Chapter 1

Sound

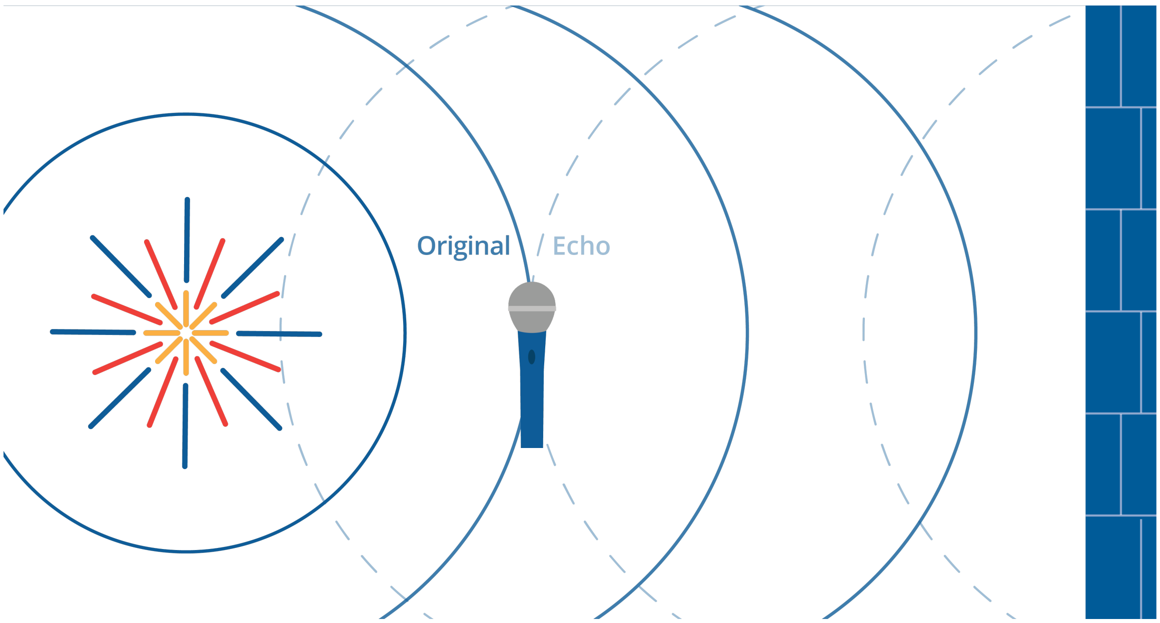
When you set off a firecracker, it makes a sound.

Let's break that down a little more: Inside the cardboard wrapper of the firecracker, there is potassium nitrate (KNO_3), sulfur (S), and carbon(C). These are all solids. When you trigger the chemical reactions with a little heat, these atoms rearrange themselves to be potassium carbonate (K_2CO_3), potassium sulfate (K_2SO_4), carbon dioxide (CO_2), and nitrogen (N_2). Note that the last two are gasses.

The molecules of a solid are much more tightly packed than the molecules of a gas. So after the chemical reaction, the molecules expand to fill a much bigger volume. The air molecules nearby get pushed away from the firecracker. They compress the molecules beyond them, and those compress the molecules beyond them.

This compression wave radiates out as a sphere; its radius growing at about 343 meters per second ("The speed of sound").

The energy of the explosion is distributed around the surface of this sphere. As the radius increases, the energy is spread more and more thinly around. This is why the firecracker seems louder when you are closer to it. (If you set off a firecracker in a sewer pipe, the sound will travel much, much farther.)



This compression wave will bounce off of hard surfaces. If you set off a firecracker 50 meters from a big wall, you will hear the explosion twice. We call the second one "an echo."

The compression wave will be absorbed by soft surfaces. If you covered that wall with pillows, there would be almost no echo.

The study of how these compression waves move and bounce is called *acoustics*. Before you build a concert hall, you hire an acoustician to look at your plans and tell you how to make it sound better.

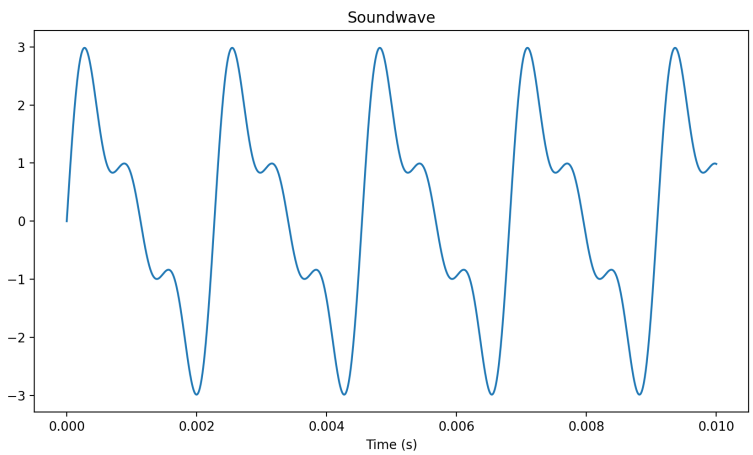
1.1 Pitch and frequency

The string on a guitar is very similar to the weighted spring example. The farther the string is displaced, the more force it feels pushing it back to equilibrium. Thus, it moves back and forth in a sine wave. (OK, it isn't a pure sine wave, but we will get to that later.)

