the white blobs, it takes each of the white regions and it expands that white region. Mathematically, the way you implement that is, if you look at the image on the right, what we're doing to create the image on the right is, for every pixel we are going to ask, is it withing some distance of a white pixel in the left image. And so, if a specific pixel is within, say, five pixels or ten pixels of a white pixel in the leftmost image, then we'll also color that pixel white in the rightmost image.

And so, the effect of this is, we'll take each of the white blobs in the leftmost image and expand them a bit, grow them a little bit, by seeing whether the nearby pixels, the white pixels, and then coloring those nearby pixels in white as well. Finally, we are just about done. We can now look at this right most image and just look at the connecting components and look at the as white regions and draw bounding boxes around them. And in particular, if we look at all the white regions, like this one, this one, this one, and so on, and if we use a simple heuristic to rule out rectangles whose aspect ratios look funny because we know that boxes around text should be much wider than they are tall. And so if we ignore the thin, tall blobs like this one and this one, and we discard these ones because they are too tall and thin, and we then draw a the rectangles around the ones whose aspect ratio thats a height to what ratio looks like for text regions, then we can draw rectangles, the bounding boxes around this text region, this text region, and that text region, corresponding to the Lula B's antique mall logo, the Lula B's, and this little open sign.





2

Of over there.

This example by the actually misses one piece of text. This is very hard to read, but there is actually one piece of text there. That says [xx] are corresponding to this but the aspect ratio looks wrong so we discarded that one.

So you know it's ok on this image, but in this particular example the classifier actually missed one piece of text. It's very hard to read because there's a piece of text written against a transparent window.

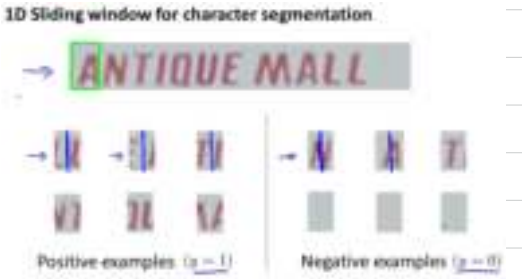
So that's text detection

using sliding windows. And having found these rectangles with the text in it, we can now just cut out these image regions and then use later stages of pipeline to try to meet the texts.

Suppose you are running a text detector using 20x20 image patches. You run the classifier on a 200x200 image and when using sliding window, you "step" the detector by 4 pixels each time. (For this problem assume you apply the algorithm at only one scale.) About how many times will you end up running your classifier on a single image? (Pick the closest answer.)

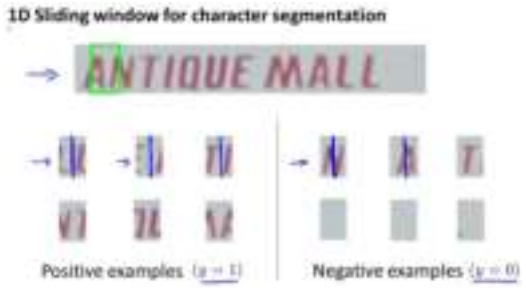
- [ ] About 100 times.
- [ ] About 400 times.
- [x] About 2,500 times.
- [ ] About 40,000 times.

✓ Correct



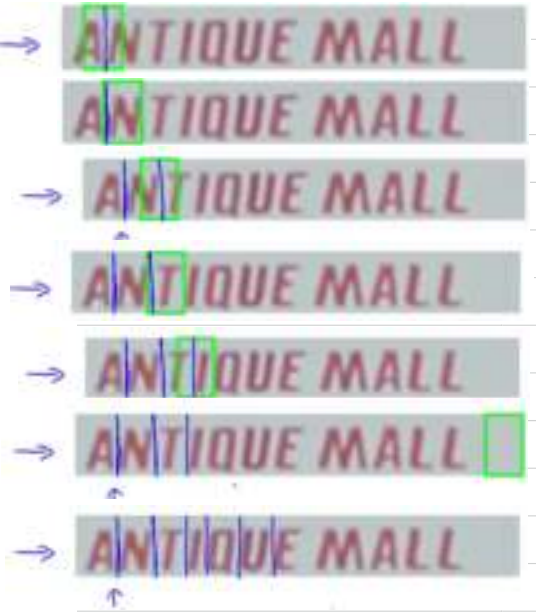
Now, you recall that the second stage of pipeline was character segmentation, so given an image like that shown on top, how do we segment out the individual characters in this image? So what we can do is again use a supervised learning algorithm with some set of positive and some set of negative examples, what were going to do is look in the image patch and try to decide if there is split between two characters

right in the middle of that image match. So for initial positive examples. This first cross example, this image patch looks like the middle of it is indeed



the middle has splits between two characters and the second example again this looks like a positive example, because if I split two characters by putting a line right down the middle, that's the right thing to do. So, these are positive examples, where the middle of the image represents a gap or a split

between two distinct characters, whereas the negative examples, well, you know, you don't want to split two characters right in the middle, and so these are negative examples because they don't represent the midpoint between two characters.

