CHAPTER 1

# Compound Interest

When you loan money to someone, you typically charge them some sort of interest. The most common loan of this sort is what the bank calls a "savings account". Any money you put in the account is loaned to the bank. The bank then lends it to someone else, who pays interest to the bank. And the bank gives some of that interest to you. However, what if you leave the interest in your account? And you start making *interest on the interest*? This is known as *compound interest*.

## 1.1 An example with annual interest payments

Let's say that you put $1000 in a savings account that pays 6% interest every year. How much money would you have after 12 years? Let's make a spreadsheet.

| | A | B | C |
|---|---|---|---|
| 1 | **Interest Rate** | 6.00% | |
| 2 | | | |
| 3 | **After year:** | **Interest** | **Balance** |
| 4 | 0 | $0.00 | 1000 |
| 5 | 1 | $60.00 | $1,060.00 |

Create a new spreadsheet and edit the cells to look like this. All the cells in rows 1 - 4 are just values: just type in what you see.

The fifth row is all formulas:

| After year | Interest | Balance |
|---|---|---|
| = A4 + 1 | = B$1 * C4 | = C4 + B5 |

The interest rate field should be formatted as a percentage. One thing to know when dealing with percentages in the spreadsheet: if the field says "600%", its value is 6.

The cells in the Interest and Balance column should be formatted as currency.

You are about to make a bunch of copies of the cells in the fifth row, so make sure they look right.

Click on A5 and shift-click on C5 to select all three cells. Drag the lower-right corner down to fill the rows 6 - 15.

A5:C16 fx = A4 + 1

| | A | B | C |
|---|---|---|---|
| 1 | **Interest Rate** | 6.00% | |
| 2 | | | |
| 3 | **After year** | **Interest** | **Balance** |
| 4 | 0 | $0.00 | 1000 |
| 5 | 1 | $60.00 | $1,060.00 |
| 6 | 2 | $63.60 | $1,123.60 |
| 7 | 3 | $67.42 | $1,191.02 |
| 8 | 4 | $71.46 | $1,262.48 |
| 9 | 5 | $75.75 | $1,338.23 |
| 10 | 6 | $80.29 | $1,418.52 |
| 11 | 7 | $85.11 | $1,503.63 |
| 12 | 8 | $90.22 | $1,593.85 |
| 13 | 9 | $95.63 | $1,689.48 |
| 14 | 10 | $101.37 | $1,790.85 |
| 15 | 11 | $107.45 | $1,898.30 |
| 16 | 12 | $113.90 | $2,012.20 |
| 17 | | | |

Look at the numbers. The first interest payment is $60, but the last is $113.90. Your balance has more than doubled!

## 1.2 Exponential Growth

We figured this out numerically by repeatedly multiplying the balance by the interest rate. What if you wanted to know what the balance would be $n$ years after investing $P_0$ dollars with an annual interest rate of $r$? (Note that $r$ in our example would be 0.06, not 6.0.)

Each year, the balance is multiplied by $1+r$, so after one year, $P_0$ would become $P_0 \times (1+r)$. The next year you would multiply this number by $(1+r)$ again: $P_0 \times (1+r) \times (1+r)$. The next year? $P_0 \times (1+r) \times (1+r) \times (1+r)$ See the pattern? $P_n$ is this balance after $n$ years, then

$$P_n = P_0(1+r)^n$$

Because $n$ is an exponent, we call this *exponential growth*. And there are few things as terrifying to a scientist as the phrase "The population is undergoing exponential growth".

## 1.3 Sensitivity to interest rate

For most people, the first surprising thing about compound interest is how quickly your money grows after a few years. The second thing that is surprising is how much difference a small change in the percentage rate makes.

Let's add another set of columns that shows what happens to your money if you convince the bank to pay you 8% instead of 6%.

Copy everything from columns B and C:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Interest Rate | 6.00% | | 6.00% | |
| 2 | | | | | |
| 3 | After year | Interest | Balance | Interest | Balance |
| 4 | 0 | $0.00 | 1000 | $0.00 | 1000 |
| 5 | 1 | $60.00 | $1,060.00 | $60.00 | $1,060.00 |
| 6 | 2 | $63.60 | $1,123.60 | $63.60 | $1,123.60 |
| 7 | 3 | $67.42 | $1,191.02 | $67.42 | $1,191.02 |
| 8 | 4 | $71.46 | $1,262.48 | $71.46 | $1,262.48 |
| 9 | 5 | $75.75 | $1,338.23 | $75.75 | $1,338.23 |
| 10 | 6 | $80.29 | $1,418.52 | $80.29 | $1,418.52 |
| 11 | 7 | $85.11 | $1,503.63 | $85.11 | $1,503.63 |
| 12 | 8 | $90.22 | $1,593.85 | $90.22 | $1,593.85 |
| 13 | 9 | $95.63 | $1,689.48 | $95.63 | $1,689.48 |
| 14 | 10 | $101.37 | $1,790.85 | $101.37 | $1,790.85 |
| 15 | 11 | $107.45 | $1,898.30 | $107.45 | $1,898.30 |
| 16 | 12 | $113.90 | $2,012.20 | $113.90 | $2,012.20 |
| 17 | | | | | |

Now edit the second interest rate to be 8%:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Interest Rate | 6.00% | | 8.00% | |
| 2 | | | | | |
| 3 | After year | Interest | Balance | Interest | Balance |
| 4 | 0 | $0.00 | 1000 | $0.00 | 1000 |
| 5 | 1 | $60.00 | $1,060.00 | $80.00 | $1,080.00 |
| 6 | 2 | $63.60 | $1,123.60 | $86.40 | $1,166.40 |
| 7 | 3 | $67.42 | $1,191.02 | $93.31 | $1,259.71 |
| 8 | 4 | $71.46 | $1,262.48 | $100.78 | $1,360.49 |
| 9 | 5 | $75.75 | $1,338.23 | $108.84 | $1,469.33 |
| 10 | 6 | $80.29 | $1,418.52 | $117.55 | $1,586.87 |
| 11 | 7 | $85.11 | $1,503.63 | $126.95 | $1,713.82 |
| 12 | 8 | $90.22 | $1,593.85 | $137.11 | $1,850.93 |
| 13 | 9 | $95.63 | $1,689.48 | $148.07 | $1,999.00 |
| 14 | 10 | $101.37 | $1,790.85 | $159.92 | $2,158.92 |
| 15 | 11 | $107.45 | $1,898.30 | $172.71 | $2,331.64 |
| 16 | 12 | $113.90 | $2,012.20 | $186.53 | $2,518.17 |
| 17 | | | | | |

Appendix A

# Answers to Exercises

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