MAT 3101 – NOTES ON REAL NUMBER PROPERTIES AND PROVING

REAL & COMPLEX NUMBERS

> FRACTION

DF → CF

CF → DF } TASK : C-PROG

> RATIONAL / IRRATIONAL

↳ PROOF

> LOG – PROOF

> PRIME NOS. – FACTORS : 1 & ITSELF

THE ORDERING OF THE SET R

IF $x-y$ IS A POSITIVE NO. WE WRITE

$$x > y$$

IF $x-y$ IS A NON-NEGATIVE NO.

$$x \geq y$$

IF $x-y$ IS NEGATIVE WE WRITE

$$x < y$$

IF $x-y$ IS NON-POSITIVE

$$x \leq y$$

FOR ANY x, y WE HAVE $x=y$ OR $x<y$ OR $x>y$.

SYMBOLS

$\forall$ FOR ALL

$\exists$ THERE EXISTS

$\leftrightarrow$ IF & ONLY IF (IFF)

$\rightarrow$ IMPLIES

$\in$ ELEMENT

$\subset$ SUBSET

$\wedge$ AND

$\vee$ OR

ARCHIMEDEAN PROPERTY

IF x & y ARE POSITIVE REALS WITH $0 < x < y$, THEN THERE IS AN INTEGER n SUCH THAT

$$nx > y$$

PROOF :

WE SHALL CONSIDER THE CASE WHERE

$$x < y$$

OUR PROOF IS BY CONTRADICTION.

SUPPOSE NO SUCH INTEGER n EXISTS

THEN

$$nx \leq y \quad , \text{ ALL } n$$

THUS

$$n \leq \frac{y}{x}$$

THIS CONTRADICTS THE FACT THAT THE INTEGERS DO NOT HAVE AN UPPER BOUND.

LUB PROPERTY

IF S IS A NONEMPTY SET OF REAL NOS. WHICH HAS AN UPPER BOUND, THEN S HAS A LEAST UPPER BOUND.

(DENOTED BY $\sup S$)

GLB

EVERY NONEMPTY SET OF REAL NOS WHICH IS BOUNDED BELOW HAS A GREATEST LOWER BOUND.

(DENOTED BY $\inf S$)

SHOW THAT IF $a < b$, THEN

$$a < \frac{a+b}{2} < b.$$

SOLN :

$a < b$	$a < b$
$a + a < a+b$	$a+b < b+b$
$2a < a+b$	$a+b < 2b$
$a < \frac{a+b}{2}$	$\frac{a+b}{2} < b$

THUS

$$a < \frac{a+b}{2} < b$$

TASK :

> BETWEEN ANY TWO REALS, THERE EXISTS INFINITELY MANY RATIONAL.

> BETWEEN ANY TWO REALS IS AN IRRATIONAL.

EXERCISES :

DETERMINE

1. $\sup \{x \mid x < 1\}$
2. $\sup \{x \mid x \leq 1\}$
3. $\sup \{1, 1.1, 0.9, 1.01, 0.99, 1.001, 0.999, \ldots\}$
4. $\sqrt{5}$ IS IRRATIONAL
5. $\log_2 3$ IS NOT RATIONAL

NEXT TOPIC : INTERVALS

ABSOLUTE VALUE – PROPERTIES

MATHEMATICAL INDUCTION

EQUALITY / INEQUALITY

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