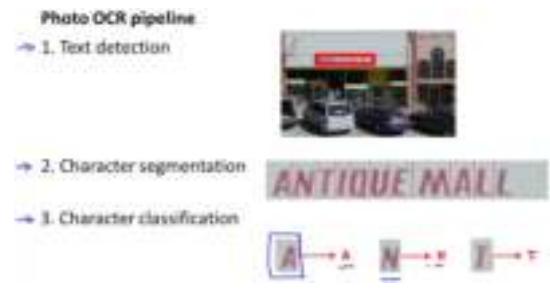


So what we will do is, we will train a classifier, maybe using new network, maybe using a different learning algorithm, to try to classify between the positive and negative examples.

Having trained such a classifier, we can then run this on this sort of text that our text detection system has pulled out. As we start by looking at that rectangle, and we ask, "Gee, does it look like the middle of that green rectangle, does it look like the midpoint between two characters?". And hopefully, the classifier will say no, then we slide the window over and this is a one dimensional sliding window classifier, because were going to slide the window only in one straight line from left to right, theres no different rows here. There's only one row here. But now, with the classifier in this position, we ask, well, should we split those two characters or should we put a split right down the middle of this rectangle. And hopefully, the classifier will output y equals one, in which case we will decide to draw a line down there, to try to split two characters.

Then we slide the window over again, optic process, don't close the gap, slide over again, optic says yes, do split there and so on, and we slowly slide the classifier over to the right and hopefully it will classify this as another positive example and so on. And we will slide this window over to the right, running the classifier at every step, and hopefully it will tell us, you know, what are the right locations

to split these characters up into, just split this image up into individual characters. And so thats 1D sliding windows for character segmentation.



So, here's the overall photo OCR pipe line again. In this video we've talked about the text detection step, where we use sliding windows to detect text. And we also use a one-dimensional sliding windows to do character segmentation to segment out, you know, this text image in division of characters.

The final step through the pipeline is the character qualification step and that step you might already be much more familiar with the early videos on supervised learning where you can apply a standard supervised learning within maybe on your network or maybe something else in order to take it's input, an image like that and classify which alphabet or which 26 characters A to Z, or maybe we should have 36 characters if you have the numerical digits as well, the multi class classification problem where you take it's input and image contained a character and decide what is the character that appears in that image? So that was the photo OCR

pipeline and how you can use ideas like sliding windows classifiers in order to put these different components to develop a photo OCR system. In the next few videos we keep on using the problem of photo OCR to explore somewhat interesting issues surrounding building an application like this.

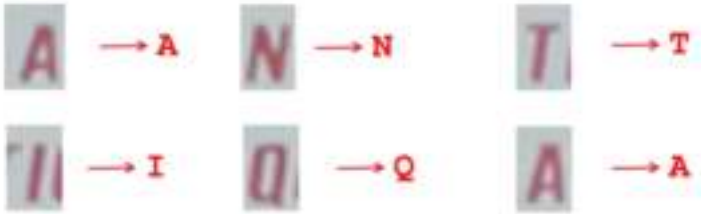
Application example:
Photo OCR

I've seen over and over that one of the most reliable ways to get a high performance machine learning system is to take a low bias learning algorithm and to train it on a massive training set.

But where did you get so much training data from? Turns out that the machine earnings there's a fascinating idea called artificial data synthesis, this doesn't apply to every single problem, and to apply to a specific problem, often takes some thought and innovation and insight. But if this idea applies to your machine, only problem, it can sometimes be a an easy way to get a huge training set to give to your learning algorithm. The idea of artificial data synthesis comprises of two variations, main the first is if we are essentially creating data from [xx], creating new data from scratch. And the second is if we already have it's small label training set and we somehow have amplify that training set or use a small training set to turn that into a larger training set and in this video we'll go over both those ideas.

Getting lots of data:
Artificial data synthesis

Character recognition



Artificial data synthesis for photo OCR

Real data

①

To talk about the artificial data synthesis idea, let's use the character portion of the photo OCR pipeline, we want to take it's input image and recognize what character it is.

If we go out and collect a large label data set, here's what it is and what it look like. For this particular example, I've chosen a square aspect ratio. So we're taking square image patches. And the goal is to take an image patch and recognize the character in the middle of that image patch.

And for the sake of simplicity, I'm going to treat these images as grey scale images, rather than color images. It turns out that using color doesn't seem to help that much for this particular problem.

So given this image patch, we'd like to recognize that that's a T. Given this image patch, we'd like to recognize that it's an 'S'.

Given that image patch we would like to recognize that as an 'I' and so on.

