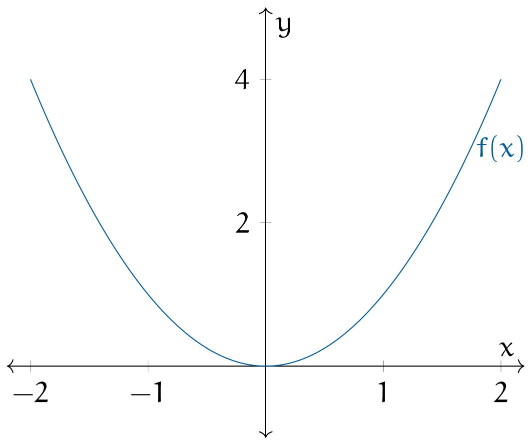
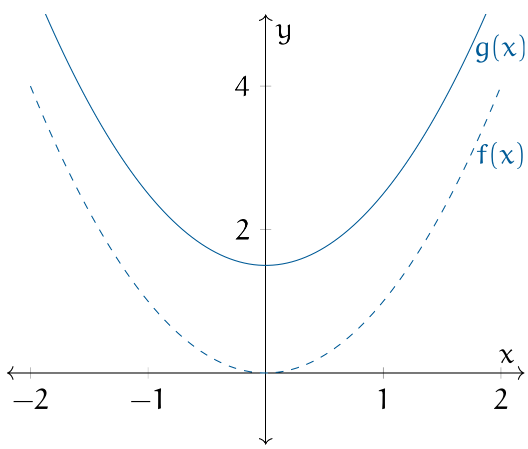
Chapter 1

Transforming Functions

Let's say I gave you the graph of a function f, like this:



And then I tell you that the function $g(x) = f(x) + 1.5$. Can you guess what the graph of g would look like? It is the same graph, just translated up 1.5:



There are four kinds of transformations that we do all the time:

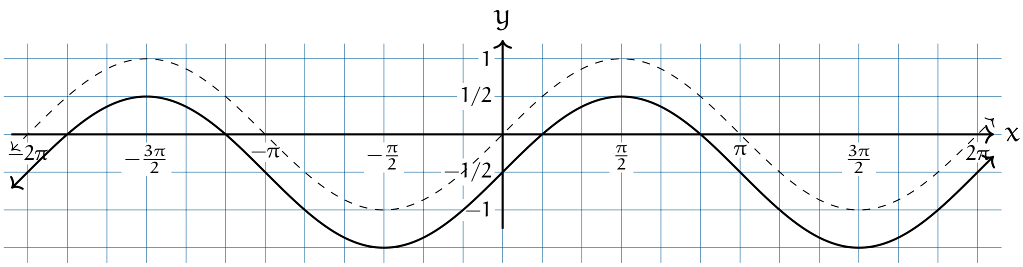
- Translation up and down in the direction of y axis (the one you just saw)
- Translation left and right in the direction of the x axis
- Scaling up and down along the y axis
- Scaling up and down along the x axis

Now I will demonstrate each of the four using the graph of $\sin(x)$.

1.1 Translation up and down

When you add a positive constant to a function, you translate the whole graph up that much. A negative constant translates it down.

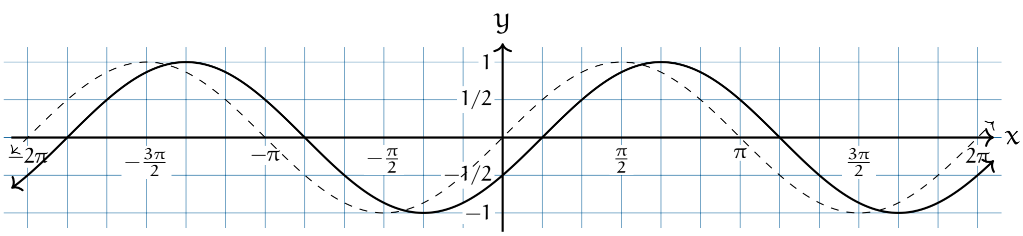
Here is the graph of $\sin(x) - 0.5$:



1.2 Translation left and right

When you add a positive number to x before running it through f, you translate the graph to the left that much. Adding a negative number translates the graph to the right.

