CHAPTER 1

Multivariate Functions

A real-valued multivariate function is a function that takes multiple real variables as input and produces a single real output.

We generally denote such a function as $f : \mathbb{R}^n \to \mathbb{R}$, where $\mathbb{R}^n$ is the domain and $\mathbb{R}$ is the co-domain.

For example, consider a function f that takes two variables x and y:

$$f(x, y) = x^2 + y^2$$

Here, $f : \mathbb{R}^2 \to \mathbb{R}$ takes an ordered pair (x, y) from the 2-dimensional real coordinate space, squares each, and adds them to produce a real number.

In a similar way, a function $g : \mathbb{R}^3 \to \mathbb{R}$ could take three variables x, y, and z, and might be defined as:

$$g(x, y, z) = x^2 + y^2 + z^2$$

Here, the function squares each of the input variables and then adds them to produce a real number.

These functions are ”real-valued” because their outputs are real numbers, and ”multivariate” because they take multiple variables as inputs.

The concepts of limits, continuity, differentiability, and integrability can all be extended to multivariate functions, although they become more complex because we now have to consider different directions in which we approach a point, not just from the left or right as in the univariate case. For example, the partial derivative is the derivative of the function with respect to one variable, holding the others constant. It is one of the basic concepts in the calculus of multivariate functions.

For example, given the function $f(x, y) = x^2 + y^2$, the partial derivatives of f are computed as:

$$\frac{\partial f}{\partial x}(x, y) = 2x$$

$$\frac{\partial f}{\partial y}(x, y) = 2y$$

Appendix A

Answers to Exercises