So all of these, our examples of row images, how can we come up with a much larger training set? Modern computers often have a huge font library and if you use a word processing software, depending on what word processor you use, you might have all of these fonts and many, many more Already stored inside. And, in fact, if you go different websites, there are, again, huge, free font libraries on the internet we can download many, many different types of fonts, hundreds or perhaps thousands of different fonts.

②

So if you want more training examples, one thing you can do is just take characters from different fonts

and paste these characters against different random backgrounds. So you might take this — and paste that c against a random background.

If you do that you now have a training example of an image of the character C.

So after some amount of work, you know this, and it is a little bit of work to synthisize realistic looking data. But after some amount of work, you can get a synthetic training set like that.

Every image shown on the right was actually a synthesized image. Where you take a font, maybe a random font downloaded off the web and you paste an image of one character or a few characters from that font against this other random background image. And then apply maybe a little blurring operators —-of app finder, distortions that app finder, meaning just the sharing and scaling and little rotation operations and if you do that you get a synthetic training set, on what the one shown here. And this is work, grade, it is, it takes thought at work, in order to make the synthetic data look realistic, and if you do a sloppy job in terms of how you create the synthetic data then it actually won't work well. But if you look at the synthetic data looks remarkably similar to the real data.

And so by using synthetic data you have essentially an unlimited supply of training examples for artificial training synthesis And so, if you use this source synthetic data, you have essentially unlimited supply of label data to create a improvised learning algorithm for the character recognition problem.

So this is an example of artificial data synthesis where youre basically creating new data from scratch, you just generating brand new images from scratch.

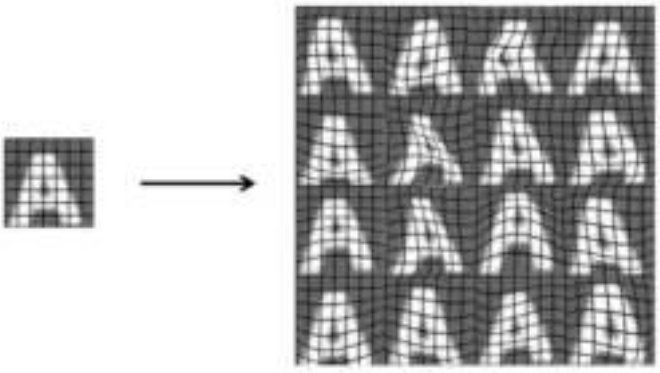
Artificial data synthesis for photo OCR

Real data

Synthetic data

→ شکل مربوط به این دو ستون بالا

Synthesizing data by introducing distortions



The other main approach to artificial data synthesis is where you take a examples that you currently have, that we take a real example, maybe from real image, and you create additional data, so as to amplify your training set. So here is an image of a compared to a from a real image, not a synthesized image, and I have overlayed this with the grid lines just for the purpose of illustration. Actually have these ---. So what you can do is then take this alphabet here, take this image and introduce artificial warpings(sp?) or artificial distortions into the image so they can take the image a and turn that into 16 new examples.

So in this way you can take a small label training set and amplify your training set to suddenly get a lot more examples, all of it.

Again, in order to do this for application, it does take thought and it does take insight to figure out what our reasonable sets of distortions, or whether these are ways that amplify and multiply your training set, and for the specific example of character recognition, introducing these warping seems like a natural choice, but for a different learning machine application, there may be different the distortions that might make more sense. Let me just show one example from the totally different domain of speech recognition.

Synthesizing data by introducing distortions: Speech recognition

- Original audio:
- Audio on bad cellphone connection
- Noisy background: Crowd
- Noisy background: Machinery

So the speech recognition, let's say you have audio clips and you want to learn from the audio clip to recognize what were the words spoken in that clip. So let's see how one labeled training example. So let's say you have one labeled training example, of someone saying a few specific words. So let's play that audio clip here. 0 -1-2-3-4-5. Alright, so someone counting from 0 to 5, and so you want to try to apply a learning algorithm to try to recognize the words said in that. So, how can we amplify the data set? Well, one thing we do is introduce additional audio distortions into the data set. So here I'm going to add background sounds to simulate a bad cell phone connection. When you hear beeping sounds, that's actually part of the audio track, that's nothing wrong with the speakers, I'm going to play this now. 0-1-2-3-4-5. Right, so you can listen to that sort of audio clip and recognize the sounds, that seems like another useful training example to have, here's another example, noisy background.

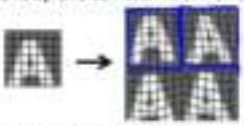
Zero, one, two, three four five you know of cars driving past, people walking in the background, here's another one, so taking the original clean audio clip so taking the clean audio of someone saying 0 1 2 3 4 5 we can then automatically synthesize these additional training examples and thus amplify one training example into maybe four different training examples.

So let me play this final example, as well. 0-1 3-4-5 So by taking just one labelled example, we have to go through the effort to collect just one labelled example fall of the 01205, and by synthesizing additional distortions, by introducing different background sounds, we've now multiplied this one example into many more examples.

