Chapter 1

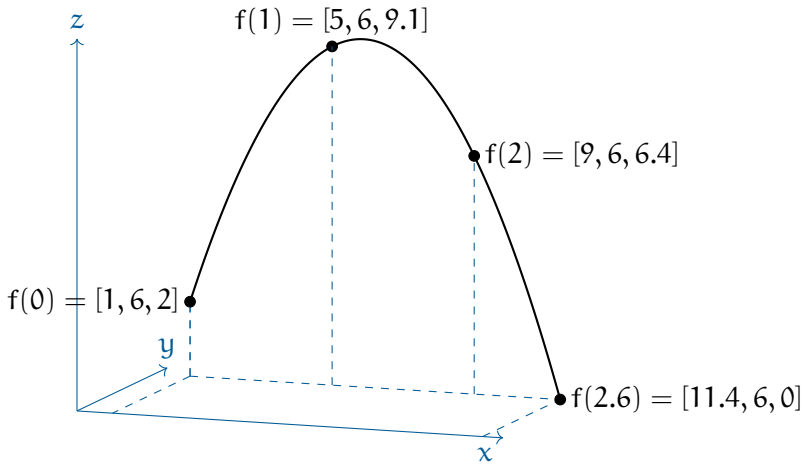
Vector-valued Functions

In the last chapter, you calculated the flight of the shell. For any time t, you could find a vector $[\mathtt{distance}, \mathtt{height}]$. This can be thought of as a function f that takes a number and returns a 2-dimensional vector. We call this a *vector-valued* function from $\mathbb{R} \to \mathbb{R}^2$.

We often make a vector-valued function by defining several real-valued functions. For example, if you threw a hammer with an initial upward speed of 12 m/2 and a horizontal speed of 4 m/s along the x axis from the point $(1, 6, 2)$, its position at time t (during its flight) would be given by:

$$f(t) = [4t + 1, 6, -4.8t^2 + 12t + 2]$$

That is, x is increasing with t, y is constant, and z is a parabola.

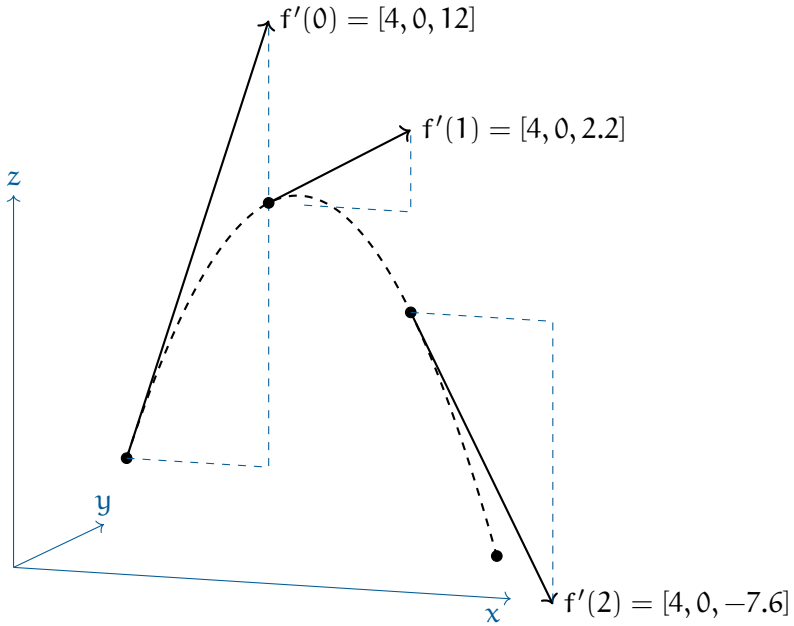


1.1 Finding the velocity vector

Now that we have its position vector, we can differentiate each component separately to get its velocity as a vector-valued function:

$$f'(t) = [4, 0, -9.8t + 12]$$

That is, the velocity is constant along the x-axis, zero along the y-axis, and decreasing with time along the z axis.

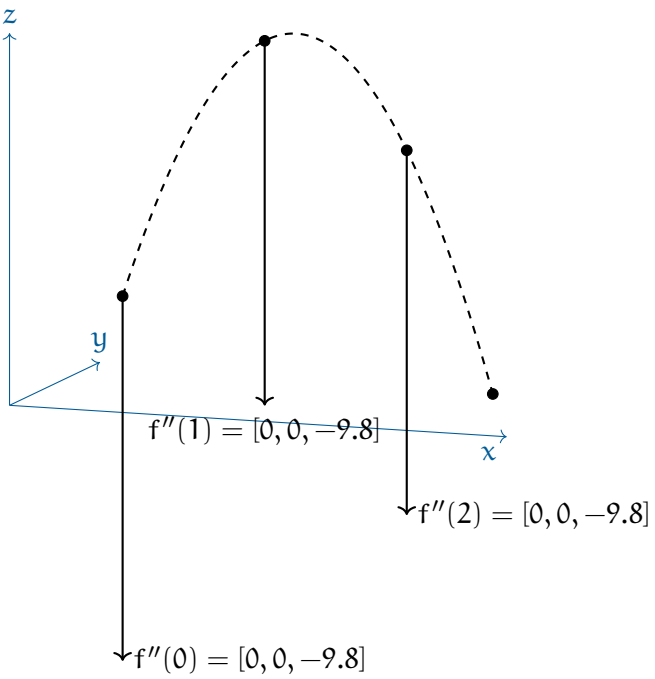


1.2 Finding the acceleration vector

Now that we have its velocity, we can get its acceleration as a vector-valued function:

$$f''(t) = [0, 0, -9.8]$$

There is no acceleration along the x or y axes. It is accelerating down at a constant $9.8 m/s^2$.



Appendix A

Answers to Exercises