MAT 3101 – NOTES ON ABSOLUTE VALUE PROPERTIES

TEXTBOOK : ADV. CALCULUS

BY : BUCK

INTERVALS

WE DENOTE INTERVALS IN THE FF'g WAY :

$(a,b) = \{x \mid a < x < b\}$ OPEN INTERVAL

$[a,b] = \{x \mid a \le x \le b\}$ CLOSED INTERVAL

$(a,b] = \{x \mid a < x \le b\}$ LEFT-HAND OPEN INT

$[a,b) = \{x \mid a \le x < b\}$ RIGHT-HAND OPEN INT

OR

$(a,b) \Rightarrow (-\infty, \infty)$

IF a, b & c ARE ANY GIVEN REAL NUMBERS,

THEN

> EITHER $a>b$, $a=b$, $a<b$ LAW OF TRICHOTOMY

> IF $a>b$ AND $b>c$, THEN $a>c$ LAW OF TRANSITIVITY

> IF $a>b$, THEN $a+c > b+c$

> IF $a>b$, $c>0$, THEN $ac>bc$

> IF $a>b$ AND $c<0$, THEN $ac<bc$

WE DEFINE THE ABSOLUTE VALUE OF A REAL NUMBER IN A FOLLOWING WAY

$$|x| = \begin{cases} x & \text{IF } x \ge 0 \\ -x & \text{IF } x < 0 \end{cases}$$

PROPERTIES

1. $|-x| = |x|$
2. $-|x| \le x \le |x|$
3. $|x+y| \le |x| + |y|$ TRIANGLE INEQUALITY
4. $\big||x|-|y|\big| \le |x-y|$
5. $|x-y|$ REPRESENTS THE DISTANCE FROM x TO y ON THE REAL LINE.
6. $|x \cdot y| = |x| \cdot |y|$
7. $|x| = \sqrt{x^2}$
8. $|x| \ge 0$ AND $|x| = 0 \Leftrightarrow x = 0$
9. $x \le y \wedge -x \le y \Rightarrow |x| \le y$

EXAMPLE

CASE 1 : x AND y HAVE THE SAME SIGN $\pm$

$|x+y| = \pm(x+y) = \pm x + \pm y = |x| + |y|$

$\rightarrow$ TRIANGLE INEQUALITY IS AN EQUALITY

CASE 2 : x AND y HAVE OPPOSITE SIGNS.

ASSUME $x \le 0 \le y$

* IF $x+y \ge 0$ THEN

$|x+y| = x+y \le -x+y = |x|+|y|$

* IF $x+y < 0$ THEN

$|x+y| = -(x+y) = -x-y \le -x+y = |x|+|y|$

$\therefore$ IN THIS CASE, IF NONE OF x AND y IS ZERO, THE INEQUALITY IS EVIDENT.

IF $p = (x_1, \dots, x_n)$ AND $q = (y_1, \dots, y_n)$

THE DOT PRODUCT OR INNER PRODUCT IS

$p \cdot q = x_1y_1 + \dots + x_ny_n$ AND

$p \cdot p = |p|^2$

SCHWARZ' INEQUALITY

FOR ANY p, q IN $\mathbb{R}^n$,

$$p \cdot q \le |p||q|$$

Pf :

LET $Q = \alpha p - \beta q$

WHERE $p, q \in \mathbb{R}^2$ AND $\alpha, \beta \in \mathbb{R}$

$0 \le Q \cdot Q$

$\le (\alpha p - \beta q)(\alpha p - \beta q)$

$\alpha^2 (p \cdot p) - 2\alpha\beta(p \cdot q) + \beta^2 (q \cdot q)$

$\alpha^2 |p|^2 - 2\alpha\beta(p \cdot q) + \beta^2 |q|^2$

THUS

$2\alpha\beta(p \cdot q) \le \alpha^2 |p|^2 + \beta^2 |q|^2$

WHICH IS TRUE FOR ANY NOS. α AND β.

NOW TAKE

$\alpha = |q|$ AND $\beta = |p|$

IF $|p| \ne 0$, $|q| \ne 0$ THEN WE GET

$2|p||q|(p \cdot q) \le |q|^2|p|^2 + |p|^2|q|^2$

$2|p||q|(p \cdot q) \le 2|p|^2|q|^2$

$p \cdot q \le |p||q|$

By: Valerie Fernandez | 10/6/2022