report

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0.1 Assignment 1

0.1.1 Name: Sourodeep Datta

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[ ]:
```
# import all the necessary libraries here
import pandas as pd
import numpy as np
from matplotlib import pyplot as plt
```

[ ]:
```
df = pd.read_csv('../../dataset/linear-regression.csv')
print(df.shape)
```

```
(1599, 12)
```

[ ]:
```
df.head()
```

[ ]:

	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	free sulfur dioxide	total sulfur dioxide	density	pH	sulphates	alcohol	quality
0	7.4	0.70	0.00	1.9	0.076	11.0	34.0	0.9978	3.51	0.56	9.4	5
1	7.8	0.88	0.00	2.6	0.098	25.0	67.0	0.9968	3.20	0.68	9.8	5
2	7.8	0.76	0.04	2.3	0.092	15.0	54.0	0.9970	3.26	0.65	9.8	5
3	11.2	0.28	0.56	1.9	0.075	17.0	60.0	0.9980	3.16	0.58	9.8	6
4	7.4	0.70	0.00	1.9	0.076	11.0	34.0	0.9978	3.51	0.56	9.4	5

Separating y from Dataset

[ ]:
```
y = df['quality']
y.head()
```

[ ]:
```
0    5
1    5
2    5
3    6
4    5
Name: quality, dtype: int64
```

[ ]:
```
df = df.drop(['quality'], axis = 1)
df.head()
```

[ ]:

	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	\
0	7.4	0.70	0.00	1.9	0.076	
1	7.8	0.88	0.00	2.6	0.098	
2	7.8	0.76	0.04	2.3	0.092	
3	11.2	0.28	0.56	1.9	0.075	
4	7.4	0.70	0.00	1.9	0.076	

	free sulfur dioxide	total sulfur dioxide	density	pH	sulphates	\
0	11.0	34.0	0.9978	3.51	0.56	
1	25.0	67.0	0.9968	3.20	0.68	
2	15.0	54.0	0.9970	3.26	0.65	
3	17.0	60.0	0.9980	3.16	0.58	
4	11.0	34.0	0.9978	3.51	0.56	

	alcohol
0	9.4
1	9.8
2	9.8
3	9.8
4	9.4

Normalizing Dataset

[ ]:
```
df_mean = df.mean(axis = 0)
df_mean.head(n = 12)
```

[ ]:
```
fixed acidity           8.319637
volatile acidity        0.527821
citric acid             0.270976
residual sugar          2.538806
chlorides               0.087467
free sulfur dioxide    15.874922
total sulfur dioxide   46.467792
density                 0.996747
```

```
pH                     3.311113
sulphates              0.658149
alcohol               10.422983
dtype: float64
```

[ ]:
```
df_std = df.std()
df_std.head(n = 12)
```

[ ]:
```
fixed acidity           1.741096
volatile acidity        0.179060
citric acid             0.194801
residual sugar          1.409928
chlorides               0.047065
free sulfur dioxide    10.460157
total sulfur dioxide   32.895324
density                 0.001887
pH                      0.154386
sulphates               0.169507
alcohol                 1.065668
dtype: float64
```

[ ]:
```
df_normalized = (df - df_mean) / df_std
df_normalized.head()
```

[ ]:
```
   fixed acidity  volatile acidity  citric acid  residual sugar  chlorides  \
0      -0.528194          0.961576    -1.391037       -0.453077  -0.243630
1      -0.298454          1.966827    -1.391037        0.043403   0.223805
2      -0.298454          1.296660    -1.185699       -0.169374   0.096323
3       1.654339         -1.384011     1.483689       -0.453077  -0.264878
4      -0.528194          0.961576    -1.391037       -0.453077  -0.243630

   free sulfur dioxide  total sulfur dioxide   density        pH  sulphates  \
0            -0.466047             -0.379014  0.558100  1.288240  -0.579025
1             0.872365              0.624168  0.028252 -0.719708   0.128910
2            -0.083643              0.228975  0.134222 -0.331073  -0.048074
3             0.107558              0.411372  0.664069 -0.978798  -0.461036
4            -0.466047             -0.379014  0.558100  1.288240  -0.579025

    alcohol
0 -0.959946
1 -0.584594
2 -0.584594
3 -0.584594
4 -0.959946
```

