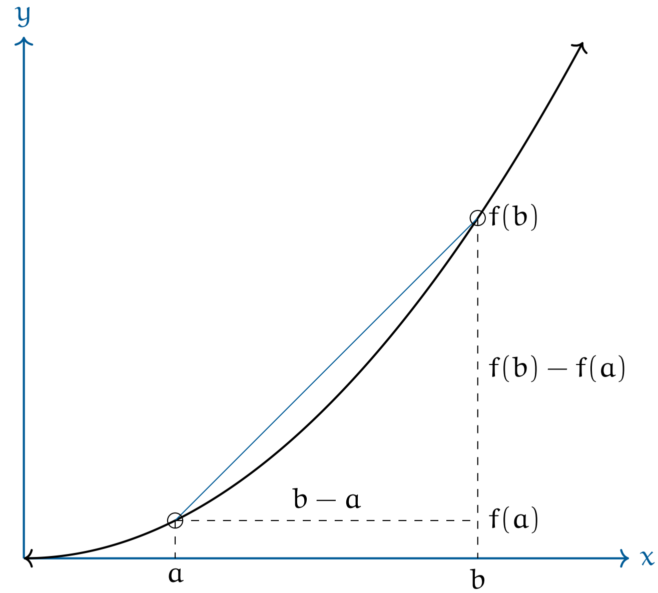
## Chapter 1

# Differentiation

We have done some differentiation, but you haven't been given the real definition because it is based on limits.

The idea is that we can find the slope between two points on the graph $a$ and $b$ like this:

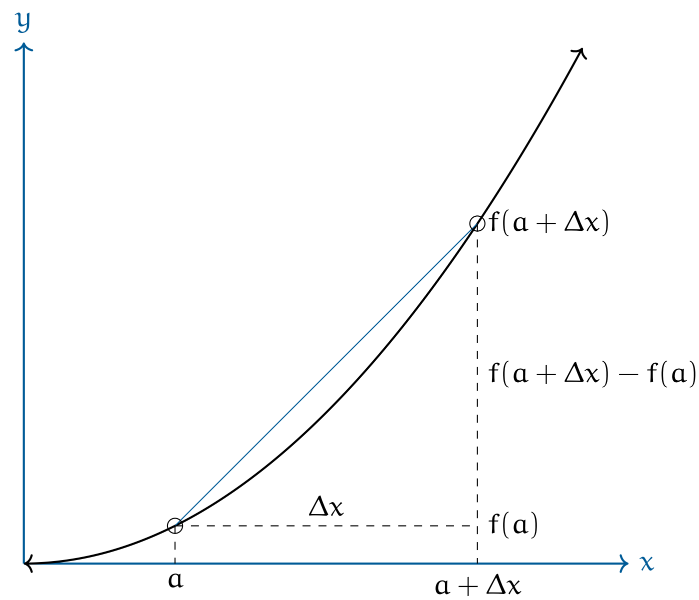
$$m = \frac{f(b) - f(a)}{b - a}$$




If we want to find the slope at $a$ we take the limit of this as the $b$ goes to $a$:

$$f'(a) = \lim_{b \to b} \frac{f(b) - f(a)}{b - a}$$

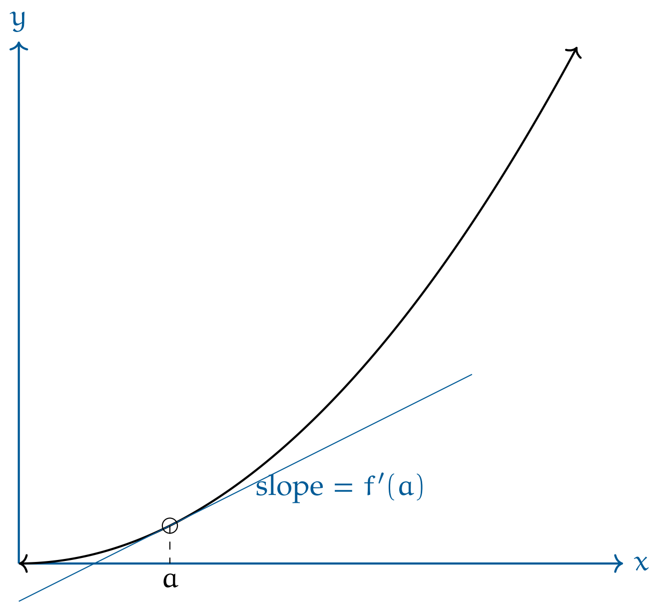
This idea is usually expressed using $\Delta x$ as the difference between $b$ and $a$:




Then the formula becomes:

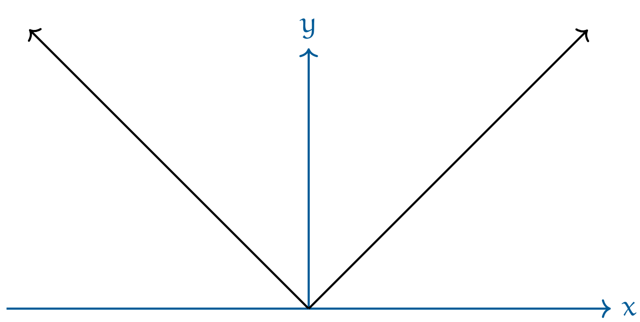
$$f'(a) = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

Now, at any point $a$ we can compute the slope of the line tangent to the function at $a$:




## 1.1 Differentiability

Warning: Not every function is differentiable everywhere. For example, if $f(x) = |x|$, you get a corner at zero.




To the left of zero, the slope is -1. To the right of zero, the slope is 1. At zero? The derivative is not defined.

If a function has a derivative everywhere, it is said to be *differentiable*. Generally, you can think of differentiable functions as smooth – their graphs have no corners.

## 1.2 Using the definition of derivative

Let's say that you want to know the slope of $f(x) = -3x^2$ at $x = 2$. Using the definition of the derivative, that would be:

$$f'(2) = \lim_{\Delta x \to 0} \frac{f(2+\Delta x) - f(2)}{\Delta x} = \lim_{\Delta x \to 0} \frac{-3(2+\Delta x)^2 - \left(-3(2)^2\right)}{\Delta x} = \lim_{\Delta x \to 0} \frac{-12 - 12\Delta x + -3(\Delta x)^2 + 12}{\Delta x} = -12$$

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*This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.*

Appendix A

# Answers to Exercises