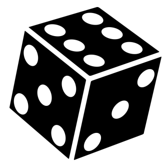
Kontinua 26 (3/5)

The Physics of Gases

Answer Key

1. (3 points) In a particular game, a fair die is tossed. If the number of spots showing is either 4 or 5 you win \$1, if the number of spots showing is 6 you win \$4, and if the number of spots showing is 1, 2, or 3 you win nothing. Let X be the amount that you win. What is the expected value of X?



Answer: You must enumerate the possibilities:

- There is a probability of 1/3 that you will win \$1.
- There is a probability of 1/6 that you will win \$4.
- There is a probability of 1/2 that you will win \$0.

So:

$$\mathbb{E}[X] = \frac{1}{3}(1) + \frac{1}{6}(4) + \frac{1}{2}(0) = \frac{6}{6} = 1$$

2. (4 points) The weight of written reports produced in a certain department has a Normal distribution with mean 60 g and standard deviation 12 g. What probability that the next report will weigh less than 48 g?

Answer: 50% of the reports will weigh more than 60 g. 48g is one standard deviation from the mean, so 34.1% of the reports will weigh between 48 and 60 grams. Thus, 84.1% of the reports weigh more than 48 grams. Thus 15.9% of the reports will weigh less than 48 grams.

There is a 15.9% chance that the next report will weigh less than 48 grams.

3. (4 points) Complete these equations: (2 points per equation)

(a) (2 points)

$$\frac{2}{5} + \frac{3}{7} =$$

Answer: First you must find the common divisor:

$$\frac{2}{5} + \frac{3}{7} = \frac{14}{35} + \frac{15}{35} = \frac{29}{35}$$

(b) (2 points)

$$\frac{2}{5} + \frac{3}{7} =$$

Answer: First you must find the common divisor:

$$\frac{2}{5} + \frac{3}{7} = \frac{14}{35} + \frac{15}{35} = \frac{29}{35}$$

4. A reservation service receives requests according to a Poisson process with an mean rate of 6 per minute.

(a) (3 points) What is the probability that during a given 1-min period the service center receives 3 requests?

Answer: The Poisson distribution for k events in a period, given the average rate of events is λ is given by:

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Substituting in:

$$P(3) = \frac{6^3 e^{-6}}{3!} = \frac{216e^{-6}}{6} = \frac{36}{e^6}$$

That is a perfectly good answer. If you used a calculator, you could give an approximation:

$$P(3) = \frac{6^3 e^{-6}}{3!} = \frac{36}{e^6} \approx 0.089$$

The answer, then, would be "There is an 8.9% chance that the center would get exactly 3 calls in a minute.

(b) (3 points) What is the probability that during a given 1-min period, exactly four of the five operators receive no requests?(*Hint*: treat either as a binomial process of 5 trials with 4 successes or consider 5 combinations of Poisson processes, e.g. only 1st operation receives a request or only 2nd operation receives a request and so on)