Adding a bias feature to dataset, to simplify calculations later

[ ]:
```
df_normalized.insert(11, 'bias', 1)
df_normalized.head()
```

[ ]:
```
   fixed acidity  volatile acidity  citric acid  residual sugar  chlorides  \
0      -0.528194          0.961576    -1.391037       -0.453077  -0.243630
1      -0.298454          1.966827    -1.391037        0.043403   0.223805
2      -0.298454          1.296660    -1.185699       -0.169374   0.096323
3       1.654339         -1.384011     1.483689       -0.453077  -0.264878
4      -0.528194          0.961576    -1.391037       -0.453077  -0.243630

   free sulfur dioxide  total sulfur dioxide   density        pH  sulphates  \
0            -0.466047             -0.379014  0.558100  1.288240  -0.579025
1             0.872365              0.624168  0.028252 -0.719708   0.128910
2            -0.083643              0.228975  0.134222 -0.331073  -0.048074
3             0.107558              0.411372  0.664069 -0.978798  -0.461036
4            -0.466047             -0.379014  0.558100  1.288240  -0.579025

    alcohol  bias
0 -0.959946     1
1 -0.584594     1
2 -0.584594     1
3 -0.584594     1
4 -0.959946     1
```

[ ]:
```
y_normalized = (y - y.mean()) / y.std()
y_normalized.head()
```

[ ]:
```
0   -0.787576
1   -0.787576
2   -0.787576
3    0.450707
4   -0.787576
Name: quality, dtype: float64
```

[ ]:
```
train_df = df_normalized.sample(frac = 0.5, random_state = 218)
val_df = df_normalized.drop(train_df.index).sample(frac = 0.6, random_state =␣
 ↪218)
test_df = df_normalized.drop(train_df.index).drop(val_df.index)
```

[ ]:
```
print(train_df.shape)
print(val_df.shape)
print(test_df.shape)
```

```
(800, 12)
(479, 12)
(320, 12)
```

[ ]:
```
y_train = y_normalized[train_df.index]
y_val = y_normalized[val_df.index]
y_test = y_normalized[test_df.index]
```

[ ]:
```
train_df.reset_index(drop = True, inplace=True)
val_df.reset_index(drop = True, inplace=True)
test_df.reset_index(drop = True, inplace=True)
y_train.reset_index(drop = True, inplace=True)
y_val.reset_index(drop = True, inplace=True)
y_test.reset_index(drop = True, inplace=True)
```

[ ]:
```
X_train = np.array(train_df)
X_val = np.array(val_df)
X_test = np.array(test_df)
y_train = np.array(y_train)
y_val = np.array(y_val)
y_test = np.array(y_test)
```

0.2 Analytical Solution

[ ]:
```
theta = np.matmul(np.matmul(np.linalg.inv(np.matmul(X_train.T, X_train)),␣
↪X_train.T), y_train)
```

0.2.1 Defining functions for calculating R^2 and RMSE

[ ]:
```
def R_squared(y, y_hat):
    SS_res = np.square(y - y_hat).sum()
    SS_tot = np.square(y - y.mean()).sum()
    ret = 1 - SS_res / SS_tot
    return ret
```

[ ]:
```
def RMSE(y, y_hat):
    return np.sqrt(np.mean((y - y_hat) ** 2))
```

0.2.2 Calculating Metrics for Analytical Solution

[ ]:
```
y_hat = np.matmul(X_test, theta)
```

[ ]:
```
print("The R-squared value for the test set is: " + str(R_squared(y_test,␣
↪y_hat)))
print("The RMSE value for the test set is: " + str(RMSE(y_test, y_hat)))
```

```
The R-squared value for the test set is: 0.32014246008040725
The RMSE value for the test set is: 0.8568483269730971
```

