

$$I. 1.) \text{ given } A^9 = \begin{bmatrix} 1 & 1 \\ -5 & -2 \end{bmatrix}$$

$$(2A)^9 = \begin{bmatrix} 8 & 8 \\ -40 & -16 \end{bmatrix}$$

$$2.) a.) A^8 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$b.) A^2 - A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$3.) a.) (AB)^T = \begin{pmatrix} 11 & 1 \\ 11 & -6 \end{pmatrix} \\ = \underline{\underline{\begin{bmatrix} 11 & 11 \\ 1 & -6 \end{bmatrix}}}$$

$$b.) 3A - 2B$$

wrong size, cannot subtract

$$c.) -A^T = \begin{bmatrix} -1 & -2 \\ -2 & -1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -2 & -1 \\ -3 & -4 \end{bmatrix}$$

$$d.) (A^T)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$$

$$\text{II. } 2X + B = -3A + C$$

$$= 2X + \begin{bmatrix} 0 & 1 \\ 5 & 2 \end{bmatrix} = -3 \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 8 & -2 \\ -6 & 4 \end{bmatrix}$$

$$= 2X + \begin{bmatrix} 0 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 6 \\ 9 & -12 \end{bmatrix} + \begin{bmatrix} 8 & -2 \\ -6 & 4 \end{bmatrix}$$

$$= 2X + \begin{bmatrix} 0 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & -8 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 5 & 2 \end{bmatrix}$$

$$= \underline{2X = \begin{bmatrix} 5 & 3 \\ -2 & -6 \end{bmatrix}}$$

2

$$= \underline{\underline{X = \begin{bmatrix} 5/2 & 3/2 \\ -1 & -3 \end{bmatrix}}}$$

$$\text{III. inverse } A = \begin{bmatrix} 1 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}^{-1}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & 4 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/7 & 4/7 & 0 \\ 0 & 1 & 0 & 2/7 & -1/7 & 0 \\ 0 & 0 & 1 & -5/7 & -8/7 & 1 \end{array} \right]$$

$$= \underline{\underline{\begin{bmatrix} -1/7 & 4/7 & 0 \\ -2/7 & -1/7 & 0 \\ -5/7 & -8/7 & 1 \end{bmatrix}}}$$

$$= 1 \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} - 4 \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} + 0 \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= 1(1-0) - 4(12-0) + 0(8-3)$$

$$= \underline{\underline{-7}}$$

IV.

no solution

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}$$

$$= \frac{1}{2} = \frac{1}{-2} \neq \frac{\boxed{3}}{\boxed{6}}$$

one solution

- not possible -

infinite solution

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

$$\frac{1}{2} = \frac{1}{2} = \frac{\boxed{3}}{\boxed{6}}$$

$$\underline{\underline{k = 6}}$$

$$x - y = 3$$

$$2x - 2y = k$$

$$\underline{\underline{k \neq 6}}$$

$$1(-2) - (2) - 1$$

$$-2 + 2 \neq 0$$

↓
false

$$V. \left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{array} \right]$$

solution \rightarrow next

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$x = 2$$

$$y = -1$$

$$\underline{\underline{z = 3}}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{array} \right]$$

$$= R_2 \rightarrow R_2 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & -5 & -5 & -10 \\ 3 & 0 & -1 & 3 \end{array} \right]$$

$$= R_3 \rightarrow R_3 - 3R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & -5 & -5 & -10 \\ 0 & -6 & -10 & -24 \end{array} \right]$$

$$= R_2 \rightarrow \frac{-R_2}{5}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & -6 & -10 & -24 \end{array} \right]$$

$$= R_1 \rightarrow R_1 - 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & -6 & -10 & -24 \end{array} \right]$$

$$= R_3 \rightarrow R_3 + 6R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -4 & -12 \end{array} \right]$$

$$= R_3 \rightarrow \frac{-R_3}{4}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$= R_1 \rightarrow R_1 - R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$= R_2 \rightarrow R_2 - R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right]$$