Representing Natural Numbers

The natural numbers are 1, 2, 3, and so on. -5 is not a natural number. π is not a natural number. $\frac{1}{2}$ is not a natural number.

You are used to seeing the natural numbers represented in a base-10 *Hindu-Arabic* numeral system. That is, when you see 2531 you think "2 thousands, 5 hundreds, 3 tens, and 1 one." Rewritten this is

$$2 \times 10^3 + 5 \times 10^2 + 3 \times 10^1 + 1 \times 10^0$$

In any Hindu-Arabic system, the location of the digits is meaningful: 101 is different from 110. Here are those numbers in Roman numerals: CI and CX. Roman numerals didn't have a symbol for zero at all.

The Hindu-Arabic system gave us really straightforward algorithms for addition and multiplication. For addition, you memorized the following table:

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15 16	17	18	

Then when you multiplied two number together, you just multiplied each pair of digits. 254×26 might look like this:

```
        2   5   4
    ×       2   6
    ─────────────────
            2   4 │ 6 × 4
        3   0     │ 6 × 5
    1   2         │ 6 × 2
            8     │ 2 × 4
    1   0         │ 2 × 5
+   4             │ 2 × 2
─────────────────
6   6   0   4
```

For multiplication, you memorized this table:

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
9	0	9	18	27	36	45	54	63	72	81

APPENDIX A

Answers to Exercises