Here is the graph of $\sin(x - \pi/6)$:



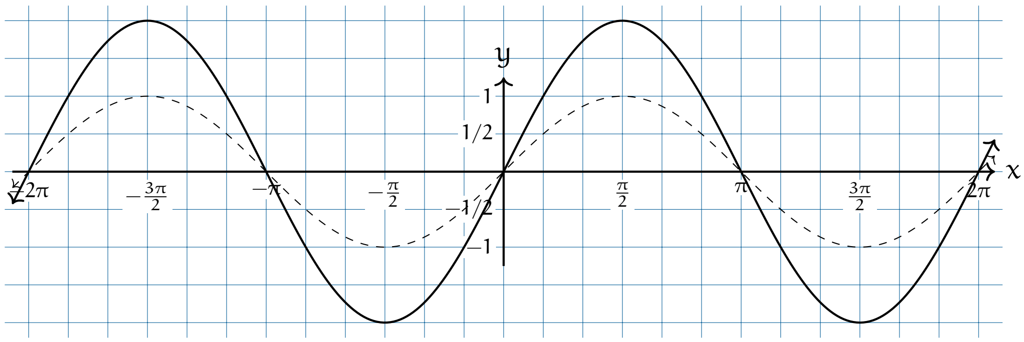
Notice the sign:

- Add to x before processing with the function translates the graph to the *left*.
- Subtract from x before processing with the function translates the graph to the *right*

1.3 Scaling up and down in the y direction

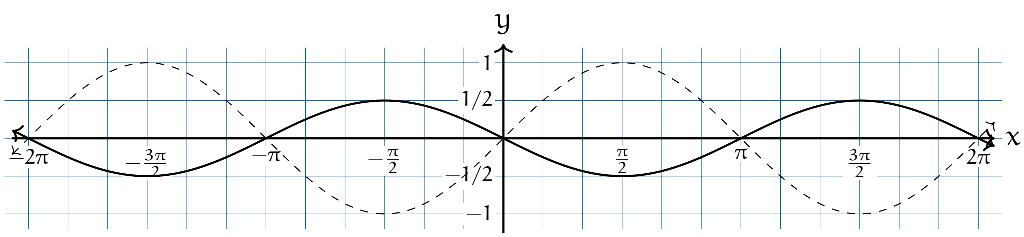
To scale the function up and down, you multiply the result of the function by a constant. If the constant is larger than 1, it stretches the function up and down.

Here is $y = 2\sin(x)$:



With a wave like this, we speak of its *Amplitude*, which you can think of as its height. The baseline that this wave oscillates around is zero. The maximum distance that it gets from that baseline is its amplitude. Thus, the amplitude here has been increased from 1 to 2.

If you multiply by a negative number, the function gets flipped. Here is $y = -0.5\sin(x)$

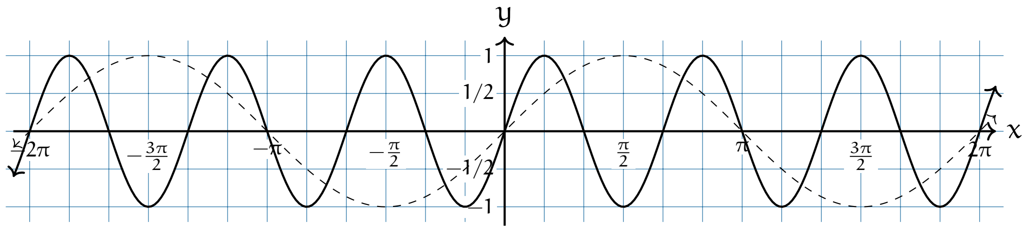


Amplitude is never negative. Thus, the amplitude of this wave is 0.5.

1.4 Scaling up and down in the x direction

If you multiply x by a number larger than 1 before running it through the function, the graph gets compressed toward zero.