0.3 Iterative Solution

Defining Model

[ ]:
```
class LinearRegressionModel:
    def __init__(self, X_train, y_train, X_val, y_val, alpha):
        self.X_train = X_train
        self.y_train = y_train
        self.X_val = X_val
        self.y_val = y_val
        self.theta = np.random.rand(X_train.shape[1])
        self.alpha = alpha
        self.train_loss = []
        self.val_loss = []

    def gradient(self):
        y_hat = np.matmul(self.X_train, self.theta)
        gradient = np.sum((y_hat - self.y_train)[:, np.newaxis] * self.X_train,␣
 ↪axis = 0) / self.X_train.shape[0]
        return gradient.T

    def step(self):
        self.theta = self.theta - self.alpha * self.gradient()

    def loss(self, X, y):
        y_hat = np.matmul(X, self.theta)
        loss = np.sum((y_hat - y) ** 2) / (2 * X.shape[0])
        return loss

    def train(self, epochs):
        for i in range(epochs):
            self.step()
            train_loss = self.loss(self.X_train, self.y_train)
            val_loss = self.loss(self.X_val, self.y_val)
            self.train_loss.append(train_loss)
            self.val_loss.append(val_loss)
            if i % 1000 == 0:
                print("Epoch: {}, Train Loss: {}, Val Loss: {}".format(i,␣
 ↪train_loss, val_loss))

    def predict(self, X):
        y_hat = np.matmul(X, self.theta)
        return y_hat
```

Function for plotting model

[ ]:
```
def plot(model):
```

```
    plt.plot(np.arange(1, len(model.train_loss) + 1), model.train_loss,␣
↪label='Trainging Loss')
    plt.plot(np.arange(1, len(model.val_loss) + 1), model.val_loss,␣
↪label='Validation Loss')
    plt.title('Training and Validation Loss')
    plt.xlabel('Epochs')
    plt.ylabel('Loss')
    plt.legend(loc='best')
    plt.show()
```