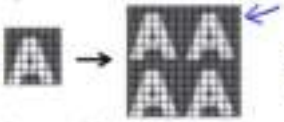
Much work by just automatically adding these different background sounds to the clean audio Just one word of warning about synthesizing

Synthesizing data by introducing distortions

- Distortion introduced should be representation of the type of noise/distortions in the test set.



- Audio: Background noise, bad cellphone connection
- Usually does not help to add purely random/meaningless noise to your data.



- x_i = intensity (brightness) of pixel i
- $x_i \leftarrow x_i +$ random noise

[Adam Coates and Tao Wang]

Just one word of warning about synthesizing

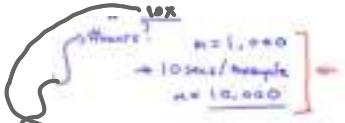
data by introducing distortions: if you try to do this yourself, the distortions you introduce should be representative the source of noises, or distortions, that you might see in the test set. So, for the character recognition example, you know, the working things begin introduced are actually kind of reasonable, because an image A that looks like that, that's, could be an image that we could actually see in a test set. Reflect a fact And, you know, that image on the upper right, that could be an image that we could imagine seeing.

And for audio, well, we do wanna recognize speech, even against a bad self internal connection, against different types of background noise, and so for the audio, we're again synthesizing examples are actually representative of the sorts of examples that we want to classify, that we want to recognize correctly.

In contrast, usually it does not help perhaps you actually a meaning as noise to your data. I'm not sure you can see this, but what we've done here is taken the image, and for each pixel, in each of these 4 images, has just added some random Gaussian noise to each pixel. To each pixel, is the pixel brightness. It would just add some, you know, maybe Gaussian random noise to each pixel. So it's just a totally meaningless noise, right? And so, unless you're expecting to see these sorts of pixel wise noise in your test set, this sort of purely random meaningless noise is less likely to be useful.

But the process of artificial data synthesis it is you know a little bit of an art as well and sometimes you just have to try it and see if it works.

But if you're trying to decide what sorts of distortions to add, you know, do think about what other meaningful distortions you might add that will cause you to generate additional training examples that are at least somewhat representative of the sorts of images you expect to see in your test sets.



Discussion on getting more data

1. Make sure you have a low bias classifier before expending the effort. (Plot learning curves). E.g. keep increasing the number of features/number of hidden units in neural network until you have a low bias classifier.
2. "How much work would it be to get 10x as much data as we currently have?"
   - Artificial data synthesis
   - Collect/label it yourself
   - "Crowd source" (E.g. Amazon Mechanical Turk)

(1) Finally, to wrap up this video, I just wanna say a couple of words, more about this idea of getting lots of data via artificial data synthesis.

As always, before expending a lot of effort, you know, figuring out how to create artificial training examples, it's often a good practice is to make sure that you really have a low biased crossfire, and having a lot more training data will be of help. And standard way to do this is to plot the learning curves, and make sure that you only have a low as well, high variance falsifier. Or if you don't have a low bias falsifier, you know, one other thing that's worth trying is to keep increasing the number of features that your classifier has, increasing the number of hidden units in your network, saying, until you actually have a low bias falsifier, and only then, should you put the effort into creating a large, artificial training set, so what you really want to avoid is to, you know, spend a whole week or spend a few months figuring out how to get a great artificially synthesized data set. Only to realize afterward, that, you know, your learning algorithm, performance doesn't improve that much, even when you're given a huge training set.

(2) So that's about my usual advice about of a testing that you really can make use of a large training set before spending a lot of effort going out to get that large training set.

Second is, when I'm working on machine learning problems, one question I often ask the team I'm working with, often ask my students, which is, how much work would it be to get 10 times as much date as we currently had.

When I face a new machine learning application very often I will sit down with a team and ask exactly this question, I've asked this question over and over and over and I've been very surprised how often this answer has been that. You know, it's really not that hard, maybe a few days of work at most, to get ten times as much data as we currently have for a machine running application and very often if you can get ten times as much data there will be a way to make your algorithm do much better. So, you know, if you ever join the product team

working on some machine learning application product this is a very good questions ask yourself ask the team don't be too surprised if after a few minutes of brainstorming if your team comes up with a way to get literally ten times this much data, in which case, I think you would be a hero to that team, because with 10 times as much data, I think you'll really get much better performance, just from learning from so much data.

(3) So there are several waysand

that comprised both the ideas of generating data from scratch using random fonts and so on. As well as the second idea of taking an existing example and and introducing distortions that amplify to enlarge the training set.A couple of other examples of ways to get a lot more data are to collect the data or to label them yourself.

So one useful calculation that I often do is, you know, how many minutes, how many hours does it take to get a certain number of examples, so actually sit down and figure out, you know, suppose it takes me ten seconds to label one example then and, suppose that, for our application, currently we have 1000 labeled examples examples so ten times as much of that would be if n were equal to ten thousand.

