MAT 3101 – NOTES ON THE MODULUS OF COMPLEX NUMBERS AND TRIGONOMETRIC FORM

THE MODULUS

THE MODULUS OF A COMPLEX NUMBER IS DEFINED AS $z = \sqrt{R(z)^2 + I(z)^2}$

EXAMPLE:

$|-1+2i| = \sqrt{(-1)^2+(2)^2} = \sqrt{5}$

$|i| = |0+1i| = 1$

PROPERTIES:

> $|z| \geq 0$; $|z| = 0 \iff z = 0$

> $|z| = |-z| = |\bar{z}|$, $|\alpha z| = |\alpha| \cdot |z|$ FOR $\alpha \in \mathbb{R}$.

> $|z \cdot w| = |z| \cdot |w|$

> $|z+w| \leq |z| + |w|$

> $|z-w| \geq \big||z|-|w|\big|$

RECALL: COMPLEX NUMBERS

THE SET OF COMPLEX NUMBERS $\mathbb{C}$ IS THE SET OF SYMBOLS $a+bi$, WHERE $a, b \in \mathbb{R}$.

EX: $(a+bi) \pm (c+di) = (a \pm c) + (b \pm d)i$

$(a+bi) \cdot (c+di) = (ac-bd)+(ad+bc)i$

$(a+bi) \div (c+di) = \dfrac{(ac+bd)+(-ad+bc)i}{c^2+d^2}$ where $c^2+d^2 > 0$

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ADDITIONAL PROPERTIES

> $\overline{(\bar{z})} = z$, $\overline{z+w} = \bar{z} + \bar{w}$, $\overline{z \cdot w} = \bar{z} \cdot \bar{w}$

> $R(z) = \dfrac{z+\bar{z}}{2}$, $I(z) = \dfrac{z-\bar{z}}{2i}$

> $z = \bar{z} \iff z \in \mathbb{R}$

> $z \cdot \bar{z} = R(z)^2 + I(z)^2$ – A NONNEGATIVE REAL NUMBER

* $\overline{a+bi}$ => MEANS THE CONJUGATE OF $a+bi$ $= a - bi$

THE TRIGONOMETRIC FORM

$$z = a+bi = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}} + \frac{b}{\sqrt{a^2+b^2}}i\right)$$

WE CAN FIND A NUMBER $\phi \in [0, 2\pi) \ni$

$$\cos \phi = \frac{a}{\sqrt{a^2+b^2}}, \quad \sin \phi = \frac{b}{\sqrt{a^2+b^2}}$$

EXAMPLE:

① $z = 1-i \rightarrow \sqrt{a^2+b^2} = \sqrt{1^2+(-1)^2} = \sqrt{2}$

$\downarrow$

$\sqrt{2}\left(\dfrac{1}{\sqrt{2}} + \dfrac{-1}{\sqrt{2}}i\right)$ WE ARE LOOKING FOR $\phi \in [0, 2\pi)$

$\cos \phi = \dfrac{1}{\sqrt{2}}$ $\quad \sin \phi = \dfrac{-1}{\sqrt{2}}$

THEREFORE,

$\phi = 7/4\, \pi$

② FIND ALL ROOTS OF ORDER 4

GIVEN: $\sqrt[4]{1-i}$

$z = \sqrt[8]{2}\,(\cos \phi_K + i \sin \phi_K)$

WHERE

$$\phi_K = \frac{\frac{7}{4}\pi + 2k\pi}{4}$$

$= \dfrac{7}{16} + \dfrac{k\pi}{2}$ AND $k = 0,1,2,3$

THEREFORE:

$$\phi_K = \frac{\frac{7}{4}\pi + 0}{4} = \frac{7\pi}{16}$$

$$\phi_1 = \frac{\frac{7}{4}\pi + 2\pi}{4} \quad \frac{15\pi}{16}$$

$$\phi_2 = \frac{\frac{7}{4}\pi + 4\pi}{4} \quad \frac{23\pi}{16}$$

$$\phi_3 = \frac{\frac{7}{4}\pi + 6\pi}{4} \quad \frac{31\pi}{16}$$

By: Valerie Fernandez | 10/6/2022