Here is $y = \sin(3x)$:

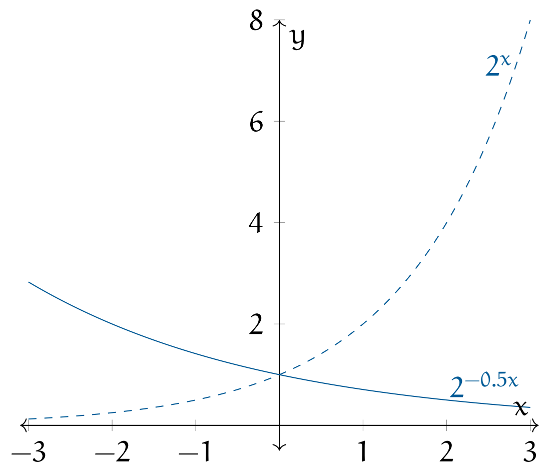


The distance between two peaks of a wave is known as its *wavelength*. The original wave had a wavelength of 2π. The compressed wave has a wavelength of $2\pi/3$.

If you multiply x by a number smaller than 1, it will stretch the function out, away from the y axis.

If you multiply x by a negative number, it will flip the function around the y axis.

Here is $y = 2^{(-0.5x)}$. Notice that it has flipped around the y axis and is stretched out along the x axis.

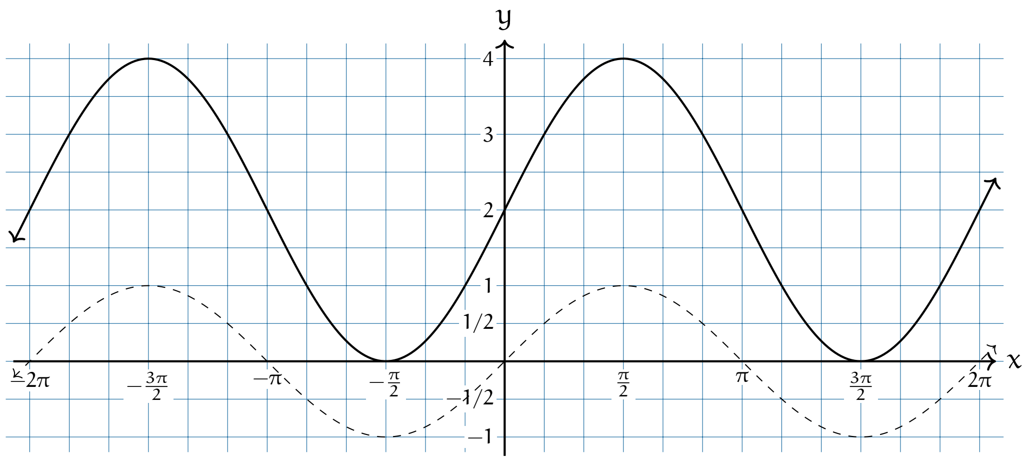


Reflection over *x*-axis	$(x, y) \to (x, -y)$
Reflection over *y*-axis	$(x, y) \to (-x, y)$
Translation	$(x, y) \to (x + a, y + b)$
Dilation	$(x, y) \to (kx, ky)$
Rotation 90° counterclockwise	$(x, y) \to (-y, x)$
Rotation 180°	$(x, y) \to (-x, -y)$

1.5 Order is important!

We can combine these transformations. This allows us, for example, to translate a function up 2 and then scale along the y axis by 3.

Here is $y = 2.0(\sin(x) + 1)$:

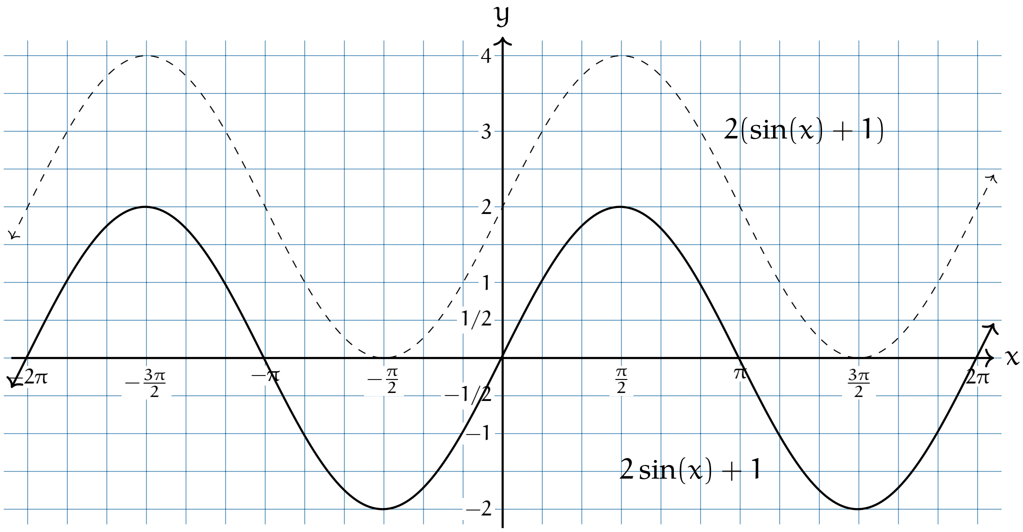


A function is often a series of steps. Here are the steps in $f(x) = 2(\sin(x) + 1)$:

1. Take the sine of x
2. Add 1 to that
3. Multiply that by 2

What if we change the order? Here are the steps in $g(x) = 2\sin(x) + 1$:

1. Take the sine of x
2. Multiply that by 2
3. Add 1 to that

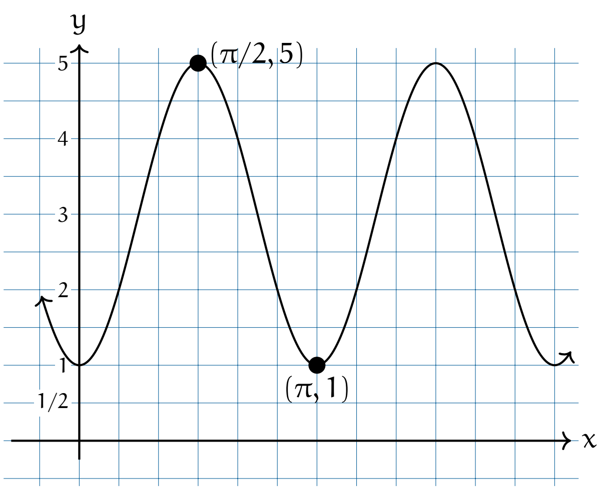


The moral: You can do multiple transformations of your function, but the order in which you do them is important.

Exercise 1 Transforms

Working Space

Find a function that creates a sine wave such that the top of the first crest is at the point $(\frac{\pi}{2}, 5)$ and the bottom of the trough that follows is at $(\pi, 1)$.



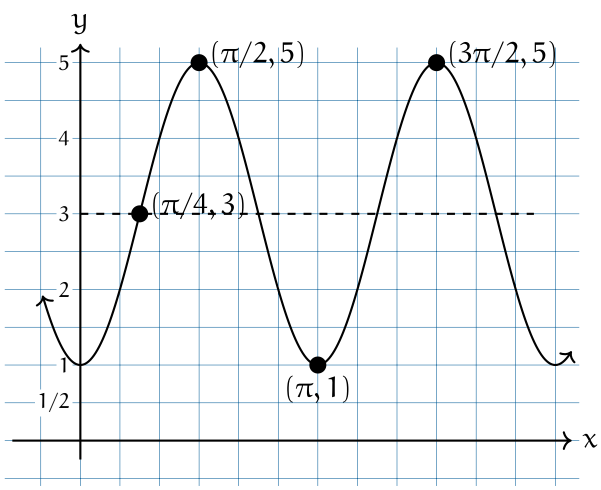
Answer on Page 9

This is a draft chapter from the Kontinua Project. Please see our website (https://kontinua.org/) for more details.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 6)



This wave has an amplitude of 2. Its baseline has been translated up to 3.

This wave has wavelength of π. A sine wave usually has a wavelength of 2π, so we need to compress the x axis by a factor of 2.

The wave first crosses its baseline at $pi/4$. The sine wave starts by crossing its baseline, so we need to translate the curve right by $\pi/4$.

$$f(x) = 2\sin(2x - \frac{\pi}{4}) + 3$$