A second way to get a lot of data is to just collect the data and you label it yourself. So what I mean by this is I will often set down and do a calculation to figure out how much time, you know just like how many hours

will it take, how many hours or how many days will it take for me or for someone else to just sit down and collect ten times as much data, as we have currently, by collecting the data ourselves and labeling them ourselves.

(4) So, for example, that, for our machine learning application, currently we have 1,000 examples, so M 1,000.

That what we do is sit down and ask, how long does it take me really to collect and label one example. And sometimes maybe it will take you, you know ten seconds to label

one new example, and so if I want 10 X as many examples, I'd do a calculation. If it takes me 10 seconds to get one training example. If I wanted to get 10 times as much data, then I need 10,000 examples. So I do the calculation, how long is it gonna take to label, to manually label 10,000 examples, if it takes me 10 seconds to label 1 example.

So when you do this calculation, often I've seen many you would be surprised, you know, how little, or sometimes a few days at work, sometimes a small number of days of work, well I've seen many teams be very surprised that sometimes how little work it could be, to just get a lot more data, and let that be a way to give your learning app to give you a huge boost in performance, and necessarily, you know, sometimes when you've just managed to do this, you will be a hero and whatever product development, whatever team you're working on, because this can be a great way to get much better performance.

(5) Third and finally, one sometimes good way to get a lot of data is to use what's now called crowd sourcing. So today, there are a few websites or a few services that allow you to hire people on the web to, you know, fairly inexpensively label large training sets for you. So this idea of crowd sourcing, or crowd sourced data labeling, is something that has, is obviously, like an entire academic literature, has some of it's own complications and so on, pertaining to labeler reliability.

Maybe, you know, hundreds of thousands of labelers, around the world, working fairly inexpensively to help label data for you, and that I've just had mentioned, there's this one alternative as well. And probably Amazon Mechanical Turk systems is probably the most popular crowd sourcing option right now.

This is often quite a bit of work to get to work, if you want to get very high quality labels, but is sometimes an option worth considering as well.

If you want to try to hire many people, fairly inexpensively on the web, our labels launch miles of data for you.

(6) So this video, we talked about the idea of artificial data synthesis of either creating new data from scratch, looking, using the ramming funds as an example, or by amplifying an existing training set, by taking existing label examples and introducing distortions to it, to sort of create extra label examples.

And finally, one thing that I hope you remember from this video this idea of if you are facing a machine learning problem, it is often worth doing two things. One just a sanity check, with learning curves, that having more data would help. And second, assuming that that's the case, I will often seat down and ask yourself seriously: what would it take to get ten times as much creative data as you currently have, and not always, but sometimes, you may be surprised by how easy that turns out to be, maybe a few days, a few weeks at work, and that can be a great way to give your learning algorithm a huge boost in performance.

You've just joined a product group that has been developing a machine learning application for the last 12 months using 1,000 training examples. Suppose that by manually collecting and labeling examples, it takes you an average of 10 seconds to obtain one extra training example. Suppose you work 8 hours a day. How many days will it take you to get 10,000 examples? (Pick the closest answer.)

- About 1 day
- About 3.3 days
- About 29 days
- About 200 days

✓ Correct

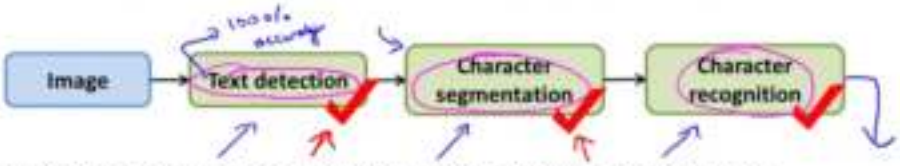
Application example: Photo OCR

Ceiling Analysis: What Part of Pipeline to work on Next

In earlier videos, I've said over and over that, when you're developing a machine learning system, one of the most valuable resources is your time as the developer, in terms of picking what to work on next. Or, if you have a team of developers or a team of engineers working together on a machine learning system. Again, one of the most valuable resources is the time of the engineers or the developers working on the system. And what you really want to avoid is that you or your colleagues your friends spend a lot of time working on some component. Only to realize after weeks or months of time spent, that all that worked just doesn't make a huge difference on the performance of the final system. In this video what I'd like to do is something called ceiling analysis.

When you're the team working on the pipeline machine on your system, this can sometimes give you a very strong signal, a very strong guidance on what parts of the pipeline might be the best use of your time to work on.

Estimating the errors due to each component (ceiling analysis)



What part of the pipeline should you spend the most time trying to improve?