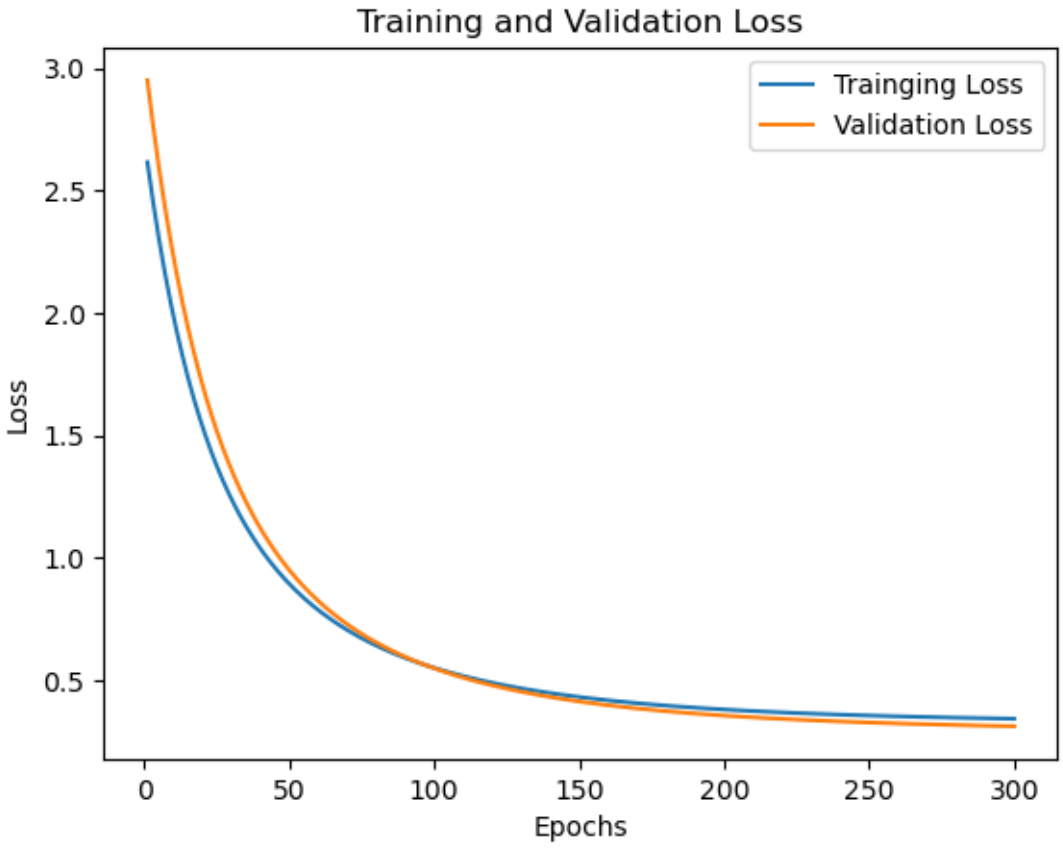
0.3.1 Model with Alpha = 0.01

[ ]:
```
np.random.seed(218)
model = LinearRegressionModel(X_train, y_train, X_val, y_val, 0.01)
```

[ ]:
```
model.train(300)
```

```
Epoch: 0, Train Loss: 2.614857389730282, Val Loss: 2.9497743937910412
```

[ ]:
```
plot(model)
```



[ ]:
```
print("The R-squared value for the test set is: " + str(R_squared(y_test, model.
  ↪predict(X_test))))
print("The RMSE value for the test set is: " + str(RMSE(y_test, model.
  ↪predict(X_test))))
```

```
The R-squared value for the test set is: 0.2947959138295665
The RMSE value for the test set is: 0.8726747372512642
```

