Chapter 1

Implicit Differentiation

Implicit differentiation is a technique in calculus for finding the derivative of a relation defined implicitly, that is, a relation between variables x and y that is not explicitly solved for one variable in terms of the other.

1.1 Implicit Differentiation Procedure

Consider an equation that defines a relationship between x and y:

$$F(x, y) = 0$$

To find the derivative of y with respect to x, we differentiate both sides of this equation with respect to x, treating y as an implicit function of x:

$$\frac{d}{dx}F(x, y) = \frac{d}{dx}0$$

Applying the chain rule during the differentiation on the left side of the equation gives:

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx} = 0$$

Finally, we solve for $\frac{dy}{dx}$ to find the derivative of y with respect to x:

$$\frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

This result is obtained using the implicit differentiation method.

1.2 Example

Consider the equation of a circle with radius r:

$$x^2 + y^2 = r^2$$

Differentiating both sides with respect to x, we get:

$$2x + 2y\frac{dy}{dx} = 0$$

Solving for $\frac{dy}{dx}$ gives:

$$\frac{dy}{dx} = -\frac{x}{y}$$

which is the slope of the tangent line to the circle at any point (x, y).

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Appendix A

Answers to Exercises

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