

Units and Conversions

Accurate measurements are at the heart of good data and good problem solving. Engineers need to be able to describe many different types of phenomena – distance, sound, light, force, and so on.

At this point, you are working with a lot of units: grams for weight, joules for energy, newtons for force, meters for distance, seconds for time, etc. For each type of measurement, there are several different units; for example, distance can be measured in feet, miles, and light-years.

Some Equalencies

Distance	
1 mile	1.6093 kilometers
1 foot	0.3048 meters
1 inch	2.54 centimeters
1 light-year	9.461×10^{12} kilometers
Volume	
1 milliliter	1 cubic centimeter
1 quart	0.9461 liters
1 gallon	3.7854 liters
1 fluid ounce	29.6 milliliters
Mass	
1 pound	0.4535924 kilograms
1 ounce	0.4535924 grams
1 metric ton	1000 kilograms
Force	
1 newton	1 kilogram meter per sec ²
Pressure	
1 pascal	1 newton per square meter
1 bar	0.98692 atmosphere
1 pound per square inch	6897 pascals
Energy	
1 joule	1 newton meter
1 calorie	4.184 joules
1 kilowatt-hour	3.6×10^6 joules

(You don't need to memorize these! Just remember that this page is here.)

In the metric system, prefixes are often used to express a multiple. Here are the common prefixes:

Common Prefixes for Metric Units

giga	$\times 10^9$
mega	$\times 10^6$
kilo	$\times 10^3$
milli	$\div 10^3$
micro	$\div 10^6$
nano	$\div 10^9$

(These are worth memorizing. Here's a mnemonic: "King Henry Doesn't Usually Drink Chocolate Milk." Or Kilo, Hecto, Deca, Unit (for example: gram), Deci, Centi, Mili.

1.1 Conversion Factors

Here is a really handy trick to remembering how to do conversions between units.

Often, you will be given a table like the one above, and someone will ask you "How many miles are in 0.23 light-years?" You know that 1 mile = 1.6093 kilometers and that 1 light-year is 9.461×10^{12} kilometers. How do you do the conversion?

The trick is to treat the two parts of the equality as a fraction that equals 1. That is, you think:

$$\frac{1 \text{ miles}}{1.6093 \text{ km}} = \frac{1.6093 \text{ km}}{1 \text{ miles}} = 1$$

and

$$\frac{1 \text{ light-years}}{9.461 \times 10^{12} \text{ km}} = \frac{9.461 \times 10^{12} \text{ km}}{1 \text{ light-years}} = 1$$

We call these fractions *conversion factors*.

Now, your problem is

$$0.23 \text{ light-years} \times \text{Some conversion factors} = ? \text{ miles}$$

Note that when you multiply fractions together, things in the numerators can cancel with things in the denominator:

$$\left(\frac{31\pi}{47}\right)\left(\frac{11}{37\pi}\right) = \left(\frac{31\cancel{\pi}}{47}\right)\left(\frac{11}{37\cancel{\pi}}\right) = \left(\frac{31}{47}\right)\left(\frac{11}{37}\right)$$

When working with conversion factors, you will do the same with the units:

$$\begin{aligned} 0.23 \text{ light-years} \left(\frac{9.461 \times 10^{12} \text{ km}}{1 \text{ light-years}}\right) \left(\frac{1 \text{ miles}}{1.6093 \text{ km}}\right) &= \\ 0.23 \cancel{\text{ light-years}} \left(\times \frac{9.461 \times 10^{12} \cancel{\text{ km}}}{1 \cancel{\text{ light-years}}}\right) \left(\frac{1 \text{ miles}}{1.6093 \cancel{\text{ km}}}\right) &= \frac{(0.23)(9.461 \times 10^{12})}{1.6093} \text{ miles} \end{aligned}$$

Exercise 1 Simple Conversion Factors

Working Space

How many calories are in 4.5 kilowatt-hours?

Answer on Page 5

1.2 Conversion Factors and Ratios

Conversion factors also work on ratios. For example, if you are told that a bug is moving 0.5 feet every 120 milliseconds. What is that in meters per second?

The problem then is

$$\frac{0.5 \text{ feet}}{120 \text{ milliseconds}} = \frac{? \text{ m}}{\text{second}}$$

So you will need conversion factors to replace the “feet” with “meters” and to replace “milliseconds” with “seconds”:

$$\left(\frac{0.5 \cancel{\text{feet}}}{120 \cancel{\text{milliseconds}}}\right) \left(\frac{0.3048 \text{ meters}}{1 \cancel{\text{feet}}}\right) \left(\frac{1000 \cancel{\text{milliseconds}}}{1 \text{ second}}\right) = \frac{(0.5)(0.3048)(1000)}{120} \text{ m/second}$$

Exercise 2 Conversion Factors

Working Space

The hole in the bottom of the boat lets in 0.1 gallons every 2 minutes. How many milliliters per second is that?

Answer on Page 5

1.3 When Conversion Factors Don't Work

Conversion factors only work when the units being converted are proportional to each other. Gallons and liters, for example, are proportional to each other: If you have n gallons, you have $n \times 3.7854$ liters.

Degrees celsius and degrees fahrenheit are *not* proportional to each other. If your food is n degrees celsius, it is $n \times \frac{9}{5} + 32$ degrees fahrenheit. You can't use conversion factors to convert celsius to fahrenheit.

Watch Khan Academy's video on this at <https://www.khanacademy.org/test-prep/sat/x0a8c2e5f:untitled-652/x0a8c2e5f:problem-solving-and-data-analysis-lessons-by-skill/a/gtp--sat-math--article--units--lesson>

This is a draft chapter from the Kontinua Project. Please see our website (<https://kontinua.org/>) for more details.

Answers to Exercises

Answer to Exercise 1 (on page 3)

$$4.5 \text{ kWh} \left(\frac{3.6 \times 10^6 \text{ joules}}{1 \text{ kWh}} \right) \left(\frac{1 \text{ calories}}{4.184 \text{ joules}} \right) = \frac{(4.5)(3.6 \times 10^6)}{4.184} = 1.08 \times 10^6 \text{ calories}$$

Answer to Exercise 2 (on page 4)

$$\frac{0.1 \text{ gallons}}{2 \text{ minutes}} \left(\frac{3.7854 \text{ liters}}{1 \text{ gallons}} \right) \left(\frac{1000 \text{ milliliters}}{1 \text{ liters}} \right) \left(\frac{1 \text{ minutes}}{60 \text{ seconds}} \right) =$$
$$\frac{(0.1)(3.7854)(1000)}{(2)(60)} \text{ ml/second} = 3.1545 \text{ ml/second}$$



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