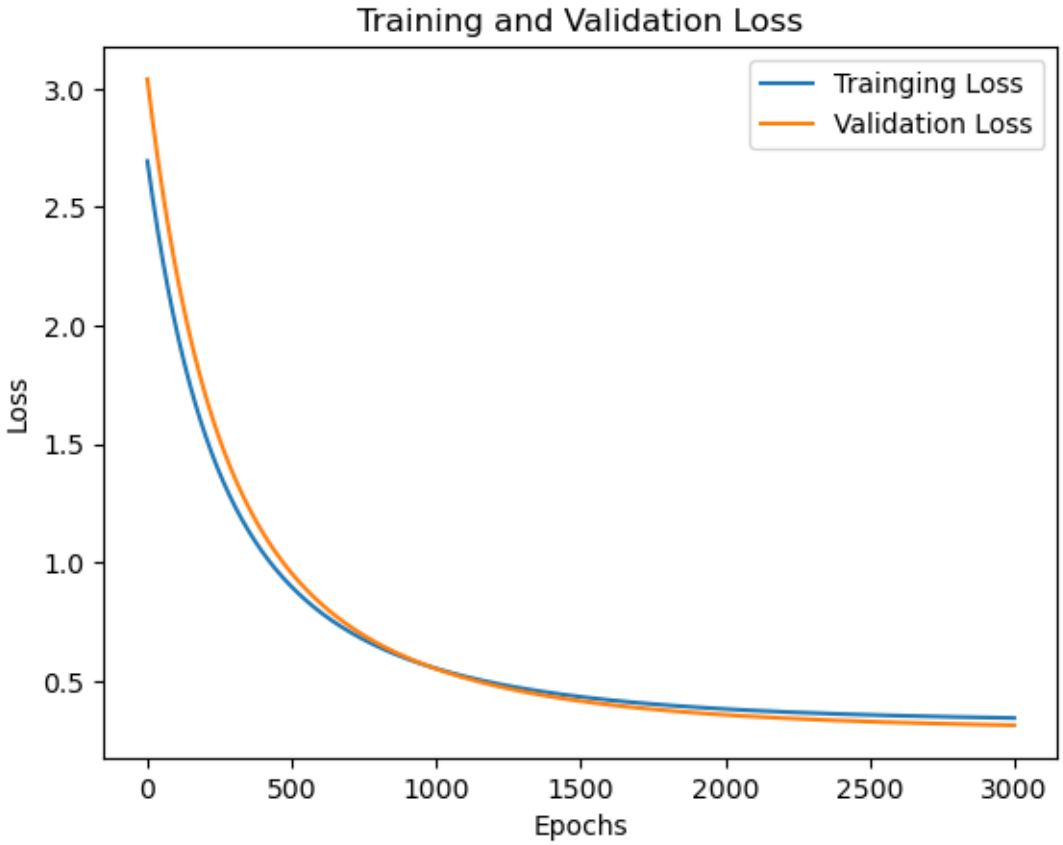
0.3.2 Model with Alpha = 0.001

[ ]:
```
np.random.seed(218)
model = LinearRegressionModel(X_train, y_train, X_val, y_val, 0.001)
```

[ ]:
```
model.train(3000)
```

```
Epoch: 0, Train Loss: 2.6944119572308356, Val Loss: 3.040288619490561
Epoch: 1000, Train Loss: 0.5515119413398927, Val Loss: 0.5501250190430178
Epoch: 2000, Train Loss: 0.38084694281625286, Val Loss: 0.3560731798383467
```

[ ]:
```
plot(model)
```



[ ]:
```
print("The R-squared value for the test set is: " + str(R_squared(y_test, model.
  ↪predict(X_test))))
print("The RMSE value for the test set is: " + str(RMSE(y_test, model.
  ↪predict(X_test))))
```

```
The R-squared value for the test set is: 0.2944313094180363
The RMSE value for the test set is: 0.8729003031189544
```

