Chapter 1

The Multivariate Normal Distribution

The multivariate normal distribution is a generalization of the one-dimensional (univariate) normal distribution to higher dimensions. It is used in statistics to describe any set of correlated real-valued random variables.

1.1 Multivariate Normal Distribution

A random vector $X = [X_1, X_2, ..., X_n]^T$ follows a multivariate normal distribution if every linear combination of its components has a univariate normal distribution. The distribution is parameterized by a mean vector and a covariance matrix.

The probability density function (pdf) of an n-dimensional multivariate normal distribution is given by:

$$f(\mathbf{x}|\boldsymbol{\mu}, \Sigma) = \frac{1}{\sqrt{(2\pi)^n|\Sigma|}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$$

where:

- $\mathbf{x} = [x_1, x_2, ..., x_n]^T$ is the point up to which the function is integrated,
- $\boldsymbol{\mu} = [\mu_1, \mu_2, ..., \mu_n]^T$ is the mean vector,
- Σ is the covariance matrix,
- $|\Sigma|$ denotes the determinant of the covariance matrix,
- T denotes the matrix transpose.

1.2 Covariance Matrix

The covariance matrix, Σ, is a symmetric matrix that contains information about the variance of each variable and the covariance between every pair of variables in the distribution.

The element Σ_{ij} is the covariance between the i-th and the j-th random variable, and Σ_{ii} is the variance of the i-th random variable.

The covariance matrix provides a measure of how much each of the dimensions varies from the mean with respect to each other. A positive covariance between two variables indicates that the variables increase or decrease together, whereas a negative covariance indicates that one variable increases when the other decreases.

Appendix A

Answers to Exercises

Index

covariance matrix, 2

Gaussian distribution, 1

multivariate normal, 1