Chapter 1

The Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus is a theorem that connects the concept of differentiating a function with the concept of integrating a function. This theorem is divided into two parts:

1.1 First Part

The first part of the Fundamental Theorem of Calculus states that if f is a continuous real-valued function defined on a closed interval $[a, b]$ and F is the function defined, for all x in $[a, b]$, by:

$$F(x) = \int_a^x f(t)\,dt \tag{1.1}$$

Then, F is uniformly continuous and differentiable on the open interval (a, b), and $F'(x) =$

$f(x)$ for all x in (a, b).

1.2 Second Part

The second part of the Fundamental Theorem of Calculus states that if f is a real-valued function defined on a closed interval $[a, b]$ that admits an antiderivative F on $[a, b]$, and f is integrable on $[a, b]$ (it need not be continuous), then

$$\int_a^b f(t)\,dt = F(b) - F(a). \tag{1.2}$$

Appendix A

Answers to Exercises

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