Chapter 1

Derivatives

In calculus, the derivative of a function represents the rate at which the function is changing at a particular point. It is a fundamental concept that has vast applications in various fields, including physics.

1.1 Definition

The derivative of a function $f(x)$ at a point x is defined as the limit:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{1.1}$$

provided this limit exists. In words, the derivative of f at x is the limit of the rate of change of f at x as the change in x approaches zero.

1.2 Applications in Physics

In physics, derivatives play a vital role in describing how quantities change with respect to one another.

1.2.1 Velocity and Acceleration

In kinematics, the derivative of the position function with respect to time gives the velocity function, and further taking the derivative of the velocity function gives the acceleration function. For example, if $s(t)$ represents the position of an object at time t, then the velocity $v(t)$ and acceleration $a(t)$ are given by:

$$v(t) = \frac{ds}{dt} \quad \text{and} \quad a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2} \tag{1.2}$$

1.2.2 Force and Momentum

In mechanics, the derivative of the momentum of an object with respect to time gives the net force acting on the object, as stated by Newton's second law of motion:

$$F = \frac{dp}{dt} \tag{1.3}$$

where F is the force, p is the momentum, and t is the time.

Appendix A

Answers to Exercises

Index