The string is connected to the center of the boxy part of the guitar, which is pushed and pulled by the string. That creates compression waves in the air around it.

If you are in the room with the guitar, those compression waves enter your ear, push and pull your eardrum, which is attached to bones that move a fluid that tickles tiny hairs, called *cilia* in your inner ear. That is how you hear.

We sometimes see plots of sound waveforms. The x-axis represents time. The y-axis represents the amount the air is compressed at the microphone that converted the air pressure into an electrical signal.



If the guitar string is made tighter (by the tuning pegs) or shorter (by the guitarist's fingers on the strings), the string vibrates more times per second. We measure the number of waves per second and we call it the *frequency* of the tone. The unit for frequency is *Hertz*: cycles per second.

Musicians have given the different frequencies names. If the guitarist plucks the lowest note on his guitar, it will vibrate at 82.4 Hertz. The guitarist will say "That pitch is low E." If the string is made half as long (by a finger on the 12th fret), the frequency will be twice as fast (164.8 Hertz), and the guitarist will say "That is E an octave up."

For any note, the note that has twice the frequency is one octave up. The note that has half the frequency is one octave down.

The octave is a very big jump in pitch, so musicians break it up into 12 smaller steps. If the guitarist shortens the E string by one fret, the frequency will be $82.4 \times 1.059463 \approx 87.3$ Hertz.

Shortening the string one fret always increases the frequency by a factor of 1.059463. Why?

Because $1.059463^1 2 = 2$. That is, if you take 12 of these hops, you end up an octave higher.

This, the smallest hop in western music, is referred to as *half step*.

Exercise 1 Notes and frequencies

The note A near the middle of the piano, is 440Hz. The note E is 7 half steps above A. What is its frequency?

Working Space

Answer on Page ??

1.2 Chords and harmonics

Of course, a guitarist seldom plays only one string at a time. Instead, he uses the frets to pick a pitch for each string and strums all six strings.

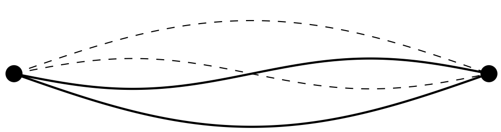
Some combinations of frequencies sound better than others. We have already talked about the octave: if one string vibrates twice for each vibration of another, they sound sweet together.

Musicians speak of "the fifth". If one string vibrates three times and the other vibrates twice in the same amount of time, they sound sweet together.

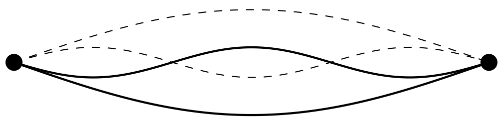
If one string vibrates 4 times while the other vibrates 3 times, they sound sweet together. Musicians call this "the third."

Each of these different frequencies tickle different cilia in the inner ear, so you can hear all six notes at the same time when the guitarist strums his guitar.

When a string vibrates, it doesn't create a single sine wave. Yes, the string vibrates from end-to-end and this generates a sine wave at what we call *the fundamental frequency*. However, there are also "standing waves" on the string. One of these standing waves is still at the centerpoint of the string, but everything to the left of the centerpoint is going up while everything to the right is going down. This creates *an overtone* that is twice the frequency of the fundamental.



The next overtone has two still points – it divides the string into three parts. The outer parts are up while the inner part is down. Its frequency is three times the fundamental frequency.



And so on: 4 times the fundamental, 5 times the fundamental, etc.

In general, tones with a lot of overtones tend to sound bright. Tones with just the fundamental sound thin.

Humans can generally hear frequencies from 20Hz to 20,000Hz (or 20kHz). Young people tend to be able to hear very high sounds better than older people.

Dogs can generally hear sounds in the 65Hz to 45kHz range.