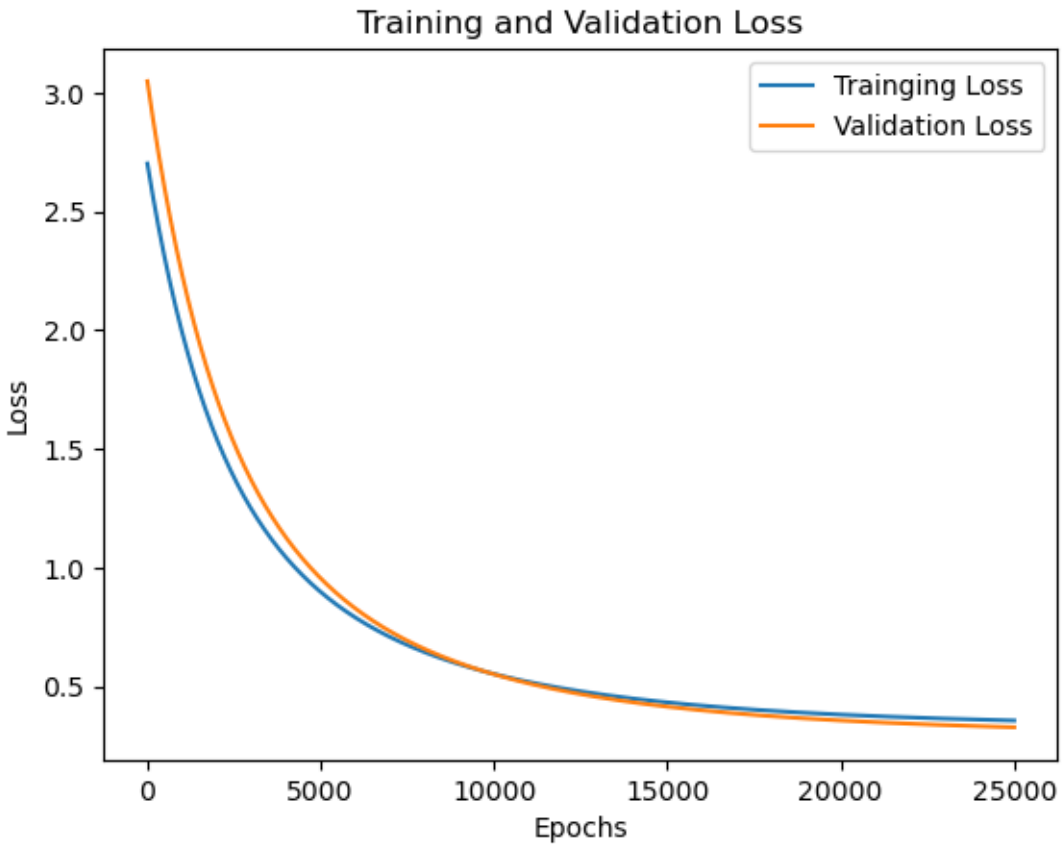
0.3.3 Model with Alpha = 0.0001

[ ]:
```
np.random.seed(218)
model = LinearRegressionModel(X_train, y_train, X_val, y_val, 0.0001)
```

[ ]:
```
model.train(25000)
```

```
Epoch: 0, Train Loss: 2.7024662621660274, Val Loss: 3.0494456301906565
Epoch: 1000, Train Loss: 1.992074220179704, Val Loss: 2.2360660199322036
Epoch: 2000, Train Loss: 1.5426920786398222, Val Loss: 1.7143931850020224
Epoch: 3000, Train Loss: 1.2464319052687862, Val Loss: 1.3669497003957207
Epoch: 4000, Train Loss: 1.042836021638482, Val Loss: 1.1267632666266088
Epoch: 5000, Train Loss: 0.8973489711342372, Val Loss: 0.9548385713525701
Epoch: 6000, Train Loss: 0.7897284540484989, Val Loss: 0.827894663236715
Epoch: 7000, Train Loss: 0.7077630996657638, Val Loss: 0.7316299451861125
Epoch: 8000, Train Loss: 0.6438356097126328, Val Loss: 0.6569809790978351
Epoch: 9000, Train Loss: 0.5930199477663399, Val Loss: 0.5980148344314871
Epoch: 10000, Train Loss: 0.5520116202197042, Val Loss: 0.5507211490213487
Epoch: 11000, Train Loss: 0.5185146908024693, Val Loss: 0.5123055525328194
Epoch: 12000, Train Loss: 0.49088226850444466, Val Loss: 0.48076651291831507
Epoch: 13000, Train Loss: 0.46789985303512904, Val Loss: 0.4546351951554992
Epoch: 14000, Train Loss: 0.4486507436009923, Val Loss: 0.4328109959925091
Epoch: 15000, Train Loss: 0.43242967686029077, Val Loss: 0.4144545418264282
Epoch: 16000, Train Loss: 0.4186855668252934, Val Loss: 0.39891608278841006
Epoch: 17000, Train Loss: 0.40698232522197386, Val Loss: 0.38568628044724296
Epoch: 18000, Train Loss: 0.396971266571866, Val Loss: 0.3743615524038621
Epoch: 19000, Train Loss: 0.38837116987025894, Val Loss: 0.3646191334843259
Epoch: 20000, Train Loss: 0.3809535536756411, Val Loss: 0.35619878659734183
Epoch: 21000, Train Loss: 0.3745315996602369, Val Loss: 0.3488891694159037
Epoch: 22000, Train Loss: 0.3689516923161833, Val Loss: 0.34251752798874363
Epoch: 23000, Train Loss: 0.36408687444969035, Val Loss: 0.3369418106279275
Epoch: 24000, Train Loss: 0.3598317308458753, Val Loss: 0.3320445701461536
```

[ ]:
```
plot(model)
```



[ ]:
```
print("The R-squared value for the test set is: " + str(R_squared(y_test, model.
  ↪predict(X_test))))
print("The RMSE value for the test set is: " + str(RMSE(y_test, model.
  ↪predict(X_test))))
```

```
The R-squared value for the test set is: 0.26755934850049523
The RMSE value for the test set is: 0.8893674159401217
```