PHYS 598 SDA RECITATION 10 - PROBLEM SET

1. Conditional distribution: Discuss the proof of the following theorem (Theorem A.11 in the notes). Let $\boldsymbol{X} = (X_1, \ldots, X_n)$ be a multivariate normal random vector with mean $\boldsymbol{\mu}$, and $\boldsymbol{X}_a = (X_1, \ldots, X_k)$ and $\boldsymbol{X}_b = (X_{k+1}, \ldots, X_n)$. Then, the conditional distribution of $\boldsymbol{X}_a$ given $\boldsymbol{X}_b = \boldsymbol{x}_b$ is multivariate normal with mean

$$E[\boldsymbol{X}_a|\boldsymbol{X}_b = \boldsymbol{x}_b] = \boldsymbol{\mu}_a + \boldsymbol{\Sigma_{ab}\Sigma_{bb}^{-1}(x_b - \mu_b)}$$

and variance

$$Var[\boldsymbol{X}_a|\boldsymbol{x}_b] = \boldsymbol{\Sigma_{aa}} - \boldsymbol{\Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba}} = \boldsymbol{\Lambda_{aa}^{-1}}.$$

Are the above answers are intuitive in the limit $k \to 1$ and $k \to n$?

2. Information theory I: Think about problems 3,4, 5 and 11 from this TU München problem set. *Rényi entropy (prob 3) is a widely used entropy measure. It is prevalent in the computation of entanglement entropy in QFTs, by resorting to the so-called replica trick. Come talk to me if you like any of the other problems!*

3*. Information theory II: There are some very cool exercises (with hints, sometimes solutions) in here. Browse through the set, and discuss with your group if you find some of them particularly interesting.

4. Shannon entropy: On your tablet/laptop open to this PDF from UWarwick and go through the theorems and proofs in Pg. 1-5. You may already be familiar with some of the results. If you feel particularly inclined, a pedagogical and through treatment is (of course!) Preskill's PH229 notes at Caltech.