Sequences

Use the definition to prove that the following sequence has a limit.

1. $\{\frac{2n^2+1}{3n^2-n}\} \rightarrow \frac{\mathbf{2}}{\mathbf{3}}$

$$\left|\frac{2n^2+1}{3n^2-n} - \frac{2}{3}\right| < \in$$

$$\left|\frac{6n^2+3-6n^2+2n}{9n^2-3n}\right| < \in$$

$$\left|\frac{2n+3}{9n^2-3n}\right| < \in$$

$$\frac{\frac{2n^2}{n^2}+\frac{1}{n^2}}{\frac{3n^2}{n^2}-\frac{n}{n^2}} \rightarrow \frac{2+0}{3+0} \rightarrow \frac{2}{3}$$

$$\left|\frac{2n+3}{9n^2-3n}\right| < \left|\frac{2n+3n}{n(9n-3)}\right| < \in$$

$$\left|\frac{5n}{n(9n-3)}\right| < \in$$

$$\left|\frac{5}{9n-3}\right| < \in$$

$$\frac{5}{\in} < 9n-3$$

$$\boxed{\frac{\mathbf{5}+\mathbf{3}\in}{\mathbf{9}\in} < \boldsymbol{n}}$$

2. $\{\frac{n+1}{2n-1}\} \rightarrow \boldsymbol{\frac{1}{2}}$

$$\left|\frac{n+1}{2n\ -\ 1}+\frac{1}{2}\right| < \in$$

$$\left|\frac{2n+2-2n+1}{4n-2}\right| < \in$$

$$\left|\frac{3}{4n-2}\right| < \in$$

$$\frac{3}{\in} < 4n-2$$

$$\boxed{\frac{\mathbf{3+2\in}}{\mathbf{4\in}} < \boldsymbol{n}}$$

3. $\{\frac{\ln n}{n^2}\} \rightarrow 0$

$$\left|\frac{\ln n}{n^2}\right| < \in$$

$$\left|\frac{\ln n}{n^2}\right| < \left|\frac{\ln n}{n^2}\right| < \in$$

$$\boxed{\frac{\mathbf{1}}{\in} < \boldsymbol{n}}$$

4. $\{\frac{3}{n-1}\} \rightarrow 0$

$$\left|\frac{3}{n-1} - 0\right| < \in$$

$$\left|\frac{3}{n-1}\right| < \in$$

$$\frac{3}{\in} < n-1$$

$$\boxed{\frac{\mathbf{3+\in}}{\in} < \boldsymbol{n}}$$

5. $\{r^{1/n}\}$ $and\ r > 0 \rightarrow r$

(Hint: Consider two
cases: r ≤ 1 and r > 1.)

$$\left|r^{\frac{1}{n}} - r\right| < \in$$

$$r^{\frac{1}{n}} < \in + r$$

$$\ln r^{\frac{1}{n}} < \ln(\in + r)$$

$$\frac{1}{n}\ln r < \ln(\in + r)$$

$$\boxed{\frac{\mathbf{ln}\,\boldsymbol{r}}{\mathbf{ln}\,(\in + \boldsymbol{r})} < \boldsymbol{n}}$$