Chapter 1

Linear Combinations

A linear combination of vectors is the addition of two or more scaled vectors. For example, given two vectors, v_1, v_2 and two scalars a_1, a_2, you'd write their linear combination as:

$$x\mathbf{w} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2$$

The scalars can be any real number. The vectors can be of any dimension.

Let's take a more generalized approach. Given vectors $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n \in \mathbb{R}^m$ and scalars $a_1, a_2, ..., a_n \in \mathbb{R}$, a linear combination of these vectors is any vector of the form

$$\mathbf{w} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + ... + a_n\mathbf{v}_n$$

Each scalar a_i scales the corresponding vector $\mathbf{v}_i$, and added together, the results are produce a new vector $\mathbf{w}$.

Let's look at an example that has 4 vectors and their scalars.

$$a_1 = 1, v_1 = [9, 1, 2]$$

$$a_2 = -1, v_2 = [8, -3, 4]$$

$$a_3 = 3, v_3 = [6, 0, 1]$$

$$a_4 = -4, v_4 = [3, 7, 2]$$

As a linear combination:

$$\mathbf{w} = 1 * [9, 1, 2] + (-1) * [8, -3, 4] + 3 * [6, 0, 1] + (-4) * [3, 7, 2]$$

After multiplying each vector by its associated scalar.

$$\mathbf{w} = [9, 1, 2] + [-8, 3, -4] + [18, 0, 3] + [-12, -28, -8]$$

When combined:

$$\mathbf{w} = [7, -24, -7]$$

Exercise 1 Linear Combination

Calculate the linear combination for vectors v_1, v_2, v_3 and scalars a_1, a_2, a_3 where:

$$a1 = 2, v1 = [2, 4, 8]$$

$$a2 = -2, v2 = [8, -6, 3]$$

$$a3 = 4, v3 = [7, 9, 2]$$

Make sure to show all your work.

Working Space

Answer on Page ??

1.1 Weighted Averages of Vectors

A weighted average of vectors is a specific type of linear combination where the coefficients (or weights) a_i are non-negative and sum to 1:

$$\sum_{i=1}^{n} a_i = 1, \quad a_i \geq 0$$

A weighted average of vectors $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n$ is then defined as

$$\mathbf{w} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + ... + a_n\mathbf{v}_n$$

In this case, each a_i not only scales the corresponding vector $\mathbf{v}_i$, but also represents the proportion of that vector in the final average vector $\mathbf{w}$.

Weighted averages are useful when you want to attribute the contribution of one feature or item over another. For example, a teacher might figure a student's final grade using exam scores, class participation, and a final project. The exam scores might make up 65% of the final grade, class participation 10%, and a final project 25%. Thus giving the formula for a grade as:

$$\textbf{Grade} = .65 * ExamScores + .10 * Participation + .25 * FinalProject$$

The teacher defines the weights, making sure they sum to 1.0.

Let's look at an example where the weights don't sum to 1.0. A store that sells umbrellas might have to get the umbrella stock from three different manufacturers. The store owner buys 100 umbrellas at a cost of $2.10 each, 50 umbrellas cost $1.85 each, and 200 umbrellas cost $2.00.

$$\textbf{TotalCost} = 2.10 * 100 + 1.85 * 50 + 2.00 * 200 = 702.5$$

To calculate the weighted average, divide the total cost by the number of items.

$$\textbf{WeightedAverage} = 702.5/350 = 2.01$$

Exercise 2 Weighted Average

A concert sells 300 tickets in the balcony at $50 each, 100 tickets on the main floor at $75 each, and 50 tickets in the section closest to the stage at $150 each. What's the weighted average?

Working Space

Answer on Page ??

1.2 Weighted Averages of Vectors in Python

Create a file called `linearCombos.py` and enter this code:

```
// import the python module that supports matrices
import numpy as np

// an array for number of umbrellas by manufacturer
items = np.array([100, 50, 200])

// weights are the cost of item by manufacturer
weights = np.array([2.10, 1.85, 2.00])

// create an array for total cost for each manufacturer
costPerManufacturer=items * weights

// sum the individuals costs to get the total
totalCost = np.sum(costPerManufacturer)

// get number of items
numItems = np.sum(items)

// you are ready to calculated the weighted average
weightedAverage = totalCost/numItems
print(weightedAverage)
```

When you run this code, you should get a weighted average of $2.01 when rounded to the nearest cent.

Appendix A

Answers to Exercises

Answer to Exercise ?? (on page ??)

$$\mathbf{w} = 2 * [2, 4, 8] + (-2) * [8, -6, 3] + 4 * [7, 9, 2]$$
$$\mathbf{w} = [4, 8, 16] + [-16, 12, -6] + [28, 36, 8]$$
$$\mathbf{w} = [16, 56, 18]$$

Answer to Exercise ?? (on page ??)

$$\mathbf{TotalSales} = 50 * 300 + 75 * 100 + 150 * 50 = 30,000$$
$$\mathbf{NumberTickets} = 300 + 100 + 50 = 450$$
$$\mathbf{WeightedAverage} = 30,000/450 = 66.67$$

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