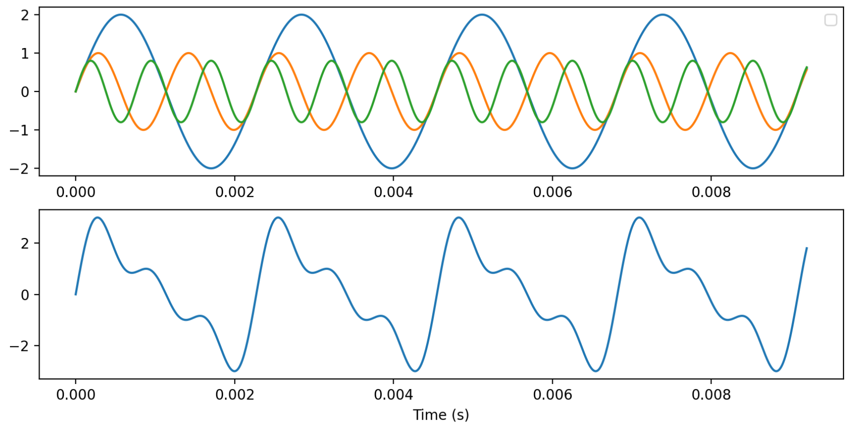
1.3 Making waves in Python

Let's make a sine wave and add some overtones to it. Create a file `harmonics.py`

```
import matplotlib.pyplot as plt
import math

# Constants: frequency and amplitude
fundamental_freq = 440.0 # A = 440 Hz
fundamental_amp = 2.0

# Up an octave
first_freq = fundamental_freq * 2.0 # Hz
first_amp = fundamental_amp * 0.5

# Up a fifth more
second_freq = fundamental_freq * 3.0 # Hz
second_amp = fundamental_amp * 0.4

# How much time to show
max_time = 0.0092 # seconds

# Calculate the values 10,000 times per second
time_step = 0.00001 # seconds

# Initialize
time = 0.0
times = []
```

```
totals = []
fundamentals = []
firsts = []
seconds = []

while time <= max_time:
    # Store the time
    times.append(time)

    # Compute value each harmonic
    fundamental = fundamental_amp * math.sin(2.0 * math.pi * fundamental_freq * time)
    first = first_amp * math.sin(2.0 * math.pi * first_freq * time)
    second = second_amp * math.sin(2.0 * math.pi * second_freq * time)

    # Sum them up
    total = fundamental + first + second

    # Store the values
    fundamentals.append(fundamental)
    firsts.append(first)
    seconds.append(second)
    totals.append(total)

    # Increment time
    time += time_step

# Plot the data
fig, ax = plt.subplots(2, 1)

# Show each component
ax[0].plot(times, fundamentals)
ax[0].plot(times, firsts)
ax[0].plot(times, seconds)
ax[0].legend()

# Show the totals
ax[1].plot(times, totals)
ax[1].set_xlabel("Time (s)")

plt.show()
```

When you run it, you should see a plot of all three sine waves and another plot of their sum:



1.3.1 Making a sound file

The graph is pretty to look at, but make a file that we can listen to.

The WAV audio file format is supported on pretty much any device, and a library for writing WAV files comes with Python. Let's write some sine waves and some noise into a WAV file.

Create a file called `soundmaker.py`

```
import wave
import math
import random

# Constants
frame_rate = 16000 # samples per second
duration_per = 0.3 # seconds per sound
frequencies = [220, 440, 880, 392] # Hz
amplitudes = [20, 125]
baseline = 127 # Values will be between 0 and 255, so 127 is the baseline
samples_per = int(frame_rate * duration_per) # number of samples per sound

# Open a file
wave_writer = wave.open('sound.wav', 'wb')

# Not stereo, just one channel
wave_writer.setnchannels(1)

# 1 byte audio means everything is in the range 0 to 255
```

```
wave_writer.setsampwidth(1)

# Set the frame rate
wave_writer.setframerate(frame_rate)

# Loop over the amplitudes and frequencies
for amplitude in amplitudes:
    for frequency in frequencies:
        time = 0.0
        # Write a sine wave
        for sample in range(samples_per):
            s = baseline + int(amplitude * math.sin(2.0 * math.pi * frequency * time))
            wave_writer.writeframes(bytes([s]))
            time += 1.0 / frame_rate

        # Write some noise after each sine wave
        for sample in range(samples_per):
            s = baseline + random.randint(0, 15)
            wave_writer.writeframes(bytes([s]))

# Close the file
wave_writer.close()
```

When you run it, it should create a sound file with several tones of different frequencies and volumes. Each tone should be followed by some noise.

Appendix A

Answers to Exercises

Answer to Exercise ?? (on page ??)

A is 440 Hz. Each half-step is a multiplication by $\sqrt[12]{2} = 1.059463094359295$ So the frequency of E is $(440)(2^{7/12}) = 659.255113825739859$