PHYS 598 SDA RECITATION 8 - PROBLEM SET

1. The Moore-Penrose Pseudoinverse: Given an $m \times n$ matrix $\mathcal{B}$, the Moore-Penrose (generalized) matrix inverse is a *unique* $n \times m$ matrix pseudoinverse $\mathcal{B}^+$. This matrix was independently defined by Moore in 1920 and Penrose (1955), and variously known as the generalized inverse, pseudoinverse, or Moore-Penrose inverse.

Browse through the theorems and examples in `this Math@UCLA link`. Your task then is to solve the exercises at end of the PDF. Please verify with me your proof of either part (c) or (d).

2. The Khatri-Rao product: Let $\mathcal{A}$ and $\mathcal{B}$ be $I \times K$ and $J \times K$ matrices, respectively. The Khatri-Rao product $\mathcal{A} \odot \mathcal{B}$ satisfies the following properties

- $(\mathcal{A} \odot \mathcal{B})^t(\mathcal{A} \odot \mathcal{B}) = (\mathcal{A}^t\mathcal{A}) * (\mathcal{B}^t\mathcal{B})$, where $*$ denotes the Hadamard product, and
- $(\mathcal{A} \odot \mathcal{B})^+ = ((\mathcal{A}^t\mathcal{A}) * (\mathcal{B}^t\mathcal{B}))^+(\mathcal{A} \odot \mathcal{B})^t$, where $^+$ denotes the Moore-Penrose pseudoinverse.

(a) The proofs of the above properties are worked out in the lecture notes (or just google!). Your task is to work out explicitly the first property for generic matrices with $I = J = 2$ and $K = 3$, preferably on a computer. (*Prof. Song used a different notation for these products than what is more commonly used. We shall stick to the notation used in class, i.e. the ones above.*)

(b) Complete the proof of theorem **A.6.** in the lecture notes. For your convenience, the statement is

$$(\mathcal{A} \otimes \mathcal{B})^t = (\mathcal{A}^t \otimes \mathcal{B}^t).$$

(c) Let $\mathcal{A}_1, \ldots, \mathcal{A}_p$ be orthogonal matrices. Then prove that $\mathcal{A}_1 \otimes \ldots \otimes \mathcal{A}_p$ is an orthogonal matrix.