Component	Accuracy
Overall system	72%
Text detection	89%
Character segmentation	90%
Character recognition	100%

(1)

To talk about ceiling analysis I'm going to keep on using the example of the photo OCR pipeline. And see right here each of these boxes, text detection, character segmentation, character recognition, each of these boxes can have even a small engineering team working on it. Or maybe the entire system is just built by you, either way. But the question is where should you allocate resources? Which of these boxes is most worth your effort of trying to improve the performance of. In order to explain the idea of ceiling analysis, I'm going to keep using the example of our photo OCR pipeline. As I mentioned earlier, each of these boxes here, each of these machines and components could be the work of a small team of engineers, or the whole system could be built by just one person. But the question is, where should you allocate scarce resources? That is, which of these components, which one or two or maybe all three of these components is most worth your time, to try to improve the performance of. So here's the idea of ceiling analysis. As in the development process for other machine learning systems as well, in order to make decisions on what to do for developing the system is going to be very helpful to have a single rolled number evaluation metric for this learning system. So let's say we pick character level accuracy. So if you're given a test set image, what is the fraction of alphabets or characters in a test image that we recognize correctly?

Or you can pick some other single road number evaluation that you could, if you want. But let's say for whatever evaluation measure we pick, we find that the overall system currently has 72% accuracy. So in other words, we have some set of test set images. And from each test set images, we run it through text detection, then character segmentation, then character recognition. And we find that on our test set the overall accuracy of the entire system was 72% on whatever metric you chose.

(2)

Now here's the idea behind ceiling analysis, which is that we're going to go through, let's say the first module of our machinery pipeline, say text detection. And what we're going to do, is we're going to monkey around with the test set. We're gonna go to the test set.

For every test example, which is going to provide it the correct text detection outputs, so in other words, we're going to go to the test set and just manually tell the algorithm where the text is in each of the test examples. So in other words gonna simulate what happens if you have a text detection system with a hundred percent accuracy, for the purpose of detecting text in an image. And really the way you do that's pretty simple, right? Instead of letting your learning algorithm detect the text in the images. You wouldn't say go to the images and just manually label what is the location of the text in my test set image. And you would then let these correct or let these ground truth labels of where is the text be part of your test set. And just use these ground truth labels as what you feed in to the next stage of the pipeline, so the character segmentation pipeline. Okay? So just to say that again. By putting a checkmark over here, what I mean is I'm going to go to my test set and just give it the correct answers. Give it the correct labels for the text detection part of the pipeline. So that as if I have a perfect test detection system on my test set.

(3)

What we need to do then is run this data through the rest of the pipeline. Through character segmentation and character recognition. And then use the same evaluation metric as before, to measure what was the overall accuracy of the entire system. And with perfect text detection, hopefully the performance will go up. And in this example, it goes up by by 89%. And then we're gonna keep going, let's got o the next stage of the pipeline, so character segmentation. So again, I'm gonna go to my test set, and now I'm going to give it the correct text detection output and give it the correct character segmentation output. So go to the test set and manually label the correct segmentations of the text into individual characters, and see how much that helps. And let's say it goes up to 90% accuracy for the overall system. Right? So as always the accuracy of the overall system. So is whatever the final output of the character recognition system is. Whatever the final output of the overall pipeline, is going to measure the accuracy of that. And finally I'm going to build a character recognition system and give that correct labels as well, and if I do that too then no surprise I should get 100% accuracy.

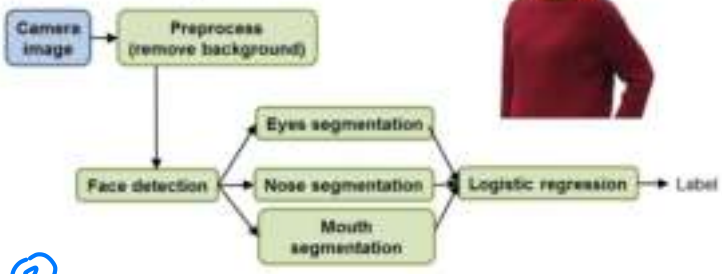
(4)

Now the nice thing about having done this analysis is, we can now understand what is the upside potential of improving each of these components? So we see that if we get perfect text detection, our performance went up from 72 to 89%. So that's a 17% performance gain. So this means that if we take our current system we spend a lot of time improving text detection, that means that we could potentially improve our system's performance by 17%. It seems like it's well worth our while. Whereas in contrast, when going from text detection when we gave it perfect character segmentation, performance went up only by 1%, so that's a more sobering message. It means that no matter how much time you spend on character segmentation. Maybe the upside potential is going to be pretty small, and maybe you do not want to have a large team of engineers working on character segmentation. This sort of analysis shows that even when you give it the perfect character segmentation, you performance goes up by only one percent. That really estimates what is the ceiling, or what is an upper bound on how much you can improve the performance of your system and working on one of these components.

And finally, going from character, when we get better character recognition with the forms went up by ten percent. So again you can decide is ten percent improvement, how much is worth your while? This tells you that maybe with more effort spent on the last stage of the pipeline, you can improve the performance of the systems as well. Another way of thinking about this, is that by going through these sort of analysis you're trying to think about what is the upside potential of improving each of these components. Or how much could you possibly gain if one of these components became absolutely perfect? And this really places an upper bound on the performance of that system. So the idea of ceiling analysis is pretty important, let me just answer this idea again but with a different example but more complex one. Let's say that you want to do

Another ceiling analysis example

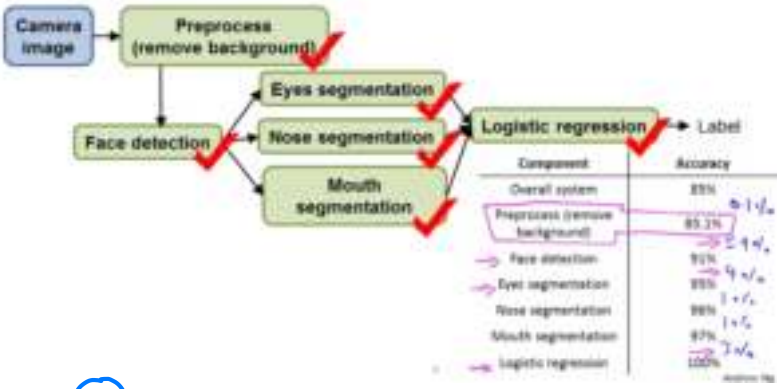
Face recognition from images
(Artificial example)



this idea again but with a different example but more complex one. Let's say that you want to do face recognition from images. You want to look at the picture and recognize whether or not the person in this picture is a particular friend of yours, and try to recognize the person Shown in this image. This is a slightly artificial example, this isn't actually how face recognition is done in practice. But we're going to set for an example, what a pipeline might look like to give you another example of how a ceiling analysis process might look. So we have a camera image, and let's say that we design a pipeline as follows, the first thing you wanna do is pre-processing of the image. So let's take this image like we have shown on the upper right, and let's say we want to remove the background. So do pre-processing and the background disappears. Next we want to say detect the face of the person, that's usually done on the learning So we'll run a sliding Windows crossfire to draw a box around a person's face. Having detected the face, it turns out that if you want to recognize people, it turns out that the eyes is a highly useful cue. We actually are, in terms of recognizing your friends the appearance of their eyes is actually one of the most important cues that you use. So lets run another crossfire to detect the eyes of the person. So the segment of the eyes and then since this will give us useful features to recognize the person. And then other parts of the face of physical interest. Maybe segment of the nose, segment of the mouth. And then having found the eyes, the nose, and the mouth, all of these give us useful features to maybe feed into a logistic regression classifier. And there's a job with a cost priority, they'd give us the overall label, to find the label for who we think is the identity of this person.

So this is a kind of complicated pipeline, it's actually probably more complicated than you should be using if you actually want to recognize people, but there's an illustrative example that's useful to think about for ceiling analysis.

Another ceiling analysis example



So how do you go through ceiling analysis for this pipeline. Well se step through these pieces one at a time. Let's say your overall system has 85% accuracy. The first thing I do is go to my test set and manually give it the full background segmentation. So manually go to the test set. And use Photoshop or something to just tell it where's the background and just manually remove the graph background, so this is a ground true background, and see how much the accuracy changes. In this example the accuracy goes up by 0.1%. So this is a strong sign that even if you have perfect background segmentation, the form is, even with perfect background removal the performance or your system isn't going to go up that much. So it's maybe not worth a huge effort to work on pre-processing on background removal.

Then quickly goes to test set give it the correct face detection images then again step though the eyes nose and mouth segmentation in some order just pick one order. Just give the correct location of the eyes. Correct location in noses, correct location in mouth, and then finally if I just give it the correct overall label I can get 100% accuracy. And so as I go through the system and just give more and more components, the correct labels in the test set, the performance of the overall system goes up and you can look at how much the performance went up on different steps. So from giving it the perfect face detection, it looks like the overall performance of the system went up by 5.9%. So that's a pretty big jump. It means that maybe it's worth quite a bit effort on better face detection. Went up 4% there, it went up 1% there. 1% there, and 3% there.

So it looks like the components that most work are while are, when I gave it perfect face detection system went up by 5.9 performance when given perfect eyes segmentation went to four percent. And then my final which is cost for well there's another three percent, gap there maybe. And so this tells maybe whether the components are most worthwhile working on. And by the way I want to tell you a true cautionary story. The reason I put this is in this in preprocessing background removal is because I actually know of a true story where there was a research team that actually literally had to people spend about a year and a half, spend 18 months working on better background removal. But actually I'm obscuring the details for obvious reasons, but there was a computer vision application where there's a team of two engineers that literally spent about a year and a half working on better background removal, actually worked out really complicated algorithms and ended up publishing one research paper. But after all that work they found that it just did not make huge difference to the overall performance of the actual application they were working on and if only someone were to do ceiling analysis before hand maybe they could have realized.

And one of them said to me afterward. If only you've did this sort of analysis like this maybe they could have realized before their 18 months of work. That they should have spend their effort focusing on some different component then literally spending 18 months working on background removal.

So to summarize, pipelines are pretty pervasive in complex machine learning applications. And when you're working on a big machine learning application, your time as developer is so valuable, so just don't waste your time working on something that ultimately isn't going to matter. And in this video we'll talk about this idea of ceiling analysis, which I've often found to be a very good tool for identifying the component of a video as you put focus on that component and make a big difference. Will actually have a huge effect on the overall performance of your final system. So over the years working machine learning, I've actually learned to not trust my own gut feeling about what components to work on. So very often, I've work on machine learning for a long time, but often I look at a machine learning problem, and I may have some gut feeling about oh, let's jump on that component and just spend all the time on that. But over the years, I've come to even trust my own gut feelings and learn not to trust gut feelings that much. And instead, if you have a sort of machine learning problem where it's possible to structure things and do a ceiling analysis, often there's a much better and much more reliable way for deciding where to put a focused effort, to really improve the performance of some component. And be kind of reassured that, when you do that, it won't actually have a huge effect on the final performance of the overall system.

Summary & thank you

Welcome to the final video of this Machine Learning class. We've been through a lot of different videos together. In this video I would like to just quickly summarize the main topics of this course and then say a few words at the end and that will wrap up the class.

Summary: Main topics

- Supervised Learning $(x^{(i)}, y^{(i)})$
  - Linear regression, logistic regression, neural networks, SVMs
- Unsupervised Learning $x^{(i)}$
  - K-means, PCA, Anomaly detection
- Special applications/special topics
  - Recommender systems, large scale machine learning.
- Advice on building a machine learning system
  - Bias/variance, regularization; deciding what to work on next: evaluation of learning algorithms, learning curves, error analysis, ceiling analysis.

So what have we done? In this class we spent a lot of time talking about supervised learning algorithms like linear regression, logistic regression, neural networks, SVMs. for problems where you have labelled data and labelled examples like x(i), y(i) And we also spent quite a lot of time talking about unsupervised learning like K-means clustering, Principal Components Analysis for dimensionality reduction and Anomaly Detection algorithms for when you have only unlabelled data x(i) Although Anomaly Detection can also use some labelled data to evaluate the algorithm. We also spent some time talking about special applications or special topics like Recommender Systems and large scale machine learning systems including parallelized and rapid-use systems as well as some special applications like sliding windows object classification for computer vision. And finally we also spent a lot of time talking about different aspects of, sort of, advice on building a machine learning system. And this involved both trying to understand what is it that makes a machine learning algorithm work or not work. So we talked about things like bias and variance, and how regularization can help with some variance problems. And we also spent a little bit of time talking about this question of how to decide what to work on next. So, how to prioritize how you spend your time when you're developing a machine learning system. So we talked about evaluation of learning algorithms, evaluation metrics like precision recall, F1 score as well as practical aspects of evaluation like the training, cross-validation and test sets. And we also spent a lot of time talking about debugging learning algorithms and making sure the learning algorithm is working. So we talked about diagnostics like learning curves and also talked about things like error analysis and ceiling analysis. And so all of these were different tools for helping you to decide what to do next and how to spend your valuable time when you're developing a machine learning system. And in addition to having the tools of machine learning at your disposal so knowing the tools of machine learning like supervised learning and unsupervised learning and so on, I hope that you now not only have the tools, but that you know how to apply these tools really well to build powerful machine learning systems.

So, that's it. Those were the topics of this class and if you worked all the way through this course you should now consider yourself an expert in machine learning. As you know, machine learning is a technology that's having huge impact on science, technology and industry. And you're now well qualified to use these tools of machine learning to great effect. I hope that many of you in this class will find ways to use machine learning to build cool systems and cool applications and cool products. And I hope that you find ways to use machine learning not only to make things work better but maybe someday to use it to make many other people's lives better as well. I also wanted to let you know that this class has been great fun for me to teach. So, thank you for that. And before wrapping up, there's just one last thing I wanted to say. Which is that it was maybe not so long ago, that I was a student myself. And even today, you know, I still try to take different courses when I have time to try to learn new things. And so I know how time-consuming it is to learn this stuff. I know that you're probably a busy person with many, many other things going on in your life. And so the fact that you still found the time or took the time to watch these videos and, you know, many of these videos just went on for hours, right? And the fact many of you took the time to go through the review questions and that many of you took the time to work through the programming exercises. And those were long and complicate programming exercises. I wanted to say thank you for that. And I know that many of you have worked hard on this class and that many of you have put a lot of time into this class, that many of you have put a lot of yourselves into this class. So I hope that you also got a lot out this class. And I wanted to say: Thank you very much for having been a student in this class.